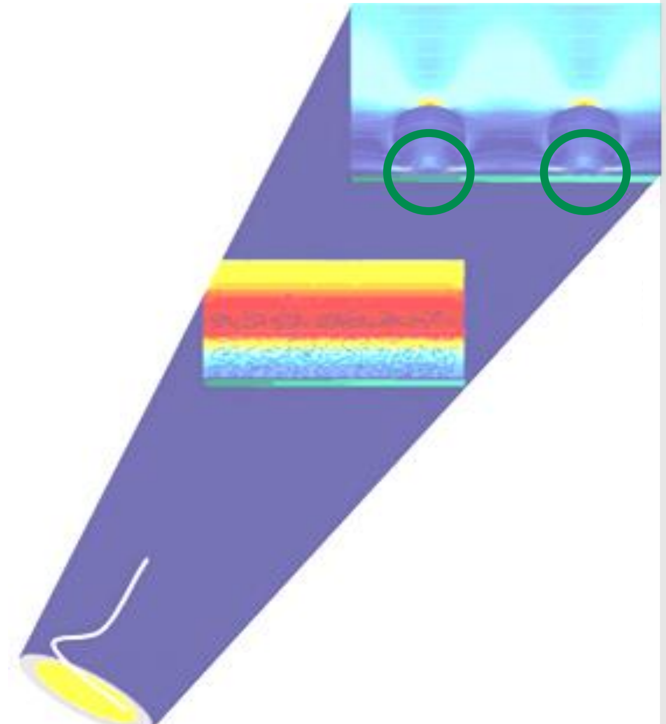
Is Partial Slip Under Transverse Oscillatory Loading a Transient Event?

Frederick Meyer,[1] *Daniel Forchheimer,*[2] *Arne Langhoff,*[1] *Diethelm Johannsmann**[1]

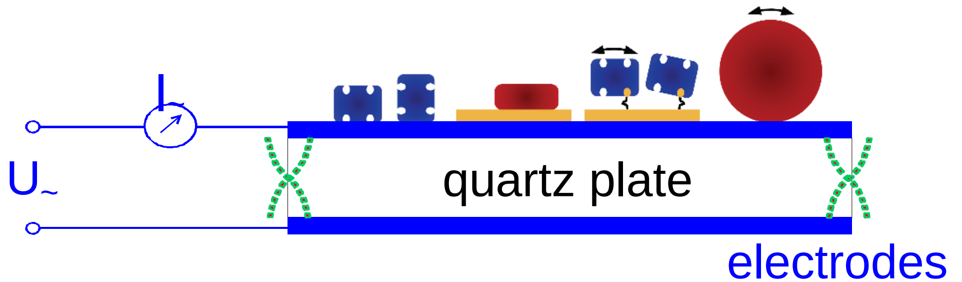
[1] *Institute of Physical Chemistry, TU Clausthal, Germany*
[2] Intermodulation Products AB, Sweden

- High-frequency nonlinear contact mechanics
- Intermodulation products
- Sudden impacts
- Crystallization

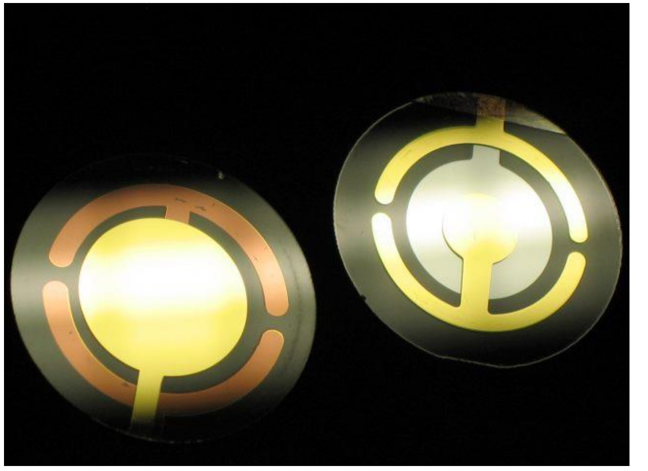


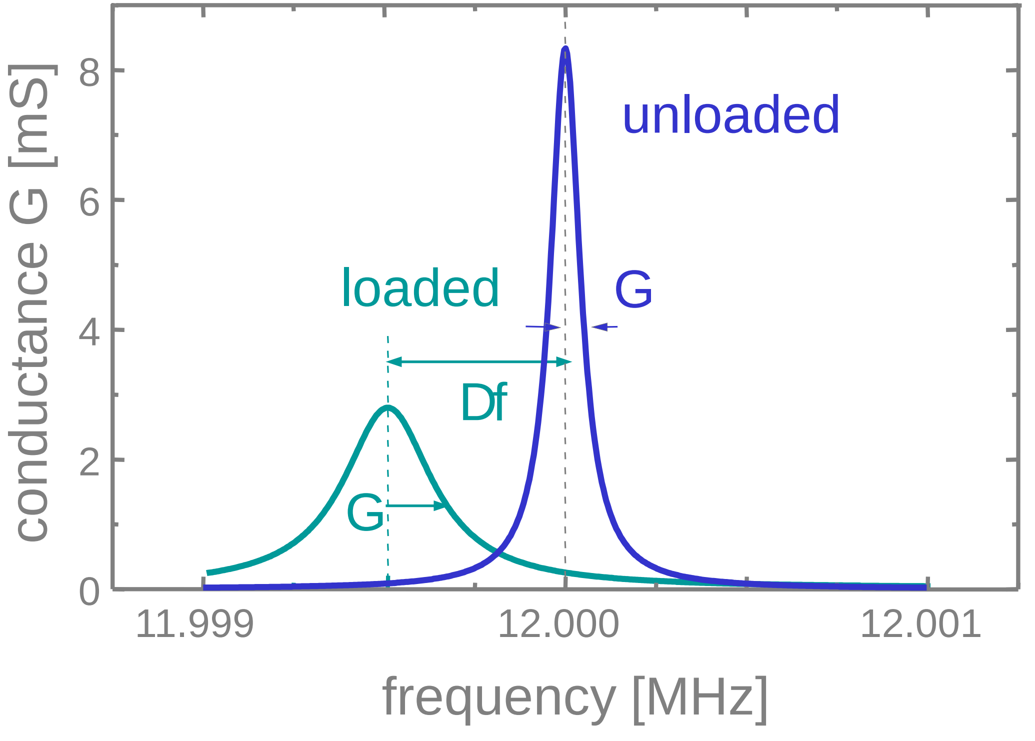
- High-frequency micromechanics: *Sylvia Hanke, Rebekka König, Judith Petri, Jana Vlachová, Frederick Meyer*
- Intermodulation: *Daniel Forchheimer*
- QCM work in general: *Arne Langhoff*

The 2nd-Generation Quartz Crystal Microbalance



f = f_{res}: large amplitude of motion
$\Rightarrow$ large current



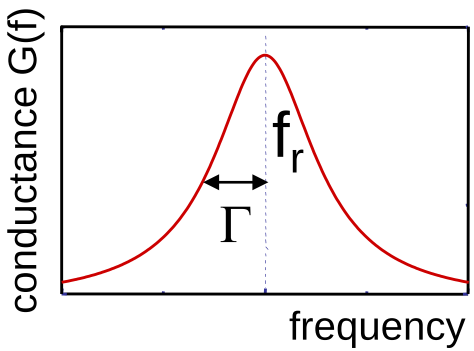


- Shifts in frequency and bandwidth: Δf, $\Delta\Gamma$
- many overtones $\Delta f(n)$, $\Delta\Gamma(n)$

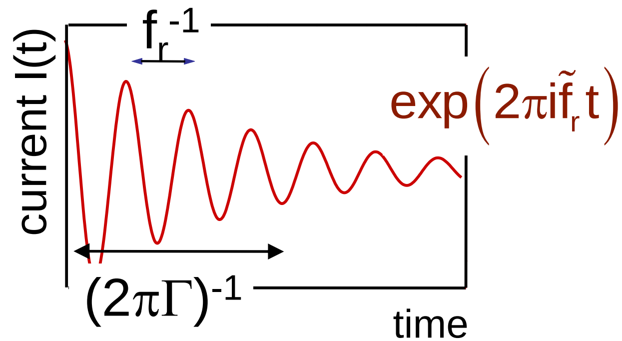
- dependence on amplitude $\Delta f(n, u_0)$, $\Delta\Gamma\ (n\ ,\ u_0)$
- Higher harmonics
- Intermodulation products

