Quantum transport in graphene

L1 Disordered graphene (G)

L2 Ballistic electrons in graphene (G/hBN)

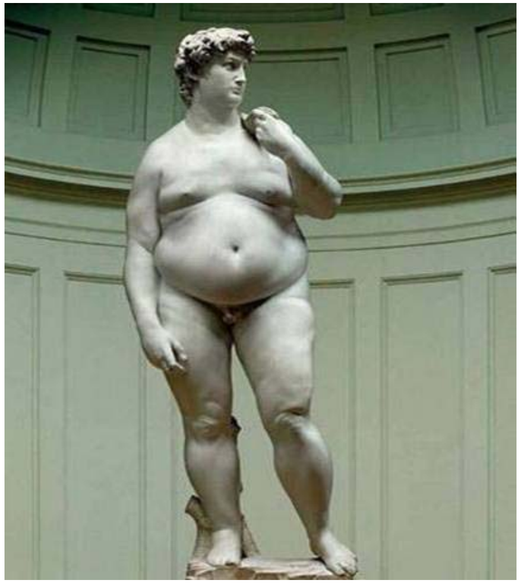
making graphene ballistic

PN junctions and Veselago lens in graphene

Andreev reflection in ballistic SGS devices

Lifshitz transition and QHE in bilayer graphene

L3 Moiré superlattice effects in G/hBN heterostructures

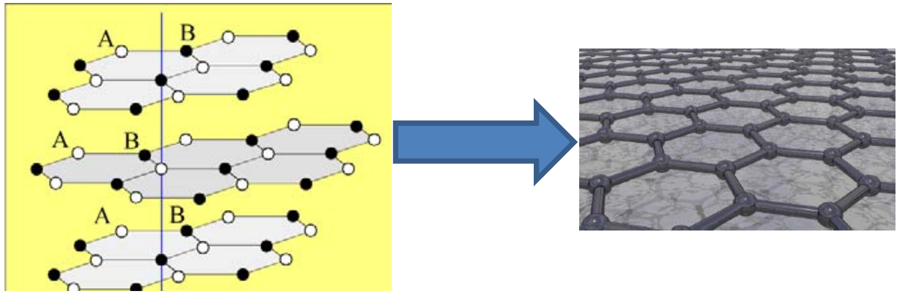


how?

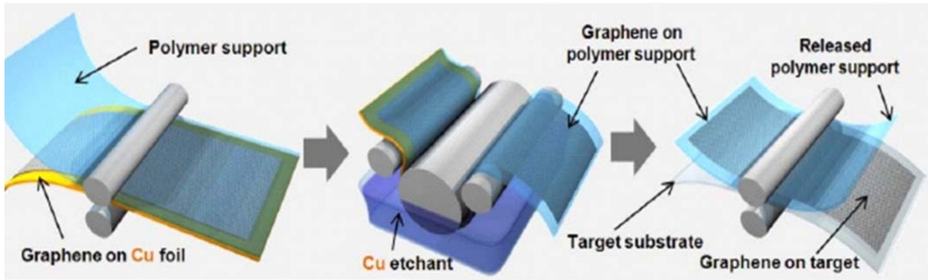
$$\hat{H} = v\vec{p}\cdot\vec{\sigma} + \hat{V}_{disorder}$$

$$\hat{H} = v\vec{p}\cdot\vec{\sigma}$$

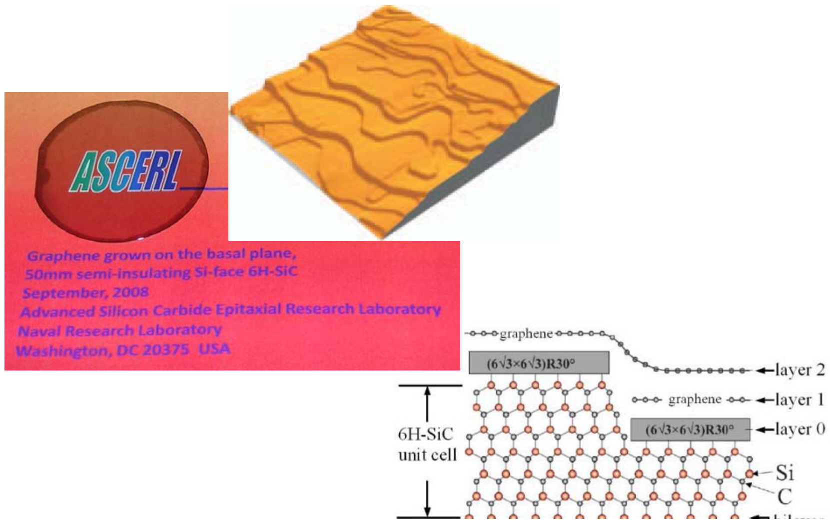
How to get best-quality graphene



Exfoliated from bulk graphite onto a substrate, or hanged suspended



Grown using chemical vapor deposition (CVD) on metals (Cu, Ni), or insulators: polycrystalline and strained $(\tau_{iv}, \tau_* \sim \tau)$

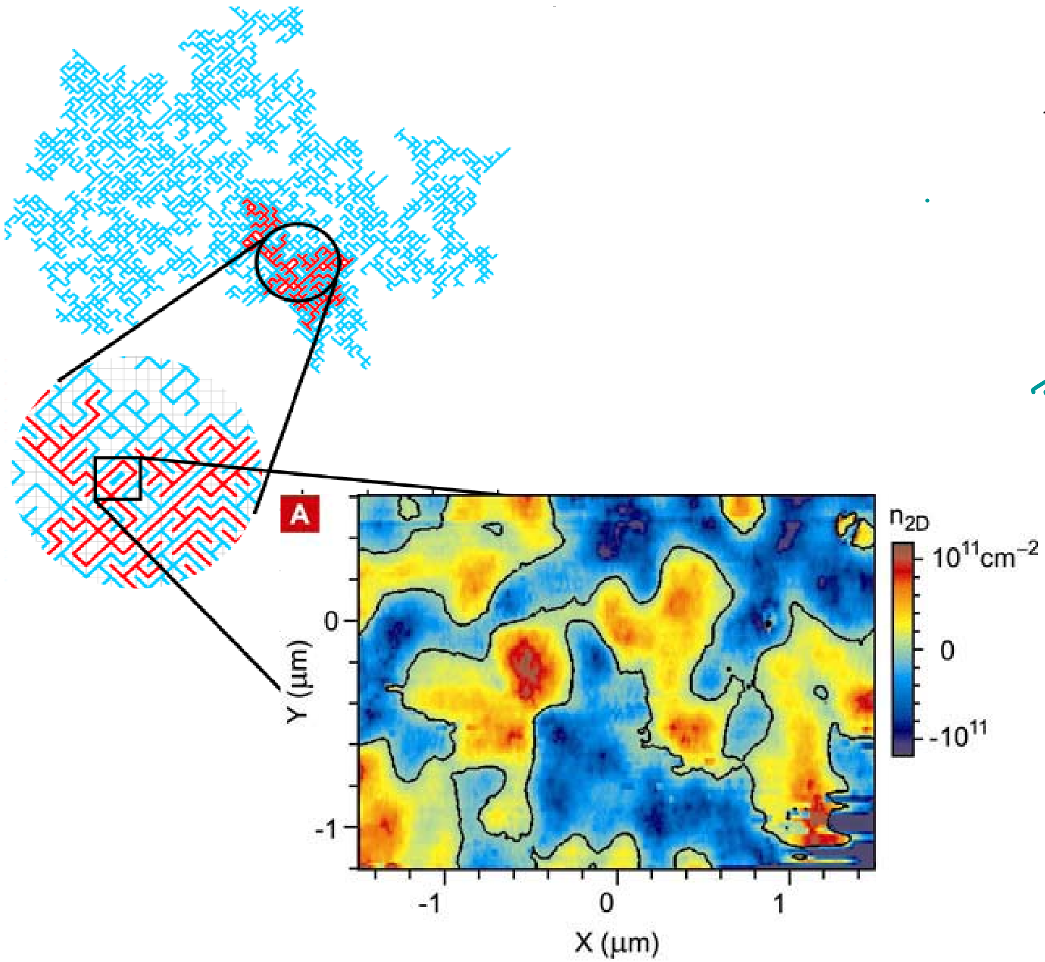


Epitaxial graphene sublimated on Si-terminated surface of SiC: heavily doped by the charge transfer from C-dead layer leaving charge disorder on SiC surface

charge inhomogeneity and electron-hole puddles at ‘n_e=0’

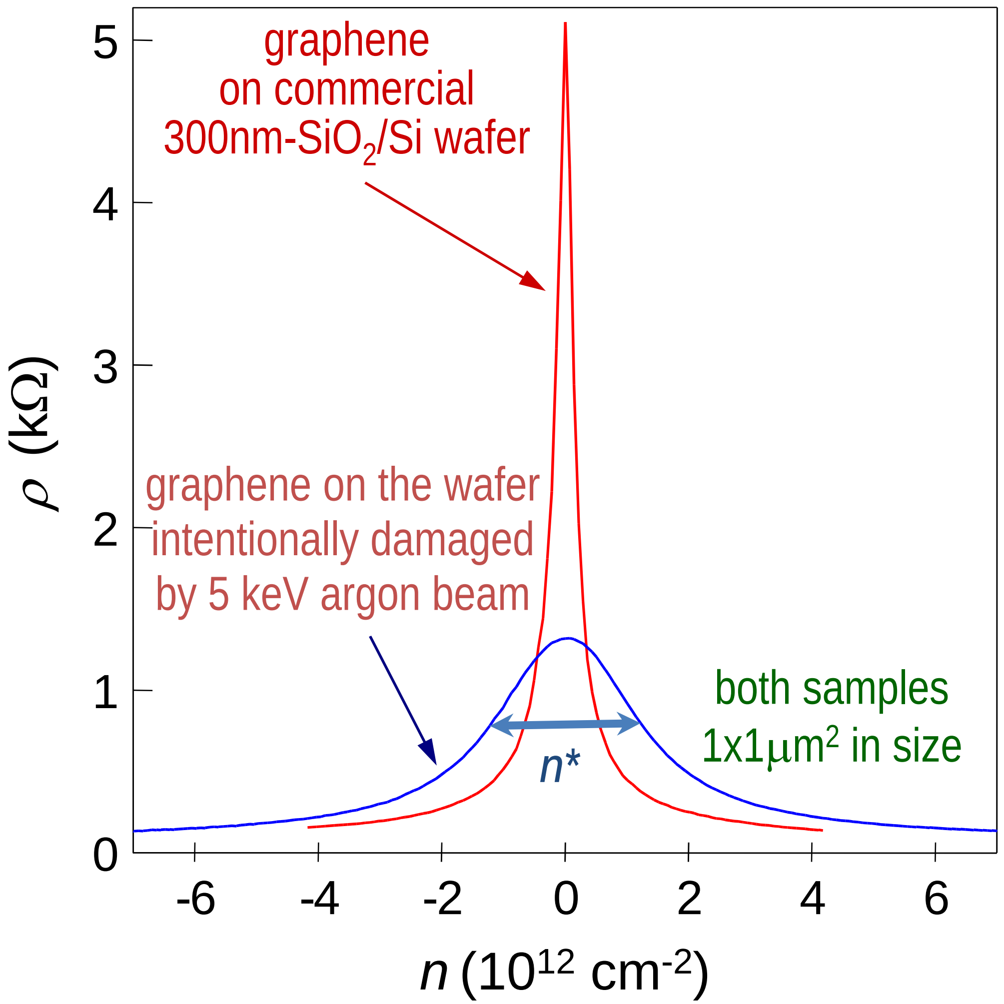
charged impurities in the substrate or deposits on its surface
deformations of graphene due to surface roughness

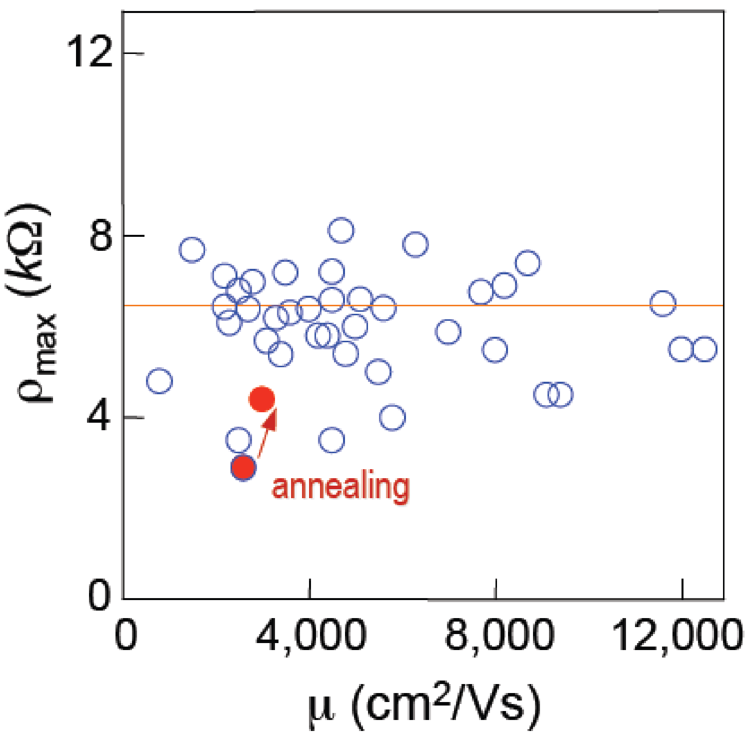
Cheianov, Falko, Altshuler, Aleiner
PRL 99, 176801 (2007)
Adam, Hwang, Galitski, Das Sarma PNAS
104, 18392 (2007)



Martin, Akerman, Ulbricht, Lohmann,
Smet, von Klitzing, Yacoby
Nature Physics 4, 144 (2008)

charge inhomogeneity and electron-hole puddles at ‘n_e=0’



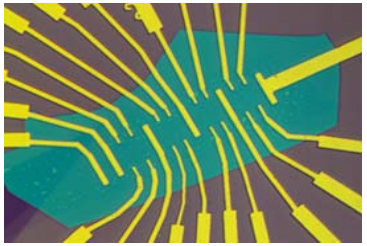


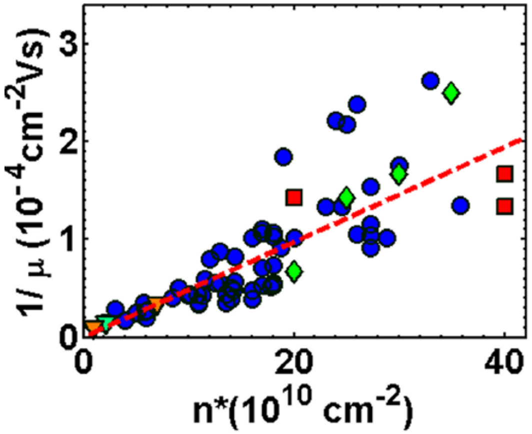
Geim, Novoselov - Nature Materials (2007)

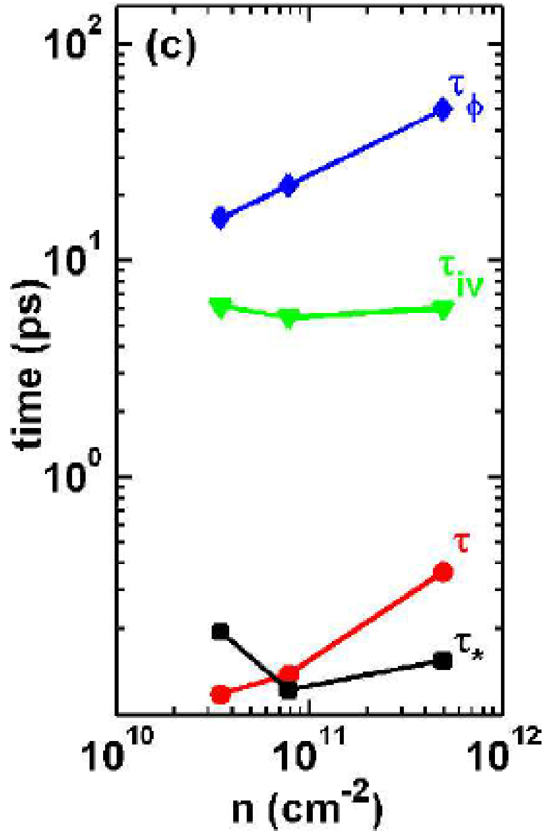
Random strain fluctuations are the limiting factor for quality of exfoliated graphene

Correlation between μ and n^*

Characteristic times from weak loc.





