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(Non-) Abelian Discrete Symmetries in String (F-) Theory

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Outline (Summary)

Progress report since F-theory'16, Caltech

I. Abelian discrete gauge symmetries in F-theory

multi-sections &Tate-Shafarevich group – highlight Z_3

highlight Heterotic duality and Mirror symmetry

II. Non-Abelian discrete gauge symmetries in F-theory

relatively unexplored

stoop down to weakly coupled regime

Non-Abelian discrete symmetries in Type IIB String

Explicit construction of CY-threefold, resulting in a four-dimensional Heisenberg-type discrete symmetry

Abelian discrete symmetries in Heterotic/F-theory

M.C., A. Grassi and M. Poretschkin,
``Discrete Symmetries in Heterotic/F-theory Duality and Mirror Symmetry,'' arXiv:1607.03176 [hep-th]

Non-Abelian discrete symmetries in Type IIB string

V. Braun, M.C., R. Donagi and M.Poretschkin,
``Type II String Theory on Calabi-Yau Manifolds with Torsion and Non-Abelian Discrete Gauge Symmetries,'' arXiv:1702.08071 [hep-th]

Abelian Discrete Symmetries in F-theory

Calabi-Yau geometries with genus-one fibrations

These geometries do not admit a section, but a multi-section

Earlier work: [Witten; deBoer, Dijkgraaf, Hori, Keurentjes, Morgan, Morrison, Sethi;...]

Recent extensive efforts'14-'16: [Braun, Morrison; Morrison, Taylor; Klevers, Mayorga-Pena, Oehlmann, Piragua, Reuter; Anderson,Garcia-Etxebarria, Grimm; Braun, Grimm, Keitel; Mayrhofer, Palti, Till, Weigand; M.C., Donagi, Klevers, Piragua, Poretschkin; Grimm, Pugh, Regalado; M.C., Grassi, Poretschkin;...]

Key features:

Higgsing models w/U(1), charge-n $\langle\Phi\rangle \neq 0$ – conifold transition

Geometries with n-section ⟺ Tate-Shafarevich Group Z_n

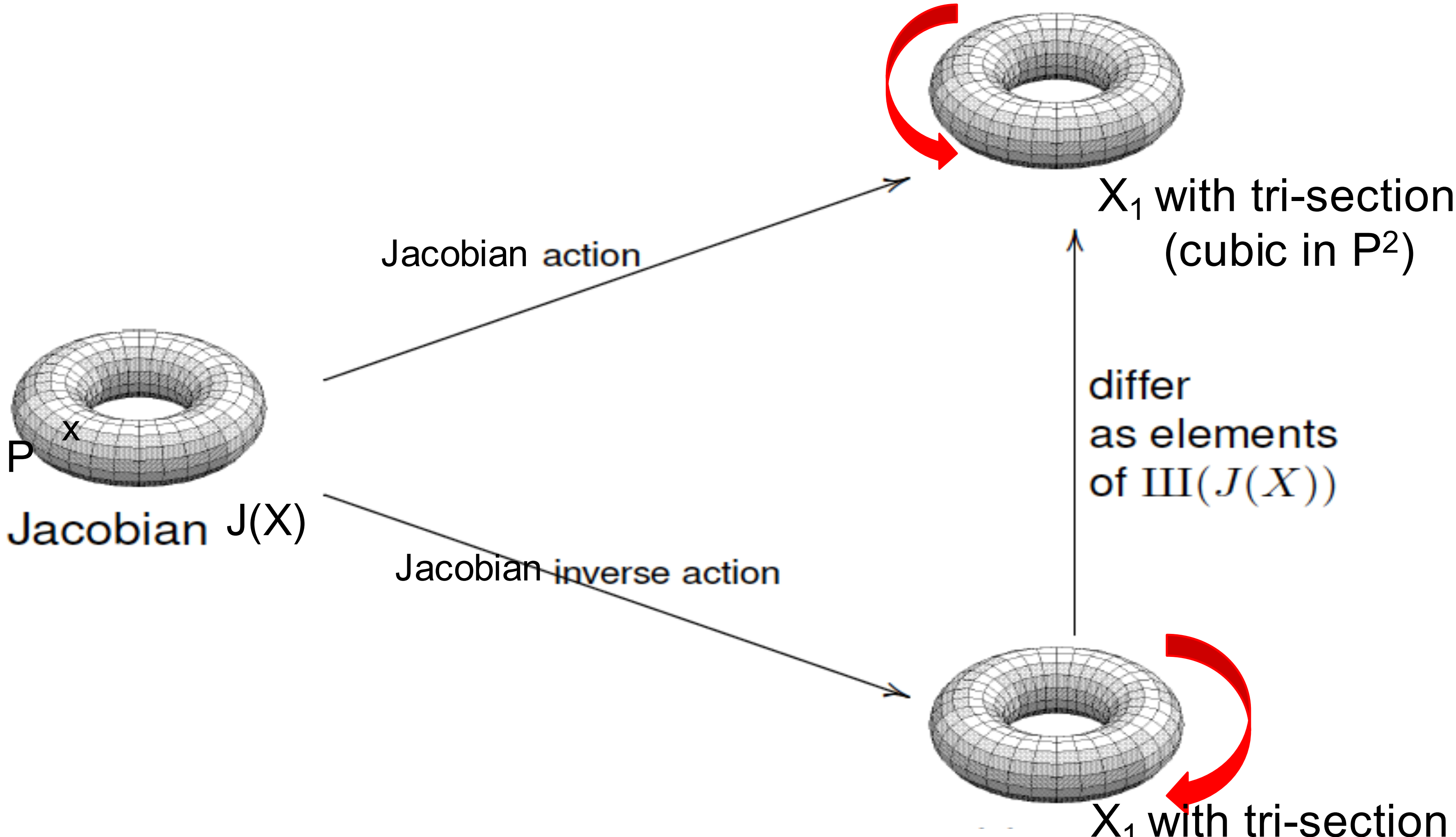
Z_2 [Anderson,Garcia-Etxebarria, Grimm; Braun, Grimm, Keitel; Mayrhofer, Palti, Till, Weigand'14]

