6d theories and nilpotent orbits in Lie algebras

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(mainly) based on
1407.6359 with M. del Zotto, J.Heckman, C.Vafa;
1601.04078 with J. Heckman, T. Rudelius;
1612.06399 with N. Mekareeya, T. Rudelius

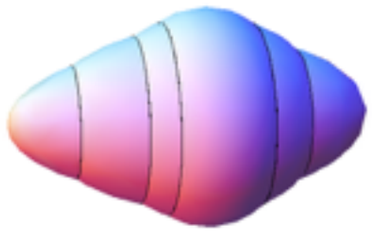
Summary

We will consider M5s at ADE singularities

with M2s, this strategy led to
[Aharony, Bergman, Jafferis, Maldacena '08]

- Described by 6d $\mathcal{N} = (1, 0)$ theories

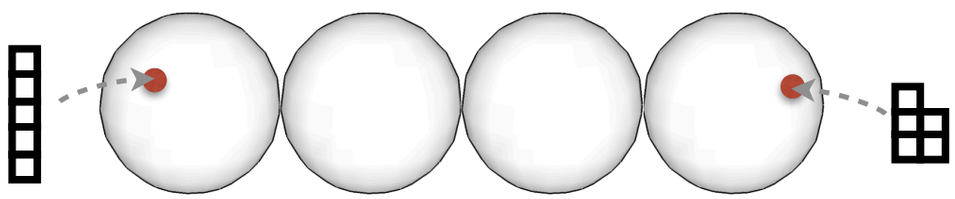
- They can be Higgsed to many other theories



  - 'A' case: IIA realization, AdS7 duals

  - 'D, E' cases: F-theory

  - two 'punctures' instead of the 3 for Class S in 4d

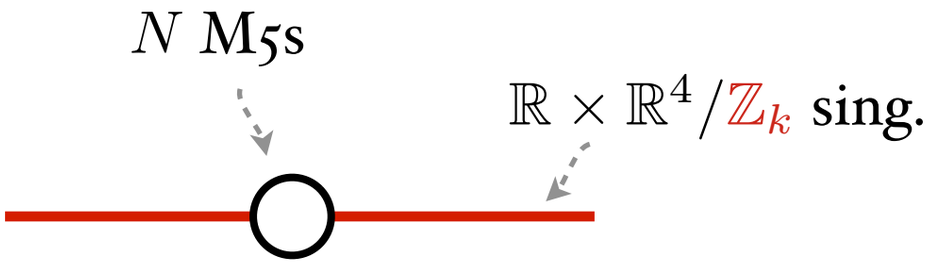


in general, nilpotent elements!

Plan

- 'A' case: IIA realization
  - Conformal matter theories
  - T-brane theories

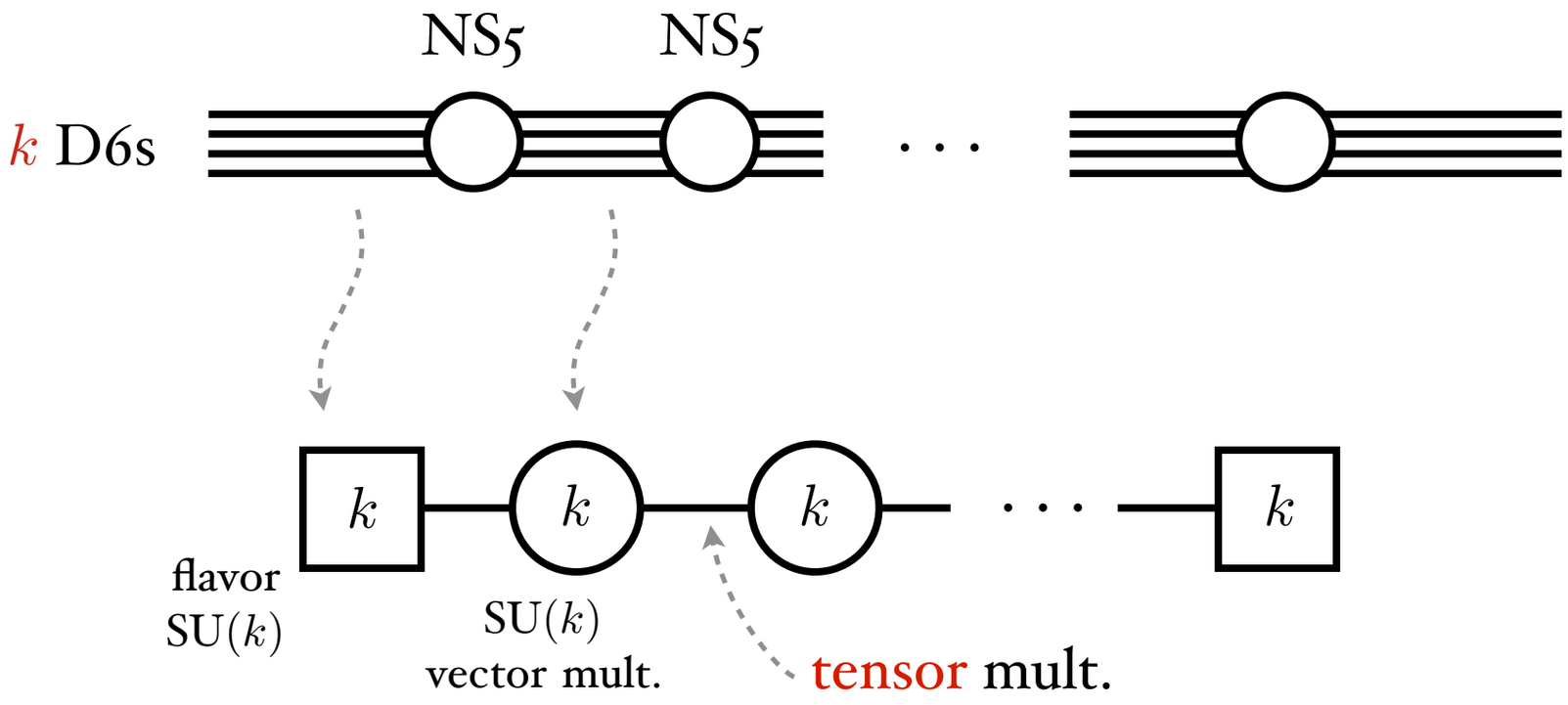
I. M5 branes at a $\mathbb{Z}_k$ singularity



