Transport in 1D

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SCIENCE TO SHAPE THE FUTURE OF THE ARMY
ARMY RESEARCH OFFICE
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MaNEP
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Plan of the lectures (1)

- Lecture 1:
  Tomonaga-Luttinger liquids
  - What are one dimensional systems
  - Universal physics in one dimension (TLL)
  - Fractionalization of excitations
  - Experimental realizations

  Mobile impurity in a TLL
  - Free impurity and polaron
  - Ferromagnetic liquid; Anderson orthogonality

Plan of the lectures (2)

- Lecture 2:
  Clean TLL
  - Fundamentals of transport in 1D and methods
  - Periodic structure: Mott and band insulators
  - A.c. and d.c. conductivity
  - Experimental realizations

  Transverse conductivity

  Magnetic field effects
  - Bosonic ladders; Meissner effect
  - Hall effect

Plan of the lectures (3)

- Lecture 3:
  Effects of disorder
  - Disorder and interactions in 1D systems
  - T=0 properties: localization of interacting particles
  - Finite temperature: transport with a bath (VRH)
  - Other types of ``disorder'' (quasiperiodics, etc.)
  - Experimental realizations

Why one dimension ?

Three urban legends about 1D

- It is a toy model to understand higher dimensional systems.
- It does not exist in nature ! This is only for theorists !
- Everything is understood there anyway !

Why 1D

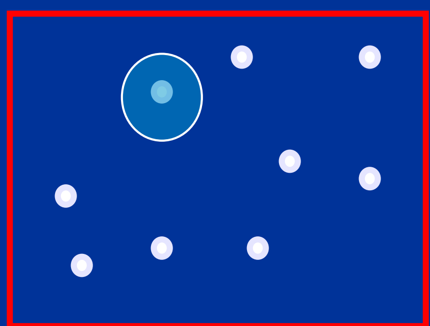
A good reason to work on 1D

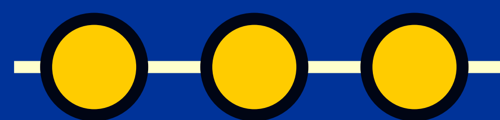
However, my personal reason for working on one-dimensional problems is merely that they are fun. A man grows stale if he works all the time on the insoluble and a trip to the beautiful work of one dimension will refresh his imagination better than a dose of LSD.

Freeman Dyson (1967)

One dimension is specially interesting

- No individual excitation can exist (only collective ones)



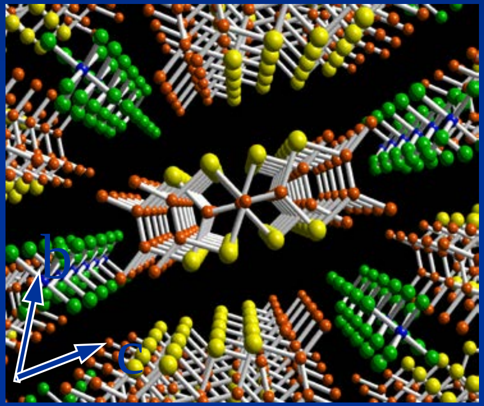


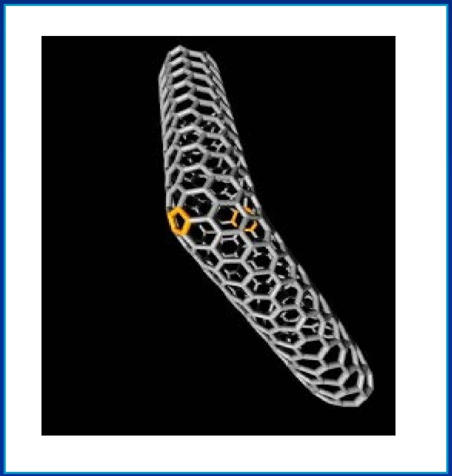
- Strong quantum fluctuations

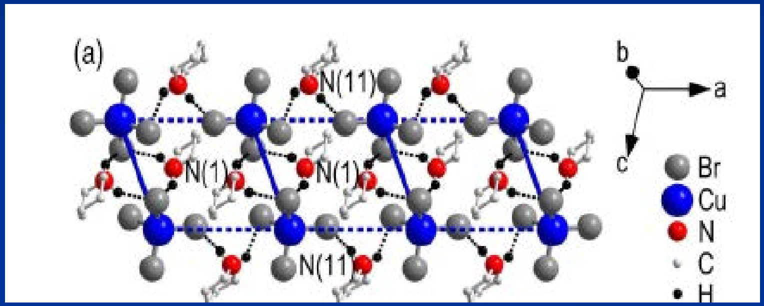
$$\psi = |\psi| e^{i\theta}$$

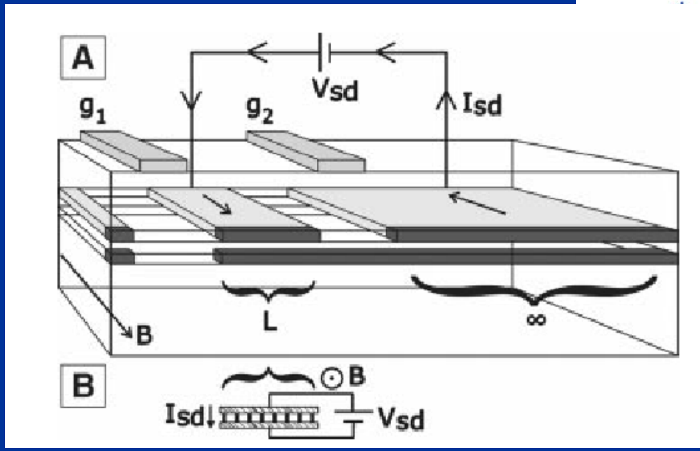
Difficult to order

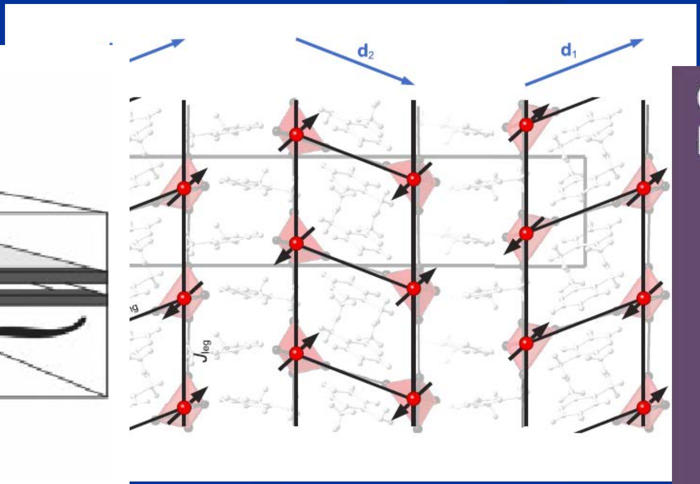
Many CM or cold atoms Systems

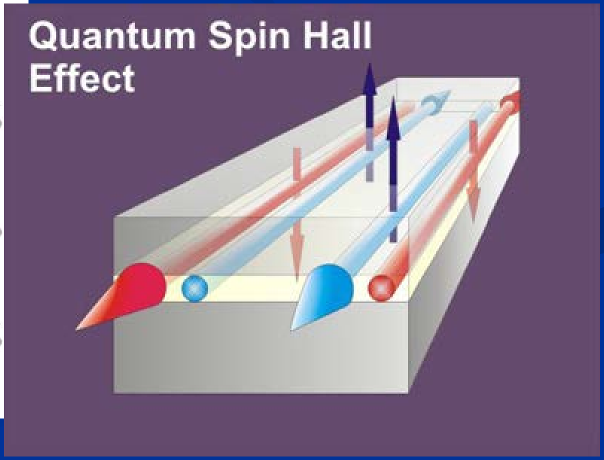




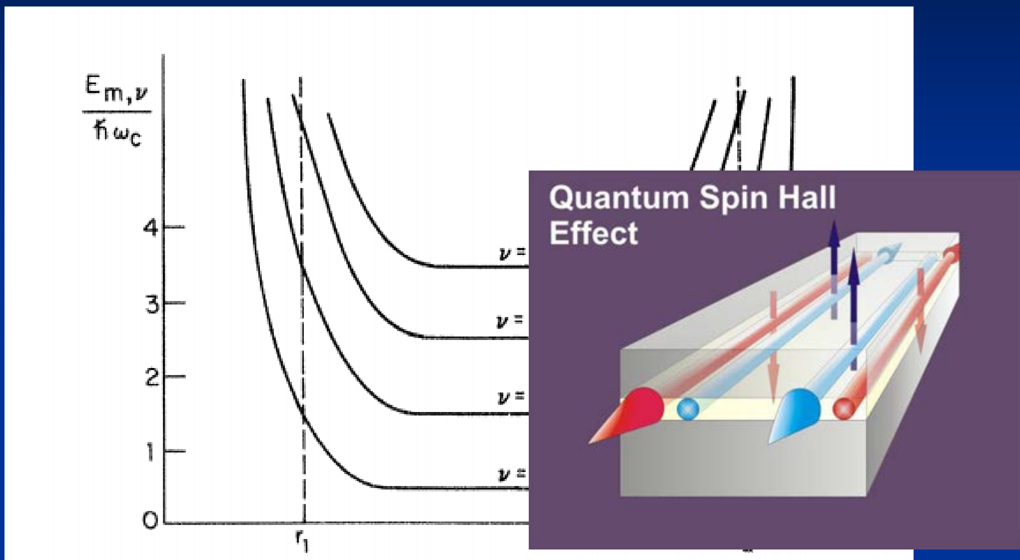


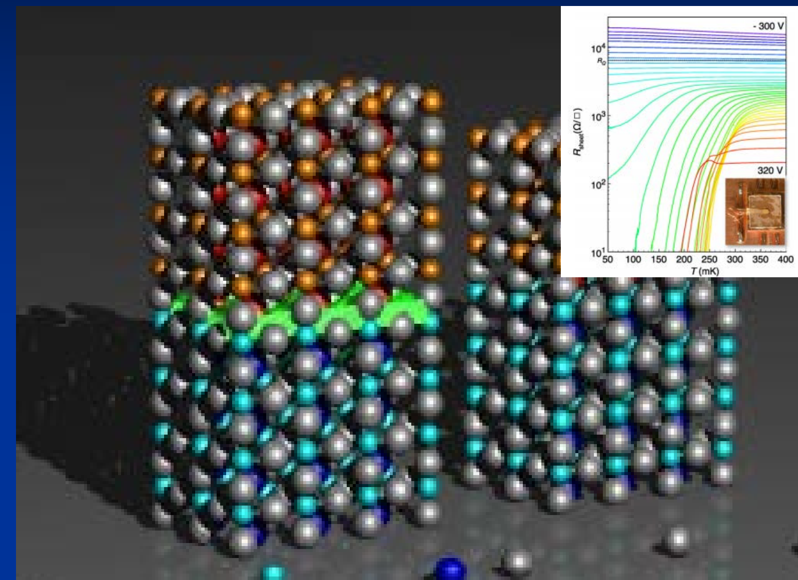






Physics at the edge





Presence of edge
(B. I. Halperin)

LaO/StO interface
(JM Triscone et al.)

Quantum hall effect
Topological insulators….

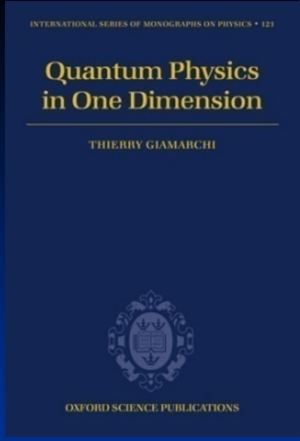
Superconductivity
between insulators…

Drastic evolution of the 1d world

- New methods (DMRG, correlations from BA, etc.)
- New systems (cold atoms, magnetic insulators, etc.)
- New questions (strong SOC, out of equilibrium, etc)

One dimension

General References



TG, Quantum physics in one dimension, Oxford (2004)

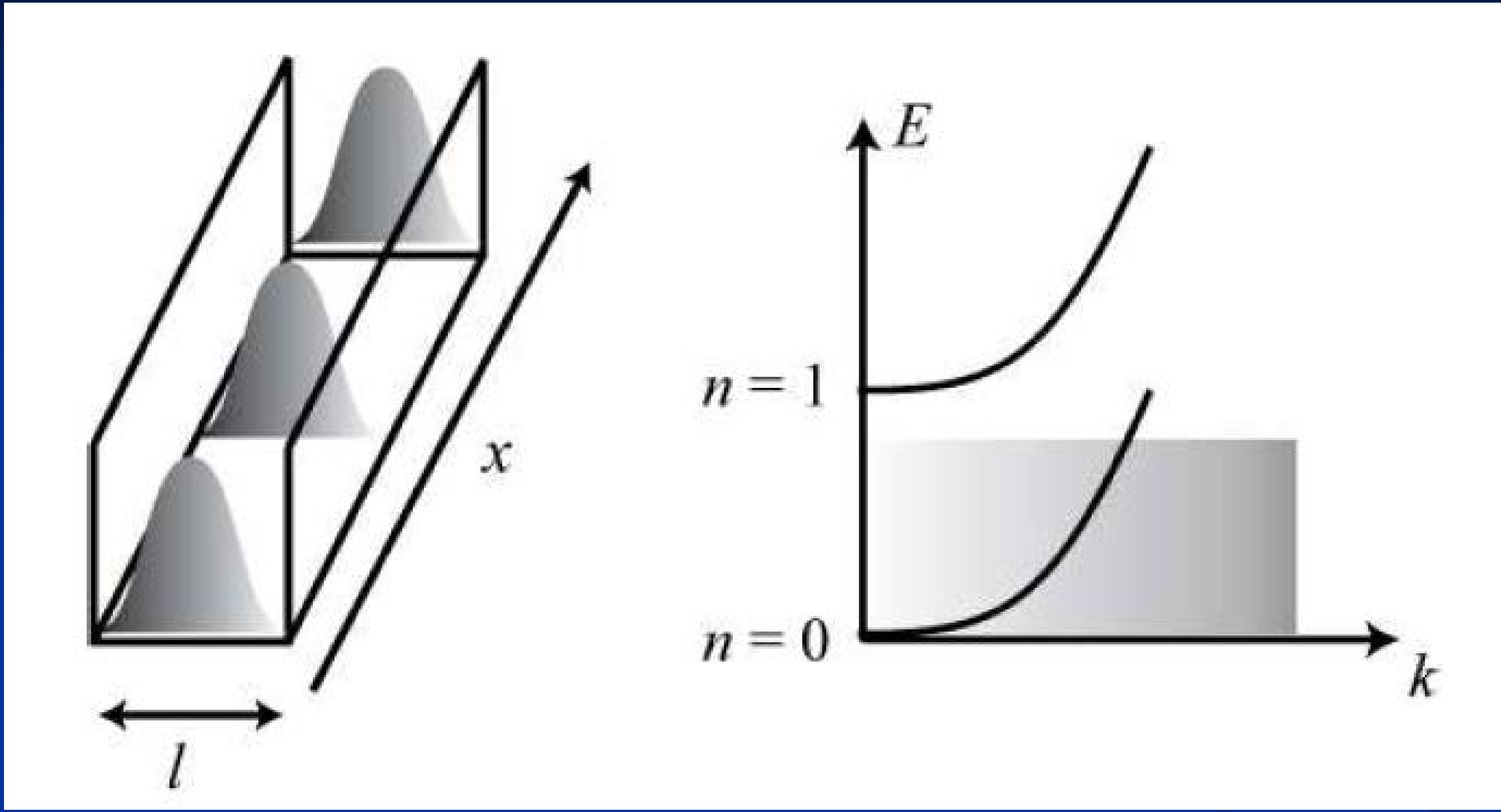
TG in ``Understanding Q. Phase Transitions", Ed. L. Carr, CRC Press (2010)

M. Cazalilla et al.,
Rev. Mod. Phys.83 1405 (2011)

TG, Int J. Mod. Phys. B 26 1244004 (2012)

TG, C. R. Acad. Sci. 17 322 (2016)

How to treat ?



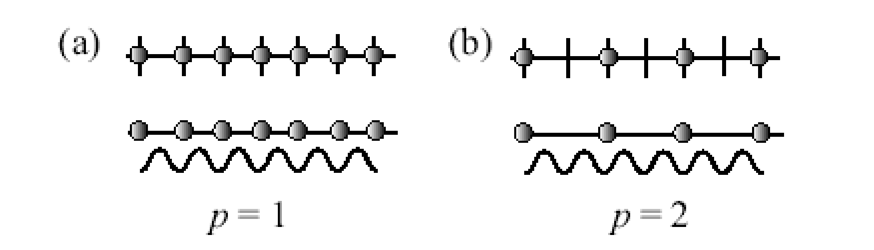
x
l
E
n = 1
n = 0
k

Typical problem (e.g. Bosons)

- Continuum:

$$H = \int dx \frac{(\nabla\psi)^\dagger(\nabla\psi)}{2M} + \frac{1}{2}\int dx\, dx'\, V(x-x')\rho(x)\rho(x') - \mu\int dx\, \rho(x)$$

- Lattice:



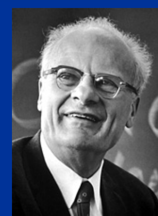
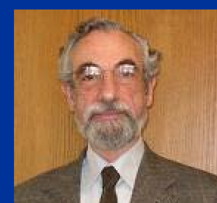
$$H = -J\sum_{\langle i,j\rangle} b_i^\dagger b_j + U\sum_i n_i(n_i-1) - \mu\sum_i n_i$$

- ``Standard" many body theory



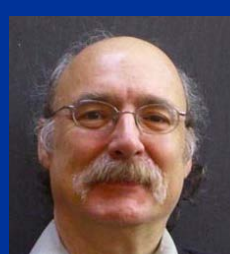


- Exact Solutions (Bethe ansatz)

- Field theories (bosonization, CFT)





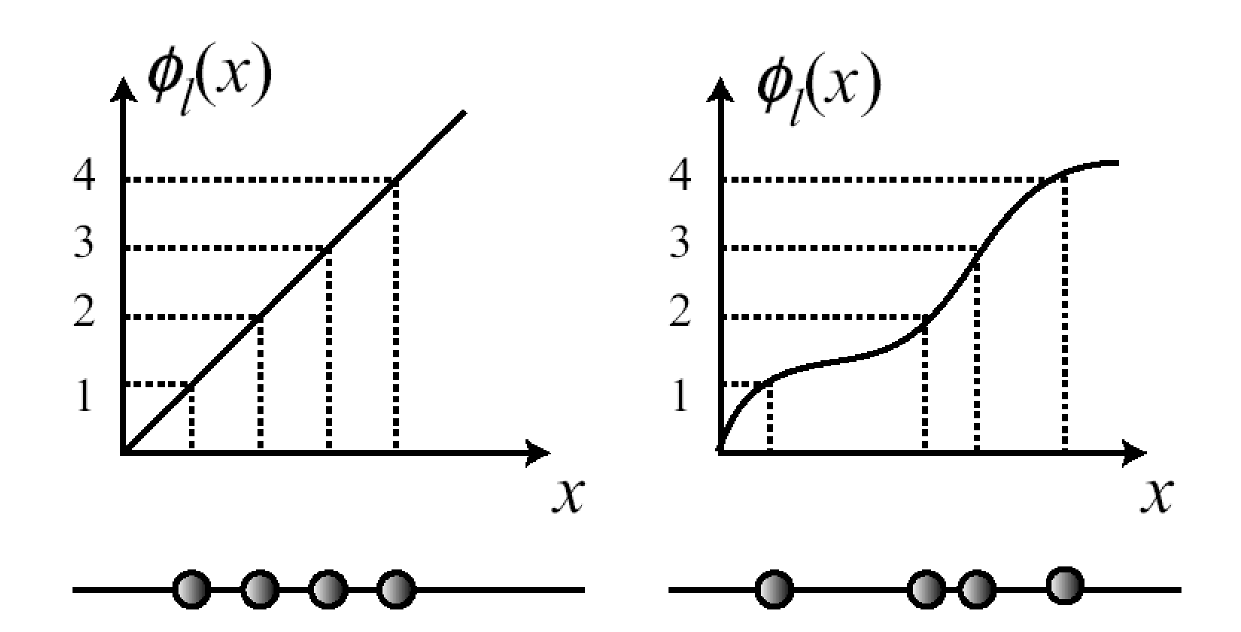
- Numerics (DMRG, MC, etc.)


Luttinger liquid physics

Labelling the particles

$$\rho(x) = \sum_i \delta(x - x_i)$$
$$= \sum_n |\nabla\phi_l(x)|\delta(\phi_l(x) - 2\pi n)$$

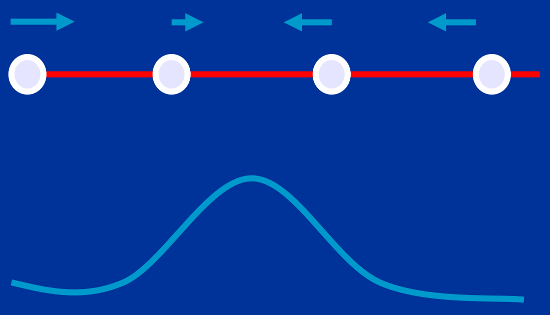
1D: unique way of labelling

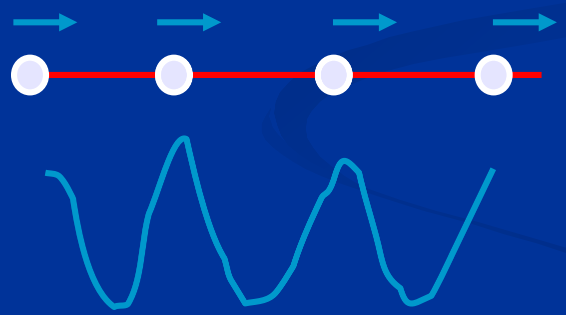


$$\phi_l(x) = 2\pi\rho_0 x - 2\phi(x)$$

$$\rho(x) = \left[\rho_0 - \frac{1}{\pi}\nabla\phi(x)\right]\sum_p e^{i2p(\pi\rho_0 x - \phi(x))}$$

ϕ(x) varies slowly





CDW

$q \sim 0$ $q \sim 2\pi\rho_0$

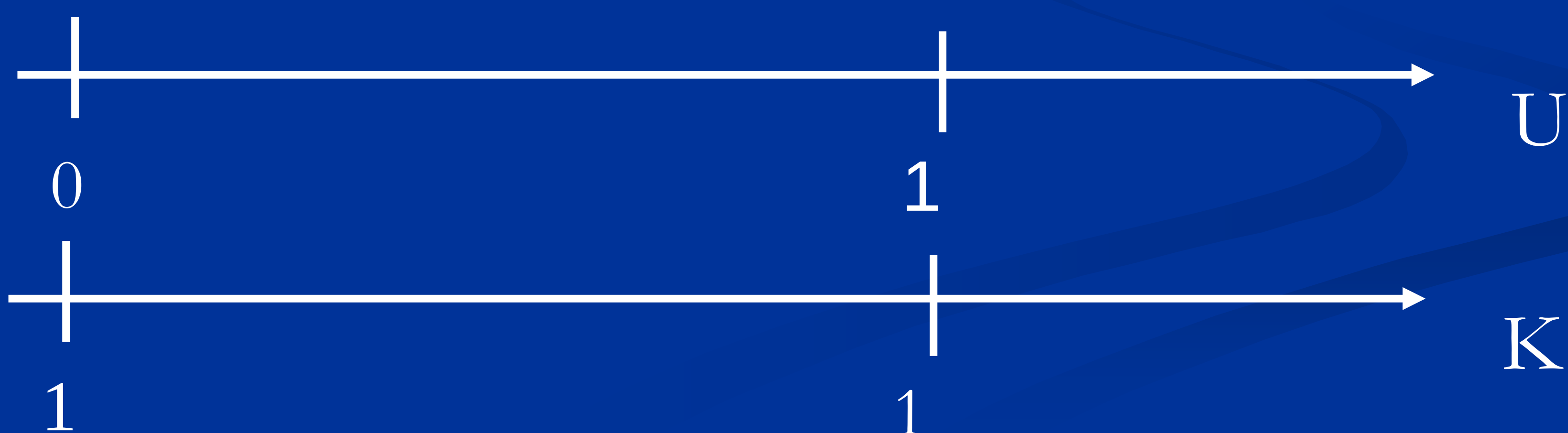
$$\psi^\dagger(x) = [\rho(x)]^{1/2} e^{-i\theta(x)}$$

θ: superfluid phase

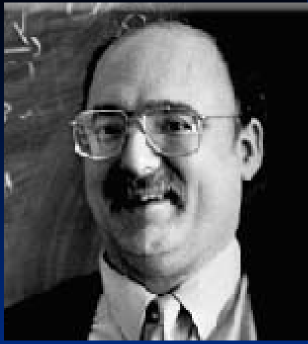
$$[\frac{1}{\pi}\nabla\phi(x), \theta(x')] = -i\delta(x - x')$$

Quantum fluctuations

$$H = \frac{\hbar}{2\pi}\int dx[\frac{uK}{\hbar^2}(\pi\Pi(x))^2 + \frac{u}{K}(\nabla\phi(x))^2]$$



Luttinger liquid concept

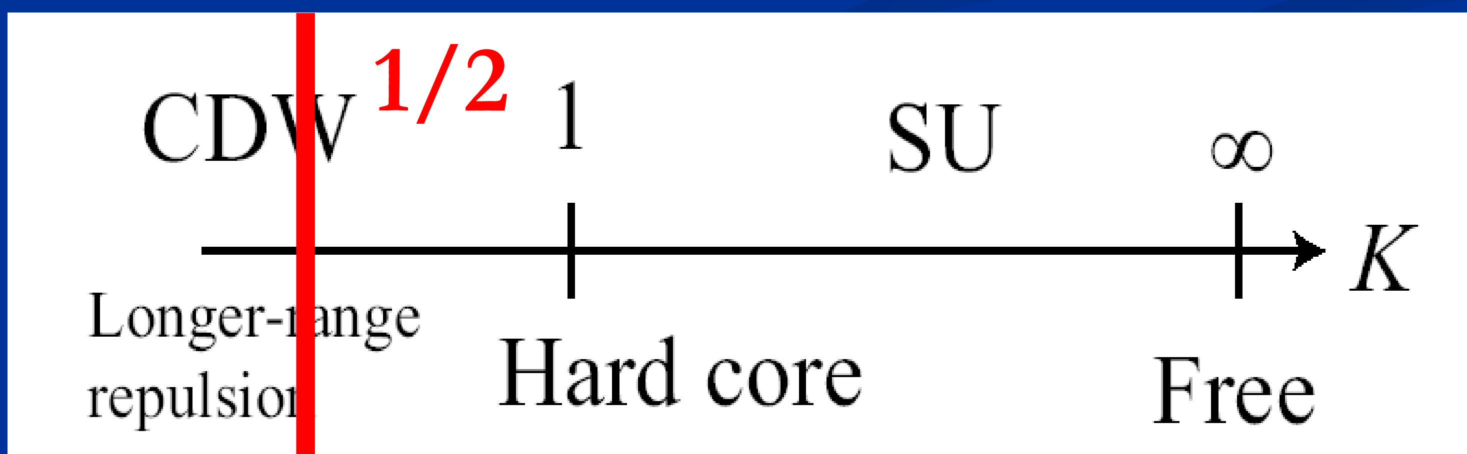


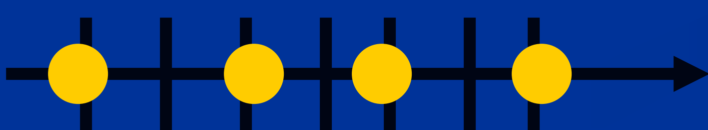
- How much is perturbative ?
- Nothing (Haldane):
  provided the correct u,K are used
- Low energy properties: Luttinger liquid
  (fermions, bosons, spins…)

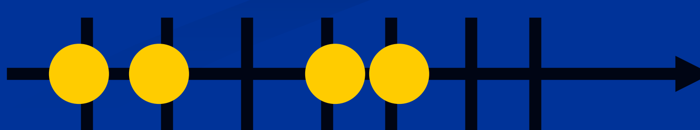
Correlations

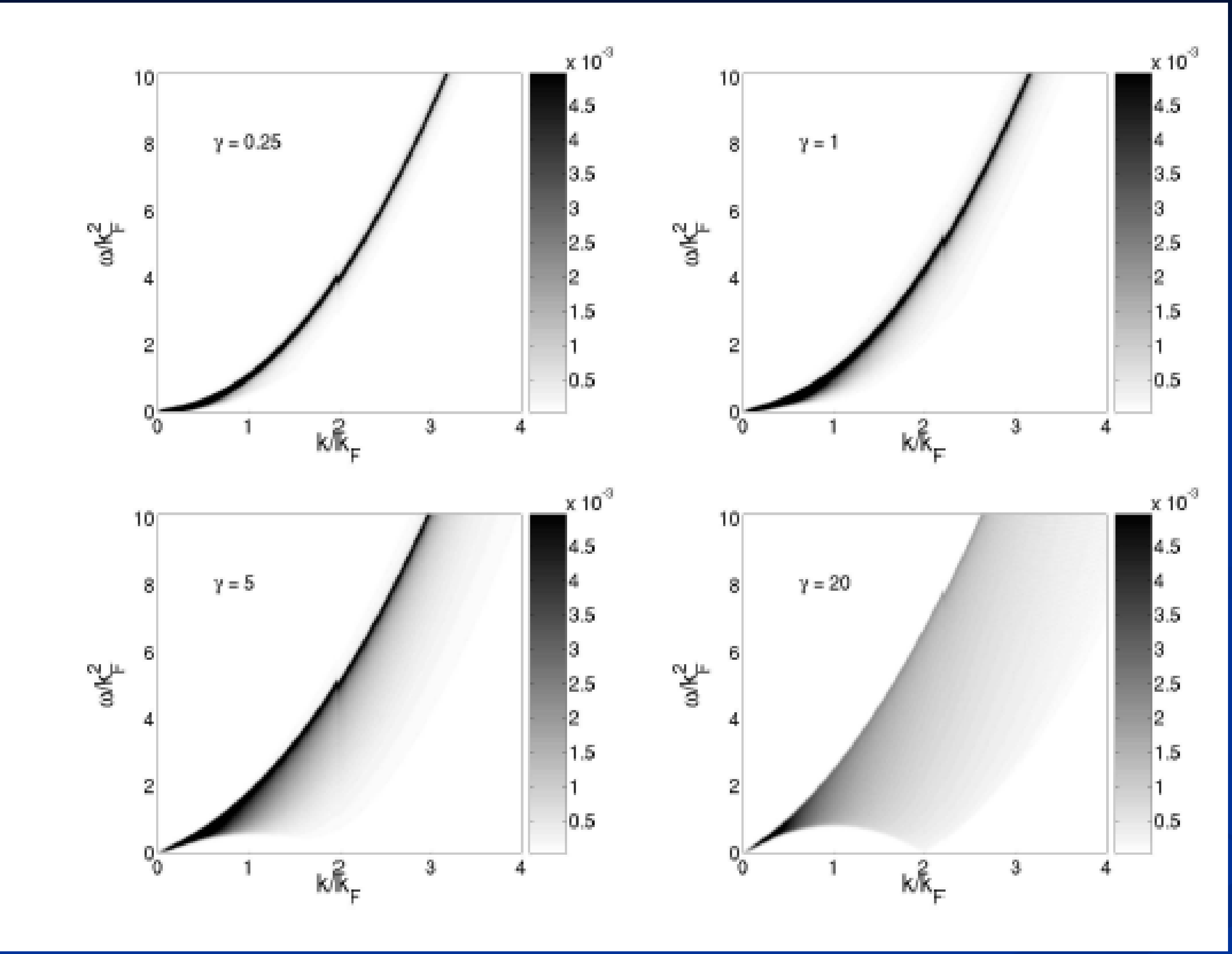
$$\langle \psi(r)\psi^\dagger(0)\rangle = A_1 \left(\frac{\alpha}{r}\right)^{\frac{1}{2K}} + \cdots$$

$$\langle \rho(r)\rho(0)\rangle = \rho_0^2 + \frac{K}{2\pi^2}\frac{y_\alpha^2 - x^2}{(y_\alpha^2 + x^2)^2} + A_3 \cos(2\pi\rho_0 x)\left(\frac{1}{r}\right)^{2K} + \cdots$$





