F-theory, M5-branes and N=4 SYM with Varying Coupling

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Plan

Goal:
Understanding 4d $N = 4$ SYM with varying coupling, i.e. D3-branes in F-theory, via M5-branes on elliptic three-folds.

I. D3s in F/M5s in M

II. 4d N=4 SYM with varying coupling and Duality Defects

III. New chiral 2d $(0, 2)$ Theories

I. D3s in F/M5s in M

4d $N = 4$ SYM with varying τ

F-theory is IIB with varying τ, where there is also a self-duality group $SL_2\mathbb{Z}$, which descends upon D3-branes to the Montonen-Olive duality group of $N = 4$ SYM.

4d $N = 4$ SYM has an $SL_2\mathbb{Z}$ duality group acting on the complexified coupling

$$\tau = \frac{\theta}{2\pi} + i\frac{4\pi}{g^2}\,, \qquad \tau \to \frac{a\tau + b}{c\tau + d}\,,$$

$ad - bc = 1$ and integral. Incidentally: the gauge group G maps to the Langlands dual group $G^\vee$.

Usually, we consider τ constant in the 4d spacetime.

Coming from F-theory, it's very natural to ask whether we can define a version of $N = 4$ SYM with varying τ, compatible with the $SL_2\mathbb{Z}$ action.

$\Rightarrow$ Network of 3d walls, 2d and 0d duality defects in $N = 4$.

Duality Defects

Variation of τ without singular loci are trivial. So the interesting physics will happen along the 4d space-time where τ is singular.

$\Rightarrow$ around such singular loci, τ will undergo an $SL_2\mathbb{Z}$ monodromy.

Usual lore: τ as the complex structure of an elliptic curve $\mathbb{E}_\tau$
$\Rightarrow$ Lift to M5-branes

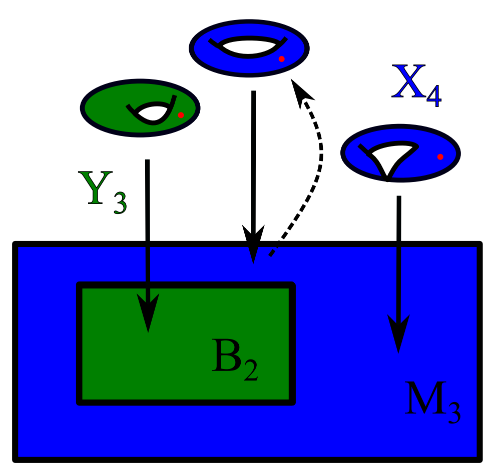
$\Rightarrow$ Setup: elliptic fibration over the 4d spacetime with $N = 4$ SYM in the bulk and duality defects (2d), which can intersect in 0d.

M5-brane point of view

$\{$6d $(2,0)$ theory on $\mathbb{E}_\tau \times \mathbb{R}^4\}$ = $\{N = 4$ SYM on $\mathbb{R}^4$ with coupling τ $\}$

So the setup that we will study is:

$\{$6d $(2,0)$ theory on a singular elliptic fibration$\}$
= $\{$ 4d $N = 4$ SYM with varying τ and duality defects$\}$



Setups:

Setup 1:

τ varies over 4d space (with B. Assel)

$\Rightarrow Y_3$ elliptic three-fold $\subset$ elliptic CY4

Setup 2:

τ varies onver a 2d space: 2d $(0,p)$ scfts (with C. Lawrie, T. Weigand)

$\Rightarrow$ D3s on curves in the base of CYn.

In both setups: M5-brane point of view will be instrumental.

Advantages of the M5-brane point of view

Various advantages in considering the M5-branes on elliptic surface $\widehat{C}$ instead of D3 on C:

- \# 3-7 modes: Automatically included as chiral modes from $\mathcal{B}_2$ reduced along $(1,1)$ forms from singular fibers.
- \# Topological Twist: 4d $N=4$ with varying τ on C requires topological duality twists (TDT) [Martucci]
  Will see: corresponds to M5-brane on $\widehat{C}$ with standard 'geometric' topological twist. [Assel, SSN]
- \# Non-abelianization:
  Bonus symmetry, and so TDT, exists for $U(1)$ $N=4$ SYM From M5-brane: 6d→ 5d + non-abelianization approch exists see e.g. [Kugo], [Cordova, Jafferis], [Assel, SSN, Wong], [Luo, Tan, Vasko, Zhao]

Similar considerations apply to the M2-brane duals, which give rise to a 1d $N=2,4$ SQM. Non-abelianization possible there using BLG theory. For K3: [Okazaki]

