

OUT OF EQUILIBRIUM QFTS & DISSIPATIVE HYDRODYNAMICS

OUTLINE

Lecture 1

- QFT dynamics out of equilibrium
- out-of-time order diagnostics
- oto path integrals & generalized Keldysh formalism
- Wightman, LR, Keldysh rules & canonical embeddings

Lecture 2:

- contour microscopic constraints
- top. susy encoding for EFTs
- thermal QFTs & Keldysh rules
- discrete symmetries in SK.
- Weldon sum rules
- Motivating thermal equivariance

Lecture 3

- Phenomenology of hydrodynamic EFTs
- equilibrium & class L
- adiabatic classification
- interlude: holography & kinetic theory

Lecture 4:

- hydrodynamic effective actions
- thermal superspace
- example: Langevin superparticle
- dissipative effective actions.
- susy ward identities. 2nd law.

Q1. What is the general path integral contour which computes correlators w/ all possible operator orderings?

Q2. Renormalization in mixed states? Open Quantum Systems and unitary evolution in closed mixed states.

formalism: generalized Keldysh, timefolds, oto contours.

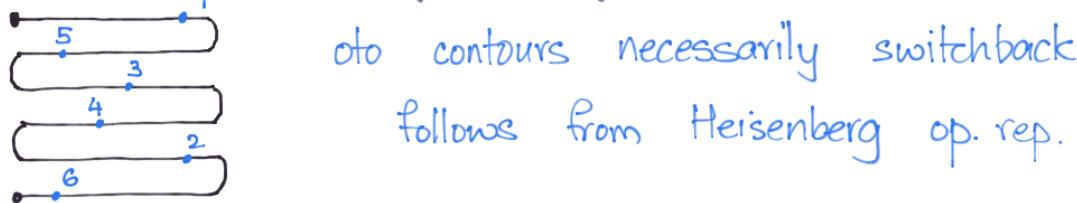
focus on closed quantum systems. applicable to arbitrary initial state which unitarily evolves.

$$\rho \rightarrow U \rho U^\dagger$$

nb $| \psi \rangle \rightarrow U | \psi \rangle$ and $\Theta \rightarrow U^\dagger \Theta U$

convention: $t_1 > t_2 > t_3 \dots > t_n$ $\Theta_i = \Theta_i(t_i) \equiv i$

$n!$ time orderings, only one $12\dots n$ is time-ordered.



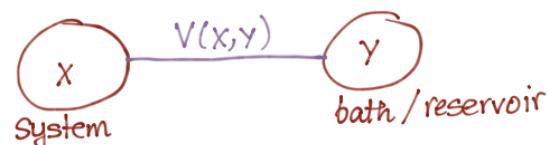
oto contours necessarily switchback

follows from Heisenberg op. rep.

1-oto: SK contours used in linear response, open quantum systems.

closed systems in mixed state. $\rho \rightarrow |\rho\rangle \in \mathcal{H} \otimes \mathcal{H}^*$ (min purification)

Feynman-Vernon argument



$$\psi(x,y) \mapsto U(x,y) \psi(x,y)$$

$$\Phi_{x_i \rightarrow x_f} = \left| \int_f \langle x_f, y_f | x_i, y_i \rangle \right|^2$$

$$= \left| \int_i \int dx dy \psi_f^* e^{iS} \psi_i \right|^2 = \int dx dy \psi_f^* \psi_f^* e^{iS - iS'} \psi_i \psi_i'$$

$$\xrightarrow{\text{trace}} \int_i dx \psi_f^* \psi_f^* e^{i(S_x - S'_x) + iF(x,x')} \psi_i \psi_i'$$

$F(x,x')$: influence functional.