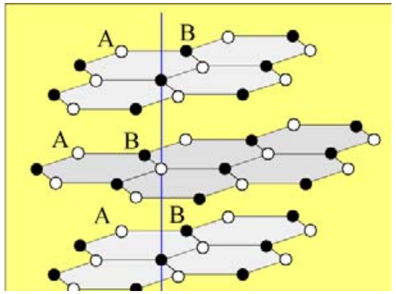


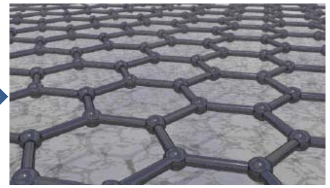
- Correlation between mobility μ and charge inhomogeneity n^* : Scattering and charge fluctuations have same microscopic origin
- Intervalley scattering time τ_{iv} >> τ elastic scattering: long-range potentials dominate
- $\tau \sim \tau^*$ time to break effective TRS in one valley: random pseudo-magnetic field due to strain dominate disorder
- Theory explains μ---n^* correlation quantitatively in terms of random strain fluctuations

data for graphene on SiO_2, $SrTiO_3$, hBN

Couto, Costanzo, Engels, Ki, Watanabe, Taniguchi, Stampfer, Guinea, Morpurgo - PRX 4, 041019 (2014)

To get best-quality graphene:





Exfoliated from bulk graphite onto
a substrate, or hanged suspended

... one needs to get rid of charge fluctuations in the substrate …

… but also to make graphene flat, avoiding strain

To get best-quality graphene:

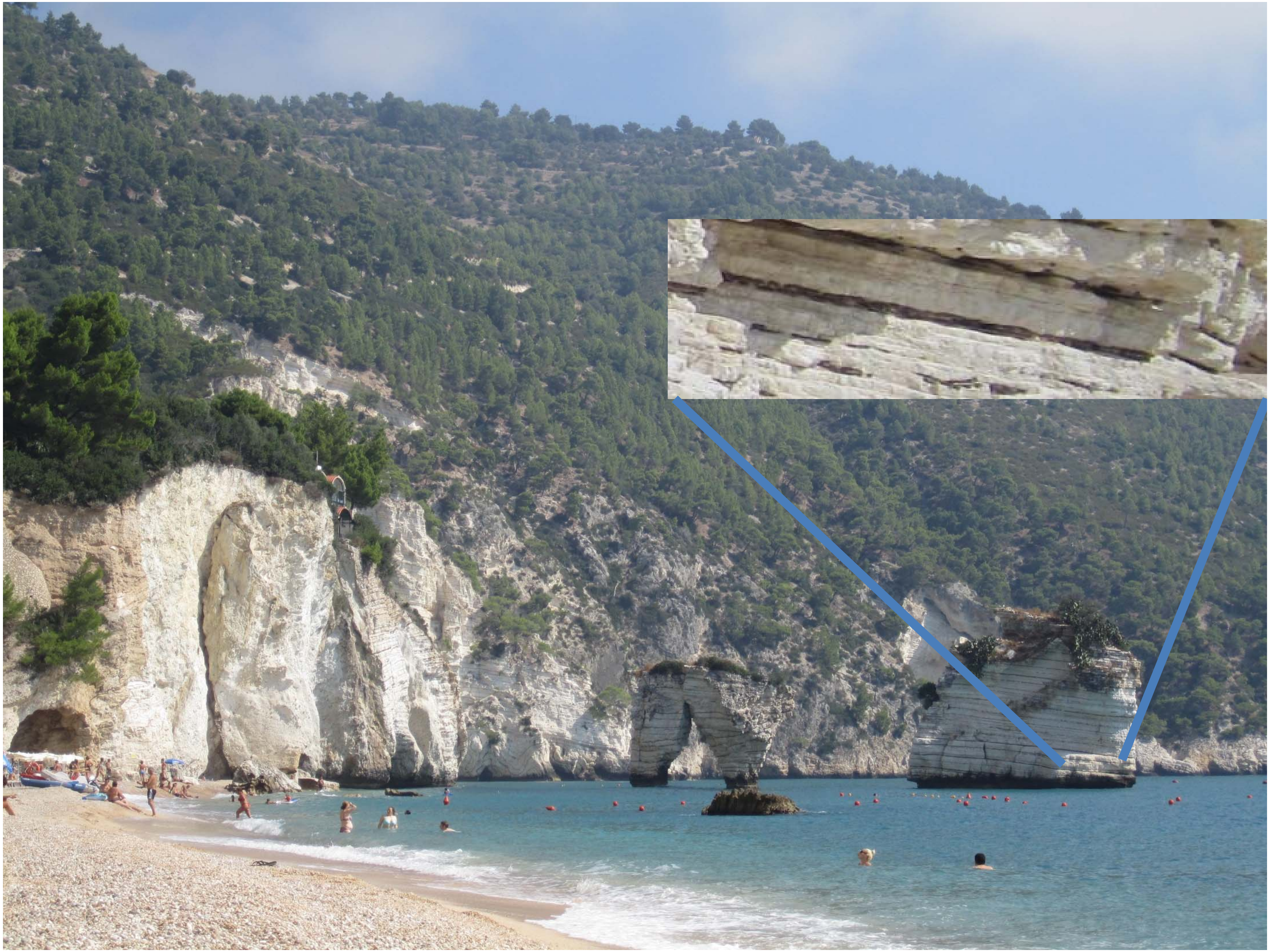
Suspending graphene does not solve the problem:

cleaning by annealing only moves dirt around

only small devices, easily strained near contacts

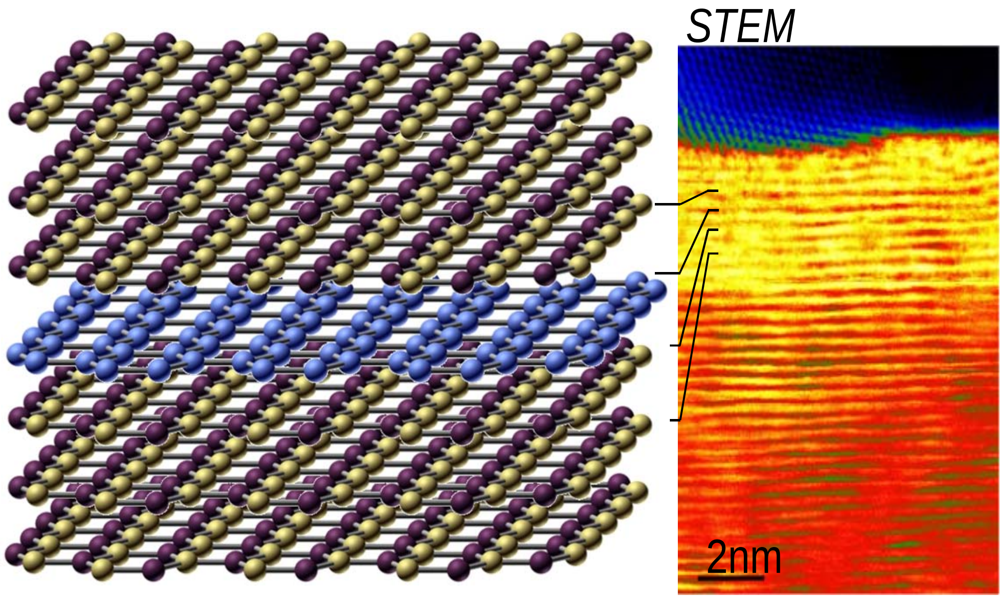
difficult to gate due to electrostatic collapse

To choose a better environment



Graphene: gapless semiconductor with Dirac electrons

$$\hat{H} = v\vec{\sigma}\cdot\vec{p}$$



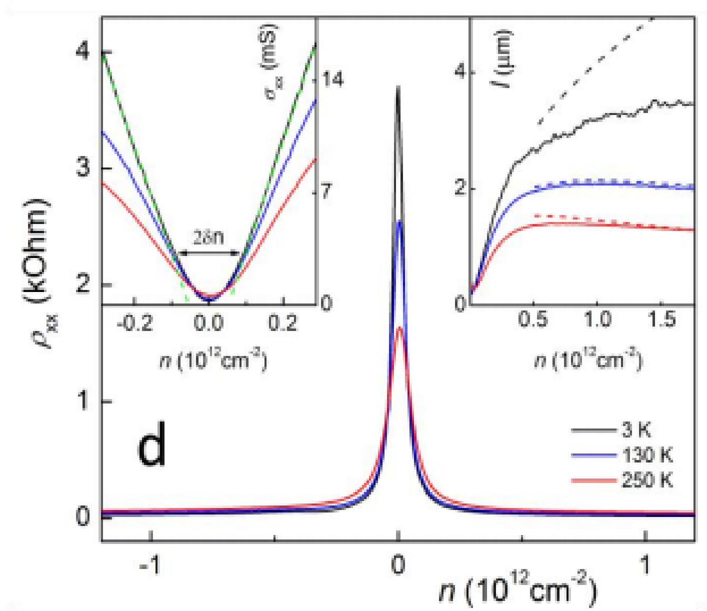
hBN ('white graphene') sp^2 – bonded insulator with a large band gap, Δ >5eV

$$\hat{H} = \Delta\sigma_z + v'\vec{\sigma}\cdot\vec{p}$$

Graphene at its best: ballistic electrons in graphene encapsulated between flakes of hexagonal boron nitride (hBN)

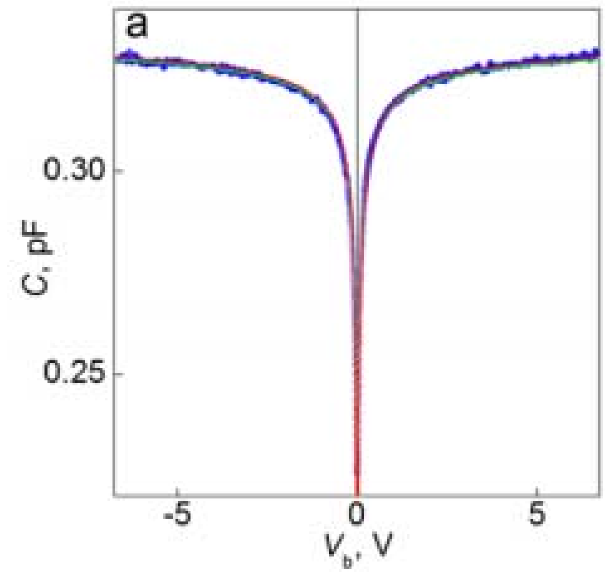
hBN-encapsulated graphene produced using dry transfer in argon: highly homogenous graphene where one can come very close to Dirac point

sharp resistivity maximum

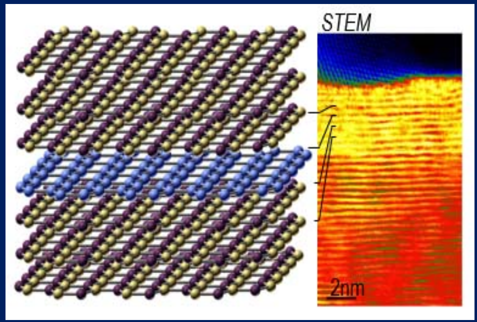


Kretinin et al - Nano Letters 14, 3270 (2014)

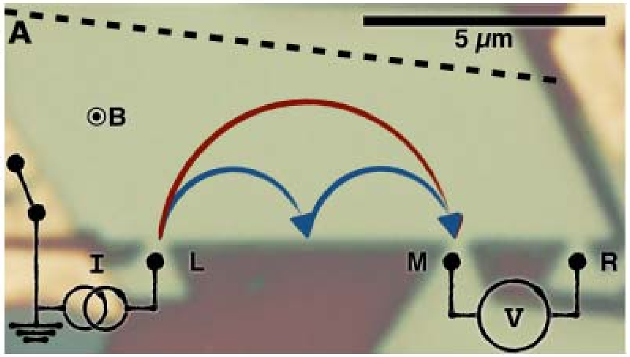
capacitance spectroscopy

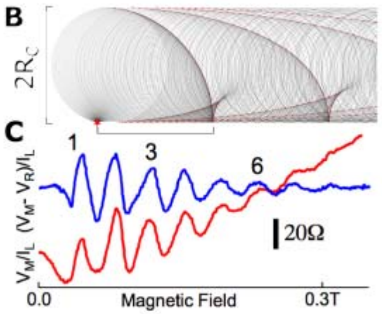


Yu et al - PNAS 110, 3282 (2013)



hBN-encapsulated graphene:
few-µm ballistic transport at high densities proven by transverse electron focusing

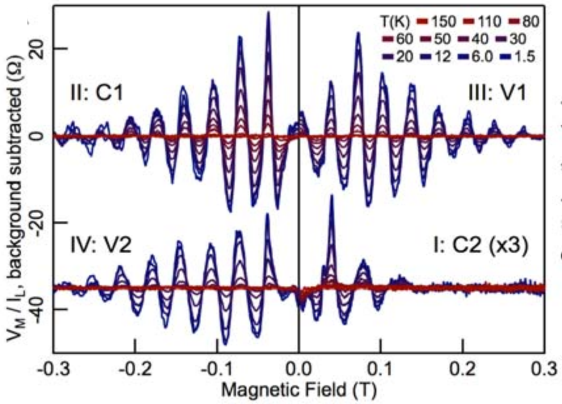


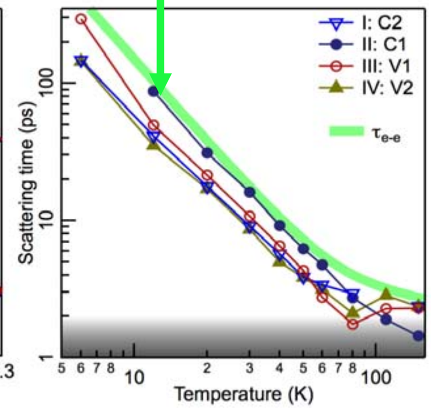


Transverse magnetic focusing (caustics of skipping orbits) of ballistic electrons

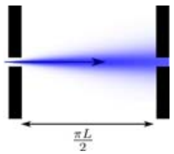
$$\frac{A(T)}{A(T_{base})} \sim e^{-\pi L/2v_F\tau}$$

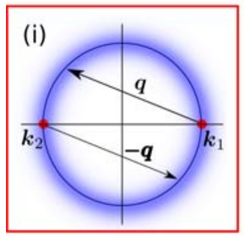
$$\tau\,(T) = -\frac{2v_F}{\pi L}\log\frac{A_1(T)}{A_1(T_{base})}$$





$$[\partial_t + v\sin(\theta_1)\partial_y]\, f(\vec{k}_1) = I\{f(\vec{k}_1)\}$$





Taychatanapat, Watanabe, Taniguchi, Jarillo-Herrero - Nature Phys 9, 225 (2013)

Lee, Wallbank, Gallagher, Watanabe, Taniguchi, Fal'ko, Goldhaber-Gordon - Science 353, 1526 (2016)

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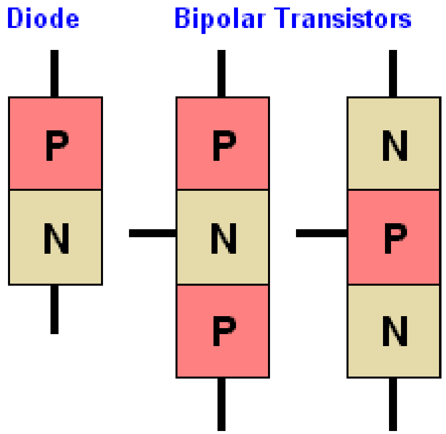
PN junctions

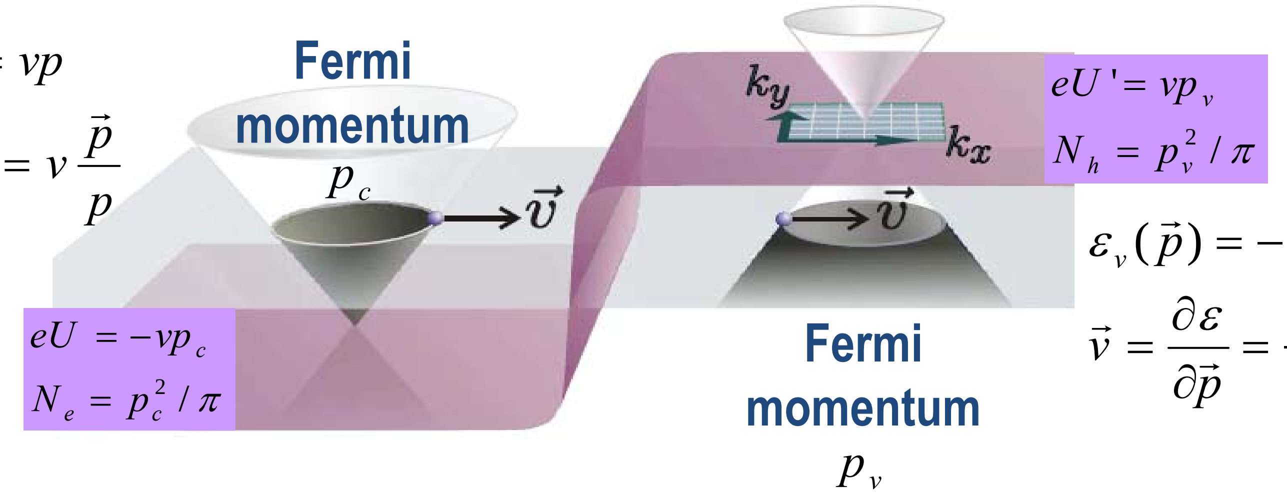


Tunneling PN junctions in semiconductors

Ballistic PN junction in graphene is highly transparent for Dirac electrons

$$\varepsilon_c(\vec{p}) = vp$$

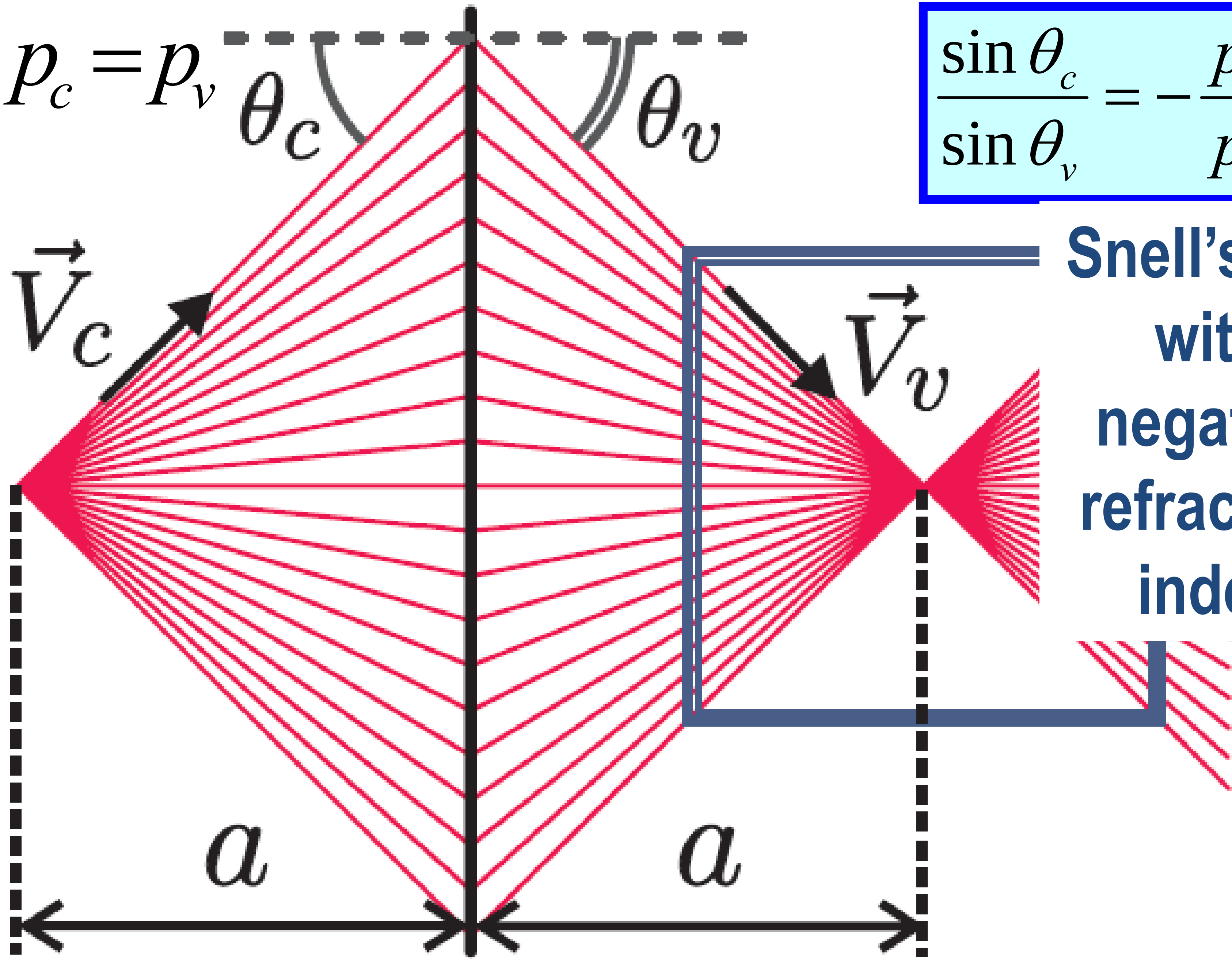
$$\vec{v} = \frac{\partial \varepsilon}{\partial \vec{p}} = v\frac{\vec{p}}{p}$$



$$\varepsilon_v(\vec{p}) = -vp$$

$$\vec{v} = \frac{\partial \varepsilon}{\partial \vec{p}} = -v\frac{\vec{p}}{p}$$

