Statistical Mechanics of Branched Polymers

Physical Virology
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My question to an anonymous referee: why do you think this is a perpetual motion machine?
To view RNA as a branched polymer is an imperfect approximation, with limited applicability ... but let us explore it.
What is known about branched polymers? Classics 1:

The Dimensions of Chain Molecules Containing Branches and Rings

Bruno H. Zimm
Department of Chemistry, University of California, Berkeley, California

Walter H. Stockmayer
Department of Chemistry, Massachusetts Institute of Technology, Cambridge, Massachusetts
(Received May 2, 1949)

Main result: gyration radius

$R_g \sim N^{1/4}$

Excluded volume becomes very important, because

$N/R_g^3 \sim N^{1/4}$ grows with $N$
"Chemical diameter"
other people call it MLD

Zimm-Stockmayer result:
“typical” chemical diameter
\( \sim N^{1/2} \)

Then its size in space is
\( (N^{1/2})^{1/2} = N^{1/4} \)

Interesting: similar measures used to characterize dendritic arbors as well as vasculature in a human body or in a plant leave...
Two ways to swell

Always possible
Quenched

May or may not be possible
Annealed

Diagram a

Diagram b

Diagram c
Flory theory: old work and recent review

*Macromolecules 1993, 26, 1293–1295*

Polymers with Annealed and Quenched Branchings Belong to Different Universality Classes

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Received September 2, 1992; Revised Manuscript Received November 27, 1992

Soft Matter

**Flory theory of randomly branched polymers**

Ralf Everaers,a Alexander Y. Grosberg,b Michael Rubinsteinc and Angelo Rosad
Flory theory

\[ R \sim N^\nu \]
\[ L \sim N^\rho \]
\[ R \sim L^{\nu_{\text{path}}} \]

\[ \nu_{\text{path}} = \nu / \rho \]

\[ \mathcal{F} = \mathcal{F}_{\text{el}} + \mathcal{F}_{\text{inter}} \]

\[ \mathcal{F}_{\text{el}} \sim \frac{R^2}{k_B T} \frac{1}{l_K L} \]

\[ \mathcal{F}_{\text{el}} \sim \frac{R^2}{k_B T} \frac{1}{l_K L} + \frac{L^2}{Nl_K^2} \]

Kramers matrix comment

Even less trivial

Highly nontrivial

Quenched

Annealed

J. Kelly developed algorithm to compute the right \( L \)

\[ \rho = \frac{1 + 2\nu}{3} \]
**Kramers theorem**

Quenched

\[
\frac{F_{\text{el}}}{k_B T} \sim \frac{R^2}{l_K L}
\]

J. Kelly developed algorithm to compute the right L

\[Q_{km} = \pm K(k)M(m)\]

\[\text{Trace}(Q_{km}) = R_g^2\] (see Ben Shaul lecture yesterday)

Maximal eigenvalue of \(Q_{km}\) gives the right L

Hendrik (Hans) Kramers (1894-1952)
Prediction works reasonably well

\[ \rho = \frac{1 + 2\nu}{3} \]
Other predictions are also OK

Green – single macromolecule in a good solvent; Magenta – single macromolecule in Q-solvent; Blue – one macromolecule among similar ones in a melt.

\[ \rho \]

\[ L \sim N^\rho \]
\[ R \sim N^v \sim L^\nu_{\text{path}} \]
Flory theory can be useful
Observation: viral RNA seem to be “compact”

Viral RNA (Qβ, ground state found by Vienna algorithm)

Randomly reshuffled RNA (typical representative)

Question: how does the work of compression depend on the sequence?

Tricky question, because RNA has to be much compressed (~1bp/nm³), so all interactions are hugely important

A.M.Yoffe, ..., A.Ben-Shaul "Predicting the sizes of large RNA molecules", PNAS, v. 105, 16153-16158, 2008
Work of compression

Our main idea:

$$\mathcal{F} = \mathcal{F}_{el} + \mathcal{F}_{inter}$$

For interactions part, we can use, any of the classical interpolation formulas, e.g., van der Waals or Flory-Huggins
Work of compression

Randomly shuffled

Dashed lines Flory-Huggins,
solid lines van der Waals
Differential work of compression: simplified formula

\[ \Delta \Delta W \equiv [\Delta W]_{\text{molecule } i} - [\Delta W]_{\text{molecule } j} = 9.1 \left( \frac{N^2}{a^3} \right)^{2/5} \left[ \frac{1}{\lambda_i^{3/5}} - \frac{1}{\lambda_j^{3/5}} \right] \]
Can we do better?

Hundreds of states within $1k_B T$ of the ground state

... 

Let us go to the opposite limit of annealed branching pattern
Confinement work: de Gennes style scaling argument

Gyration radius without confinement $R_g \sim aN^{1/4}$

Confinement entropy $\Delta S = f(R/R_g) \rightarrow (R/an^{1/4})^x$

Should be extensive, proportional to $N \rightarrow \Delta S \sim N(a/R)^4$

Different from uncertainty principle
A primer on Schrödinger equation method for a linear chain

Uncertainty principle:
To confine particle within R you need to provide kinetic energy $(\hbar/R)^2/m$

To confine polymer within R you need to provide entropy $(a/R)^2$

Interesting: another version of quantum mechanics
What is known about branched polymers? classics 2:

Statistics of Branching and Hairpin Helices for the dAT Copolymer

P.-G. de GENNES, Laboratoire de Physique des Solides, Laboratoire associé au Centre National de la Recherche Scientifique (CNRS), Faculté des Sciences, Université de Paris, Orsay, France
One step more after de Gennes

Macromolecules 1995, 28, 3718–3727

Conformational Entropy of a Branched Polymer

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\[
(1/p^*) \Phi(r) = \Lambda_1 \hat{g} e^{-\phi_1(r)} + \Lambda_3 \hat{g} \left[ e^{-\phi_3(r)} \Phi^2(r) \right]
\]

\[
(1/p^*) \Psi(r) = 2\Lambda_3 \hat{g} \left[ e^{-\phi_3(r)} \Phi(r) \Psi(r) \right]
\]

How side branches affect the "diameter"
Recent paper in an unusual journal

Low Temperature Physics/Fizika Nizkikh Temperatur, 2017, v. 43, No. 1, pp. 122–131

The confinement of an annealed branched polymer by a potential well

Alexander Y. Grosberg¹, Joshua Kelly², and Robijn Bruinsma²,³

Ilya M. Lifshitz (1917-1982) turns 100 this year
Simplest problem: confinement

Maps on classical mechanics

\[ 2(1 - \omega)\Phi = \hat{g} \left[ e^{\Phi_0 - \Phi} (1 + \Phi^2) \right] \]

Maps on quantum mechanics

\[ (1 - \omega)\Psi = \hat{g} \left[ e^{\Phi_0 - \Phi} \Phi \Psi \right] \]

\[ \hat{g} \approx 1 + \frac{a^2}{2D} \]
Main results for the confinement problem

1D
Confinement free energy is consistent with scaling

3D
“Territories” in influenza virus
(genome consists of 8 RNAs)
Acknowledgements and Conclusions

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