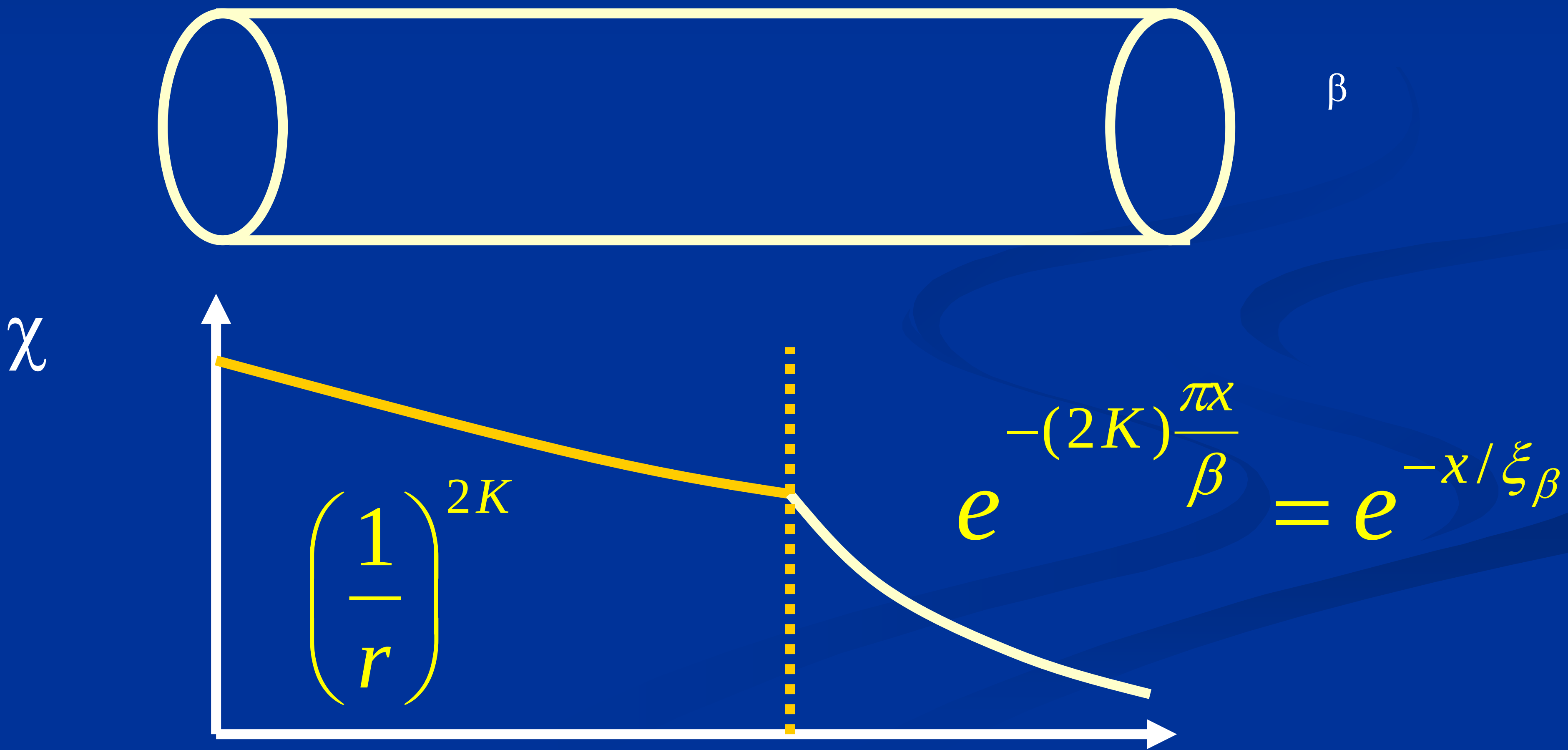




S(q,!) J.S. Caux et al PRA 74 031605 (2006)

Finite temperature

Conformal theory



Other 1D systems

Spins

Use boson or fermions mapping

$$S^{+} = (-1)^{i} e^{i\theta} + e^{i\theta} \cos(2\phi)$$

$$S^{z} = \frac{-1}{\pi} \nabla \phi + (\text{-}1)^{i} \cos(2\phi)$$

Powerlaw correlation functions

$$\langle S^z(x,0) S^z(0,0) \rangle = C_1 \frac{1}{x^2} + C_2 (-1)^x \left(\frac{1}{x}\right)^{2K}$$

$$\langle S^+(x,0) S^-(0,0) \rangle = C_3 \left(\frac{1}{x}\right)^{2K + \frac{1}{2K}} + C_4 (-1)^x \left(\frac{1}{x}\right)^{\frac{1}{2K}}$$

Non universal exponents K(h,J)

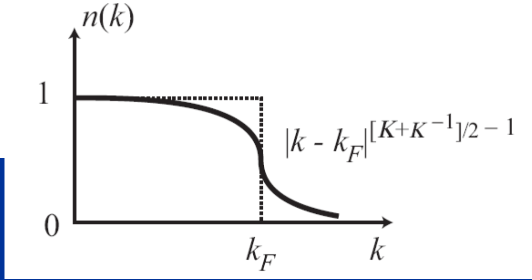
Fermions

$$\psi_F^\dagger(x) = \psi_B^\dagger(x) e^{i\frac{1}{2}\phi_l(x)}$$

$$\psi_F^\dagger(x) = [\rho_0 - \frac{1}{\pi}\nabla\phi(x)]^{1/2} \sum_p e^{i(2p+1)(\pi\rho_0 x - \phi(x))} e^{-i\theta(x)}$$

Right ($+k_F$) and left ($-k_F$) particles

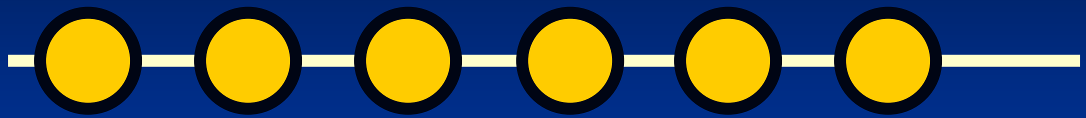
$$\langle\rho(x,\tau)\rho(0)\rangle = \rho_0^2 + \frac{K}{2\pi^2}\frac{y_\alpha^2 - x^2}{(x^2+y_\alpha^2)^2} + \rho_0^2 A_2 \cos(2\pi\rho_0 x)\left(\frac{\alpha}{r}\right)^{2K} + \rho_0^2 A_4 \cos(4\pi\rho_0 x)\left(\frac{\alpha}{r}\right)^{8K} + \cdots$$



Calculation of Luttinger parameters

- Trick: use thermodynamics and BA or numerics
- Compressibility: u/K
- Response to a twist in boundary: u K
- Specific heat : T/u
- Etc.

Tonks limit



U = 1 : spinless fermions

Not for n(k) : $\psi_F \neq \psi_B$

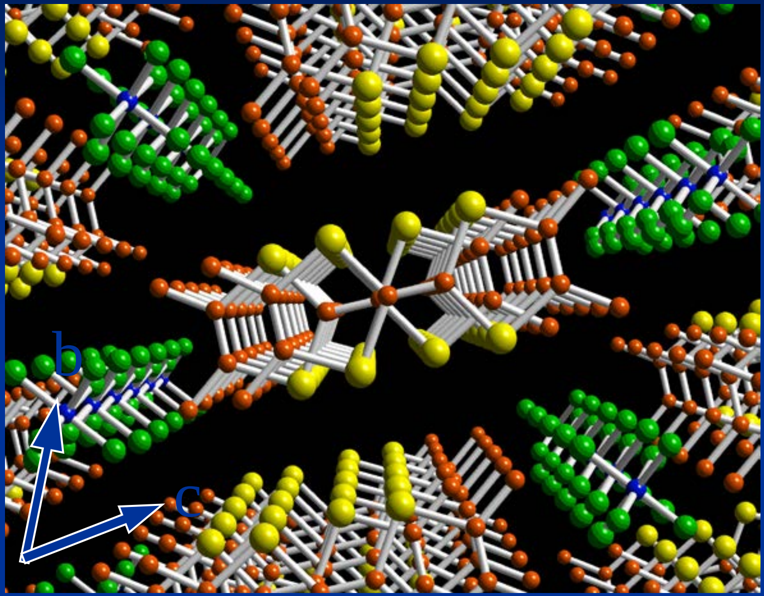
Free fermions: $$\langle \rho(x)\rho(0)\rangle \propto \cos(2k_F x)\left(\frac{1}{x}\right)^2$$

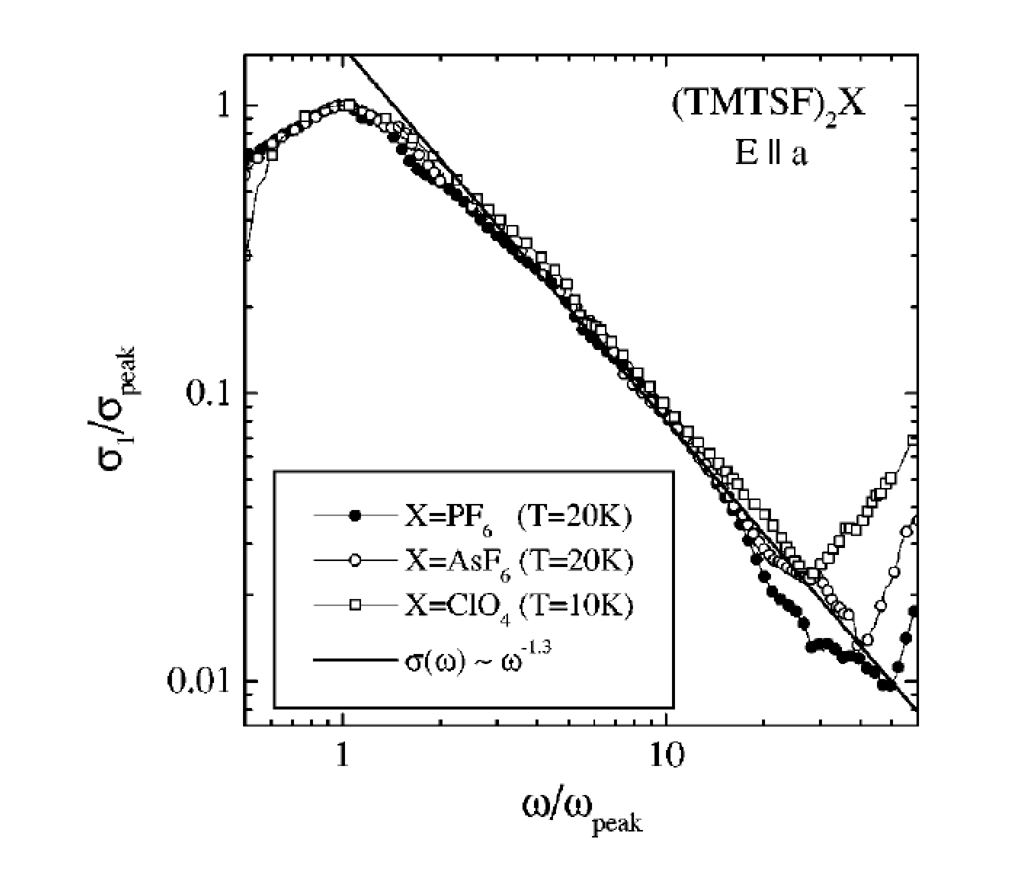
K=1

Note: $$\langle \psi_B(x)\psi_B(0)^\dagger\rangle \propto \left(\frac{1}{x}\right)^{1/2}$$

Tests of Luttinger liquids

Organic conductors



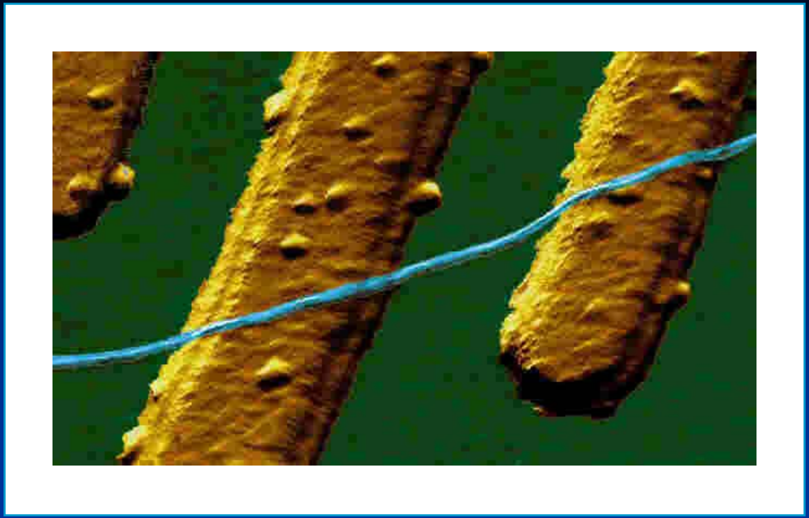


$$\sigma(\omega) \sim \omega^{-\nu}$$

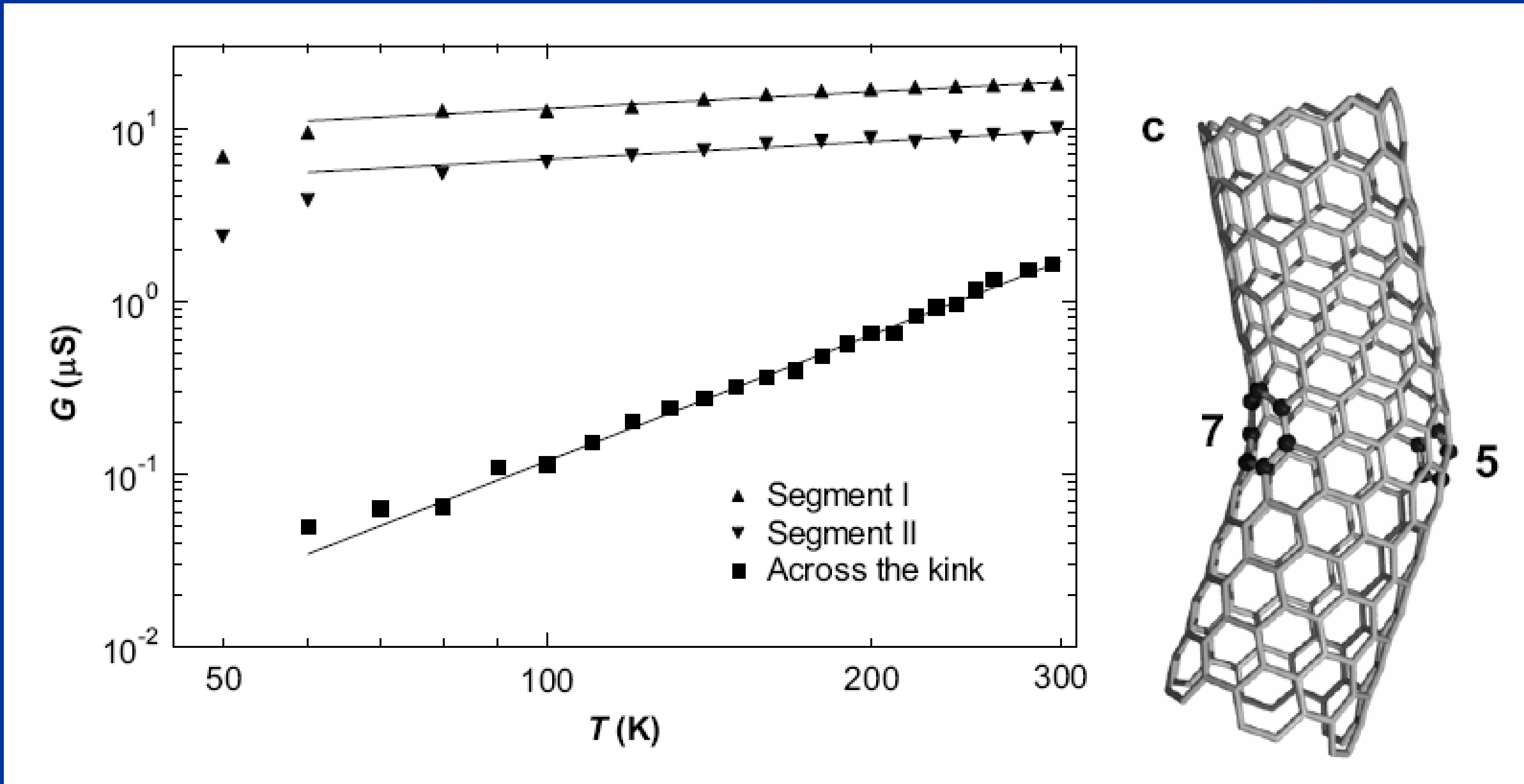
TG PRB (91) :
Physica B 230 (1996)

A. Schwartz et al. PRB 58 1261 (1998)

First observation of LL/powerlaw !!



Z. Yao et al. Nature 402 273 (1999)



And many others…..

Edge states in quantum hall effect

Josephson junction arrays

Helium in capillaries

Photons in waveguides

Adatoms on vicinal surfaces

………..

Cold atoms

General references

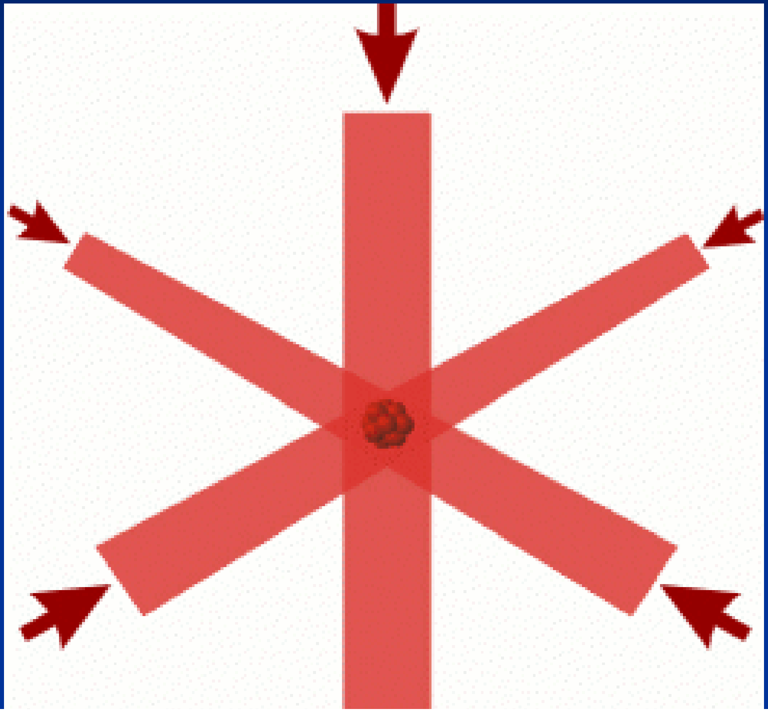
References:

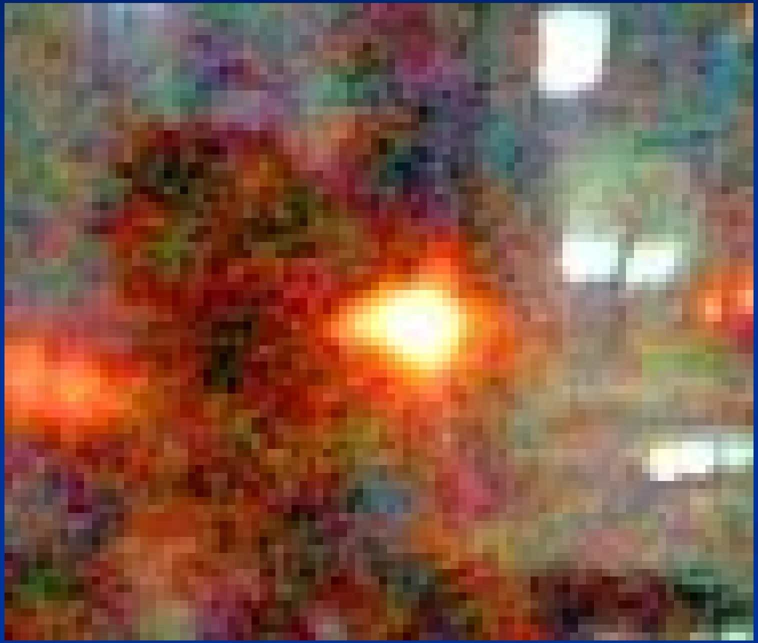
Immanuel Bloch, Jean Dalibard, and Wilhelm Zwerger
Rev. Mod. Phys. **80**, 885 (2008)

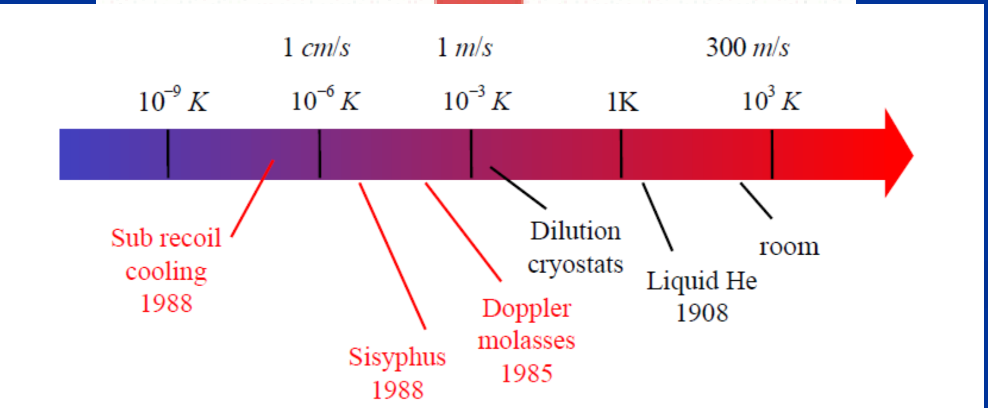
T. Esslinger:
Condensed Matter Physics 1 (2010).

A. Georges, T. Giamarchi
Les houches 2012, arXiv:1308.2684

Atom trapping and cooling

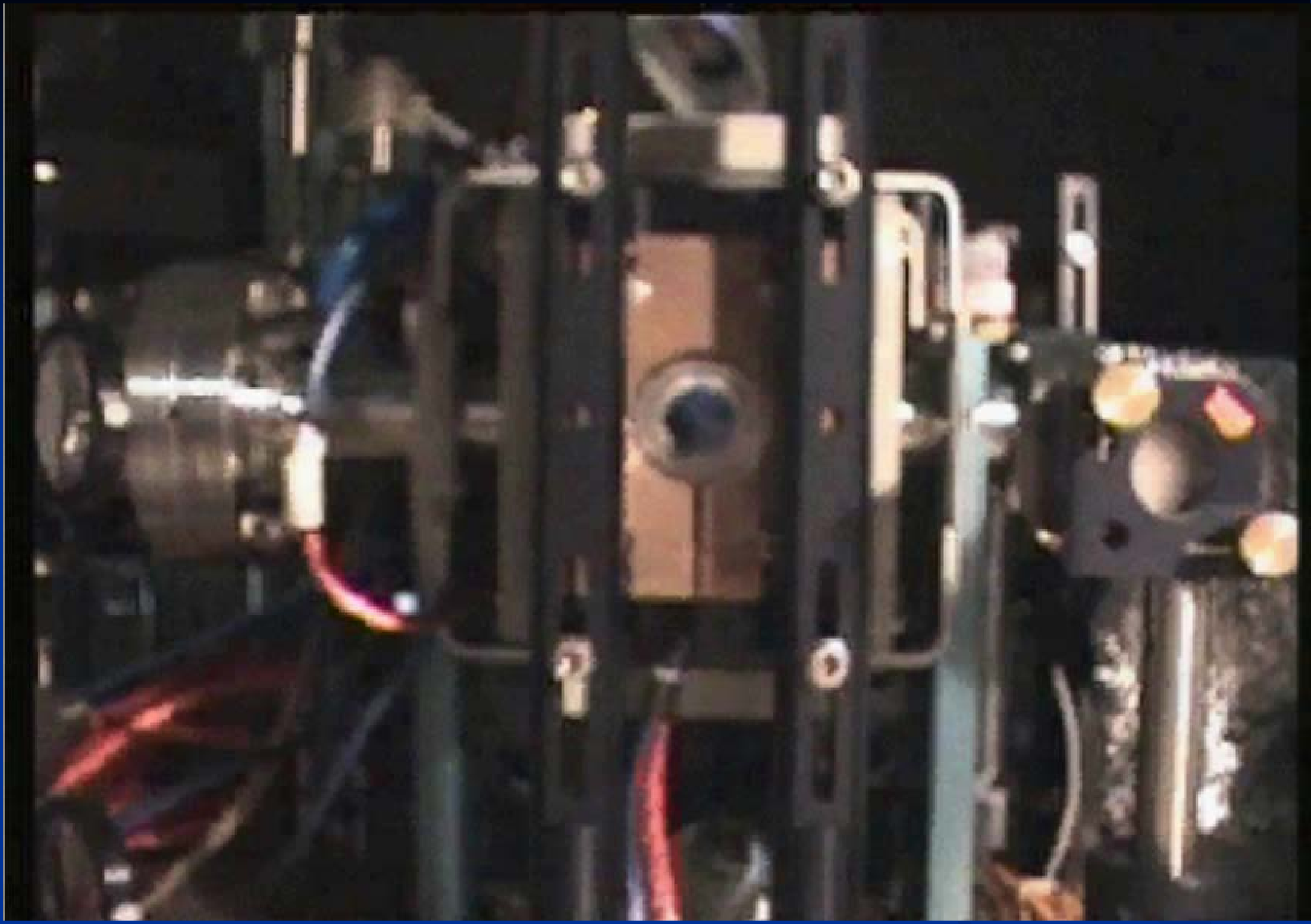






1998: Chu, Cohen-Tannoudji, Phillips

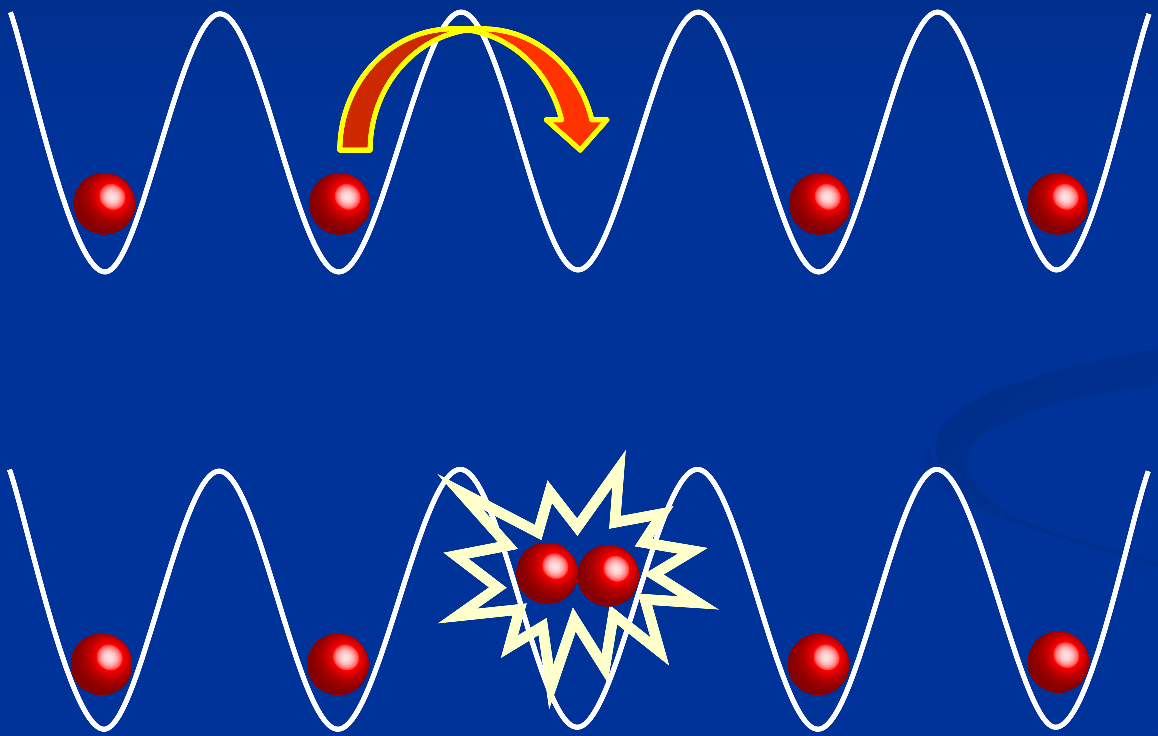
2001: Cornell, Ketterle, Wieman



Groupe: I. Bloch (Munich U.)