Cheianov, VF - PR B 74, 041403 (2006)
Katsnelson, Novoselov, Geim, Nature Physics 2, 620 (2006)

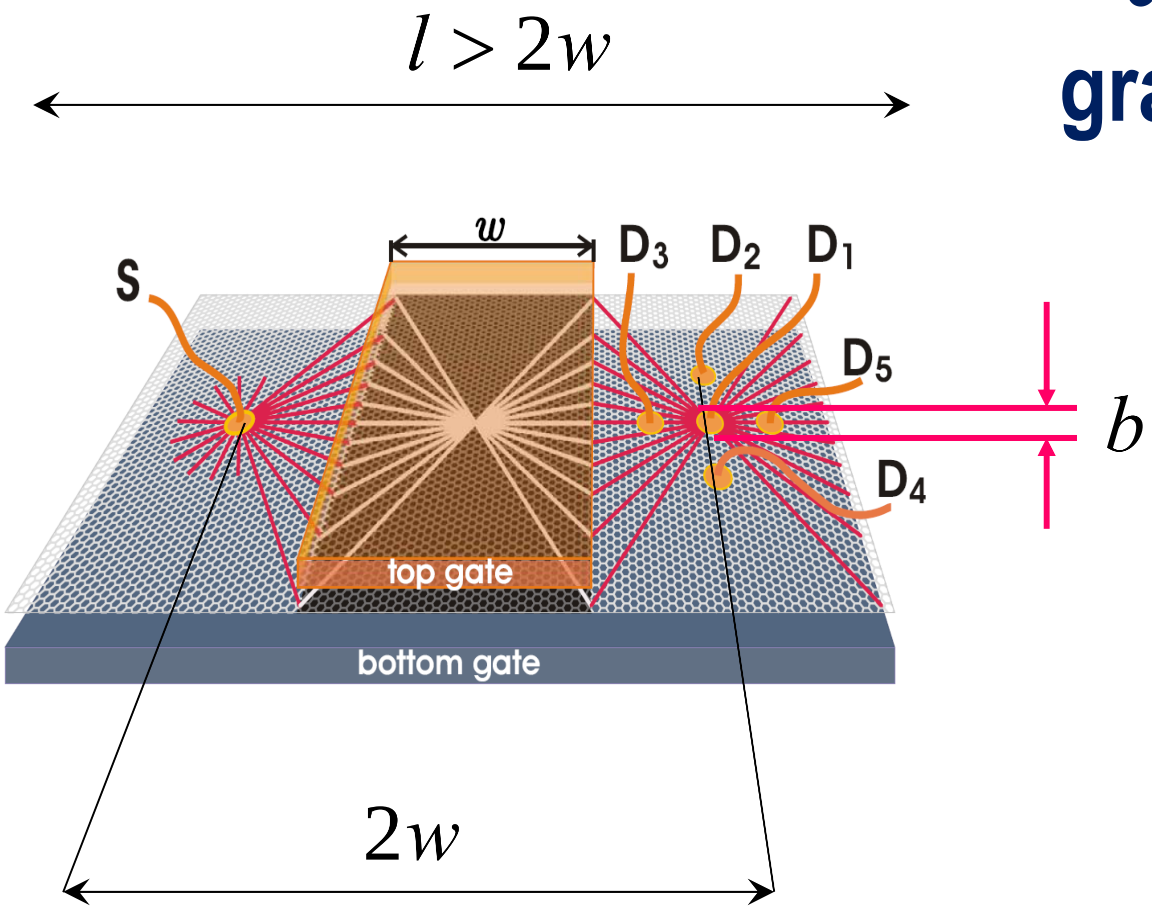


$$\frac{\sin\theta_c}{\sin\theta_v} = -\frac{p_v}{p_c} = n$$

Snell's law with negative refraction index

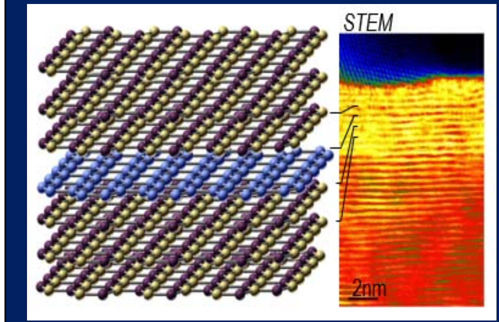
Cheianov, Fal'ko, Altshuler - Science 315, 1252 (2007)

Veselago lens for electrons in ballistic graphene using bipolar PNP graphene transistor



$$kT < \frac{b}{w}\varepsilon_F$$

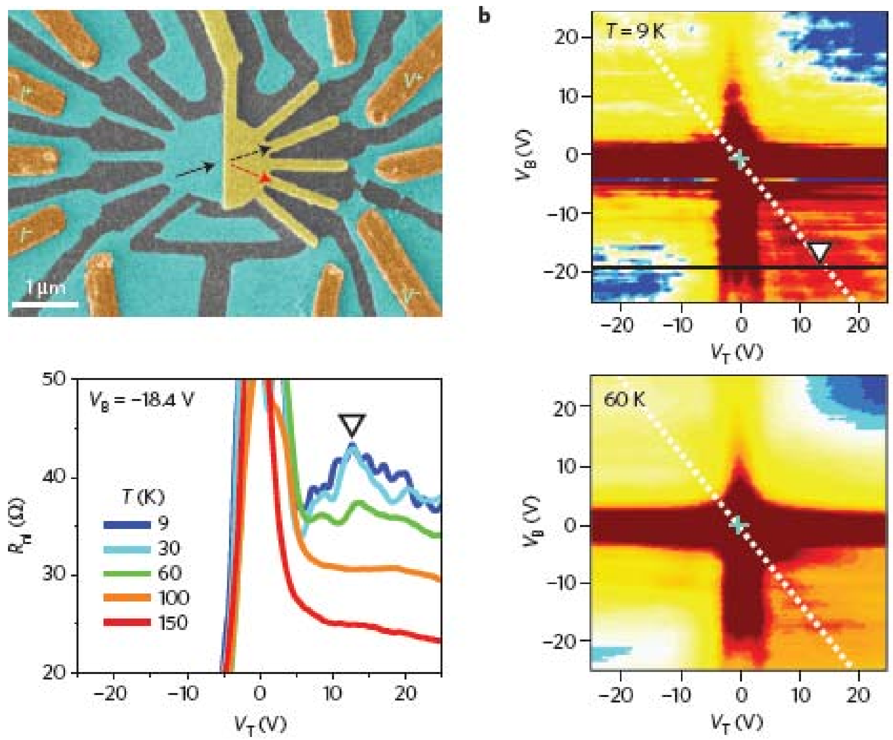
Cheianov, Fal'ko, Altshuler - Science 315, 1252 (2007)

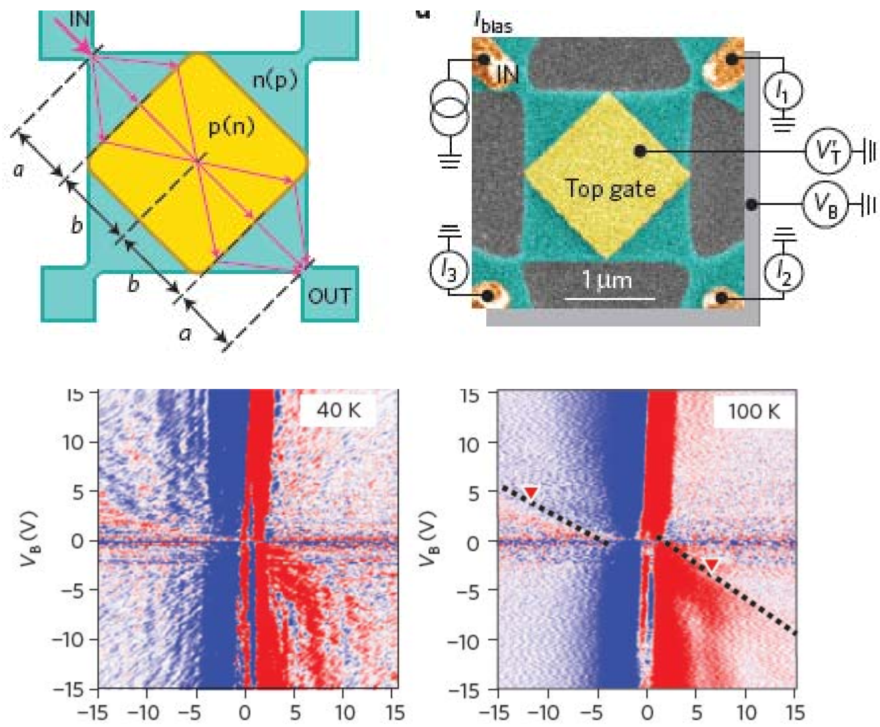


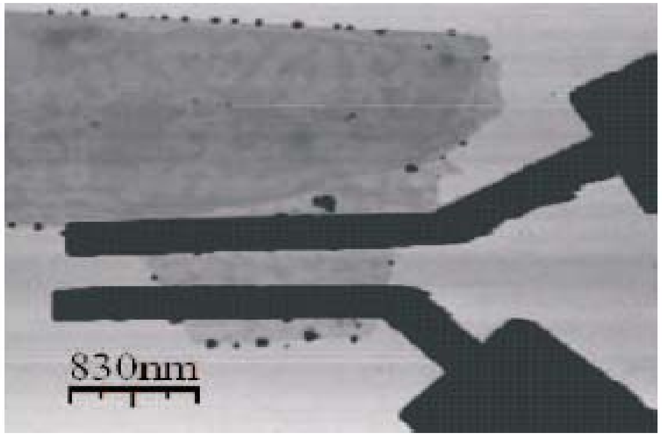
Negative refraction of Dirac electrons in hBN/G/hBN

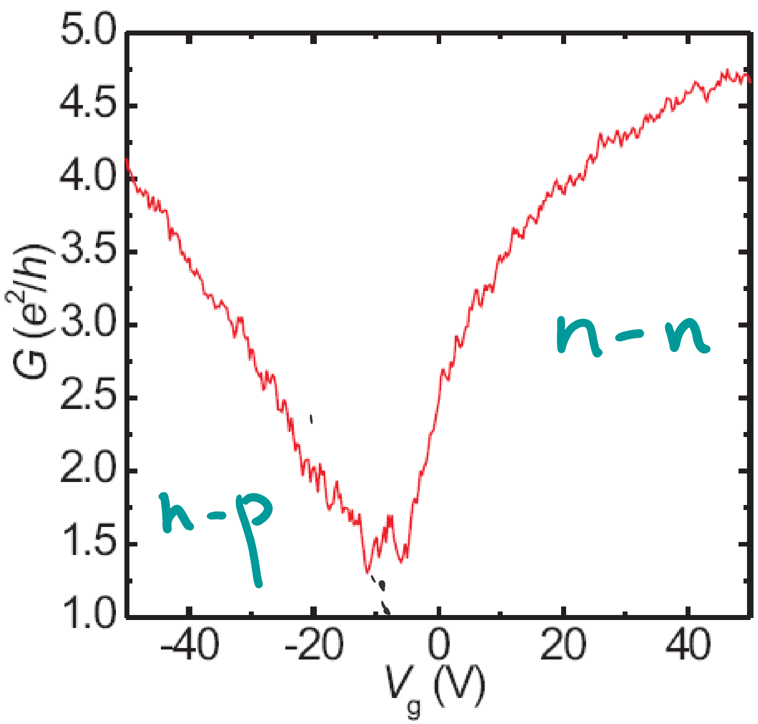
nature physics
LETTERS
PUBLISHED ONLINE: 14 SEPTEMBER 2015 | DOI: 10.1038/NPHYS3460

Gil-Ho Lee[†], Geon-Hyoung Park and Hu-Jong Lee*









PN junctions naturally form near metallic contacts to graphene, due to the charge transfer determined by the work function difference between graphene and metals used for contacts.

Heersche *et al* - Nature Physics (2007)

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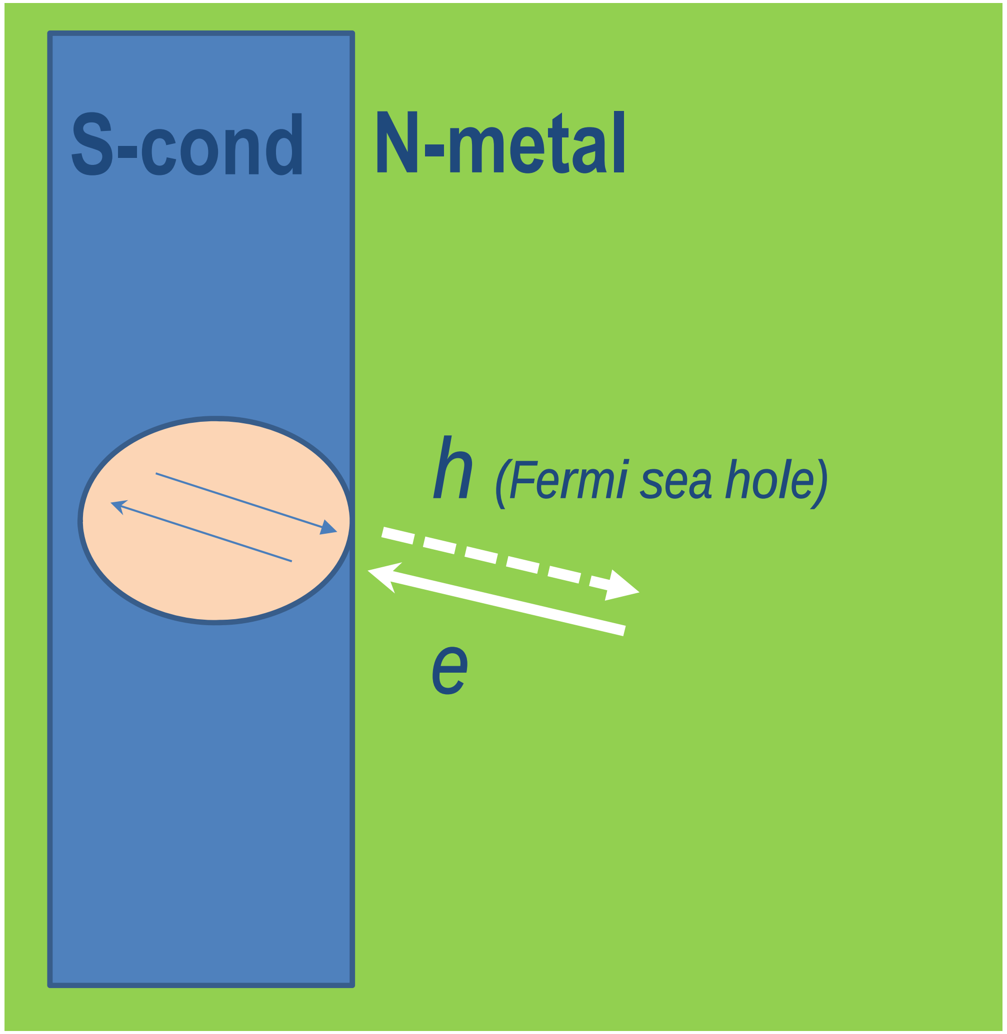
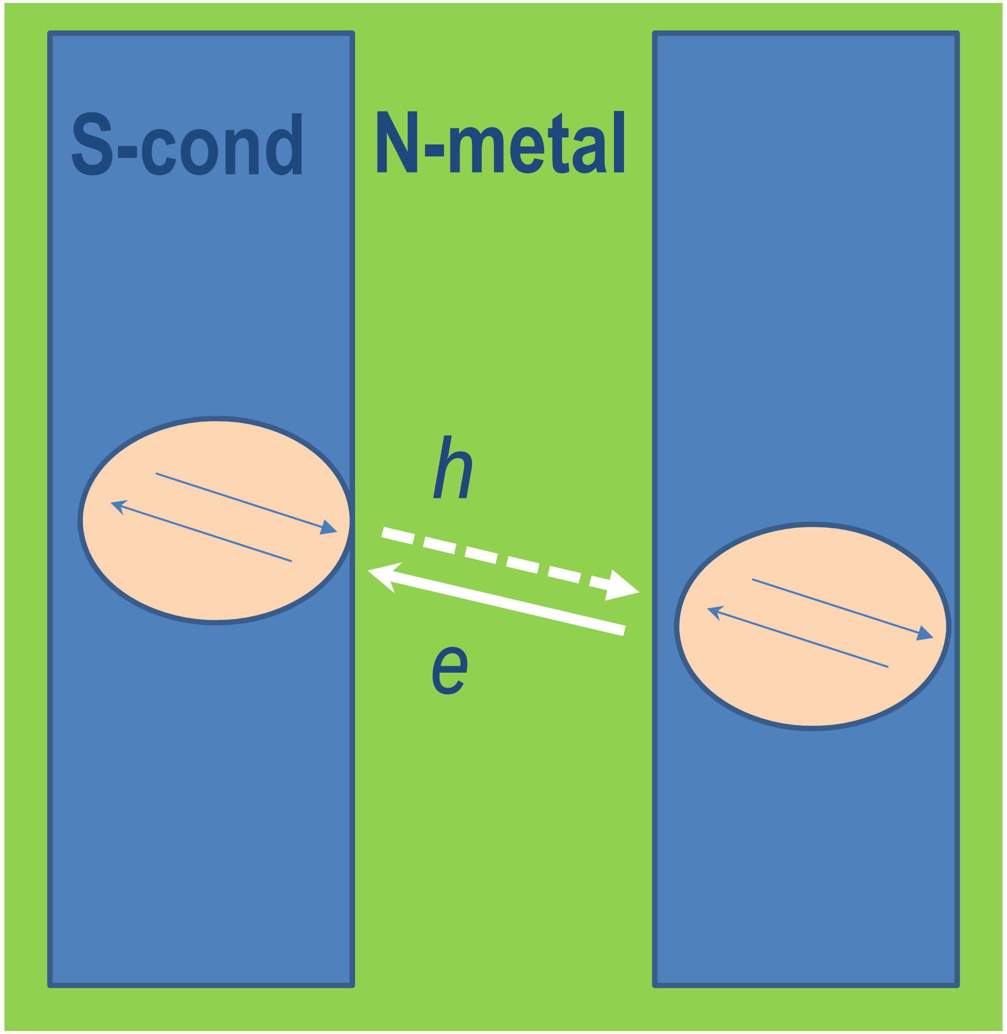
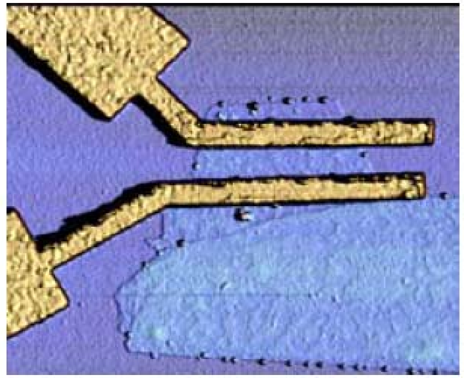
Andreev reflection

