Lecture 4 2+1D Dualities.

Free Dirac Fermion? ψ 2 component $\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}$

$$\mathcal{L}'_{Dirac} = \bar{\psi}\gamma^{\mu}(\partial_\mu - iA)\psi + m\bar{\psi}\psi + \frac{1}{8\pi}A\wedge dA$$

$\mathcal{L}_{eff} = \frac{1}{4\pi}A\wedge dA$ $\qquad$ $\mathcal{L}_{eff} = F^2$ (dielectric)

$\longrightarrow m$

$m<0$ $\quad \sigma_{xy} = 1$ $\quad M = 0$ $\quad m>0;\ \sigma_{xy} = 0$

Integrating out fermions with mass "m" leads to: $\mathcal{L}_{eff} = -\frac{\text{sign}\, m}{8\pi} A\wedge dA$.

Note:- current $j = -\delta\mathcal{L}/\delta A$ so we have

$$j^x = \frac{1}{2\pi}\epsilon^{xy0}(\partial_y A_0 - \partial_0 A_y) = \frac{1}{2\pi}E^y$$

$$j_x = \sigma_{xy}E_y \qquad \sigma_{xy} = \frac{1}{2\pi}$$

This was in units $e = \hbar = 1$

$$\sigma_{xy} = \frac{e^2}{2\pi\hbar} = \frac{e^2}{h}.$$ Quantized Hall Conductance.

Strategy ~ find a theory that captures the same two phases.

We can use the dual vortex theory with a mechanism to bind gauge charge to vorticity.

$$\mathcal{L}_v = \mathcal{L}[\varphi_v, a] - \frac{1}{2\pi}\epsilon^{\mu\nu\lambda}A_\mu \partial_\nu a_\lambda$$

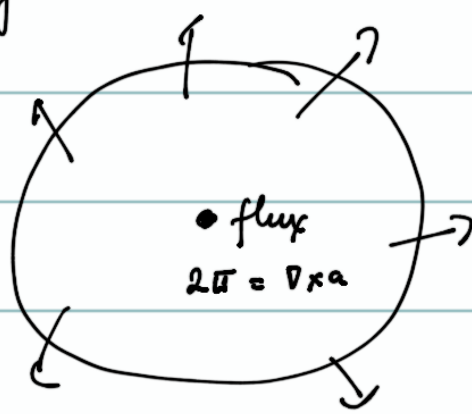
Want to ensure $j_v = j_{boson} = \frac{\epsilon^{\mu\nu\lambda}\partial_\nu a_\lambda}{2\pi}$

$$-a^\nu\left(j_\nu - \frac{\epsilon^{\mu\nu\lambda}\partial_\nu a_\lambda}{4\pi}\right)$$ will do it

$$\mathcal{L}_V = \mathcal{L}[\psi_V, a] + \frac{a\wedge da}{4\pi} - \frac{1}{2\pi} A\wedge da + \frac{A\wedge dA}{4\pi}$$

$$\boxed{= \mathcal{L}[\psi_V, a] + \frac{(a-A)\wedge(da-dA)}{4\pi}} \quad I$$

How exactly does Chern Simons term change statistics?



$\frac{1}{4\pi} a\wedge da. =$

$-j^0 = \frac{1}{2\pi} da.$

add ψ_V to neutralize gauge charge.

But now unit charge ψ_V in field of a monopole $\simeq$ monopole harmonics spin $1/2$

Now lets calculate phase diagram:

ψ_v gapped then no response absorb A in a
Gap due to chern Simons $\sigma_{xy} = 0$.

ψ_v condensed $|\partial_\mu \varphi_v - a_\mu|^2 + \mathcal{L}_{CS}$.

