

Evidence Towards a Swampland Conjecture

Eran Palti
University of Heidelberg

1602.06517 (JHEP 1608 (2016) 043) with Florent Baume

1609.00010 (JHEP 01 (2017) 088) with Daniel Klaewer

Physics and Geometry of F-theory, Trieste, February 2017

In Quantum Field Theory there is no universal connection between the vacuum expectation value of a scalar field and a physical mass scale

Does gravity behave in the same way?

Important to understand how effective theories can support $\Delta\phi > M_p$, while keeping $\Lambda < M_p$

Will present evidence towards a universal relation between $\Delta\phi$ and the mass scale of quantum gravity physics, which emerges at $\Delta\phi > M_p$

The Weak Gravity Conjecture

[Arkani-Hamed, Motl, Nicolis, Vafa '06]

In a theory with a U(1) gauge symmetry, with gauge coupling g , there must exist a state of charge q and mass m_{WGC} such that

$$qgM_p \geq m_{\text{WGC}}$$

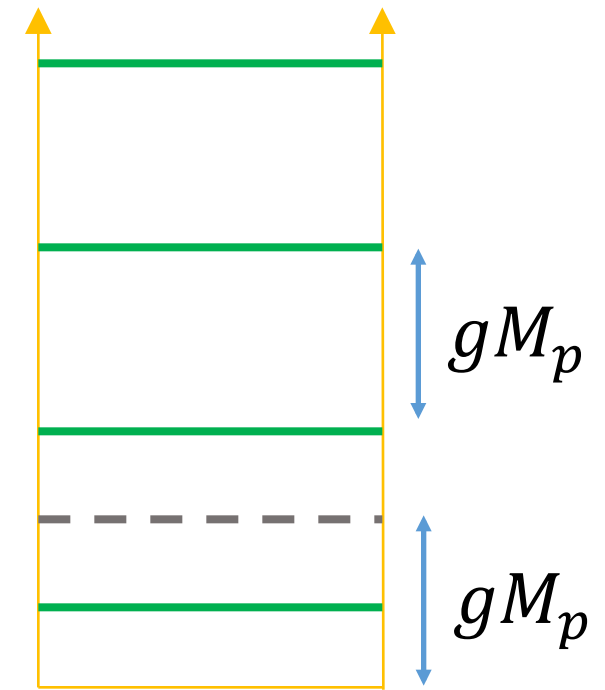
It is natural to associate gM_p to the scale of quantum gravity physics

Lattice WGC: The state satisfying the WGC is the first in an infinite tower of states, of increasing mass and charge, all satisfying the WGC

[Heidenreich, Reece, Rudelius '15]

Evidence

- Appears to be the case in String Theory
- Black Holes charged under both KK U(1) and gauge U(1) violate the WGC unless there is such a tower
- Sharpening of Completeness Conjecture [Polchinski '03]
- Matches cut-off constraint for monopole to not be a Black Hole $\Lambda < gM_p$, $gM_p^2 > \rho(r_\Lambda)^{\frac{1}{2}}$



The Swampland Conjecture (Conjecture 2 of [\[Ooguri, Vafa '06\]](#)):

If a scalar field undergoes a variation $\Delta\phi$, then there is an infinite tower of states whose mass changes by a factor of order $e^{-\alpha\Delta\phi}$, for some constant $\alpha > 0$.

We interpret the conjecture as a statement about the asymptotic structure of moduli space $\Delta\phi \rightarrow \infty$.

In order to quantify, let us define a **Refined Swampland Conjecture**:

$$m_{\text{SC}}(\phi_0 + \Delta\phi) = m_{\text{SC}}(\phi_0) \Gamma(\phi_0, \Delta\phi) e^{-\alpha \frac{\Delta\phi}{M_p}}$$

with $\Gamma(\phi_0, \Delta\phi) e^{-\alpha \frac{\Delta\phi}{M_p}} < 1$ and monotonically decreasing at an exponential rate for

$$\Delta\phi > \mathcal{O}(1) M_p$$

(Conjecture applies to all fields, not just strict moduli)

Evidence based on String Theory

Moduli:

Moduli in string theory have approximately logarithmic canonical normalisation

They universally control the mass of infinite towers of states

$$\phi \sim \log s$$

$$M_{\text{tower}} \sim s^{-\alpha} \sim e^{-\alpha\phi}$$

Axions:

Periodic axions are incompatible with monotonic $e^{-\alpha\Delta\phi}$ behaviour. This is ok as long as $\Delta\phi < M_p$.

