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NONLINEAR STOCHASTIC MODEL OF STICK-SLIP FRICTION DUE TO ICE SURFACE SOFTENING

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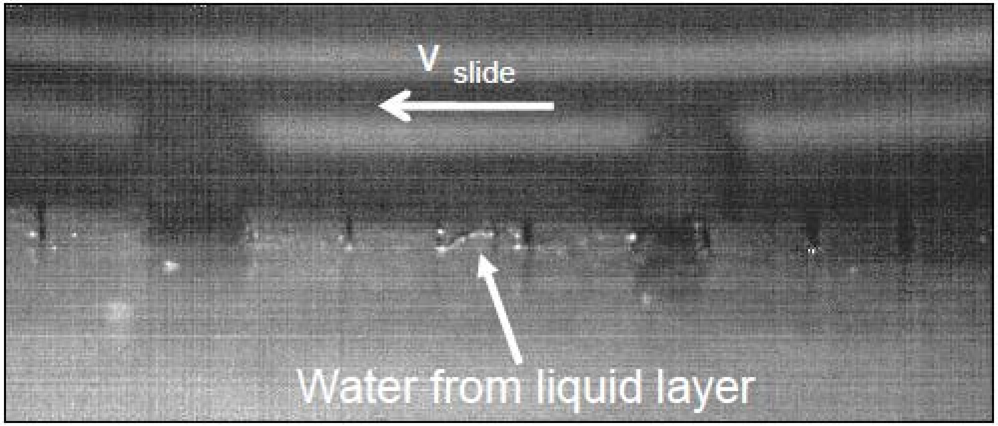
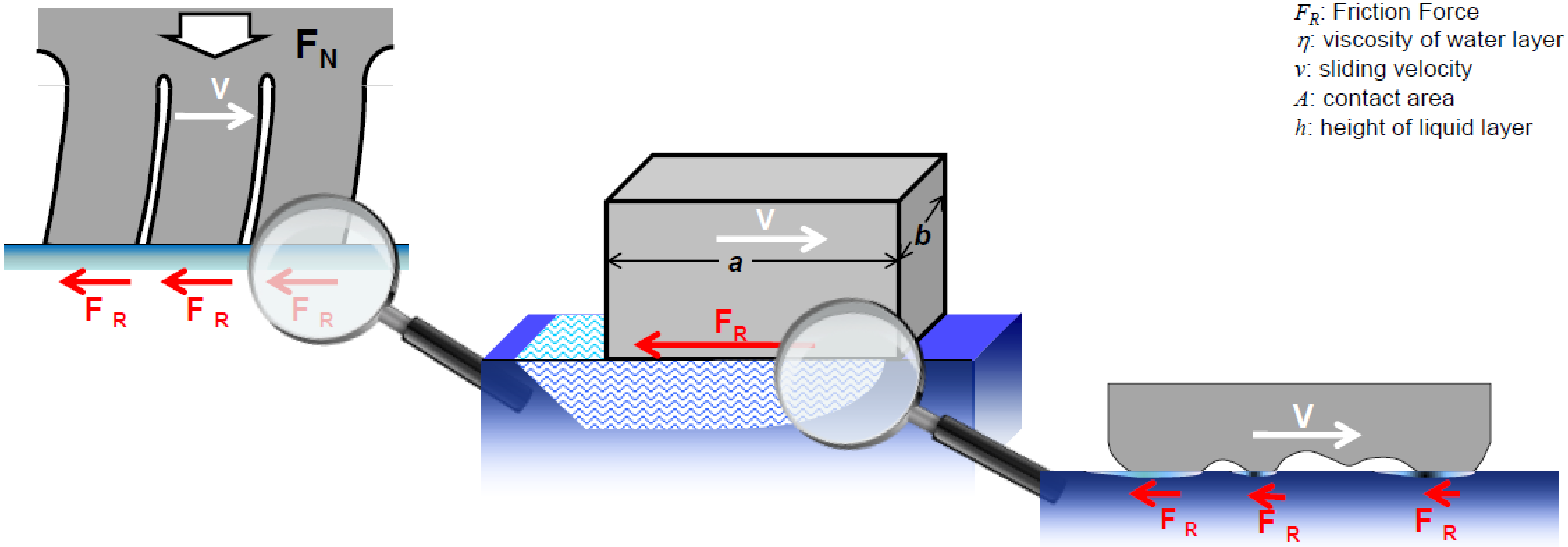
"Trends in Nanotribology 2017", ICTP, Trieste