- $\mathcal{N} = (1, 0)$ supersymmetry
- we expect $\mathrm{SU}(k) \times \mathrm{SU}(k)$ flavor symm.
- number of dof: $a \sim k^2 N^3$

- Separate the M5s:

- Analysis is more convenient if we reduce to IIA



vector mult. (λ_+, A_μ)

hypermult. (q, ψ_-)

tensor mult. $(\phi, \chi_-, b^+_{\mu\nu})$

NS5 transverse motions

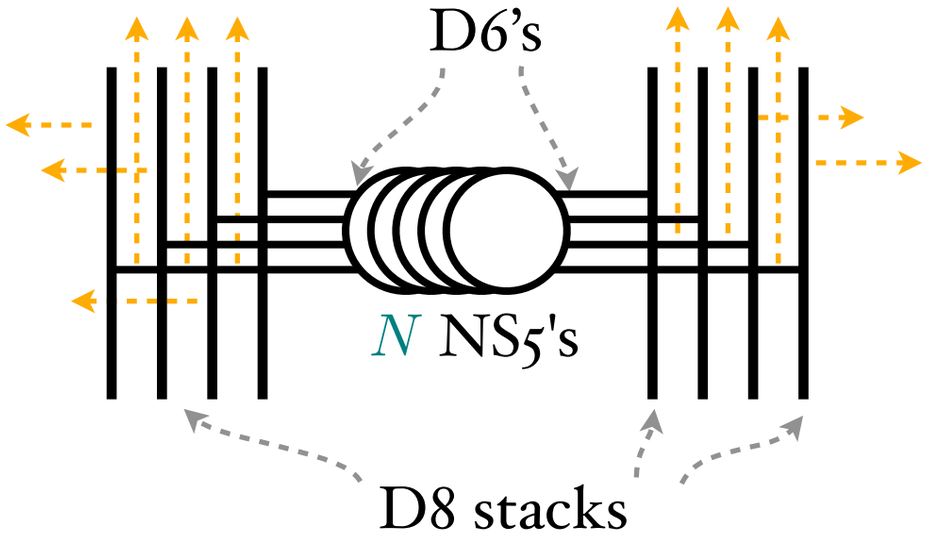
$$\dim_{\text{Higgs}} = N - 1 + k^2$$

$\mathcal{L} \supset (\phi_{i+1} - \phi_i)\text{Tr}F^2$ $\quad$ $\phi_i = x^6$ positions of NS5's

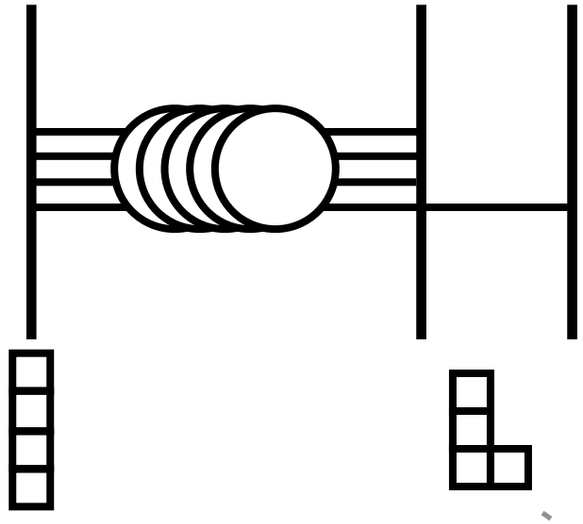
coincident NS5s =
strong coupling point; CFT?

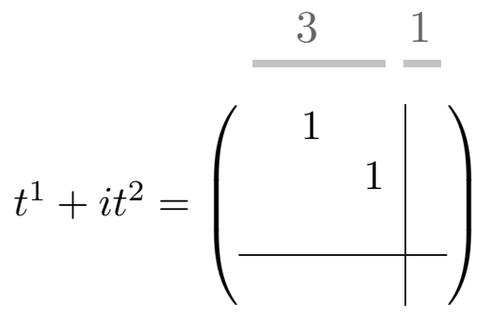
- alternative realization: each D6 ends on a single D8

[Gaiotto, Witten '08; Gaiotto, AT '14]



this suggests
Higgs RG flows:





$t^1 + it^2 =$

- These D8's can be thought of as
  Nahm poles for the D6's.

BPS equations on D6:
Nahm equations

$\partial_z X^1 = [X^2, X^3]$ etc.

$$X^i \sim \frac{t_i}{z}$$

$$[t_i, t_j] = \epsilon_{ijk} t_k$$

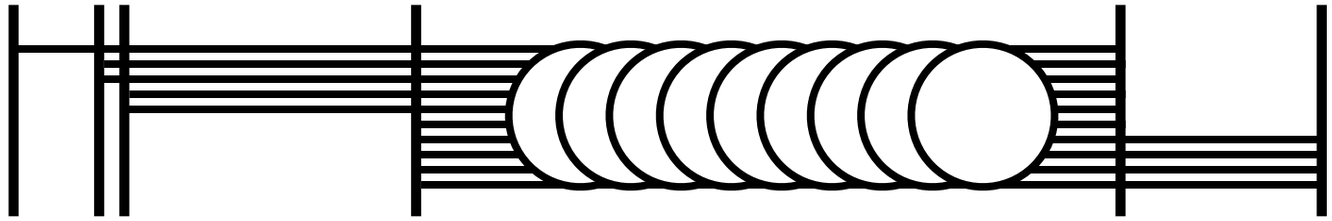
$\mathrm{su}(2)$ subalgebra of $\mathrm{su}(k)$

$\wr$

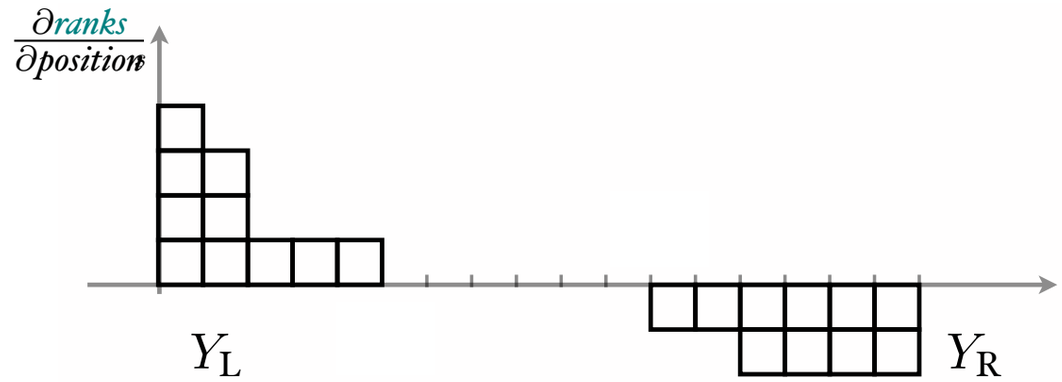
partition

[Young diagram]