Z_3 [M.C.,Donagi,Klevers,Piragua,Poretschkin 1502.06953]

Tate-Shafarevich group and Z_3

[M.C., Donagi, Klevers, Piragua, Poretschkin 1502.06953]

Only two geometries: X_1 w/ trisection and Jacobian $J(X_1)$



There are three different elements of TS group!

Shown to be in one-to-one correspondence with three M-theory vacua.

Discrete Symmetries & Heterotic/F-theory Duality

[Morrison,Vafa '96; Friedman,Morgan,Witten '97]

Basic Duality (8D):

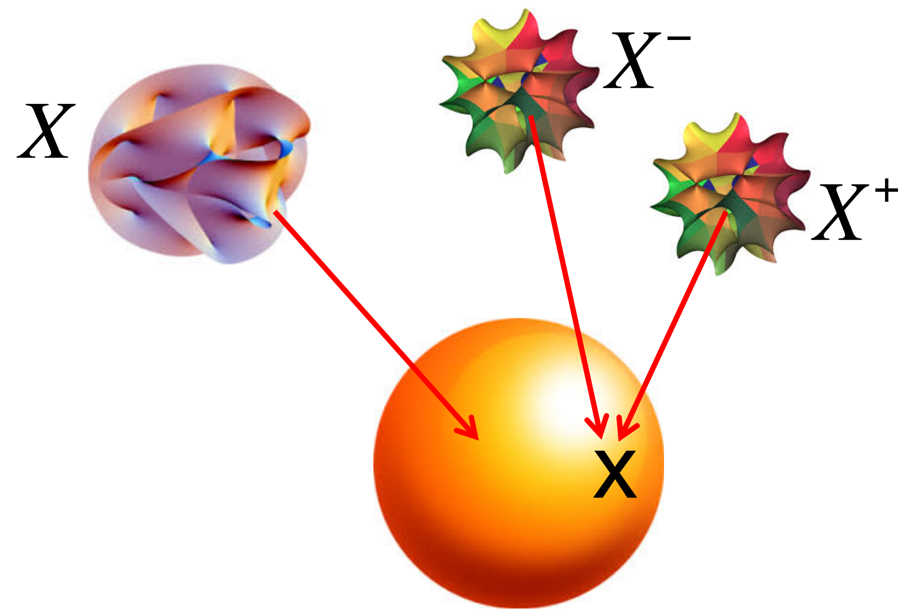
Heterotic $E_8 \times E_8$ String on T^2

