Maps, Moduli Spaces and Higher Order Constraints

James Gray
(Virginia Tech)

New stuff to appear in:

J.G. and Hadi Parsian arXiv:17??.?????

Lara Anderson, J.G., Magdalena Larfors and Andre Lukas arXiv:17??.?????

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stringpheno2017@phys.vt.edu

VirginiaTech
Invent the Future

Local Organizers:

Lara Anderson
Xin Gao
James Gray
Seung-Joo Lee
Eric Sharpe
Jonathan Heckman
Jonathan Link
Djordje Minic
Ilarion Melnikov

NSF

Warm up: Bundles on Calabi-Yau

- The moduli space of a Calabi-Yau compactification in the presence of a gauge bundle is *not* described in terms of

$$H^1(TX) \oplus H^1(TX^\vee) \oplus H^1(\mathrm{End}_0(V))$$

  - It is described in terms of a subspace of these cohomology groups determined by the kernel of certain maps
  - Those maps are determined by the supergravity data of the solution.
- To see this we can analyze the supersymmetry conditions.

- Supersymmetry conditions:

$$\mathcal{J}^2 = -1 \qquad F_{ab} = F_{\overline{a}\overline{b}} = 0$$

$$N(\mathcal{J}) = 0 \qquad g^{a\overline{b}} F_{a\overline{b}} = 0$$

- Perturb all of the fields:

$$\mathcal{J} = \mathcal{J}^{(0)} + \delta\mathcal{J} \qquad A = A^{(0)} + \delta A$$

- From the left two equations we obtain the usual:

$$\delta\mathcal{J}_{\overline{a}}{}^{b} \in H^1(TX)$$

- To perturb the remaining equations use projectors:

$$\overline{P}_I{}^J = \frac{1}{2}(1 + i\mathcal{J})_I{}^J$$

and rewrite our equation in a more usable form

$$F_{\overline{a}\overline{b}} = 0 \quad \Rightarrow \quad \overline{P}_I{}^{I'}\overline{P}_J{}^{J'}F_{I'J'} = 0$$

- And work out the perturbed equation to first order:

$$i\delta\mathcal{J}_{[\overline{a}}{}^{d}F_{\overline{b}]d} = 2D_{[\overline{a}}\delta A_{\overline{b}]}$$

What are the moduli according to this equation?

- Bundle moduli are still in the game:

$$H^1(\mathrm{End}_0(V))$$

- But not all of the complex structure. The allowed moduli are in the following kernel:

$$\ker(H^1(TX) \xrightarrow{F} H^2(\mathrm{End}_0(V)))$$

(notice the map is defined by the supergravity data).

- We can also rewrite this result in terms of a single sheaf cohomology group. Define:

$$0 \to \mathrm{End}_0(V) \to \mathcal{Q} \to TX \to 0$$

- Then a little sequence chasing and facts about Calabi-Yau/stable bundles reveals:

$$H^1(\mathcal{Q}) = \left\{ \begin{array}{c} H^1(\mathrm{End}_0(V)) \\ \oplus \\ \ker(H^1(TX) \to H^2(\mathrm{End}_0(V))) \end{array} \right.$$

- So the moduli of heterotic Calabi-Yau compactifications are given by this, not the naïve complex structure and bundle moduli.

Anderson, JG, Lukas Ovrut arXiv:1107.5076

- Can we see similar structure in other cases?...

Non-Kahler Heterotic Compactifications:

- Follow the same procedure with the Strominger system:

Hull, Strominger

$$F_{ab} = F_{\overline{a}\overline{b}} = 0 \qquad H = i/2(\overline{\partial} - \partial)J$$

$$dH = -\frac{1}{30}\alpha' \mathrm{tr} F \wedge F + \alpha' \mathrm{tr} R \wedge R$$

$$g^{a\overline{b}} F_{a\overline{b}} = 0 \qquad H_{\overline{b}c\overline{a}} g^{\overline{b}c} = -6\overline{\partial}_{\overline{a}}\phi$$

Gillard, Papadopoulos and Tsimpis hep-th/0304126

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Assumption:

Lemma: Let X be a compact Kähler manifold. For A a d-closed (p,q) form, the following statements are equivalent.

$$A = \overline{\partial} C \Leftrightarrow A = \partial C' \Leftrightarrow A = dC''$$
$$\Leftrightarrow A = \partial\overline{\partial}\tilde{C} \Leftrightarrow A = \partial\hat{C} + \overline{\partial}\check{C}$$

For some $C, C', C'', \tilde{C}$ and $\check{C}$.