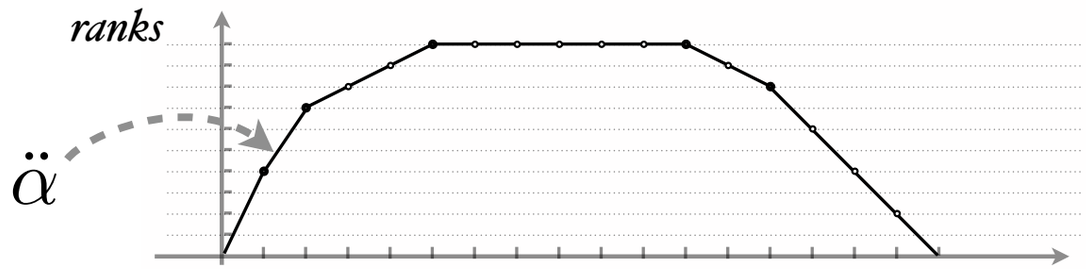
Here is how to reconstruct the theory.

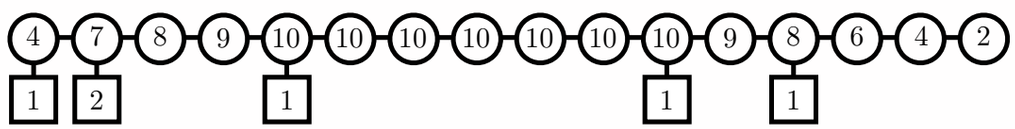


- Start with partitions:



- ‘integrate’: gauge groups.

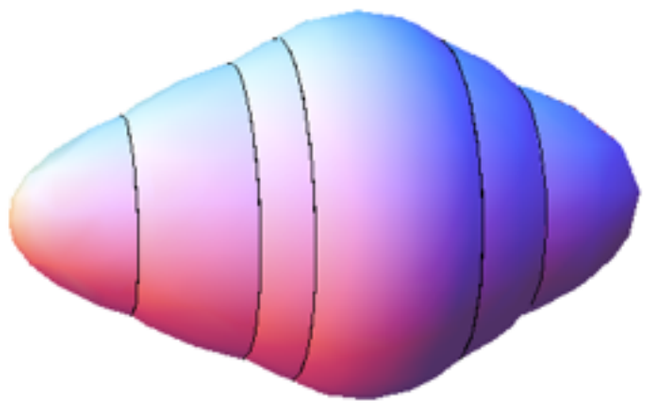




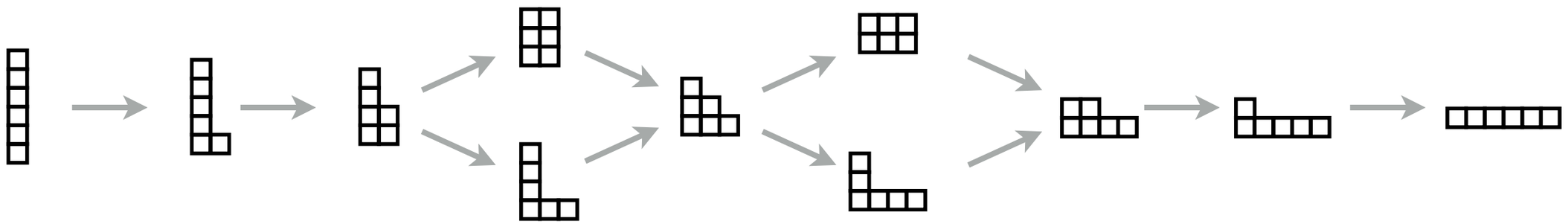
Gravity dual:

$$ds^2 = 8\sqrt{-\frac{\ddot{\alpha}}{\alpha}}ds^2_{\text{AdS}_7} + \sqrt{-\frac{\alpha}{\ddot{\alpha}}}dz^2 + \frac{\alpha^{3/2}(-\ddot{\alpha})^{1/2}}{\sqrt{2\alpha\ddot{\alpha}-\dot{\alpha}^2}}ds^2_{S^2}$$

[Apruzzi,Fazzi,Rosa,AT’13; Gaiotto, AT ’14; Apruzzi,Fazzi,Passias,Rota’15;Cremonesi,AT’15]



- We expect a hierarchy of RG flows
  corresponding to the hierarchy of Y.d.