Groupe: T. Esslinger (ETH, Zurich)

Virtual solids



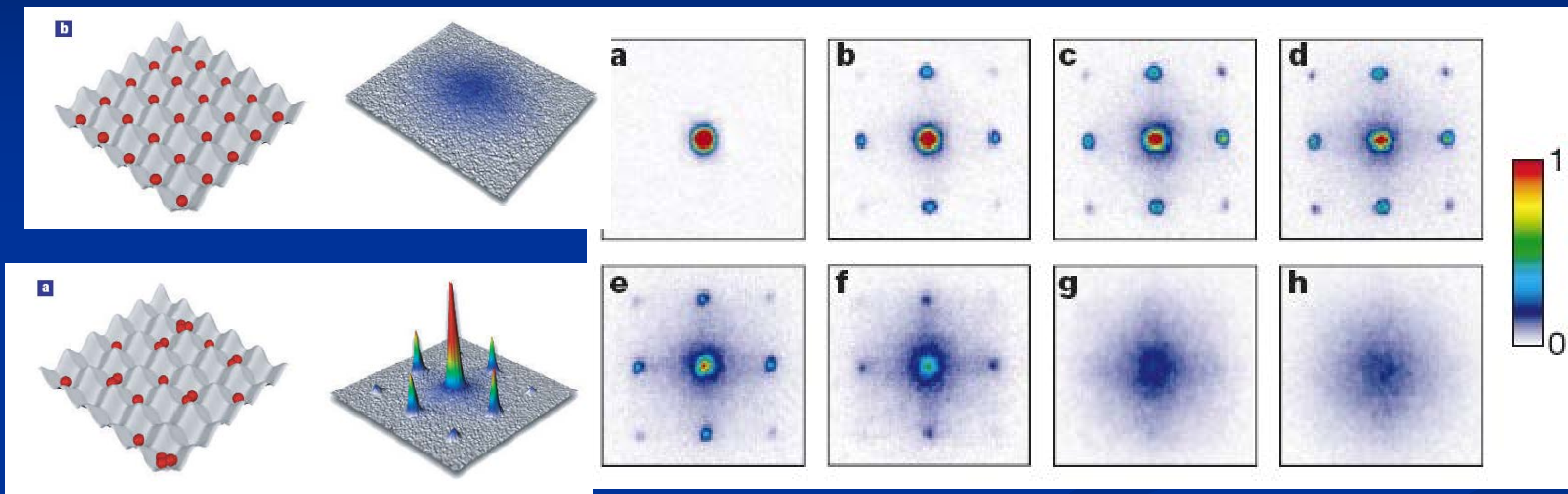
Tunnelling

Short range interaction

P. Zoller

Proposal: D. Jaksch et al PRL81 3108 (98)

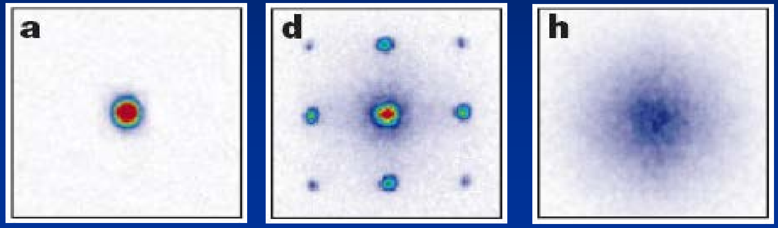
Simulators for condensed matter



M. Greiner, O. Mandel, T. Esslinger, T. W. Hansch, I. Bloch, Nature 415 39 (2002)

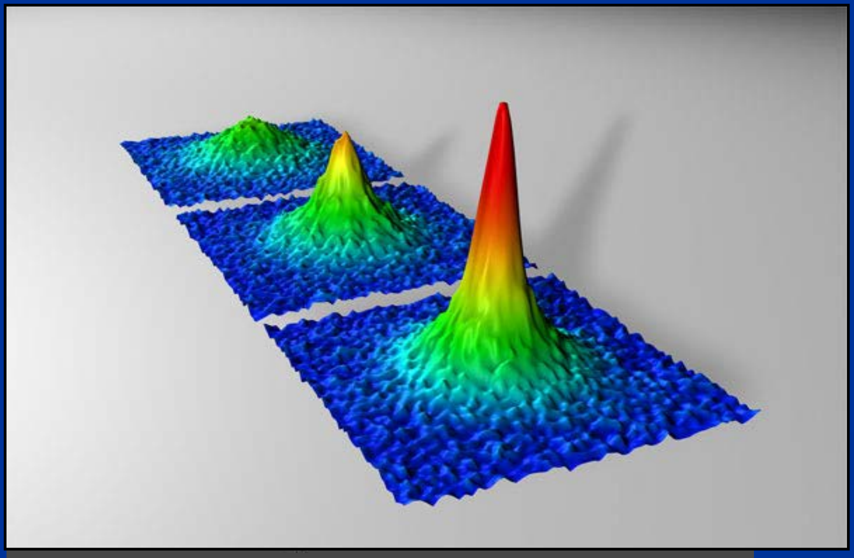
Good control on the system

Interactions
(Lattice, Feschbach resonnance)

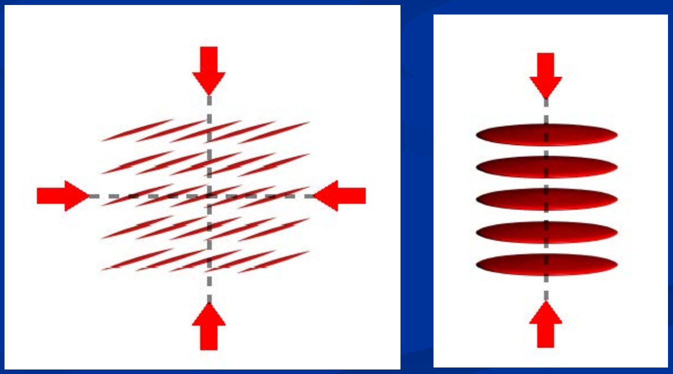


Statistics

Fermions
Bosons



Dimensionality



Bosons (continuum)

Observation of a One-Dimensional Tonks-Girardeau Gas

Toshiya Kinoshita, Trevor Wenger, David S. Weiss*

SCIENCE VOL 305 20 AUGUST 2004

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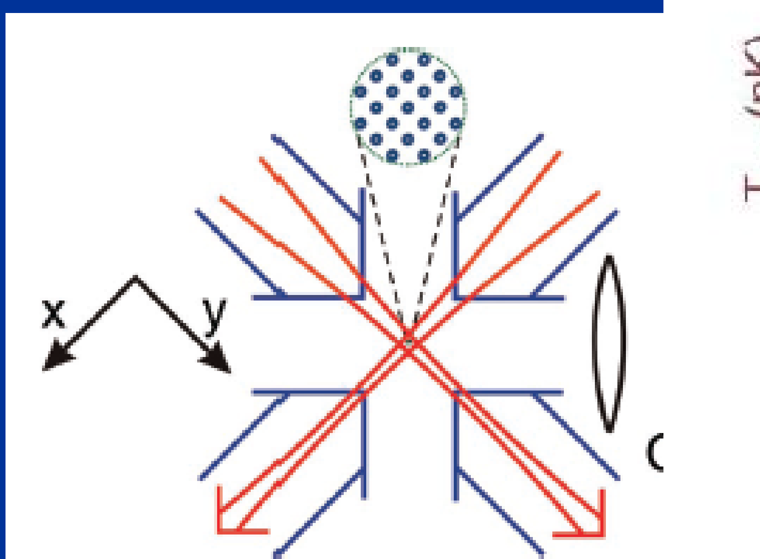
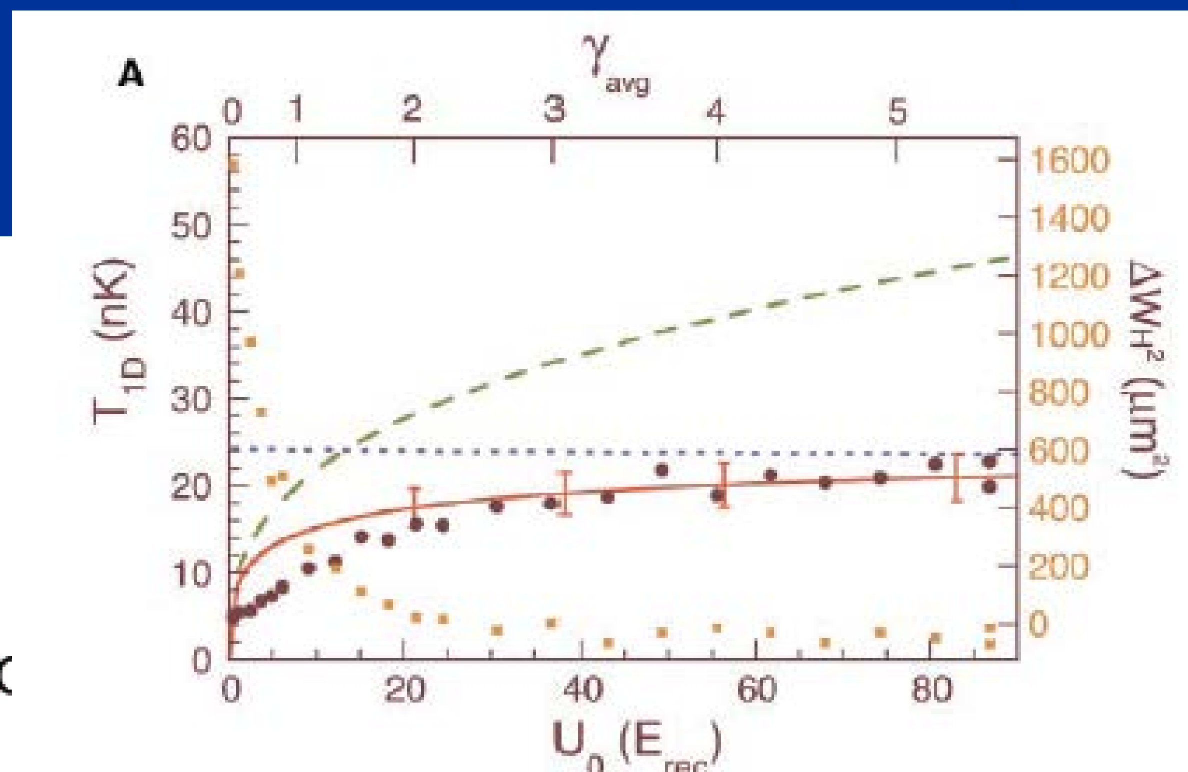
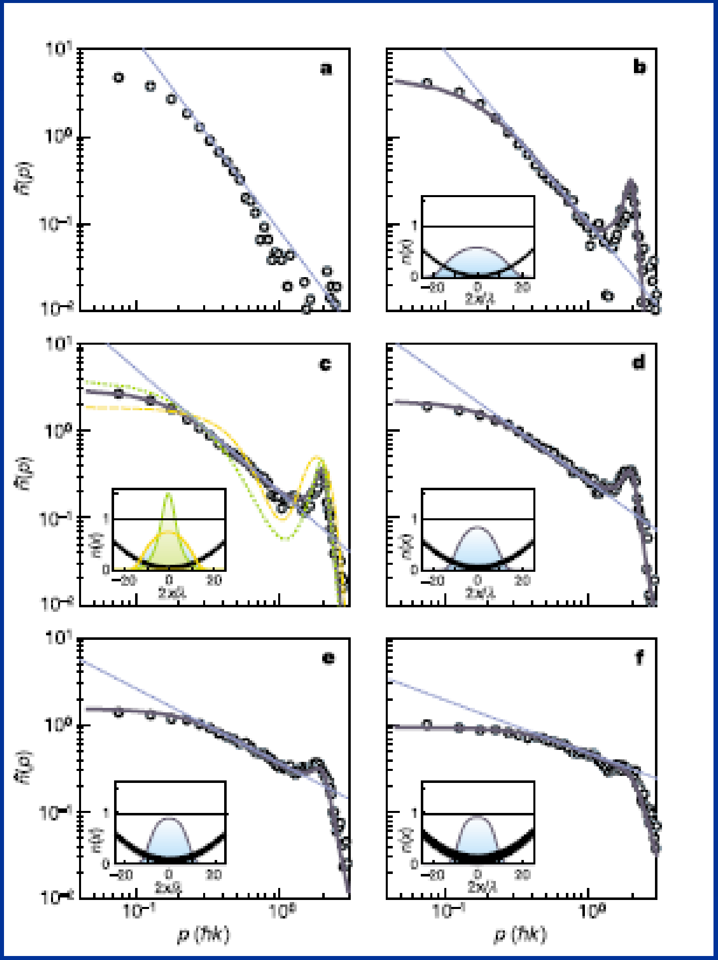
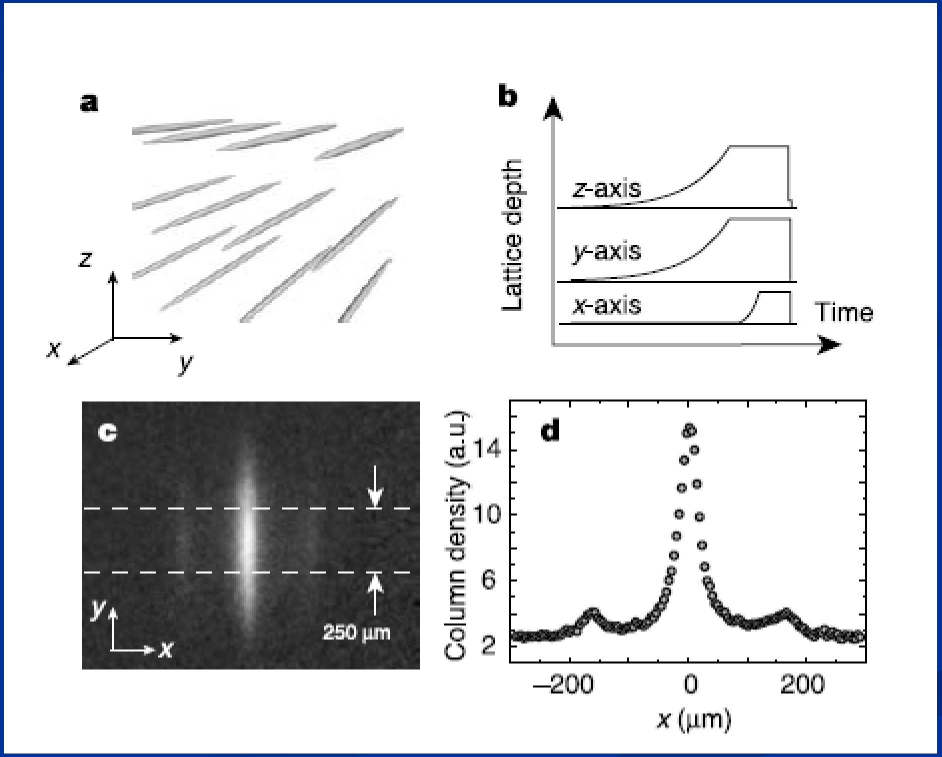




Optical lattices (dilute)

B. Paredes et al., Nature 429 277 (2004)

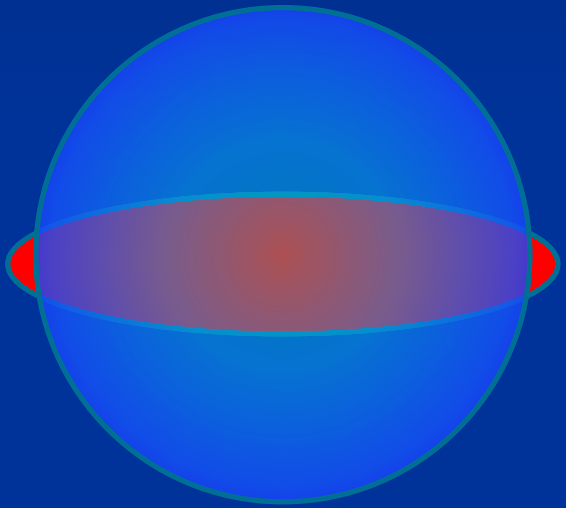




$$n(k) = \int dx e^{ikx} \langle \psi^{\dagger}(x) \psi(0) \rangle$$

Confining potential / Trap

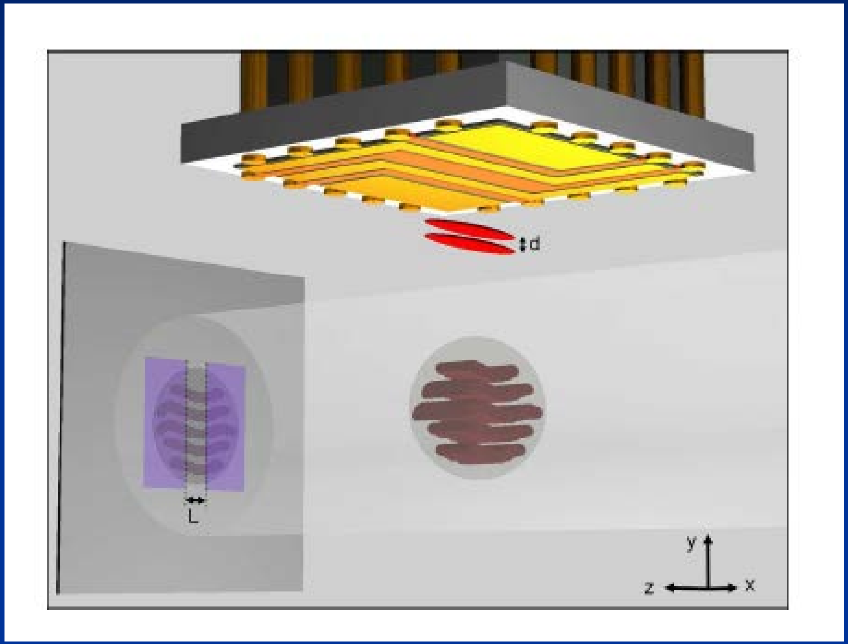
- No homogeneous phase !

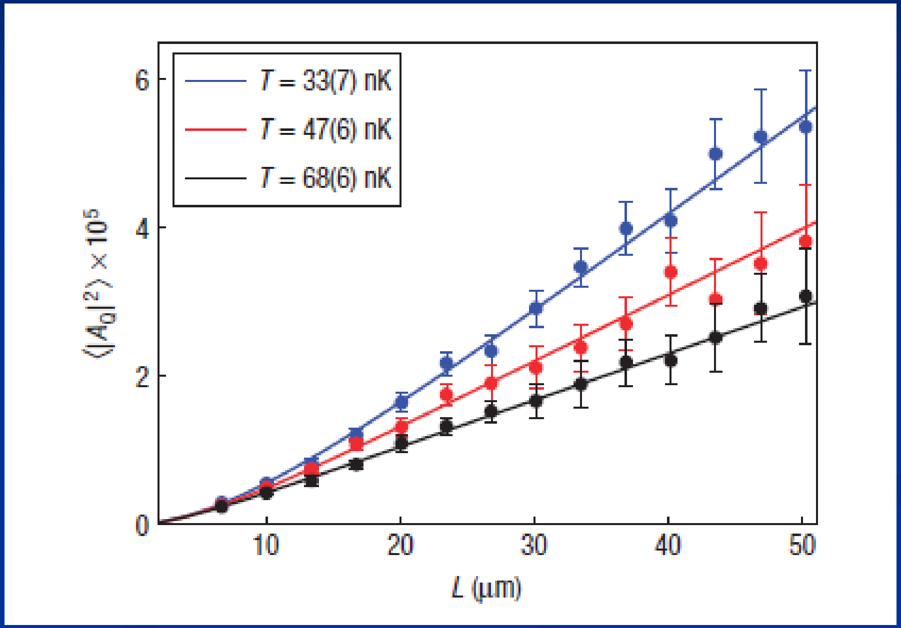


$$H = \int d^3r \frac{1}{2}\omega_0^2 r^2 \rho(r)$$

I am your worst nightmare

Atom chips

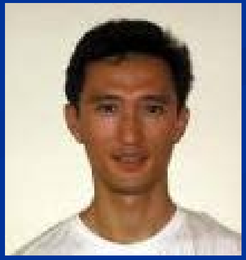




$$\int_0^L dr \langle \psi(r) \psi^\dagger(0) \rangle$$

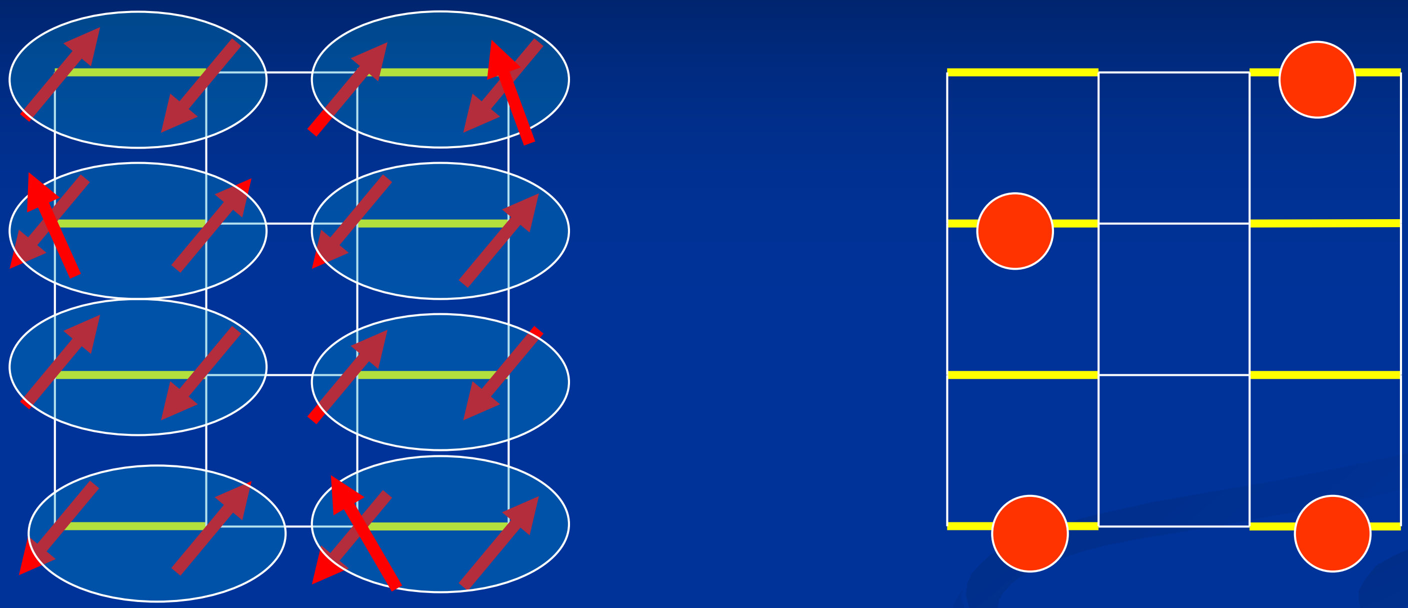
S. Hofferberth et al. Nat. Phys 4 489 (2008)

K large (42)



Magnetic insulators

triplon = hard core boson



h ~ h_c dilute limit: « free » bosons

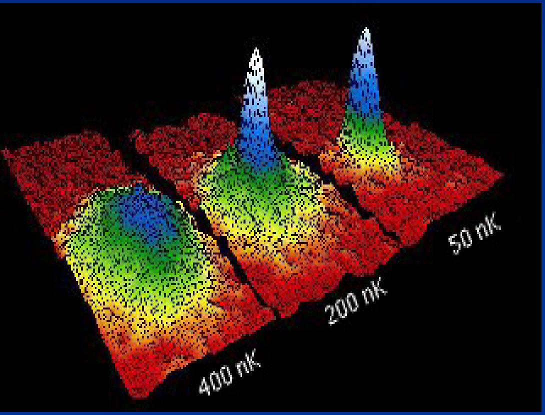
Bose Einstein Condensation

(TG and A. M. Tsvelik PRB 59 11398 (1999))

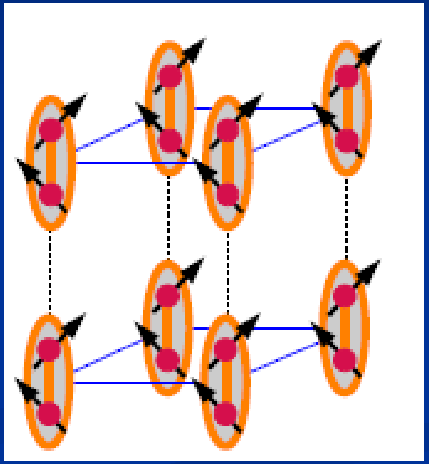
BEC vs BEC

Cold atoms

Dimers/Spins



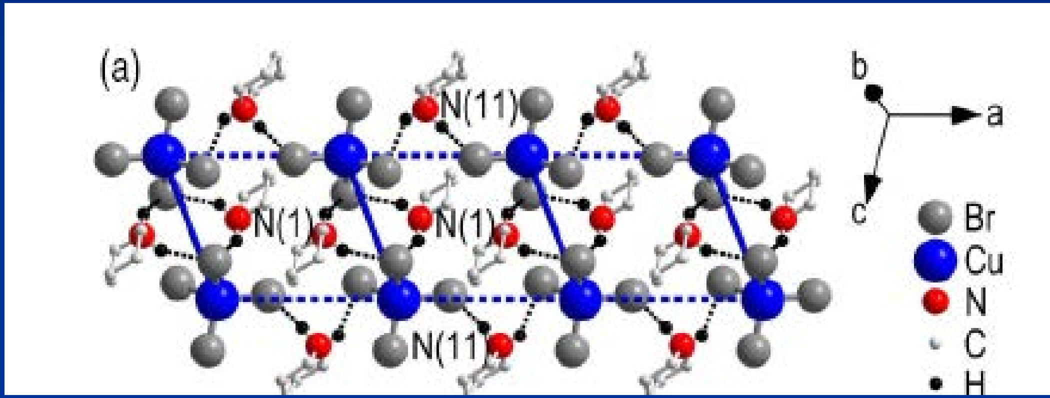
TG, Ch. Rüegg, O. Tchernyshyov, Nat. Phys. 4 198 (08)



Bose gas	Antiferromagnet
Particles	Spin excitations ($\Delta S^z = \pm 1$)
Boson number N	Spin component S^z
Charge conservation U(1)	Rotational invariance O(2)
Condensate wavefunction $\langle \psi(\mathbf{r}) \rangle$	Transverse magnetic order $\langle s_i^x + i s_i^y \rangle$
Chemical potential μ	Magnetic field B
Superfluid density ρ_s	Transverse spin stiffness
Mott insulating state	Integer magnetization plateau

Spin dimer systems

B. C. Watson et al., PRL 86 5168 (2001)

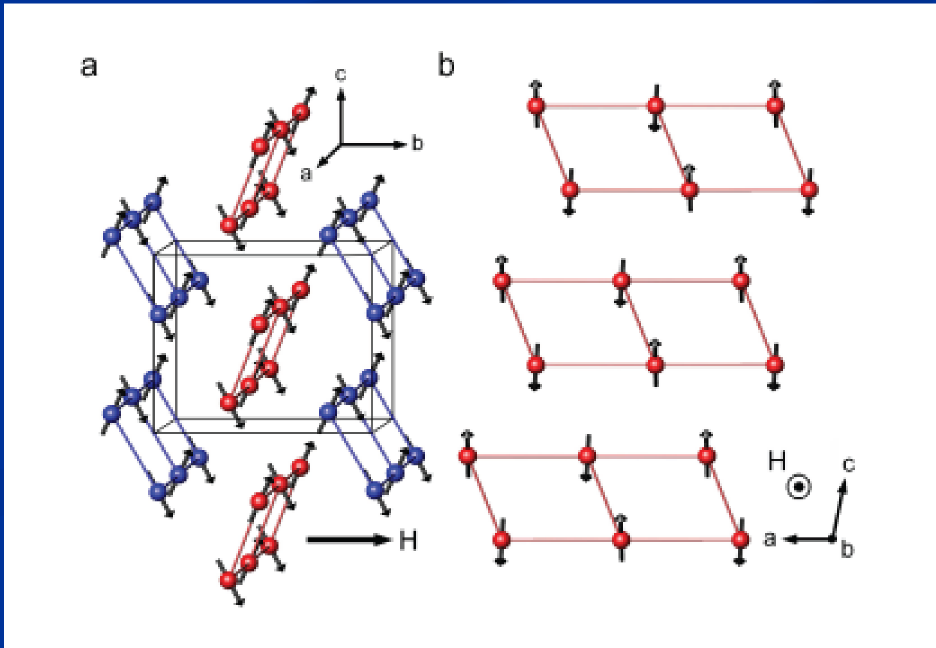


M. Klanjsek et al.,

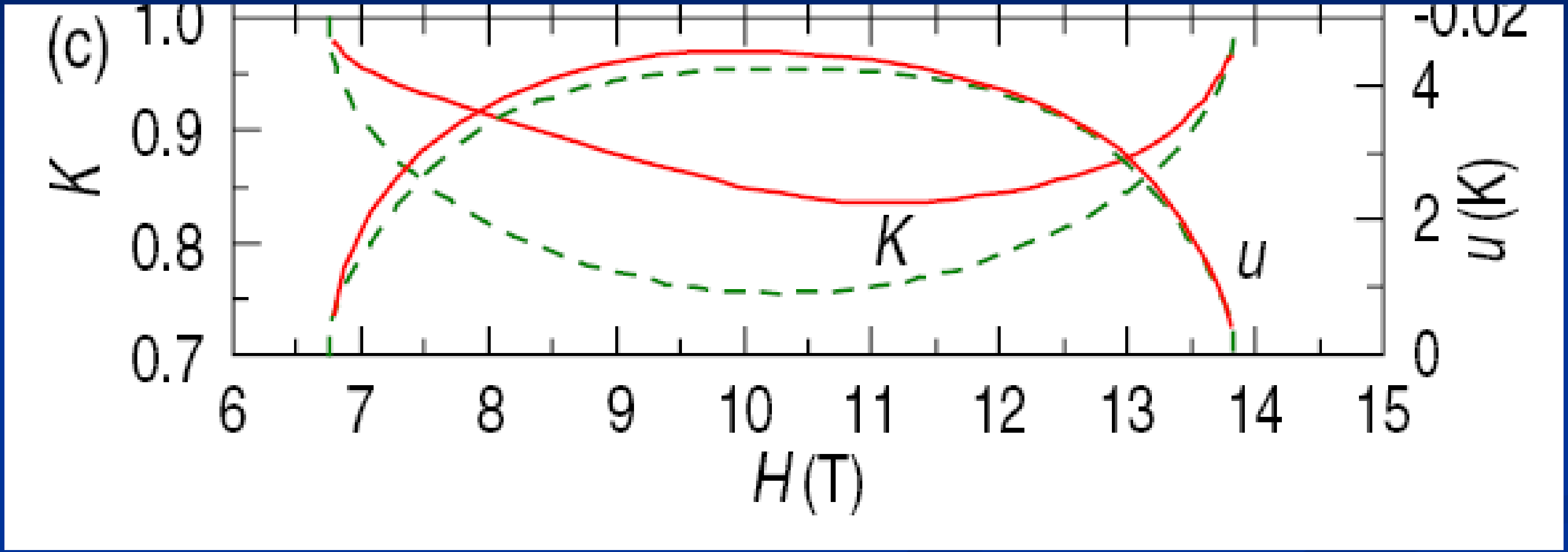
PRL 101 137207 (2008)

B. Thielemann et al.,

PRB 79 020408 ® (2009)



Luttinger parameters



M. Klanjsek et al., PRL 101 137207 (2008)

Red : Ladder (DMRG)

Green: Strong coupling ($J_r \rightarrow 1$) (BA)

Correlation functions

M. Klanjsek et al., PRL 101 137207 (2008)

R. Chitra, TG PRB 55 5816 (97); TG, AM Tsvelik PRB 59 11398 (99)

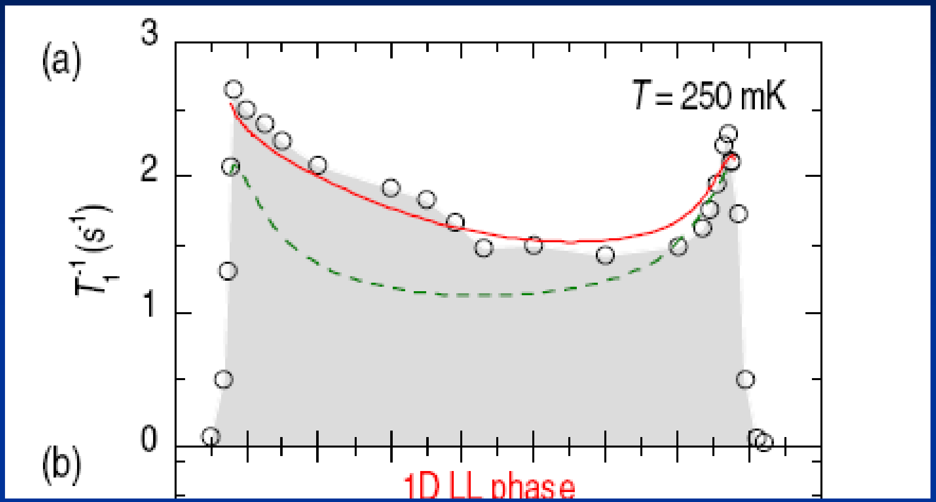
- NMR relaxation rate:

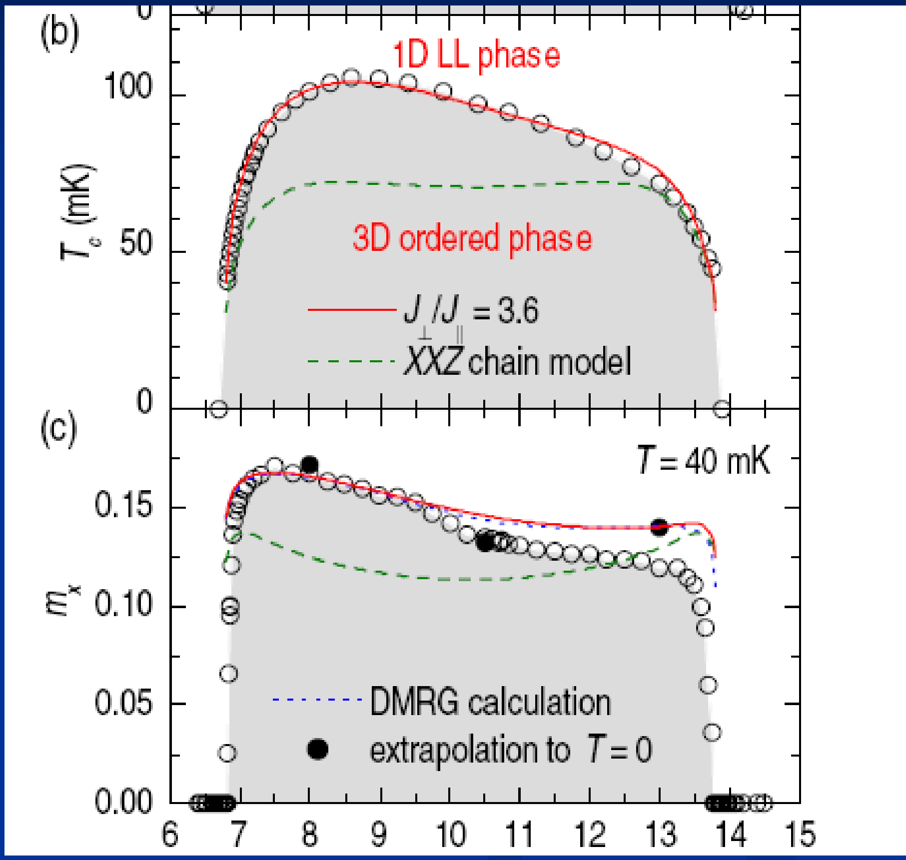
$$T_1^{-1} = \frac{\hbar \gamma^2 A_\perp^2 A_0^x}{k_B u} \cos\left(\frac{\pi}{4K}\right) B\left(\frac{1}{4K}, 1 - \frac{1}{2K}\right)\left(\frac{2\pi T}{u}\right)^{(1/2K)-1},$$

- Tc to ordered phase: $1/J' = \chi_{1D}(Tc)$

$$T_c = \frac{u}{2\pi}\left[\sin\left(\frac{\pi}{4K}\right) B^2\left(\frac{1}{8K}, 1 - \frac{1}{4K}\right)\frac{zJ' A_0^x}{2u}\right]^{2K/(4K-1)}.$$

NMR





M. Klanjsek et al.,

PRL 101 137207 (2008)

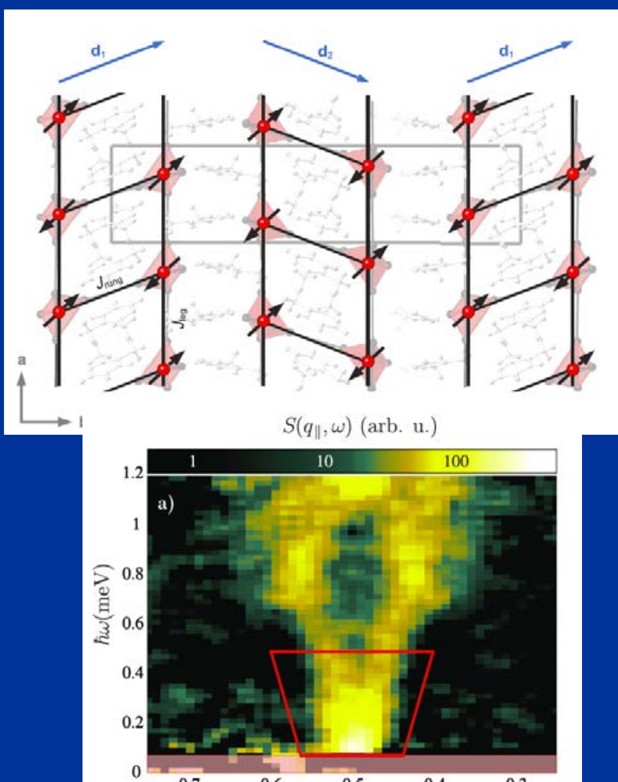
Quantitative test of TLL

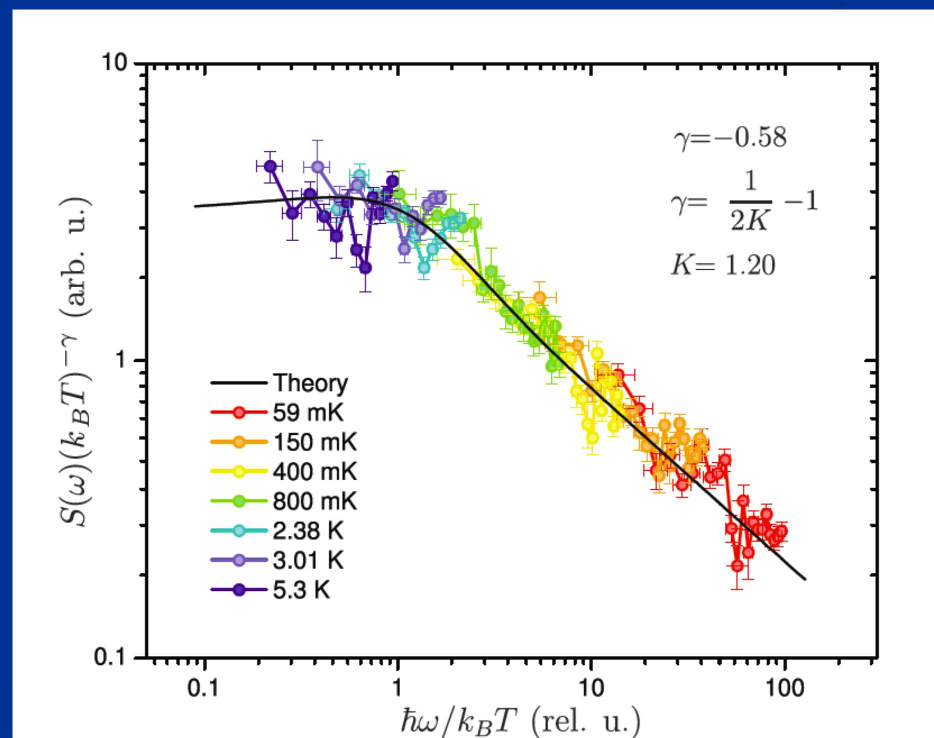
Spin ladders

D. Schmidiger et al. PRL 108 167201 (2012):

K. Yu et al. PRB 91 020406(R) (2015)

$$\langle S^- S^+ \rangle_{q,\omega} = \langle \psi \psi^\dagger \rangle_{q,\omega}$$

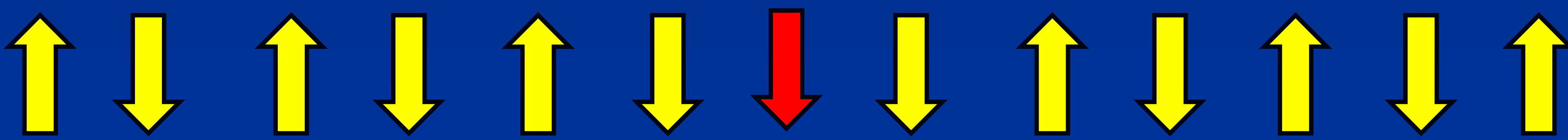




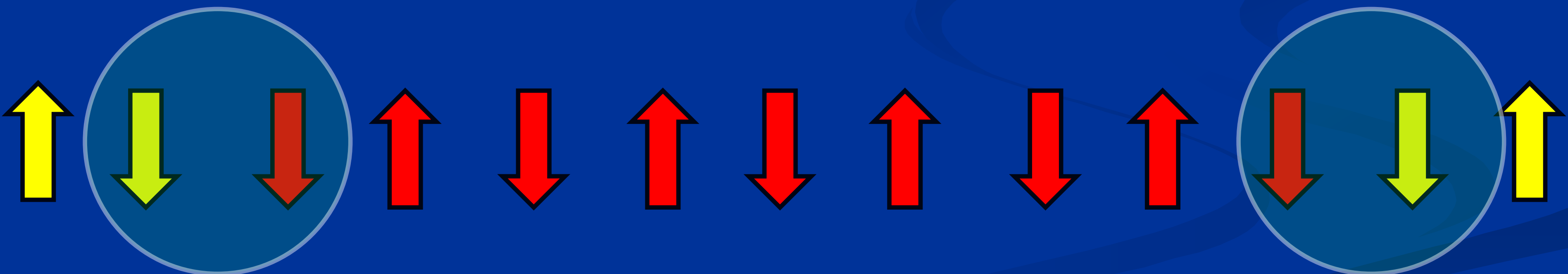
Other important 1D properties Fractionalization of excitations

Fractionalization of excitations

$$H = J\sum_i \vec{S}_i \cdot \vec{S}_{i+1}$$

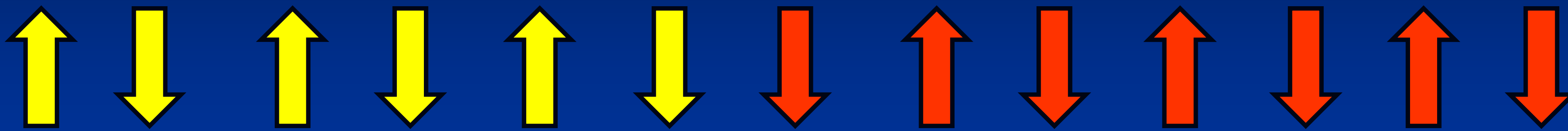


$\Delta S = -1$ $E = \epsilon(q)$ Magnon



$\Delta S = -1/2$ Spinons $\Delta S = -1/2$

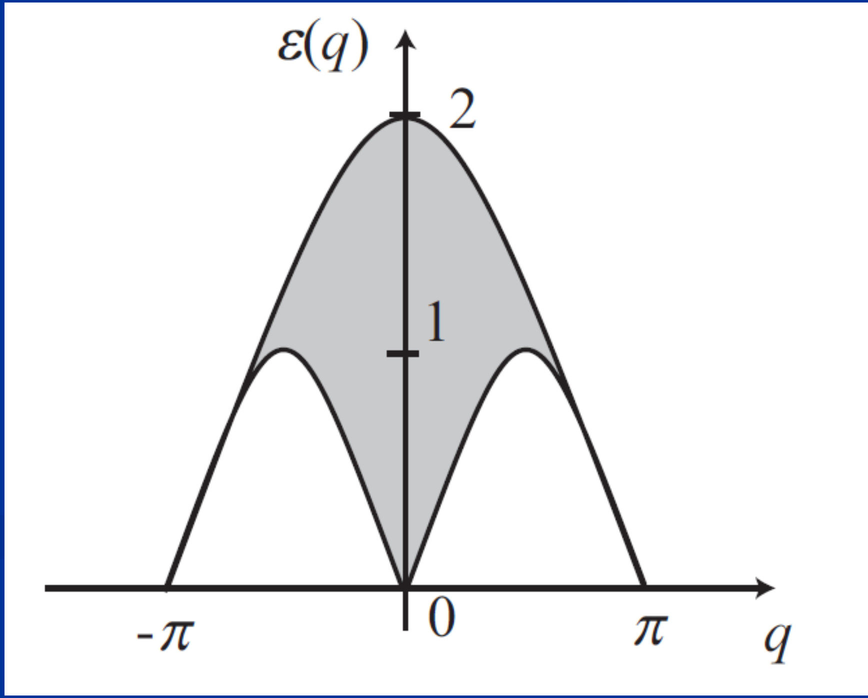
Magnons and spinons: 1 = ½ + ½



- Hidden (topological) order parameters
- Continuum of excitations

$$E(k) = \cos(k_1) + \cos(k_2)$$

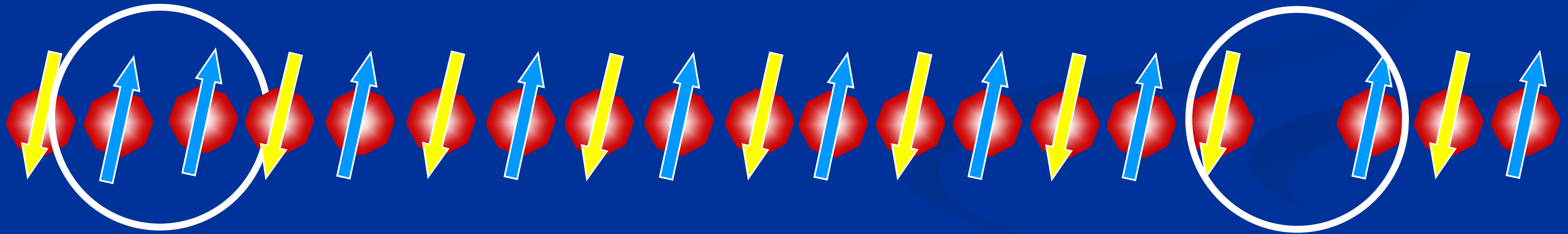
$$k = k_1 + k_2$$



Deconstruction of the electron: spin-charge separation

Spin

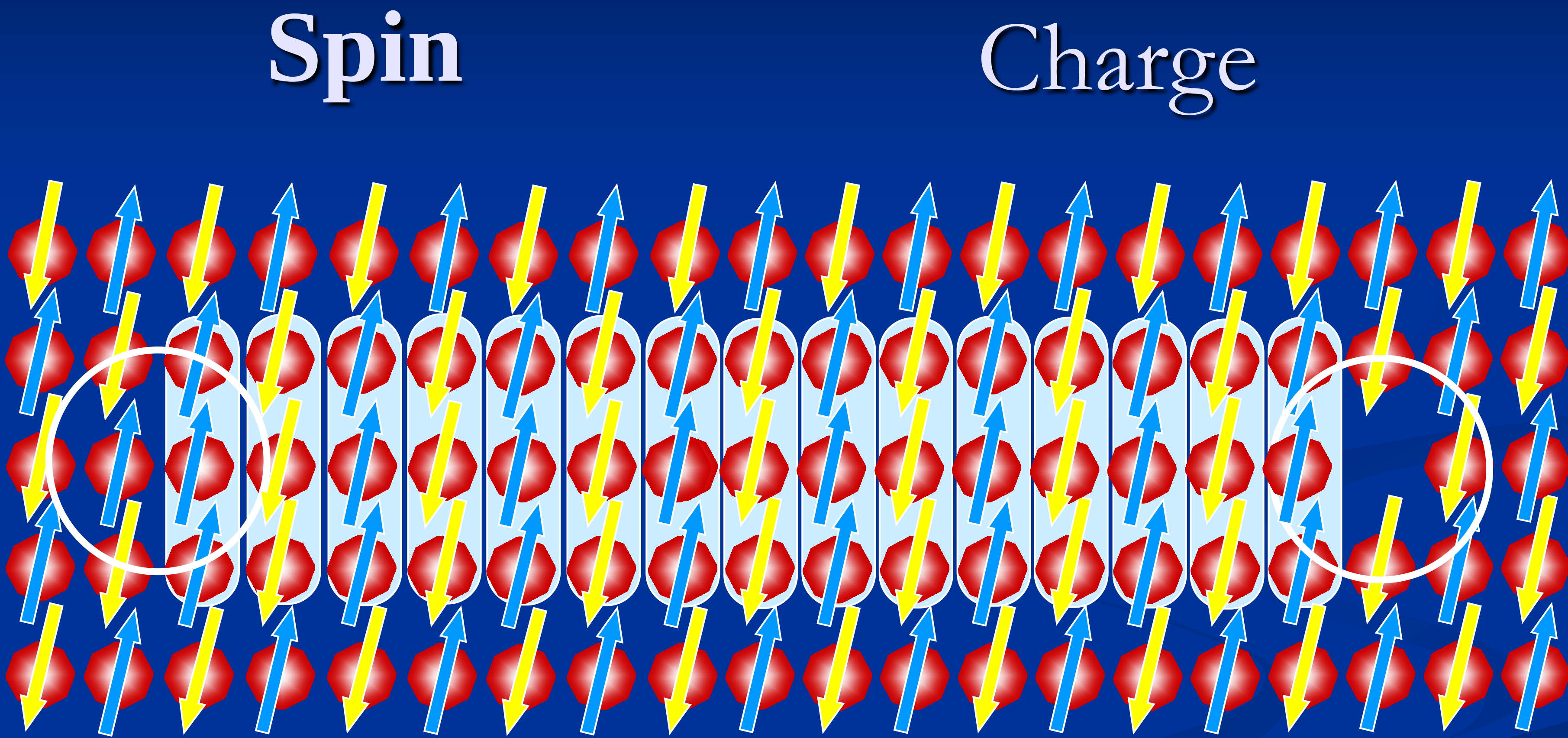
Charge



Spinon

Holon

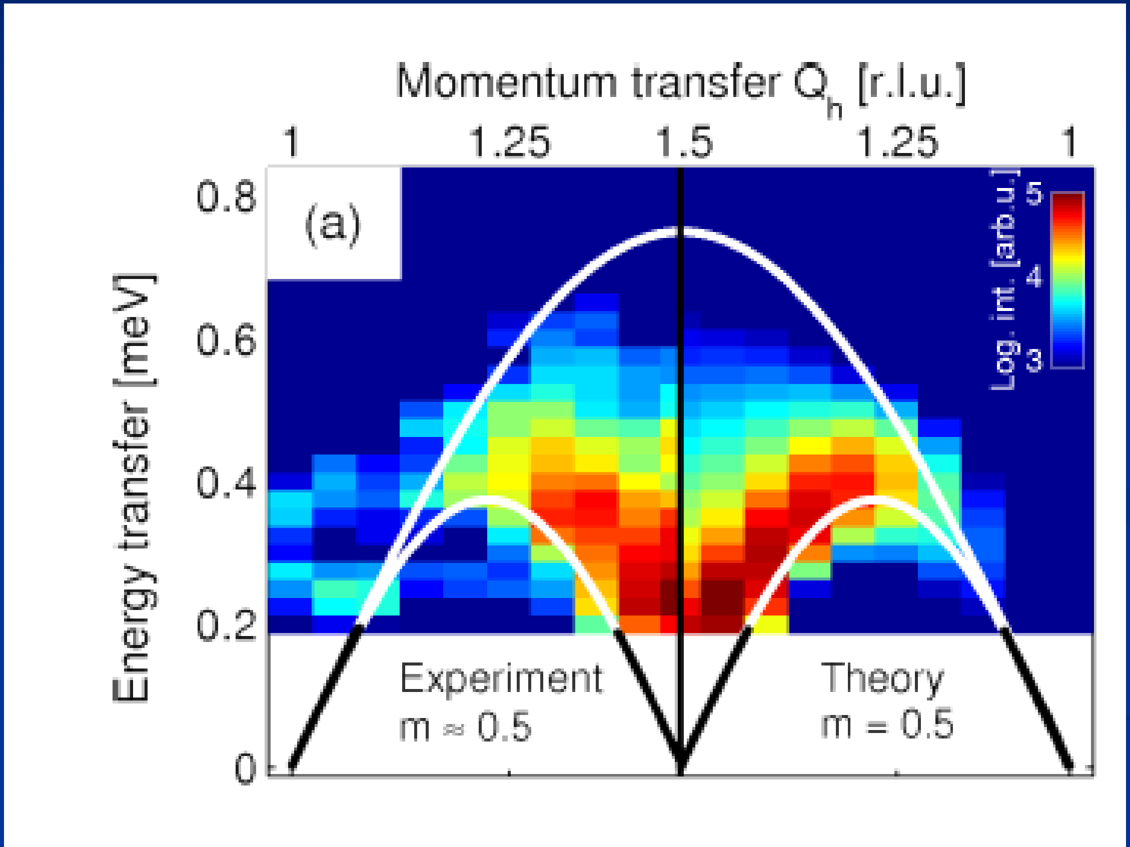
Spin-Charge Separation higher D ?



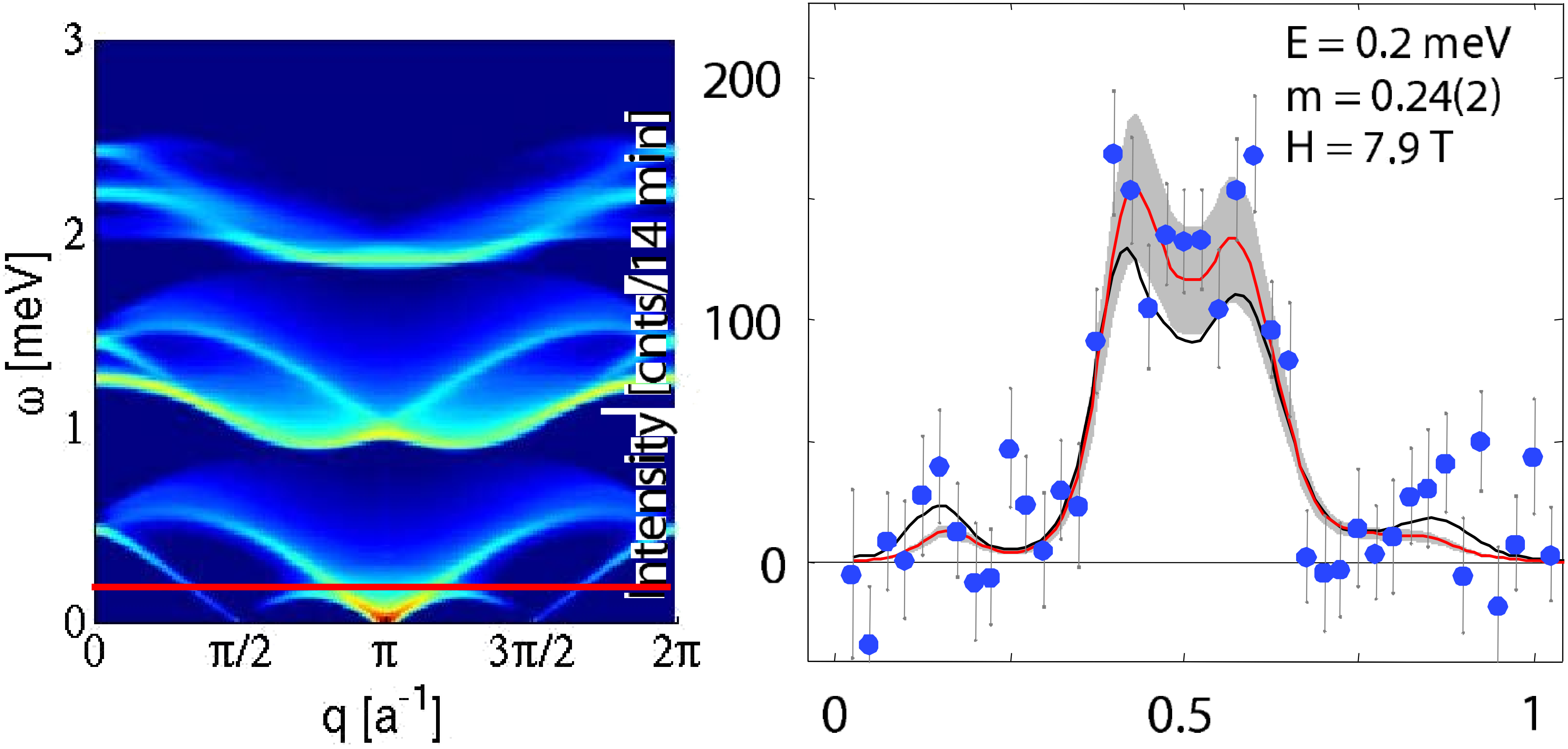
Energy increases with spin-charge separation

Confinement of spin-charge: « quasiparticle »

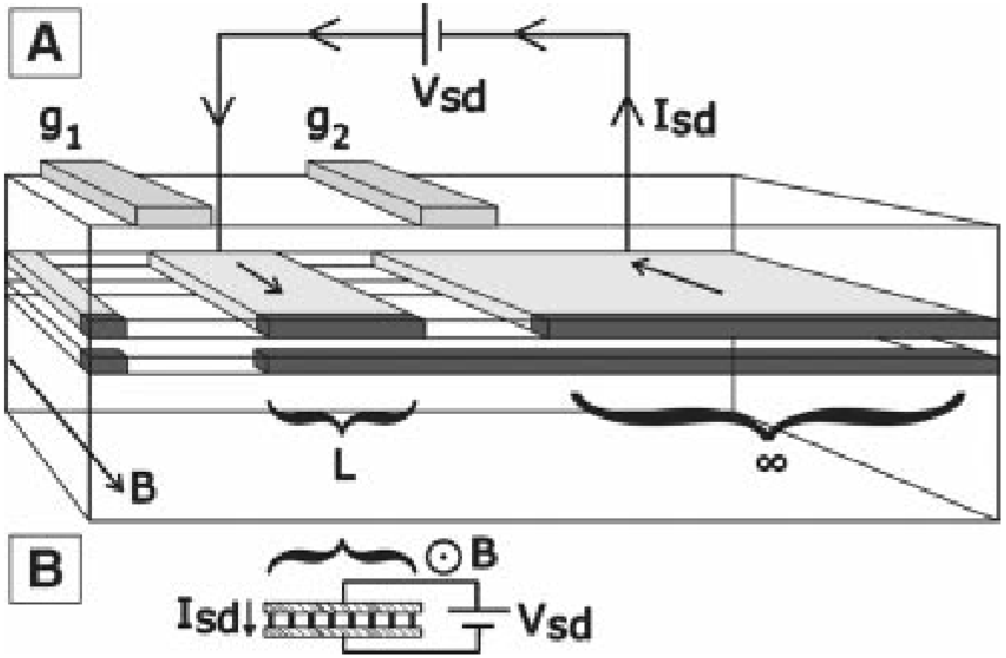
Neutron scattering

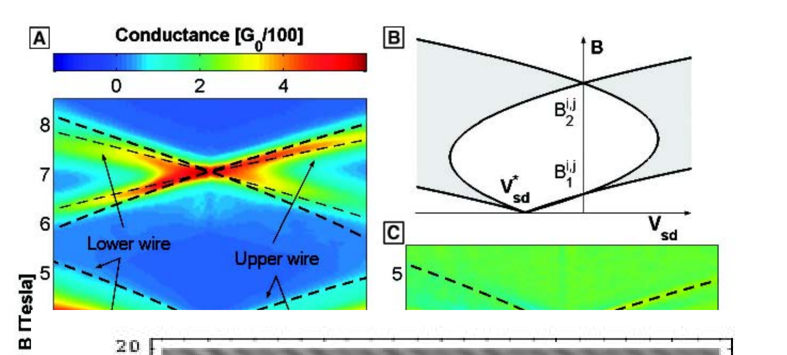


B. Thieleman et al. PRL 102, 107204 (2009)



P. Bouillot et al. PRB 83, 054407 (2011)

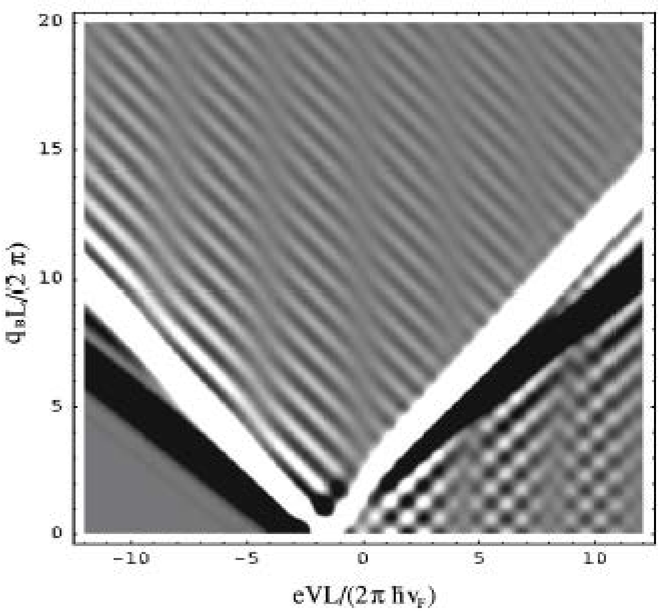




O.M Ausslander et al., Science **298** 1354 (2001)

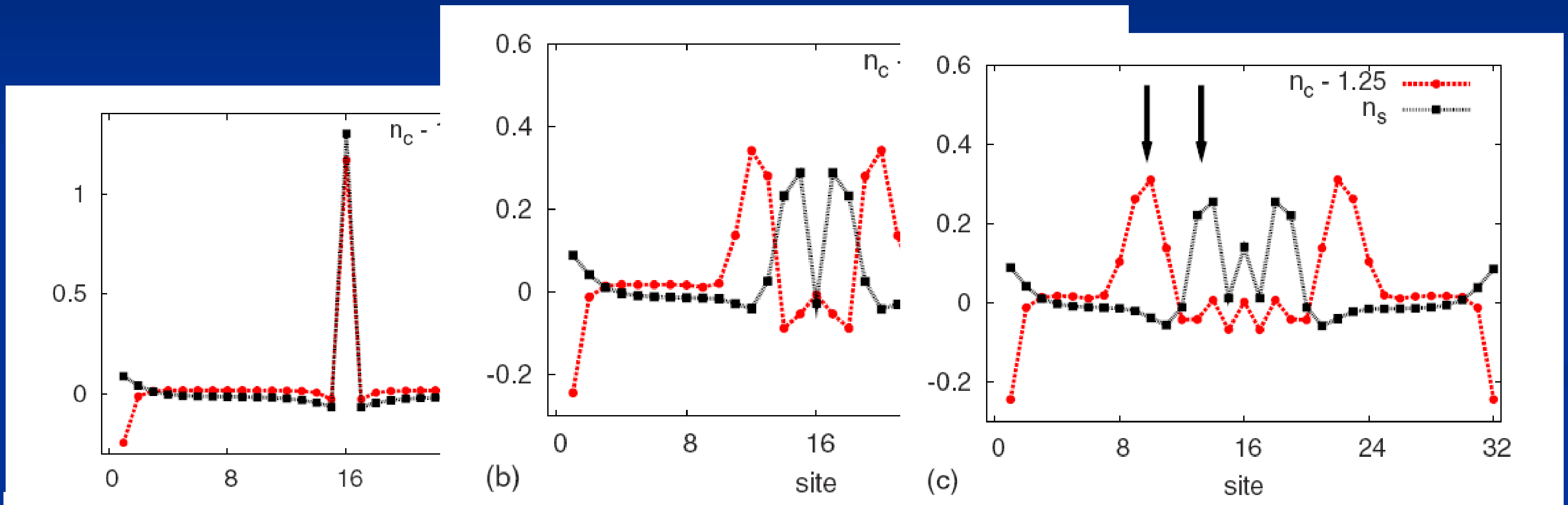
Y. Tserkovnyak et al., PRL **89** 136805 (2002)