- For the perturbation analysis the Atiyah computation goes through unchanged.
- The other equations are somewhat more messy.

Anderson, JG, Sharpe arXiv:1402.1532

- Here is what you find for the moduli:

$$H^1(\mathcal{H}) = \left\{ \begin{array}{c} \ker\left(\ker\{H^1(TX) \xrightarrow{[F],[R]} H^2(\mathrm{End}_0(V)) \oplus H^2(\mathrm{End}_0(TX))\} \xrightarrow{M} H^2(TX^\vee)\right) \\ \oplus \\ \ker\left(H^1(\mathrm{End}_0(V)) \xrightarrow{-\frac{4}{30}\alpha'[F]} H^2(TX^\vee)\right) \oplus \ker\left(H^1(\mathrm{End}_0(TX)) \xrightarrow{4\alpha'[R]} H^2(TX^\vee)\right) \\ \oplus \\ H^1(TX^\vee)\ . \end{array} \right.$$

- Again this is a subset of what you might very naively write down (up to some subtleties).

- The relevant subset is picked out as nested kernels of maps where the maps are defined by the supergravity data.

see also: de la Ossa, Svanes arXiv:1402.1725

Does this work in non-heterotic cases?

Becker, Becker hep-th/9605053

- Consider M-theory on a Calabi-Yau fourfold:

$$G^{(0,4)} = 0 \Rightarrow \qquad (\overline{\partial}\delta C)_{\overline{abcd}} = 0$$

$$G^{(1,3)} = 0 \Rightarrow$$

$$\frac{3}{2} i \delta \mathcal{J}^{b}_{[\bar{b}} G_{a\bar{c}\bar{d}]b} + (d\delta C)_{a\bar{b}\bar{c}\bar{d}} = 0$$

- So the allowed complex structure are: $\ker\left(H^1(TX) \stackrel{G^{(2,2)}}{\longrightarrow} H^3(TX^\vee) \right)$

- How about type IIB?

IIB	$a = 0$ or $b = 0$ (A)	$a = \pm ib$ (B)	$a = \pm b$ (C)	(ABC)
1	$W_1 = F_3^{(1)} = H_3^{(1)} = 0$			
8	$W_2 = 0$			
6	$F_3^{(6)} = 0$ $W_3 =$ $\pm * H_3^{(6)}$	$W_3 = 0$ $e^{\phi} F_3^{(6)} =$ $\mp * H_3^{(6)}$	$H_3^{(6)} = 0$ $W_3 =$ $\pm e^{\phi} * F_3^{(6)}$	(3.20)
3	$\bar{W}_5 = 2W_4 =$ $\mp 2iH_3^{(\bar{3})} =$ $2\bar{\partial}\phi$ $\bar{\partial}A = \bar{\partial}a = 0$	$e^{\phi} F_5^{(\bar{3})} = \frac{2}{3} i\bar{W}_5 =$ $iW_4 = -2i\bar{\partial}A =$ $-4i\bar{\partial}\log a$ $\bar{\partial}\phi = 0$ F: $e^{\phi} F_1^{(\bar{3})} = 2e^{\phi} F_5^{(\bar{3})} =$ $i\bar{W}_5 = iW_4 = i\bar{\partial}\phi$	$\pm e^{\phi} F_3^{(\bar{3})} = 2i\bar{W}_5 =$ $-2i\bar{\partial}A =$ $-4i\bar{\partial}\log a =$ $-i\bar{\partial}\phi$	(3.21)

Table 3.4: Possible $\mathcal{N} = 1$ vacua in IIB.

Grana, Minasian, Petrini, Tomasiello hep-th/0406137

- First column just is just Strominger again and we know that works…

- For example

$$W_3 = \pm * H_3^{(6)} \quad \text{and} \quad W_4 = \mp i H_3^{(\bar{3})}$$

together imply $\quad H = i/2(\overline{\partial} - \partial)J$

- In fact a map structure can be written for all three of these columns (although perhaps not in general for a type IIB vacuum).

- Lets look at how this works in a case everyone is very familiar with.

- Take a Calabi-Yau and imaginary self dual flux.