⬍ dual to

F-Theory on elliptically fibered K3 surface X

Manifest in stable degeneration limit:

K3 surface X splits into two half-K3 surfaces X^+ and X^-



K3-fibration over P^1
(moduli)

Dictionary:

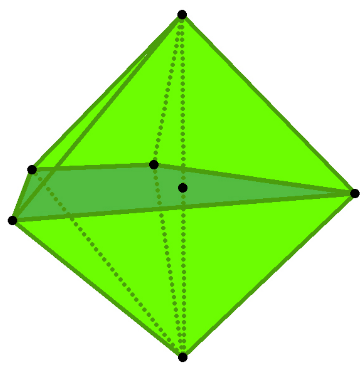
- X^+ and X^- → background bundles V_1 and V_2
- Heterotic gauge group $G = G_1 \times G_2$ $G_i = [E_8, V_i]$
- The Heterotic geometry T^2: at intersection of X^+ and X^-

Heterotic/F-theory Duality

[Morrison, Vafa '96], [Berglund, Mayr '98]

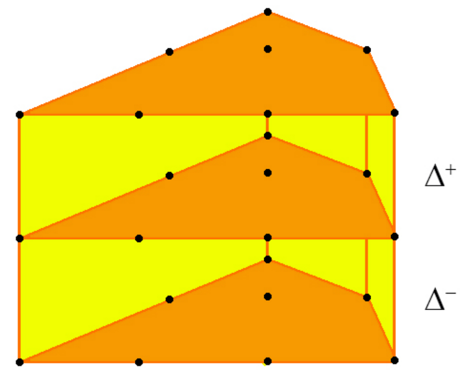
Employ toric geometry techniques in 8D/6D to study stable degeneration limit of F-theory

Toric polytope:



specifies the ambient space X

Dual polytope:



specifies the elements of $O(-K_X)$ - monomials in ambient space

6D: fiber this construction over another P^1

Study:

U(1)'s [M.C., Grassi, Klevers, Poretschkin, Song 1511.08208]
at F-theory'16, Caltech

Discrete symmetries [M.C., Grassi, Poretschkin 1607.03176]
highlights here

Discrete Symmetry in Heterotic/F-theory Duality

[M.C., Grassi, Poretschkin 1607.03176]

Goal: Trace the origin of discrete symmetry D

- Conjecture for $P^2(1,2,3)$ fibration [Berglund, Mayr '98]

  X_2 elliptically fibered, toric K3 with singularities (gauge groups) of type G_1 in X^+ and G_2 in X^-

  ⇕

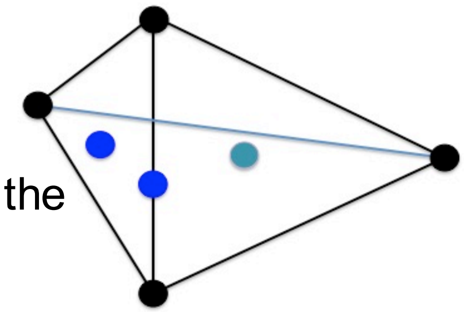
  its mirror dual Y_2 with singularities (gauge groups) of type H_1 in X^+ and H_2 in X^- with $H_i=[E_8, G_i]$

- Employ the conjecture to construct background bundles with structure group G where $D=[E_8, G]$ beyond $P^2(1,2,3)$
- Explore ``symmetric'' stable degeneration with $G_1=G_2$

  → symmetric appearance of discrete symmetry D

Example with Z_2 symmetry

Polytope:



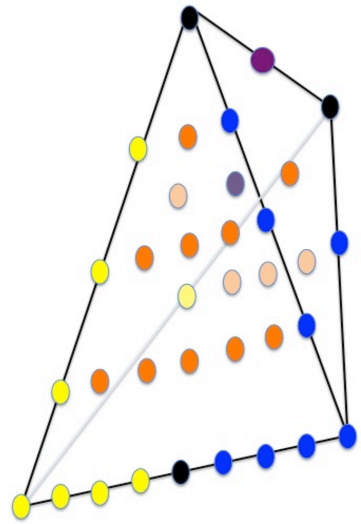
(monominals of the ambient space)