Small Load Approximation

Complex Resonance Frequency $\tilde{f}_r = f_r + i\Gamma$



Fourier Transform ⟷



Small-load approximation

$$\frac{\Delta\tilde{f}}{f_0} \approx \frac{i}{\pi Z_q}\tilde{Z}_L = \frac{i}{\pi Z_q}\frac{\hat{\sigma}}{\hat{v}}$$

$Z_L = \frac{\hat{\sigma}}{\hat{v}}$ "load impedance"

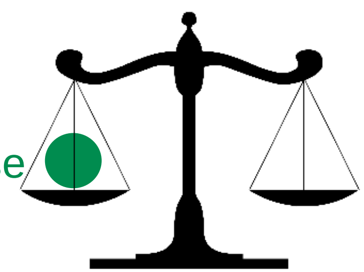
$\hat{\sigma}$ stress

$\hat{v}$ velocity

^: complex amplitude ($\sigma(t) = \hat{\sigma}\exp(i\omega t)$)

QCM: The Quartz Crystal
Micro *Stress*-Balance

periodic stress,
in-phase, out-of-phase



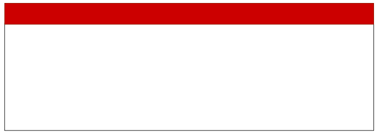
Mason, W.P., *Piezoelectric Crystals and their Applications to Ultrasonics* **1948**
Pechhold, W *Acustica* **1959,** 9, 48
Johannsmann, D., *The Quartz Crystal Microbalance in Soft Matter Research, Springer* **2014**

Analogous equations exist in atomic force microscopy, valid **if** the perturbations are small

Small Load Approximation

Sauerbrey

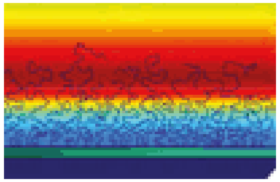
$$\frac{\Delta\tilde{f}}{f_0} = \frac{i}{\pi Z_q}\frac{\text{inertial stress}}{\text{velocity}} = \frac{i}{\pi Z_q}\frac{i\omega\hat{v}\,\Delta m}{\hat{v}}$$



$$\Rightarrow \frac{\Delta f}{f} \approx -\frac{\Delta m}{m_q} \qquad \Leftarrow \begin{cases} \Delta m: \text{Mass per unit area of film} \\ m_q = Z_q/(2f_0): \text{Mass per unit area of crystal} \end{cases}$$

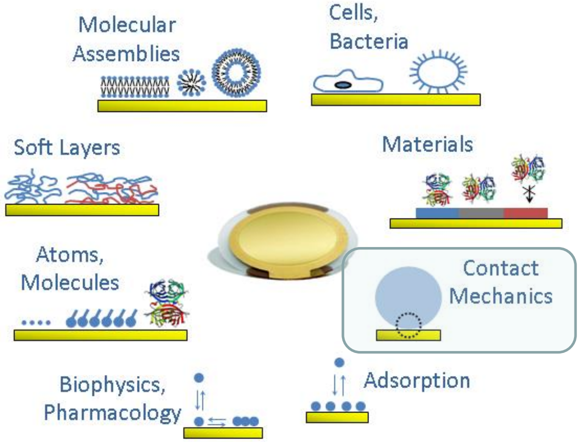
Stress might go back to **viscous drag**, **elastic forces** ...

acoustic multilayers, interfackal high-frequency rheology



The stress can be averaged over **area**:
$$\frac{\Delta\tilde{f}}{f_0} = \frac{i}{\pi Z_q}\frac{\langle\hat{\sigma}\rangle_{area}}{\hat{v}}$$

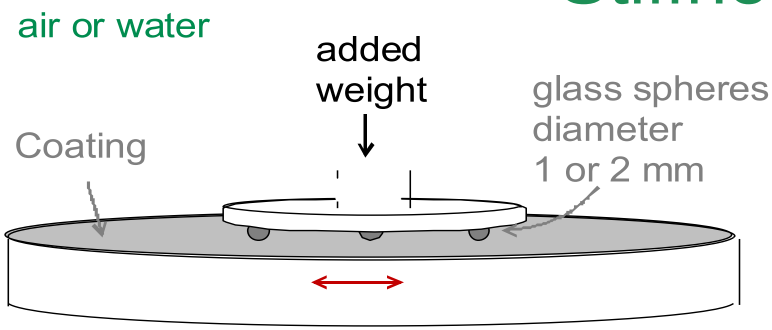
Discrete objects: $\langle\hat{\sigma}\rangle_{area} = \frac{N}{A}\hat{F}$ with $\hat{F}$ the periodic force

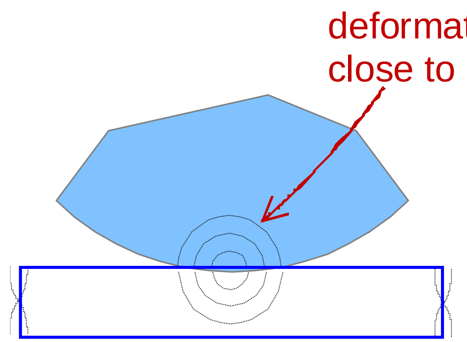


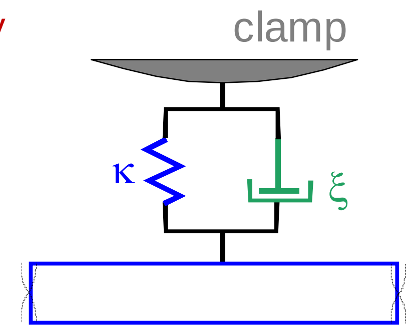
The force can be averaged over **time**:
$$\frac{\Delta\tilde{f}}{f_0} = \frac{i}{\pi Z_q}\frac{N}{A}\frac{2\langle F(t)\exp(i\omega t)\rangle_{time}}{\hat{v}}$$

QCM covers **nonlinear** force-displacement relations

Stiffness of Sphere-Plate Contacts







soft link, heavy sphere

$\rightarrow \tilde{\omega}_P << \omega$

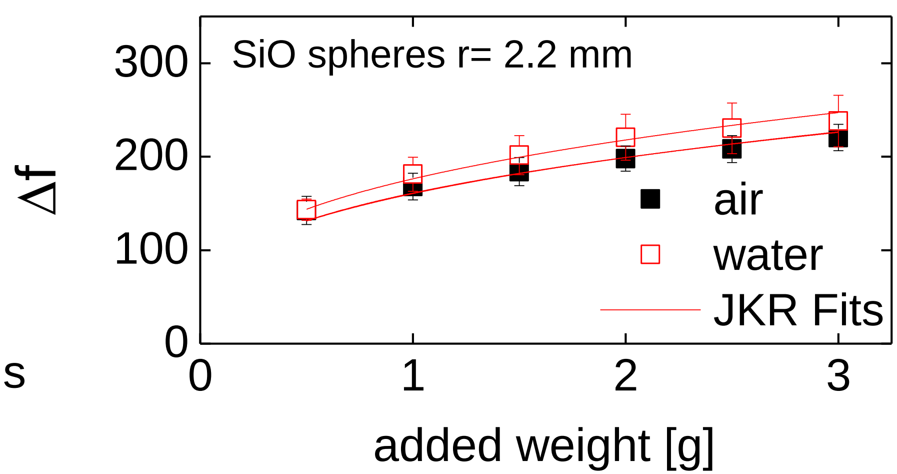
$$\frac{\Delta \tilde{f}}{f_0} = \frac{N}{A\pi Z_q}\frac{\tilde{\kappa}}{\omega}$$

Elastic Load

Hertz-Mindlin: $\kappa = 2G^{*}a$

a: contact radius

$$\frac{1}{G^{*}} = \frac{1-2\nu_1}{4G_1} + \frac{1-2\nu_2}{4G_2}$$: effective modulus