  Define: $$G_3 = F_3 - ie^{-\phi}H_3$$

  Then we have:

$$*G_3 = iG_3 \qquad G_{(0,3)} = 0$$

- Perform the perturbation analysis and we obtain for the allowed complex structure moduli:

$$\ker(H^1(TX) \xrightarrow{G_{(2,1)}} H^2(TX^\vee))$$

- In this case note that

$$h^1(TX) = h^2(TX^\vee)$$

by Serre duality.

- So if your map is sufficiently general it will be surjective, and all of the complex structure would be fixed.

- This is the analogue in this picture of the counting matching the number of F-terms to the number of moduli to be fixed.

Higher order obstructions: Warm up.

- How do higher order obstructions appear for bundle moduli in Calabi-Yau compactifications?
- Just perturb to second order:

$$A = A^{(0)} + \delta^{(1)} A + \delta^{(2)} A$$

- And expand the equations of motion as before:

$$f_{xyz} \delta A_{\overline{a}}^{(1)y} \delta A_{\overline{b}}^{(1)z} = 2 \left(D_{[\overline{a}}^{(0)} \delta A_{\overline{b}]}^{(2)} \right)_x$$

- This can be viewed as a non-linear map structure. The allowed bundle moduli at second order are:

$$\ker\left(H^1(\mathrm{End}_0(V)) \longrightarrow H^2(\mathrm{End}_0(V))\right)$$

(the kernel of the Kuranishi map).

- In terms of superpotentials, this is associated with a cubic interaction:

$$\int_X \Omega \wedge \mathrm{Tr}\left(\delta^{(1)}A\left[\delta^{(1)}A, \delta^{(1)}A\right]\right)$$

Berglund, Candelas, de la Ossa, Derrick, Distler, Hubsch: arXiv:9505164

- What about the case of heterotic compactifications where we are interested in $H^1(\mathcal{Q})$ rather than $H^1(\mathrm{End}_0(V))$?
- We have the sequence:

$$0 \to \mathrm{End}_0(V) \to Q \to TX \to 0$$

- So we can compute:

$$H^1(\mathcal{Q}) = \begin{cases} H^1(\mathrm{End}_0(V)) \\ \oplus \\ \ker\left(H^1(TX) \to H^2(\mathrm{End}_0(V))\right) \end{cases}$$

$$H^2(\mathcal{Q}) = \begin{cases} \mathrm{coker}\left(H^1(TX) \to H^2(\mathrm{End}_0(V))\right) \\ \oplus \\ H^2(TX) \end{cases}$$

- And you would think:

$$\alpha \cong \ker(H^1(\mathcal{Q}) \to H^2(\mathcal{Q}))$$

- Does the low energy supergravity description of the string agree? We might look for a map structure something like:

$$\begin{pmatrix} \delta A^{(1)}_M \\ \delta J^{(1)c}_{\overline{a}} \end{pmatrix} \to \begin{pmatrix} f_{xyz}\delta A^{(1)y}_{\overline{a}} \delta A^{(1)z}_{\overline{b}} + i(\delta J^{(1)c}_{[\overline{a}} D^{(0)}_{|c|} \delta A^{(1)}_{\overline{b}]})^x \\ \delta J^{(1)c}_{[\overline{b}} \partial_{|c|} \delta J^{(1)a}_{\overline{c}]} \end{pmatrix}$$

- Then, for example, we would look for something like:

$$f_{xyz}\delta A^{(1)y}_{\overline{a}} \delta A^{(1)z}_{\overline{b}} + i(\delta J^{(1)c}_{[\overline{a}} D^{(0)}_{|c|} \delta A^{(1)}_{\overline{b}]})^x \quad = \quad (D^{(0)}_{[\overline{a}} \Gamma_{\overline{b}]})^x + \Delta^c_{[\overline{b}} F^x_{\overline{a}]c}$$

in rearranging the perturbed equations of motion.

- Indeed you do find something like this – albeit it a little bit more complicated (due to a piece in the second order correction to $\delta\mathcal{J}$ mixing in).

- So indeed the obstructions are captured by

$$\alpha \cong \ker(H^1(\mathcal{Q}) \to H^2(\mathcal{Q}))$$

Final Comments

- It would be nice to perform a similar second order analysis for the Strominger system to look at cubic interactions in that case.

- In the future we would like to map some of this across to F-theory using Het/F duality (for example).

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