8D: $((\mathrm{E}_7 \times \mathrm{SU}(2))/\mathbb{Z}_2)^2$ - gauge symmetry

$\mathbb{Z}_2{}^2$ - vector bundle

6D: $(\mathrm{E}_7 \times \mathrm{SU}(2))/\mathbb{Z}_2$ - gauge symmetry

Dual polytope:



$\mathbb{Z}_2{}^2$ - gauge symmetry

$((\mathrm{E}_7 \times \mathrm{SU}(2))/\mathbb{Z}_2)^2$ - vector bundle

$\mathbb{Z}_2$ - gauge symmetry

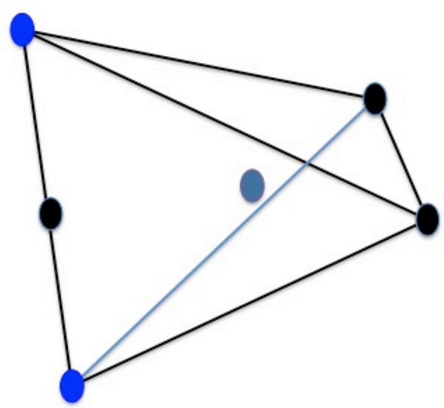
Field theory: Higgsing symmetric U(1) model:

only one (symm. comb.) U(1)-massless

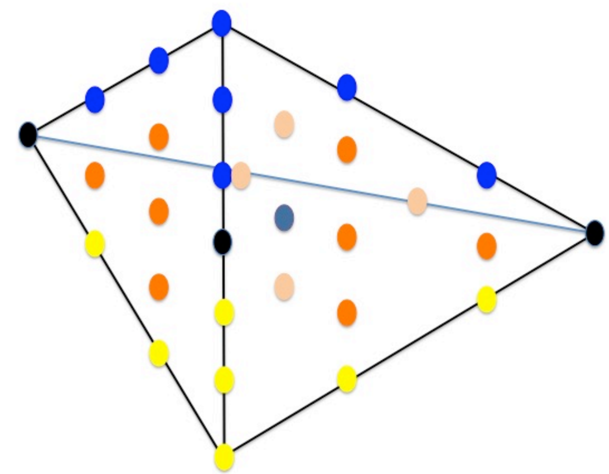
→ only one Z_2 -``massless"

Example with Z_3 symmetry

Polytope:



Dual polytope:



6D: $(E_6 \times E_6 \times SU(3))/\mathbb{Z}_3$ - gauge symmetry

$\mathbb{Z}_3$ - gauge symmetry

These examples demonstrate:

toric CY's with MW torsion of order-n,

via Heterotic duality related to

mirror dual toric CY's with n-section.

Related: [Klevers, Peña, Piragua, Oehlmann, Reuter '14]

Non-Abelian Discrete Symmetries – less understood

F-theory - limited exploration

[Grimm, Pugh,Regalado '15], c.f., T. Grimm's talk

[M.C., Lawrie, Lin, work in progress] [M.C., Donagi, Lin, work progress]

stoop down to weak coupling

Type II string compactification

Important progress in these directions builds on the work

[Camara, Ibanez, Marchesano '11]

Abelian discrete gauge symmetries realized on Calabi-Yau threefolds with torsion.

Non-Abelian Heisenberg-type discrete symmetries realized on Calabi-Yau threefolds with torsion classes that have specific non-trivial cup-products.