Appears to be the case in string theory*

*body of work on possible ways around this, though no explicit example

Evidence based on String Theory

Monodromy axions have their periodic symmetry spontaneously broken

$$L = f^2 (\partial a)^2 - m^2 a^2$$

De-compactify the axion field space allowing $\Delta a \rightarrow \infty$

The axion decay constant f is independent of the axion a , appears to contradict the Swampland Conjecture

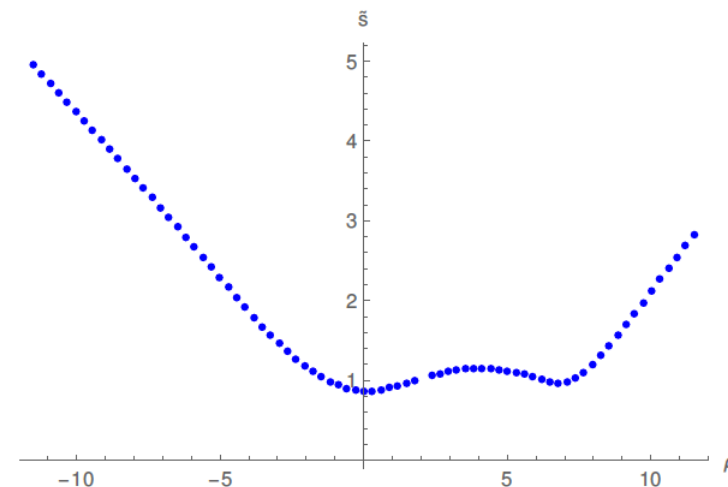
Can test in string theory in compactifications of type IIA string theory on a Calabi-Yau in presence of fluxes

[Baume, EP '16]

Evidence based on String Theory

- Find that as the axion develops a large vev, the gravitational backreaction of its potential $m^2 a^2$ causes moduli fields to track the axion $s = a$

- This modifies its own field space metric $f(s) \rightarrow f(a)$, leading to logarithmic normalisation $L = \left(\frac{\partial a}{a}\right)^2$



- Induces a power-law dependence of the mass of a tower of states on a

Find that the SC behavior emerges at $\Delta\phi > M_p$, independently of fluxes

Generality of result in String Theory / F-theory under investigation

(eg. [Valenzuela '16; Bielleman, Ibanez, Pedro, Valenzuela, Wieck '16; Hebecker et al. '15; ...])

Evidence not based on String Theory

Consider a theory with gravity, gauge field, and scalar field

$$S = \frac{1}{2} \int \sqrt{g} d^4x \left[R - 2 (\partial\phi)^2 - \frac{1}{2g(\phi)^2} F^2 \right] \quad (M_p = 1)$$