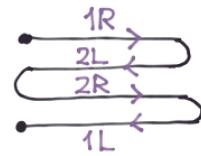
Usually deal w/ closed q.systems w/ entanglement specified in initial state. unchanging

oto generating fn: single copy \hat{O} , \mathcal{H} embedded into $2k$ -fold tensor product $\mathcal{H}_{\alpha R} \otimes \mathcal{H}_{\alpha L}^*$ $\alpha = 1, \dots, k$ $O_{\alpha R}, O_{\alpha L}$

$$\xrightarrow{R} U \xleftarrow{L} U^\dagger \quad) \quad \begin{matrix} 11 & & & & : & p \end{matrix}$$

$$Z_{\text{oto}}[J_{\alpha R}, J_{\alpha L}] = \text{Tr} \left(\dots U_{2L}^\dagger U_{1R} \rho U_{1L}^\dagger U_{2R} \dots \right)$$

$$Z_{\text{SK}}[J_R, J_L] = \text{Tr} (U_R \rho U_L^\dagger)$$



correlator groupings: Wightman basis $G_\sigma = \langle \hat{O}_{\sigma(1)} \hat{O}_{\sigma(2)} \dots \hat{O}_{\sigma(n)} \rangle$

natural from oto perspective: $n!$ elements.

- LR correlators aligned to the contour $(2k)^n$

- Av/Dif correlators $O_{\alpha \text{Av}} = \frac{1}{2} (O_{\alpha R} + O_{\alpha L})$

$$O_{\alpha \text{Dif}} = (-1)^{\alpha+1} (O_{\alpha R} - O_{\alpha L})$$

- Nested correlators $\{[\hat{O}_1, \hat{O}_2], \hat{O}_3\}$ $n! 2^{n-2}$

1. Nested \rightarrow Wightman by sJacobi involving $\{, \}$ & $[,]$

$(2^{n-2} - 2) n!$ improper & $n!$ proper sJacobi relations.

2. LR \leftrightarrow Av/Dif basis rotation.

3. Av/Dif \rightarrow Nested via Keldysh rules *

4. LR \rightarrow Wightman canonical embedding *

Exemplify these in 1:oto (SK theory) & then indicate generalizations

SK formalism: $O_R = \hat{O} \otimes 11$ $O_L = 11 \otimes \hat{O}$ $O_{\text{av}} = \frac{1}{2} (O_R + O_L)$

source coupling $J_R O_R - J_L O_L = J_{\text{av}} O_{\text{dif}} + J_{\text{dif}} O_{\text{av}}$ $O_{\text{dif}} = O_R - O_L$

$$Z[J_R, J_L] = \text{Tr} (U_R \rho U_L^\dagger)$$

$$\langle \bar{T}(\hat{O}_1 \hat{O}_2 \dots \hat{O}_p) T(\hat{O}_{p+1} \dots \hat{O}_{p+q}) \rangle = \langle \tau_{\text{SK}} (O_L^1 O_L^2 \dots O_L^p O_R^{p+1} \dots O_R^{p+q}) \rangle$$

Time ordering step fn: $\Theta_{A_1 \dots A_n} = \Theta_{A_1 > A_2 > \dots > A_n}$

$$= \Theta_{A_1 A_2} \Theta_{A_2 A_3} \dots \Theta_{A_{n-1} A_n}$$

$$\text{useful identity } \sum_{\sigma \in S_n} \Theta_{A_{\sigma(1)} \dots A_{\sigma(n)}} = 1$$

LR & Av/Dif are easily related by rotation. So lets simplify

$$\langle \tau_{SK} O_{av}^1 \dots O_{av}^p O_{dif}^1 \dots O_{dif}^{p+q} \rangle = \xi^{\#_1} \eta^{\#_2} \langle \tau_{SK} O^1 \dots O^{p+q} \rangle = \sum_{\sigma \in S_n} \Theta_{\sigma[n]} \xi^{\#_{\sigma_1}} \eta^{\#_{\sigma_2}} \langle \tau_{SK} O_{\sigma[n]} \rangle$$

Examine each ordering and see if the average ξ or dif η is earliest for that ordering.