The 6d $(2,0)$ Theory

\# Lorentz and R-symmetry:

$$SO(1,5)_L \times Sp(4)_R \subset OSp(6|4)$$

\# Tensor multiplet:

$$\begin{aligned} \mathcal{B}_{MN}: &\quad (\mathbf{15},\mathbf{1}) \quad \text{with selfduality } \mathcal{H} = d\mathcal{B} = *_6\mathcal{H} \\ \Phi^{\widehat{m}\widehat{n}}: &\quad (\mathbf{1},\mathbf{5}) \\ \rho^{\widehat{m}}: &\quad (\bar{\mathbf{4}},\mathbf{4}) \end{aligned}$$

\# Abelian EOMs:

$$\mathcal{H}^- = d\mathcal{H} = 0\,, \qquad \partial^2\Phi^{\widehat{m}\widehat{n}} = 0\,, \qquad \partial\!\!\!/\rho^{\widehat{m}} = 0\,.$$

II. 4d $N = 4$ SYM with varying coupling and Duality Defects

[Assel, SSN]

M5-branes on Elliptic 3-folds

An elliptic fibration $\mathbb{E}_\tau \to Y_3 \to B$ (Y not CY) has metric

$$ds^2_Y = \frac{1}{\tau_2}\left((dx + \tau_1 dy)^2 + \tau_2^2 dy^2\right) + g^B_{\mu\nu} db^\mu db^\nu \,.$$

Pick a frame e^a for the base B and

$$e^4 = \frac{1}{\sqrt{\tau_2}}(dx + \tau_1 dy)\,, \qquad e^5 = \sqrt{\tau_2}dy\,.$$

Let Y_3 be a Kähler three-fold, so the holonomy is reduced to $U(3)_L$:

$$SO(6)_L \to U(3)_L$$
$$\mathbf{4} \to \mathbf{3}_1 \oplus \mathbf{1}_{-3}\,.$$

On a curved space: Killing spinor equation with ∇_M connection

$$(\nabla_M - A^R_M)\eta = 0$$

R-symmetry background $\Rightarrow$ constant spinor wrt twisted connection.

M5-branes on Elliptic 3-folds: Twist

\# Standard geometric twist: $U(1)_L$ with $U(1)_R$

$$Sp(4)_R \rightarrow SU(2)_R \times U(1)_R$$
$$\mathbf{4} \rightarrow \mathbf{2}_1 \oplus \mathbf{2}_{-1} \,.$$

\# Topological Twist

$$T_{U(1)_{\text{twist}}} = (T_{U(1)_L} - 3T_{U(1)_R})$$

implies that the supercharge decomposes as

$$\begin{array}{ccc} SO(6)_L \times Sp(4)_R & \rightarrow & SU(3)_L \times SU(2)_R \times U(1)_{\text{twist}} \times U(1)_R \\ (\mathbf{4},\mathbf{4}) & \rightarrow & (\mathbf{3},\mathbf{2})_{-2,1} \oplus (\mathbf{3},\mathbf{2})_{4,-1} \oplus (\mathbf{1},\mathbf{2})_{-6,1} \oplus (\mathbf{1},\mathbf{2})_{0,-1} \end{array}$$

$\Rightarrow (\mathbf{1},\mathbf{2})_{0,-1}$ give two scalar supercharges

Now specialize the 6d spacetime to be $\mathbb{E}_\tau \to Y_3 \to B_2$ with coordinates $x^0, \cdots, x^5$, and (x^4, x^5) the directions of the elliptic fiber.

The spin connection along $U(1)_L$ is

$$\Omega^{U(1)_L} = -\frac{1}{6}(\Omega^{01} + \Omega^{23} + \Omega^{45}),$$

and the twist corresponds to turning on the background gauge field

$$A^{U(1)_R} = -3\Omega^{U(1)_L}.$$

The base B_2 is Kähler as well, so the holonomy lies in $U(1)_\ell \times SU(2)_\ell \subset U(3)_L$ with the $U(1)$ generators given by

$$T_L = T_\ell + 2T_{45}$$

Key: $SO(2)_{45}$ rotation is along the fiber, and the non-trivial fibration is characterized through a connection in this $SO(2)_{45}$ direction and the spin connection is

$$\mathcal{A}_D = \omega^D = -\frac{\partial_a \tau_1}{4\tau_2} e^a$$

Duality Twist

This means: from the 4d point of view the topological twisting requires

$$\mathcal{A}_D = \omega^D = -\frac{\partial_a \tau_1}{4\tau_2} e^a$$

The associated $U(1)$ is in fact what is known as the "bonus symmetry" of abelian $N = 4$ SYM [Intrilligator][Kapustin, Witten] and we recovered the duality twist of N=4 SYM [Martucci] from the M5-brane theory.