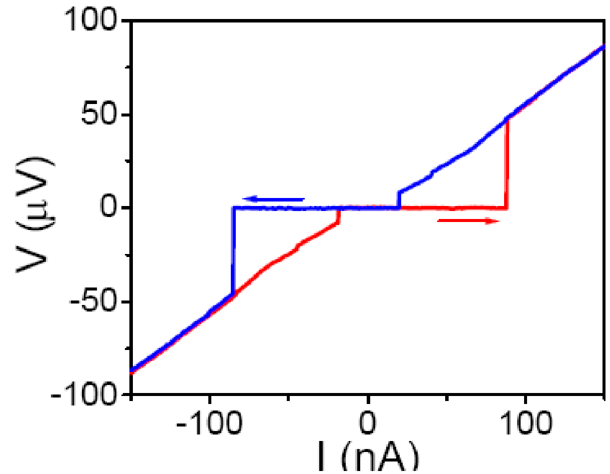


Andreev reflection in S-graphene-S junctions



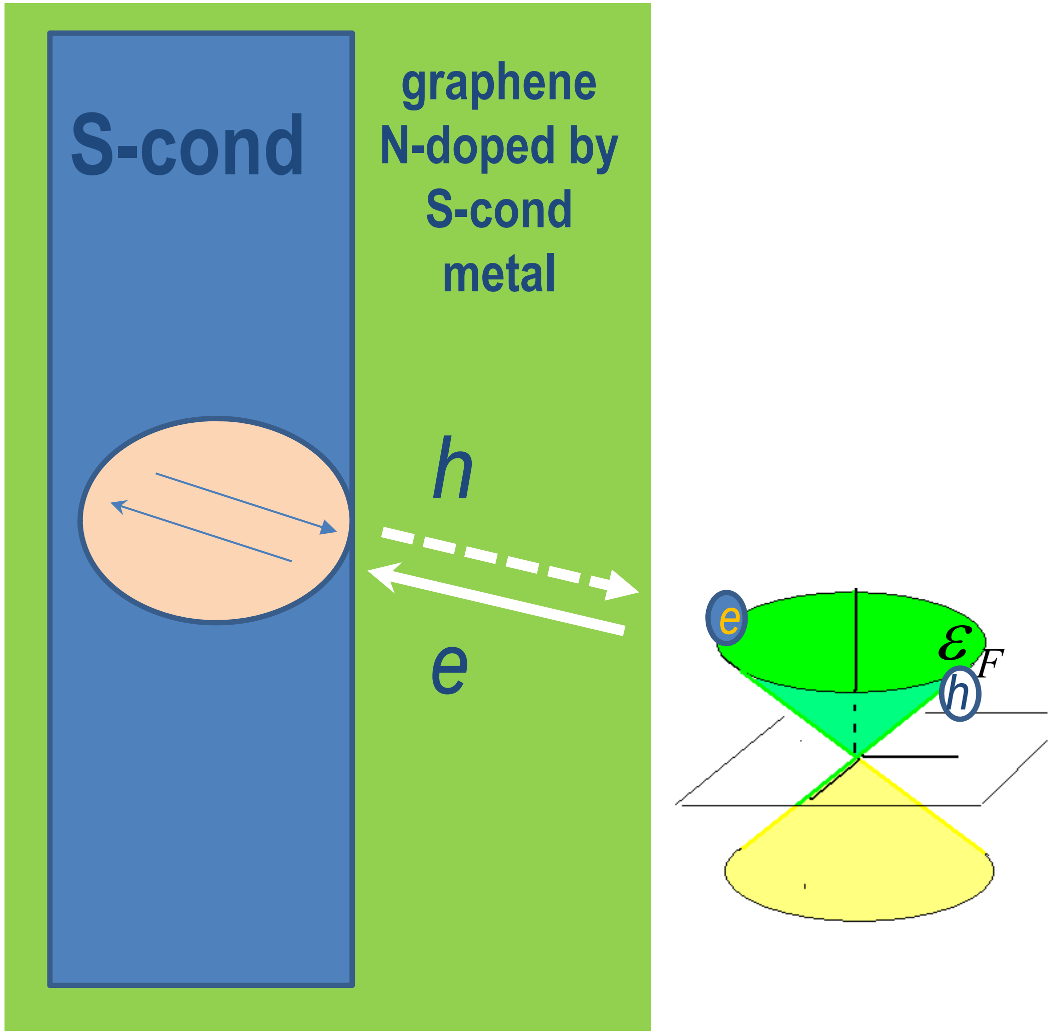


Heersche et al - Nature 446, 56-59 (2007)

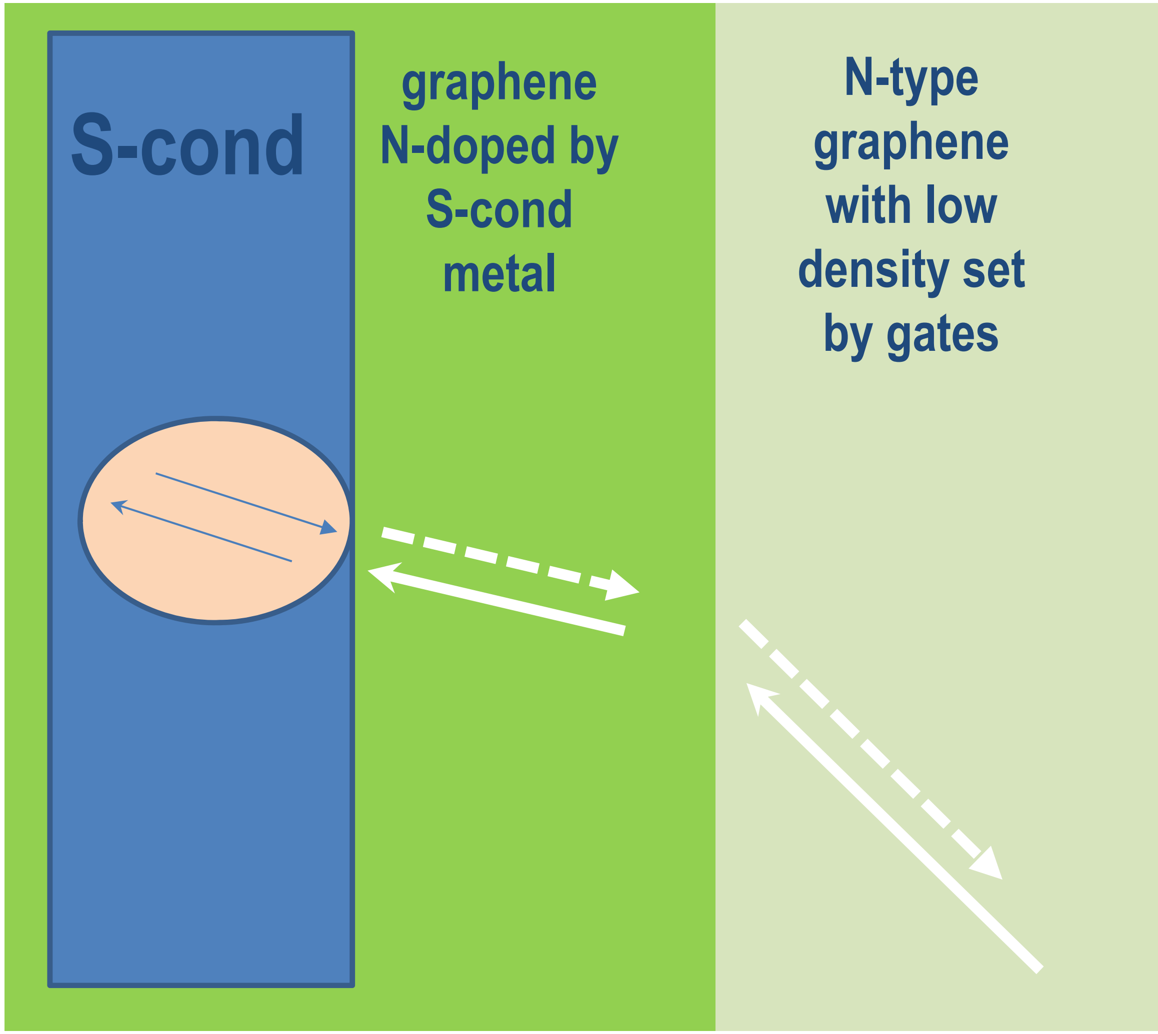


superconducting proximity effect transistor (using disordered graphene)

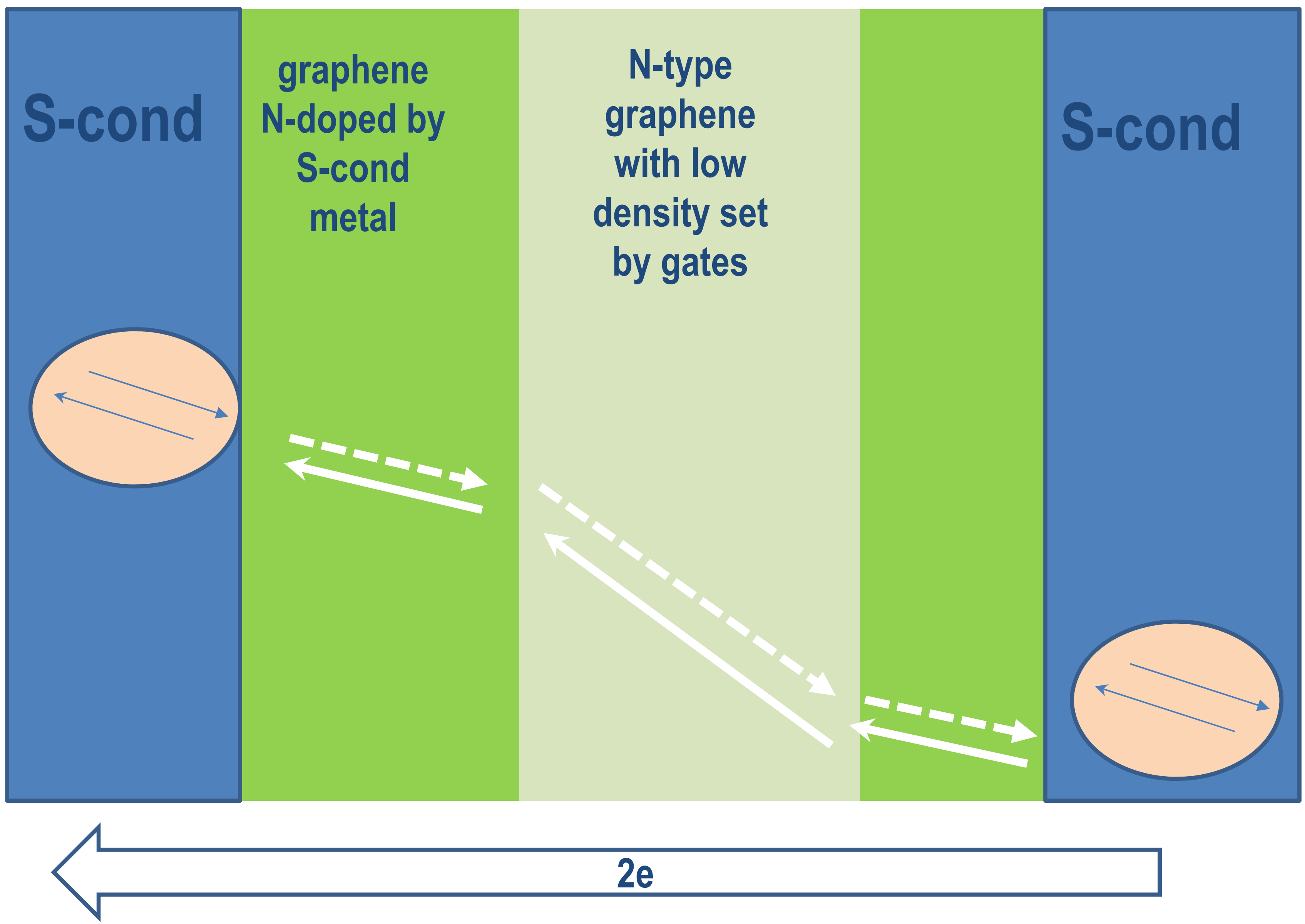
Andreev reflection at graphene/S-cond contact



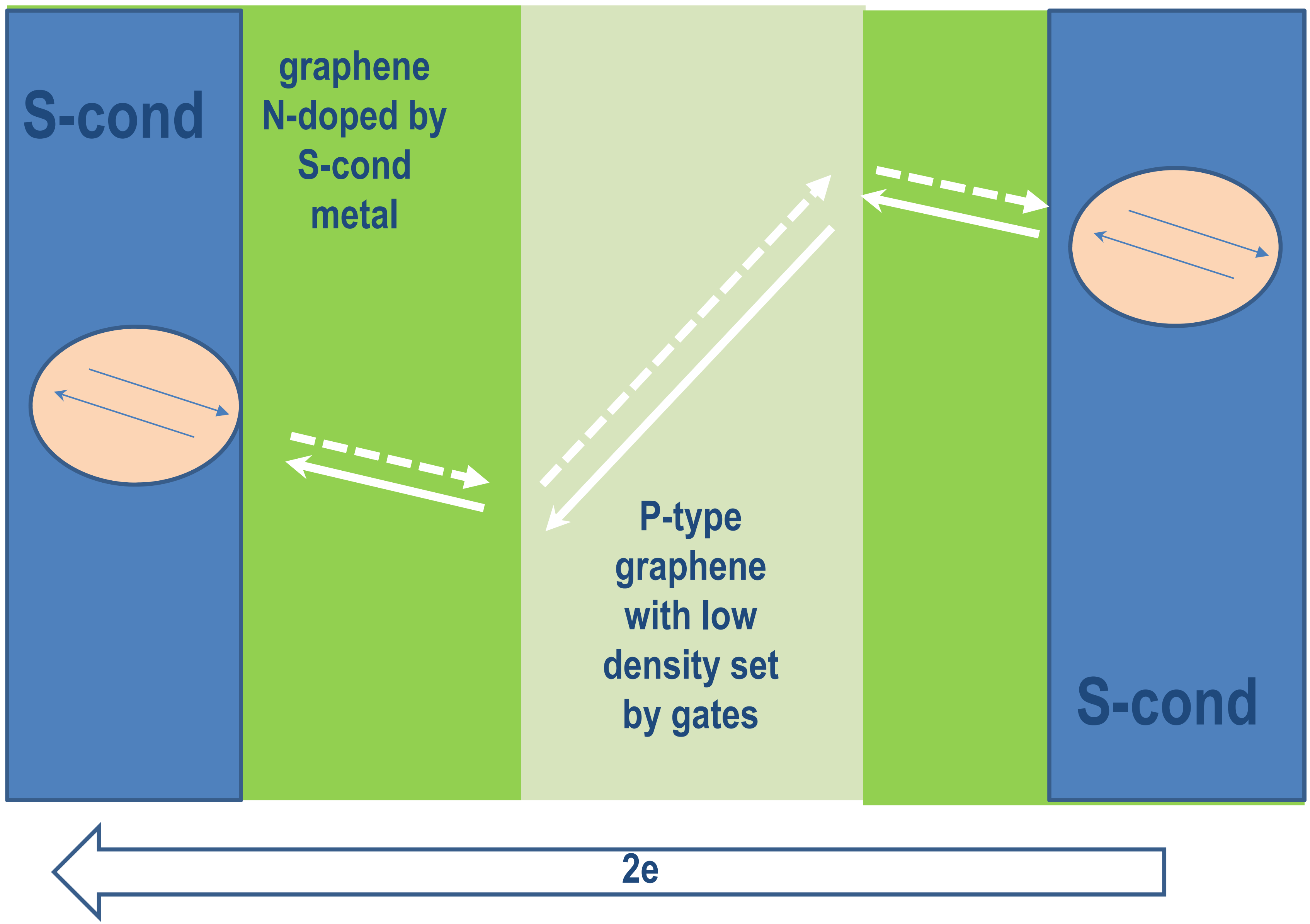
Andreev reflection at graphene/S-cond contact

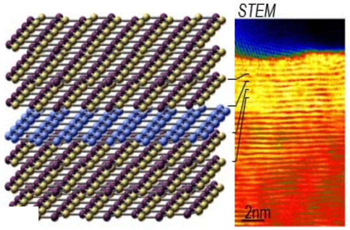


Supercurrent in monopolar GraFET (NN'N)



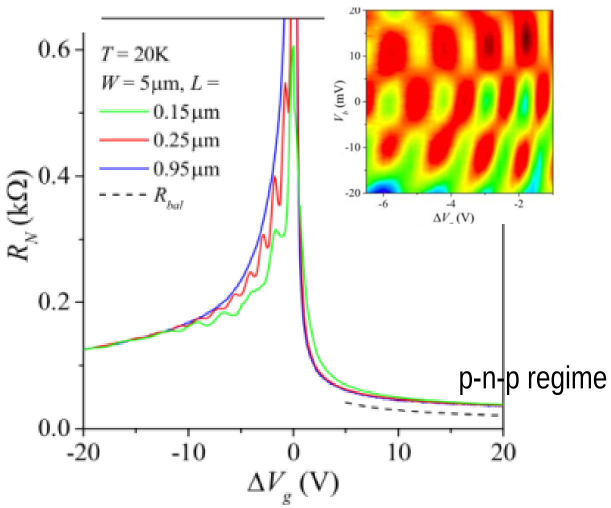
Supercurrent in bipolar GraFET (NPN)