$\Rightarrow$ $a_\mu^\perp = 0$ minimize. $\mathcal{L}_{eff} = \dfrac{A \wedge dA}{4\pi}$ $\sigma_{xy} = 1$.

therefore both phases reproduced (need other checks like chiral central charge).
Likely that the phase transition is also captured. (WF-CS)

And finally a Fermi-Fermi duality $\bullet\!\uparrow\uparrow$ 2 vortices bound to a charge (composite fermions)

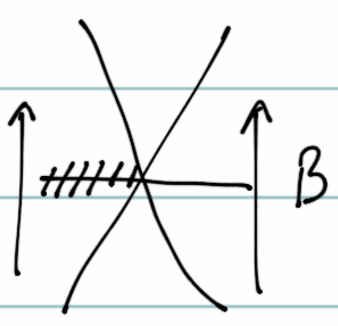
$$\boxed{\bar{\Psi}\gamma^\mu(\partial_\mu - iA_\mu^{ext})\Psi \longleftrightarrow \bar{\Psi}_{CF}\gamma^\mu(\partial_\mu - i2a_\mu)\Psi. + \frac{1}{2\pi} a \wedge dA.}$$

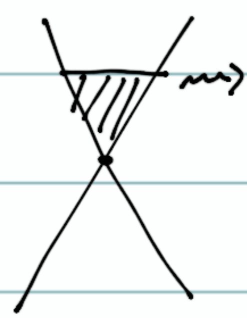
II

(No $\frac{1}{8\pi} 2a\, d\, 2a$. - Naively parity anomaly, can be fixed by (i) Surface theory & forbidding unit monopoles in bulk (ii) Introducing extra fields.).

Magnetic field $\nabla \times A_{ext}$. on LHS.

On RHS → chemical potential for composite fermions. $N_{CF} = \frac{1}{2} N_B$.





Composite Fermions - Dirac Metal.

Prediction $\rightarrow$ 

$$H = \vec{\alpha}\cdot\vec{p} = p_x \sigma_x + p_y \sigma_y. + V(x)$$

Dirac - opposite spin

No scattering!

Slave Particle description.

A useful technique to obtain several of these dualities is the "slave particle" description, analogous to the CP_1 representation of the $O(3)$ σ model. There, the $O(3)$ vector $\hat{n}$ is written interms of a 2comp. z, $\hat{n} = z^\dagger \vec{\sigma} z$ where $|\hat{n}| = 1$ is garunteed by $z^\dagger z = 1$, and $\vec{\sigma}$ are Pauli matrices. Then, there is a $U(1)$ redundancy in the choice of z to represent a physical field which must be gauged.

We can follow a similar proceedure for the Free Dirac fermion duality, writing the physical fermion field ψ_f as the product of a Bose field ψ_v & fermion f.

$$\psi_f = \psi_v \cdot f$$

We want to capture transition between Chern number (Hall conductance) $C = 1$ for ψ_f & $C = 0$.

We can obtain $C=1$ by putting the 'f' fermions in a $C_f = 1$ band structure and condensing $\langle \psi_v \rangle \neq 0$. Then there is no real distinction between ψ_f & f and we obtain one phase. Now we would like to show that the other phase $C=0$ occurs when ψ_v is gapped.

First assume f carries global $U(1)$ $[A]$ and from the $U(1)$ redundancy

$$f \to e^{-i\theta} f \; ; \; \psi_v \to e^{+i\theta} \psi_v$$

should also carry $U(1)$ gauge charge. $[a]$.

Integrating out fermions with minimal coupling $D = \partial + (a - A)$ leads to

$$\mathcal{L}_f = \frac{1}{4\pi} (a-A) \wedge d(a-A)$$

Adding this to $\mathcal{L}_v$

$$\mathcal{L}_{tot} = \left| (\partial_\mu - i a_\mu) \phi_v \right|^2 + r' |\phi_v|^2 + \lambda' |\phi_v|^4$$

$$+ \frac{1}{4\pi} (a - A) \wedge d(a - A)$$

which is I and the identical analysis reproduces the two phases.

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