$$av \Rightarrow \frac{1}{2} \tau_{SK}(y O_R^{\sigma_1}) + \tau_{SK}(O_L^{\sigma_1} y) = \tau_{SK}\{y, \hat{o}\}$$

$$dif \Rightarrow \tau_{SK}\langle y O_R^{\sigma_1} \rangle - \tau_{SK}(O_L^{\sigma_1} y) = \tau_{SK}[y, \hat{o}]$$

$$\text{Keldysh bracket: } (\hat{A}, B_a)_{SK} = [\hat{A}, \hat{B}] \quad (\hat{A}, B_{av}) = \{\hat{A}, \hat{B}\}$$

Identity annihilates differences.

$$\langle \tau_{SK} O_1 \dots O_{p+q} \rangle = \sum_{\text{orderings}} \Theta_\sigma \langle (\dots ((\hat{1}, O_{\sigma_1}), O_{\sigma_2}) \dots, O_{\sigma_{p+q}}) \rangle$$

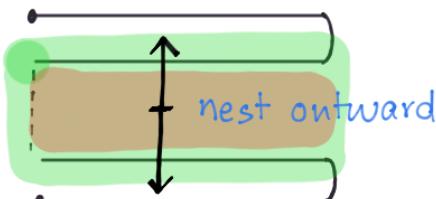
$$\langle \tau_{SK} A_{av} B_{av} \rangle = \Theta_{AB} ((\hat{1}, A_{av}), B_{av}) + \Theta_{BA} ((\hat{1}, B_{av}), A_{av}) = \{\hat{A}, \hat{B}\}$$

$$\langle \tau_{SK} A_{av} B_{dif} \rangle = \Theta_{AB} [\hat{A}, \hat{B}]$$

$$\Rightarrow \langle \hat{1}, O_{dif} \rangle = 0 \quad \& \quad O_{dif} \text{ not futuremost.}$$

generalized Keldysh rules

$$G_{k-oto}(t_1, \dots, t_{n_k}) = \sum_{\sigma_1 \in S_{n_1}} \Theta_{\sigma_1} \sum_{\sigma_2 \in S_{n_2}} \Theta_{\sigma_2} \dots \langle (\dots (\dots ((\gamma_s, O_{\sigma_2}^2), \dots O_{\sigma_2}^2), \dots O_{\sigma_1}^1), \dots O_{\sigma_1}^1) \rangle$$

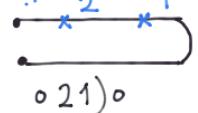


Ex) Keldysh rules for 3 pt. fns.

Mapping LR k-oto \mapsto Wightman basis $(2k)^n \rightarrow n!$

$n!$ time orderings should fit into various contours.

proper-oto : max q need to compute n -pt \underline{f}_n $1 \leq q \leq \lfloor \frac{n+1}{2} \rfloor$

- 2 point \underline{f}_n : 1-oto suffices $12_{\text{to}} \quad 21_{\text{oto}}$ 

- 3 pt \underline{f}_n require at most 2oto.

$$123 \quad \circ 321) \circ$$

$$213 \quad \circ 31)2 \circ$$

$$312 \quad \circ 21)3 \circ$$

$$321 \quad \circ 1)23 \circ$$

$$132 \quad \circ 2)3(1) \circ$$

$$231 \quad \circ 1)3(2) \circ$$

Ex) 4-pt \underline{f}_n

Some useful decompositions: $n! = \sum_{q=1}^{\lfloor \frac{n+1}{2} \rfloor} g_{n,q}$

$$g_{n,q} = \text{cf } \mu^q \text{ in } (2\sqrt{1-\mu})^{n+1} \det_n \left(\frac{2}{1+\sqrt{1-\mu}} - 1 \right)$$

every proper q -oto appears $h_{n,k}^{(q)}$ times in k -oto contour

$$h_{n,k}^{(q)} = \text{cf } t^k \text{ in } 2^{2q-1} t^q \frac{(1+t)^{n-(2q-1)}}{(1-t)^{n+1}} \Theta(n-(2q-1)).$$

Lecture 2 : microscopic constraints .