[Berasaluce-Gonzales, Camara, Marchesano, Regalado, Uranga '12]

Calabi-Yau threefold X_6 with Torsion

$$\begin{aligned} \text{Torsion}\,(H^5(X_6,\mathbb{Z})) &\simeq \text{Torsion}\,(H^2(X_6,\mathbb{Z})) = \mathbb{Z}_k \\ \text{Torsion}\,(H^4(X_6,\mathbb{Z})) &\simeq \text{Torsion}\,(H^3(X_6,\mathbb{Z})) = \mathbb{Z}_{k'} \end{aligned}$$

example

[Camara, Ibanez, Marchesano '11]

ρ_2, β_3, $\tilde{\omega}_4$, and ζ_5 represent the generators of the torsion cohomologies w/

$$\begin{aligned} d\gamma_1 &= k\rho_2, & d\tilde{\rho}_4 &= k\zeta_5, \\ d\alpha_3 &= k'\tilde{\omega}_4, & d\omega_2 &= k'\beta_3 \end{aligned}$$

k^{-1} and k'^{-1} torsion linking numbers

(consequence of expressions for torsion linking numbers)

γ_1, ω_2, α_3 and $\tilde{\rho}_4$ are non-closed forms satisfying:

$$\int_{X_6} \gamma_1 \wedge \zeta_5 = \int_{X_6} \rho_2 \wedge \tilde{\rho}_4 = \int_{X_6} \alpha_3 \wedge \beta_3 = \int_{X_6} \omega_2 \wedge \tilde{\omega}_4 = 1$$

Upon Type IIB KK reduction of C_2, B_2, C_4 gauge potentials on X_6
→ Z_k x $Z_{k'}$ discrete symmetry, realized in the Stückelberg mass $\mathcal{G}_{ij}\,\eta^i_\mu \eta^{\mu\, j}$

[Berasaluce-Gonzales, Camara, Marchesano, Regalado, Uranga '12]

When: $\rho_2 \wedge \rho_2 = M\,\tilde{\omega}_4\,, \qquad M \in \mathbb{Z}\,,$ M non-vanishing

Upon KK reduction, Heisenberg discrete symmetry specified by k, k', M:

$$\mathcal{G}_{ij}\,\eta^i_\mu \eta^{\mu\, j} \quad \text{w/} \quad \begin{aligned} \eta^i_\mu &= \partial_\mu b^i - k\,A^i_\mu\,, \qquad i = 1,2\,, \\ \eta^3_\mu &= \partial_\mu b^3 - k' A^3_\mu - M b^2(\partial_\mu b^1 - k\,A^1_\mu) \end{aligned}$$

Calabi-Yau threefold X_6 with Torsion

$$\begin{aligned} \mathrm{Torsion}\,(H^5(X_6,\mathbb{Z})) &\simeq \mathrm{Torsion}\,(H^2(X_6,\mathbb{Z})) = \mathbb{Z}_{k_1}\times\mathbb{Z}_{k_2} \\ \mathrm{Torsion}\,(H^4(X_6,\mathbb{Z})) &\simeq \mathrm{Torsion}\,(H^3(X_6,\mathbb{Z})) = \mathbb{Z}_{k_3} \end{aligned}$$

another example

When: $\rho_2\wedge\rho_2 = M\,\tilde\omega_4\,, \qquad M\in\mathbb{Z}\,,$ w/ M non-vanishing

→ Heisenberg discrete symmetry specified by k_1, k_2, k_3 and M:

$$\mathcal{G}_{IJ*}\,\eta^I_\mu\eta^{\mu\,J*} \quad \text{w/} \quad \begin{aligned} \eta_{\mu(i)} &= \partial_\mu b^2_{(i)} - \tau\partial_\mu b^1_{(i)} + k_i\left(A^2_{\mu(i)} - \tau A^1_{\mu(i)}\right) \qquad i=1,2 \\ \eta^3_\mu &= \partial_\mu b^3 + k_3 A^3_\mu - M\left(b^2_{(1)} - \tau b^1_{(1)}\right) k_2\, A^1_{\mu(2)} \end{aligned}$$

$\tau = C_0 + \mathrm{i}e^{-\phi}$

Dilaton-axion coupling

[Grimm, Pugh, Regalado '15]

Non-Abelian Discrete Symmetry in Type IIB

Requires the study of Calabi-Yau threefolds with torsion by determining torsion cohomology groups and their cup-products
→ technically challenging

Choose a specific Calabi-Yau threefold X_6:
free quotient of T^6 by a fixed point free action of $Z_2 \times Z_2$:
[part of a general classification [Donagi, Wendland'09]]

$$g_1 : (z_0, z_1, z_2) \mapsto \left(z_0 + \tfrac{1}{2},\ -z_1,\ -z_2\right),$$
$$g_2 : (z_0, z_1, z_2) \mapsto \left(-z_0,\ z_1 + \tfrac{1}{2},\ -z_2 + \tfrac{1}{2}\right)$$