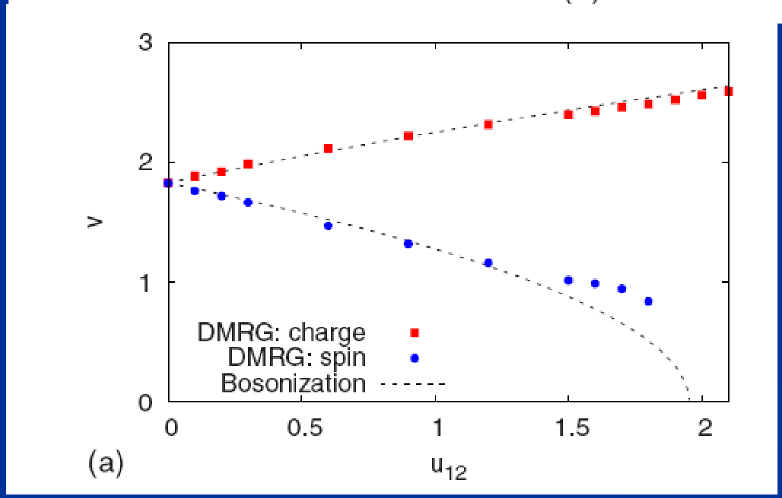
Y. Tserkovnyak et al., PRB **68** 125312 (2003)

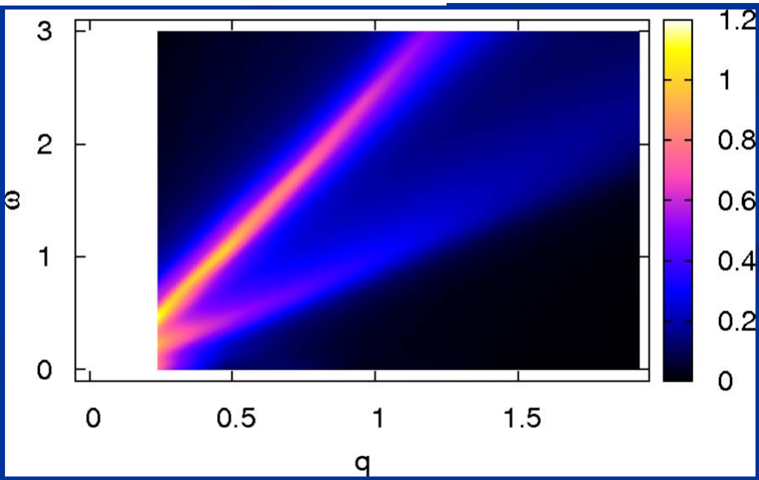


Proposal for cold atoms (Rb)

A. Kleine, C. Kollath et al. PRA 77 013607 (2008); NJP 10 045025 (2008)





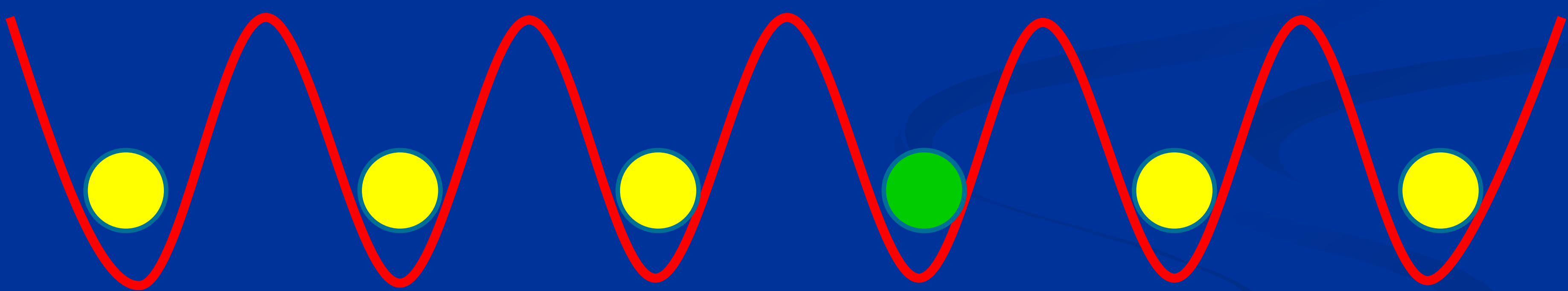


And now …. To transport

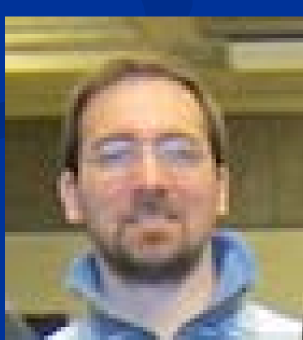
One impurity; no bath

Mobile impurity

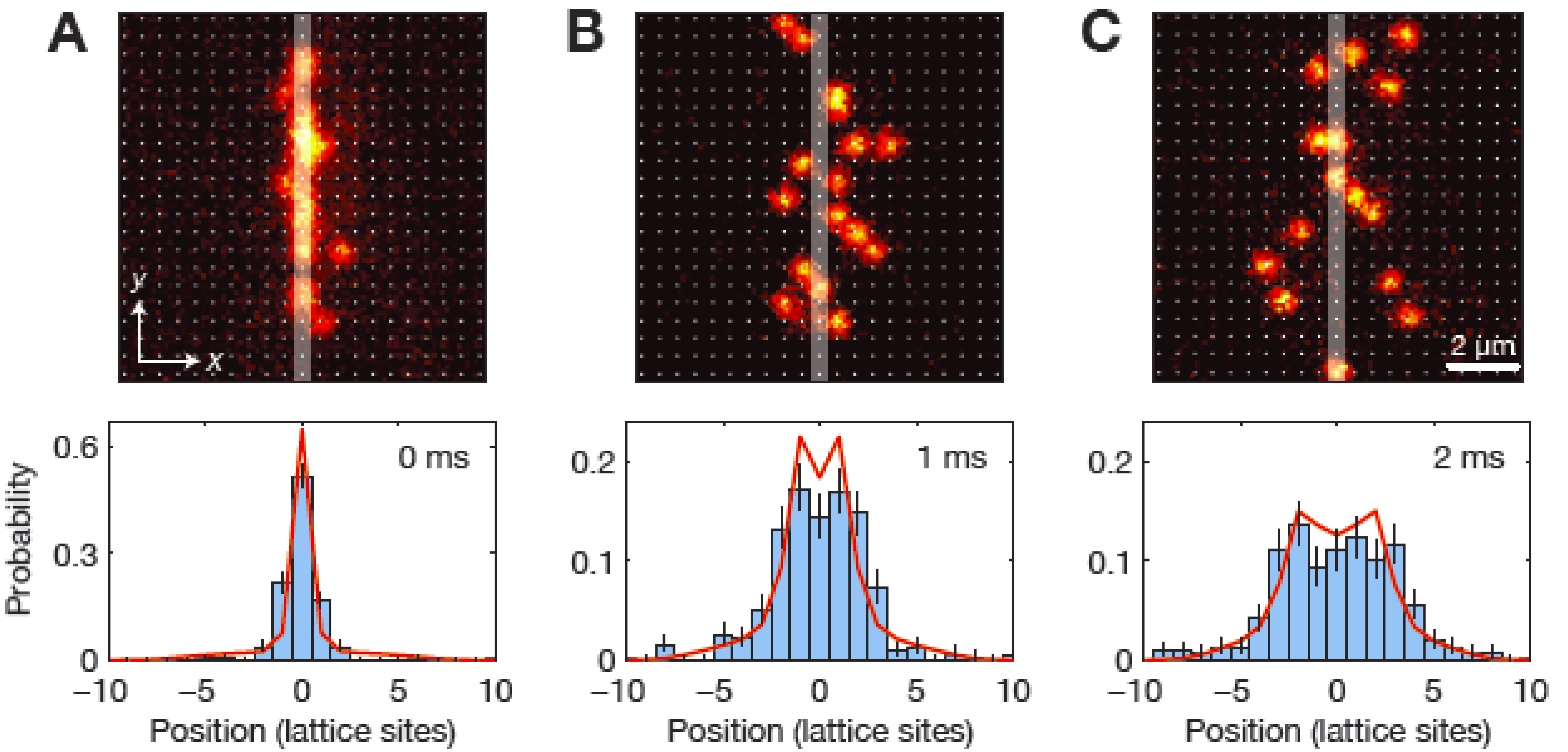
T. Fukuhara, A. Kantian, M. Endres, M. Cheneau, P. Schauss, S. Hild, D. Bellem, U. Schollwock, TG, C. Gross, I. Bloch, S. Kuhr , Nat. Phys. (2013)



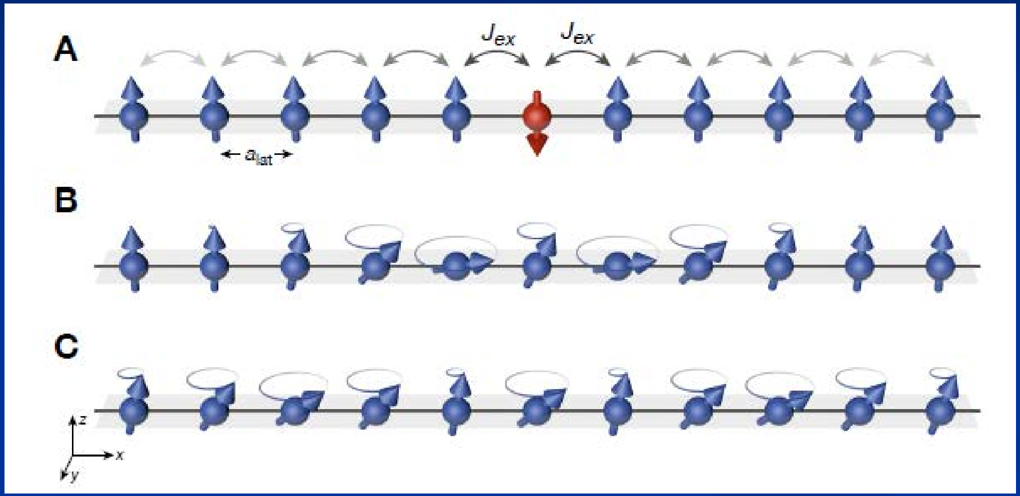


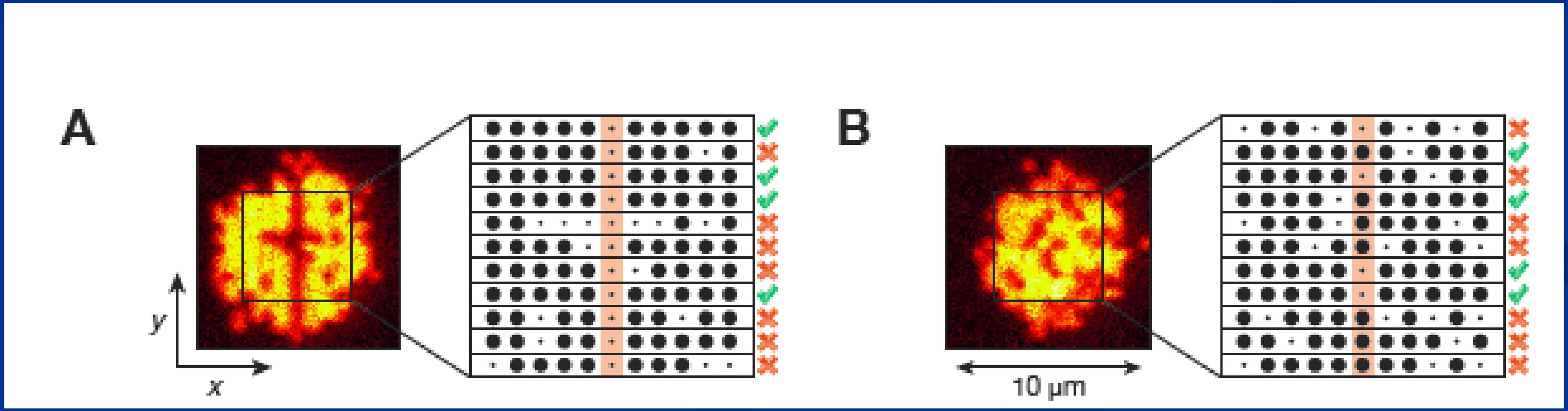



Mobile impurity

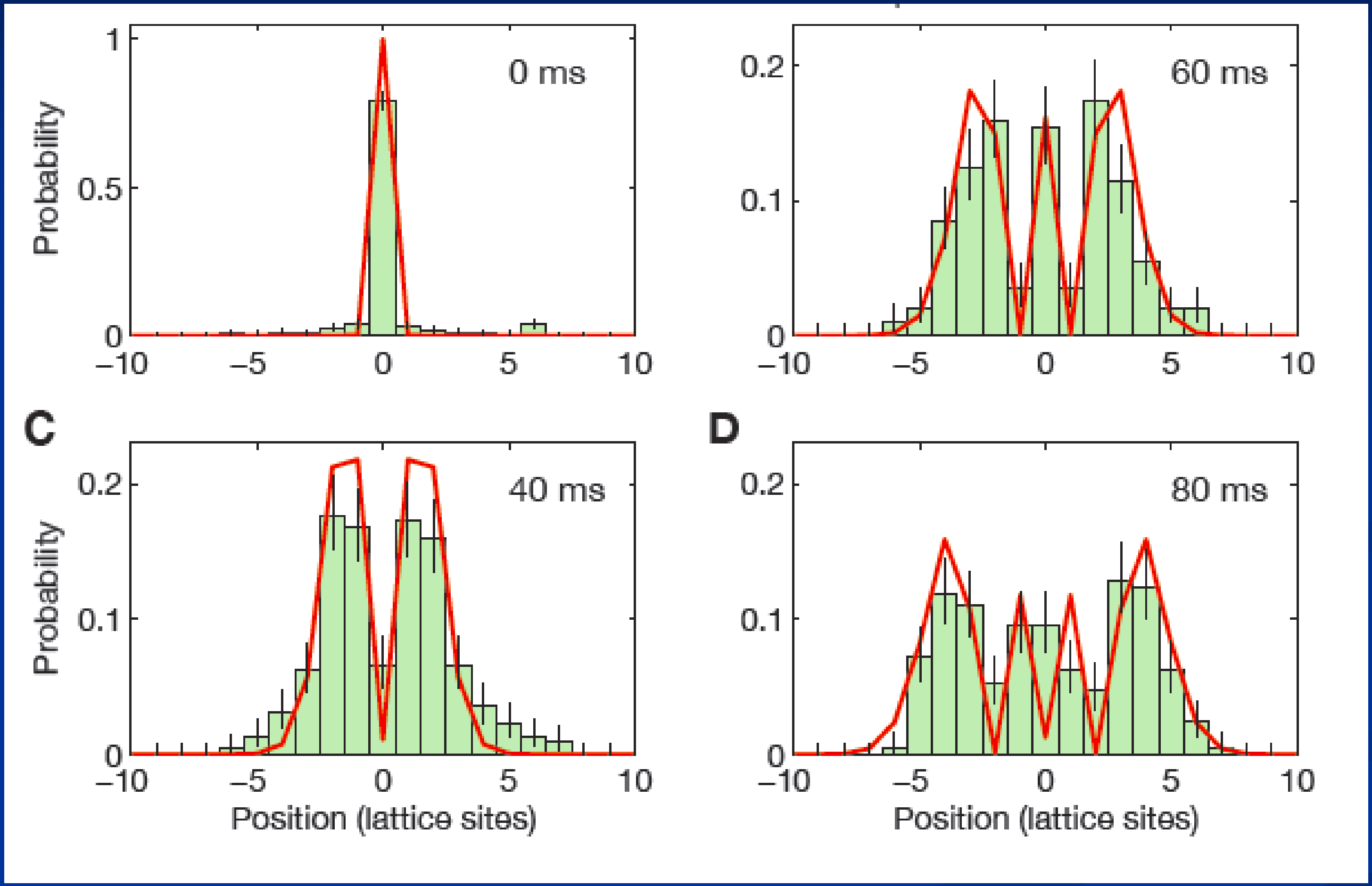


Ferromagnetic Heisenberg



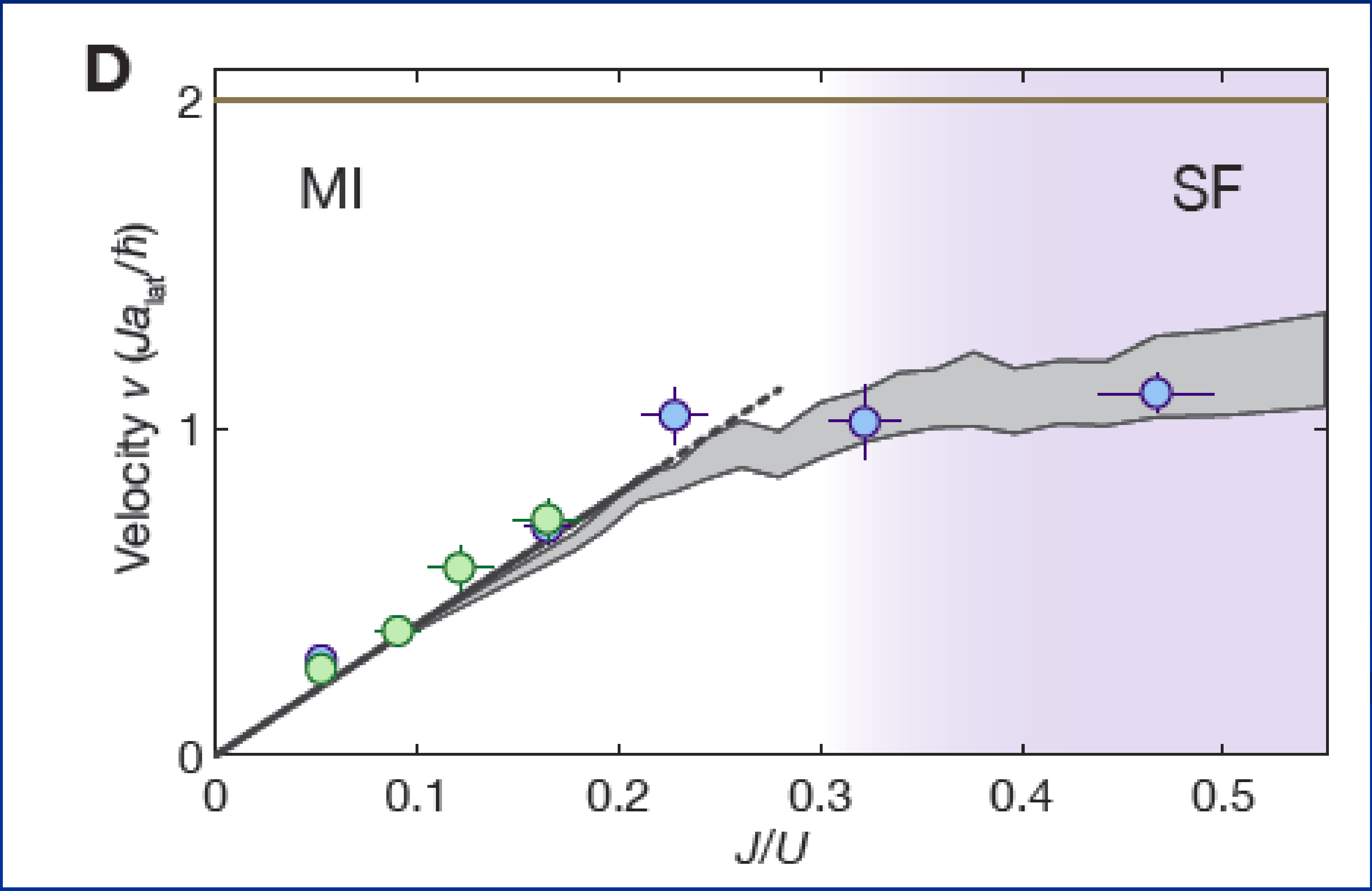


Coherent propagation of a magnon



Heisenberg model : $H = J_{ex} \sum_i \vec{S}_i \cdot \vec{S}_{i+1}$ $\qquad J_{ex} = \frac{4t^2}{U}$

Bath: from MI to superfluid



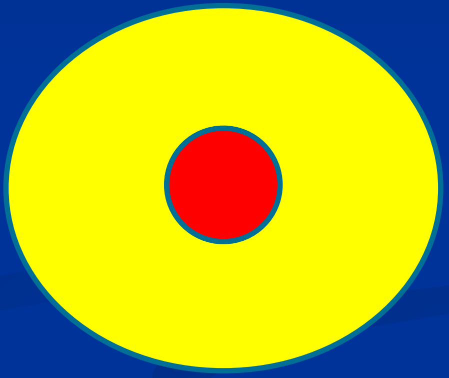
More general problem:

Impurity in a quantum bath

Naïve answer: Feynman polaron

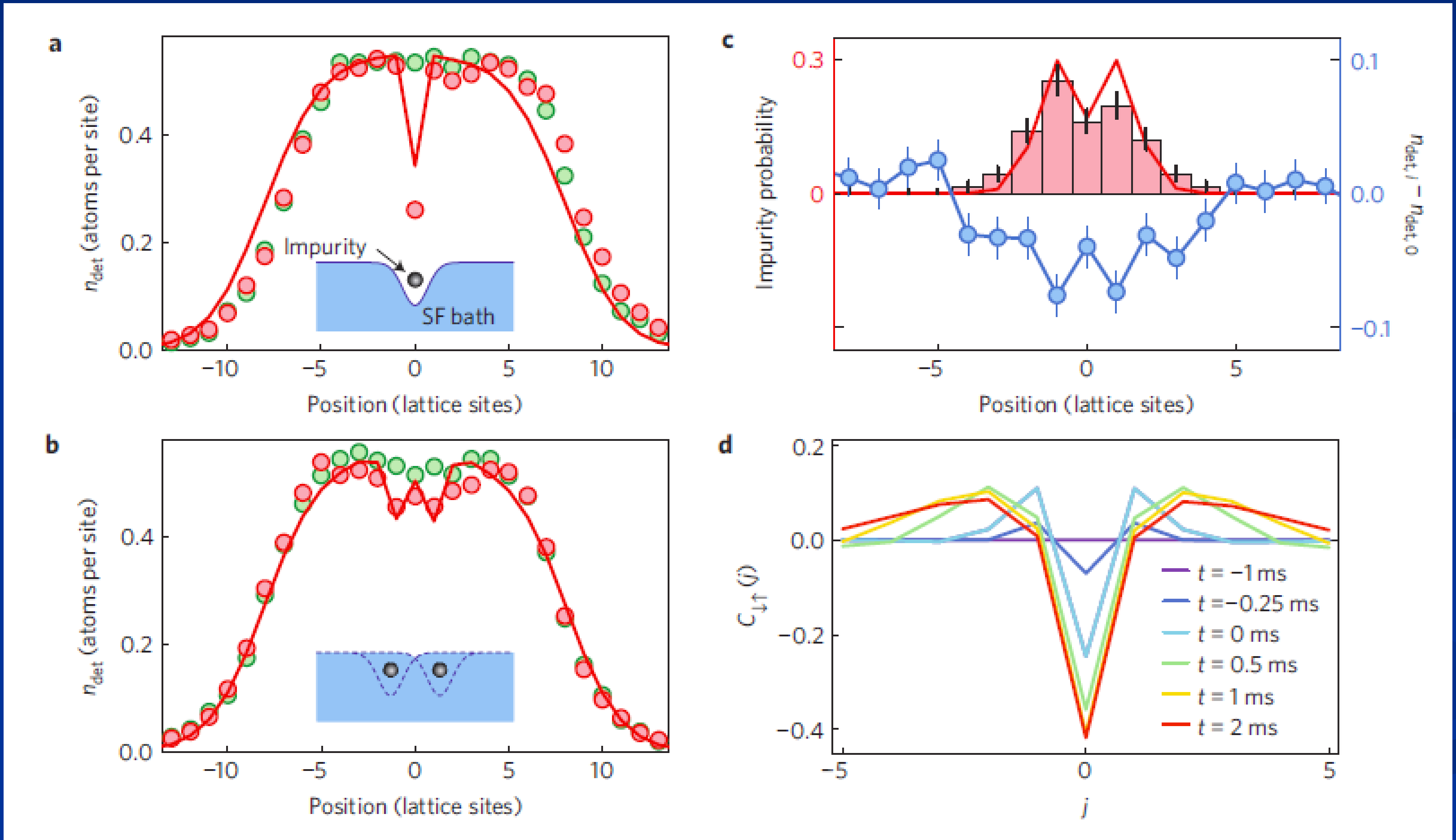
$$\sum_q \epsilon(q) c_q^\dagger c_q + \sum_k u|k| b_k^\dagger b_k + g \sum_k A_k (b_k + b_{-k}^\dagger) \rho_\downarrow(-k)$$

- Dressed Quantum particle



- Renormalization of the mass m -> m*

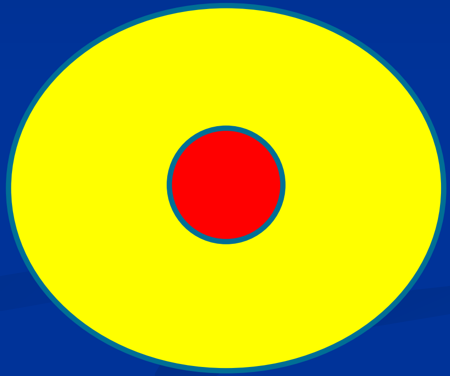
Polaronic effect



Naïve answer: Feynman polaron

$$\sum_q \epsilon(q) c_q^\dagger c_q + \sum_k u|k| b_k^\dagger b_k + g \sum_k A_k (b_k + b^\dagger_{-k}) \rho_\downarrow(-k)$$

- Dressed Quantum particle



- Renormalization of the mass m -> m*
- Not so simple in 1d (Anderson orthogonality)

Mobile impurity

M. B. Zvonarev, V. V. Cheianov, TG, PRL 99 240404 (2007);

$$H = \sum_{j=1}^{N} \frac{p_j^2}{2m} + \sum_{i<j} [g\delta(x_i - x_j) + U(x_i - x_j)]$$

$$\gamma = mg/\hbar^2 \rho_0$$

$$\frac{m^*}{m} = \frac{3\gamma}{2\pi^2}$$

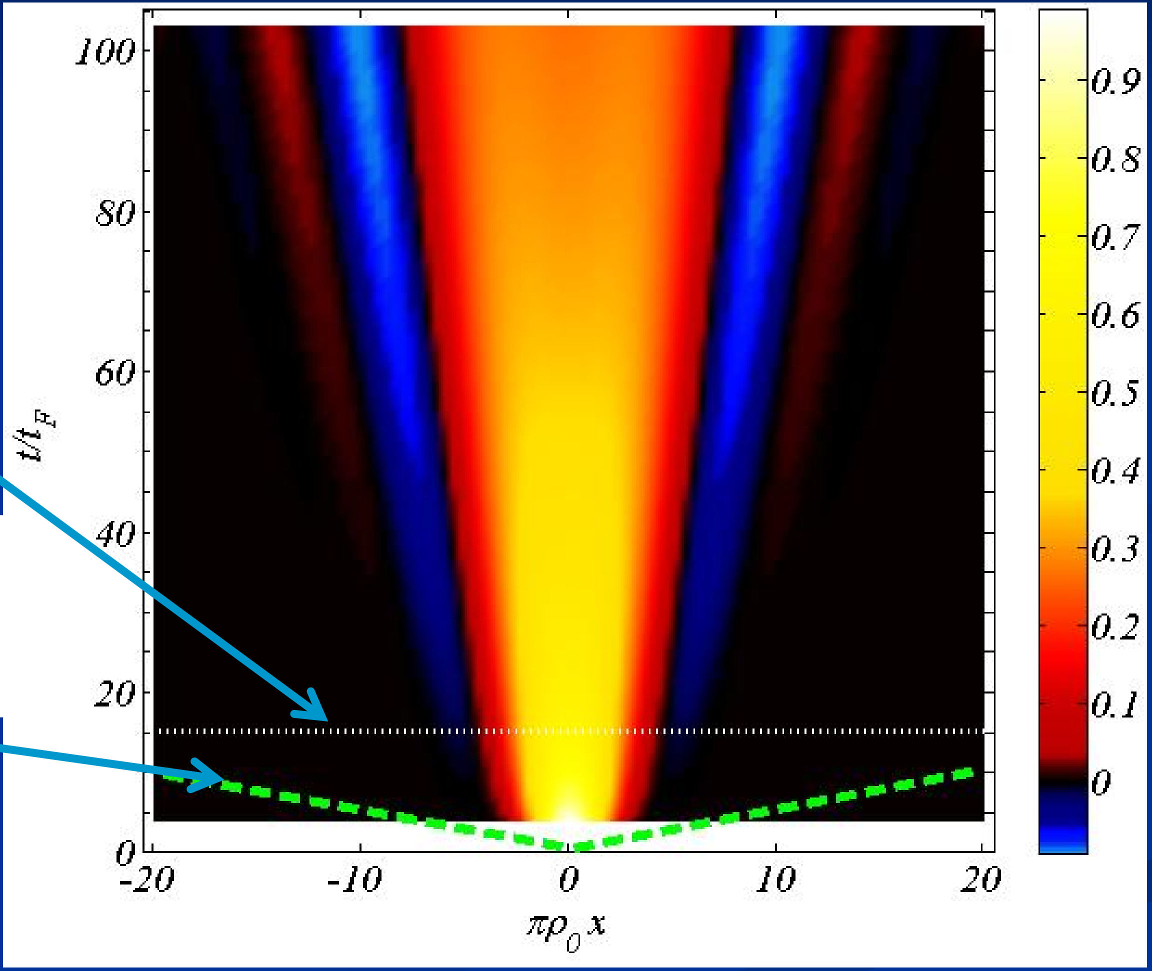
Single spin down particle

$$G_\perp(x,t) = \langle \Uparrow | s_+(x,t) s_-(0,0) | \Uparrow \rangle$$

Propagation of the impurity

Trapped/open regimes

Light cone of spinless bosons

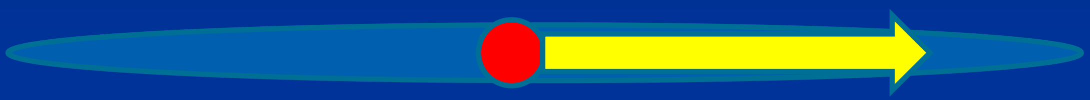


$$G_\perp(x,t) \simeq \frac{1}{\sqrt{\ln(t/t_F)}} \exp\left\{-\frac{1}{K}\frac{(\pi\rho_0 x)^2}{2\ln(t/t_F)}\right\}.$$

$$G_\perp \simeq e^{-(x^2/2\ell^2)} t^{-\alpha} G_\perp^H, \qquad \ell(t) = \frac{2K^{-(1/2)}}{\pi\rho_0} \frac{t/t_F}{\sqrt{\ln t/t_F}} \frac{m}{m_*}.$$