$\text{SK} \Rightarrow \langle \Pi \partial_{\text{dif}} \rangle = 0$ & largest time eqn.

k-oto \Rightarrow generalized localizations partial: $2R=2L$, $1R=2L$, $1L=2R$

total localizations: $2R=1R=1L=2L$ & $2R=1L$ & $1R=2L$

precursor localizations: $1R=1L$

most general constraints are turning point relations

largest/ smallest time eqns.

→ these capture unitarity. Require efficient encoding.

Introduce topological invariance (henceforth $k=1$).

∂_{dif} are BRST exact under Q_{SK} , \bar{Q}_{SK} (GLR require only " Q_{SK} ").

$$\begin{array}{ccc} O_R, O_L & & \\ \swarrow Q_{\text{SK}} \quad \searrow \bar{Q}_{\text{SK}} & & \\ O_G & & O_{\bar{G}} \\ \swarrow \bar{Q}_{\text{SK}} \quad \searrow Q_{\text{SK}} & & \\ & O_{\text{dif}} & \end{array}$$

$$[Q_{\text{SK}}, O_{av}] = O_G \quad [Q_{\text{SK}}, O_{\bar{G}}] = -O_{\text{dif}}$$

$$[\bar{Q}_{\text{SK}}, O_{av}] = O_{\bar{G}} \quad [\bar{Q}_{\text{SK}}, O_G] = O_{\text{dif}}$$

$$Q_{\text{SK}}^2 = \bar{Q}_{\text{SK}}^2 = \{Q_{\text{SK}}, \bar{Q}_{\text{SK}}\} = 0$$

$$\text{eg. } -S_{\text{scalar}} = \int d^d x \sqrt{-g} \left(\frac{1}{2} \partial_\mu \phi_R^\dagger \partial^\mu \phi_R - \frac{1}{2} \partial_\mu \phi_L^\dagger \partial^\mu \phi_L \right)$$

$$-S_{\text{SK}} = \int d^d x \sqrt{-g} \left(\frac{1}{2} \partial_\mu \phi_{av}^\dagger \partial^\mu \phi_{\text{dif}} + \frac{1}{2} \partial_\mu \phi_{\text{dif}}^\dagger \partial^\mu \phi_{av} + c^\dagger \nabla^2 c - \bar{c}^\dagger \nabla^2 \bar{c} \right)$$

SK superfields $\overset{\circ}{\phi} \rightarrow \overset{\circ}{\phi}$

$$\overset{\circ}{\phi} = O_{av} + \theta O_{\bar{G}} + \bar{\theta} O_G + \bar{\theta}\theta Q_{\text{dif}}$$

$$[Q_{\text{SK}}, \overset{\circ}{\phi}] = \frac{\partial \overset{\circ}{\phi}}{\partial \theta} \quad [\bar{Q}_{\text{SK}}, \overset{\circ}{\phi}] = \frac{\partial \overset{\circ}{\phi}}{\partial \bar{\theta}}$$

$$[Q_{\text{SK}}, O_{av}] = \left. \frac{\partial \overset{\circ}{\phi}}{\partial \theta} \right| = O_G + \theta O_{\text{dif}} \Big| = O_G$$

$$[Q_{\text{SK}}, O_{\bar{G}}] = \left. \left[Q_{\text{SK}}, \frac{\partial \overset{\circ}{\phi}}{\partial \theta} \right| \right] = \left. \frac{\partial}{\partial \bar{\theta}} \left(\frac{\partial \overset{\circ}{\phi}}{\partial \theta} \right) \right| = -O_{\text{dif}}$$

Thermal physics: $\rho_\beta = e^{-\beta H}$ chemical pot etc are easy to add.

Euclidean evolution by β $\mathcal{Z}_T(\beta) = \text{Tr}(\rho_\beta)$

general thermal equilibrium: QFT on a background w/ timelike killing vector $K^\mu = (\partial/\partial t)^\mu$

$$ds^2 = -e^{2\sigma(x^m)}(dt + a_i dx^i)^2 + g_{ij} dx^i dx^j$$

Euclidean geometry: thermal circle fibration over space. Σ_u .