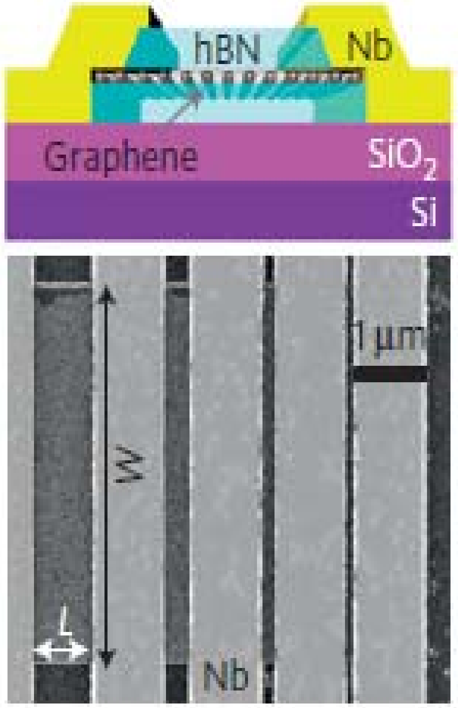




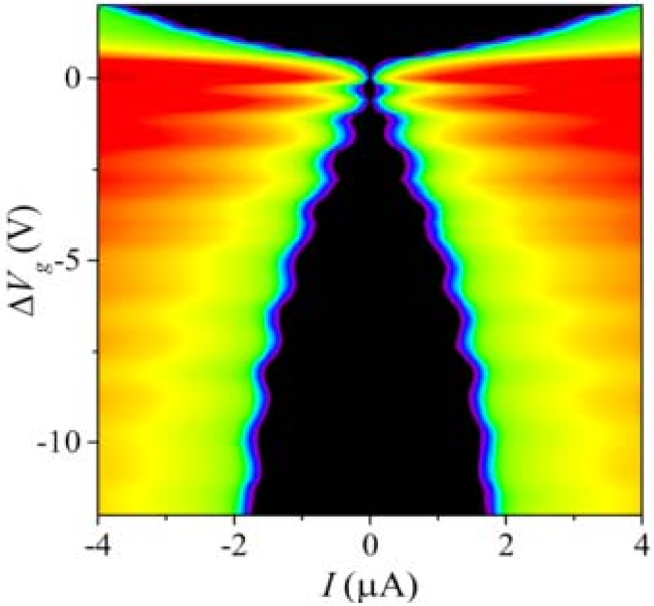
Fabry-Perot oscillations of I(V) and critical supercurrent in hBN/G/hBN with S-leads

Ballistic graphene: Fabry-Perot oscillations of *dI/dV* at $T>T_c$



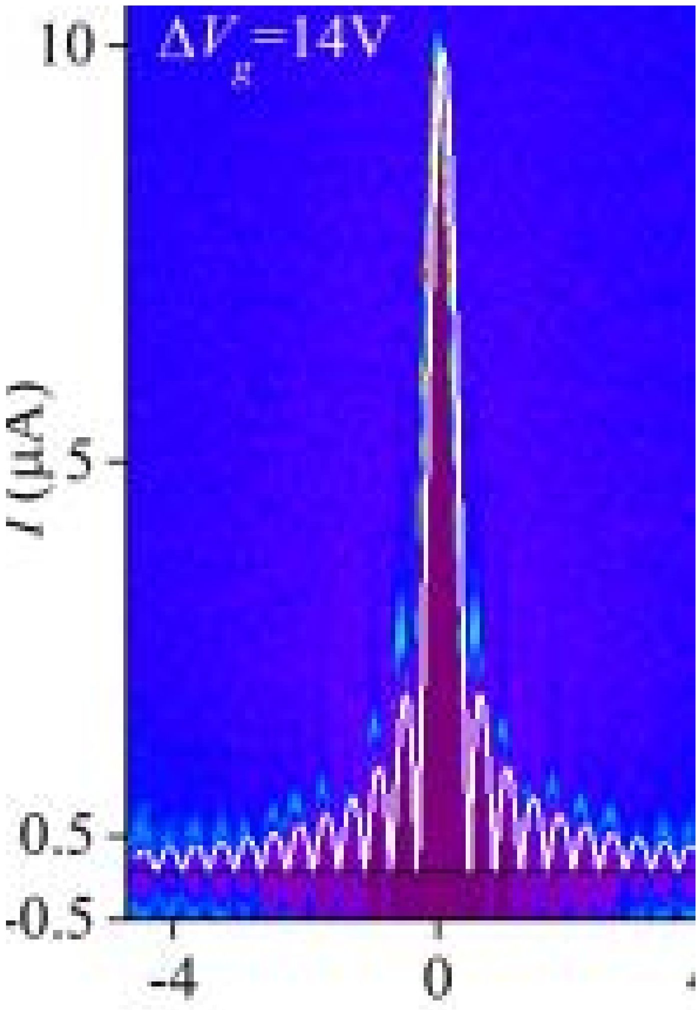


Ballistic SGS: Fabry-Perot oscillations of critical supercurrent current at $T<T_c$

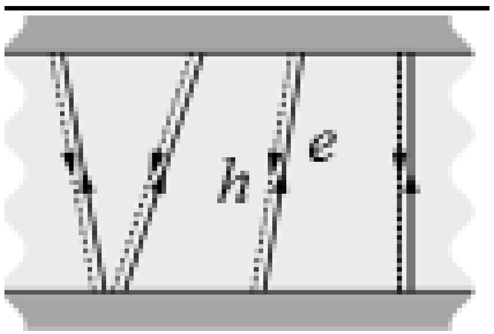


Ben-Shalom, Zhu, Fal'ko, Mishchenko, Kretinin, Novoselov, Woods, Watanabe, Taniguchi, Geim, Prance
Nature Physics 12, 318 (2016)

Magneto-oscillations:
low-B Fraunhofer pattern



$$I_c(\Phi) = \frac{I_{c0}}{\Phi/\Phi_0}\,\eta\left(\left\{\frac{\Phi}{\Phi_0}\right\}\right)$$

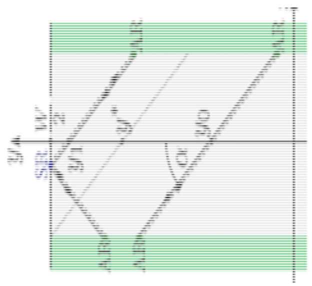


wide ($d \ll W$) ballistic SNS junction in a ‘strong’ magnetic field

$$I_c(\Phi) = \frac{I_{c0}}{\Phi/\Phi_0}\,\eta\left(\left\{\frac{\Phi}{\Phi_0}\right\}, \frac{d^2}{\ell_B^2}\right)$$

$$B > \frac{\Phi_0}{d^2}$$

‘high’ magnetic fields:
edge supercurrent



$$I_c(B) \sim \frac{I_{c0}}{\varphi}\frac{\ell_B^2}{d^2} \propto B^{-2}$$

$$\delta I_c \sim \frac{I_{c0}}{\varphi}\frac{b_0}{d} \propto B^{-1}$$

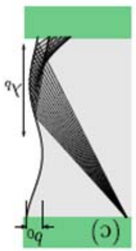
random caustics of
retracing Andreev paths
near a disordered edge

Meier, Fal’ko, Glazman – PRB 93, 184506 (2016)

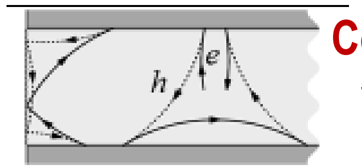
Reentrant mesoscopic proximity effect due to edges in a wide ($d \ll W$) ballistic SNS junction

$$\delta I_c \sim \frac{I_{c0}}{\varphi}\frac{b_0}{d} \propto B^{-1}$$

random caustics of retracing Andreev paths near a disordered edge



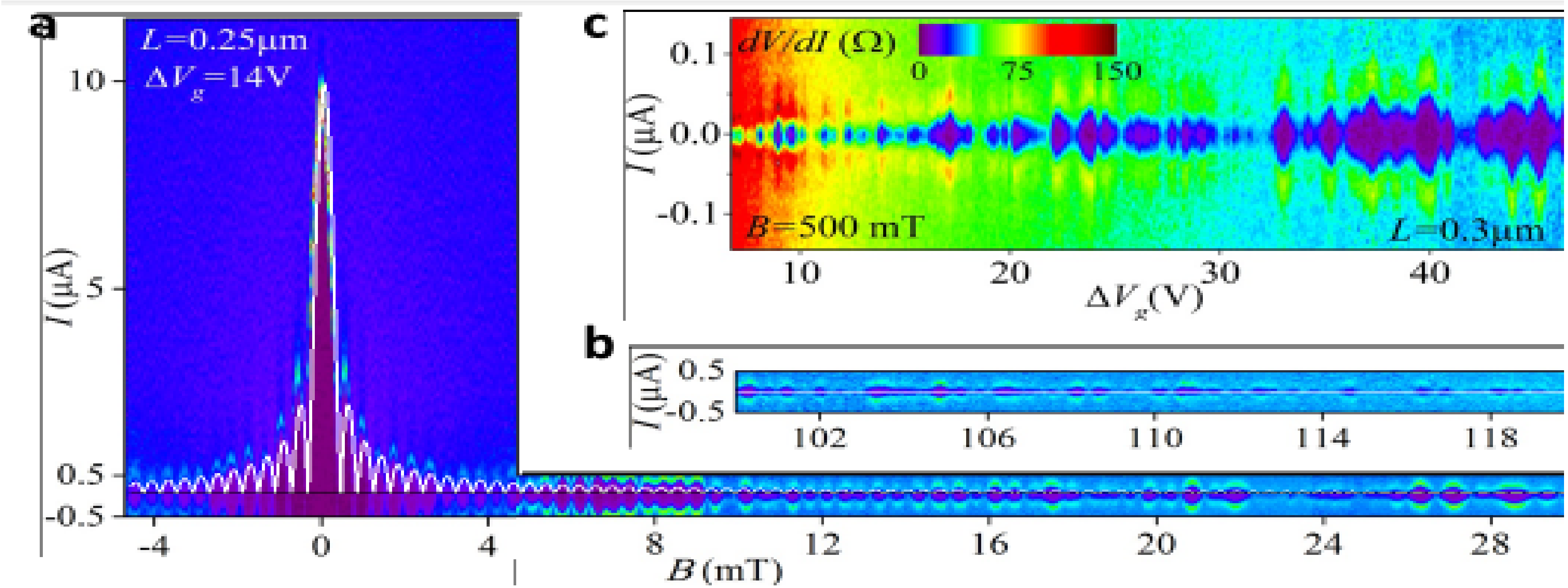
$$B > \frac{\Phi_0}{\xi\, d}$$



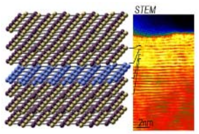
$$\delta I_c \sim \frac{e}{\tau_{Th}} \sim \frac{ev}{d}$$

Cooper pair transfer via non-retracing Andreev paths (e-h loops)

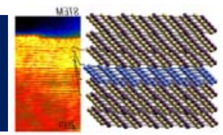
(up to $r_c < \frac{d}{2}$)

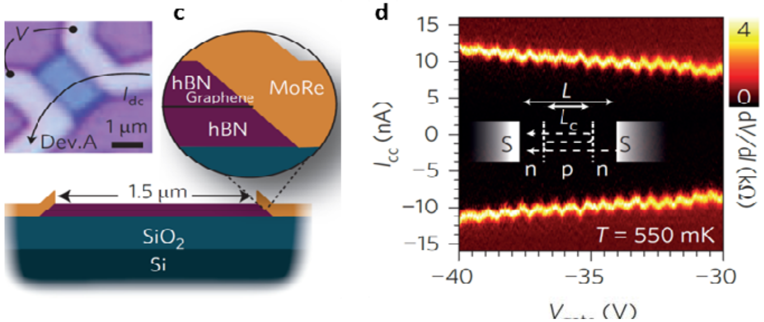


Ben Shalom, Zhu, Fal'ko, Mishchenko, Kretinin, Novoselov, Woods, Watanabe, Taniguchi, Geim, Prance
Nature Physics 12, 318 (2016)



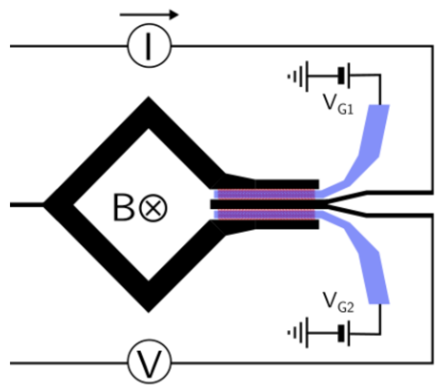
QT devices using ballistic SGS

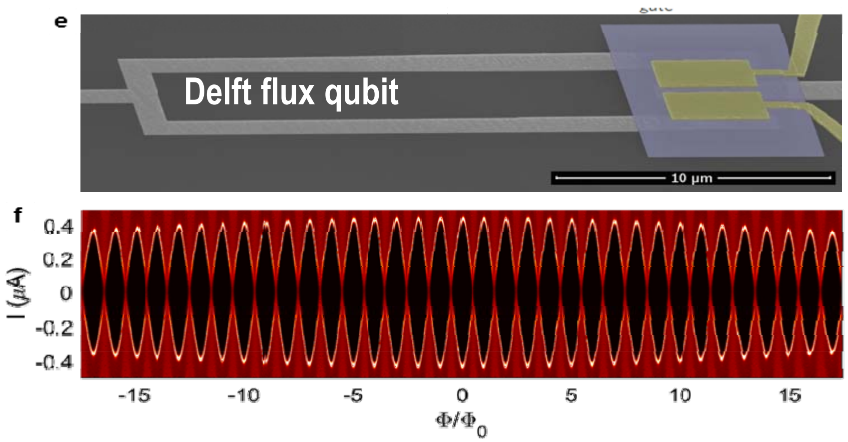


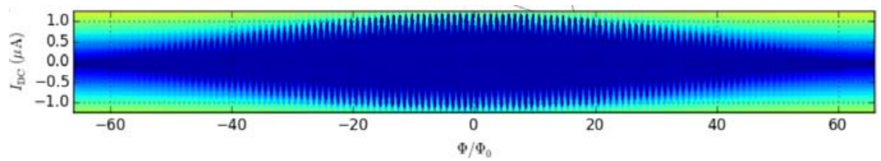


Calado, Goswami, Nanda, Diez, Akhmerov, Watanabe, Taniguchi, Klapwijk, Vandersypen
Nature Nanotechnology 10, 761 (2015)

Lancaster graphene FET-based SQUID: supercurrent can be switched on/off fast using electrostatic gates:



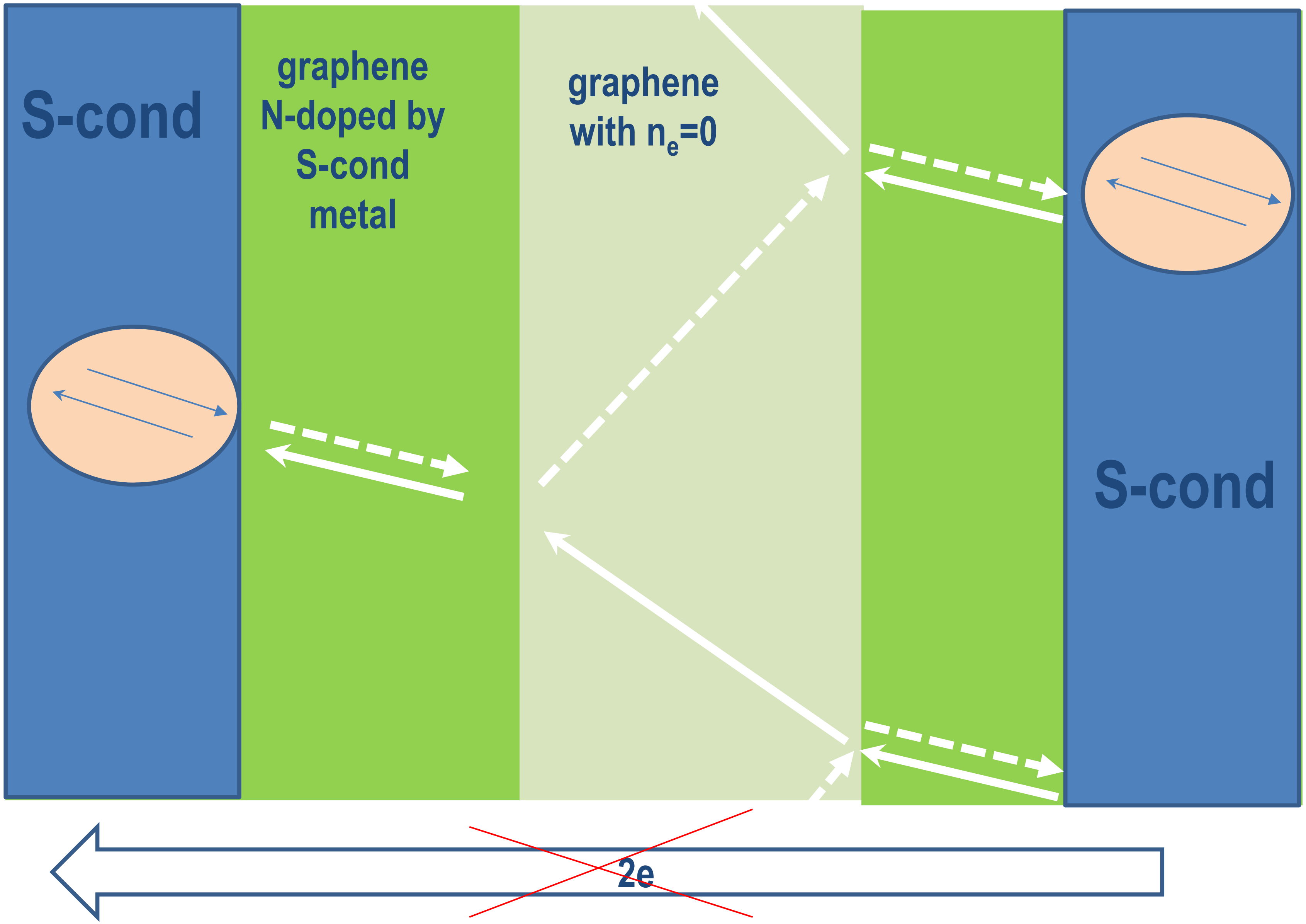




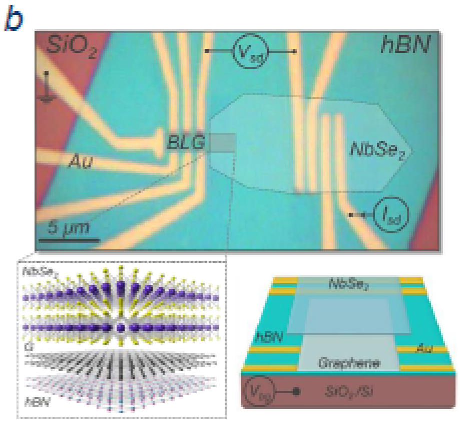
quantum device for magnetic field measurement

