Ocean circulation models and modelling: A General Overview

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Two pillars of numerical oceanography

OCEAN MODELS:

- Repository for mechanistic theories of how the ocean works, with numerical methods transforming theories into a computational tool for scientific investigations.
- Mathematically formulated physical theories and numerical methods provide the foundation.
- OCEAN MODELLING:
 - The use of numerical simulations as an experimental tool to help deduce mechanisms for emergent space-time patterns of ocean phenomena.
 - Math physical theories and analysis methods provide the foundation



Topics to be discussed

Science of ocean models

- continuum thermo-hydrodynamical equations
- algorithms for hydrostatic primitive equations
- subgrid scale parameterizations
- Science from ocean models (ocean modelling)
 - analysis of heat budget according to physical processes
 - analysis of sea level patterns
 - analysis of high/low-frequency variability
 - mechanistic understanding of physical processes (eddy-induced mixing, heat uptake, ...)



Some Published background

Science of ocean models

- Griffies and Treguier (2013): chapter from the 2nd edition of Ocean **Circulation and Climate**
- Griffies (2009): chapter in Encyclopedia of Ocean Sciences
- Griffies and Adcroft (2008): chapter discussing the formulation of ocean model equations from an AGU monograph.
- Griffies (2005): "Some ocean model fundamentals"
- Griffies (2004): monograph on ocean climate model fundamentals
- Ocean models and ocean modelling
 - Griffies et al. (2000): review of ocean climate model development
 - Griffies et al (2009): research article on Coordinated-Ocean ice Reference Experiments (COREs)
 - Griffies et al (2010): White Paper from the OceanObs09 conference summarizing the status of ocean models



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Ocean Circulation and Climate, 2nd Edition (2013)



A 21st CENTURY PERSPECTIVE



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General motivational comments

- Ocean model fundamentals and the use of ocean models as a tool for science involves some of the most difficult problems in classical and computational physics.
 - turbulence closures and subgrid scale parameterizations
 - analysis and rationalization of massive datasets
 - efficient methods for discretizing continuous media.
- We are also touching on elements of the most important environmental and societal problem facing the planet.
 - Climate warming is happening and humans are the key reason.
 - The ocean's role in the earth climate is significant.
 - Providing rational and robust models for understanding and predicting climate is a central element of oceanography and climate science.
 - Now is an incredibly exciting time to enter this field, particularly for those who feel passionate about diving deep into some of the most difficult intellectual problems in science while addressing some of the most important problems for the biosphere.









Why use ocean models?

Ocean models are ubiquitous in ocean/climate science. Why?

- Numerical models are the primary means available for probing, in a nearly controlled manner, the ocean/climate system.
 - There is one natural ocean, yet many numerical oceans.
- Model foundations have improved through better understanding of the ocean (theory, observations, laboratory) and enhanced numerical methods.
- Computer power has increased to allow for refined resolution incorporating more details resulting in improved realism.



Types of ocean and climate models





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A Q-G approach

Q-G equations

For a flat-bottomed three-layer configuration, the QG potential vorticity (QGPV) equation is

$$\partial_t q_i + J(\psi_i, q_i) = f_0 \mathbf{A} \mathbf{e} + A_2 \nabla^4 \psi_i - A_4 \nabla^6 \psi_i, \quad (1)$$

A mixed-layer

$$\partial_t T + \boldsymbol{\nabla} \cdot (\mathbf{u}T) = \frac{T}{H_m} \begin{cases} w_{\text{oek}} \\ -w_{\text{aek}} \end{cases} + K_2 \nabla^2 T - K_4 \nabla^4 T \\ + \frac{1}{H_m} \begin{cases} -\frac{F_{\text{o0}} - F_{\text{om}}}{\rho_o C_{\text{po}}} \\ \frac{F_{\text{a0}} - F_{\text{om}}}{\rho_a C_{\text{pa}}} \end{cases}, \qquad (2)$$