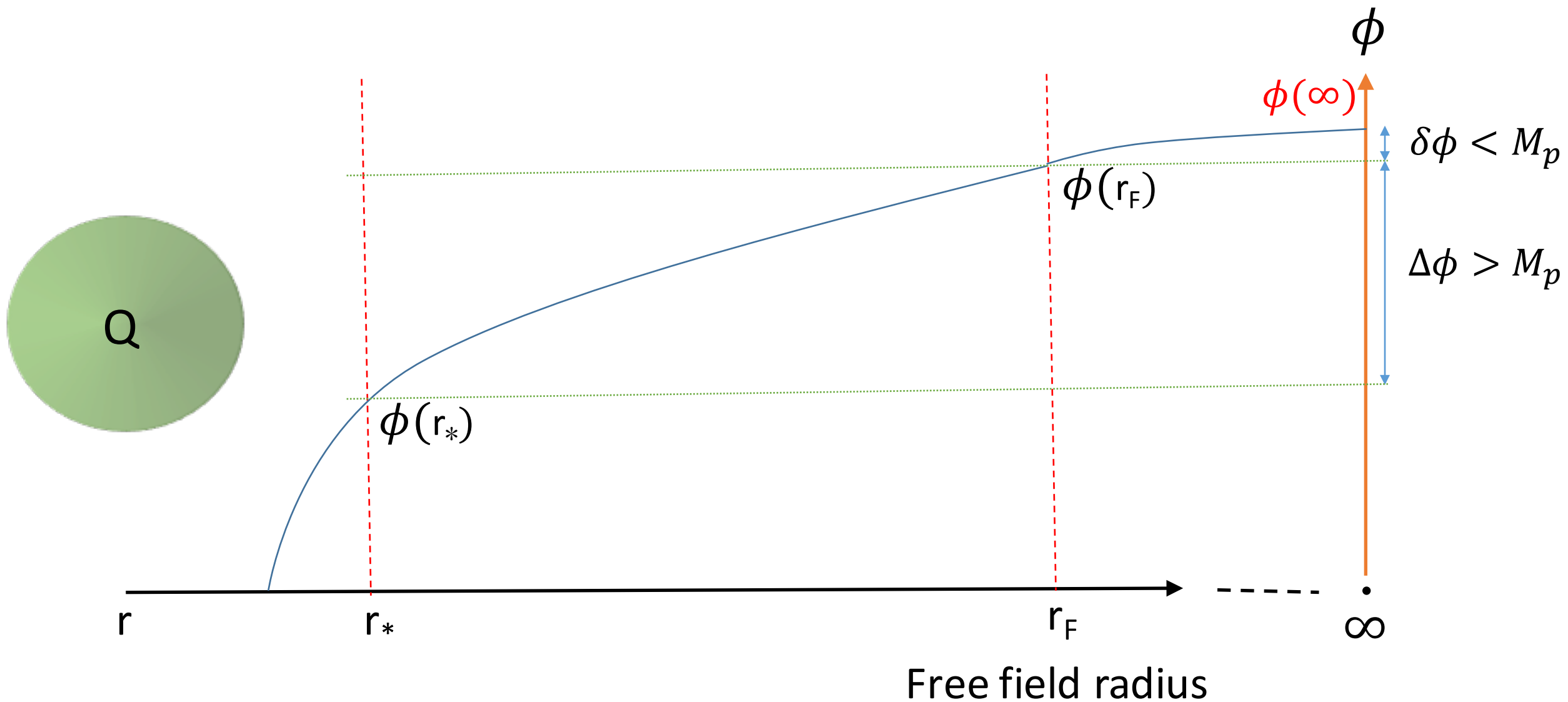
Can utilise the Weak Gravity Conjecture to write the Swampland Conjecture as

$$g(\phi_0 + \Delta\phi) = g(\phi_0) \Gamma(\phi_0, \Delta\phi) e^{-\alpha \frac{\Delta\phi}{M_p}}$$

Consider spherical charged sources in this theory: Black Holes, Monopoles, charge distributions.