The bonus symmetry exists for the abelian $N = 4$ SYM and acts as follows on the supercharges for $ab - cd = 1$

$$\begin{aligned} Q^{\dot{m}} &\rightarrow e^{-\frac{i}{2}\alpha(\gamma)} Q^{\dot{m}} \\ \tilde{Q}^{m} &\rightarrow e^{\frac{i}{2}\alpha(\gamma)} \tilde{Q}^{m} \end{aligned} \qquad \text{where} \qquad e^{i\alpha(\gamma)} = \frac{c\tau + d}{|c\tau + d|}$$

$$\phi^{\hat{i}} \rightarrow \phi^{\hat{i}}, \qquad \lambda_+^{\dot{m}} \rightarrow e^{-\frac{i}{2}\alpha(\gamma)} \lambda_+^{\dot{m}}, \qquad \lambda_-^{m} \rightarrow e^{\frac{i}{2}\alpha(\gamma)} \lambda_-^{m}$$

$$F_{\mu\nu}^{(\pm)} \rightarrow e^{\mp i\alpha(\gamma)} F_{\mu\nu}^{(\pm)} \qquad F^{(\pm)} \equiv \sqrt{\tau_2} \left(\frac{F \pm \star F}{2} \right)$$

Duality Twisted $N = 4$ SYM from 6d

6d topological twist + dim reduction to B gives an $N = 4$ SYM with varying τ over a Kähler base B

$$
\begin{aligned}
S^{U(1)}_{\text{total}} &= \frac{1}{4\pi} \int_B \tau_2 F_2 \wedge \star F_2 - i\tau_1 F_2 \wedge F_2 \\
&+ \frac{8}{\pi} \int_B \bar{\partial} \star \psi^{\alpha}_{(1,0)}\, \chi_{(0,0)\alpha} - \partial \psi^{\alpha}_{(1,0)} \wedge \rho_{(0,2)\alpha} - \partial_{\mathcal{A}} \star \tilde{\psi}^{\dot{\alpha}}_{(0,1)}\, \tilde{\chi}_{(0,0)\dot{\alpha}} + \bar{\partial}_{\mathcal{A}} \tilde{\psi}^{\dot{\alpha}}_{(0,1)} \wedge \tilde{\rho}_{(2,0)\dot{\alpha}} \\
&- \frac{1}{4\pi} \int_B \bar{\partial} \varphi^{\alpha\dot{\alpha}} \wedge \star \partial \varphi_{\alpha\dot{\alpha}} + 2\bar{\partial}_{\mathcal{A}} \sigma_{(2,0)} \wedge \star \partial_{\mathcal{A}} \tilde{\sigma}_{(0,2)}
\end{aligned}
$$

and non-abelian extension (see paper with Ben Assel).

The twisted fields are form fields and sections of the $\mathcal{A}_D$ bundle specified by the charges:

	$F_2^{(\pm)}$	$\varphi^{\alpha\dot{\alpha}}$	$\sigma_{(2,0)}$	$\tilde{\sigma}_{(0,2)}$	$\chi^{\alpha}_{(0,0)}$	$\tilde{\chi}^{\dot{\alpha}}_{(0,0)}$	$\psi^{\alpha}_{(1,0)}$	$\tilde{\psi}^{\dot{\alpha}}_{(0,1)}$	$\rho^{\alpha}_{(0,2)}$	$\tilde{\rho}^{\dot{\alpha}}_{(2,0)}$
$L_D^{q/2}$	∓ 2	0	-2	2	0	-2	0	2	0	-2

$$
\begin{aligned}
S^{na} = \int_B \frac{8}{\pi\sqrt{\tau_2}} \mathrm{Tr}\Big[&- \frac{i}{16} f_{(0,0)} [\sigma_{(2,0)} \wedge \tilde{\sigma}_{(0,2)}] - [\tilde{\psi}^{\dot\alpha}_{(0,1)} \wedge \star \psi^{\alpha}_{(1,0)}] \varphi_{\alpha\dot\alpha} \\
&+ \frac{1}{4} [\tilde{\psi}^{\dot\alpha}_{(0,1)} \wedge \tilde{\psi}_{(0,1)\dot\alpha}] \wedge \sigma_{(2,0)} - \frac{1}{4} [\psi^{\alpha}_{(1,0)} \wedge \psi_{(1,0)\alpha}] \wedge \tilde{\sigma}_{(0,2)} \\
&+ [\tilde{\chi}^{\dot\alpha}_{(0,0)} \wedge \star \chi^{\alpha}_{(0,0)}] \varphi_{\alpha\dot\alpha} + [\tilde{\rho}^{\dot\alpha}_{(2,0)} \wedge \rho^{\alpha}_{(0,2)}] \varphi_{\alpha\dot\alpha} \\
&- [\tilde{\chi}^{\dot\alpha}_{(0,0)}, \tilde{\rho}_{(2,0)\dot\alpha}] \wedge \tilde{\sigma}_{(0,2)} + [\chi^{\alpha}_{(0,0)}, \rho_{(0,2)\alpha}] \wedge \sigma_{(2,0)} \Big] \\
+ \frac{1}{16\pi\tau_2} \mathrm{Tr} \Big[&2 [\varphi^{\alpha\dot\alpha}, \sigma_{(2,0)}] \wedge [\varphi_{\alpha\dot\alpha}, \tilde{\sigma}_{(0,2)}] + [\varphi^{\alpha\dot\alpha}, \varphi_{\beta\dot\alpha}][\varphi^{\beta\dot\beta}, \star\varphi_{\alpha\dot\beta}] \\
&+ [\varphi^{\alpha\dot\alpha}, \varphi^{\beta\dot\beta}][\varphi_{\beta\dot\alpha}, \star\varphi_{\alpha\dot\beta}] + [\sigma_{(2,0)} \wedge \tilde{\sigma}_{(0,2)}] \star ([\sigma_{(2,0)} \wedge \tilde{\sigma}_{(0,2)}]) \Big] \,,
\end{aligned}
$$