Liquid layer between tread and ice

T. Kessel, R. Mundl, B. Wies, K. Wiese, Tire Society Meeting, September 2011



- **Viscous friction in the liquid layer**
- **Ice melting as a result of heating at friction**



F_R: Friction Force
η: viscosity of water layer
v: sliding velocity
A: contact area
h: height of liquid layer

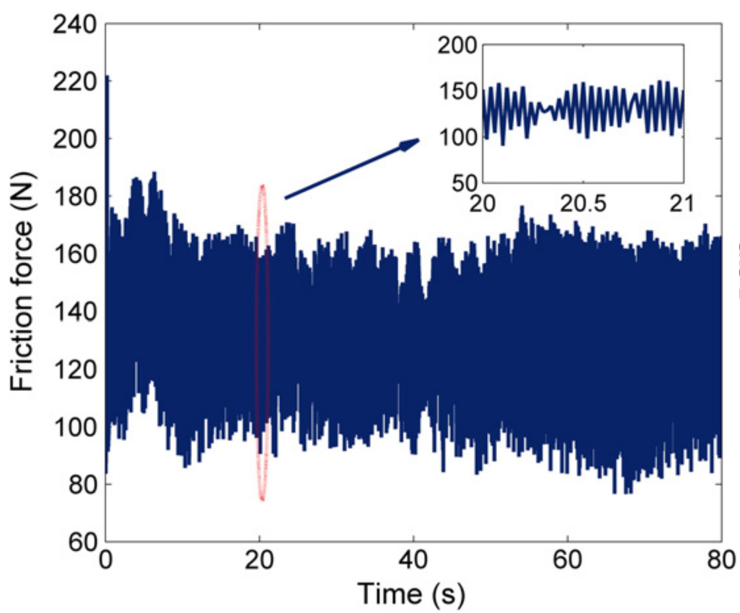
Interrupted friction mode - «stick-slip»

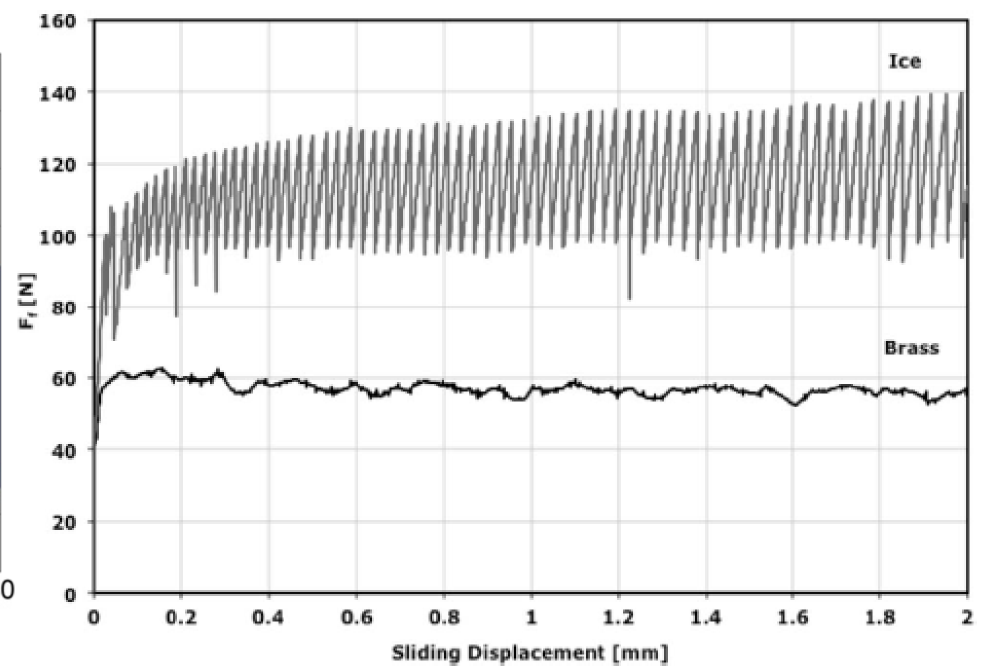
Time dependence of friction force

S. Sukhorukov, S. Løset, Cold Regions Science and Technology. – 2013. – Vol. 94. – P. 1–12.

Dependence of shear force on displacement

E.M.Schulson, A.L.Fortt, Journal of Geophysical Research: Solid Earth. – 2012. – Vol.117. – P. B12204.





