Exotic matter in F-theory and the 6D swamp

Physics and Geometry of F-theory '17, ICTP Trieste

March 2, 2017

Washington (Wati) Taylor, MIT

Based in part on:
D. Morrison, WT: arXiv:1106.3563,
M. Cvetic, D. Klevers, H. Piragua, WT: arXiv:1507.05954,
L. Anderson, J. Gray, N. Raghuram, WT: arXiv:1512.05791,
D. Klevers, WT: arXiv:1604.01030,
D. Klevers, D. Morrison, N. Raghuram, WT: arXiv:1703.nnnnn
A. Turner, WT: arXiv:170m.nnnnn

Goals: classify matter representations in F-theory and 6D supergravity

— Generalize Kodaira classification/dictionary to codimension 2

— Systematically understand range of theories possible in F/string theory

— Identify and clear out "swamp" [cf. Rudelius talk]

— Classify Calabi-Yau threefolds and fourfolds (+5-folds, ...?)

"Generic" SU(N) matter in F-theory:

$N\,(\square)\,;\;\; N(N-1)/2\,(\begin{smallmatrix}\square\\\square\end{smallmatrix})\,;\;\; N^2-1$ (adjoint)

- Low-energy theory: anomaly cancellation

$$a \cdot b_i = \frac{1}{6}\lambda_i\left(A^i_{Adj} - \sum_R x^i_R A^i_R\right) \qquad \mathrm{tr}_R F^2 = A_R \mathrm{tr} F^2$$

$$b_i \cdot b_i = \frac{1}{3}\lambda_i^2\left(\sum_R x^i_R C^i_R - C^i_{Adj}\right) \qquad \mathrm{tr}_R F^4 = B_R \mathrm{tr} F^4 + C_R (\mathrm{tr} F^2)^2$$

$$0 = B^i_{Adj} - \sum_R x^i_R B^i_R \qquad a, b \in \Gamma(1, T)$$

(A_R, B_R, C_R) of generic reps independent; can always solve w/ these 3 types.

- Weierstrass tuning (using unique factorization, matches Tate $N < 6$)

SU(2): $f = -\phi^2/48 + f_1\sigma + f_2\sigma^2 + \cdots, \quad g = \phi^3/864 - \phi f_1/12\sigma + g_2\sigma^2 + \cdots$

SU(N): $\phi \to \phi_0^2$ ("split condition"), cancel at higher orders

$\Delta \sim \phi_0^k \tilde{\Delta} \sigma^N + \cdots$

$\phi_0 \to D_{N+1}(\begin{smallmatrix}\square\\\square\end{smallmatrix})$; $\tilde{\Delta} \to I_{N+1}(\square)$; adjoint nonlocal

Exotic $SU(N)$ matter

$\yng(1,1,1)$: $SU(6), SU(7), SU(8)$ $\qquad g = 0$

$\yng(2)$: $SU(N)$ $\qquad g = 1$

$\yng(3)$: $SU(2)$ $\qquad g = 3$

Organizing principle: $g_R = 1 + \frac{1}{2}(a \cdot b + b \cdot b) = \frac{1}{12}(2C_R + B_R - A_R)$ [KPT]
(From anomalies; F-theory: arithmetic genus contribution of singular curve)

For U(1): generic matter $q = 1, 2$ [Morrison-Park form]
Exotic U(1) matter: $q > 2$

Questions:

1) What matter spectra are consistent in low-energy theory?
2) What can we realize through Weierstrass?
3) Connecting to other matter: Higgsing and “matter transitions”

Antisymmetric matter: $\yng(1,1,1)$ of SU(6), SU(7), SU(8)

- Realized by exotic forms of Weierstrass models $A_{N-1} \to E_6, E_7, E_8$ [MT]

- Anomaly equivalences [MT, Grassi/Morrison], e.g.

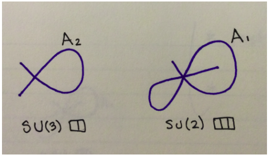
$$\frac{1}{2}\mathbf{20}\left(\frac{1}{2}\yng(1,1,1)\right) + \mathbf{6}\,(\yng(1)) \quad \leftrightarrow \quad \mathbf{15}\,(\yng(1,1)) + \mathbf{1}\,.$$

- Realized through matter transitions (no change in tensors, vectors) [AGRT]

- SU(6) $\yng(1,1,1)$ appears in KS database [Huang/WT]
  Tate SU(6) $(0, 1, 3, 3, 6) \to (0, 2, 2, 4, 6)$

- SU(9) $\yng(1,1,1)$, SU(8) $\yng(1,1,1,1)$ appear ok from anomalies
  but resist Weierstrass formulation [explain later]

SU(N) matter on singular curves



Sadov: SU(N) $\square\square$ from double point?
But: smooth deformation of Tate
SU(N) $\rightarrow \sigma = \xi^2 - B\eta^2$
gives adjoint [no transition].

Need something more exotic

Examples found:

- UnHiggsing $U(1) \times U(1) \rightarrow SU(3)$ $\square\square$ [CKPT]
- Higgsing SU(6) w/ $\begin{smallmatrix}\square\\\square\\\square\end{smallmatrix}$ $\rightarrow SU(3)$ $\square\square$ [AGRT]
- UnHiggsing $U(1)$ w/ $q = 3 \rightarrow SU(2)$ $\square\square\square$ [KT]

Subtle Weierstrass models using singular σ,
nontrivial cancellation in expansion of Δ.