Here $A = a_{(1,0)} + a_{(0,1)}$ and $dA = F_2$ implies

$$
f_{(2,0)} = \sqrt{\tau}_2 \partial a_{(1,0)} \,, \quad f_{(0,2)} = \sqrt{\tau}_2 \bar\partial a_{(0,1)} \,, \quad f_{(1,1)} + f_{(0,0)} \wedge j = \sqrt{\tau}_2 (\bar\partial a_{(1,0)} + \partial a_{(0,1)})
$$

So far: this describes the '4d bulk' theory on B_2 with varying τ. Loci of interest: singularities in the fiber, which give duality defects.

Singular Elliptic Curves and Defects

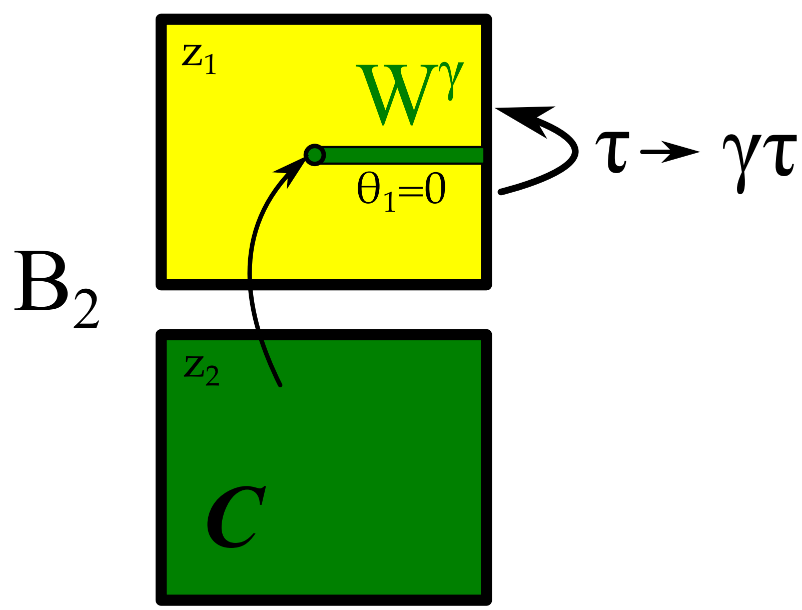
We can describe the elliptic fibration by $\mathbb{E}_\tau$ in terms of a Weierstrass model

$$y^2 = x^3 + fx + g$$

f and g sections $K_B^{-2/-3}$ and the singular loci are

$$\Delta = 4f^3 + 27g^2 = 0\,.$$

Close to a singular locus $z_1 = 0$, $\tau \sim i \log z_1 + \cdots$ with a branch-cut in the complex plane z_1. For the M5 this is relevant along $\Delta \cap B$:



Gauge theoretic description of walls and defects

Locally we can cut up $B = \cup B_i$ and W_{ij} 3d walls between these regions, where τ has a branch-cut.

Define

$$F_D = \tau_1 F + i\tau_2 \star F$$

then the action of $\gamma \in SL_2\mathbb{Z}$ monodromy on the gauge field is

$$(F_D^{(j)}, F^{(j)})\Big|_{W_{ij}} = \gamma(F_D^{(i)}, F^{(i)})\Big|_{W_{ij}}$$

This maps the gauge part $S_F = -\frac{i}{4\pi}\int_B F \wedge F_D$ to itself, except for an offset on the 3d wall (see also [Ganor])

$$S^{\gamma}_{W_{ij}} = -\frac{i}{4\pi}\int_{W_{ij}} \left(A^{(i)} \wedge F_D^{(i)} - A^{(j)} \wedge F_D^{(j)}\right)$$

E.g. $\gamma = T^k$ this is a level k CS term.