KMS condition in equilibrium: correlation f_{ns} are analytic in

$$0 < \text{Im}(t) < \beta \quad \text{Tr}(\hat{\rho}_\beta \hat{A}(t-i\beta) \hat{B}(0)) = \text{Tr}(\hat{\rho}_\beta \hat{B}(0) \hat{A}(t))$$

$$\hat{A}(t-i\beta) = \hat{\rho}_\beta^{-1} \hat{A}(t) \hat{\rho}_\beta = e^{-i\beta \hat{\rho}_\beta} \hat{A}$$

$$i\Delta_\beta = 1 - e^{-i\beta \hat{\rho}_\beta}$$

not the only implementation: some authors prefer to allow evolution by $+i\beta/2$

KMS condition as an operator statement: $\Delta_\beta \hat{O} = 0 \equiv [\mathcal{L}_{\text{KMS}}, \hat{O}]$

Using KMS one can derive thermal sum rules

WELDON $\langle T_{SK} \prod_{k=1}^n (O_R^{(k)} - \tilde{O}_L^{(k)}) \rangle = 0 \quad \tilde{O}_L = e^{-i\beta \hat{\rho}_\beta} O_L$

+ smallest time egn. $O_R - \tilde{O}_L$ can't be next to density matrix

not the only implementation: some authors prefer to allow evolution by $+i\beta/2$

$$\begin{pmatrix} O_R \\ O_L \end{pmatrix} \xrightarrow[\text{involution}]{\mathcal{K}_\beta} \begin{pmatrix} O_L(t - \frac{i}{2}\beta) \\ O_R(t + \frac{i}{2}\beta) \end{pmatrix}$$

$$\begin{pmatrix} O_R(t) \\ O_L(t) \end{pmatrix} \xrightarrow{} \begin{pmatrix} O_L(t - i\beta) \\ O_R(t) \end{pmatrix}$$

Thermo field double

$\text{Tr}(\rho_\beta^{1/2} U_R S_R^{1/2} U_L^\dagger)$ obscures unitarity (Rényi like).

encoding KMS in SK superspace: already have bosonic generator \mathcal{L}_{KMS}
 partners Q_{KMS} , \bar{Q}_{KMS} , Q°_{KMS} .

want a descent that argues $\partial_R - \tilde{\partial}_L$ is BRST exact.

$$\begin{array}{ccc} & O_L & \\ Q_{\text{KMS}} \swarrow & & \searrow Q^{\circ}_{\text{KMS}} \\ iO_G & & -i\bar{O}_{\bar{G}} \\ \swarrow Q^{\circ}_{\text{KMS}} & \partial_R - \tilde{\partial}_L & \searrow Q_{\text{KMS}} \end{array}$$

$$\begin{aligned} [Q_{\text{KMS}}, O_{av}] &= -(i + \frac{1}{2}\Delta_B) O_G & [Q_{\text{KMS}}, \bar{O}_{\bar{G}}] &= -\Delta_B O_{av} + (i + \frac{1}{2}\Delta_B) O_{\text{dif}} \\ [Q_{\text{KMS}}, O_{\bar{G}}] &= 0 & [Q_{\text{KMS}}, O_{\text{dif}}] &= -\Delta_B O_G \\ [\bar{Q}_{\text{KMS}}, O_{av}] &= (i + \frac{1}{2}\Delta_B) \bar{O}_G & [\bar{Q}_{\text{KMS}}, \bar{O}_{\bar{G}}] &= 0 \\ [\bar{Q}_{\text{KMS}}, O_{\bar{G}}] &= \Delta_B O_{av} - (i + \frac{1}{2}\Delta_B) O_{\text{dif}} & [\bar{Q}_{\text{KMS}}, O_{\text{dif}}] &= \Delta_B \bar{O}_{\bar{G}} \end{aligned}$$

general SK-KMS algebra:

$$\begin{array}{ccc} & Q^{\circ}_{\text{KMS}} & \\ Q_{SK} \swarrow & & \searrow \bar{Q}_{SK} \\ Q_{\text{KMS}} & & -\bar{Q}_{\text{KMS}} \\ \bar{Q}_{SK} \swarrow & \searrow -Q_{SK} & \end{array}$$

$$\begin{aligned} Q_I^2 &= \bar{Q}_I^2 = 0 & I \in \{SK, KMS\} \\ \{Q_I, \bar{Q}_I\} &= 0 \\ \{Q_{SK}, \bar{Q}_{KMS}\} &= \{Q_{SK}, Q_{KMS}\} = \mathcal{L}_{\text{KMS}} \\ [Q_{SK}, Q^{\circ}_{\text{KMS}}] &= Q_{\text{KMS}} & [\bar{Q}_{SK}, Q^{\circ}_{\text{KMS}}] &= \bar{Q}_{\text{KMS}} \end{aligned}$$

general superspace analysis: Q_{SK}, \bar{Q}_{SK} : super derivations

$Q_{\text{KMS}}, \bar{Q}_{\text{KMS}}, Q^{\circ}_{\text{KMS}} \hookrightarrow$ super interior contractions

\mathcal{L}_{KMS} : super lie derivations

Eg $\mathcal{N}=1$ equivariance as $\{d, i, f\}$

generalization to $\mathcal{N}=2$ w/ thermal equivariance.

Lecture 3: Hydrodynamics as an effective field theory.

hydro: long wavelength, near-equilibrium dynamics. local equilibrium dynamical data: conservation of conserved currents.

phenomenology: constrained by 2nd law. usually on-shell $\nabla_\mu J_s^\mu \geq 0$

off-shell formalism very helpful: $\beta^M = \frac{u^\mu}{T}$ (Killing in eq) $g_{\mu\nu}$

$$\nabla_\mu J_s^\mu + \beta_\mu \nabla_\nu T^{\mu\nu} = \Delta \geq 0 \quad (\text{changes easy; includes anomalies})$$

$$E_T^M: \nabla_\nu T^{\mu\nu} - J_\nu F^{\mu\nu} - T_H^{J\mu} = 0 \quad E_J = D_\mu J^\mu - J_H^{-1} = 0$$

$$\nabla_\mu J_s^\mu + \beta_\mu E_T^\mu + (\Lambda_B + \beta^\alpha A_\alpha) E_J = \Delta \geq 0$$

$$-\frac{G^\sigma}{T} = J_S^\sigma + \beta_\nu T^{\nu\sigma} + (\Lambda_B + \beta^\alpha A_\alpha) J^\sigma$$

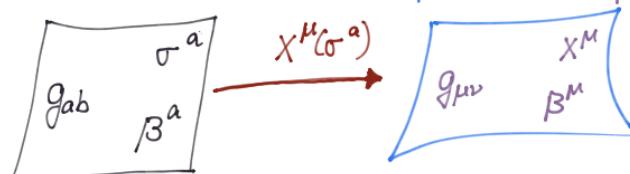
$$\nabla_\sigma \left(\frac{G^\sigma}{T} \right) - \frac{G_H^\sigma}{T} = \frac{1}{2} T^{\mu\nu} \delta_B g_{\mu\nu} - J^\mu \delta_B A_\mu + \Delta$$

Full classification exists (HLR '15)

A, B, C, **D**, $H_S, \bar{H}_S, H_V, \bar{H}_V$
 ↳ only contribution to Δ

$$G^\sigma = g \beta^\sigma + v^\sigma$$

Effective action description: simple story for $H_S \cup \bar{H}_S$



$$S_{\text{hydro}} = \int d^d x \sqrt{-g} \mathcal{L} [\beta^a, g_{ab}(x)]$$

$$\frac{\delta S}{\delta X^M} = 0 \Rightarrow \nabla_\alpha T^{\alpha\mu} = 0$$

Push-forward $\mathcal{L}[\beta^M, g_{\mu\nu}]$

$$\frac{1}{2} \delta(F g f) - \nabla_\mu (\phi \Theta)^\mu = \frac{1}{2} T^{\mu\nu} \delta g_{\mu\nu} + T \delta \phi \beta^\mu$$

$$G^\sigma = -T \left(\beta^\sigma \mathcal{L} - (\phi_B \Theta)^\sigma + \text{Komar}^\sigma \right)$$

} Entropy current is a Noether current for Euclidean thermal translations.