A Q-G approach





Some specific applications

- Mechanistic studies of ocean and climate processes
 - Process studies using fine resolution (≤ 1 km) simulations (MITgcm, SUNTANS)
 - Mechanisms for coastal and shelf processes (≤ 10 km) (ROMS)
 - Mechanisms for observed large-scale variations (< 200 km) (CLIVAR CORE)
 - Mechanisms for climate variability (≤ 200 km) (MOM, NEMO, ICCM)
- Operational predictions and state estimation
 - Coastal forecasting (BLUElink)
 - Ocean state estimation (ECCO)
- Projections for future climate change
 - IPCC-class simulations with anthropogenic forcing (CMIP)
 - Sea level changes (John Church's research group)
 - Changes in sea ice (NOAA info page)



Revolution in ocean obs requires models to interpret

Near-global observations are pushing models to improve.

Argo + satellites provide high quality near-global information relevant for predictions and climate change.





From Argo at UCSD







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Daily SST animation from GFDL CM2.6 climate model

This coupled climate model uses a 0.1° configuration of MOM5 for the ocean component, under a 50 km global atmosphere.



Theoretical foundations for ocean models

- Continuum thermo-hydrodynamical equations of the ocean
 - Seawater mass conservation
 - Tracer mass conservation
 - Momentum conservation
 - Linear irreversible thermodynamics of seawater
 - Typically assume hydrostatic balance
- Boundary conditions
 - Air-sea interactions
 - Sea ice-ocean interactions
 - Ice shelf-ocean interactions
 - Solid-earth-ocean interactions
- Subgrid scale parameterizations
 - Momentum closure: frictional stress tensor
 - Tracer closure: transport tensor
 - Boundary layer parameterizations



A sample of ocean processes

Slides in this section

- A zoo of physical ocean processes
- Space-time diagram of ocean motions
- Upper ocean boundary and wave interactions
- The marginal ice zone (MIZ)
- Southern Ocean processes
- The value of idealized Southern Ocean simulations



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A zoo of physical ocean processes



- The ocean contains a zoo of physical processes!
 - Strong coupling between processes ⇔ no spectral gap.
- Coupling means it is generally better to resolve than parameterize.
- Yet we cannot resolve everything.

From Griffies and Treguier (2013)



topographic Interactions

 Topographic wave generation is a significant sink of energy for geostrophic flows and a source of energy for turbulent mixing in the deep ocean



- About 1/3 comes from geostrophic motions flowing over rough topography in the Southern Ocean.
- Lee wave-driven mixing should be represented in ocean and climate models, but currently it is not!



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Upper ocean boundary and wave interactions



From Cavaleri et al (2012)

- New research activities in boundary layer param prompted by refined atmos and ocean resolutions that admit new dynamical regimes (e.g., mesoscale eddies, tropical cyclones).
- An increased awareness in the climate community of the importance of surface ocean gravity waves (e.g., Cavalieri et al (2012)).



The marginal ice zone (MIZ)



Questions about processes at the marginal ice zone are of prime importance as Arctic sea ice melts.



Southern Ocean processes





Southern Ocean processes



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Southern Ocean processes



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Southern Ocean processes





A Southern Ocean process study





Resolving versus parameterizing: some numbers

- Direct Numerical Simulation (DNS) of global ocean climate requires 3×10^{10} time steps of one second (1000 years).
- Setting the model's grid scale to the Kolmogorov length $\Delta=10^{-3}\text{m}$ over a global ocean domain of volume $1.3\times10^{18}\,\text{m}^3$ requires 1.3×10^{27} discrete grid cells. This is roughly $10^4\times$ Avogadro's number!
- Each model grid point has a velocity vector and tracer fields to time integrate.
- Conclude:
 - We will be dust long before DNS of global ocean climate.
 - We must use parameterizations to simulate the ocean.
 - The rationalization of a DNS simulation typically requires a coarse-grained perspective, as certainly would DNS of the World Ocean climate.