Driven impurity vs diffusion

- Normal transport



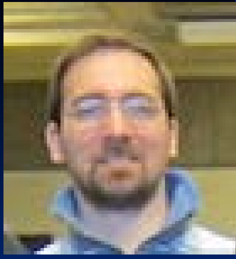
$$v = \mu F$$ $$v = f(F)$$

$$\langle x^2 \rangle \sim Dt$$ $$\langle x^2 \rangle \sim \log(t)$$

- Einstein relation: μ = D

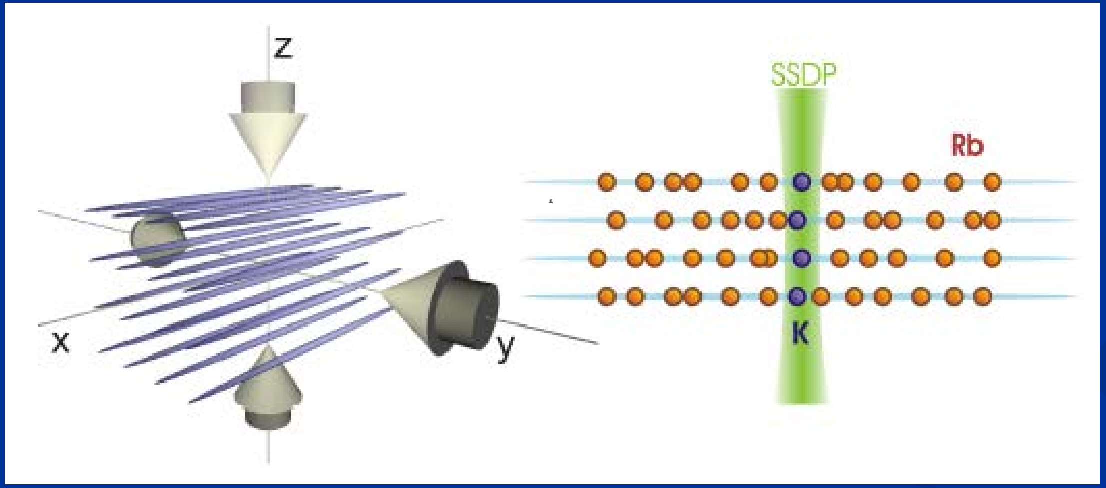
Experiments

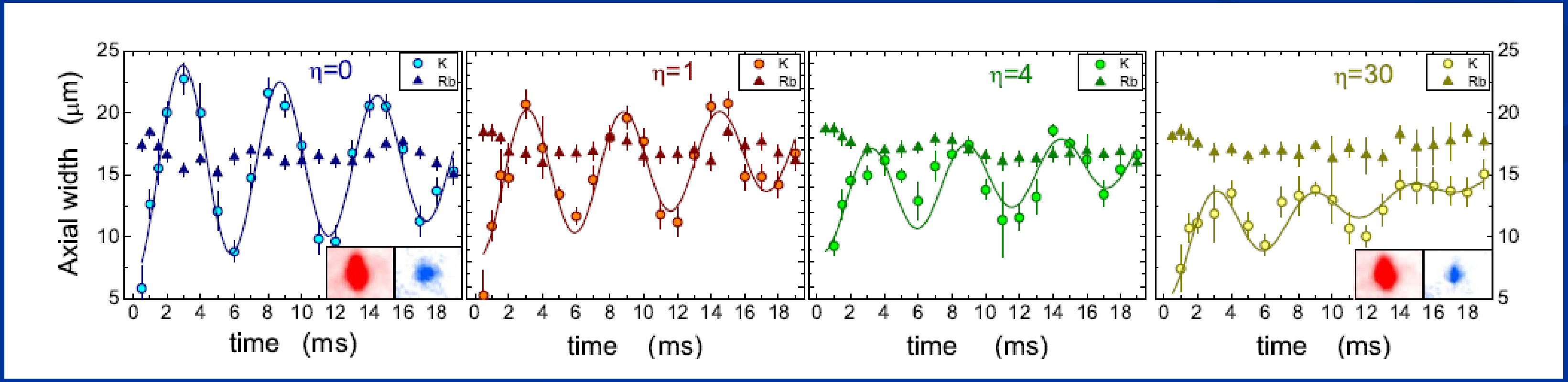
Diffusive impurity



J. Catani, G. Lamporesi, D. Naik, M. Gring, M. Inguscio, F. Minardi, A. Kantian, TG

PRA 85 023623 (2012)

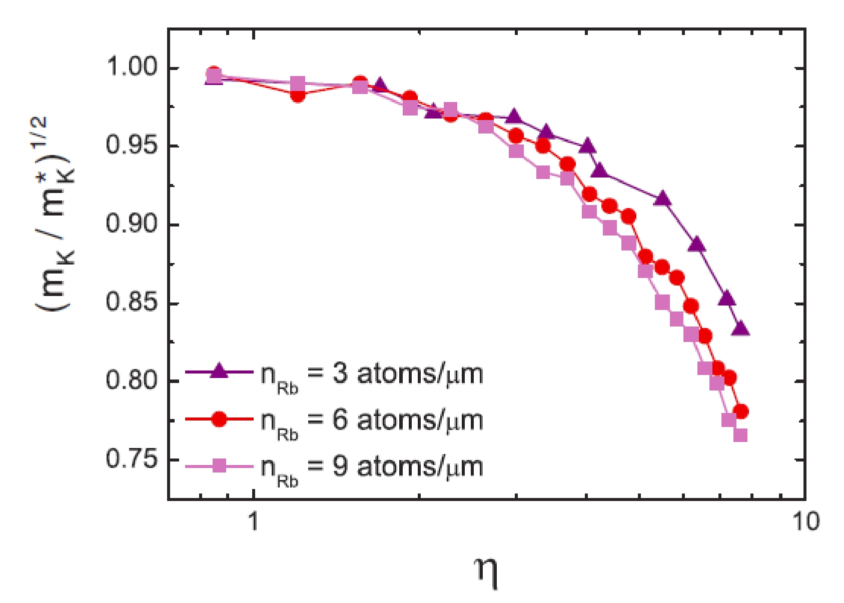




Variational method

J. Catani et al PRA 85 023623 (2012); F. Grusdt et al. arXiv:1704.02606

$$\hat{H} = \frac{\hat{p}^2}{2m_K} + \sum_{k\neq 0} \epsilon_k \hat{b}_k^\dagger \hat{b}_k + \sum_{k\neq 0} V_k e^{ik\hat{x}}(b_k + b_{-k}^\dagger),$$



$$S = \int_0^{\beta\hbar} d\tau \frac{m_K}{2}\dot{x}^2(\tau) - \sum_k \frac{V_k^2}{2\hbar}\int_0^{\beta\hbar} d\tau \times \int_0^{\beta\hbar} d\tau'\, G(k,|\tau-\tau'|)e^{ik[x(\tau)-x(\tau')]},$$

$$S_0 = \int_0^{\beta\hbar} d\tau \frac{m_K}{2}\dot{x}^2(\tau) + \frac{MW^3}{8}\int\int_0^{\beta\hbar} \times d\tau\, d\tau' \frac{\cosh(W|\tau-\tau'| - W\hbar\beta/2)}{\sinh(W\beta\hbar/2)}[x(\tau)-x(\tau')]^2,$$

Many remaining problems/questions

- Different regimes:
  A. Kantian, U. Schollwoeck, TG, PRL 113 070601 (2014)
- Fermionic baths:
  B. Horovitz, TG, P. Le Doussal, PRL 111, 115302 (2013)
- Driven impurity :
  F. Meinert et al. Science 356 945 (2017)
- Bath with a structure:
  A.M. Visuri, P. Torma TG, PRB 93 125110 (2016); PRA 95 063605 (2017)

Plan of the lectures (2)

- Lecture 2:
  Clean TLL
  - Fundamentals of transport in 1D and methods
  - Periodic structure: Mott and band insulators
  - A.c. and d.c. conductivity
  - Experimental realizations

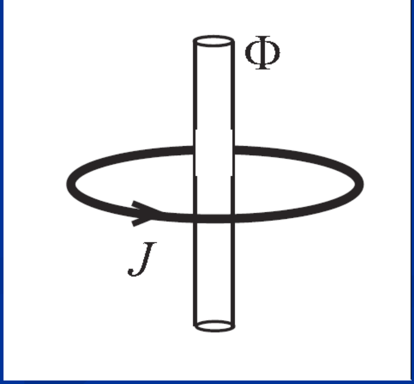
  Transverse conductivity

  Magnetic field effects
  - Bosonic ladders; Meissner effect
  - Hall effect

Transport

$$J = v(n_R - n_L) \propto \Pi \sim \partial_t \phi$$

$$\sigma(\omega) = -\frac{e^2}{\pi^2\hbar} i(\omega + i\delta)\langle \phi(q=0,\omega_n)^* \phi(q=0,\omega_n)\rangle_{i\omega_n \to \omega + i\delta}$$



$$\mathrm{Re}\,\sigma(\omega) = \mathcal{D}\delta(\omega) + \sigma_{\mathrm{reg}}(\omega)$$

$$\mathcal{D} = uK$$

- Memory function (TG PRB 44 2905 (1991))

$$\sigma(\omega) = \frac{i}{\omega}\left[\frac{2u_\rho K_\rho}{\pi} + \chi(\omega)\right]$$

$$\sigma(\omega) = \frac{i2u_\rho K_\rho}{\pi} \frac{1}{\omega + M(\omega)}$$

$$M(\omega) = \frac{\omega\chi(\omega)}{\chi(0) - \chi(\omega)}$$

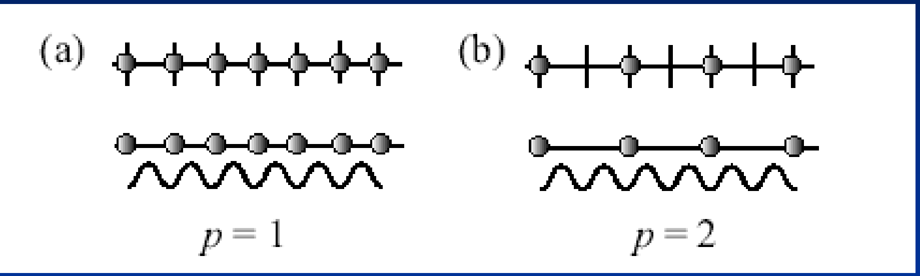
$$F = [J, H]$$

- Approximate ``hydrodynamic” solution:

$$M(\omega) \simeq \frac{[\langle F;F\rangle^0_\omega - \langle F;F\rangle^0_{\omega=0}]/\omega}{-\chi(0)}$$

- Takes into account all vertex corrections (mandatory in 1d)
- Assumes: i) perturbation; ii) independent scattering event; Not necessarily correct

Periodic lattice



$$H = \int dx V_0 \cos(Qx)\rho(x)$$

$$H = \int dx V_0 \cos(Qx)\rho_0 e^{i(2\pi\rho_0 x - 2\phi(x))}$$

- Incommensurate: $Q \neq 2\pi\rho_0$

$$H = \int dx \cos(2\phi(x) + \delta x)$$

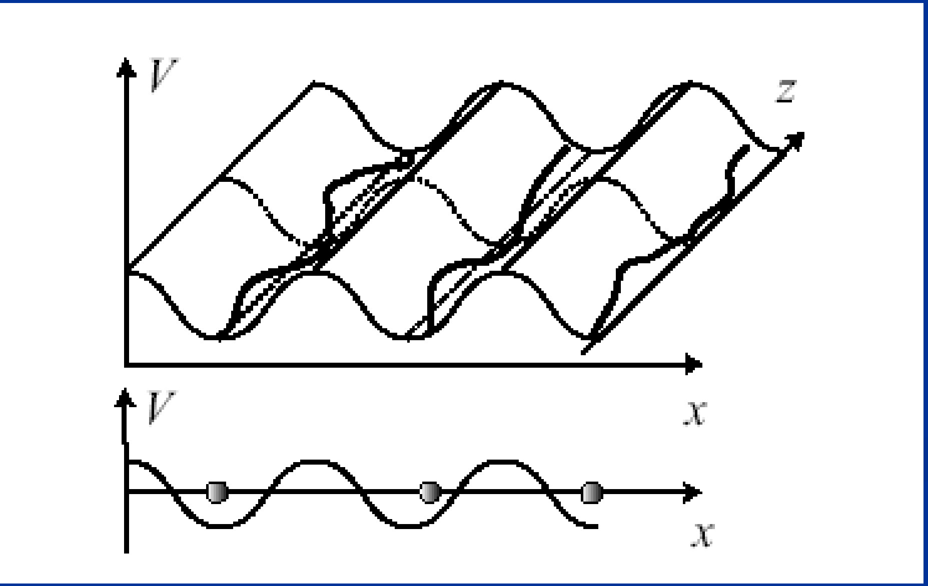
- Commensutate: $Q = 2\pi\rho_0$

$$H = \int dx \cos(2\phi(x))$$

Competition

$$S_0 = \int \frac{dx d\tau}{2\pi K}\left[\tfrac{1}{u}(\partial_\tau \varphi(x,\tau))^2 + u(\partial_x \varphi(x,\tau))^2\right]$$

$$S_L = -V_0 \rho_0 \int dx d\tau \cos(2\phi(x))$$



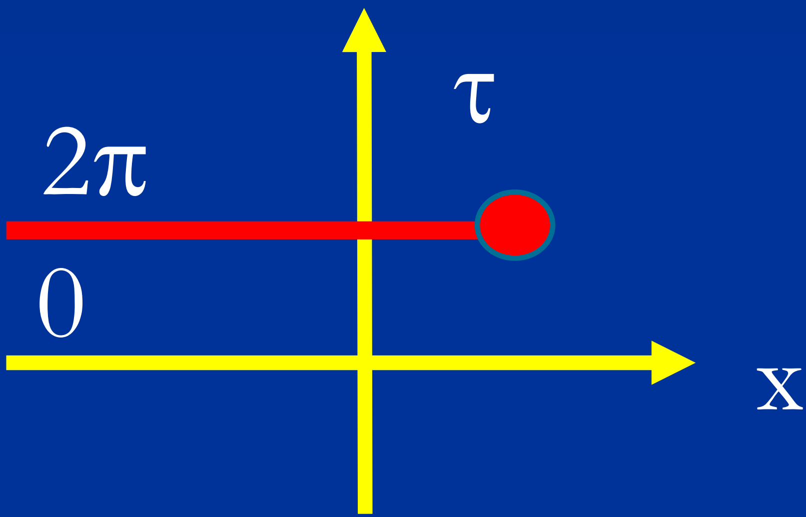
Beresinskii-Kosterlitz-Thouless transition at K=2

String order parameter

Vortex operator

$$e^{iaP}\,|\,x\rangle = |x+a\rangle$$

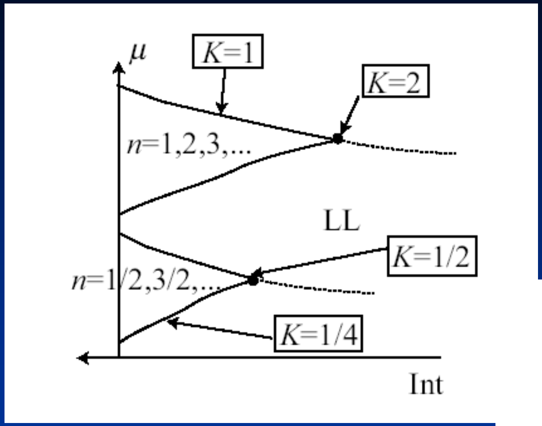
$$\phi(x,\tau) = \pi \int_{-\infty}^{x} dx' \Pi_\theta(x',\tau)$$

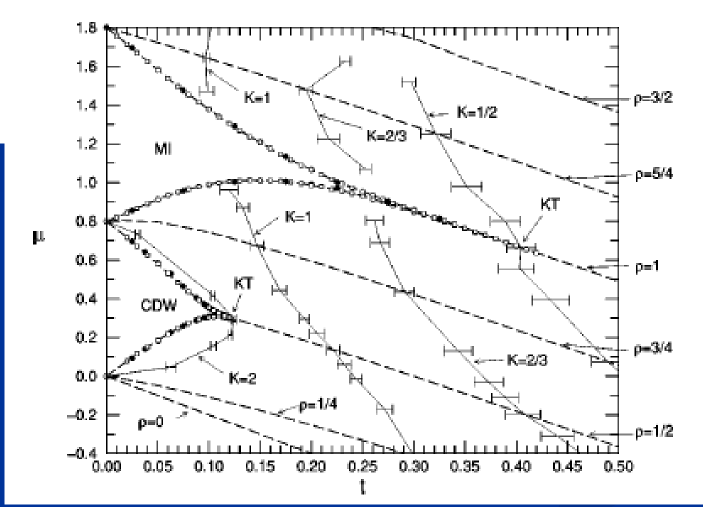


$$\cos(2\phi(x_1,\tau_1))$$

- Vortex operator for θ
- K : inverse temperature
- g : vortex fugacity

$$S = \frac{K}{2\pi}\int dx d\tau \left[\frac{1}{u}(\partial_\tau\theta) + u(\partial_x\theta)^2\right] - g\int dx \cos(2\phi)$$





Mott insulator:

ϕ is locked

Density is fixed

TG, Physica B 230 975(97): arXiv/0605472 (Salerno lectures);

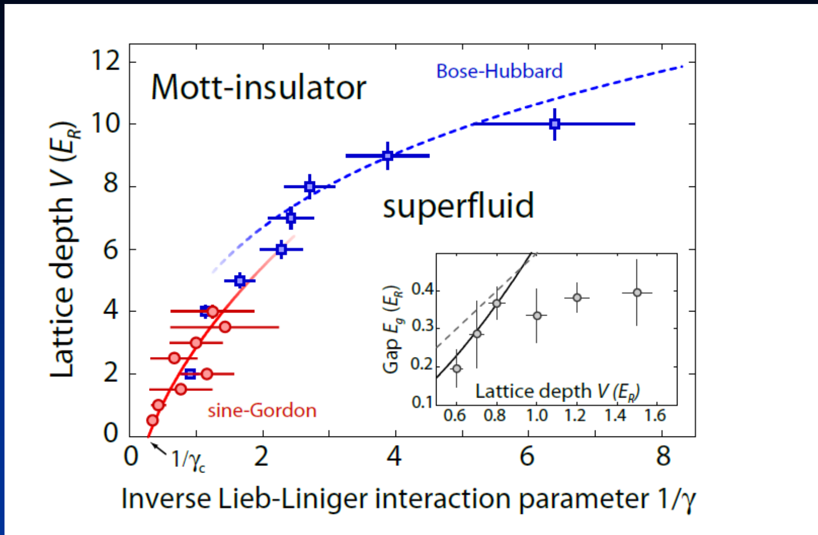
Oxford (2004);

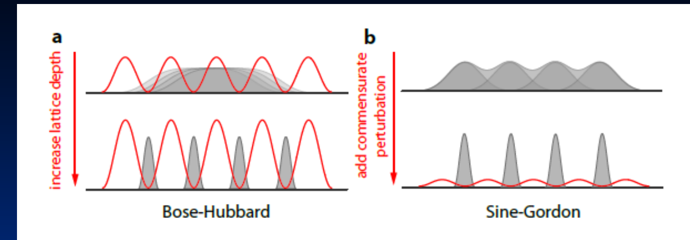
M. Cazalilla et al., Rev. Mod. Phys.83 1405 (2011)

T. Kuhner et al. PRB 61 12474 (2000)

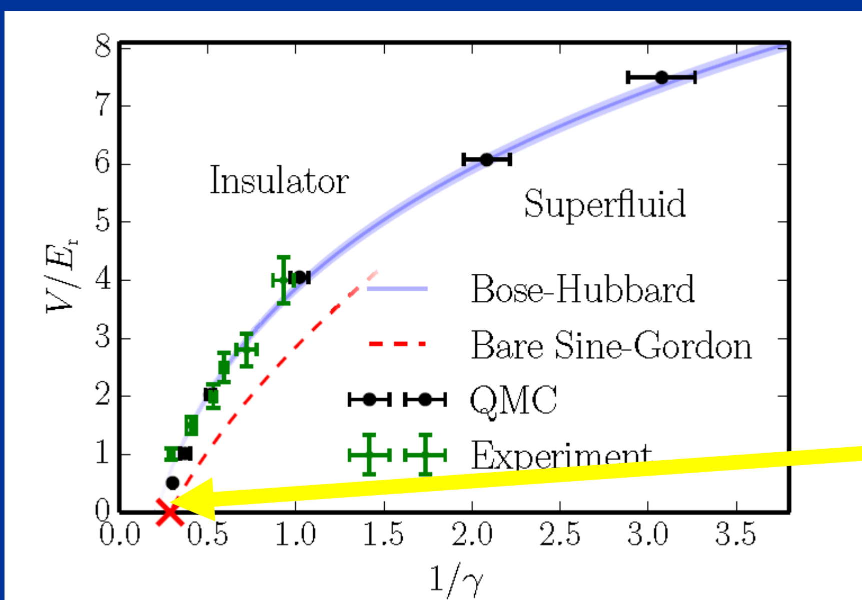
Gap in the excitation spectrum

$$G(r) \propto e^{-r/\xi}$$





E. Haller et al. Nature 466 597 (2010)



Renormalized Sine-Gordon

Shows:

K* = 2

G. Boeris et al. PRA 93 93, 011601(R) (2016)

Non local (topological) order

$$\rho(x) \sim \nabla \phi(x)$$

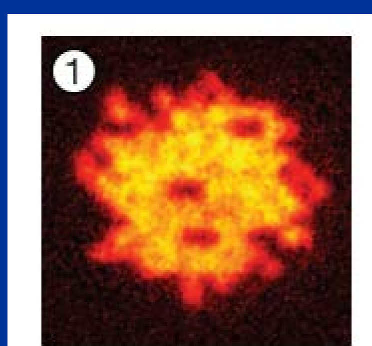
$$\mathcal{O}_{\mathrm{P}}^2 = \lim_{l \to \infty} \mathcal{O}_{\mathrm{P}}^2(l) = \lim_{l \to \infty} \left\langle \prod_{k \leq j \leq k+l} e^{i\pi\delta\hat{n}_j} \right\rangle$$

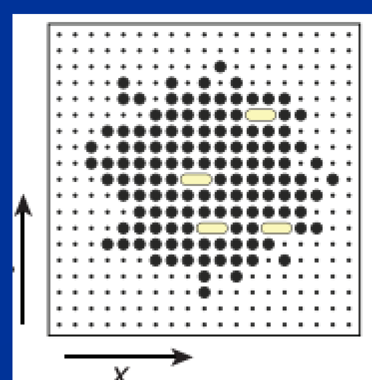
E. Berg, E. Dalla Torre, T. Giamarchi, E. Altman, *Phys. Rev. B* **77**, 245119 (2008).

Observation of Correlated Particle-Hole Pairs and String Order in Low-Dimensional Mott Insulators

M. Endres,[1*] M. Cheneau,[1] T. Fukuhara,[1] C. Weitenberg,[1] P. Schauß,[1] C. Gross,[1] L. Mazza,[1] M. C. Bañuls,[1] L. Pollet,[2] I. Bloch,[1,3] S. Kuhr[1,4]

Science (2011)



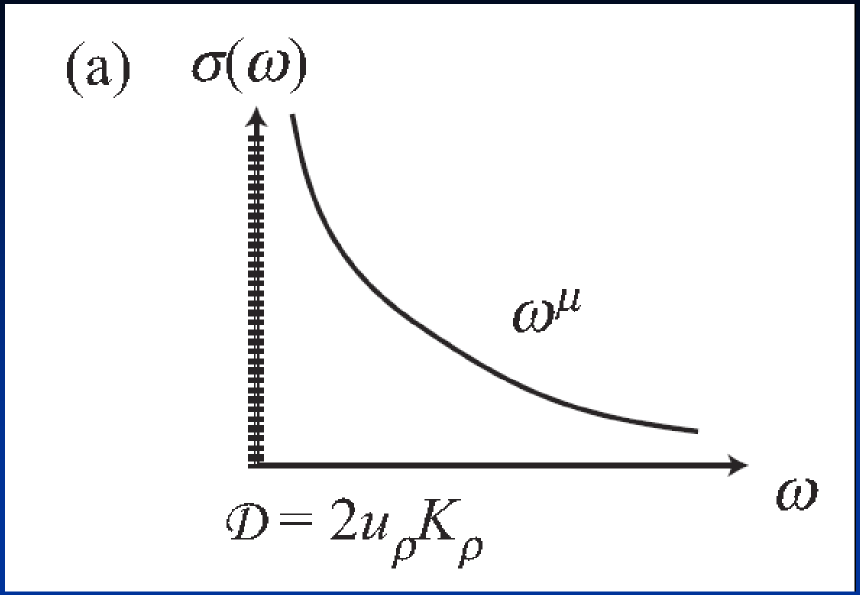


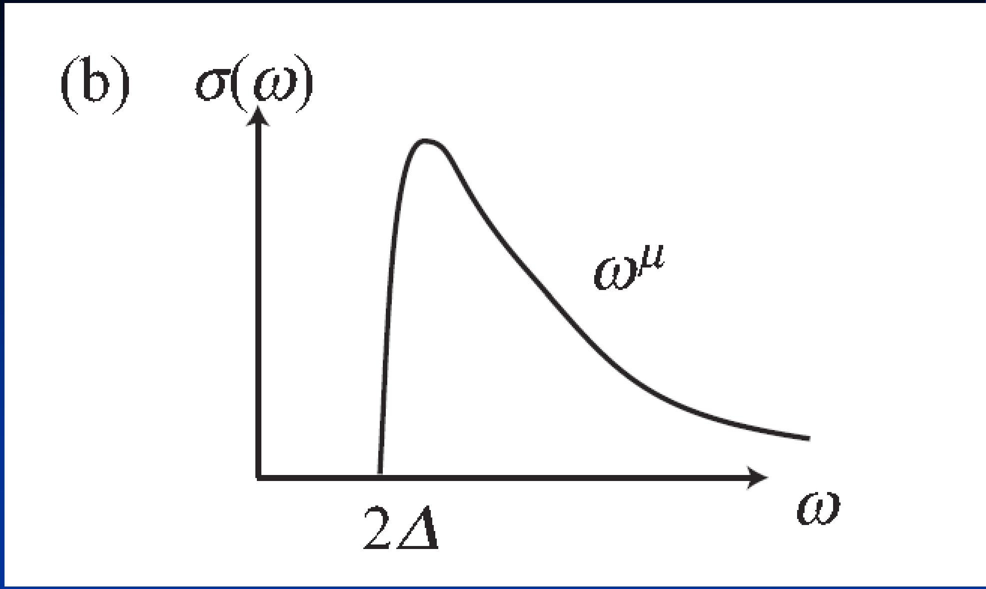
Conductivity (T=0)

$$F = [j, H] = \frac{8g_3}{(2\pi\alpha)^2}(u_\rho K_\rho)\, i\ \sin(\sqrt{8}\phi_\rho(x,\tau) - \delta x)$$

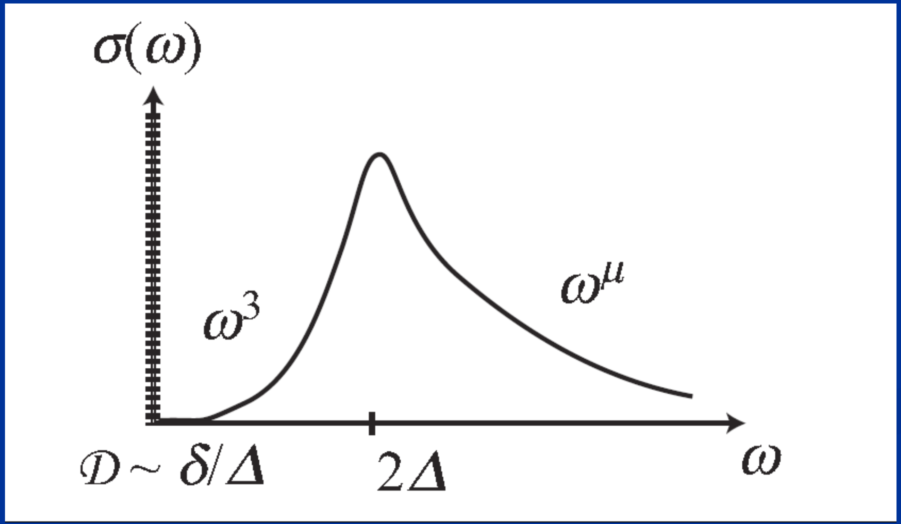
$$M(\omega) = \frac{g_3^2 K_\rho}{\pi^3\alpha^2}\left(\frac{2\pi\alpha T}{u_\rho}\right)^{4K_\rho - 2}\frac{1}{\omega}[B(K_\rho - iS_+, 1-2K_\rho)B(K_\rho - iS_-, 1-2K_\rho) - B(K_\rho - iS_+^0, 1-2K_\rho)B(K_\rho - iS_-^0, 1-2K_\rho)] \quad (7.97)$$

$$S_\pm = (\omega \pm u_\rho\delta)/(4\pi T)$$





Combine memory function and RG (TG (1991))



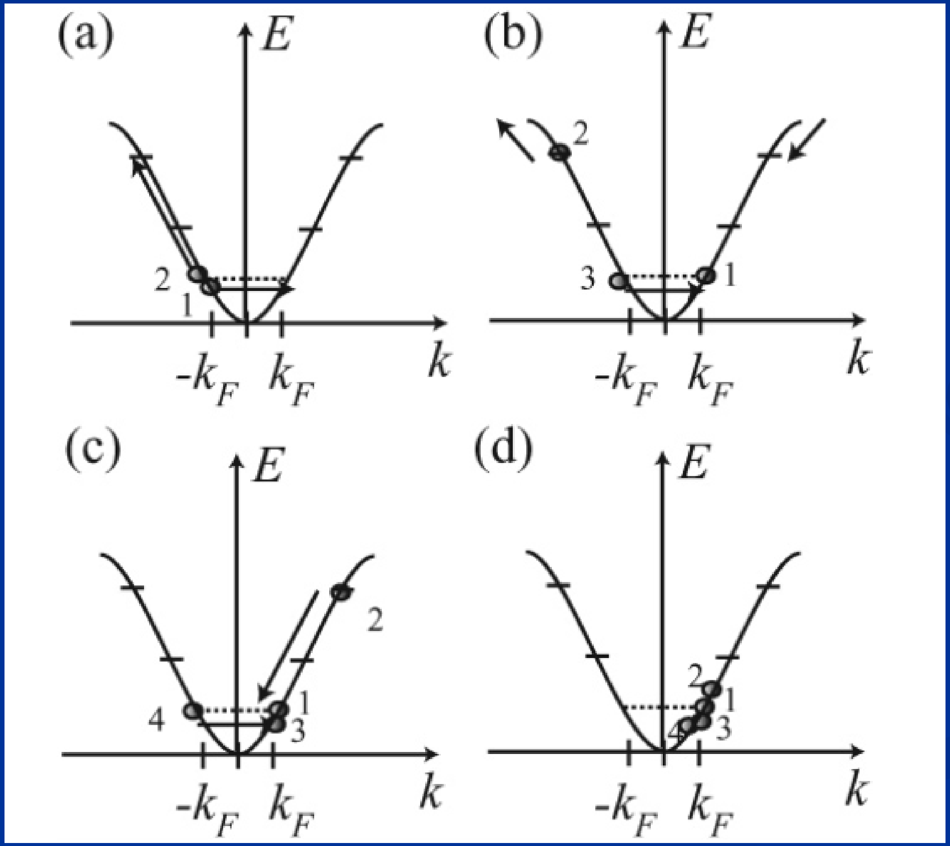
Incommensurate:

Doping δ

Organic conductors

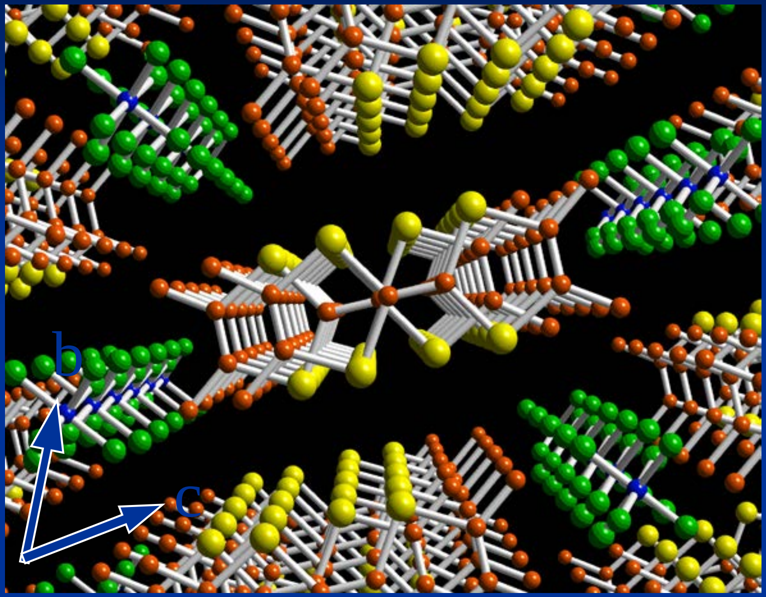
TG Chemical Review 104 5037 (2004)