- Higgs moduli space dimension

unHiggsed theory

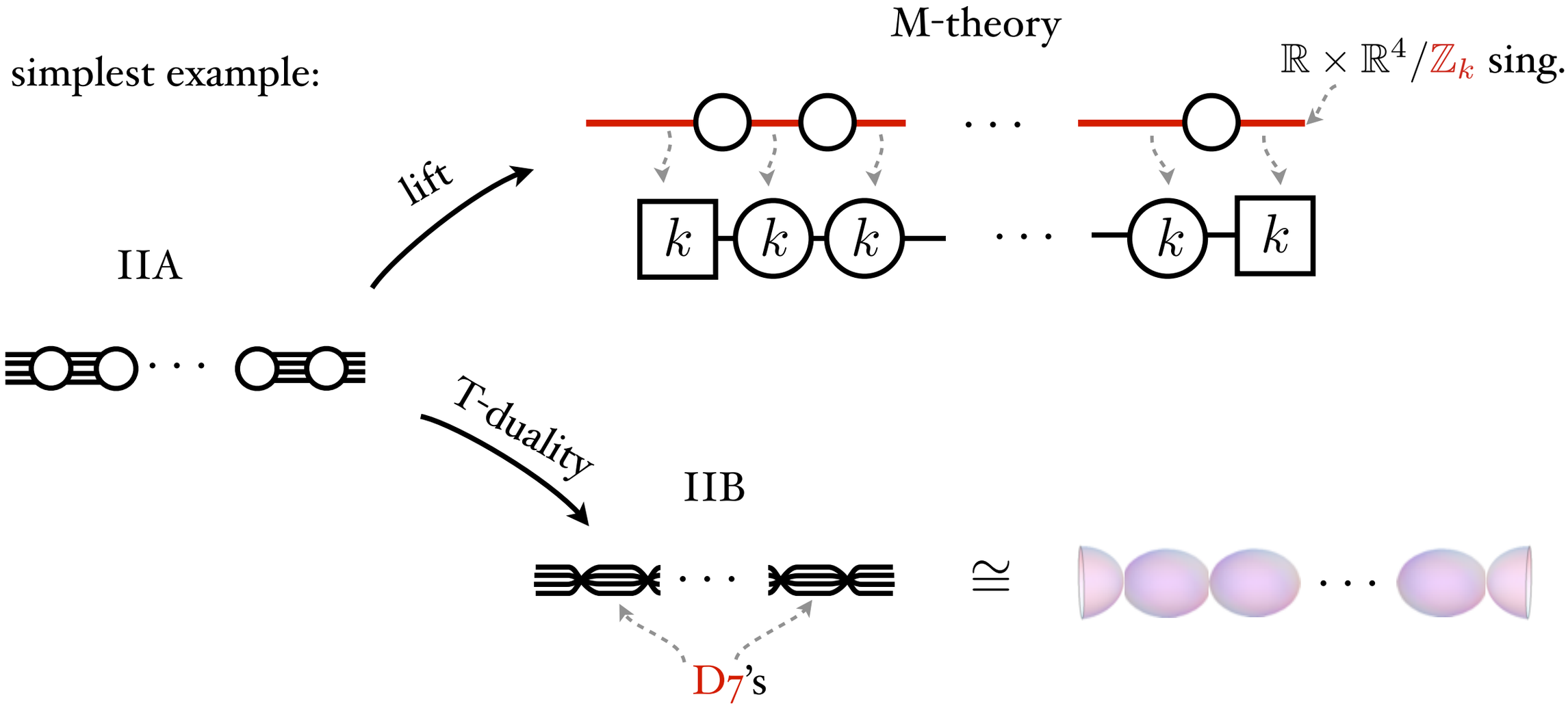
$$\dim_{\text{Higgs}} = \overbrace{N - 1 + k^2} - \dim_{\mathcal{O}_{\text{L}}} - \dim_{\mathcal{O}_{\text{R}}}$$

dimensions nilpotent orbits
associated to partitions

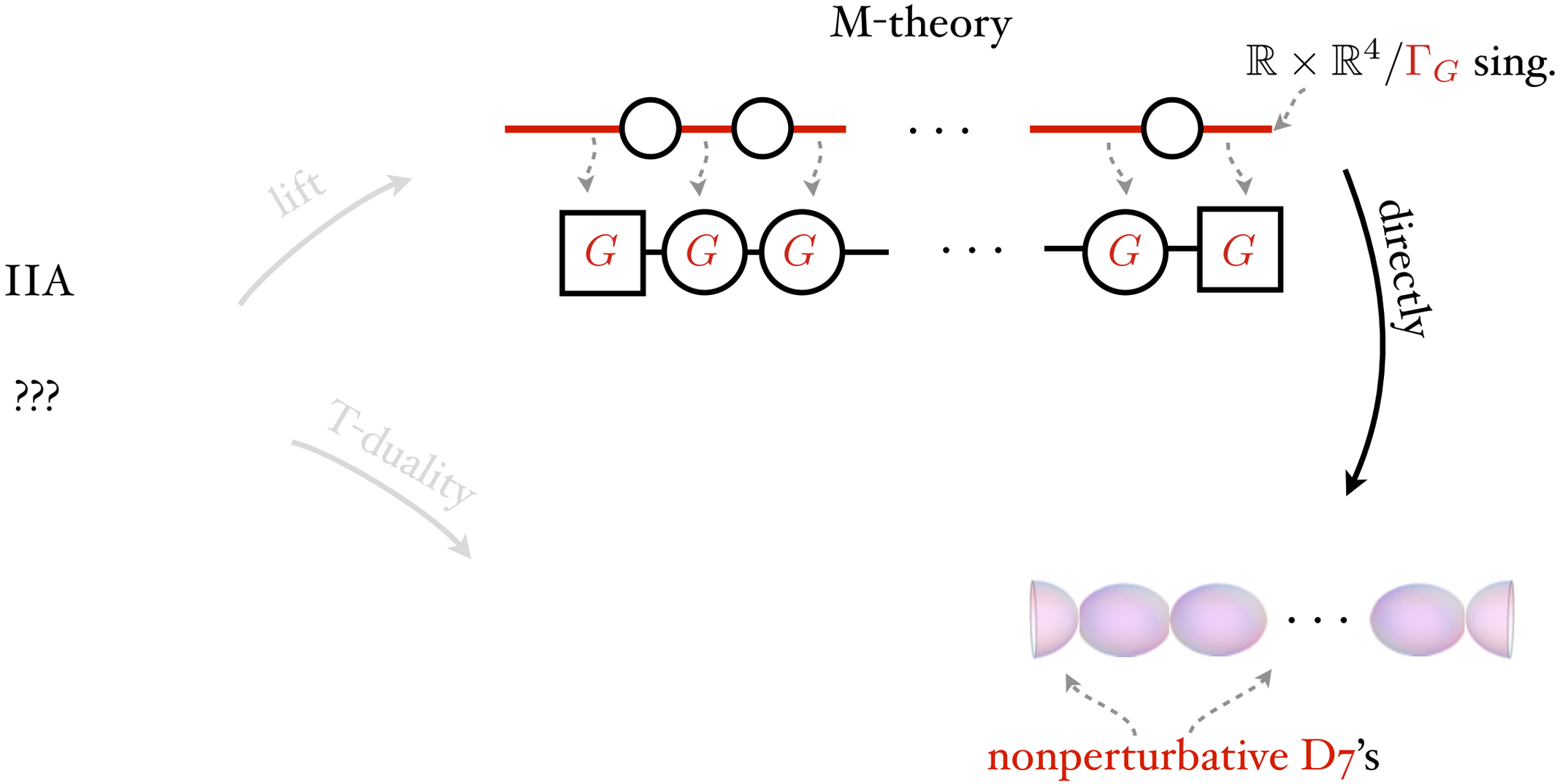
II. From IIA to Conformal Matter

So far we have seen chains of $SU(N)$ gauge groups

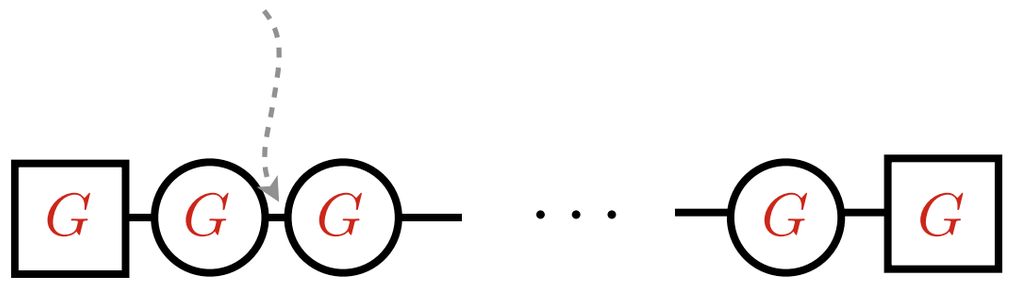
simplest example:

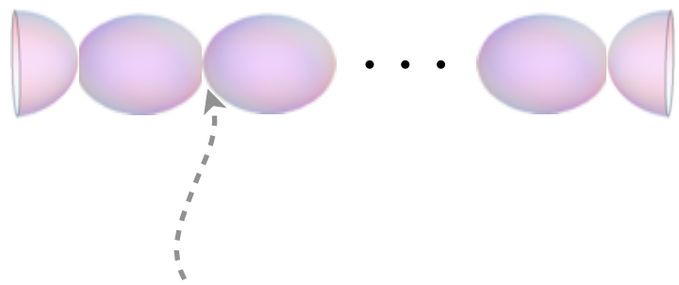


- F-theory allows to include more general gauge groups



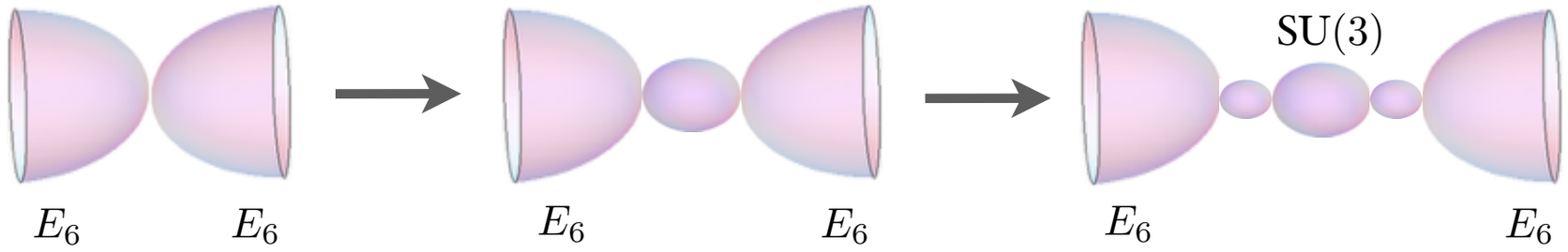
what is this ‘link’? no longer just a tensor+hyper





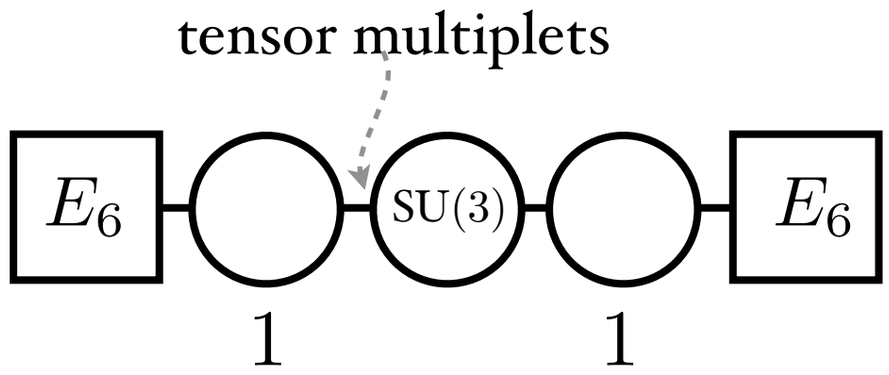
in F-theory, we can ‘blow up’ the singularity between two touching sevenbranes

- Example: E_6

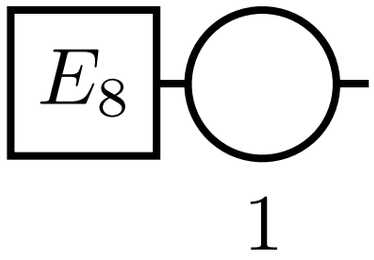


this pattern also appeared in
[Berhadsky, Johansen '96]
[Aspinwall, Morrison '97]
[Intriligator'97]...

"$E_6 \times E_6$ conformal matter"



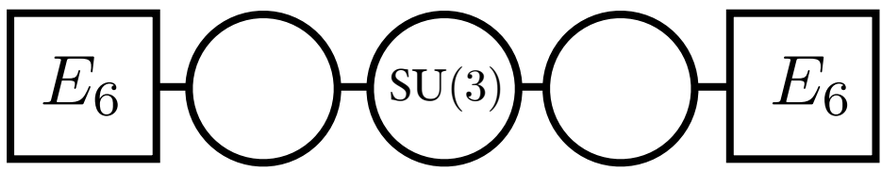
here $\mathrm{SU}(3) \subset E_8$
has been gauged



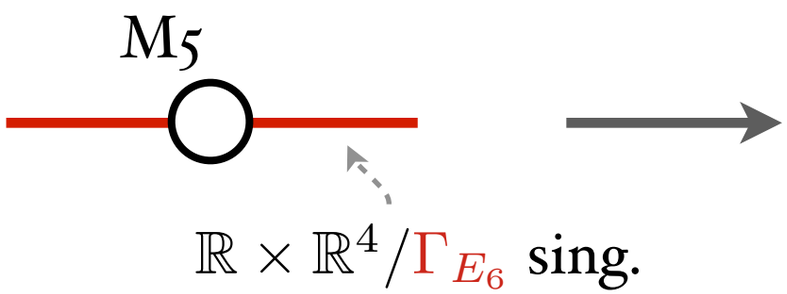
=

"E-string"
(no gauge group)
E_8 flavor symmetry

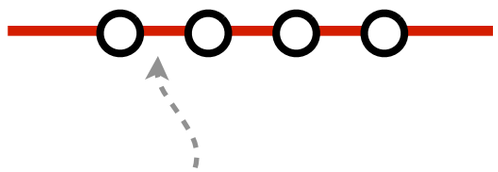
It also appears
for M5's near M9



In M-theory:



Conjecture: 4 fractional M5's



[del Zotto, Heckman, AT, Vafa '14]

a 'discrete flux' is created whenever a fractional M5 is crossed

for a nice alternative explanation

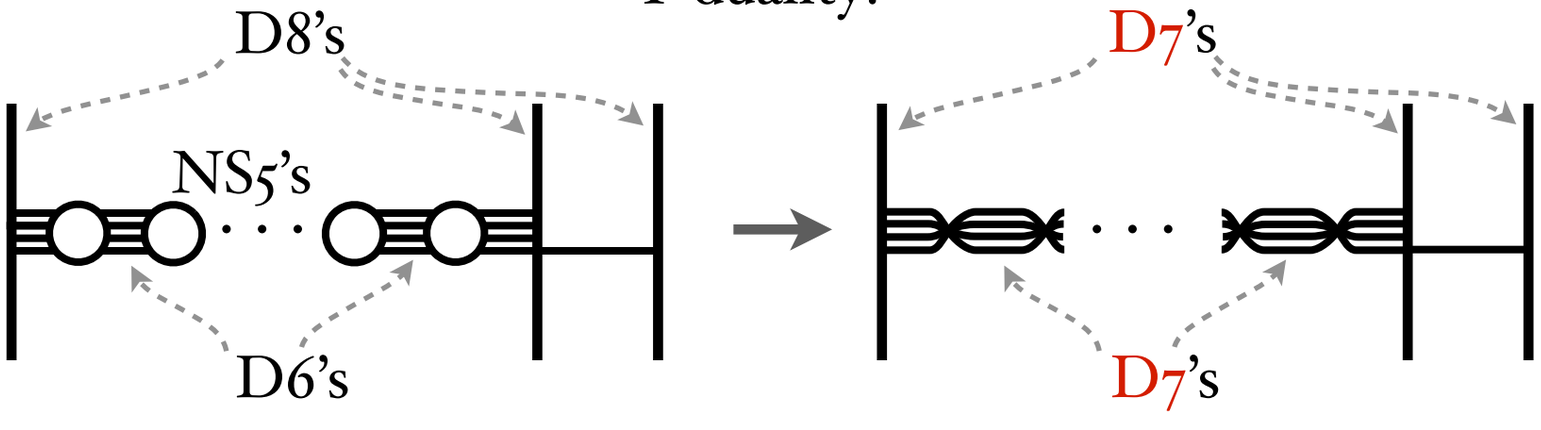
[Ohmori, Shimizu, Tachikawa, Yonekura '15, Tachikawa '15]

III. T-brane theories

What about D8s?

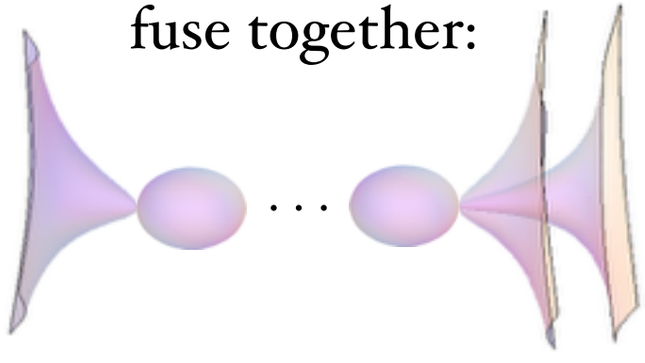
[del Zotto, Heckman, AT, Vafa '14]

T-duality:



$\cong$

but actually the D7's fuse together:



Nahm's equations → Hitchin's equations

Chain of intersecting curves, decorated by Hitchin poles

$$\phi = X^1 + iX^2 \sim \frac{t}{z - z_0}$$

residue is nilpotent

"T-brane"