Chiral Duality Defects

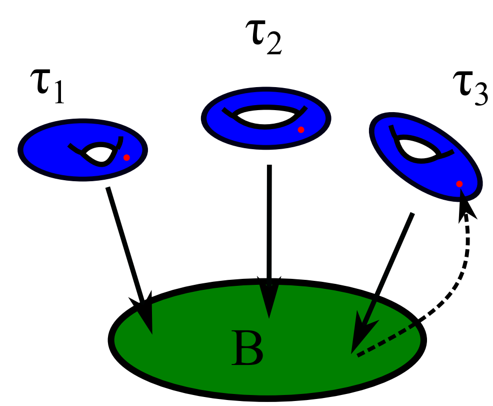
The wall action S^γ is neither supersymmetric nor gauge invariant. At the boundary of the wall $\partial W = \mathcal{C}$ this induces chiral dofs: e.g. for the T^k wall this is simply a chiral WZW model with β_i, $i = 1, \cdots, k$, with $\star_2 d\beta_i = i d\beta_i$ [Witten]

$$S_{\mathcal{C}} = \sum_{i=1}^{k} -\frac{1}{8\pi} \int_{\mathcal{C}} \star_2 (d\beta_i - A) \wedge (d\beta_i - A) - \frac{i}{4\pi} \int_{\mathcal{C}} \beta_i \, F$$

Under gauge transformations $A \to A + d\Lambda$, $\beta_i \to \beta_i + \Lambda$ this generates $\int F \Lambda$ which cancels the anomaly from the 3d wall.

Duality Defects from M5-branes

From the elliptic fibration and M5-brane we can apply this to any γ:



Singular fibers resolve into collections of $S^2 = \mathbb{P}^1$s, intersecting in affine ADE Dynkin diagrams.

Each resolution spheres gives rise to an $\omega^{(1,1)}$ form, along which we can expand $\mathcal{B}$

$$d\mathcal{B} = \sum_{i=1}^{k-1} \left(\partial_z b_i dz \wedge \omega^i_{(1,1)} + \partial_{\bar{z}} b_i d\bar{z} \wedge \omega^i_{(1,1)} \right)$$

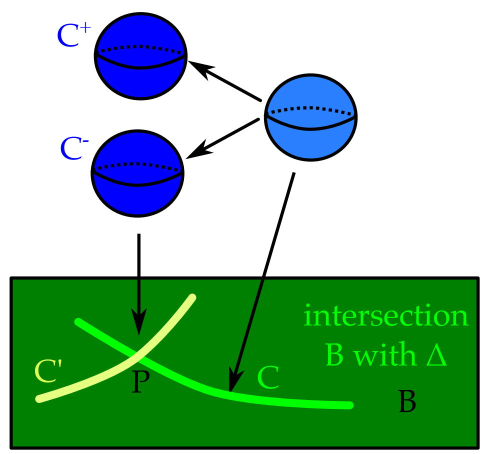
Imposing self-duality, and redefining the basis of chiral modes b_i with the "section" of the elliptic fibration, identifies these modes with β_i.

Intersections of Surface Defects: Point-defects

These chiral $(0,2)$ supersymmetric defects can intersect at points

$$P_{\alpha\beta} = \{z_\alpha = z_\beta = z = 0\} = \mathcal{C}_\alpha \cap \mathcal{C}_\beta = B \cap \Delta_\alpha \cap \Delta_\beta$$

Geometrically: Kodaira fiber $\mathbb{P}^1$s become further reducible $\mathbb{P}^1_i \to C_+ + C_-$



Duality defects form network and at intersections:

$$\left(\int_{C^+} + \int_{C^-}\right)\mathcal{B} = \int_{\mathbb{P}^1_i} \mathcal{B} \qquad \to \qquad \beta_+ + \beta_- = \beta_i$$

Such point-intersections are generic e.g. in CY4.
$\Rightarrow$ 4d-3d-2d-0d Matroshkas

Example:

D3-branes wrapping B_2 intersecting discriminant loci in

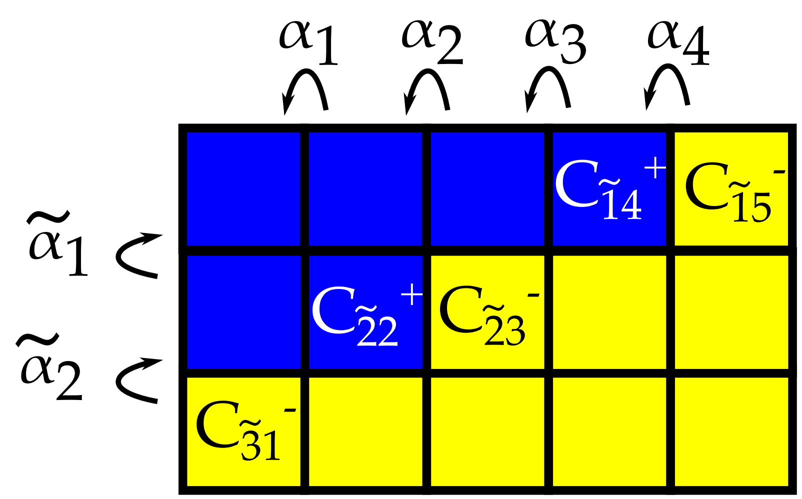
$$\Delta_1 \cap B = \mathcal{C} \leftrightarrow SU(n) \qquad \Delta_2 \cap B = \tilde{\mathcal{C}} \leftrightarrow SU(m)$$

E.g. fibers are given in terms of simple roots F_i, $i = 0, 1, \cdots, n-1$ and $\tilde{F}_j$, $j = 0, 1, \cdots, m-1$ and there are chiral modes localized on each curve

$$\mathcal{C}: \qquad \beta_i\,, \qquad i = 0, 1, 2, 3, 4\,, \qquad \tilde{\mathcal{C}}: \qquad \tilde{\beta}_i\,, \qquad i = 0, 1, 2$$

The fibers in codim 2 split as, e.g. for $SU(5)$ and $SU(3)$: into weights of the bi-fundamental:

$$C_{ij}^{\pm} \equiv \pm(L_i + \tilde{L}_j)\,.$$



$$
\mathcal{C}\ :\quad \begin{cases} F_0 & \rightarrow F_0' + C^{-}_{\tilde{3}1} \\ F_1 & \rightarrow F_1 \\ F_2 & \rightarrow C^{+}_{\tilde{2}2} + C^{-}_{\tilde{2}3} \\ F_3 & \rightarrow F_3 \\ F_4 & \rightarrow C^{-}_{\tilde{1}5} + C^{+}_{\tilde{1}4} \end{cases}
\qquad
\tilde{\mathcal{C}}\ :\quad \begin{cases} \tilde{F}_0 & \rightarrow \tilde{F}_0' + C^{-}_{\tilde{1}5} \\ \tilde{F}_1 & \rightarrow C^{+}_{\tilde{1}4} + F_3 + C^{-}_{\tilde{2}3} \\ \tilde{F}_2 & \rightarrow C^{+}_{\tilde{2}2} + F_1 + C^{-}_{\tilde{3}1} \end{cases}
$$

In codim 3 the $SU(5)$ and $SU(3)$ singularities collide at points $P = \mathcal{C} \cap \tilde{\mathcal{C}}$ in B: Local coupling along the surface defect to the bulk gauge field

$$S_{\mathcal{C}} \supset \int_{\mathcal{C}} \left(\sum_{i=0}^{k-1} \beta_i \right) F_{\mathcal{C}}$$

gives constraints:

$$\left. \left(F_{\mathcal{C}} \sum_{i=0}^{4} \beta_i \right) \right|_P = F_{\mathcal{C}} \left. \left(\beta^+_{\tilde{3}5} + \beta^-_{\tilde{3}1} + \beta_1 + \beta^+_{\tilde{2}2} + \beta^-_{\tilde{2}3} + \beta_3 + \beta^-_{\tilde{1}5} + \beta^+_{\tilde{1}4} \right) \right|_P$$
$$\left. \left(F_{\tilde{\mathcal{C}}} \sum_{i=0}^{2} \tilde{\beta}_i \right) \right|_P = F_{\tilde{\mathcal{C}}} \left. \left(\beta^+_{\tilde{3}5} + \beta^-_{\tilde{1}5} + \beta^+_{\tilde{1}4} + \beta_3 + \beta^-_{\tilde{2}3} + \beta^+_{\tilde{2}2} + \beta_1 + \beta^-_{\tilde{3}1} \right) \right|_P$$

Locally, this enhances the flavor symmetry of the 2d chiral models to $SU(n + m)$.