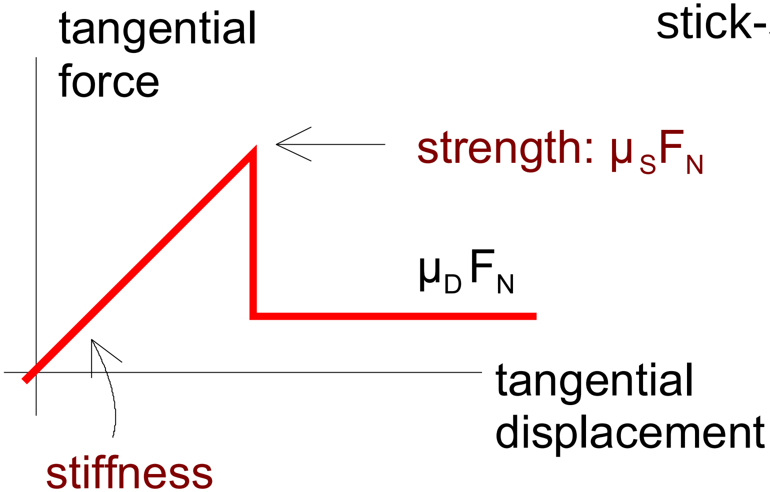


Vlachová, J.; König, R.; Johannsmann, D.
Beilstein J. Nanotechnol. ***2015, 6, 845.***

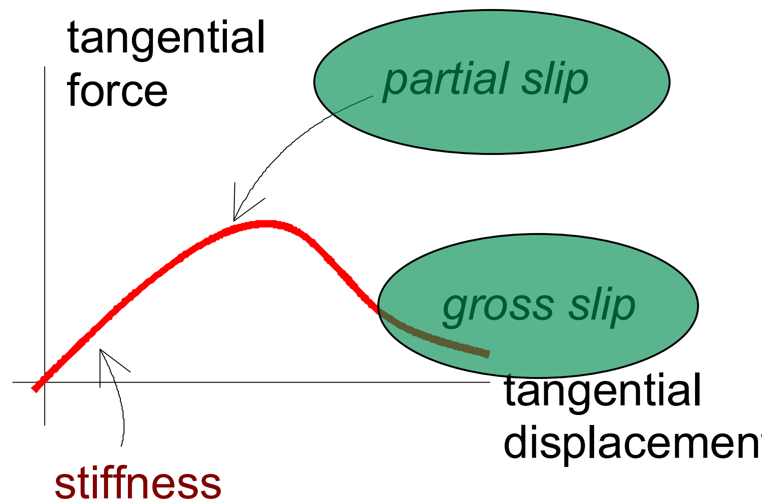
Contact Stiffness → Contact Strength

Coulomb friction

Quasi-static, force control:
stick-slip transition is an instability

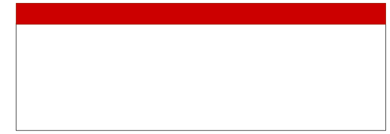


Oscillatory motion, strain control:
⇒ Partial slip *not* an instability



Small Load Approximation

Sauerbrey



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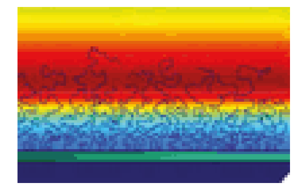
acoustic multilayers, QTM

high-frequency polymer rheology



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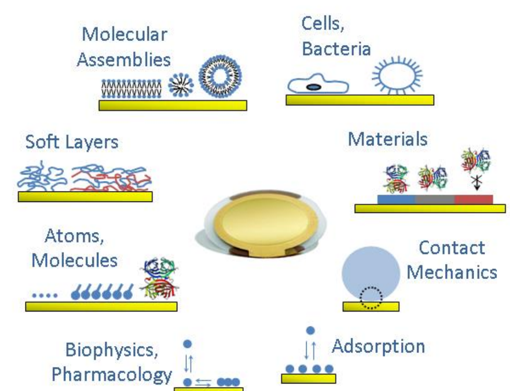


The force can be averaged over **time**:

$$\frac{\Delta\tilde{f}}{f_0} = \frac{i}{\pi Z_q}\frac{N}{A}\frac{2\langle F(t)\exp(i\omega t)\rangle_{time}}{\hat{v}}$$

QCM covers **nonlinear** force-displacement relations

The QCM and Nonlinear Response

The stress can be averaged over time: $\frac{\Delta \tilde{f}}{f_0} = \frac{i}{\pi Z_q} \frac{2\langle \sigma(t) \exp(i\omega t)\rangle_{time}}{\hat{v}}$

Loads are small $\rightarrow$ *strain – control*

$$u \approx u_0 \cos(\omega t) \Rightarrow dt = \frac{1}{\omega} \frac{1}{\sqrt{1-(u/u_0)^2}} du$$

Stress and force can be averaged over displacement, u

$$\Rightarrow \Delta f \sim \langle F(t) \cos(i\omega t)\rangle_{time}$$

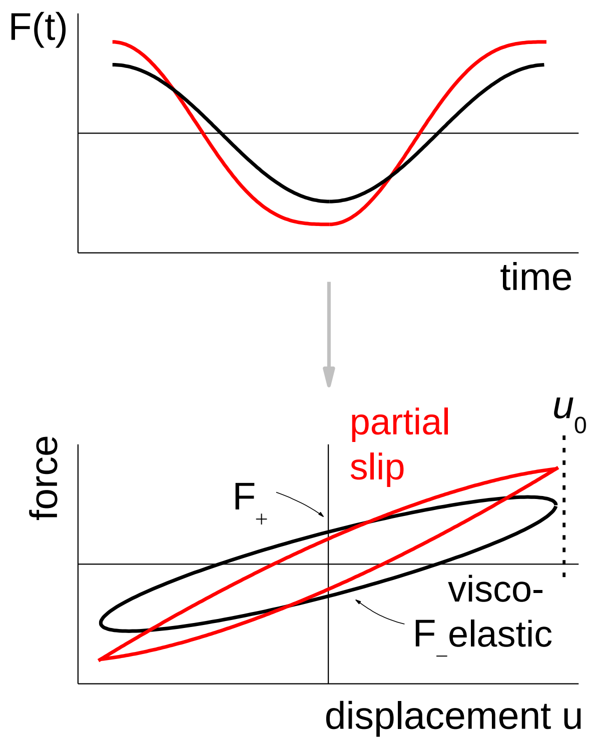
$$= \left\langle \left(F_+(u,u_0,\omega) + F_-(u,u_0,\omega)\right) \frac{u/u_0}{\sqrt{1-(u/u_0)^2}} \right\rangle_u$$

$$\Delta\Gamma \sim \langle F(t) \sin(i\omega t)\rangle_{time}$$

$$= \left\langle \left(F_+(u,u_0,\omega) - F_-(u,u_0,\omega)\right)\right\rangle_u$$

$\Delta f, \Delta\Gamma$ are weighted averages of friction loop

shape of friction loop uncertain



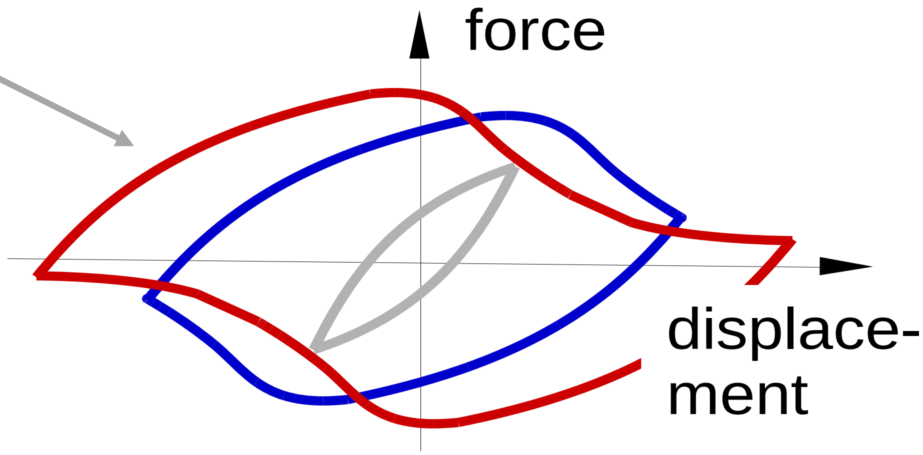
Hanke, S.; Petri, J.; Johannsmann, D., *PRE* **2013,** 032408
Johannsmann, D., *Springer* **2014**