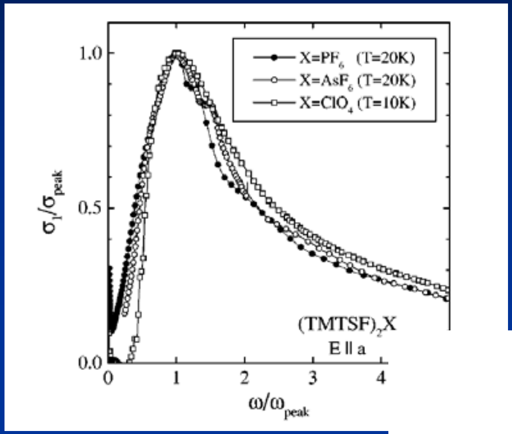
- Need to consider also ¼ filled Umklapp

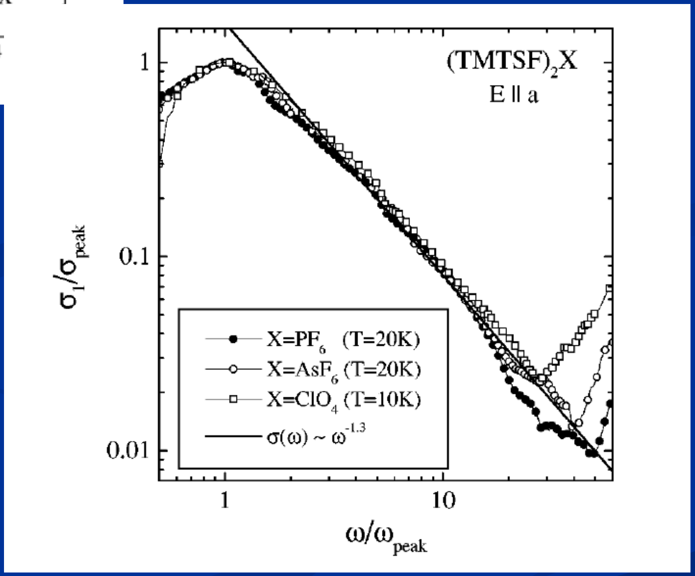


Non perturbative

Organic conductors







$$\sigma(\omega) \sim \omega^{-\nu}$$

TG PRB (91) :
Physica B 230 (1996)

A. Schwartz et al. PRB 58 1261 (1998)

Quarter filling commensurability

Finite temperature

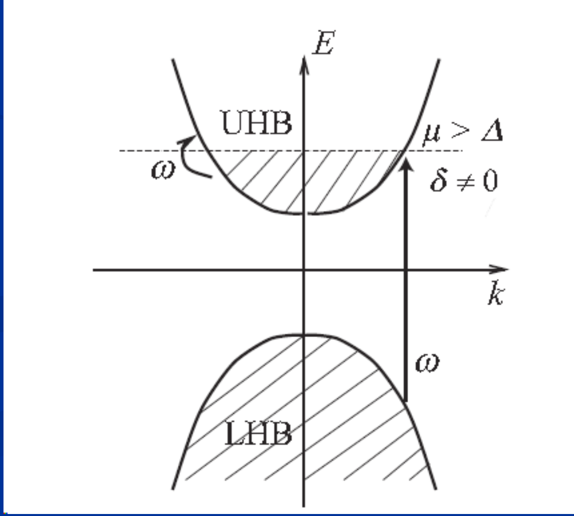
$$M(\omega) \simeq i\frac{g_3^2 K_\rho}{\pi^3\alpha^2} B^2(K_\rho, 1-2K_\rho)\cos^2(\pi K_\rho)\frac{1}{T}\left(\frac{2\pi\alpha T}{u_\rho}\right)^{4K_\rho-2}$$

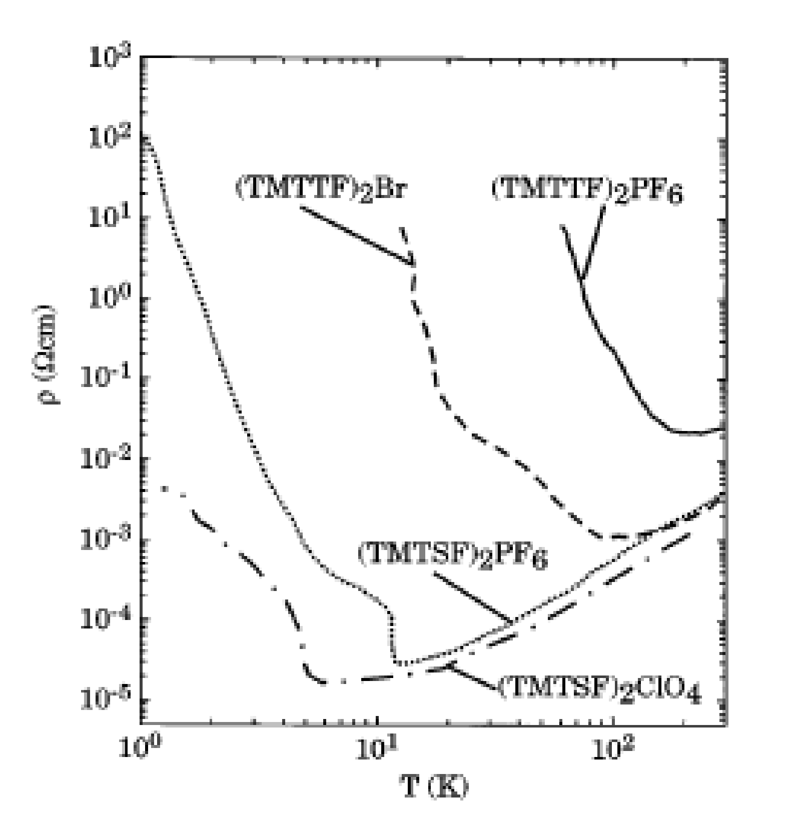
$$\sigma(T) \sim \frac{1}{g_3^2} T^{3-4K_\rho}$$

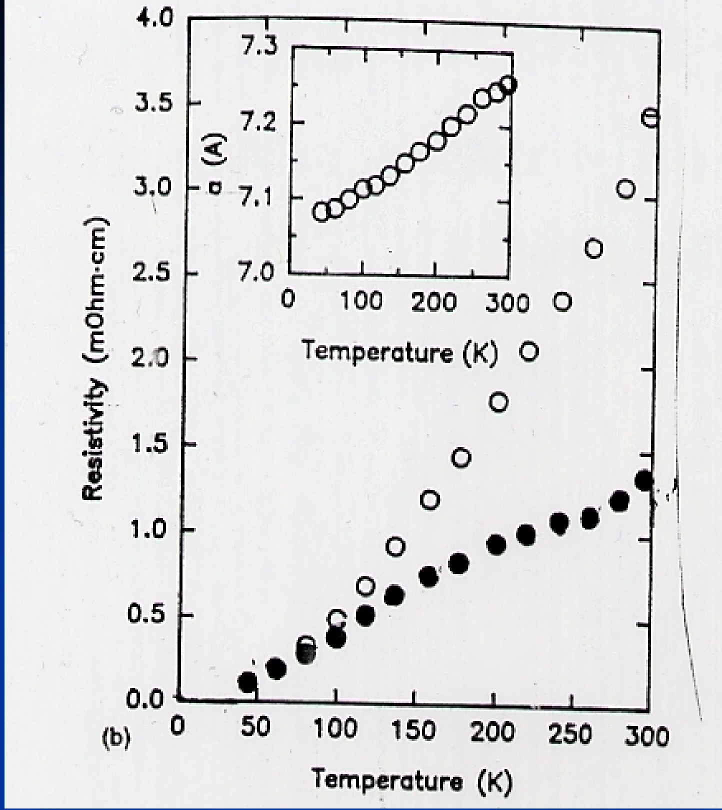
Conservation laws

$$g\cos(2\phi) = g(\psi_R^\dagger \psi_L + h.c.)$$

D finite at finite T







V. Vescoli et al. Euro Phys J B 13 503 (2000)

D. Jérome in ``Organic superconductors”, ed. J.P. Farges

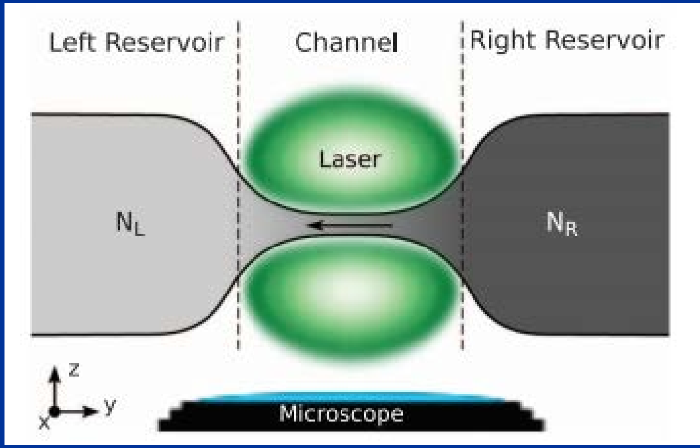
More in: The physics of organic conductors and superconductors, ed. A. Lebed, Springer (2008).
TG: arXiv:cond-mat/0702565

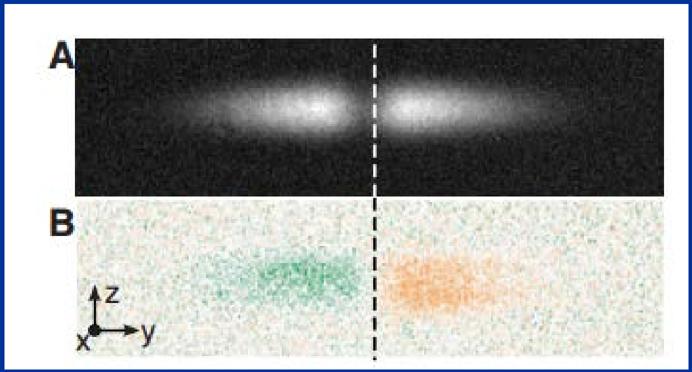
Cold atoms

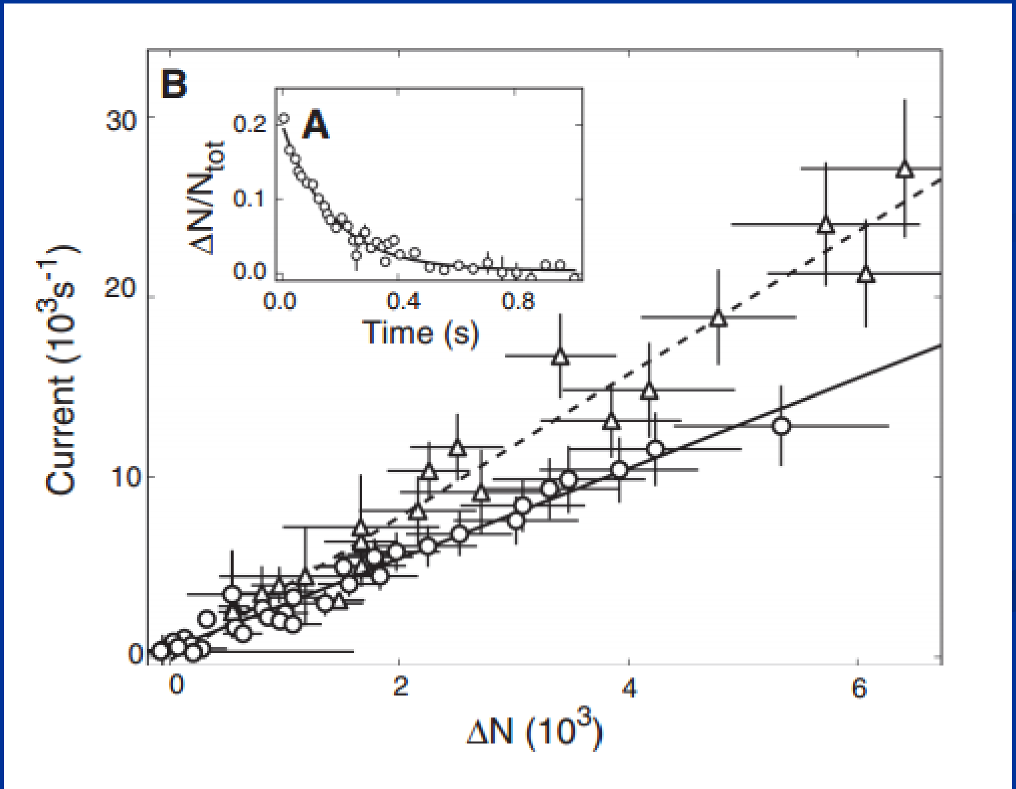
Conduction of Ultracold Fermions Through a Mesoscopic Channel

Jean-Philippe Brantut, Jakob Meineke, David Stadler, Sebastian Krinner, Tilman Esslinger*

SCIENCE VOL 337 31 AUGUST 2012



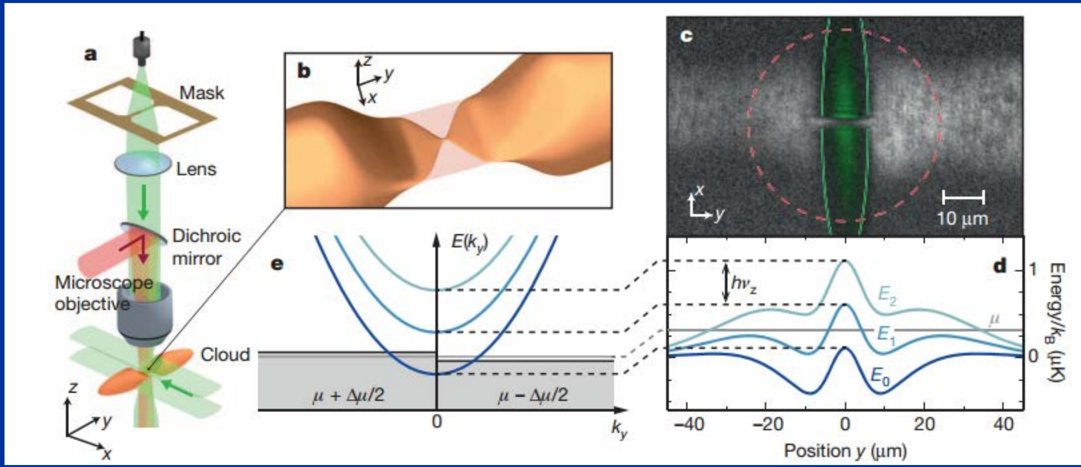


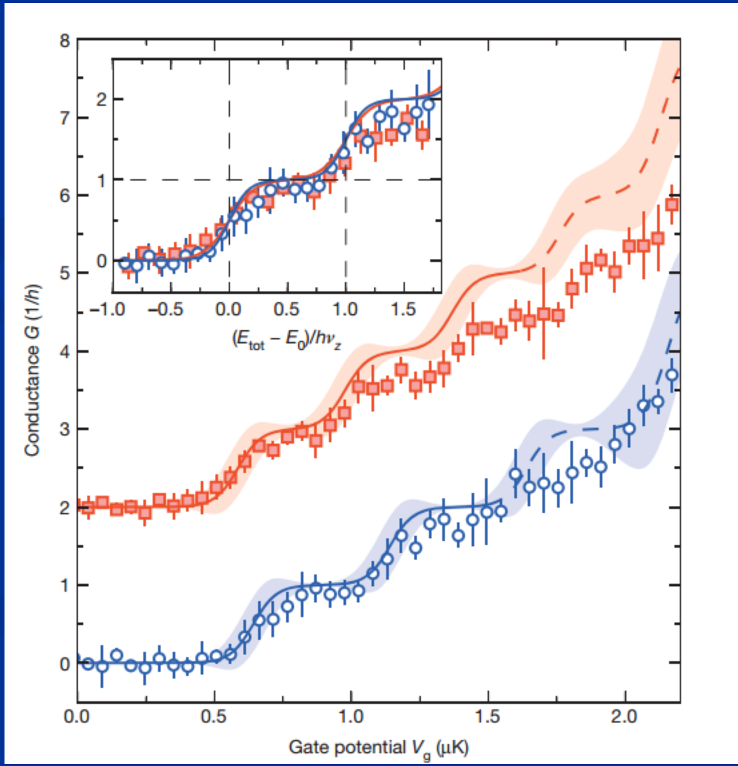


Observation of quantized conductance in neutral matter

Sebastian Krinner[1], David Stadler[1], Dominik Husmann[1], Jean-Philippe Brantut[1] & Tilman Esslinger[1]

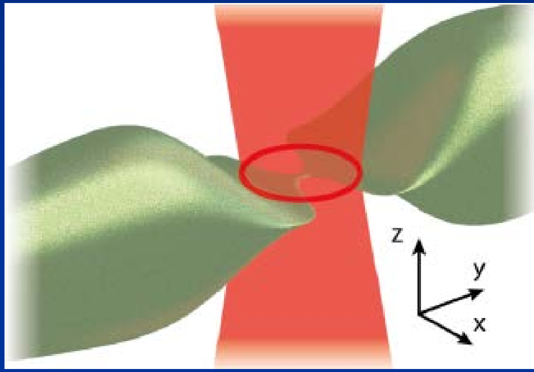
64 | NATURE | VOL 517 | 1 JANUARY 2015

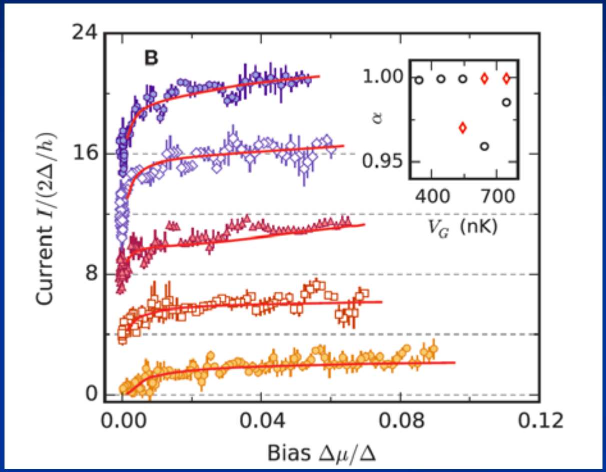




Transport through 0 and 1d

- Quantum point contact:

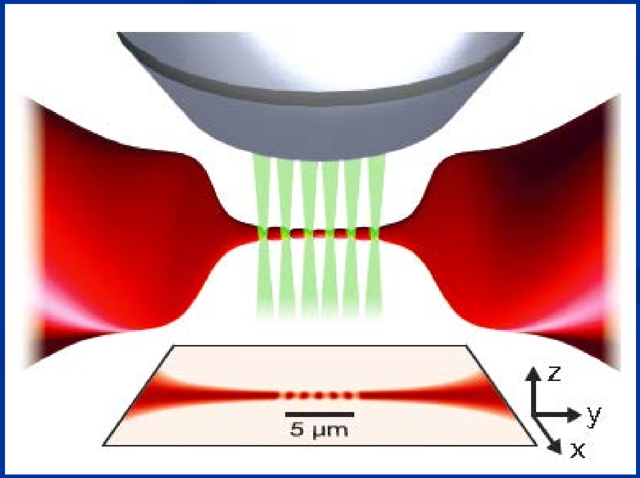


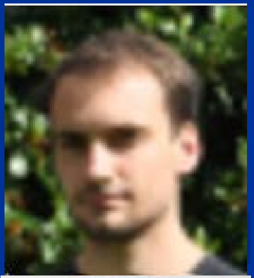


D. Husmann, S. Uchino, et al. Science 350 62667 (2015).

- Periodic 1d structure:

M. Lebrat, P. Grisins et al.





Beyond a single chain

- Largely open physics
- Strong difficulties to treat (analytics, numeric)
- Allows to incorporate SOC and magnetic field effects
- Many studies limited to non-interacting case
- Relevant for many experimental systems

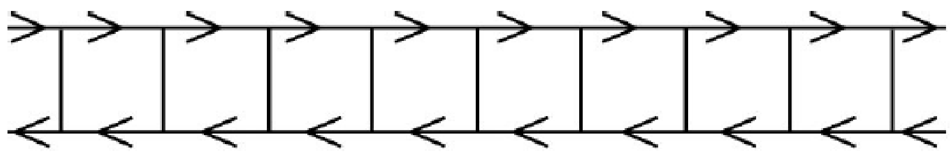
Ladder physics

- Contains the interplay of lattice vs gauge fields
- Interactions treatable by bosonization, numerics, etc.
- Additional beautiful one-dimensional physics

Magnetic field effects

Meissner effect in bosonic ladders

E. Orignac, TG, PRB 64 144515 (2001)


(a)

$$H = -t_{\|} \sum_{i,p=1,2} \left(b_{i+1,p}^{\dagger} e^{ie^* a A_{\|,p}(i)} b_{i,p} + b_{i,p}^{\dagger} e^{-ie^* a A_{\|,p}(i)} b_{i+1,p} \right)$$

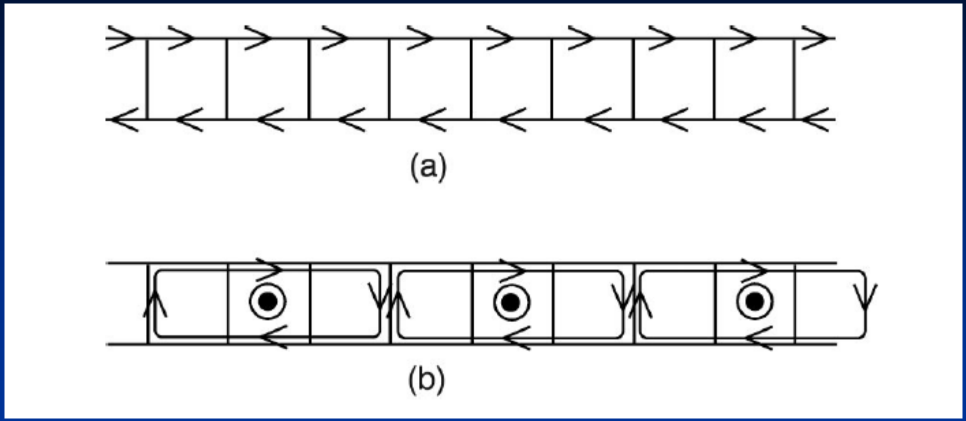
$$- t_{\perp} \sum_{i} \left(b_{i,2}^{\dagger} e^{ie^* A_{\perp}(i)} b_{i,1} + b_{i,1}^{\dagger} e^{-ie^* A_{\perp}(i)} b_{i,2} \right)$$

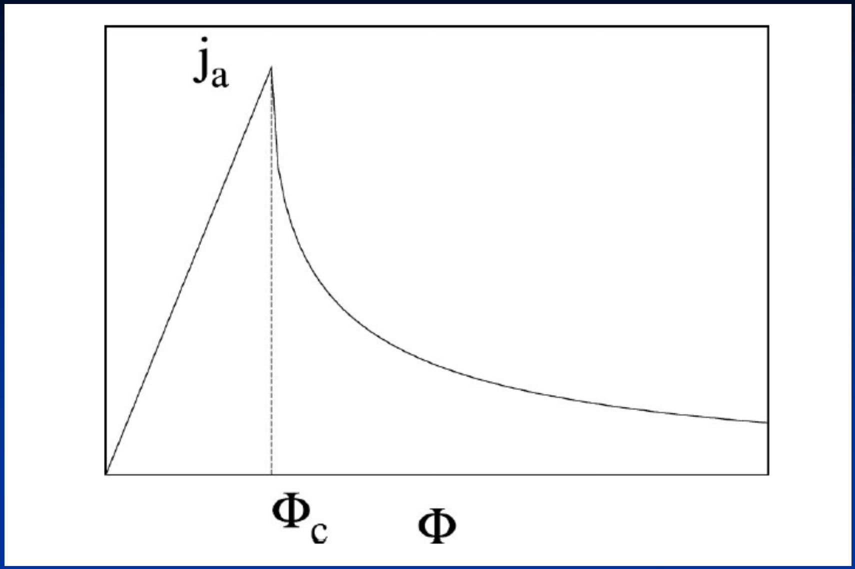
$$+ U \sum_{i,p} n_{i,p}(n_{i,p} - 1) + V n_{i,1} n_{i,2}, \qquad (1)$$

$$\int \vec{A} \cdot \vec{dl} = \Phi$$

$$H = H_s^0 + H_a^0 - \frac{t_{\perp}}{\pi a} \int dx \cos[\sqrt{2}\,\theta_a + e^* A_{\perp}(x)]$$

$$+ \frac{2Va}{(2\pi a)^2} \int dx \cos \sqrt{8}\,\phi_a ,$$



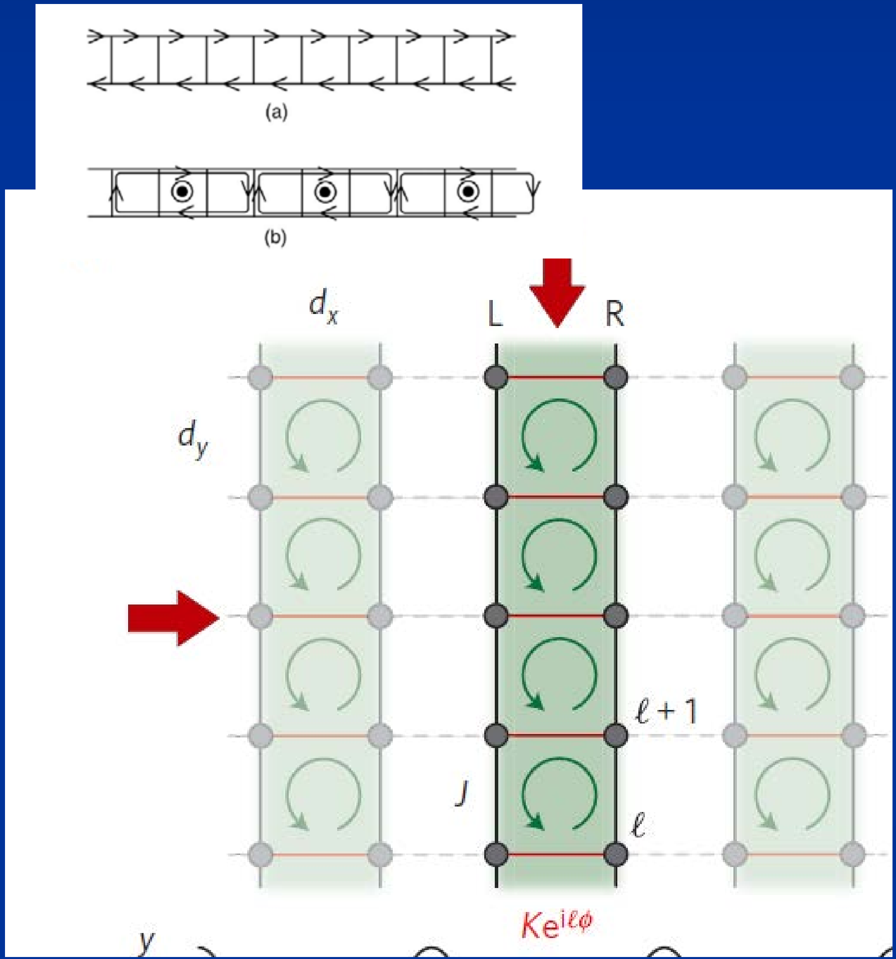


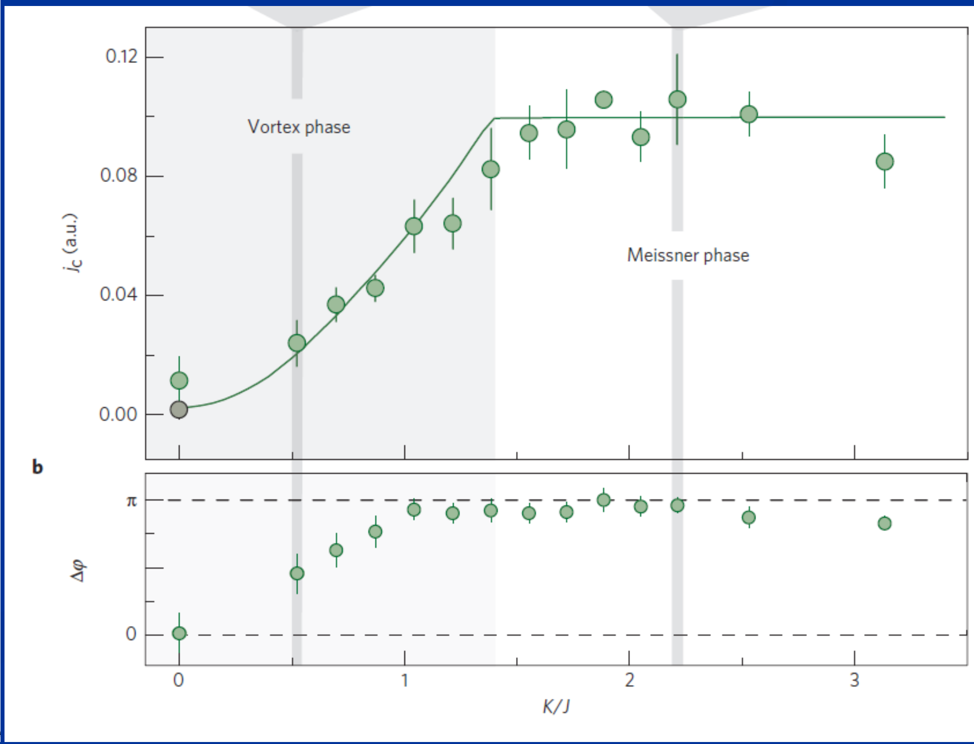
Orbital currents (``Meissner” effect)