III. New 2d $(0,2)$ Theories

[Lawrie, SSN, Weigand]

D3s on C = M5 on $\widehat{C}$

Consider now $N=4$ SYM on $\mathbb{R}^{1,1}\times C$, with τ-varying over curve C:

$$SO(1,4)_L \to SO(1,1)_L \times U(1)_L$$

and to preserve supersymmetry, consider $U(1)_R \subset SU(4)_R$:

$$SU(4)_R \quad \to \quad \begin{array}{rl} SO(4)_T \times \underline{U(1)_R} & \text{CY}_3 \text{ Duality-Twist: } (0,4) \\ SU(2)_R \times \underline{U(1)_R} \times SO(2)_T & \text{CY}_4 \text{ Duality-Twist: } (0,2) \\ SU(3)_R \times \underline{U(1)_R} & \text{CY}_5 \text{ Duality-Twist: } (0,2) \end{array}$$

Geometric embedding corresponds to D3-branes on $C \times \mathbb{R}^{1,1}$ with

$$\{C \subset B_{n-1} = \text{Base of the elliptic } CY_n\}$$
$$= \{\text{6d } (2,0) \text{ theory on a elliptic surface } \widehat{C} = \mathbb{E}_\tau \to C\ \}$$

CY_3: MSW for elliptic CY [Vafa]

$N=4$ Duality Twist as M5 Toplogical Twist

CY_n Duality-Twist = Geometric Twist of M5 on $\hat{C}$

CY_n Duality-Twist

$$T_C^{\text{twist}} = \frac{1}{2}(T_C + T_R) \qquad T_D^{\text{twist}} = \frac{1}{2}(T_D + T_R)\,.$$

M5-brane Topological Twist: e.g. for CY_4 twist

$$\begin{aligned}
Sp(4)_R &\quad\rightarrow\quad SU(2)_R \times U(1)_R \\
\mathbf{4} &\quad\rightarrow\quad \mathbf{2}_0 \oplus \mathbf{1}_1 \oplus \mathbf{1}_{-1} \\
SO(1,5)_L &\quad\rightarrow\quad SU(2)_l \times SO(1,1)_L \times U(1)_l \\
\mathbf{4} &\quad\rightarrow\quad \mathbf{2}_{1,0} \oplus \mathbf{1}_{-1,1} \oplus \mathbf{1}_{-1,-1}
\end{aligned}$$

Twist is defined as

$$T^{\text{twist,M5}} = T_l + T_R$$

Example: CY_4-Duality Twist of $N = 4$ SYM from 6d

$$
\begin{array}{rl}
 & SU(2)_l \times SU(2)_R \times SO(1,1)_L \times U(1)_{\text{twist}} \times U(1)_R \\
\rho\,,\, Q\ : & (\mathbf{2},\mathbf{2})_{-1,0,0} \oplus (\mathbf{2},\mathbf{1})_{-1,1,1} \oplus (\mathbf{2},\mathbf{1})_{-1,-1,-1} \oplus (\mathbf{1},\mathbf{2})_{1,-1,0} \oplus {\color{darkred}(\mathbf{1},\mathbf{1})_{1,0,1}} \\
 & \oplus (\mathbf{1},\mathbf{1})_{1,-2,-1} \oplus (\mathbf{1},\mathbf{2})_{1,1,0} \oplus (\mathbf{1},\mathbf{1})_{1,2,1} \oplus {\color{darkred}(\mathbf{1},\mathbf{1})_{1,0,-1}} \\
\Phi\ : & (\mathbf{1},\mathbf{2})_{0,1,1} \oplus (\mathbf{1},\mathbf{2})_{0,-1,-1} \oplus (\mathbf{1},\mathbf{1})_{0,0,0} \\
H\ : & (\mathbf{3},\mathbf{1})_{-2,0,0} \oplus (\mathbf{1},\mathbf{1})_{2,2,0} \oplus (\mathbf{1},\mathbf{1})_{2,0,0} \oplus (\mathbf{1},\mathbf{1})_{2,-2,0} \oplus (\mathbf{2},\mathbf{1})_{0,1,0} \oplus (\mathbf{2},\mathbf{1})_{0,-1,0}\,.
\end{array}
$$

Geometric identification

$$
q^{\text{twist}}(K_{\widehat{C}}) = -2, \qquad\qquad q^{\text{twist}}(N_{\widehat{C}/Y_4}) = -1
$$

Spectrum of 2d $(0,2)$ from M5 on $\widehat{C} \subset CY_4$

Multiplicity	$(0,2)$	complex scalars	R-Weyl	L-Weyl
$h^{0,0}(\widehat{C}) = 1$	Chiral	1	1	–
$h^{0,1}(\widehat{C}) = g$	Fermi	–	–	1
$h^{0,2}(\widehat{C}) = g - 1 + c_1(B_3) \cdot C$	Chiral	1	1	–
$h^0(\widehat{C}, N_{\widehat{C}/Y_4}) = h^0(C, N_{C/B_3})$	Chiral	1	1	–
$\frac{1}{2} h^1(\widehat{C}, N_{\widehat{C}/Y_4}) = h^0(C, N_{C/B_3}) - c_1(B_3) \cdot C$	Fermi	–	–	1
$h^{1,1}(\widehat{C}) - 2h^{0,2}(\widehat{C}) - 2 = 8\, c_1(B_3) \cdot C$	Fermi	–	–	1