Basic equations

Kelvin-Voigt equation for shear strain:

$$\dot{\varepsilon} = -\varepsilon / \tau_\varepsilon + \sigma / \eta_\varepsilon \quad (1)$$

τ_ε is the relaxation time

η_ε is the effective shear viscosity

In stationary case (1) is reduced to the Hooke-type relationship:

$$\dot{\varepsilon} = 0 \Rightarrow \sigma = G_\varepsilon \varepsilon, \ G_\varepsilon = \eta_\varepsilon / \tau_\varepsilon = G(\omega)\big|_{\omega \to 0}$$

Landau-Khalatnikov-type equation for shear stress:

$$\tau_\sigma \dot{\sigma} = -\sigma + G(T)\varepsilon \quad (2)$$

$\tau_\sigma = \eta / G(T)$ is the relaxation time for shear stress

$$\dot{\sigma} = 0 \Rightarrow \sigma = G(T)\varepsilon, \ G(T) = G(\omega)\big|_{\omega \to \infty}$$

Temperature dependence of shear modulus:

$$G(T) = G_0(T / T_c - 1) \quad (3)$$

Kinetic equation for temperature

Continuity equation for the heat $Q = T\delta S$:

$$T\dot{S} = -\nabla \mathbf{q} \quad (4)$$

Onsager equation for heat current:

$$\mathbf{q} = -\kappa \nabla T \quad (5)$$

Entropy in case of thermoelastic stress:

$$S = S_0(T) + K\alpha\varepsilon^0 \quad (6)$$

Relative volume expansion (dilatation):

$$\hat{\varepsilon}^0 = \varepsilon^0 \hat{I}, \quad \varepsilon^0 \equiv \alpha(T - T_0) \quad (7)$$

Transfer from the dilatational component $K\alpha\varepsilon^0$ to the elastic energy - $\sigma\varepsilon/T$ of the shear component:

$$(4) \;\Rightarrow\; T\dot{S}_0(T) - \sigma\dot{\varepsilon} = \kappa\nabla^2 T \quad (8)$$

Basic equations

Final expression for T: $(\kappa / l^2)(\tau_T Q - T) \approx \kappa \nabla^2 T, \quad c_p = T dS_0 / dT \Rightarrow (8) \Rightarrow$

$$\tau_T \dot{T} = (\tau_T Q - T) - \frac{l^2 \sigma\varepsilon}{\kappa \tau_\varepsilon}, \qquad Q = Q_0 + \sigma^2 / c_p \eta_\varepsilon, \quad T_e = \tau_T Q \qquad (9)$$

Basic equations (1), (2) and (9):

$$\tau_\varepsilon \dot{\varepsilon} = -\varepsilon + \sigma \qquad (10)$$

$$\tau_\sigma \dot{\sigma} = -\sigma + g(T-1)\varepsilon \qquad (11)$$

$$\tau_T \dot{T} = (\tau_T Q - T) - \sigma\varepsilon \qquad (12)$$

$$g = \frac{G_0}{G_\varepsilon} \qquad (13)$$

Measure units for σ, ε, T ($\tau_T = l^2 c_p / \kappa$ is the time of heat conductivity):

$$\sigma_s = (c_p \eta_\varepsilon T_c / \tau_T)^{1/2} \qquad \varepsilon_s = \sigma_s / G_\varepsilon \qquad T_c$$

Continuous transition

Adiabatic approximation:

$$\tau_\sigma << \tau_\varepsilon, \quad \tau_T << \tau_\varepsilon \quad (14)$$

$\tau_{\varepsilon\,\min} \approx 2\cdot10^{-5}$ s, $\tau_\sigma \approx a/c \sim 10^{-12}$ s, $a \sim 1$ nm, $c \sim 10^3$ m/s