$\mathbb{C}/(\mathbb{Z} + \tau_i \mathbb{Z}) \ni z_i = x_i + \tau_i y_i$

$i = 0, 1, 2$

Computation of cup-products:

Strategy: relate X_6 to a submanifold Y_0 :

$$Y_0 \hookrightarrow X, \quad (x_0,\ x_1,\ x_2,\ y_0) \mapsto (x_0 + \tau_0 y_0,\ x_1,\ x_2),$$

$z_i = x_i + \tau_i y_i$

four-dimensional sub-torus quotient, invariant under $Z_2 \times Z_2$.

Cup-products in Y_0 could be done explicitly by hand, but obtained as part of a computational scheme (cellular model → co-chains of cubical cells) that gives, among others, full integer cohomology of X_6.

Restriction i^*: $H^*(X_6, Z) \to H^*(Y_0, Z)$ - surjective, and exhibits $H^*(Y_0, Z)$ as a direct summand of $H^*(X_6, Z)$, along with the multiplicative structure of the cohomlogy ring of Y_0

Determine cup-products of $H^2(X_6, Z)$ torsion classes that are non-vanishing in $H^4(X_6, Z)$.

Results:

Full cohomology for Y_0:

$$H^d(Y_0,\mathbb{Z}) = \begin{cases} \mathbb{Z}_2 & d=4 \\ \mathbb{Z}^2 & d=3 \\ \underset{c_0}{\mathbb{Z}_2} \oplus \underset{c_1}{\mathbb{Z}_4} \oplus \underset{c_2}{\mathbb{Z}_4} \oplus \underset{c_3}{\mathbb{Z}} & d=2 \\ 0 & d=1 \\ \mathbb{Z} & d=0 \end{cases}$$

For X_6:

$$H^d(X_6,\mathbb{Z}) = \begin{cases} \mathbb{Z} & d=6 \\ \mathbb{Z}_4^2 \oplus \mathbb{Z}_2^3 & d=5 \\ \mathbb{Z}^3 \oplus \mathbb{Z}_2^3 & d=4 \\ \mathbb{Z}^8 \oplus \mathbb{Z}_2^3 & d=3 \\ \mathbb{Z}^3 \oplus \mathbb{Z}_4^2 \oplus \mathbb{Z}_2^3 & d=2 \\ 0 & d=1 \\ \mathbb{Z} & d=0 \end{cases}$$

Non-vanishing cup-products for Y_0:

$$\cup : H^2(Y_0,\mathbb{Z}) \times H^2(Y_0,\mathbb{Z}) \to H^4(Y_0,\mathbb{Z}) = \mathbb{Z}_2$$

$$c_0 \cup c_1 = c_2 \cup c_3 \neq 0$$

For X_6:

$$\bar{c}_i \in H^2(X_6,\mathbb{Z}) \quad i^*(\bar{c}_i) = c_i$$

$$\bar{c}_0 \cup \bar{c}_1,\ \bar{c}_2 \cup \bar{c}_3 \in H^4(X_6,\mathbb{Z})$$

First explicit construction of a Type IIB a Calabi-Yau manifold that exhibits a Heisenberg- type discrete symmetry

w/ k_1=2, k_2=4, k_3=2, M=1 (earlier example)

Summary

- Abelian Discrete Symmetries in F-theory
  Highlight insights into heterotic duality
- Non-Abelian discrete symmetries in
  Type IIB: Construction of CY manifold whose torsional classes have non-trivial cup-products
  → Heisenberg discrete group - first explicit example

Outlook

- Techniques presented here applicable to F-theory study of Heisenberg symmetries ([Grimm, Pugh, Regalado'15] c.f, T. Grimm's talk)
- Non-Abelian discrete symmetry in F-theory via Higgsing of higher index representations ([M.C.,Klevers,Taylor'15], [Klevers, Taylor'16], c.f., W. Taylor's talk)
  [M.C., Lawrie, Lin, work in progress]

Presented at the next meeting on Geometry of String Theory