Remaining 5 adiabatic classes require more structure

$$\mathcal{L}_T = N^\mu \tilde{A}_\mu + \frac{1}{2} T^{\mu\nu} \tilde{g}_{\mu\nu}$$

phenomenologically
introduced $U(1)_T$



"sk partner"

Recovering dissipative hydro action: microscopic theory has SK-KMS algebra has features of $N_T=2$ equivariant algebras. What is the gauge sym?

Conjecture: Low energy thy is invariant under thermal diffeos $U(1)_T$

$$Q = Q_{SK} + \text{gauge} \quad Q^2 = (\overset{\circ}{\mathcal{F}}_{\bar{\Theta}\bar{\Theta}}) L_{KMS} \quad \{ \bar{Q}, Q \} = \overset{\circ}{\mathcal{F}}_{\bar{\Theta}\Theta} | L_{KMS}$$

$$\bar{Q} = \bar{Q}_{SK} + \text{gauge} \quad \bar{Q}^2 = (\overset{\circ}{\mathcal{F}}_{\Theta\Theta}) L_{KMS}$$

Implementation: worldvolume w/ susy $\{\sigma^\alpha, \theta, \bar{\theta}\} = z^\Gamma$ $B^\alpha, \bar{B}^\beta = \bar{B}^{\bar{\beta}} = 0$

supergauge parameter λ with 0-adj fields (\dot{x}) : $\dot{x} \mapsto \dot{x} + (\lambda, \dot{x})_\beta$
 $(\lambda, \dot{x})_\beta = \lambda \mathcal{L}_\beta \dot{x}$ gauge algebra: $(\lambda, \lambda')_\beta = \lambda \mathcal{L}_\beta \lambda' - \lambda' \mathcal{L}_\beta \lambda$

gauge field data: $\overset{\circ}{A}_{IDz^\Sigma} = \overset{\circ}{A}_\alpha d\sigma^\alpha + \overset{\circ}{A}_\theta d\theta + \overset{\circ}{A}_{\bar{\theta}} d\bar{\theta}$

matter data: $\overset{\circ}{X}^\mu$: $\underset{\text{classical}}{X_{av}^\mu} + \theta \underset{\text{fluctuation}}{(\)^\mu} + \bar{\theta} \underset{\text{fluctuation}}{(\)^\mu} + \bar{\theta} \theta \underset{\text{fluctuation}}{\tilde{X}^\mu}$

Langevin dynamics $m \frac{d^2 \dot{x}}{dt^2} + \frac{\partial U}{\partial x} + \nu \Delta_B x = N$ $\Delta_B = \beta \frac{d}{dt}$

$$\mathcal{E}_x + N = 0$$

$$\Rightarrow \text{BRST action} \quad \tilde{X} \mathcal{E}_x + i\nu \tilde{X}^2 + X \bar{\psi} \frac{\delta \mathcal{E}_x}{\delta x} X \bar{\psi}$$

Can be understood as a gauge fixed version of thermal eq. + algebra

Data: \dot{x} , $\overset{\circ}{A}$ & Q, \bar{Q} implemented as D_θ & $D_{\bar{\theta}}$

$$S_{BO} = \int dt d\theta d\bar{\theta} \left\{ \frac{m}{2} \left(\frac{d\dot{x}}{dt} \right)^2 - U(x) - i\nu \overset{\circ}{D}_\theta \dot{x} \overset{\circ}{D}_{\bar{\theta}} \dot{x} \right\}$$

origin of dissipation: $\phi = \bar{\phi} = \eta = \bar{\eta} = 0$

versus for gauge invariant data.