A source induces a spatial gradient flow for $g(\phi)$ and ϕ

Evidence not based on String Theory



Gravitational effect of kinetic term

The Newtonian potential Φ sets the scale of strong gravity physics

$$ds^2 = - [1 + 2\Phi(r)] dt^2 + [1 - 2\Phi(r)] (dr^2 + r^2 d\Omega)$$

Consider an arbitrary power-law profile for a scalar field

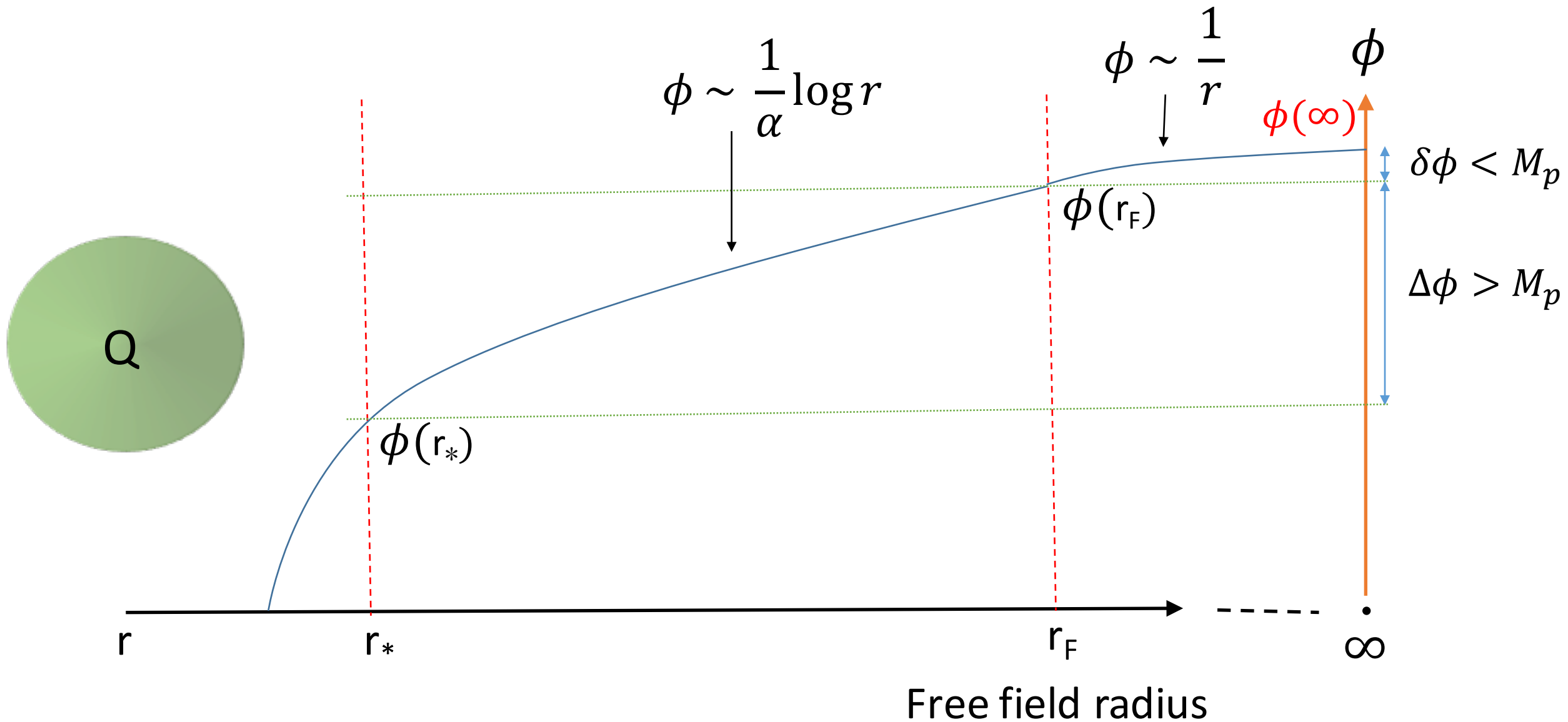
$$\phi(r) = \frac{\beta}{\alpha} \left(\frac{r}{r_F} \right)^{\frac{1}{\beta}}$$

Find that for a variation from r_* to r_F have

$$\Delta\phi = \frac{\beta}{\alpha} \left(1 - \left(\frac{r_*}{r_F} \right)^{\frac{1}{\beta}} \right) \quad \Phi > \frac{\Delta\phi^2}{\beta} \left(\frac{1 + \left(\frac{r_*}{r_F} \right)^{\frac{1}{\beta}}}{1 - \left(\frac{r_*}{r_F} \right)^{\frac{1}{\beta}}} \right) \quad |\Phi| < 1 \Rightarrow \beta > (\Delta\phi)^2$$

As $\Delta\phi \rightarrow \infty$ we have $\Delta\phi \rightarrow \frac{1}{\alpha} \log \left(\frac{r_F}{r_*} \right)$, converging rapidly for $\Delta\phi > 1$