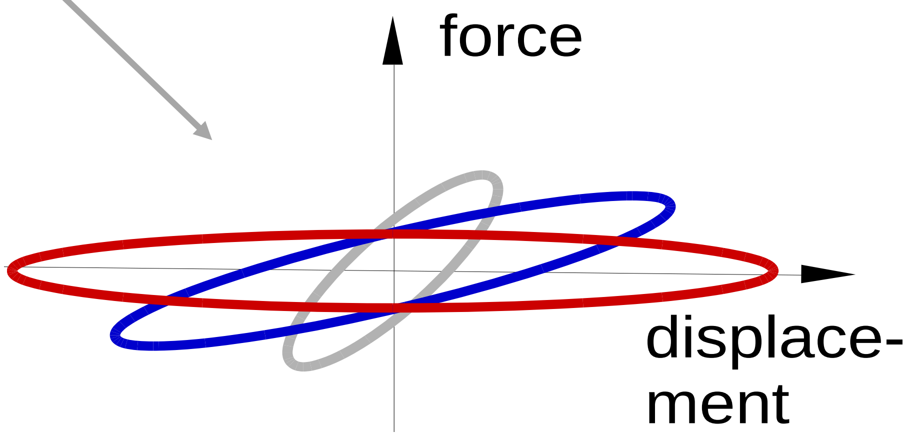
Shape of Friction Loop?

Data fit to Mindlin model

They might also be explained by a temperature-induced softening of the contact

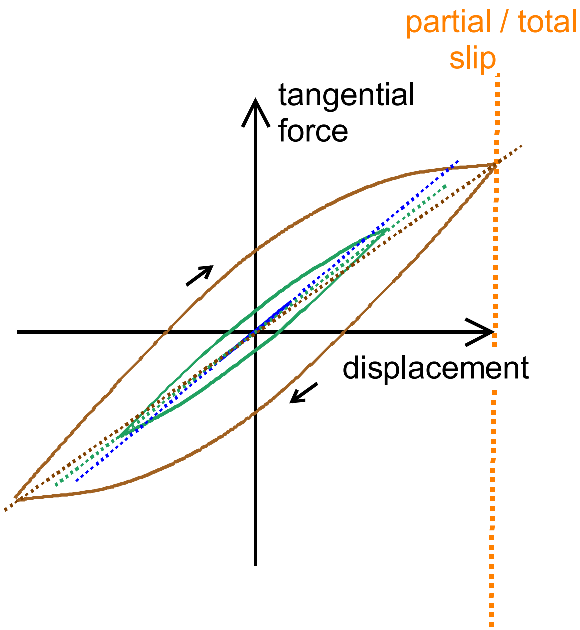
This question can be answered with 3rd harmonic generation

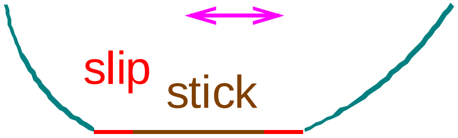




Ghosh, S. K. et al. ; *Biosensors & Bioelectronics* ***2011***, *29, 145*
Berg, S.; DJ, Surface Science ***2003***, *541, 225*

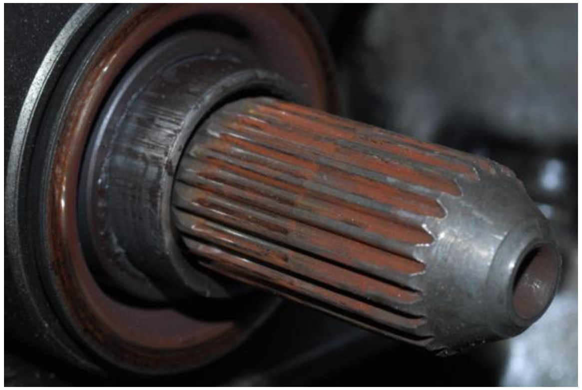
Partial Slip





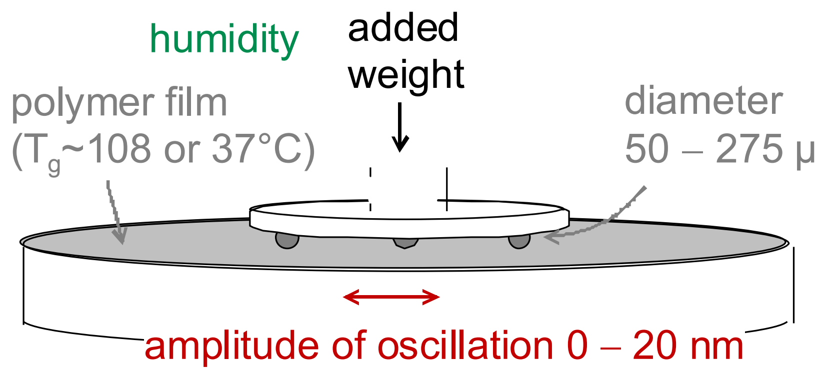
Cattaneo, C.,
Rendiconti dell' Academia Nationale dei Lincei **1938**
Mindlin, R.D.; Deresiewicz H.: *J. Appl. Mech.* **1953**
Johnson, K. L., *Contact Mechanics* **1985**
Savkoor, A. R. Tech. University Delft, **1987**
Varenberg, M.; Etsion, I.; Halperin, G.,
Tribology Letters **2005**

When transient: $\Rightarrow$ Transition state between stick and slip, mixed lubrication, …
When slow: $\Rightarrow$ Contact aging, compaction, soil mechanics, granular media, …
When oscillatory: $\Rightarrow$ **Fretting wear**



http://www.mr2oc.com/208-aef-engine-powertrain/455990-calling-m-e-s-fretting-failure-mode-clutch-hub-female-spline-e153-mr2-turbo-trans.html

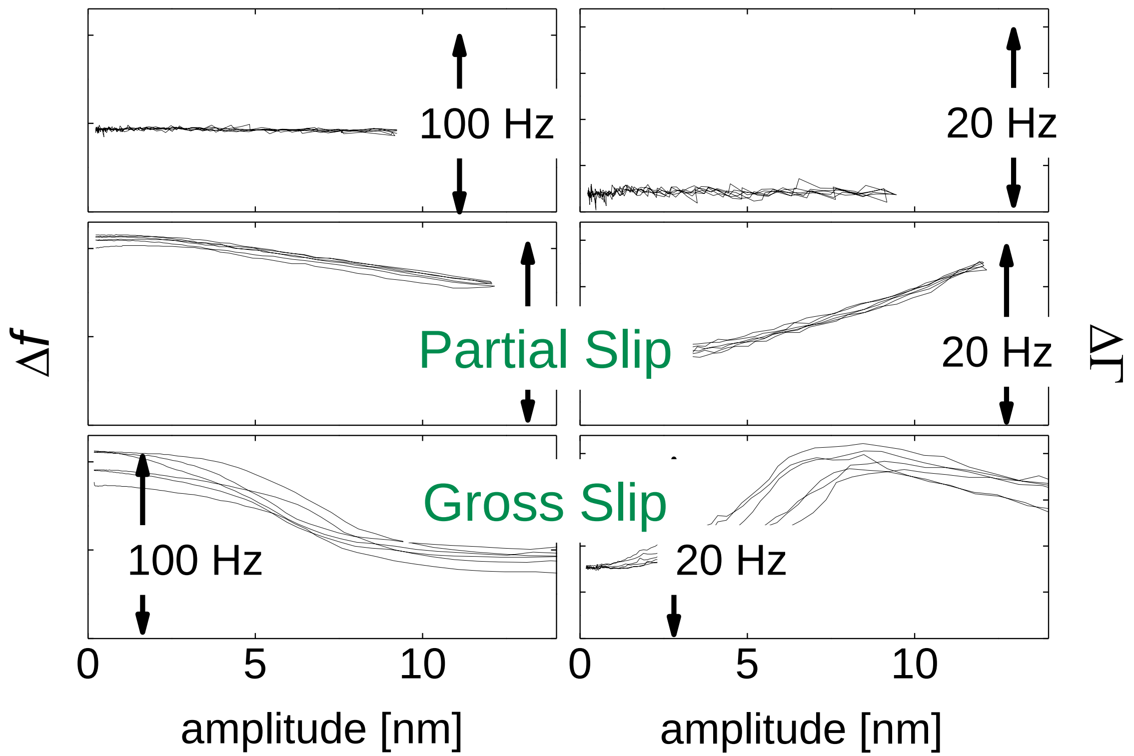
Partial Slip and Gross Slip



Small spheres (d = 50 µm)
soft substrate (T_g ~ 37°C)