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DNS of Navier-Stokes Equations



$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{\partial p}{\rho_0 \partial x_i} - g_i \frac{\rho'}{\rho_0} + \frac{\partial}{\partial x_j} \left(\nu \frac{\partial u_i}{\partial x_j}\right)$$
(1)

$$Re = \frac{UL}{\nu} = \frac{1[m/s] \times 100 \times 10^3[m]}{10^{-6}[m^2/s]} = 10^{11}$$
DNS means to resolve all scales of motion (down to molecular viscosity)

Reynolds Averaging Navier-Stokes (RANS)

take
$$u_i = U_i + u'_i$$

$$\frac{\partial U_i}{\partial t} + U_j \frac{\partial U_i}{\partial x_j} = -\frac{\partial P}{\rho_0 \partial x_i} - g_i \frac{\rho}{\rho_0} + \frac{\partial}{\partial x_j} (\nu \frac{\partial U_i}{\partial x_j}) - \frac{\partial}{\partial x_j} (\overline{u'_j u'_i})$$
(2)

- The new nonlinear term on the RHS are the velocity fluctuations appearing due to the non-linearities of the NS eqs.
- The term is called Reynolds stress (a component of the total stress tensor) $R_{ij} \equiv \rho \overline{u'_i u'_j}$.
- The system is not closed (# unknowns > # equations) leading to to the **Turbulence Closure Problem**.
- Common approach is the eddy viscosity approach $\overline{u'w'} = -K\frac{\partial U}{\partial z}$, and many ways of computing eddy viscosity/diffusivity.



Facets of what we mean by "resolution"

- general principles of resolution are the same for both atmospheric and ocean models
- there are different rules of thumb: one is that it takes 5 grid points to accurately define a feature without aliasing
- this means 1/8° global resolution with an average horizontal grid cell of 14 km can accurately depict only features larger than 56 km
- models with variable grid spacing have variable resolution – beware of resolution-dependent physics!
- resolution is not cheap because of the CFL* condition, as we shrink the horizontal grid spacing we must add vertical layers and decrease the time step



"every halving of the grid spacing requires roughly ten times as many computations"

* no transport faster than one grid cell per time step!



Spatial scale of mesocale and submesoscale eddies



MODIS satellite w/ inserts by A. Adcroft (GFDL)



Posing the ocean model problem The vertical ○○○○○○○○○○○○○○ The horizontal Where

How to bridge the gap? Hierarchical Modelling!











Vertical coordinate representation



Adapted by Chassignet et al (2006) from original figure in Griffies et al (2000)

- GEOPOTENTIAL OR PRESSURE: common for non-hydrostatic process modelling and large-scale climate modelling (MITgcm, MOM, NEMO)
- ISOPYCNAL: clean representation of interior quasi-adiabatic flows and overflows (GOLD, HYCOM)
- SIGMA OR TERRAIN FOLLOWING: common for shelf/coastal modelling (ROMS)



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Geopotential or pressure vertical coordinates



- Most common method for global models; extensive experience.
- Generalizations: $z^* = H(z \eta)/(H + \eta)$ absorbs SSH undulations; pressure (mass conserving).
- Spurious diapycnal mixing if poorly chosen numerical methods & parameter settings (e.g., Ilicak et al 2012).
- Downslope flows poorly represented absent very fine resolution (Winton et al. 1998).



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Sigma or terrain following vertical coordinates



:oordinate

- Extensive applications for coasts & shelves.
- Traditionally $\sigma = (z \eta)/(H + \eta)$, but with generalizations.
- As for geopotential, ∃ spurious diapycnal mixing with poorly chosen numerical methods & parameter settings.
- Much care is needed to handle horizontal pressure gradient calculation; generally need to smooth topography.
- There are very few global climate realizations.



Isopycncal vertical coordinates



- Quasi-adiabatic interior & flow-topography interactions (e.g., overflows)
- Inherently poor representation if weak vertical stratification (e.g., Labrador Sea, Southern Ocean, coastal regions).
- Care needed to represent realistic ocean thermodynamics and conservative transport, though proven methods now common.
- GFDL-GOLD, HYCOM, and Bergen: respectable climate efforts.









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Horizontal representation: structured finite volume



- Most common approach since 1960s; e.g., HYCOM, MITgcm, MOM, NEMO, ROMS.
- Recent advances with nesting allow for multi-scale simulations (Debreu and Blayo, 2008)



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Horizontal representation: structured finite volume





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Examples of structured finite volume grids





From Griffies et al (2005)

From Biastoch et al (2009)

- Tripolar common for global models
- Nested example has refined grid (0.1°) around South Africa embedded in global grid (0.5°) to examine Agulhas eddies impact on Atlantic circulation.