Evidence not based on String Theory

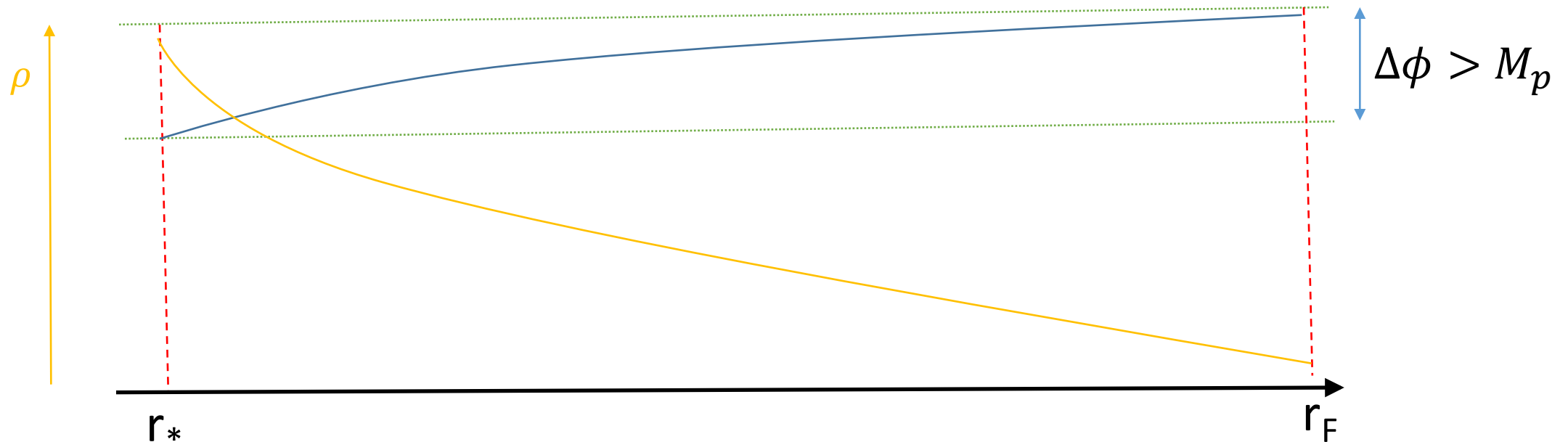


For logarithmic spatial running of the scalar field we have

$$\rho(r)^{\frac{1}{2}} > \partial\phi = \frac{1}{\alpha r}$$

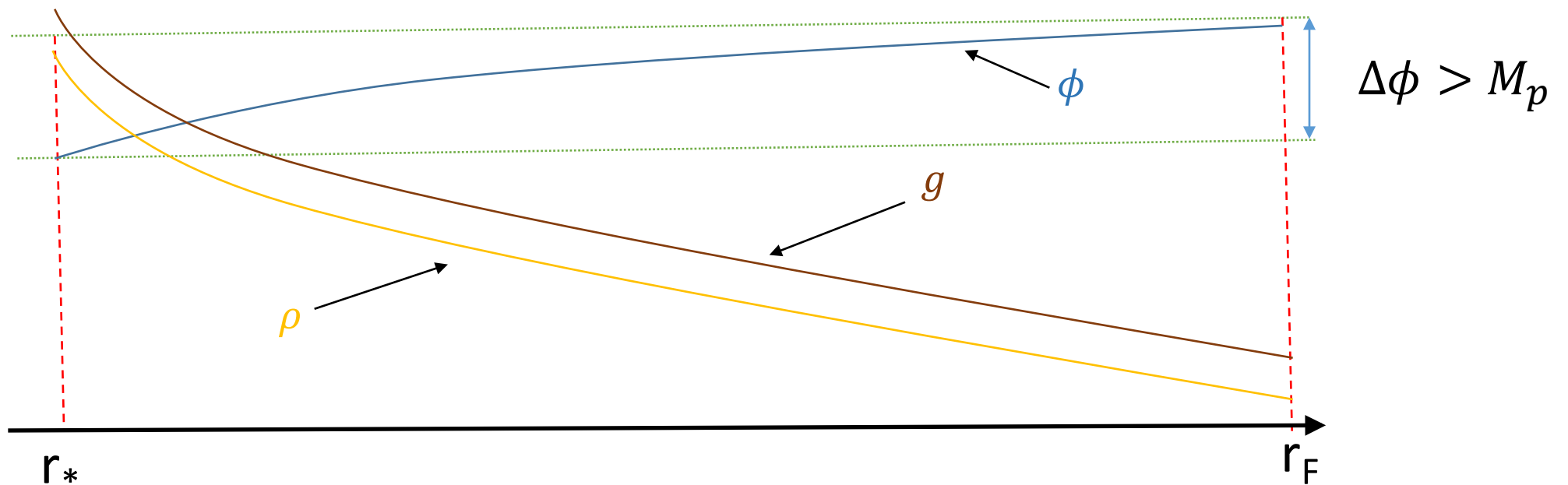
Therefore the energy density is exponentially decreasing

$$\frac{\rho(r_F)^{\frac{1}{2}}}{\rho(r_*)^{\frac{1}{2}}} = \frac{r_*}{r_F} \leq e^{-\alpha\Delta\phi}$$



The gauge coupling must track the energy density:

- The (Local) Weak Gravity Conjecture implies $g(r) > \rho(r)^{\frac{1}{2}}$
(Black Holes describable in a semi-classical gravity regime outside horizon)
- At the free-field radius can show $g(r_F) < \rho^{\frac{1}{2}}(r_F) \left(-\alpha \partial_\phi (\ln g) \big|_{\phi(r_F)} \right)$