Medium size spheres (140 µm)
soft substrate (T_g ~ 37°C)

Large spheres (275 µm)
hard substrate (T_g ~ 108°C)



Partial Slip ⇒ Nonlinear Stress-Strain Relations

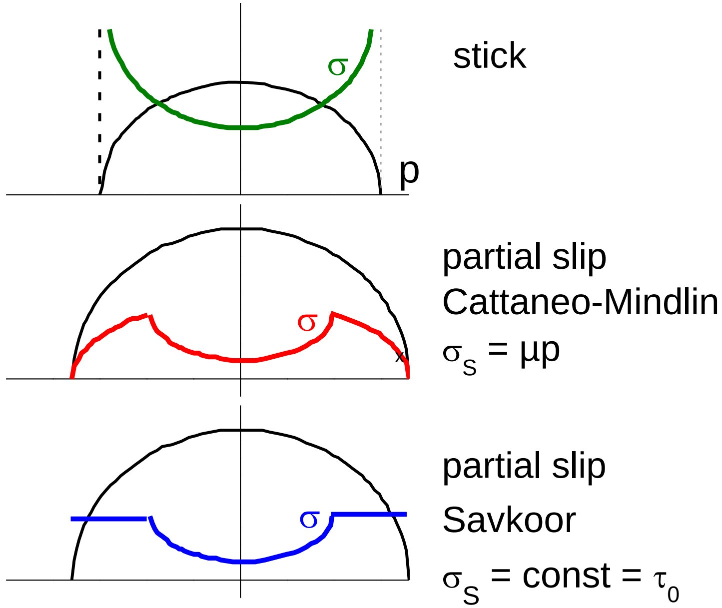
Quantitative models for the force-displacement relation exist
(but: quasi-static)

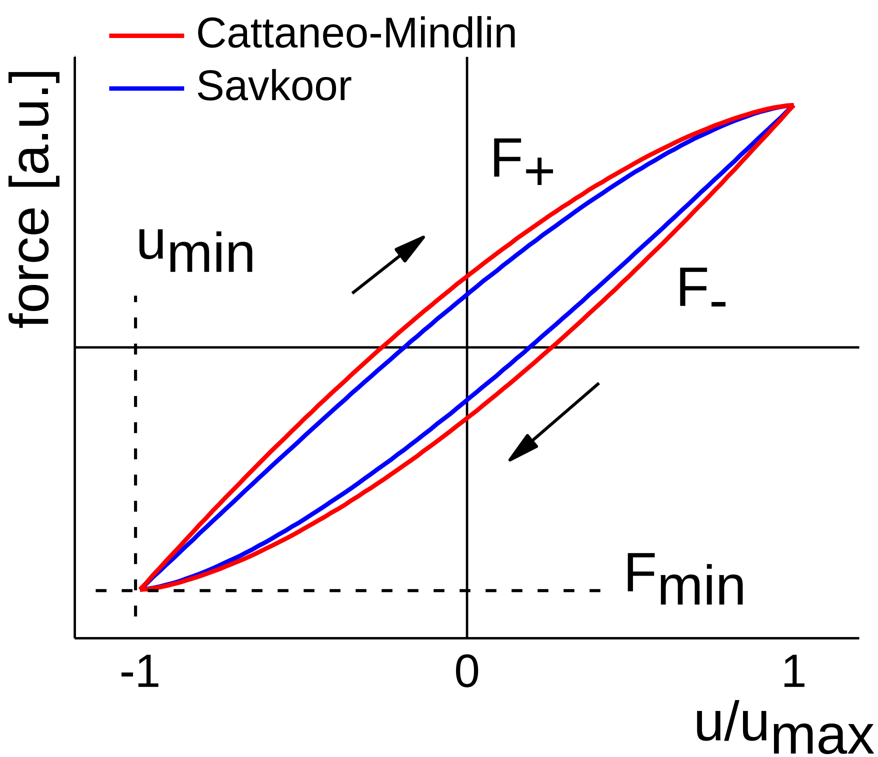
Stress in sliding zone follows Coulomb ($\sigma_S = \mu p$)

Cattaneo, C., *Rendiconti dell' Academia Nationale dei Lincei* **1938**
Mindlin, R.D.; Deresiewicz H.: *J. Appl. Mech.* **1953**

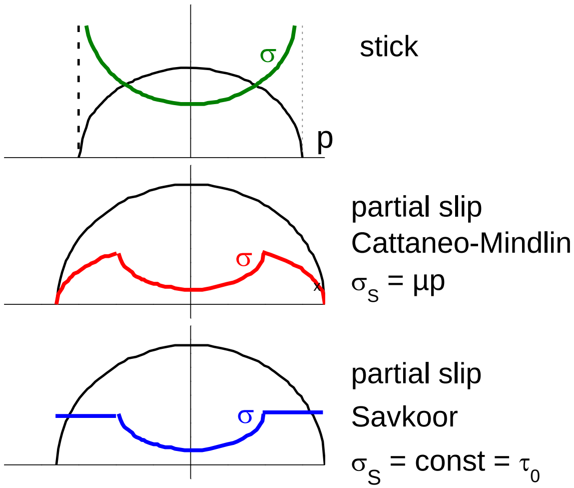
Stress in sliding zone constant (σ_S = const.)

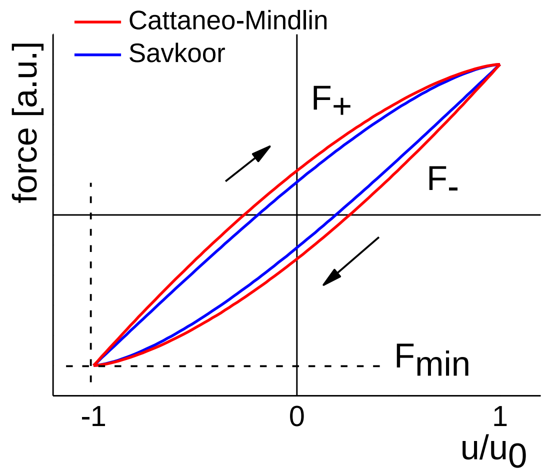
Savkoor, A. R. Tech. University Delft, 1987





Partial Slip ⇒ Nonlinear Stress-Strain Relations





Cattaneo-Mindlin

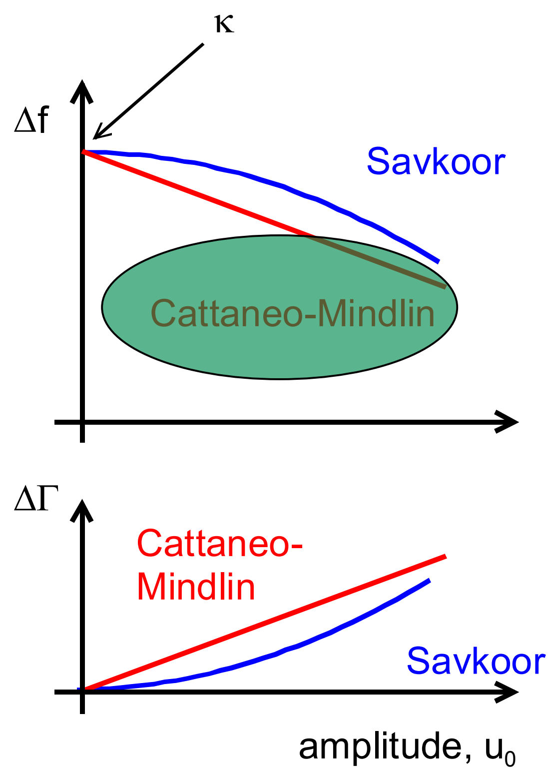
$$\Delta f(u_0) \approx \frac{N}{A2n\pi^2 Z_q}\kappa\left(1-\frac{\kappa}{3\mu F_N}u_0\right)$$