$\phi^\circ = -i$ spontaneous CPT breaking

$U(1) \circ \text{CPT}$: $iS_{BO} \mapsto iS_{BO} - i\langle \phi^\circ \rangle_B (\Delta G + W)$

Jarzynski work relation $\langle e^{-\beta W} \rangle = e^{-\beta \Delta G}$

\Rightarrow 2nd law $\Delta G \leq W$

SYMMETRIES & HYDRO 6-NODELS

* superdiffeos on target + world-volume

* CPT symmetry $Z_{SK}[J_R, J_L]^* = Z_{SK}[J_L^*, J_R^*]$

$$\begin{array}{l} \theta \leftrightarrow \bar{\theta} \\ \text{dif} \leftrightarrow -\text{dif} \end{array}$$

* gauge invariant, ghost # zero.

* $\mathcal{L}[\dot{g}_{ab}]$ captures all topological constraints on influence fncts.

physical theory $\ddot{g}_{ij} \rightarrow \ddot{g}_{ij} + \bar{\theta} \partial_i h_{ij}$

* eg Langevin particle: $S_{BO} = \int dt d\theta d\bar{\theta} \left\{ \frac{m}{2} (\dot{D}_t \dot{x})^2 - U(\dot{x}) - i \omega \dot{D}_{\theta} \dot{x} \dot{D}_{\bar{\theta}} \dot{x} \right\}$

dissipation / stochasticity arises w/ $\langle \dot{F}_{\theta\bar{\theta}} \rangle = -i$

* hydro action $S_{WV} = \int d\sigma \mathcal{L}_{WV} = \int d\sigma d\theta d\bar{\theta} \frac{\sqrt{-\dot{g}}}{1+\beta A} \left(\dot{L} - \frac{i}{4} \eta^{(ab)(cd)} \dot{D}_{\theta} \dot{g}_{ab} \dot{D}_{\bar{\theta}} \dot{g}_{cd} \right)$

$\mathcal{L}_{WV} = \frac{\sqrt{-g}}{1+\beta A} \left[\frac{1}{2} T_L^{ab} - \frac{i}{2} \eta^{(ab)(cd)} (F_{\theta\bar{\theta}}, g_{cd}) \right] \tilde{g}_{ab} - N_L^a \tilde{A}_a + \frac{i}{8} [\eta^{(ab)(cd)} + \eta^{(cd)(ab)}] \tilde{g}_{ab} \tilde{g}_{cd}$
 → Jarzynski MSR construction of KMR ; CGL

Challenge: AdS/CFT gives info about CFT_d fluid.

$$\begin{aligned} \mathcal{L} = C_{eff} \int d\theta d\bar{\theta} \frac{\sqrt{-\dot{g}}}{1+\beta A} & \left\{ \left(\frac{4\pi \dot{T}}{d} \right)^d \left(1 - \frac{id}{8\pi} \hat{P}^{ca} \hat{P}^{bd} \dot{D}_{\theta} \dot{g}_{ab} \dot{D}_{\bar{\theta}} \dot{g}_{cd} \right) \right. \\ & - \left. \left(\frac{4\pi \dot{T}}{d} \right)^{d-2} \left[\frac{w \dot{R}}{d-2} + \frac{1}{d} \text{Harmonic} \left(\frac{2}{d}-1 \right) \dot{\sigma}^2 + \frac{1}{2} \dot{\omega}^2 \right] \right\} \end{aligned}$$

References :

1. Felix Haehl, R Loganayagam, MR :

1502.00636 }
1412.1090 } hydro bgnl

1510.02494 philosophy

1511.07809

1610.01940 }

1610.01941 } SK-KMS

hydro action

2. Parallel developments (1701.07896 : comparison)

Crossley, Glorioso, Liu 1511.03646
1701.07817

Glorioso, Lin 1612.07705
Gao, Lin 1701.07445

earlier attempt : Kovtun, Moore, Romatschke (KMR)

1405.3967

Jensen, Pinzani-Fokeeva, Yarom 1701.07436