For ice: $\rho \approx 916$ kg/m^3, $\kappa \approx 2.22$ W/m$\times$K, $c_p \approx 2050$ J/kg$\times$K, $G_\varepsilon \approx 10$ GPa and water at $T = 0^\circ$C: $\eta_\varepsilon \approx 1.8\times10^{-3}$ Pa$\times$s, the second ineq. (14) $\Rightarrow l << L \Rightarrow$

The maximal distance into which heat penetrates ice:

$$L = \sqrt{\frac{\chi \nu_\varepsilon}{c_\varepsilon^2}} \approx 10 \text{ nm} \quad (15)$$

Landau-Khalatnikov equation: $\tau_\varepsilon \dot{\varepsilon} = -\partial V / \partial \varepsilon \quad (16)$

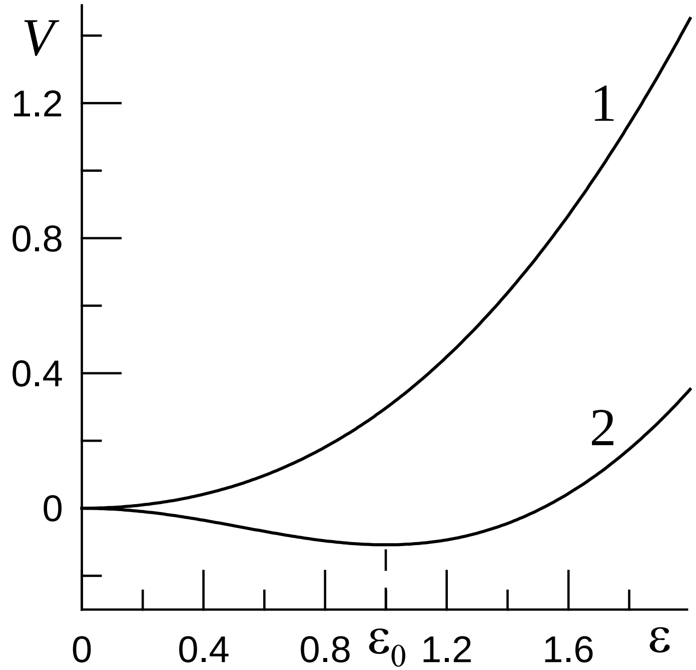
Synergetic potential: $V = \frac{1}{2}\left[\varepsilon^2 + (1 - T_e)\ln(1 + g\varepsilon^2)\right] \quad (17)$

Critical temperature: $T_{c0} = 1 + g^{-1},\ g \equiv G_0 / G_\varepsilon < 1,\ G_\varepsilon \equiv \eta_\varepsilon / \tau_\varepsilon \quad (18)$

Steady-state strain: $\varepsilon_0 = (T_e - (1 + g^{-1}))^{1/2} \quad (19)$

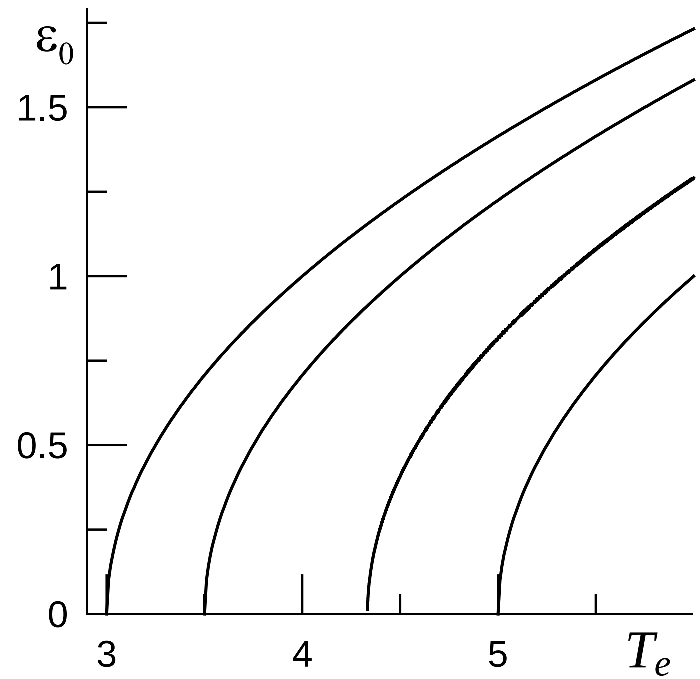
Continuous transition

Synergetic potential



1 - $T_e = 2 < T_{c0}$, 2 - $T_e = 4 > T_{c0}$

Steady-state strain



$g = 0.25, 0.3, 0.4, 0.5$
for curves from right to left

Influence of deformational defect of shear modulus

Simplest approximation:

$$G_\varepsilon(\varepsilon) = \frac{G_\varepsilon}{1+\varepsilon/\varepsilon_p} \qquad (20)$$