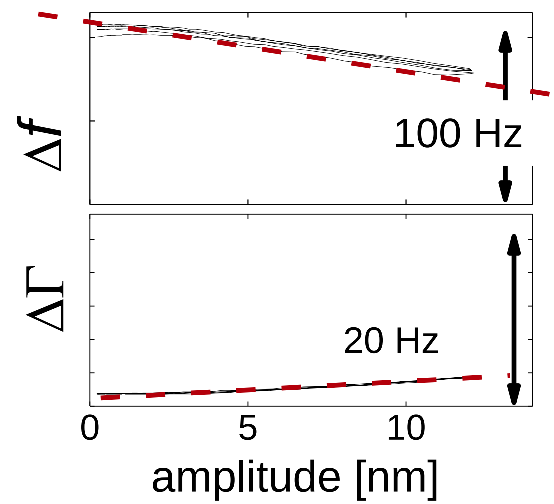
$$\Delta\Gamma(u_0) \approx \frac{N}{A2n\pi^2 Z_q}\kappa\frac{4}{9\pi}\frac{\kappa}{\mu F_N}u_0$$

Savkoor

$$\Delta f(u_0) \approx \frac{N}{A2n\pi^2 Z_q}\kappa\left(1-\frac{5}{8}\left(\frac{\kappa u_0}{4\tau_0 a^2}\right)^2\right)$$

$$\Delta\Gamma(u_0) \approx \frac{N}{A2n\pi^2 Z_q}\kappa\frac{8}{6\pi}\left(\frac{\kappa u_0}{2\tau_0 a^2}\right)^2$$

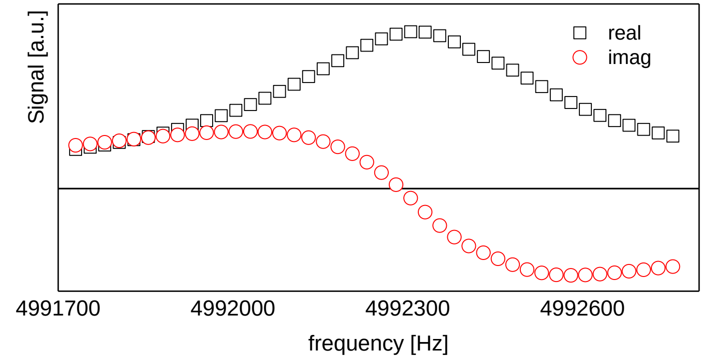




Also: Leopoldes, J.; Jia, X., *PRL* ***2010***

Multifrequency Lockin Analysis

Intermodulation Products AB, Sweden



A) **Frequency combs**

A fast (milliseconds) way to probe resonances

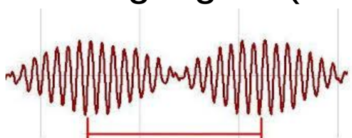
42 freqs, fit Lorentzians $\rightarrow \Delta f, \Delta\Gamma$

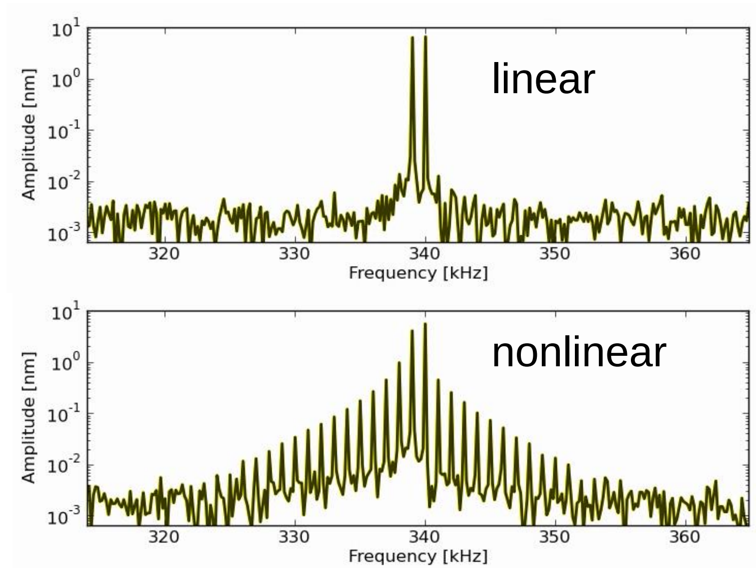
(no calibration required)

B) **Intermodulation products**

Excite with 2 frequencies

$\rightarrow$ beating signal (fast amplitude ramps)





Nonlinearities $\rightarrow$ signals at $f = 2f_1 - f_2$ and $2f_2 - f_1$

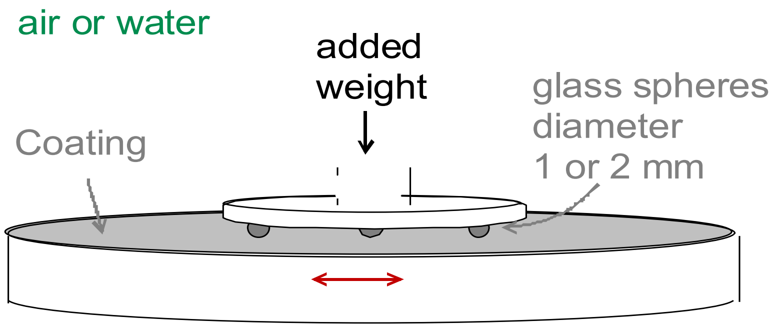
- Fast
- Signal resonantly enhanced, response function well understood

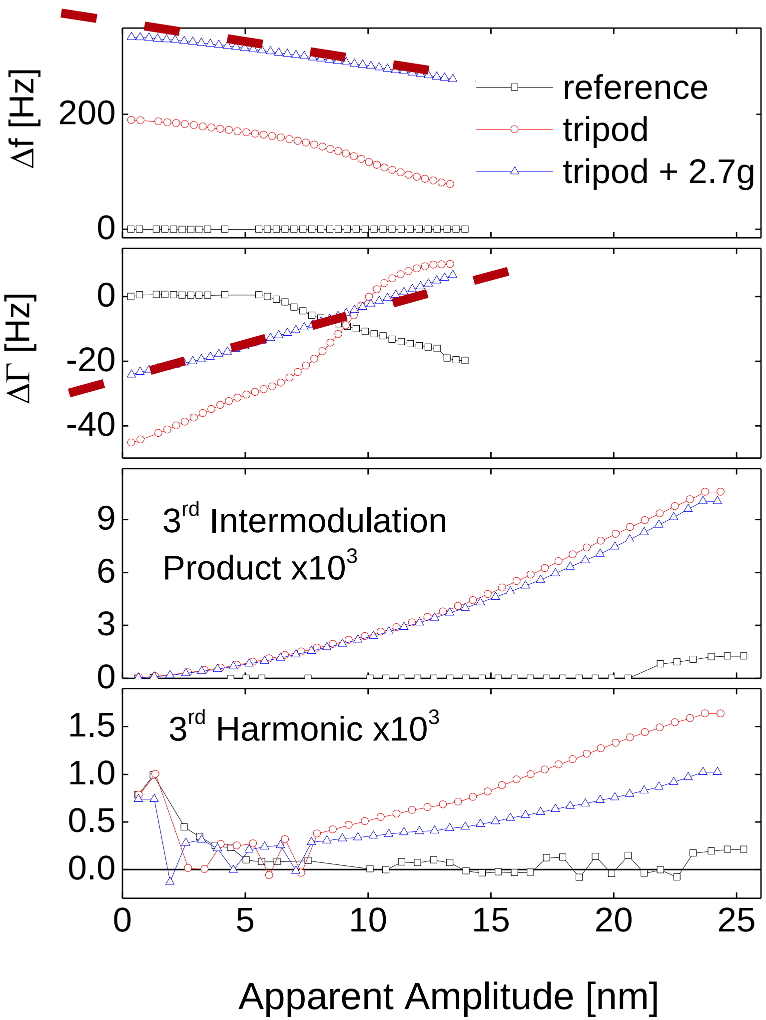
  C. Hutter *et al.* Phys. Rev. Lett. **2010**, 104, 050801
- Calibration issues