Find $g(\phi + \Delta\phi) \leq g(\phi) \Gamma(\phi, \Delta\phi) e^{-\alpha\Delta\phi}$ with $\Gamma(\phi, \Delta\phi) e^{-\alpha\Delta\phi} < 1$ for $\Delta\phi > 1$

Logarithmic spatial dependence at Strong Curvature

Extend the Newtonian analysis to an arbitrary spherically symmetric background

$$ds^2 = -e^{2U(r)} dt^2 + e^{-2U(r)} (dr^2 + f(r) r^2 d\Omega^2)$$

Re-parameterise $U = -\frac{\alpha}{1+\alpha^2} \ln \left(H_1^\alpha H_2^{\frac{1}{\alpha}} \right) + \frac{1}{2} \ln f$, $\phi = \frac{\alpha}{1+\alpha^2} \ln \left(\frac{H_1}{H_2} \right)$

Can show that if H_1 and H_2 are Eigenfunctions of the Laplacian then for large spatial variation $\Delta\phi \gg 1$ have $\phi \simeq \frac{\alpha}{1+\alpha^2} \log r$

Imposing a relativistic version of the local WGC $\sqrt{R(r)} < g(r) M_p$

We have that $\sqrt{R(r)} \sim r^{-\frac{\alpha^2}{1+\alpha^2}}$ leads to the same exponential behaviour

Summary

Introduced the Refined Swampland Conjecture

$$m_{\text{SC}}(\phi_0 + \Delta\phi) = m_{\text{SC}}(\phi_0) \Gamma(\phi_0, \Delta\phi) e^{-\alpha \frac{\Delta\phi}{M_p}}$$

Evidence for the conjecture from string theory

Evidence for the conjecture based on Quantum Gravity expectations

If true, the scale of Quantum Gravity physics is exponentially sensitive to $\Delta\phi$
for $\Delta\phi > M_p$

The physical implications are wide-ranging and not explored as yet

Thank You

Super-Planckian Field Variations in Cosmology: Inflation

If the Swampland Conjecture holds then there is a tower of states with mass

$$m = \beta M_p e^{-\alpha \Delta\phi} \quad \text{for } \Delta\phi > \gamma M_p$$

This implies an exponential tension between a high energy scale cut-off and large field variations.

Primordial tensor modes in large field inflation requires both

$$\text{Lyth bound: } \frac{\Delta\phi}{M_P} \geq 0.25 \left(\frac{r}{0.01} \right)^{\frac{1}{2}} \quad \text{Energy scale: } V^{\frac{1}{4}} \sim \left(\frac{r}{0.01} \right)^{\frac{1}{4}} 10^{16} \text{ GeV}$$

For $\beta = \gamma = 1$ we have that $\alpha = 2, 3, 4$ implies a bound on the tensor-to-scalar ratio of $r < 0.22, 0.11, 0.06$.

Super-Planckian Field Variations in Cosmology: Dark Energy

Power-law quintessence as a model of dark energy, $V \sim \frac{M^{4+p}}{\phi^p}$

Field mass: $m^2 \sim \frac{\partial^2 V}{\partial \phi^2} \sim \frac{\rho_\phi}{\phi^2}$

Hubble scale: $H^2 \sim \frac{\rho_\phi}{M_P^2}$

Onset of dark energy is at $m \sim H$ which implies $\phi \sim M_P$.

Super-Planckian fields are generic in quintessence models [\[Copeland, Sami, Tsujikawa '06\]](#)

Infrared gravity physics tied to Ultraviolet gravity physics !

Evidence for $g M_p$ as a QG cut-off

[Arkani-Hamed, Motl, Nicolis, Vafa '06]