Synergetic potential:

$$V = \frac{1}{2}\varepsilon^2 + (1-T_e)\left\{\frac{1}{2}\ln\left(1+g\varepsilon^2\right) + \frac{1}{\alpha}\left[\varepsilon - \frac{1}{\sqrt{g}}\arctan\left(\sqrt{g}\varepsilon\right)\right]\right\} \qquad (21)$$

Steady-state values of strain:

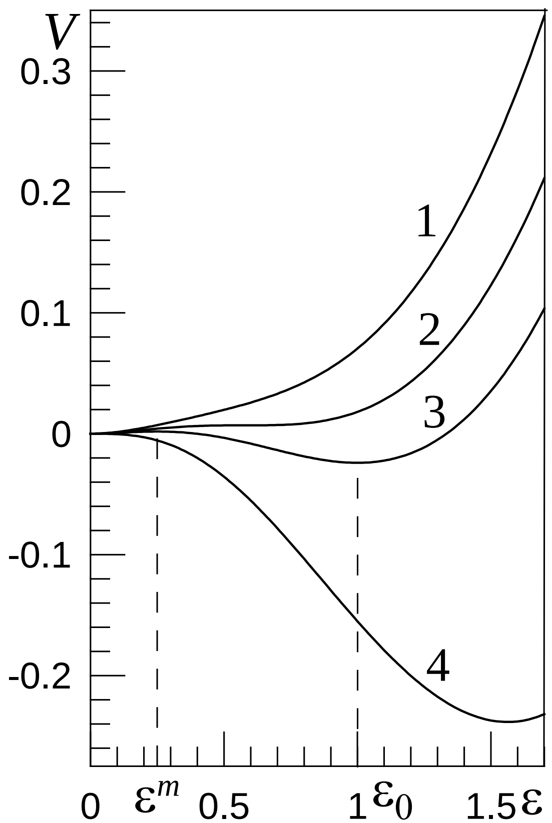
$$(\varepsilon_0^m)^2 = (2\alpha)^{-1}\left\{(T_e-1) \mp \sqrt{(T_e-1)^2 - 4g^{-1}\alpha^2\left[1-g(T_e-1)\right]}\right\} \qquad (22)$$

Temperature of absolute instability of the softened ice layer (plateau appearing):

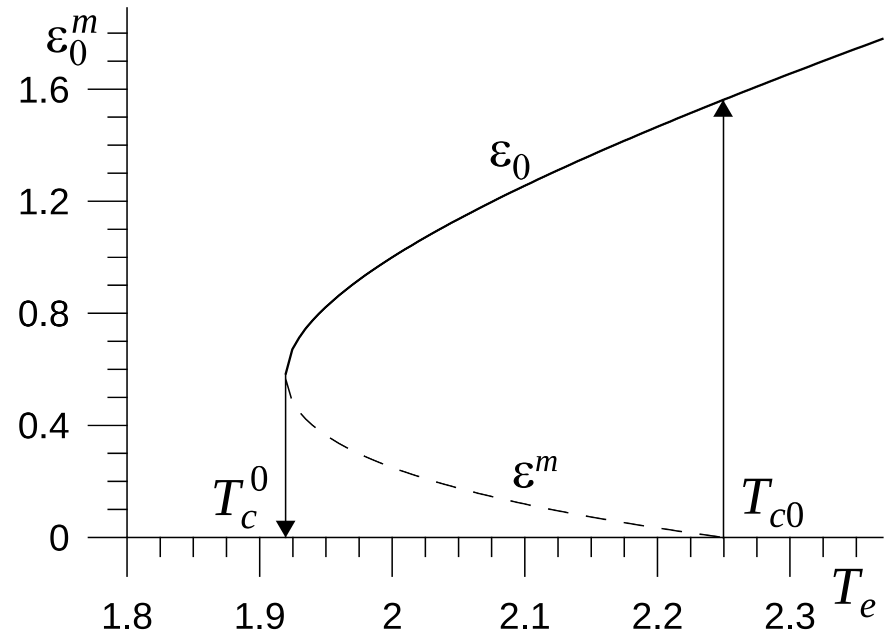
$$T_c^0 = 1 - 2\alpha^2 + 2\alpha\sqrt{\alpha^2 + g^{-1}} \qquad (23)$$