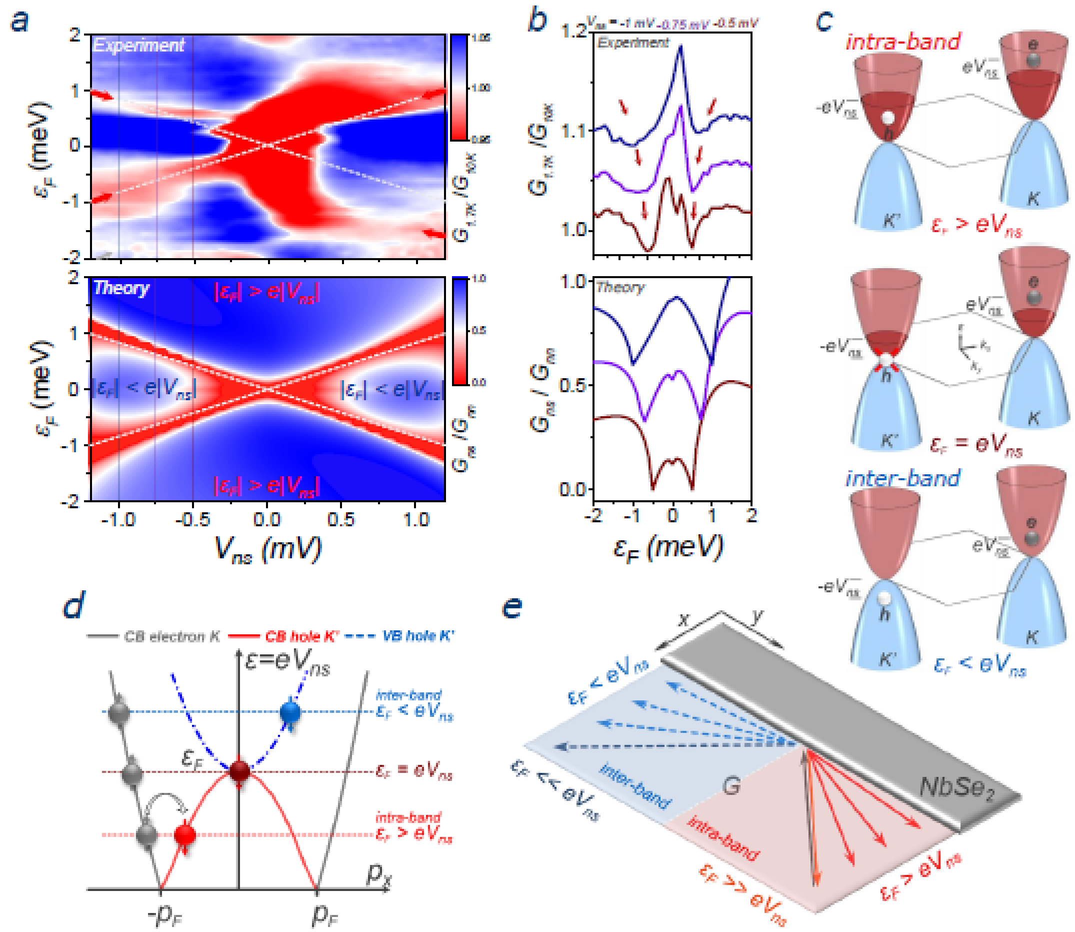
Specular Andreev reflection for graphene at neutrality point

Beenakker - PRL 97, 067007 (2006)



Specular Andreev reflection in bilayer graphene at neutrality point





Efetov, Wang, Handschin, Efetov,
Shuang, Cava, Taniguchi,
Watanabe, Hone, Dean, Kim
Nature Physics 12, 328-332 (2016)

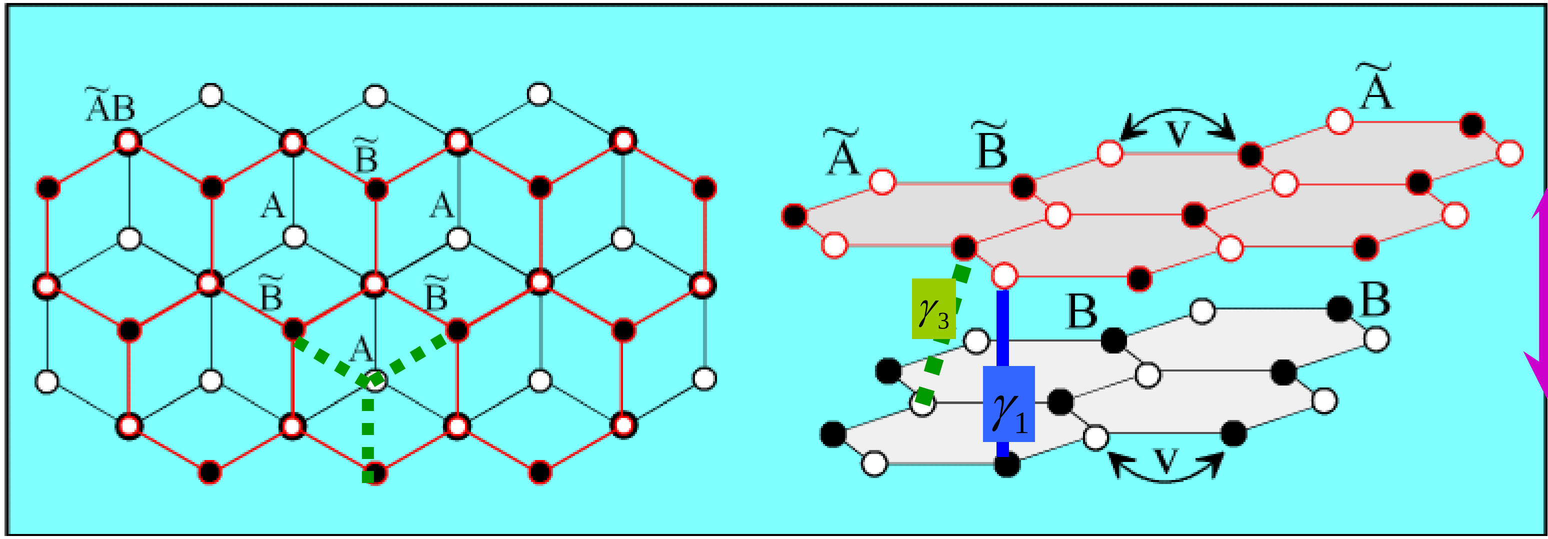
Quantum transport in graphene

L1 Disordered graphene (G)

L2 Ballistic electrons in graphene (G/hBN)

- charge inhomogeneity in graphene solved
- PN junctions and Veselago lens in graphene
- Andreev reflection in ballistic SGS devices
- Lifshitz transition detected using QHE in bilayer G

L3 Moiré superlattice effects in G/hBN heterostructures



$$u = E_z d$$

skew inter-layer $A\widetilde{B}$ hopping

$$v_3 = -\frac{\sqrt{3}}{2}\frac{\gamma_3 a}{\hbar} \sim 0.1v$$

$$\pi = p_x + ip_y = pe^{i\varphi}$$

$$H = \begin{pmatrix} \frac{1}{2}u & v_3\pi & 0 & v\pi \\ v_3\pi^+ & -\frac{1}{2}u & v\pi^+ & 0 \\ 0 & v\pi & -\frac{1}{2}u & \gamma_1 \\ v\pi^+ & 0 & \gamma_1 & \frac{1}{2}u \end{pmatrix} \begin{pmatrix} A \\ \widetilde{B} \\ \widetilde{A} \\ B \end{pmatrix}$$

$$\epsilon_\alpha^2 = \frac{\gamma_1^2}{2} + \frac{u^2}{4} + \left(v^2 + \frac{v_3^2}{2}\right)p^2 + (-1)^\alpha \sqrt{\Gamma}$$

$$\Gamma = \tfrac{1}{4}(\gamma_1^2 - v_3^2 p^2)^2 + v^2 p^2[\gamma_1^2 + u^2 + v_3^2 p^2] + 2\xi\gamma_1 v_3 v^2 p^3 \cos 3\varphi,$$

McCann, Fal'ko - PRL 96, 086805 (2006)

Dirac point generates
a 4-fold degenerate *ε=0* Landau level

McClure - PR 104, 666 (1956)

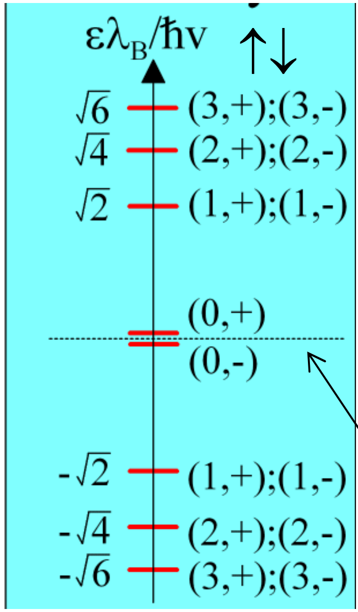
$$H = v\xi \begin{pmatrix} 0 & \pi^{+} \\ \pi & 0 \end{pmatrix}$$

energy scale $\hbar v/\lambda_B$

where $\lambda_B = \sqrt{\dfrac{\hbar}{eB}}$

state at zero energy:

$$\pi\phi_0 = 0$$



$$\varepsilon^{\pm} = \pm\sqrt{2n}\,\frac{v}{\lambda_B}$$

$$\begin{pmatrix} \phi_0 \\ 0 \end{pmatrix}, \begin{pmatrix} \phi_1 \\ 0 \end{pmatrix}$$

$$\vec{p} = -i\hbar\nabla - \tfrac{e}{c}\vec{A}, \quad rot\vec{A} = B\vec{l}_z$$

$$\pi = p_x + ip_y; \quad \pi^{+} = p_x - ip_y$$

descending/raising operators in LL orbitals

$$H = \frac{-1}{2m} \begin{pmatrix} 0 & (\pi^{+})^2 \\ \pi^2 & 0 \end{pmatrix}$$

energy scale $\hbar\omega_c$

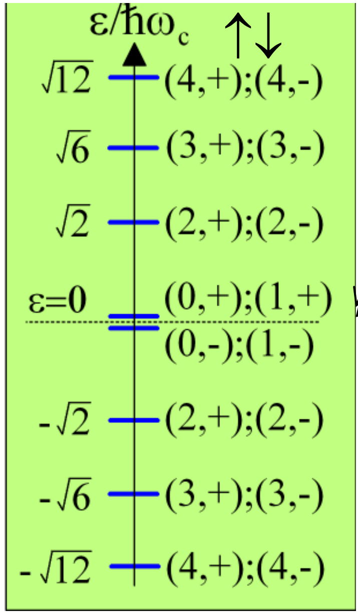
where $\omega_c = \dfrac{eB}{m}$

$m \approx 0.035 m_e$

states at zero energy:

$$\pi^2\phi_0 = 0$$

$$\pi^2\phi_1 = 0$$

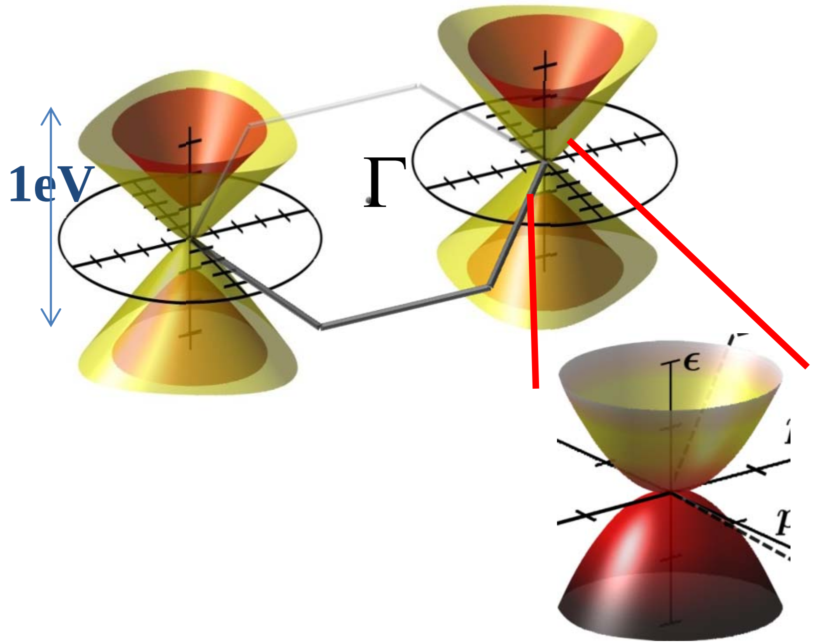


$$\varepsilon^{\pm} = \pm\hbar\omega_c\sqrt{n(n-1)}$$

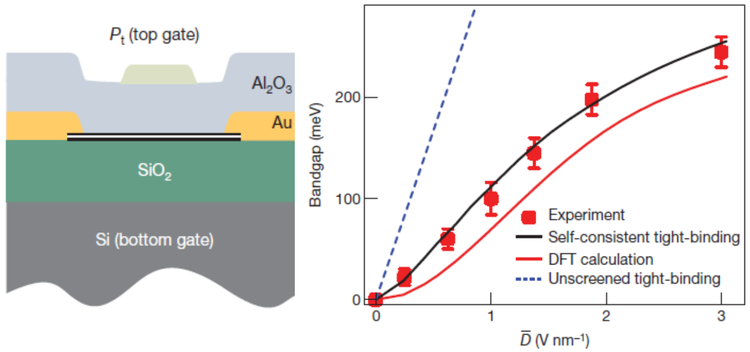
***8*-fold degenerate *ε=0* Landau level, which splits when inversion symmetry is broken and on-site energies on A and B' sublattices differ.**

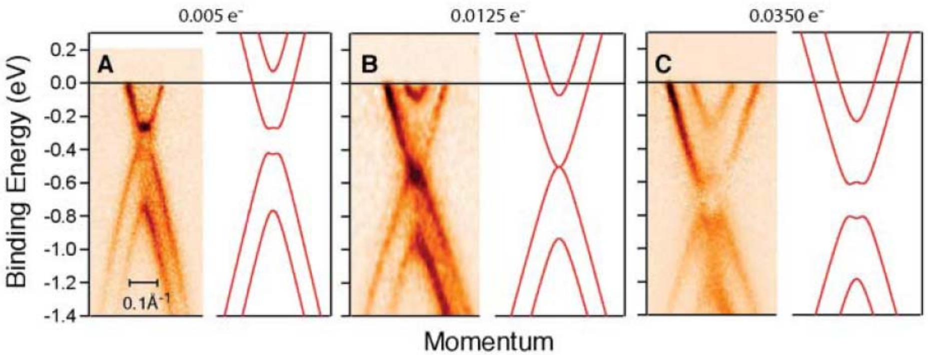
McCann, VF - PRL 96, 086805 (2006)

Electrical control of a gap in bilayer graphene



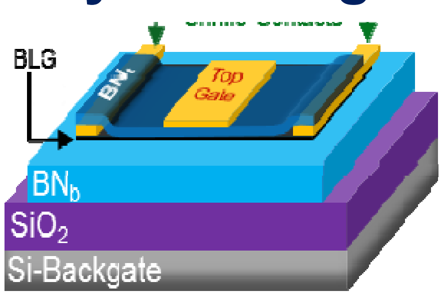
Oostinga, *et al* - Nature Mat 7, 151 (2008)
Zhang, *et al* - Nature 459, 820 (2009)





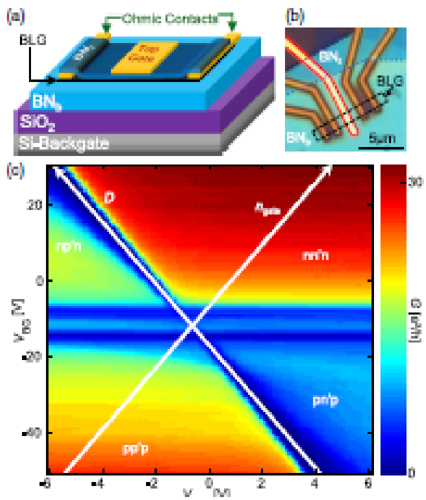
T. Ohta *et al* – Science 313, 951 (2006)
(Rotenberg's group at Berkeley NL)

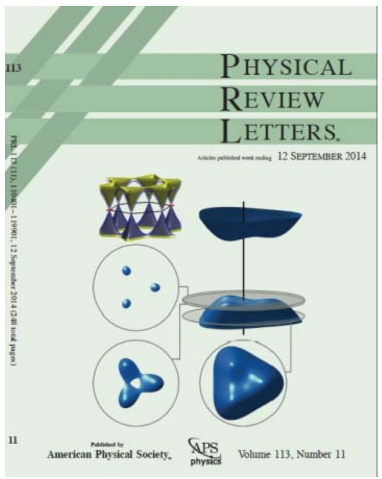
Encapsulation of BLG in hBN allows for better quality and larger E_z

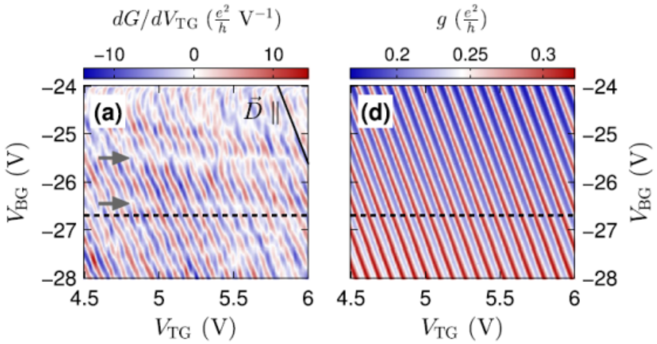


Electrically-controlled band gap in high-quality hBN/BLG/hBN structures

- Bilayer graphene encapsulated between two hBN films
- Electrostatically controlled gap in the range up to 0.2eV
- High quality/mobility has enabled to observe Fabry-Perot oscillations of conductance and ferromagnetic quantum Hall states
- Electrically tuneable topology (Lifshitz transition) has been observed