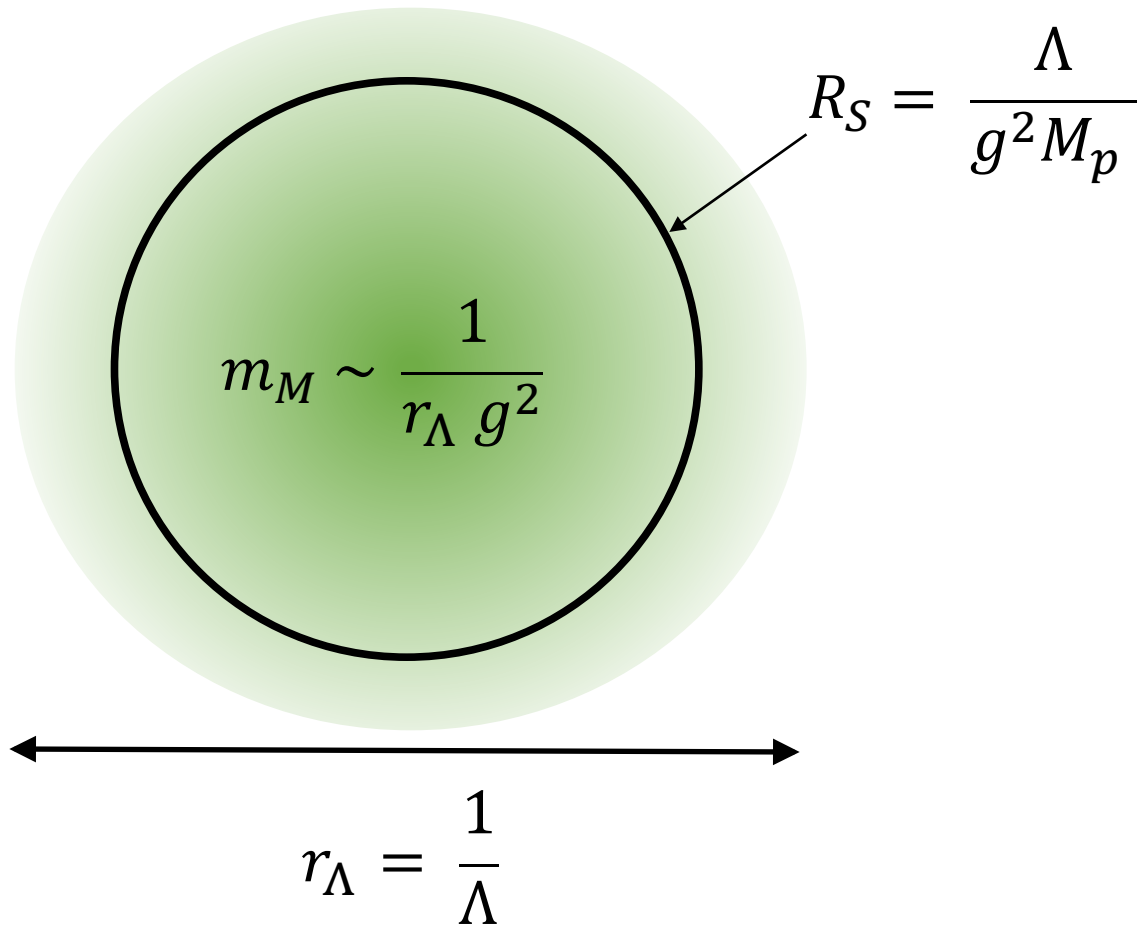
Sending $g \rightarrow 0$ turns a gauge U(1) symmetry into a global symmetry

General Black-Hole based arguments against global symmetries in Quantum Gravity

Evidence for gM_p as a QG cut-off

[Arkani-Hamed, Motl, Nicolis, Vafa '06]

Consider the magnetic dual of the WGC $\frac{q_M}{g} M_p \geq m_M$, apply to monopole:



- Apply magnetic WGC
- Require unit-charged monopole to not be a classical Black Hole

$$\Lambda < gM_p$$

$$\Rightarrow gM_p^2 > \rho(r_\Lambda)^{\frac{1}{2}}$$

Super-Planckian Field Variations in Cosmology

Interested in variations of scalar fields that are larger than the Planck mass

$$\Delta\phi > M_p$$

Arise often in scalar field cosmology and impact our understanding of contemporary observational cosmology



$$\text{Lyth bound: } \frac{\Delta\phi}{M_P} \geq 0.25 \left(\frac{r}{0.01} \right)^{\frac{1}{2}}$$

[BICEP3, Spider, CMBPol, ...]: $r \sim 0.001$

Is Quantum Gravity physics sensitive to $\Delta\phi$?

Quantum gravity physics is typically associated to energy scales of order the Planck mass

There is no general link between the energy scales of a theory and the field variations (applying QFT logic)

$$V = m^2 \phi^2 \ll M_p^4$$

Will present evidence towards a conjecture that the scale of Quantum Gravity physics is exponentially sensitive directly to $\Delta\phi$, and can lie far below the Planck scale