Influence of deformational defect of shear modulus

Synergetic potential



Steady-state strain



1 - $T_e<T_c^{\,0}$; 2 - $T_e= T_c^{\,0}$; 3 - $T_c^{\,0}<T_e<T_{c0}$; 4 - $T_e\geq T_{c0}$

Noise effect

$$\tau_\varepsilon \dot{\varepsilon} = -\varepsilon + \sigma + \sqrt{I_\varepsilon}\,\xi_1(t) \qquad (24)$$

$$\tau_\sigma \dot{\sigma} = -\sigma + g(T-1)\varepsilon + \sqrt{I_\sigma}\,\xi_2(t) \qquad (25)$$

$$\tau_T \dot{T} = \left(\tau_T Q - T\right) - \sigma\varepsilon + \sqrt{I_T}\,\xi_3(t) \qquad (26)$$

Langevin equation $(\tau_\varepsilon >> \tau_\sigma, \tau_T)$:

$$\tau_\sigma \dot{\varepsilon} = f(\varepsilon) + \sqrt{I(\varepsilon)}\,\xi\left(t\right), \quad f \equiv -\frac{\partial V}{\partial \varepsilon} \qquad (27)$$

Synergetic potential: $$V = \frac{1}{2}\left[\varepsilon^2 + (1-T_e)\ln(1+g\varepsilon^2)\right] \qquad (28)$$

$\xi(t)$ is the δ - correlated Gaussian stochastic source (white noise):

$$\left\langle \xi_i(t) \right\rangle = 0, \quad \left\langle \xi_i(t)\xi_j(t') \right\rangle = 2D\delta_{ij}\delta(t-t') \qquad (29)$$

Effective potential

Fokker-Planck equation:

$$\frac{\partial P(\varepsilon,t)}{\partial t} = -\frac{\partial}{\partial \varepsilon}[f(\varepsilon)P(\varepsilon,t)] + D\frac{\partial}{\partial \varepsilon}\left[\sqrt{I(\varepsilon)}\frac{\partial}{\partial \varepsilon}\left(\sqrt{I(\varepsilon)}P(\varepsilon,t)\right)\right] \quad (30)$$

Stationary distribution:

$$P(\varepsilon) = Z^{-1}\exp\left\{-U(\varepsilon)\right\} \quad (31)$$

Effective potential:

$$U(\varepsilon) = \frac{1}{2}\ln I(\varepsilon) - \frac{1}{D}\int_0^{\varepsilon}\frac{f(\varepsilon')}{I(\varepsilon')}d\varepsilon', \qquad f \equiv -\frac{\partial V}{\partial \varepsilon} \quad (32)$$