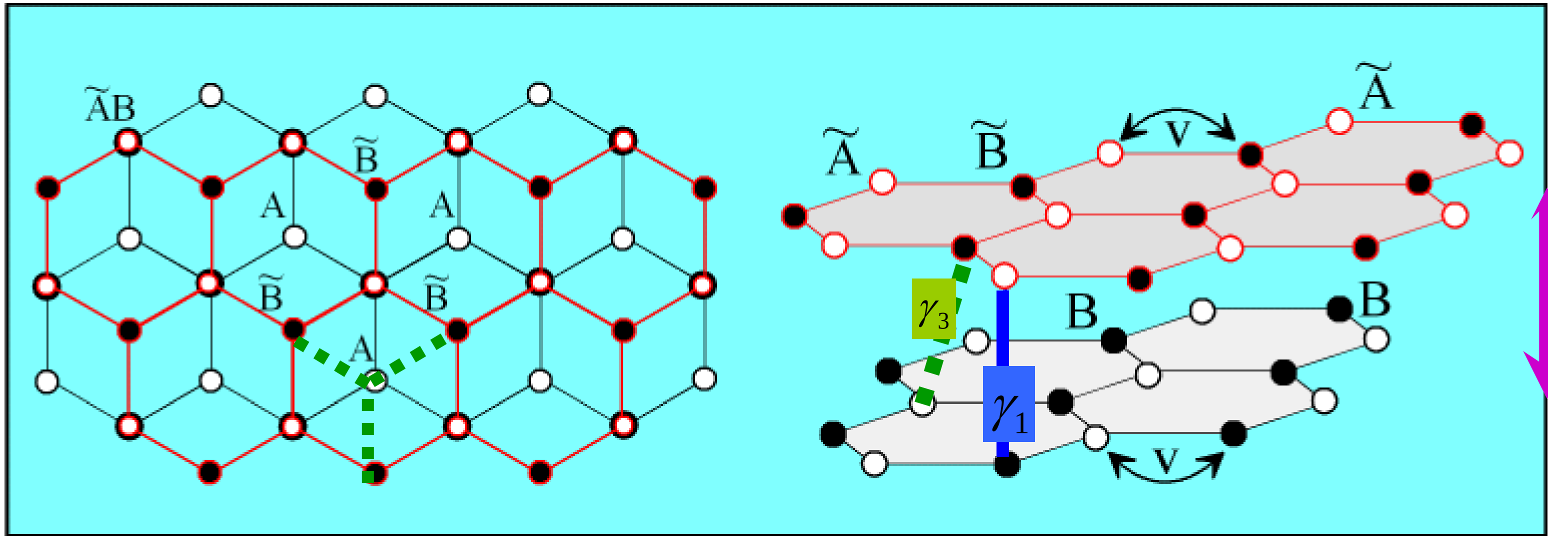




PRL 113, 116601 (2014) PHYSICAL REVIEW LETTERS week ending 12 SEPTEMBER 2014

Fabry-Pérot Interference in Gapped Bilayer Graphene with Broken Anti-Klein Tunneling

Anastasia Varlet,[1,*] Ming-Hao Liu (劉明豪),[2] Viktor Krueckl,[2] Dominik Bischoff,[1] Pauline Simonet,[1] Kenji Watanabe,[3] Takashi Taniguchi,[3] Klaus Richter,[2] Klaus Ensslin,[1] and Thomas Ihn[1]



$$u = E_z d$$

skew inter-layer $A\tilde{B}$ hopping

$$v_3 = -\frac{\sqrt{3}}{2}\frac{\gamma_3 a}{\hbar} \sim 0.1v$$

$$\pi = p_x + ip_y = pe^{i\varphi}$$

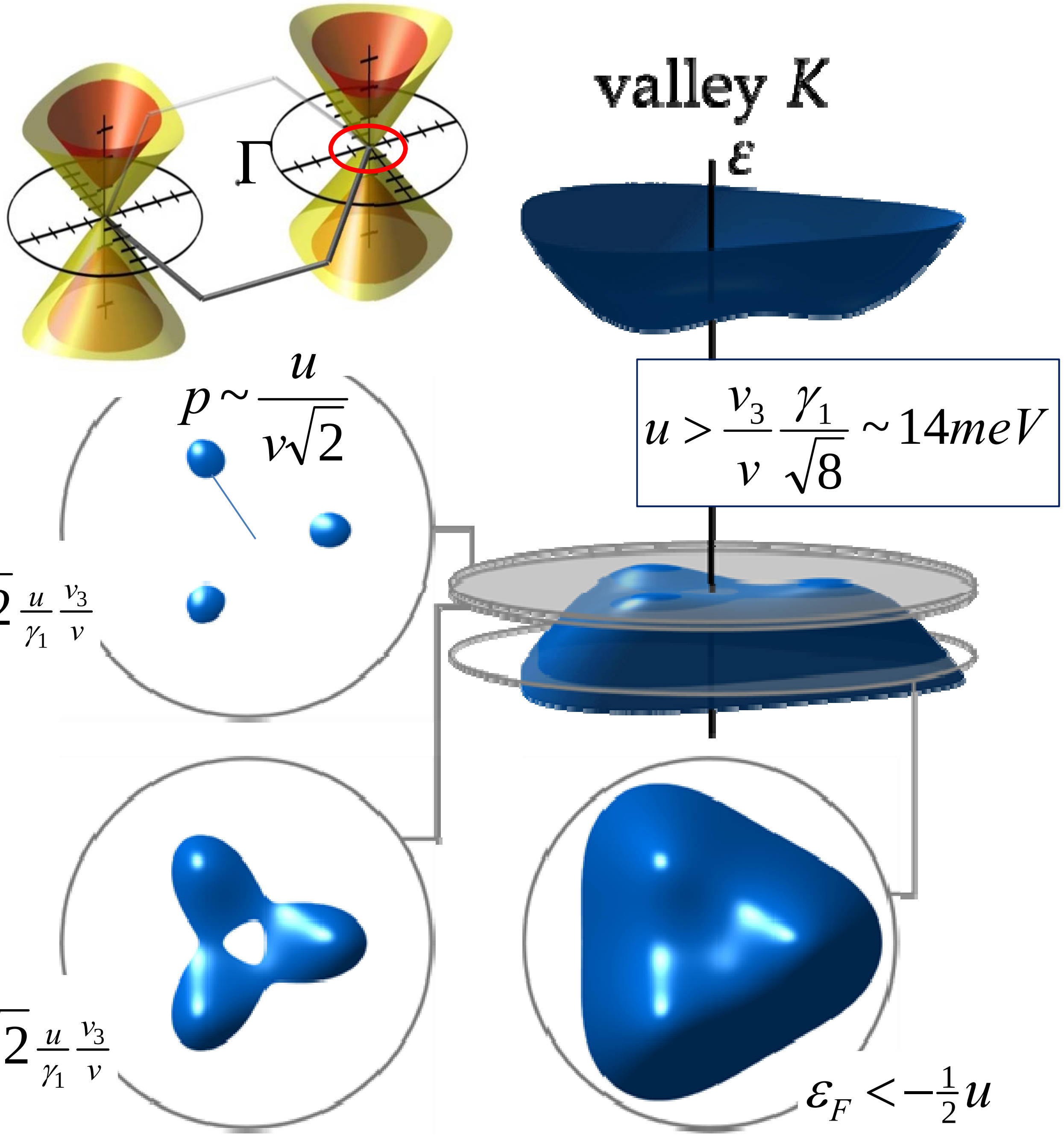
$$H = \begin{pmatrix} \frac{1}{2}u & v_3\pi & 0 & v\pi \\ v_3\pi^+ & -\frac{1}{2}u & v\pi^+ & 0 \\ 0 & v\pi & -\frac{1}{2}u & \gamma_1 \\ v\pi^+ & 0 & \gamma_1 & \frac{1}{2}u \end{pmatrix} \begin{pmatrix} A \\ \tilde{B} \\ \tilde{A} \\ B \end{pmatrix}$$

$$\epsilon_\alpha^2 = \frac{\gamma_1^2}{2} + \frac{u^2}{4} + \left(v^2 + \frac{v_3^2}{2}\right)p^2 + (-1)^\alpha\sqrt{\Gamma}$$

$$\Gamma = \tfrac{1}{4}(\gamma_1^2 - v_3^2p^2)^2 + v^2p^2[\gamma_1^2 + u^2 + v_3^2p^2] + 2\xi\gamma_1 v_3 v^2 p^3 \cos 3\varphi,$$

McCann, Fal'ko - PRL 96, 086805 (2006)

Gapped BLG: intricate band features due to trigonal warping



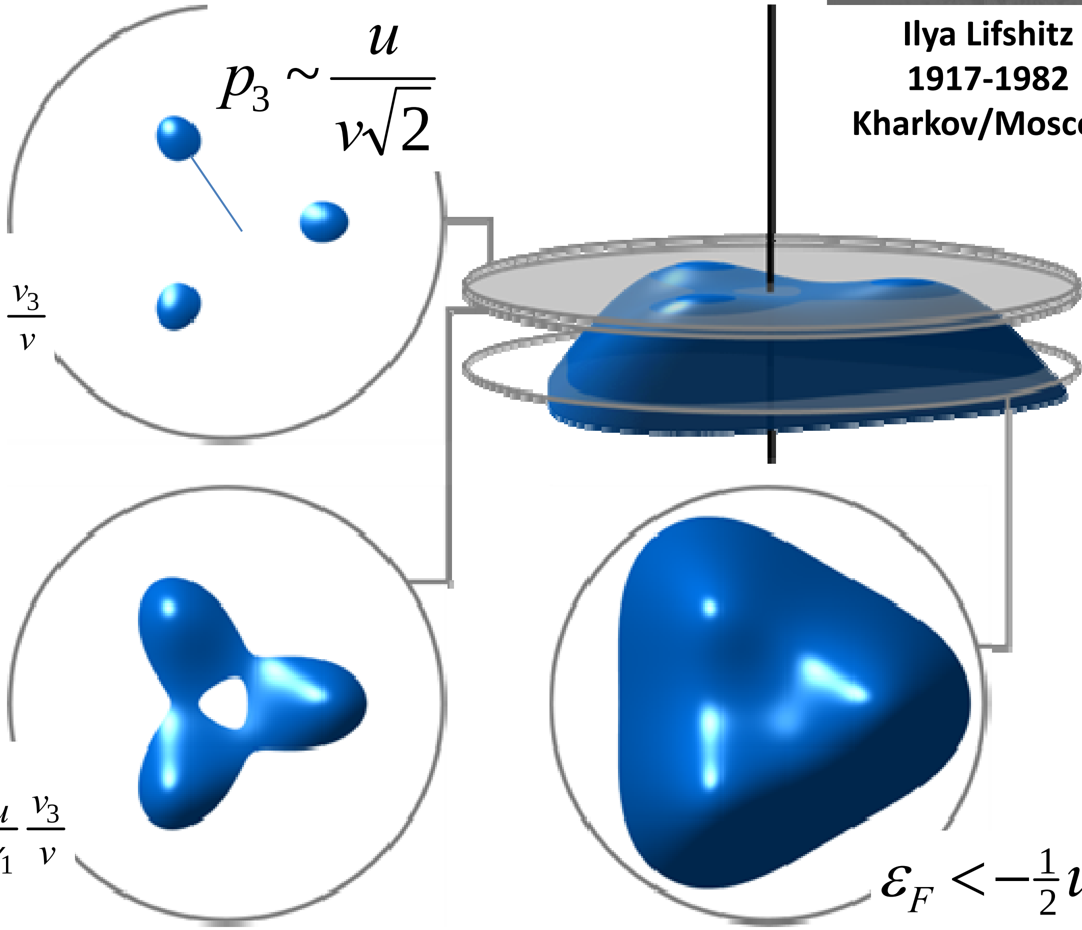
$$\frac{\varepsilon_F}{\frac{1}{2}u} = -1 + \frac{1}{2}\left(\frac{u}{\gamma_1}\right)^2 + \sqrt{2}\,\frac{u}{\gamma_1}\frac{v_3}{v}$$

$$\frac{\varepsilon_F}{\frac{1}{2}u} = -1 + \frac{1}{2}\left(\frac{u}{\gamma_1}\right)^2 - \sqrt{2}\,\frac{u}{\gamma_1}\frac{v_3}{v}$$

Lifshitz transition in metals

- Topology of the Fermi surface changes
- Cyclotron orbits in magnetic field change circulation
- Magnetic breakdown - field mixes disconnected parts of Fermi surfaces, at $\delta p \sim 1/\lambda_B$.

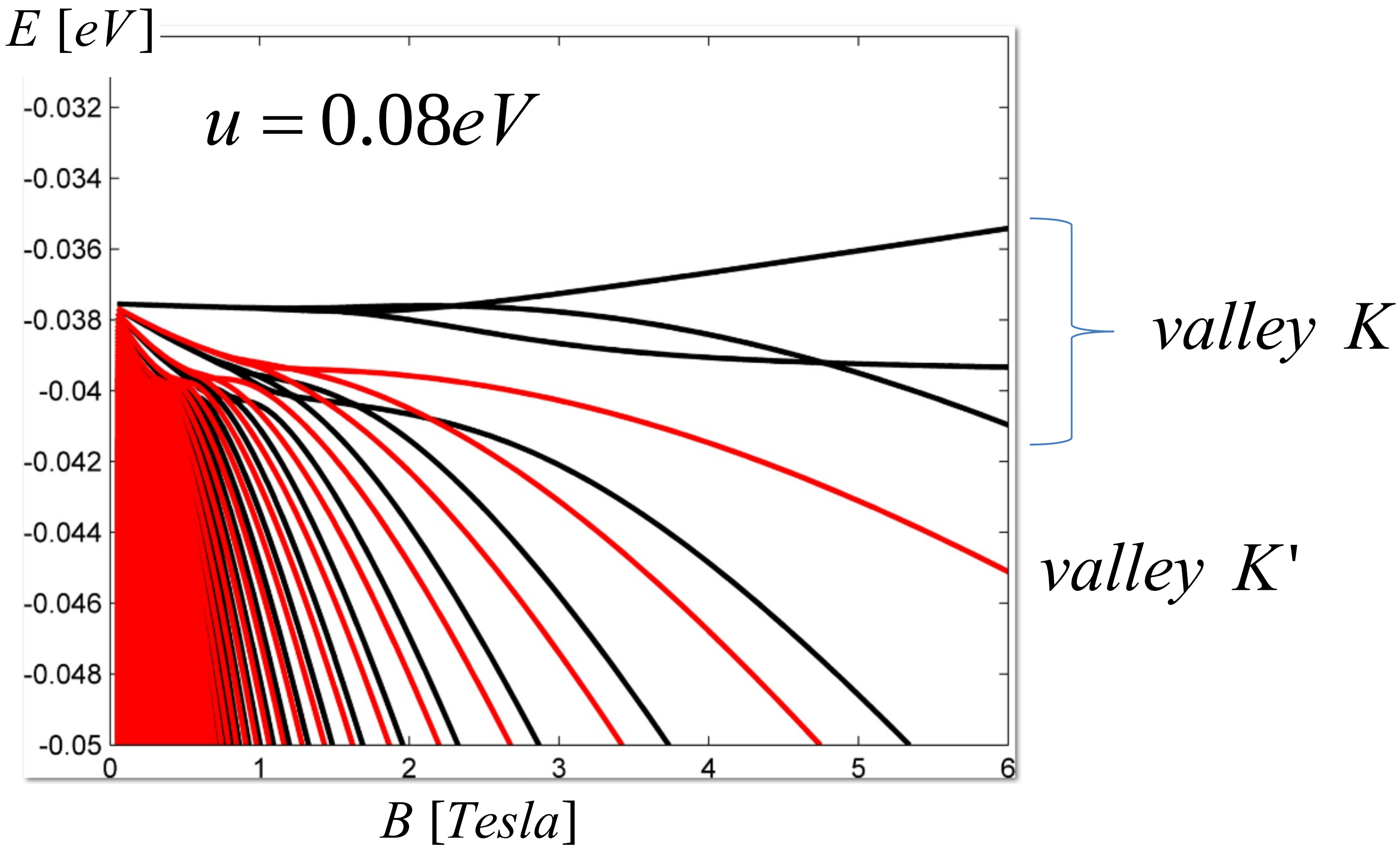
Ilya Lifshitz
1917-1982
Kharkov/Moscow



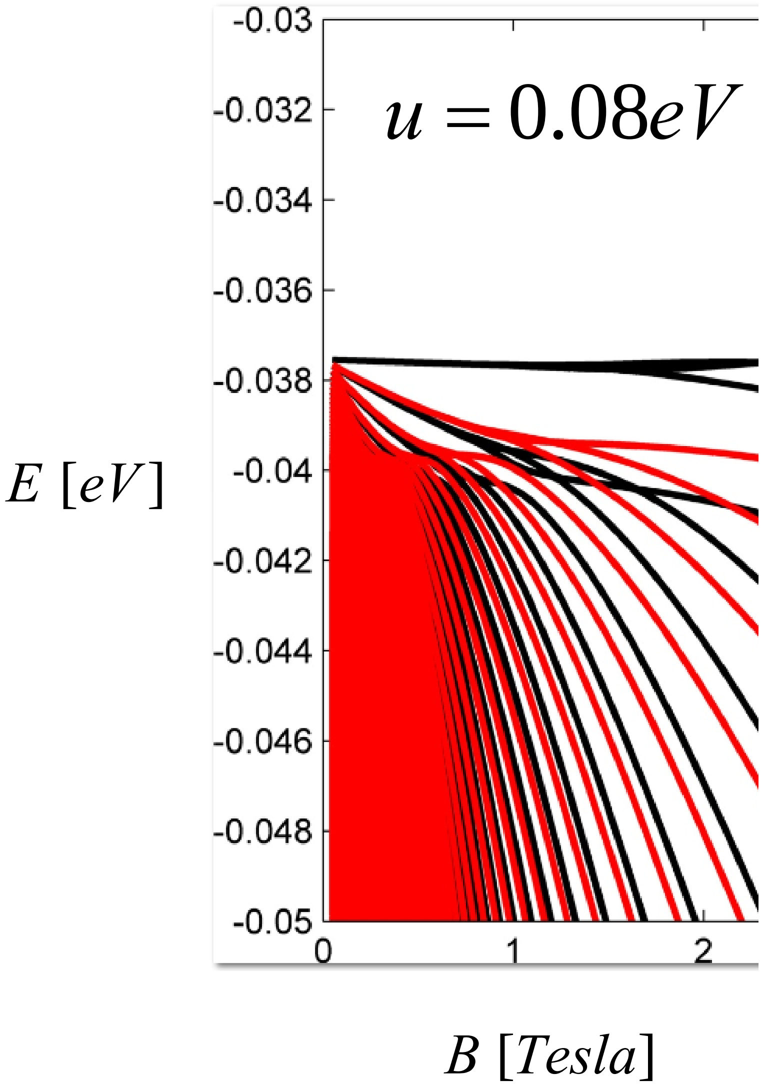
$$\frac{\varepsilon_F}{\frac{1}{2}u} = -1 + \frac{1}{2}\left(\frac{u}{\gamma_1}\right)^2 + \sqrt{2}\,\frac{u}{\gamma_1}\frac{v_3}{v}$$

$$\frac{\varepsilon_F}{\frac{1}{2}u} = -1 + \frac{1}{2}\left(\frac{u}{\gamma_1}\right)^2 - \sqrt{2}\,\frac{u}{\gamma_1}\frac{v_3}{v}$$