Central Charges

Direct computation from 6d $(2,0)$ or anomalies, on the elliptic surface $\mathbb{E}_\tau \to C$ times $\mathbb{R}^{1,1}$ (much like in the earlier discussion) yields

$$\begin{aligned} c_R =& 3(g + c_1(B_3) \cdot C + h^0(C, N_{C/B_3})) \\ c_L =& 3(g + h^0(C, N_{C/B_3})) + c_1(B_3) \cdot C + 8c_1(B) \cdot C \end{aligned}$$

From the $N = 4$ with duality twist, the zero modes do not incorporate the $3 - 7$ modes:

$$\delta c_L^{\text{defects}} = 8c_1(B) \cdot C.$$

In the 6d approach these are automatically incorporated.

Discussion of other cases:

CY_3 Duality twist $N = (0,4)$:

$$c_R = 3C \cdot C N_c^2 + 3c_1(B) \cdot C N_c + 6 \,, \qquad c_L = 3C \cdot C N_c^2 + 6c_1(B) \cdot C N_c + 6$$

This is dual to M5-branes on elliptic surfaces in CY three-folds, i.e. MSW-string, $N_c = 1$ already in [Vafa]. Computation of elliptic genera see e.g. [Haghighat, Murthy, Vandoren, Vafa].

CY_5 Duality twist: No M5 picture, but M2

$$c_L = 3(g + h^0(C, N_{C/B_4}) - 1) + 9c_1(B_4) \cdot C$$
$$c_R = 3(g + c_1(B_4) \cdot C + h^0(C, N_{C/B_4}) - 1)$$

Application to 2d $(0,2)$ vacua from CY_5 compactifications of F-theory [SSN, Weigand], [Apruzzi, Hassler, Heckman, Melnikov]. Tadpole cancellation requires D3-branes wrapped on curves in the class

$$C = \frac{1}{24} c_4(Y_5)|_{B_4}$$

BPS-equations and Hitchin moduli space

For τ constant, $N=4$ SYM on $C \times \mathbb{R}^{1,1}$ with Vafa-Witten twist, gives rise to a sigma-model into the Hitchin moduli space, which for the abelian case is just flat connections [Bershadsky, Johansen, Sadov, Vafa].

In all duality-twisted theories the BPS equations imply

$$\mathcal{F}_{\mathcal{A}} = \frac{1}{2}\Big(\bar{\partial}_{\mathcal{A}}(\sqrt{\tau_2}a) - \partial_{\mathcal{A}}(\sqrt{\tau_2}\bar{a})\Big) = 0$$

where the internal components of the gauge field $a, \bar{a}$ are

$$\sqrt{\tau_2}\bar{a} \in \Gamma(\Omega^{0,1}(C, \mathcal{L}_D^{-1}))$$
$$\sqrt{\tau_2}a \in \Gamma(\Omega^{0,0}(C, K_C \otimes \mathcal{L}_D))$$

In particular, for this abelian setup, the theory is a sigma-model into $U(1)_D$-twisted flat connections. $\rightarrow$ duality twisted Hitchin moduli space

Summary and Outlook

Matroshkas:

- M5 on an elliptic three-fold give rise to N=4 SYM with varying τ, a network of intersecting duality defects ’4d-3d-2d-0d’
- General $\gamma \in SL_2\mathbb{Z}$ duality defects with $(0,2)$ supersymmetry. Flavor symmetry dictated by the singular fiber geometry, classify duality defects, and extend to non-abelian setup [in progress]
- Localization, including defect intersections, e.g. as in [Gomis, Le Floch, Pan, Peelaers]

2d SCFTs:

D3s in F-theory on $C \subset B$ gives rise to 2d scfts with $(0, p)$ susy. Best described in terms of dual M5-brane on $\widehat{C}$.

Non-abelian generalization, sigma-model description into generalized Hitching moduli space:
D3 description so far limited to $U(1)$ gauge group. Non-abelianize starting from 6d, as in [Assel, SSN]. E.g. $C \subset K3^\tau$ get non-abelian version of the heterotic string.

M2-branes on $C \times \mathbb{R}$ give rise to Super-QM: i.e. twisted version of the Bagger-Lambert-Gustavsson theory on C.

AdS/CFT with varying τ:
These 2d $(0, p)$ SCFTs have interesting "F-theory" AdS-duals, i.e. varying-τ IIB solutions [Couzens, Martelli, SSN, Wong] (F-theoretic lift of the 6d N=1 sugra configurations in [Haghigat, Murthy, Vafa, Vandoren])

$$AdS_3 \times S^3 \times CY_3^\tau$$

Similarly: AdS_3 solutions for 2d $(0, 2)$ theories from CY4 in F-theory.