So there should be 6d SCFTs

$T^N{}_{\rho_L, \rho_R}$

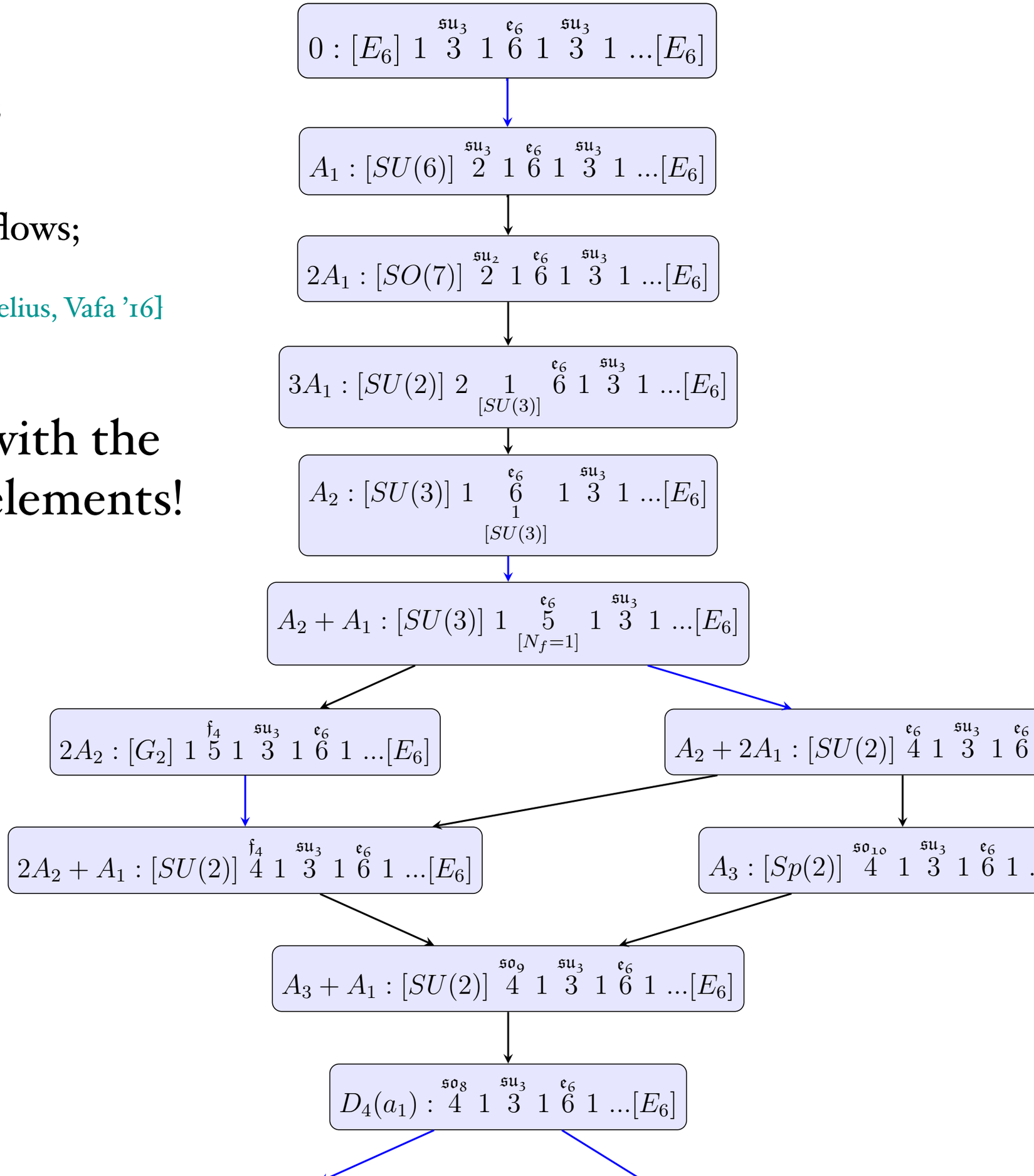
nilpotent ADE elements

how do we find them?

Starting with a chain of
'conformal matter' theories

We worked out the web of RG flows;
[Heckman, Rudelius, AT '16]
using methods in [Heckman, Morrison, Rudelius, Vafa '16]

it coincides precisely with the
hierarchy of nilpotent elements!



Higgs moduli spaces

[Mekareeya, Rudelius, AT '16]

conjecture: $\dim_{\rm H} = \dim_{\rm H}(\text{c.m. chain}) - \dim_{\mathcal{O}_{\rm L}} - \dim_{\mathcal{O}_{\rm R}}$

let's check! difference of [anomaly arguments]