Tripolar grids

 Regular latitude-longitude grids have a problem when they approach the poles: grid lines tend to converge, resulting in shrinking grid cells. And at some point, grid lines converge on a single point, which is difficult for models to handle computationally. One way ocean models deal with this problem is to lay a circular grid over the arctic polar region, thus eliminating a north pole. While this circular arctic grid has two points of grid convergence rather than one, they can be positioned over land. We refer to the resulting model grid as a tripolar grid.



Tripolar grids

Tripolar Grid





Schopf 2005 after Murray 1996



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Horizontal representation: unstructured finite volume



- Aimed at seamless representation of multiple-scales.
- Can obtain enhanced conservation properties in discrete equations.
- Indirect addressing of algorithms adds computational cost (i.e., number of neighbors unknown a priori).
- Los Alamos and NCAR have a maturing effort: MPAS-ocean (Ringler et al 2013).
- Nascent effort ongoing at MPI-ICON



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Example of unstructured finite volume grid

MPAS Horizontally Unstructured Grids Variable resolution: 120 km to 30km in Southern Ocean

Compliments of M. Petersen, LANL (2013)

- Refined resolution towards Southern Ocean.
- Note: this is not from a working model; it is a mere example of the grid capabilities.



Horizontal representation: unstructured finite element



From Danilov (2013)

- Aimed at seamless representation of multiple-scales.
- Decompose continuous equations using basis functions and ٠ matrix inversions.
- Indirect addressing adds computational cost. ۲
- Effort at AWI focused on climate: FESOM
- Effort at Louvain focused on shallow ocean: SLIM
- Non-hydrostatic process model at Stanford: SUNTANS



The horizontal Where

Examples of unstructured finite element meshes



From Danilov (2013)

- Example 1: refined resolution in coastal regions
- Example 2: refined resolution in regions of dynamical interest









Frontier issues I

Role of mesoscale eddies in climate

- Global models are increasingly being run with an explicit representation (albeit imperfect) of the mesoscales.
- How/will climate sensitivity, variability, predictability be modified with eddying ocean simulations?
- How well do our parameterized models reflect the eddying simulations?
- Parameterizations, including stochastic methods
 - Although many modelling centres can now run eddying simulations, we will need mesoscale eddy parameterizations for many decades.
 - Stochastic methods are being successfully used for atmospheric parameterizations, and they are taking root for ocean parameterizations as well.



Frontiers issues II

Multi-scale modelling

- Whether structured (with nesting) or unstructured, models are being applied to address problems with multiple scales.
- Impacts of coast on large-scale, and converse.
- Seamless modelling is a dream that is being pushed for scientific and non-scientific reasons.

Coupling circulation models to surface wave models

- As ocean and atmospheric models refine resolution, traditional methods for parameterizing air-sea interface start to break down; e.g., Monin-Obukov similarity theory shows its limitations.
- The upper ocean exhibits waves, and waves affect the coupling.
- Ocean surface waves affect air-sea interactions as well as upper ocean mixing.



Trust but verify

Models are most useful when appreciating their limitations.

- Model limitations arise from:
 - fundamentals and/or numerical methods;
 - configuration design;
 - boundary information and/or other component models;
 - computational power.
- Do not treat models as a black box.
 - Models are tools to help deduce mechanisms.
 - Use diagnostic methods to uncover reasons for particular behaviour.

Numerical errors often appear in physically interesting manners.

- Remain skeptical even if the simulation "looks right".
- Investigate why and how.
- The more one learns about models, the more one can sense whether a particular simulation is physically sensible or the artefact of faulty methods.



Lecture outline and aims Motivation for using ocean models Posing the ocean model problem The vertical

The horizontal Where

Wherein lies the truth?

Perspectives on models

- Models cannot be validated. At best, they can be evaluated (see Oreskes et al 1994).
- "All models are wrong, but some are useful." (G.E.P. Box, statistician)









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