Cosmological Moduli and Precision Cosmology

Anshuman Maharana

Harish-Chandra Research Institute
Allahabad

1409.7037 with K Dutta
1506.05745 with K Dutta and K Das
1604.08512 with M Cicoli, K Dutta and F Quevedo

ICTP, May 2016
Cosmology and Moduli Fields

• From the very early days of model building in supergravity models in was realised that moduli fields can lead to cosmological timeline distinct from the standard one.

Goncharov, Linde, Vysotsky
Dine, Fischler, Nemeschansky
Coughlan, Holman, Ramond, Ross
Coughlan, Fischler, Kolb, Raby and Ross

modular cosmology
We show that if there is a realistic 4-d string, the dilaton and moduli supermultiplets will generically acquire a small mass $\sim O(m_{3/2})$, providing the only vacuum-independent evidence of low-energy physics in string theory beyond the supersymmetric standard model. The only assumptions behind this result are (i) softly broken supersymmetry at low energies with zero cosmological constant, (ii) these particles interact with gravitational strength and the scalar components have a flat potential in perturbation theory, which are well-known properties of string theories. (iii) They acquire a vev of the order of the Planck scale (as required for the correct value of the gauge coupling constants and the expected compactification scale) after supersymmetry gets broken. We explore the cosmological implications of these particles. Similar to the gravitino, the fermionic states may overclose the Universe if they are stable or destroy nucleosynthesis if they decay unless their masses belong to a certain range or inflation dilutes them. For the scalar states it is known that the problem cannot be entirely solved by inflation, since oscillations around the minimum of the potential, rather than thermal production, are the main source for their energy and can lead to a huge entropy generation at late times. We discuss some possible ways to alleviate this entropy problem, that favour low-temperature baryogenesis, and also comment on the possible role of these particles as dark matter candidates or as sources of the baryon asymmetry through their decay.
This talk is about —

modular cosmology and inflation as the theory of inhomogeneities in the universe

$N_{\text{infl}}$ in modular cosmology.
Outline

Review of modular cosmology

$N_{infl}$ in modular cosmology.
Cosmology and Moduli

• Starting point of the analysis moduli dynamics during inflation.

• Analysis of dynamics during inflation gives, for $m_\varphi \lesssim H_{\text{infl}}$

  At the end of inflation the modulus $\varphi$ has VEV $\hat{\varphi}$,

  \[ Y = \frac{\hat{\varphi}}{M_{\text{pl}}} \lesssim 1 \]

• Single modulus approximation is often good as from then on dynamics of the lightest most relevant.
Cosmology and Moduli

Thus just after reheating, energy density has two components

- **Radiation**: To which the inflaton has dumped its energy density.

- **Modulus**: Potential energy due to displacement.

- If $m_\varphi < H_{\text{infl}}$ then the former dominates.
Cosmology and Moduli

- The energy density associated with radiation falls off as

\[ \rho_{\text{rad}}(t) \propto \frac{1}{a^4(t)} \]

- On the other hand, for the modulus

Initially, high value of Hubble friction keeps it pinned to its expectation value.
Cosmology and Moduli

- The energy density associated with radiation falls off as
  \[ \rho_{\text{rad}}(t) \propto \frac{1}{a^4(t)} \]

- As the universe expands, Hubble falls

  When \( H \lesssim m_\varphi \) the modulus begins to oscillate.

- Time average of energy density falls off as
  \[ \rho_{\text{modulus}}(t) \propto \frac{1}{a^3(t)} \]

Cosmological evolution of cold moduli particles.
Modulus Domination

- A modification the standard cosmological history

Inflation → Reheating → Radiation domination → Modulus domination ....

- Modulus domination continues until decay of modulus at

\[ \tau_{\text{mod}} \approx \frac{16\pi M_{\text{pl}}^2}{m_\phi^3} \]

the characteristic lifetime for decay via their Planck suppressed interactions.

Modulus decays ... Universe Reheats ... Thermal History
Modular Cosmology

Inflation
Reheating
Radiation Domination
Modulus Domination
Reheating (after modulus decay)
Radiation Domination
Today

Conventional Cosmology

Inflation
Reheating
Radiation Domination
Today
A Bound from Nucleosynthesis

• To account for the success of big bang nucleosynthesis, the reheat temperature after modulus decay has to be at least as large as the binding of energy of light elements.

\[ T_{\text{reheat}} \gtrsim 1 \text{ MeV} \]

• Reheat temperature in terms of width

\[ T_{\text{reheat}} \approx \sqrt{\Gamma M_{\text{pl}}} \quad \Gamma \approx \frac{m_\phi^3}{16\pi M_{\text{pl}}^2} \]

• Lighter the modulus lower the reheat temperature. Lower bound on reheat temperature translates to a lower bound for the modulus mass \( m_\phi \gtrsim 30 \text{ TeV} \).
$N_{\text{infl}}$ in modular cosmology.
Inflation and Inhomogeneities

- Inhomogeneities are a result of freezing of quantum fluctuations at the time of horizon exit; \( k/a \approx H \).

\[ k \approx 0.05 \text{ Mpc}^{-1} \] for CMB observations by the PLANCK satellite.

Properties of the universe at this stage determine the properties of the inhomogeneities of the CMB.

\[ V(\chi) \]

- CMB mode freeze
- Inflation Ends
It is conventional to keep track of the point of freezing by the number of e-folding between freezing and end of inflation.

\[ N_{\text{infl}} = \ln \left( \frac{a(t_{\text{end}})}{a(t_{\text{freeze}})} \right) \]

For e.g. \( m^2 \chi^2 \) potential (similar expressions for all models)

\[ n_s = 1 - \frac{2}{N} \quad r = \frac{8}{N} \]

Given a potential we need the value of \( N_{\text{infl}} \) to extract predictions
Inflation and Inhomogeneities

- How is $N_{\text{infl}}$ determined?