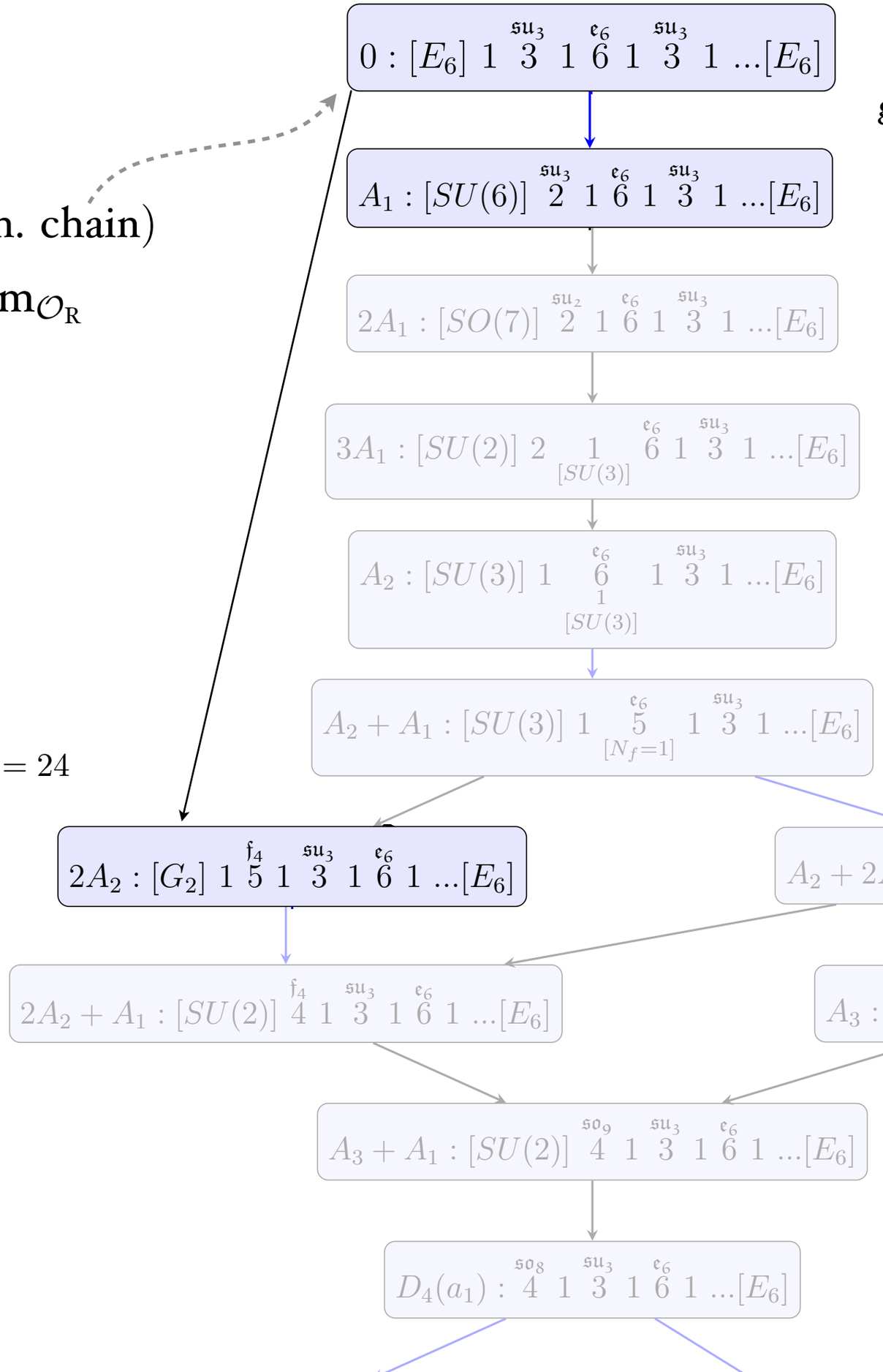
$$29n_{\rm T} + n_{\rm H} - n_{\rm V}$$

$\mathfrak{su}_3 \quad \mathfrak{e}_6 \quad \mathfrak{f}_4$

$29 \times 2 - (8 + 78 - 52) = 24$

$d_{\mathcal{O}} = 24$ ✓

...it always works
in fun ways

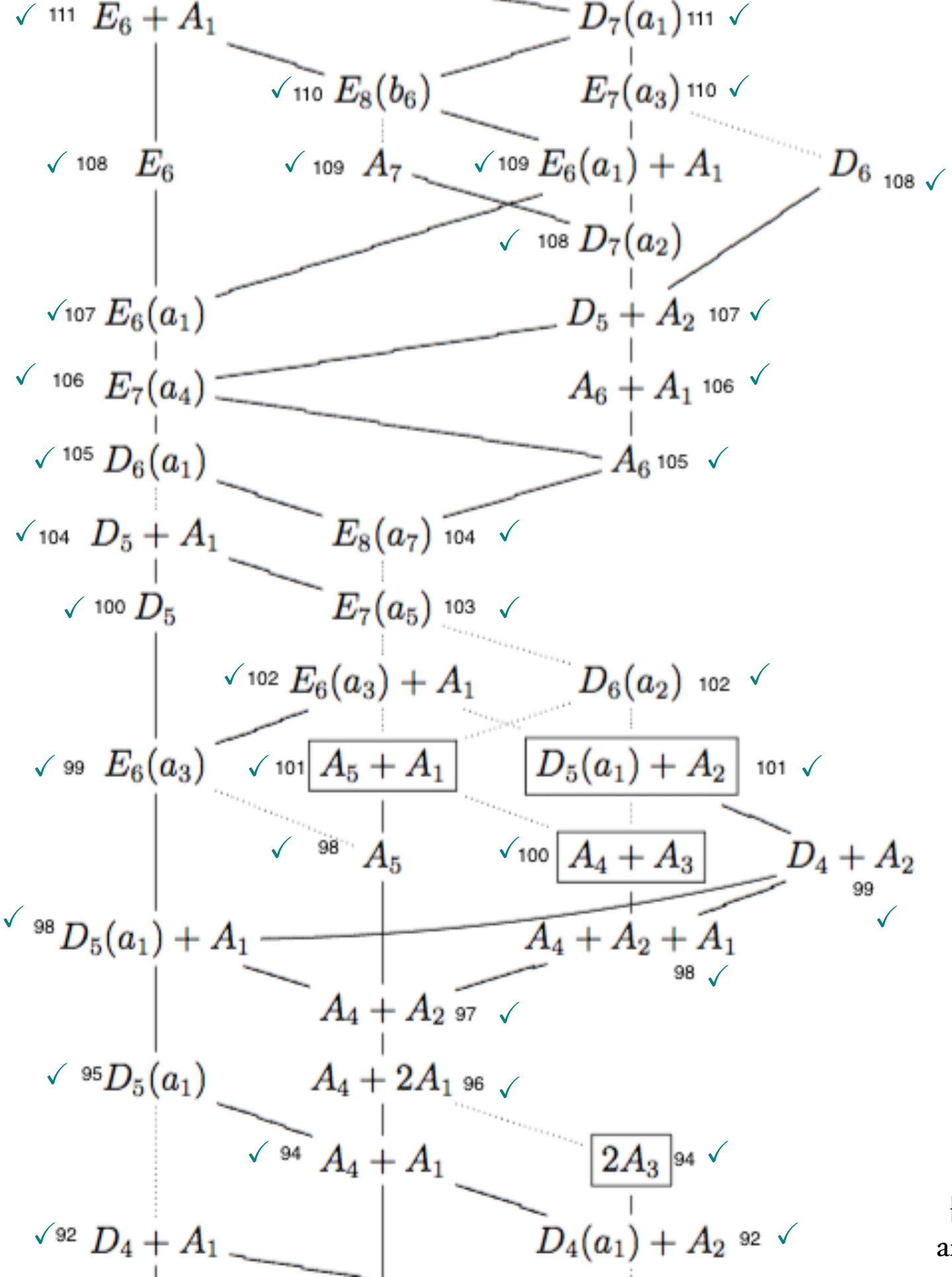


we lose one tensor,
gain 6 fundam. of $\mathfrak{su}_3$
$29 \times 1 - 18 = 11$

$d_{\mathcal{O}} = 11$ ✓

[all dimensions are quaternionic]

...really!

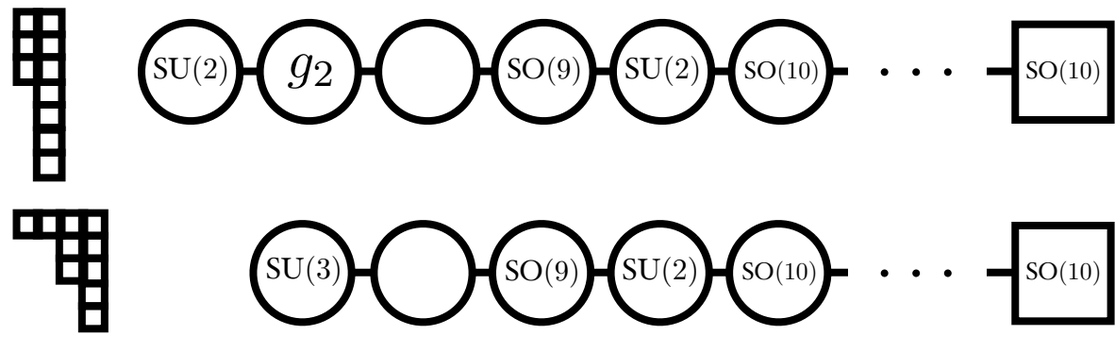


[all dimensions are quaternionic]

Even for $\mathrm{SO}(2n)$
the theories are not obvious:

IIA brane diagram ⇨ sometimes negative numbers of branes

But F-theory succeeds!



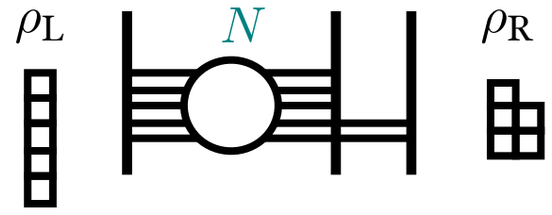
Conclusions

- Many ways to Higgs M5s at singularities $T^N_{\rho_L,\rho_R}$

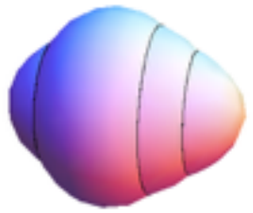
  nilpotent ADE elements

- In the A_k case:

  - nilpotent = pattern of D6s ending on D8s



  - analytically known AdS7 duals



- In the D_k, E_k cases:

  - more exotic ingredients
  - but similar overall structure!

what implications for M5-dynamics?