Can we explain systematically?

Solution: use non-UFD nature of ring on singular divisor
[Klevers/Morrison/Raghuram/WT]

Example: $\sigma = \xi^3 - B\eta^3$, B a non-factorizable function on $\mathbb{P}^2$.

intrinsic ring on σ, $R = \mathcal{R}_{\mathbb{P}^2}/(\sigma \sim 0)$ is not a UFD.

Adjoin $\alpha : \alpha^3 = B$: *normalized intrinsic ring* $\tilde{R}$ ($\sim$ Galois extension); $\xi \sim \alpha\eta$

Choose $\phi = \alpha^2\eta \in \tilde{R}$

$$\begin{aligned} f_0 &= -\phi^2/48 = -\alpha^4\eta^2/48 = -B\xi\eta/48 \in R \\ g_0 &= \phi^3/864 = \alpha^6\eta^3/864 = B^2\eta^3/864 \in R\,. \end{aligned}$$

$\Delta_0 = 4f_0^3 + 27g_0^2 \sim (-B^3\xi^3\eta^3 + B^4\eta^6)/27648 = -B^3\eta^3\sigma/27648\,.$

$\Delta_1 = g_1(B^2\eta^3)/16 + (B^2\eta^2\xi^2)f_1/192 - B^3\eta^3/27648\,.$

Solve by $f_1 = \eta\lambda$, $g_1 = -\xi^2\lambda/12 + B/1728$

$\Delta = \mathcal{O}(\sigma^2)$ gives SU(2) on σ, double points at $\xi = \eta = 0$,
non-Tate Weierstrass form gives $\square\square\square$

General non-Tate Weierstrass constructions

Systematic constructions for general forms

$A\xi^2 + B\xi\eta + C\eta^2$

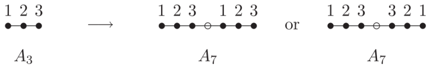
$A\xi^3 + B\xi^2\eta + C\xi\eta^2 + D\eta^3$

$\Rightarrow$ models with 2 x, 3 x points at $\xi = \eta$.

e.g. quintic with 2 double points $[\xi] = 2, [\eta] = 1$.

Geometry:

• In SU(N) models, different branches for ϕ_0 modify geometry of monodromy at branch point.



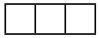
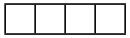
$\Rightarrow$ explains non-Tate form for SU(N) $\square\!\square$.

• Connected by matter transition

e.g. adjoint + $\cdot \leftrightarrow \square\!\square + \begin{smallmatrix}\square\\ \square\end{smallmatrix}$

Swamp 1

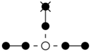
Other representations seem to arise in non-anomalous low-energy 6D supergravity models

$SU(3)$  $SU(2)$ 

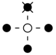
$SU(8)$  $SU(4)$ , . . .

Claim: not possible in a Weierstrass model. (Also, no other G)

Need e.g.



$A_2^3 \to \hat{D}_6$



$A_1^4 \to \hat{D}_4$

Extra node $\to$ gauge factor.

Can't happen: extra node intersects section, not shrunk in F-theory

OK in 5D theory though (?)

Questions: Why OK in low-energy theory?
New low-energy constraints? Swamp?

Swamp II

Some combinations are anomaly-OK but don't match geometry.

- SU(3) $S = 36 \times \square\square$, $A = 30 \times \begin{smallmatrix}\square\\ \square\end{smallmatrix}$:
need $A \geq S$, since every double point has $\square\square + \begin{smallmatrix}\square\\ \square\end{smallmatrix}$

- SU(2) $2 \times \square\square\square$, $b = 5$:
No quintic with 2 triple points! (could put both on a line)

Swamp? Low-energy constraints?

- In general, don't yet have Weierstrass model for all cases.

- Note: some models with $\square\square\square$ can't go to generic model via transitions (e.g. $b \geq 13, T = 0$ requires some $\square\square\square$'s.

Exotic U(1) matter

Charges $q = 3$ OK in F-theory [Klevers/MayorgaPena/Oehlmann/Piragua/Reuter]
(unHiggs to SU(2) $\square\square\square$) [Klevers/WT]

$q = 4, \ldots$ from Higgsing SU(3) and higher models

What is allowed at low energy? [Turner/WT]

At $T = 0$, $\sum q_i^2 = 18b$, $\sum q_i^4 = 3b^2$ for one U(1).

Charges 1, 2: all anomaly free models $\rightarrow$ F-theory
Higgsed from generic SU(2) models; take Morrison-Park form.

$q \leq 3, 4, \ldots$: finite solutions, some don't unHiggs. Swamp?

Infinite family of solutions:
$54 \times (q = 2n) + 54 \times (q = 2n + 1) + 54 \times (q = 4n + 1)$

# of Weierstrass models, low energy nonabelian $T = 0$ models is finite!
New low-energy inconsistency? Swamp?

Another interesting example: [Buchmüller/Dierigl/Oehlmann/Ruehle]
$G = SO(10) \times U(1)$, matter in $(16_s, 1)$. Swamp? Exotic Weierstrass model?

Conclusions

- General nonabelian exotic matter constructed by extending non-UFD ring on singular divisors

- Modest swamp contributions from nonabelian exotic matter

- Infinite apparent swamp from abelian exotic matter

- Goals:
  clear swampland,
  systematic construction of elliptic Calabi-Yau threefolds and fourfolds