- Energy density of universe at the time of horizon exit of mode relevant for CMB observations.

- Strength of gravity waves.

More precisely,

$$A_s = \frac{2}{3\pi^2 r} \left( \frac{\rho}{M_{\text{pl}}^4} \right)$$

- $\rho$ - Energy density of universe at the time of horizon exit of mode relevant for CMB observations.

- $r$ - Strength of gravity waves.
An early time and today’s energy densities known. This implies a consistency condition:

Any history we ascribe must be such that the early time energy density evolves to the energy density today.

Post-inflationary Epoch consists of **reheating** followed by **thermal history** in conventional cosmologies.
Planck 2013 results. XXII Constraints on Inflation

\[ N_{\text{infl}} + \frac{1}{4} (1 - 3w_{rh}) N_{rh} \approx 57 + \frac{1}{4} \ln r + \frac{1}{4} \ln \left( \frac{\rho_k}{\rho_{\text{end}}} \right) \]

This motivates the usual range of 50-60 for \( N_{\text{infl}} \)
Energy density at horizon exit, $\rho_k$. 

Horizon exit of CMB modes 

Inflation Ends 

$N_{\text{infl}}$ 

$\implies k \gg 0$ 

Thermal 

Post-inflationary Epoch. 

Energy density today, $\rho_0$. 

Modular Cosmology 

Horizon exit of CMB modes 

Inflation Ends 

$N_{\text{infl}}$ 

Modulus Domination 

Thermal 

Energy density today
We obtain

$$N_{\text{infl}} + \frac{1}{4} N_{\text{modulus}} + \frac{1}{4} (1 - 3w_{\text{rh1}}) N_{\text{rh1}} + \frac{1}{4} (1 - 3w_{\text{rh2}}) N_{\text{rh2}} \approx 5 \ln r + \frac{1}{4} \ln \left( \frac{\rho_k}{\rho_{\text{end}}} \right)$$

The number of e-folding during modulus domination.

$$N_{\text{modulus}} \approx \frac{4}{3} \ln \left( \frac{\sqrt{16\pi M_{\text{pl}}} Y^2}{m_\phi} \right)$$

$$Y = \frac{\hat{\phi}}{M_{\text{pl}}}$$  The initial displacement in Planck Units (generic estimate from EFT $Y \approx \mathcal{O}(1)$)

$$m_\phi$$  The post-inflationary mass of the modulus

Since the dependence is on $\ln(M_{\text{pl}}/m_\phi)$ this can significantly bring down the value of $N_{\text{infl}}$. 

K Dutta, AM 14
For activated by axion monodromy, has outside the joint 95% CL for the data at smaller scales. The constraints are given at the pivot scale Table 4.

Table 3.

N_{\text{infl}} + \frac{1}{4} N_{\text{modulus}} + \frac{1}{4} \left(1 - 3w_{\text{rh1}}\right) N_{\text{rh1}} + \frac{1}{4} \left(1 - 3w_{\text{rh2}}\right) N_{\text{rh2}} \approx 57 + \frac{1}{4} \ln r + \frac{1}{4} \ln \left(\frac{\rho_k}{\rho_{\text{end}}}\right)

For \ Y \approx 0.1, \ m_\phi \approx 10^2 \ \text{TeV} \ \Rightarrow \ \frac{1}{4} N_{\text{modulus}} \approx 10

m_\phi \approx 10^7 \ \text{TeV} \ \Rightarrow \ \frac{1}{4} N_{\text{modulus}} \approx 5
no epoch of modulus domination.

\( m_\varphi > H_{\text{infl}} \)

\[ m_\varphi = 10^6 \text{ TeV} \]

\[ Y = 1/10 \]
On the other hand, if one has understanding of moduli stabilisation then it is possible to explicitly compute

- The initial displacement of the modulus.
- The inflaton width.

It is then possible to determine $N_{\text{infl}}$ more accurately.

We have carried this out explicitly for Kahler Moduli Inflation.

M Cicoli, K Dutta, AM, F Quevedo 1604.08512
Kahler Moduli Inflation

Model of inflation set in the large volume scenario for moduli stabilisation.

\[
V = \sum_{i=2}^{n} \frac{8(a_i A_i)^2 \sqrt{\tau_i}}{3V \lambda_i} e^{-2a_i \tau_i} - \sum_{i=2}^{n} \frac{4a_i A_i W_0}{V^2} \tau_i e^{-a_i \tau_i} + \frac{3\hat{\xi}W_0^2}{4V^3} + \frac{D}{V^\gamma}
\]

\(V\) Volume Modulus

\(\tau_i\) Other Kahler Moduli

The role of the inflaton is played by one of the Kahler Moduli \((\tau_n)\) — when it is displaced from its global minimum.

The Volume modulus is the lightest geometric modulus, dominates the energy density after inflation.
We compute the magnitude of the vacuum misalignment and find

$$\hat{\phi} \approx 0.1M_{\text{pl}}$$

in keeping with EFT expectations.

In summary, we find

$$N_{\text{infl}} \approx 45$$

The effect of the epoch of modulus in this model is to affect the spectral tilt at the percent level.
Modular Cosmology is a generic feature in string and supergravity models.

The epoch of modulus domination can have a significant effect on $N_{\text{infl}}$, even if the modulus is quite heavy.

The vacuum misalignment is needed as an input, for which one needs to work in setting where there is control over moduli stabilisation e.g. Kahler moduli inflation.

Thank you Fernando, wishing you all the BEST.