C) **2^{nd} and 3^{rd} Harmonic Generation**

- Excite at f, probe at 2f, 3f, etc… (also: determine background)
- Signal *not* resonantly enhanced
- Probes MHz dynamics

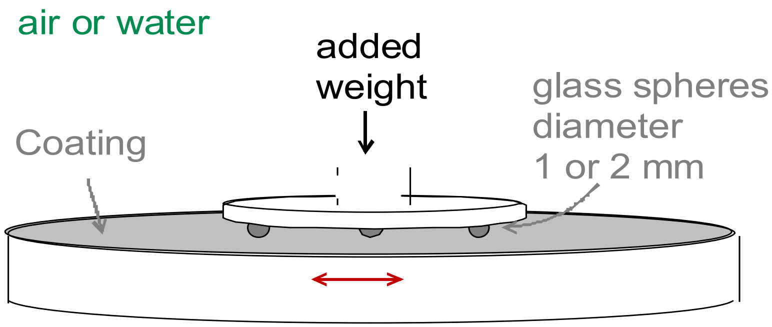
Partial Slip

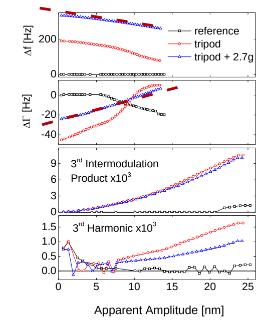




- Mindlin model works here (does not always work)
- There *is* a 3rd harmonic signal
- There *is* a weak 2nd harmonic signal (normal forces involved)
- Here: 5 MHz (fundamental mode) results are different on overtones
- Strong intermodulation products

Partial Slip



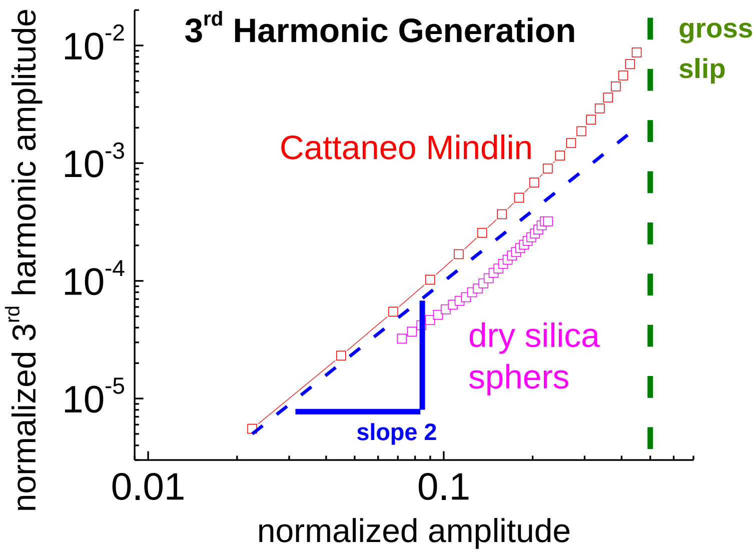


- Mindlin model works here
  (does not always work)

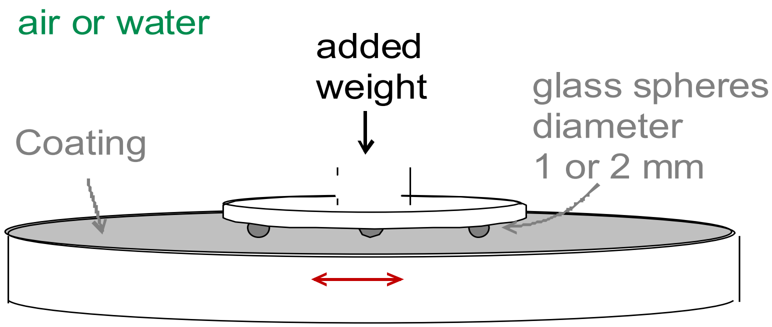
- Red: Mindin-Theory

- Calibration issues

- Partial slip is at least partially transient



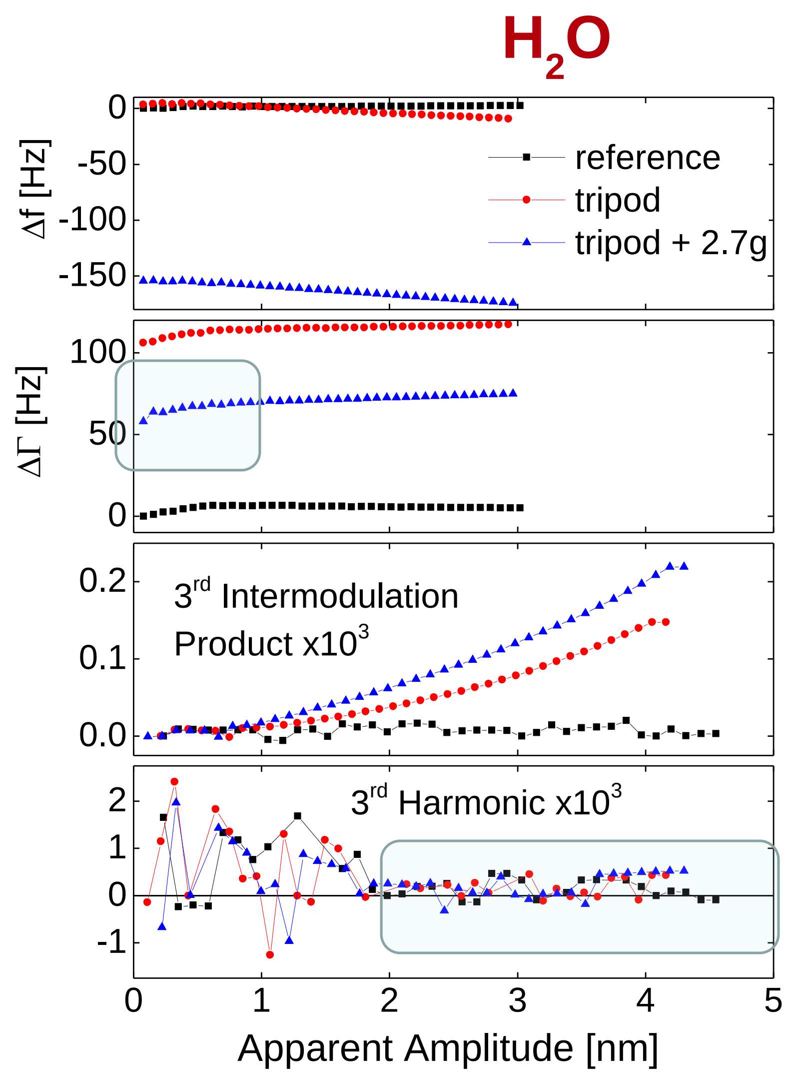
Partial Slip in Liquids



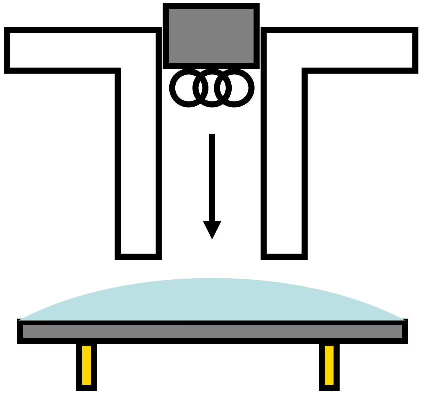
– Rich phenomenology at small amplitudes