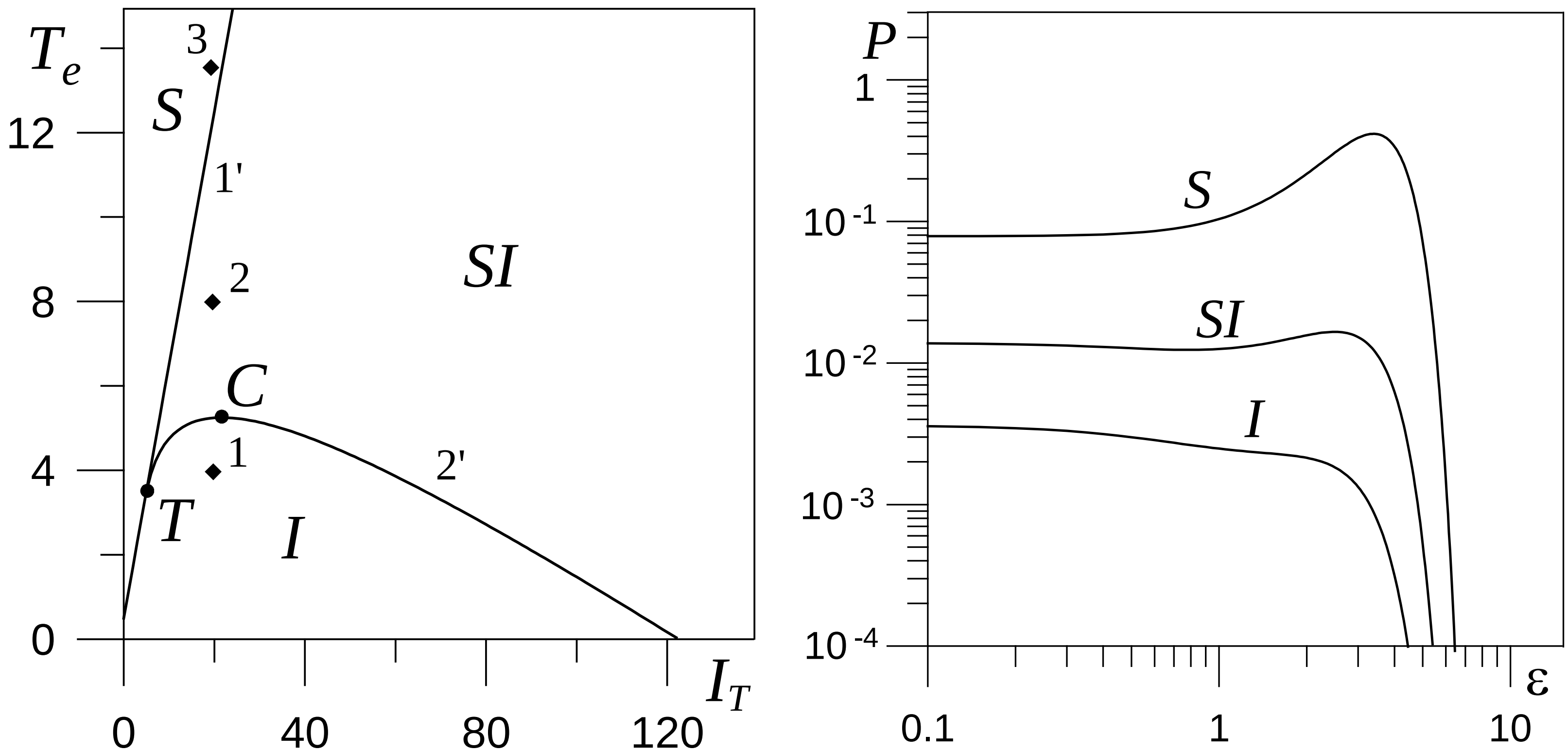
Extremums of distribution function (31) (potential (32)):

$$x^3 + g(1-T_e)x^2 - Dg^2 I_T x + 2Dg(gI_T - I_\sigma) = 0, \quad x \equiv 1 + g\varepsilon^2 \quad (33)$$

Phase diagram

Existence boundary of ice domain *I* (zero solution):

$$T^c = 1 + g^{-1} + D(gI_T - 2I_\sigma) \quad (34)$$



Time series of friction force

Euler method: $\varepsilon_{n+1} = \varepsilon_n + \left(f(\varepsilon_n) + D\sqrt{I(\varepsilon_n)} \frac{\partial}{\partial \varepsilon} \sqrt{I(\varepsilon_n)} \right) \Delta t + \sqrt{I(\varepsilon_n) \Delta t} W_n$ (35)

Box-Muller model: $W_n = \sqrt{\mu^2} \sqrt{-2 \ln r_1} \cos(2\pi r_2),\ r_n \in (0,1]$ (36)