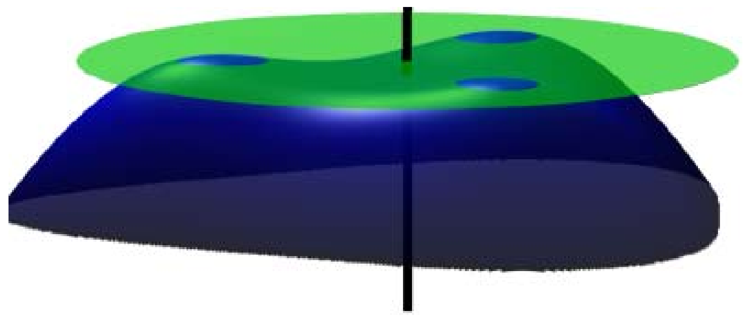
$$H = \begin{pmatrix} \frac{1}{2}u & v_3\pi & 0 & v\pi \\ v_3\pi^+ & -\frac{1}{2}u & v\pi^+ & 0 \\ 0 & v\pi & -\frac{1}{2}u & \gamma_1 \\ v\pi^+ & 0 & \gamma_1 & \frac{1}{2}u \end{pmatrix}$$



6-fold (2 x spin and 3 x orbital) degenerate LL at small magnetic fields

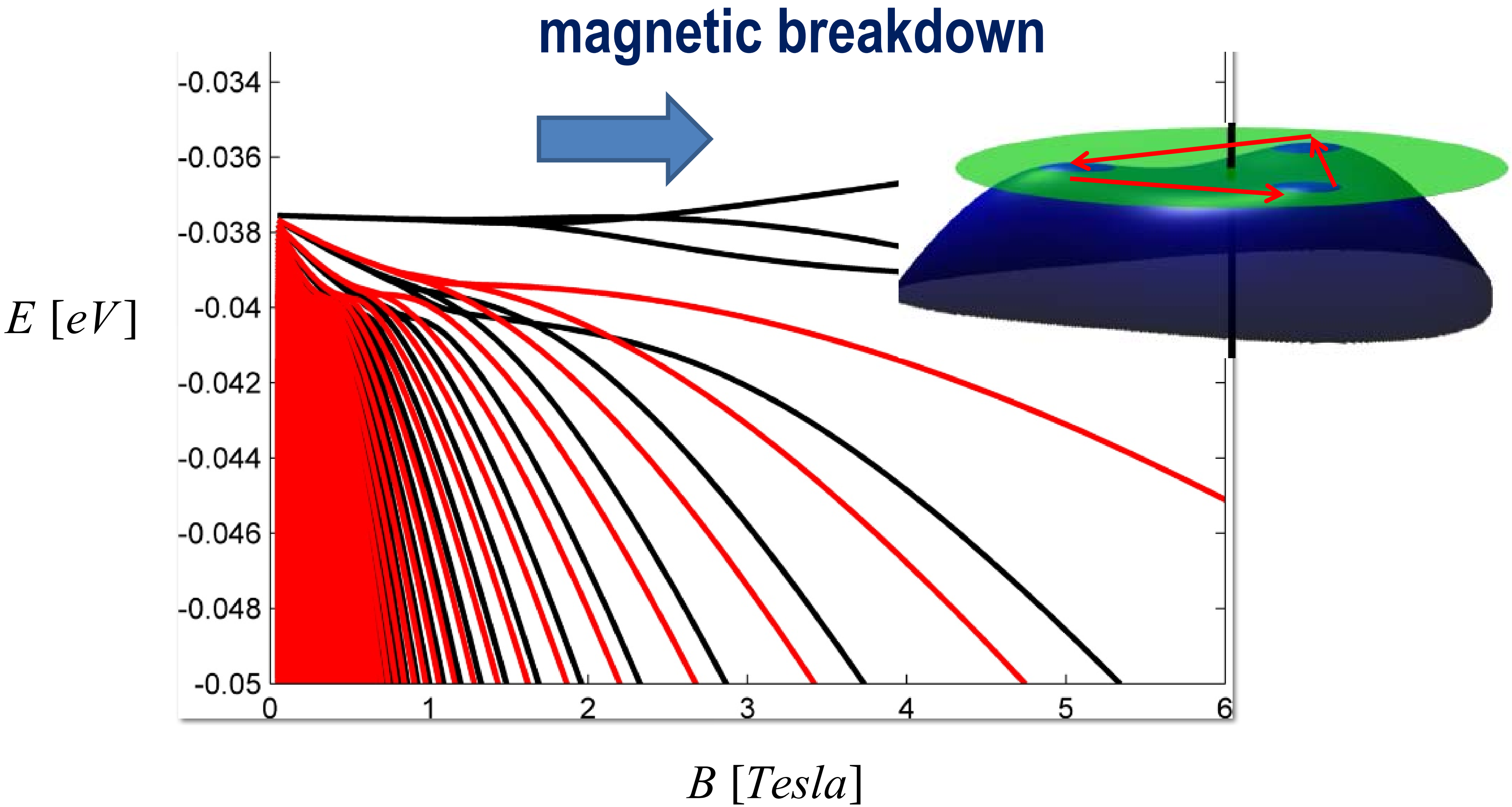


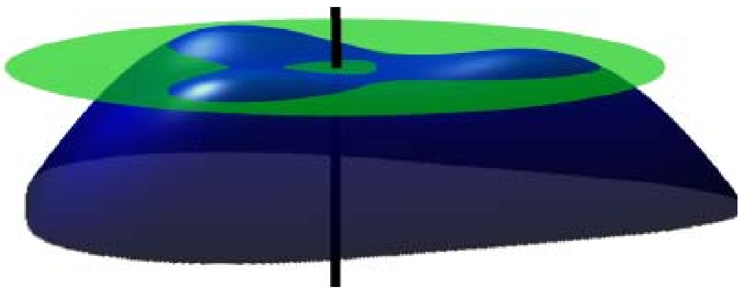
valley K



$\nu = -3$ **spin polarised (ferromagnetic) QHE state**

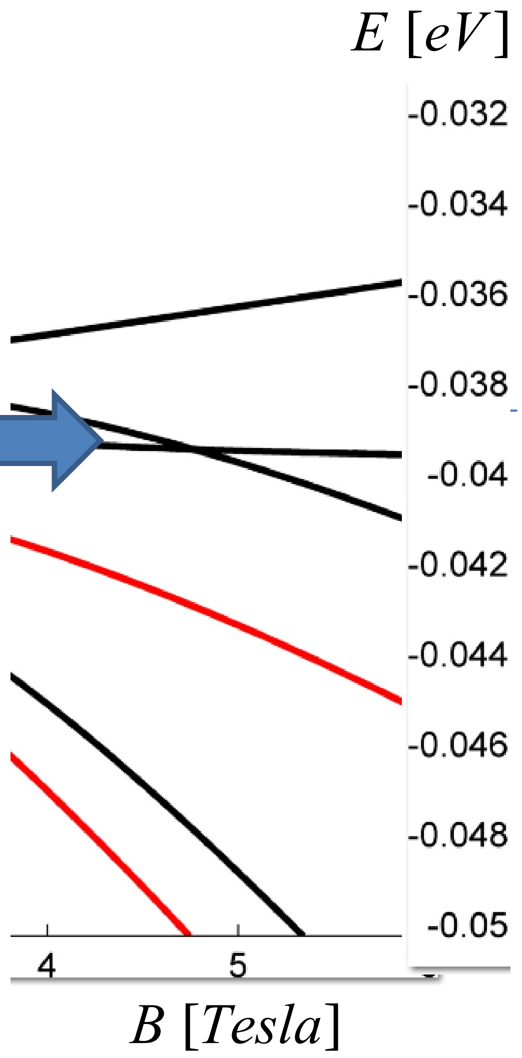
$\nu = -6$ **unpolarised QHE state**


magnetic breakdown
-0.034
-0.036
-0.038
-0.04
-0.042
-0.044
-0.046
-0.048
-0.05
0
1
2
3
4
5
6
E [eV]
B [Tesla]



Landau level crossing

ν = -3,-5 QHFM gaps vanish and ν = -4 undergoes ferromagnetic transition.



ν = 0,-1,-2

ferromagnetic and normal QHE

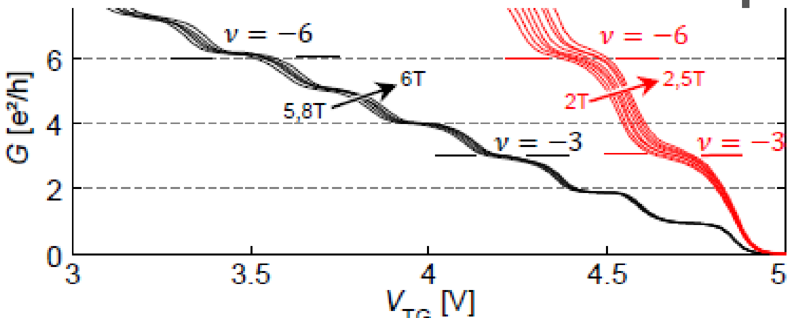
Polarised

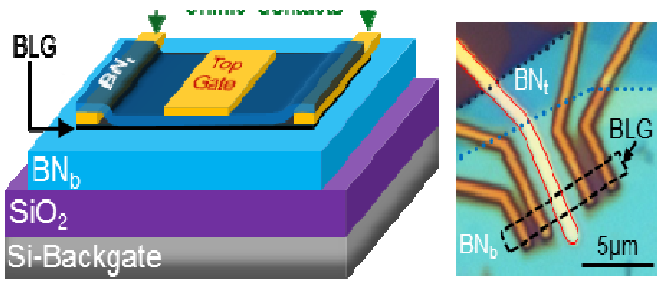
ν = -3,-5

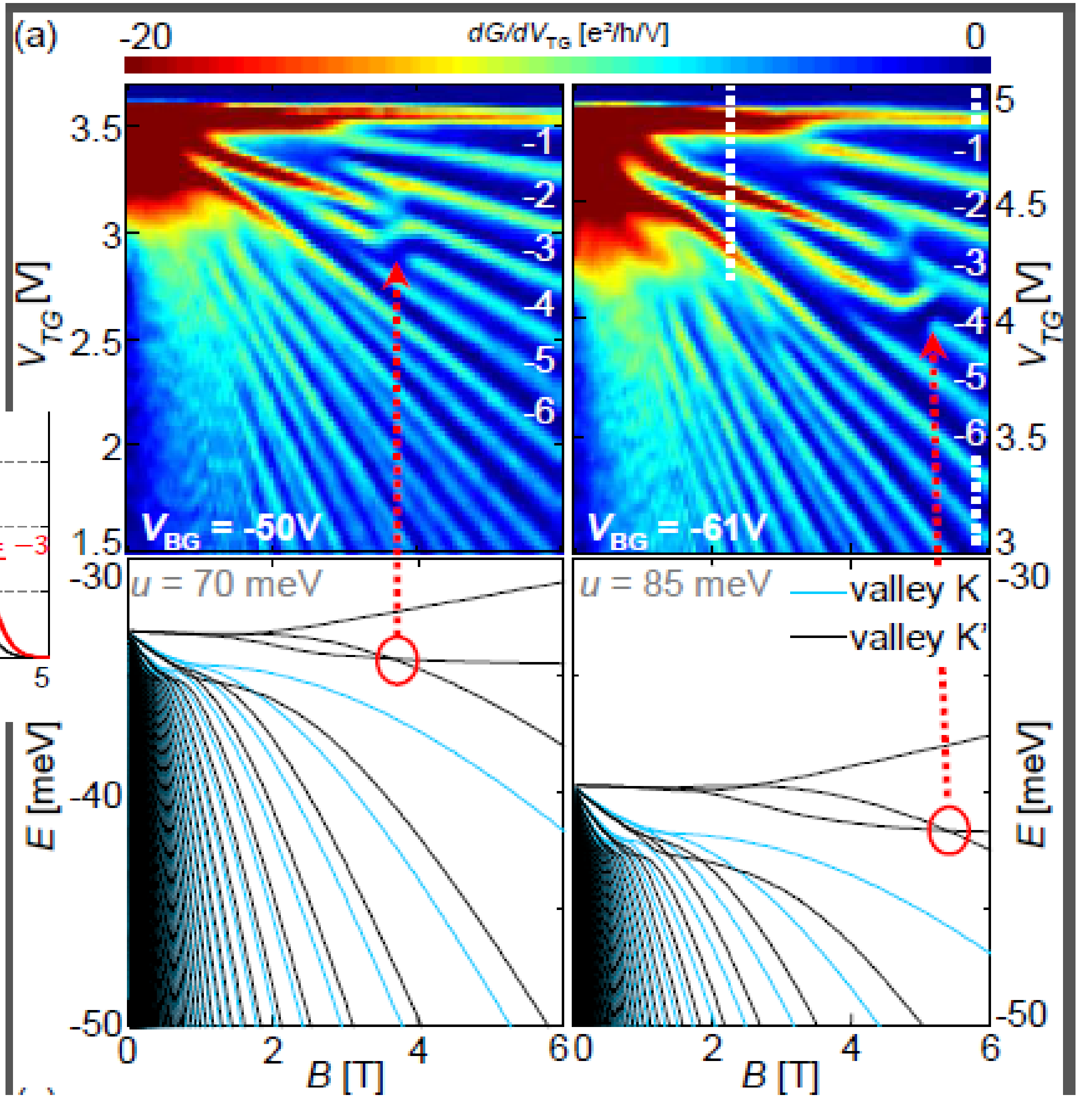
ν =-4,-6

QHE

Lifshitz transition, magnetic breakdown, and phase transitions between QHFM states







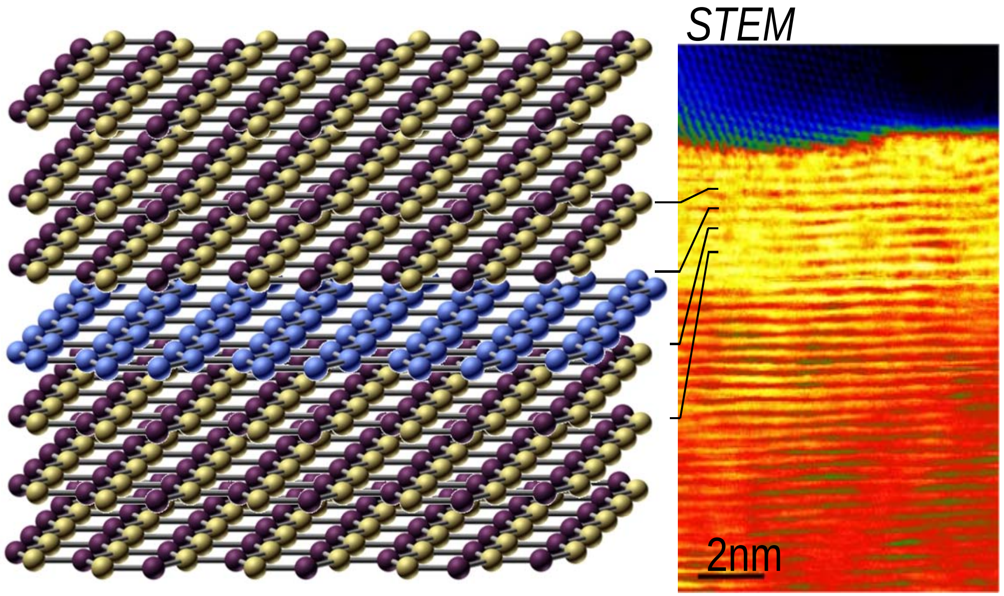
Varlet, Bischoff, Simonet, Watanabe, Taniguchi, Ihn, Ensslin, Mucha-Kruczyński, Fal'ko - PRL 113, 116602 (2014)

✓ Ballistic electrons in hBN-encapsulated graphene

John Wallbank (NGI)
Tom Lane (NGI)
Marcin Mucha-Kruczynski (Bath)
Leonid Glazman (Yale)
Boris Altshuler (Columbia)
Vadim Cheianov (Leiden)

Konstantin Novoselov (NGI)
Roman Gorbachev (NGI)
Leonid Ponomarenko (Lancaster)
Klaus Ensslin (ETH Zurich)
Marek Potemski (CNRS-Grenoble)
Takashi Taniguchi (NIMS)

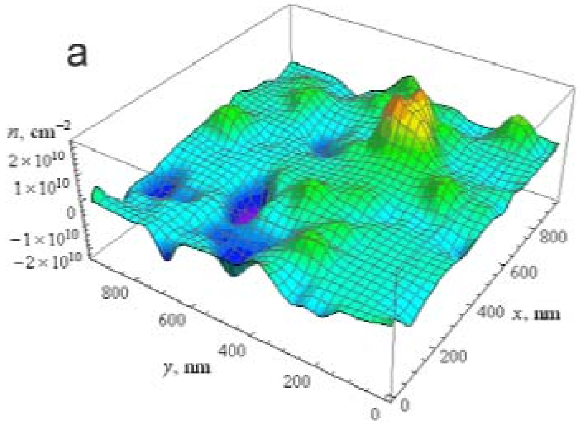
EPSRC
erc
GRAPHENE FLAGSHIP

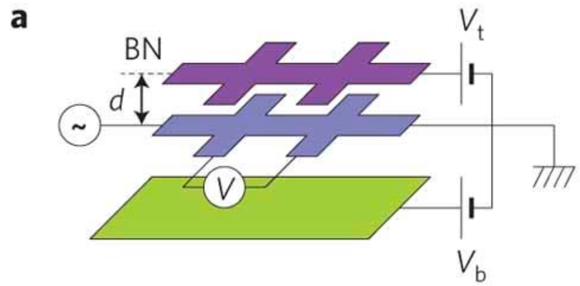


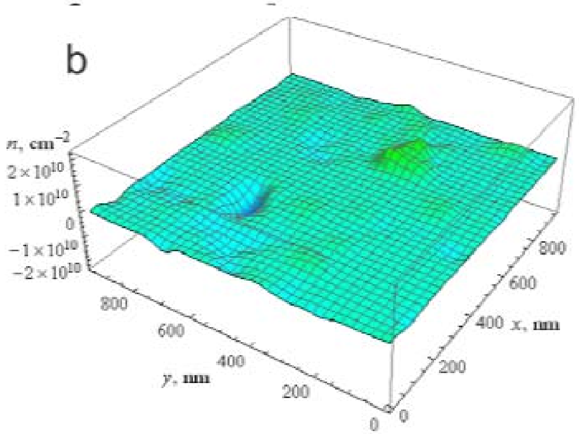
- Graphene at its best: ballistic electrons in graphene (G) encapsulated in van der Waals heterostructures with hexagonal boron nitride (hBN)

- Next lecture: moiré superlattice in aligned graphene – hBN heterostructures and moiré minibands

Graphene with carrier density n_c used as gate in G/hBN/G







Ponomarenko, Geim, Zhukov, Jalil, Morozov, Novoselov, Grigorieva, Hill, Cheianov, Fal'ko, Watanabe, Taniguchi, Gorbachev
Nature Physics 7,958 (2011)

Insulating state in closely gated graphene at *n=0*