– No 3rd harmonic generation

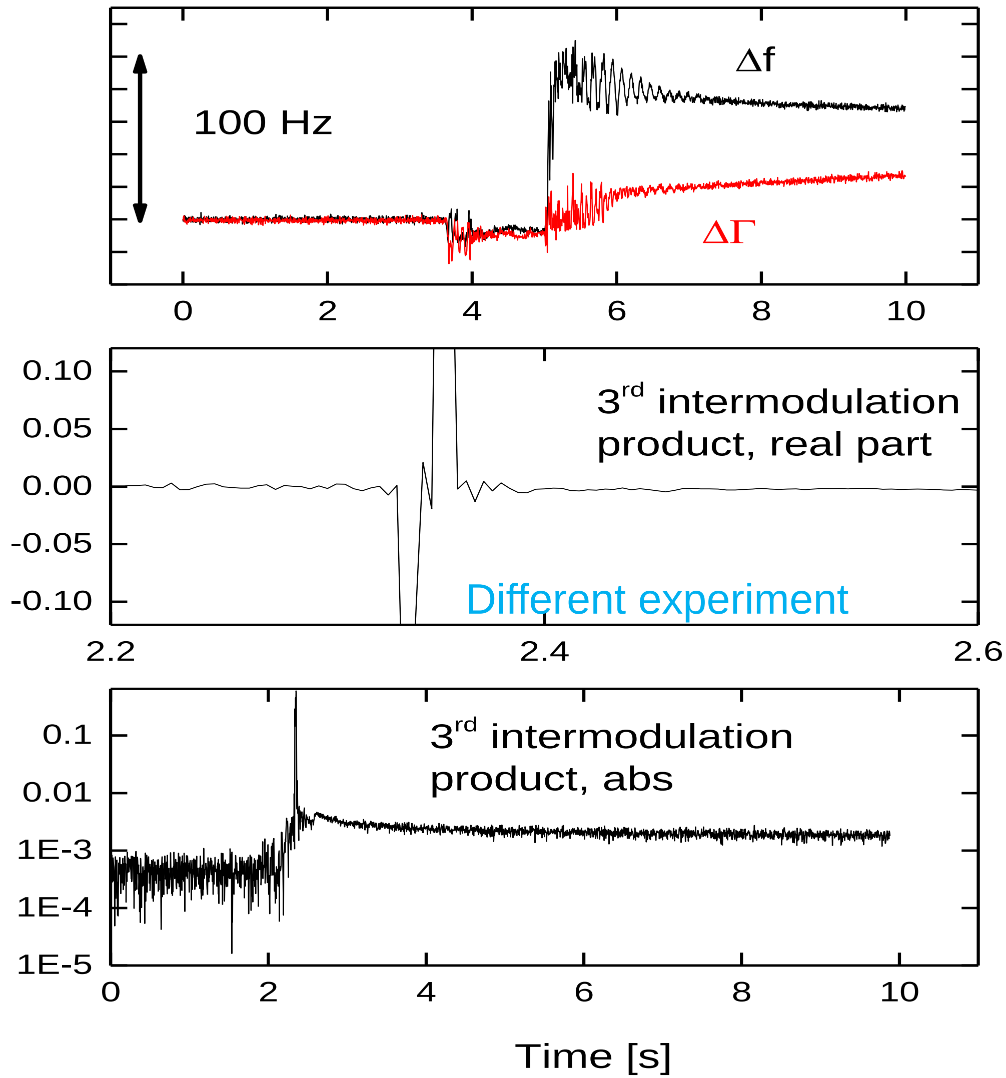
– Intermodulation products seen



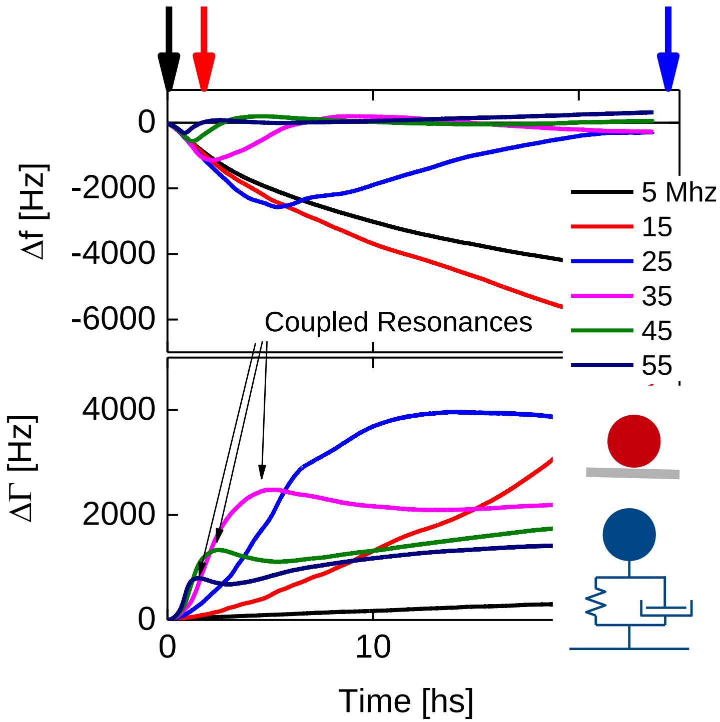
Sudden Impacts



- Transient friction events accessible
- Nonlinearities only *at* impact

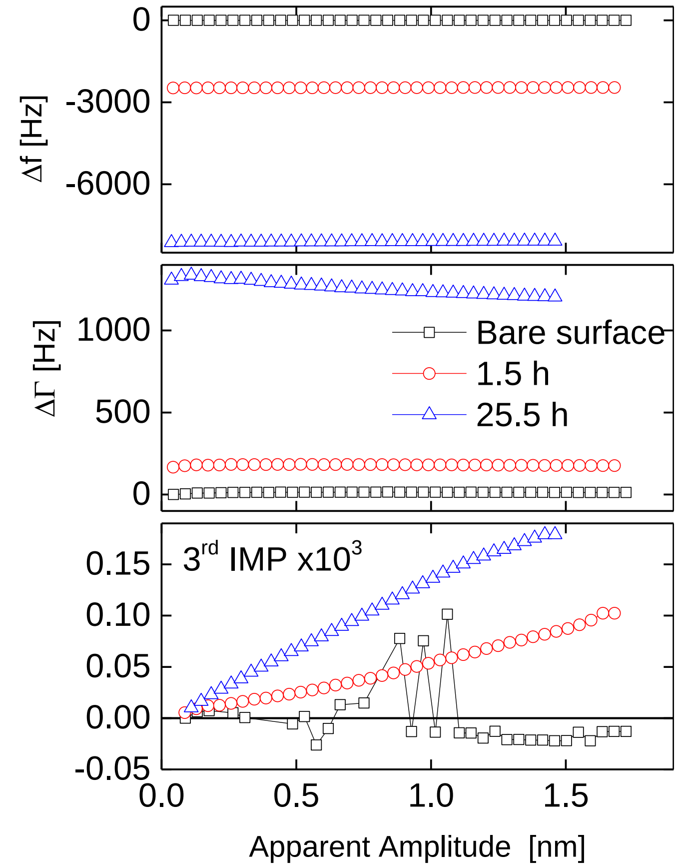


Crystallization



- CaCO$_3$(aq), T = 23°C
  thiolated gold surface, quiescent solution
  initial supersaturation S = 1.5

- Conventional QCM: Coupled Resonances



Multifrequency Lockin Amplifier
(separate experiment)

- Δf(u), ΔΓ (u) do *not* fit Mindlin model
- no 3rd harmonic generation
- Intermodulation products seen

Conclusions

- Fast QCM measurements (Δt = 4 ms)
- 3rd harmonic generation: Partial slip *is* a transient event.
- Intermodulation products probe nonlinear mechanics.
- Impact of particles on QCM surface can be monitored. Nonlinearities are most pronounced during first impact.
- $CaCO_3$ nanoparticles (crystals) formed as individual objects Undergo rocking motion with nonlinear response.