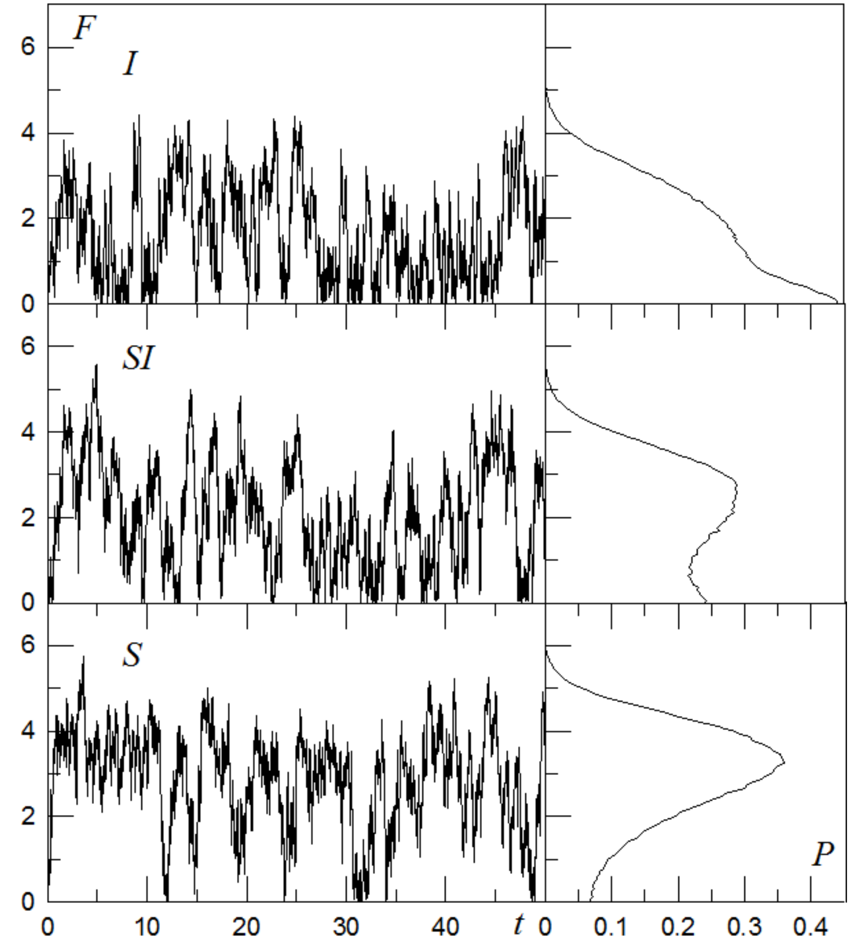
$\langle W_n \rangle = 0,\ \langle W_n W_{n'} \rangle = 0,\ \langle W_n^2 \rangle \to 2D$ (37)

For friction of PMMA, rubber, steel and ice on ice: $A \approx 10^{-6} - 10^{-1}\ \mathrm{m}^2$

Yield strength: $G_\varepsilon \approx 0.1 - 10$ MPa

Relaxation time: $\tau_\varepsilon \approx 0.1 - 5$ s

Friction force: $F(t) = AG_\varepsilon |\varepsilon|(t)$ (38)



Conclusions

1. This consideration shows that the ice surface softening during friction is conditioned by the self-organization of the strain and stress shear components, on the one hand, and the layer temperature, on the other hand. At this strain ε is the order parameter, stress σ plays the role of the conjugate field, and temperature T acts as the control parameter. The positive feedback of T and ε on σ leads to self-organization. The temperature dependence of shear modulus in equations (2) and (3) has a crucial role.

2. The assumption about shear modulus vs strain dependence allows us to acquire the relationships for the temperatures of absolute instability of the softened ice layer T_c^0 (23) and stability limit of the solid ice T_{c0} (18).

3. The real temperature of transition, laying in the (T_c^0, T_{c0}) range, can be extracted from the equality $V(0) = V(\varepsilon_0)$ of potentials in various phases. The analysis of equation (18) demonstrates that softening begins earlier in the systems with large typical G_0 and minor relaxed G_ε values of shear modulus.

4. The above research of the thermal and deformational fluctuations effect on the softening of ice surface film allows us to build the phase diagram with the domains of ice, softened ice and inhomogeneous ice surface. The increase of the ice film's temperature noise can ascent or descent the friction dependently on the initial conditions, but the increase of shear stress noise results only in growth of the softening domain. The ice friction domain is bounded by comparatively low values of the background sliding block temperature and fluctuations intensities of ice surface stress and temperature.

5. Numerical solution of the Langevin equation permits to calculate the friction force time series for each friction mode. Their comparison with experimental dependencies is carried out.

Thank you for attention!