Field ``Hc1”: appearance of vortices

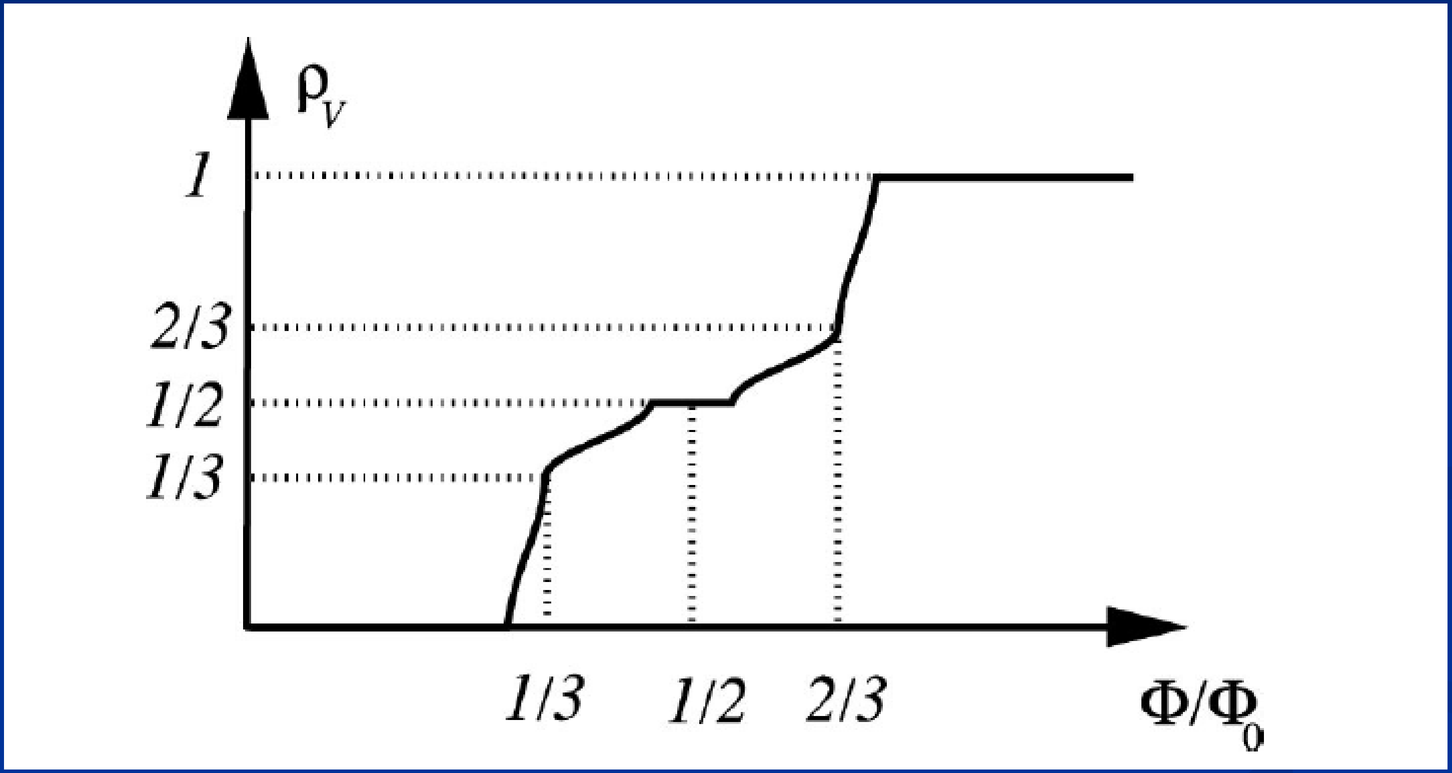
Artificial gauge field (cold atoms)

M Atala et al. Nat Phys, 10 588 (2014)





Interactions: devil's staircase



Plateau at ½ should be visible

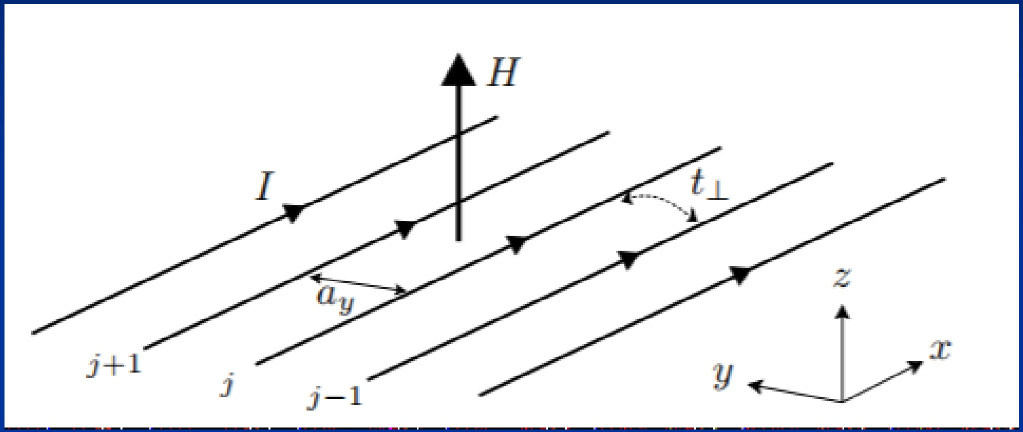
Many other works on ladders and B / SOC

- Nersesyan, Lehur, Roux, ……
- SOC: Unusual superconductivity
  S. Uchino, A. Tokuno, TG, PRA 89 023623 (2014)
  S. Uchino, TG, PRA 91 013604 (2015)

Many chains: hall effect

A. Lopatin, A. Georges, TG PRB 63 075109 (2001)
G. Leon, C. Berthod, TG PRB 75, 195123 (2007)
G. Leon, C. Berthod, TG, A.J. Millis PRB 78, 085105 (2008)

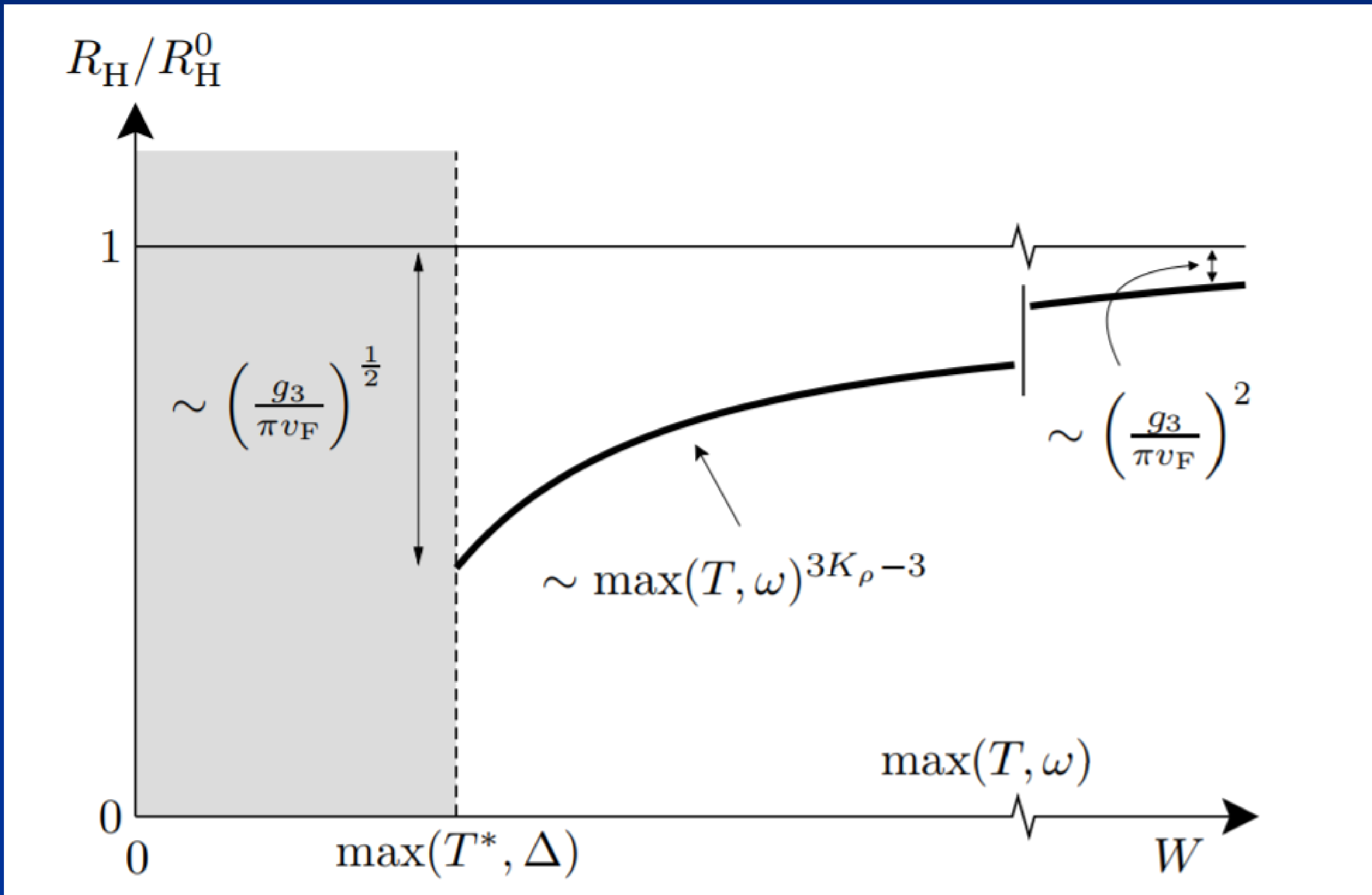
Why difficult ?

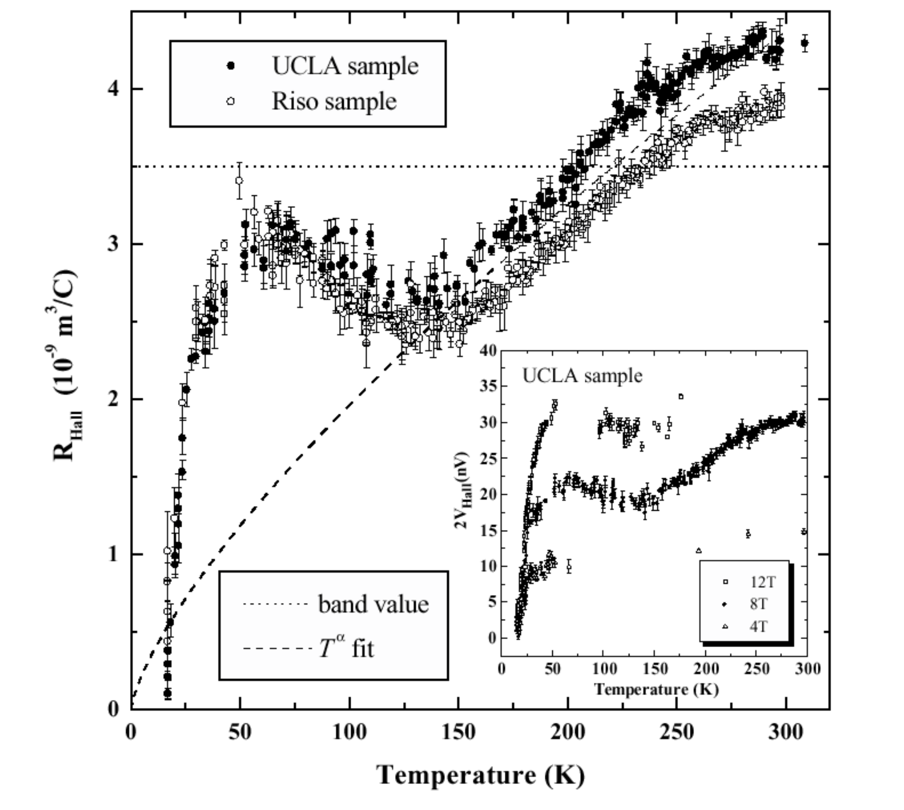


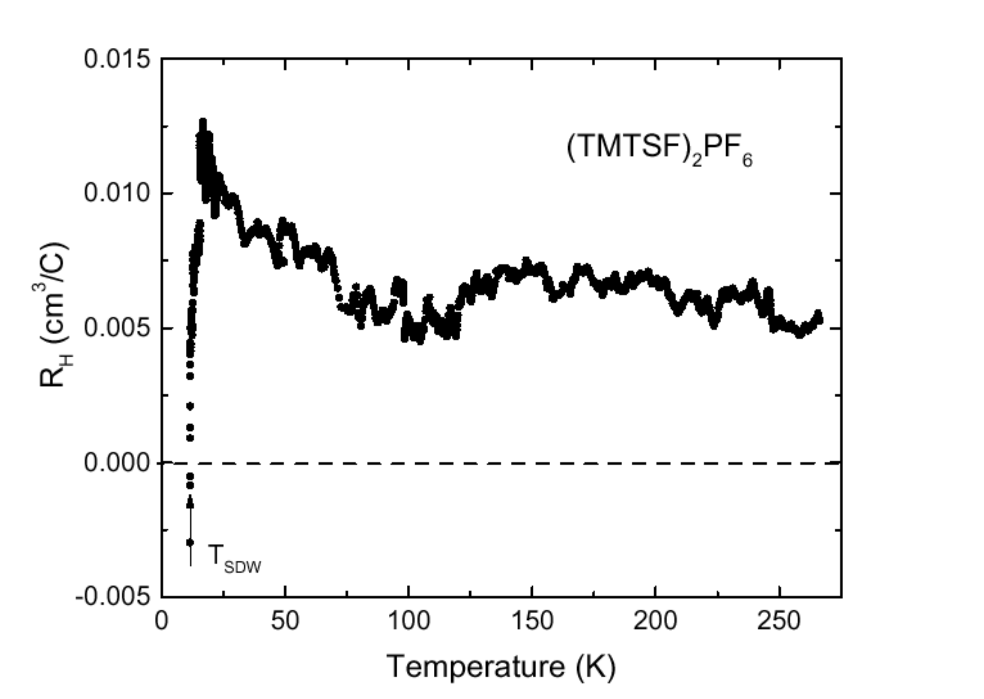
Need to go beyond TLL approximation:

- Band curvature needed (no particle-hole symmetry)
- Perturbation in : curvature, hopping, umklapp, magnetic field….

High temperature regime



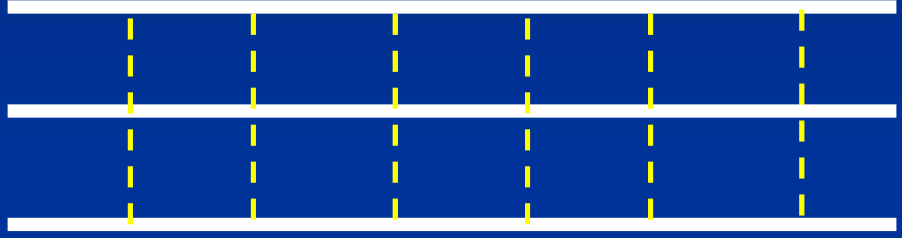




G. Mihaly et al., PRL 84 2670 (00)

J. Moser et al., PRL 84 2674 (00)

Transverse transport

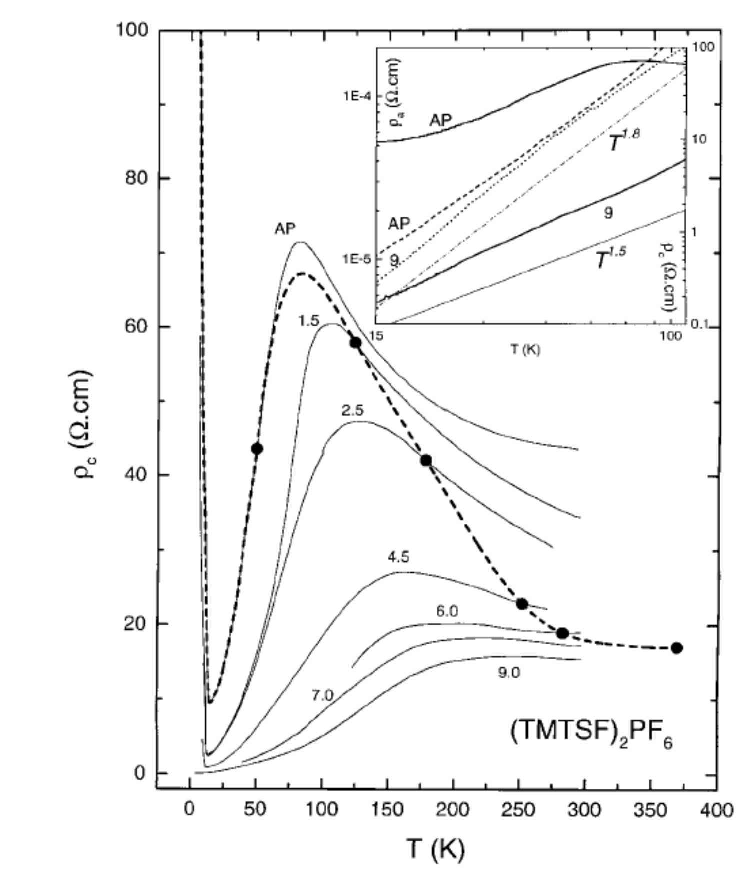


Perpendicular hopping t'

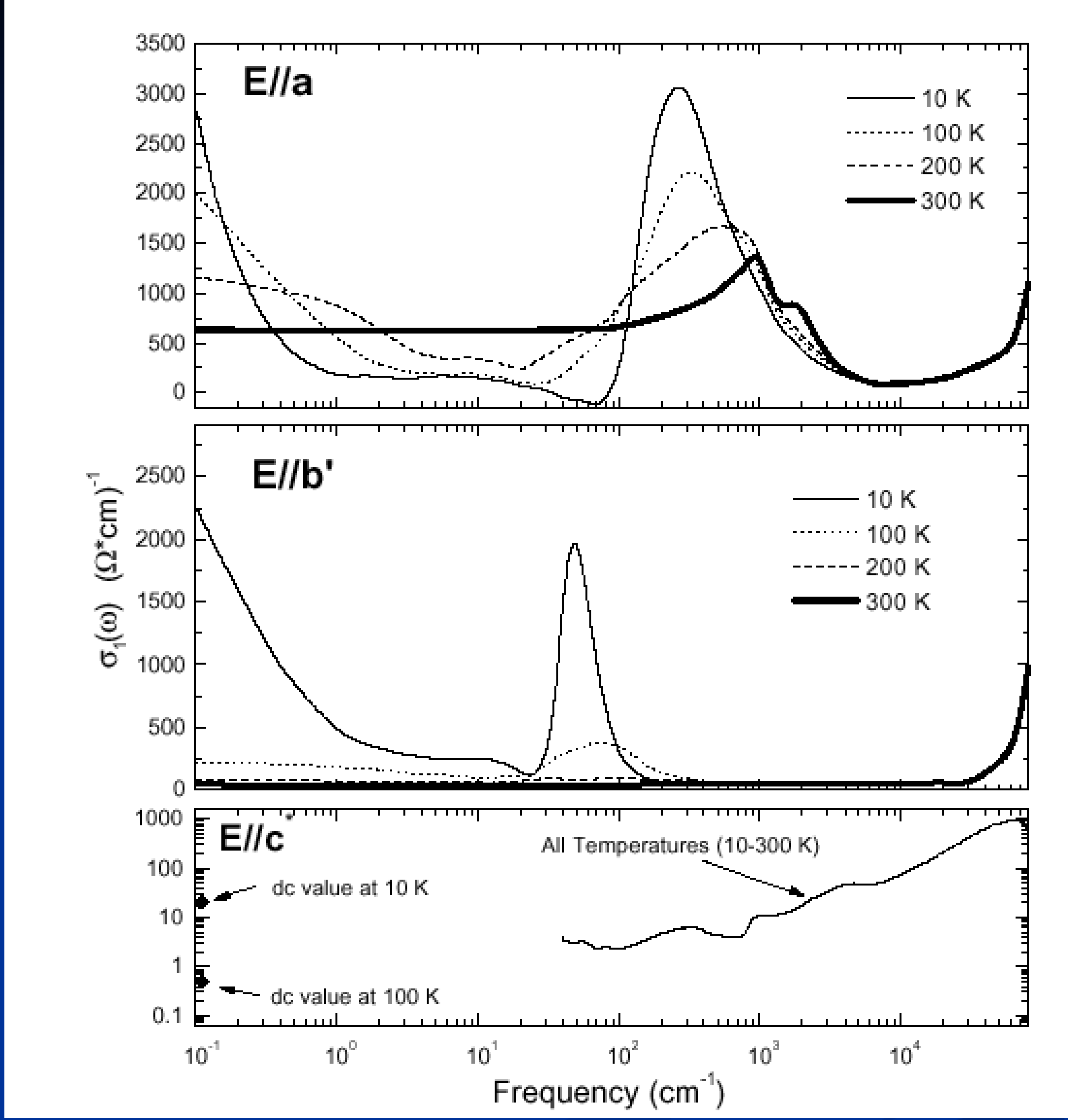
T > t': tunneling, not usual transport

$$\sigma(\omega,T) \propto (\omega,T)^{2\alpha-1}$$

$$\alpha = \tfrac{1}{4}(K + K^{-1}) - \tfrac{1}{2}$$



J. Moser et al. Euro Phys. J. B 1 39 (1998)



V. Vescoli et al. Euro Phys J B 11 365 (1999)

Plan of the lectures (3)

- Lecture 3:
  Effects of disorder
  - Disorder and interactions in 1D systems
  - T=0 properties: localization of interacting particles
  - Finite temperature: transport with a bath (VRH)
  - Other types of ``disorder" (quasiperiodics, etc.)
  - Experimental realizations

General References

TG, E. Orignac
arXiv:cond-mat/0005220

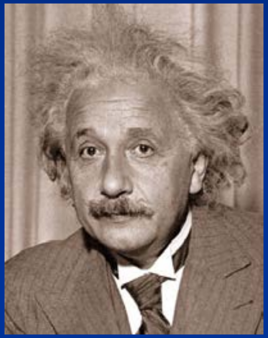
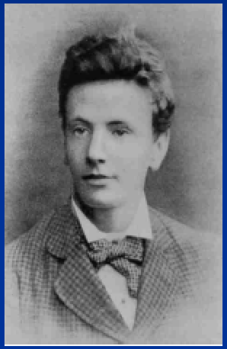
T. Nattermann, TG, P. Le doussal,
arXiv:cond-mat/0403487

TG, Salerno lectures,
arXiv:cond-mat/0605472

TG, Varenna lectures 2014

Disorder: ubiquitous in CM

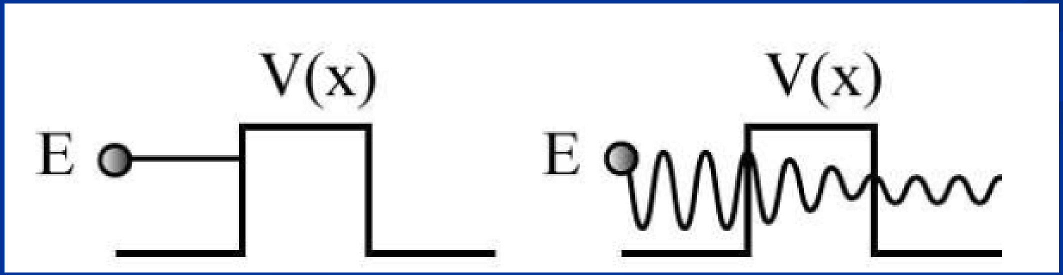
- Classical particles: diffusion $r^2 = Dt$

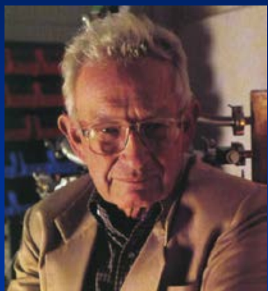
$$D = \mu k_B T$$

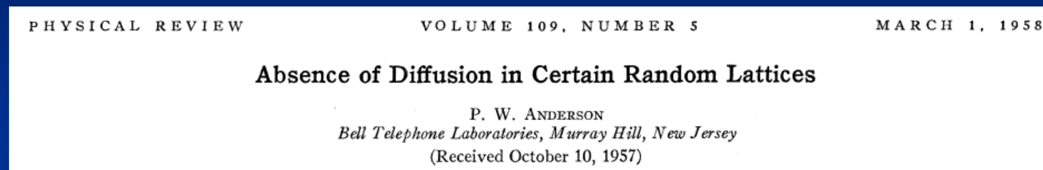
Einstein, A., *Z. Elektrochem.*, **14**, 235 (1908).
Sutherland, W., *Phil. Mag.*, **9**, 781 (1905).

- Electrons are wave: what does it change ?
- Nothing !!!! $\sigma = \dfrac{ne^2\tau}{m}$



Anderson Localization





PHYSICAL REVIEW VOLUME 109, NUMBER 5 MARCH 1, 1958

Absence of Diffusion in Certain Random Lattices

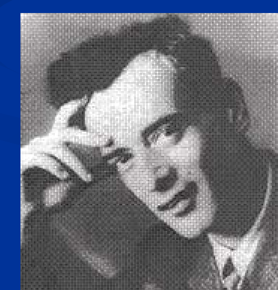
P. W. ANDERSON
Bell Telephone Laboratories, Murray Hill, New Jersey
(Received October 10, 1957)

Light, sound, electrons, etc……. waves

www.andersonlocalization.com

Interactions *and* disorder ?

- U>0 Landau Fermi liquid $m \to m^*$



- U < 0 Supraconductivity Insensitive to disorder ?

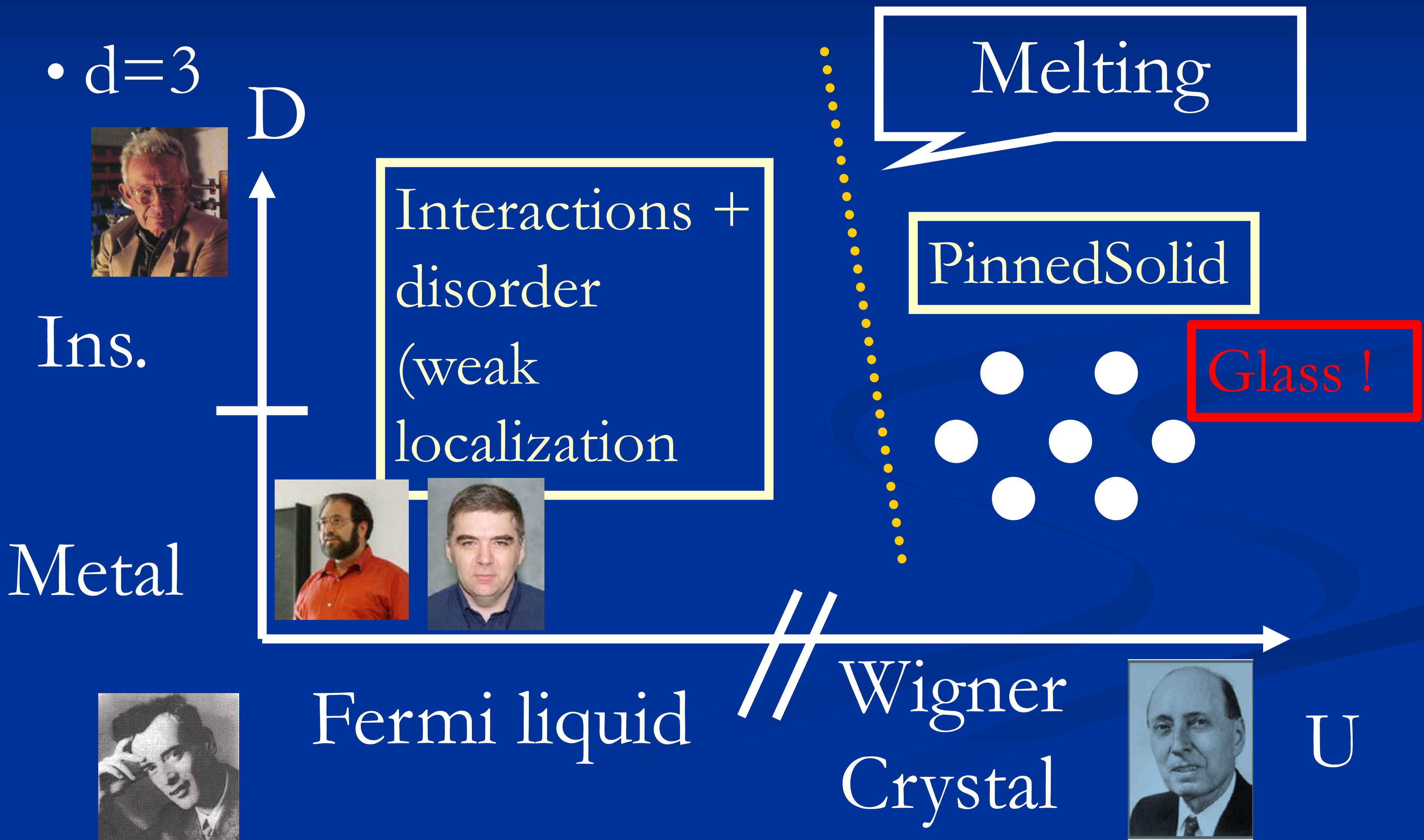
Disorder and Interactions

- Fermions: reinforcement of interactions by disorder
  perturbative: Altshuler-Aronov-Lee (80)
  RG: Finkelstein (84); TG+Schulz (88)

Localization ? Phases (electron glass) ? Transport ? …..

Phase diagram

TG arXiv/0403531, Varenna lectures (2002)



Disorder and Interactions

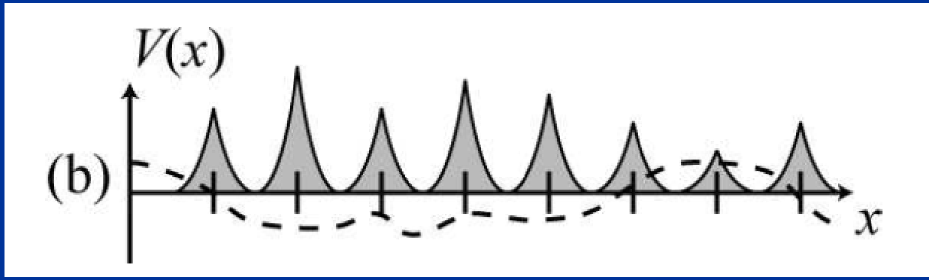
- Fermions: reinforcement of interactions by disorder
  perturbative: Altshuler-Aronov-Lee (80)
  RG: Finkelstein (84); TG+Schulz (88)

Localization ? Phases (electron glass) ? Transport ? …..

- Bosons: competition between superfluidity/localization

Free Bosons: pathological

$$H = \frac{1}{2m}\left(\frac{1}{L}\right)^2 - V_0$$



Interactions needed from the start

Example: localization of interacting bosons

Localization of Interacting Particles and Many-body localization

- Here assumption of the contact with a thermal bath
- Equilibrium is reached ``after some time”
- Will essentially not worry about ergodicity/aging

Dirty bosons

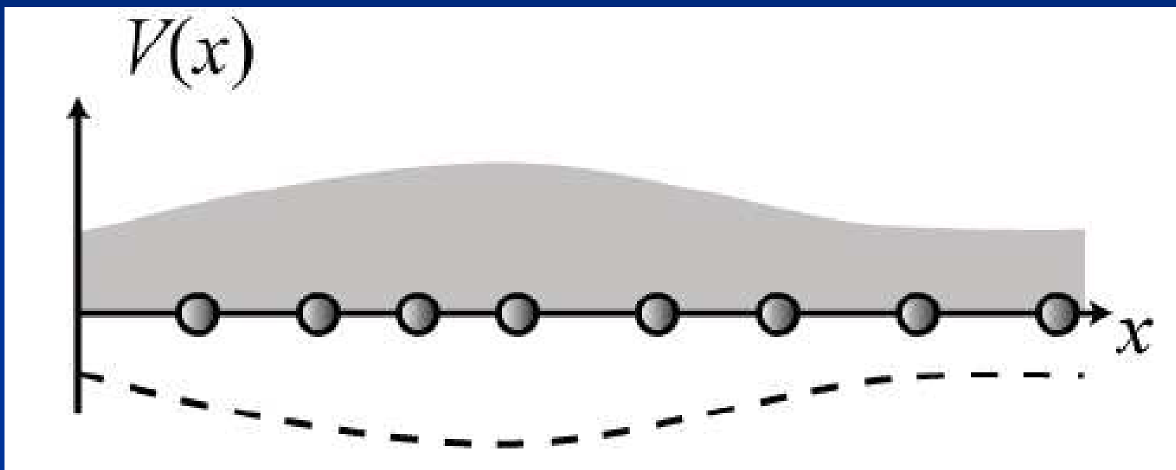
TG + H. J. Schulz EPL 3 1287 (1987); PRB 37 325 (1988)

$$H_{\text{dis}} = \int dx\, V(x)\rho(x)$$

$$H_{\text{dis}} = \int dx\, V(x)[-\frac{1}{\pi}\nabla\phi(x) + \rho_0(e^{i(2\pi\rho_0 x - 2\phi(x))} + \text{h.c.})]$$

``Two" fourier components of disorder

Forward scattering (q » 0)

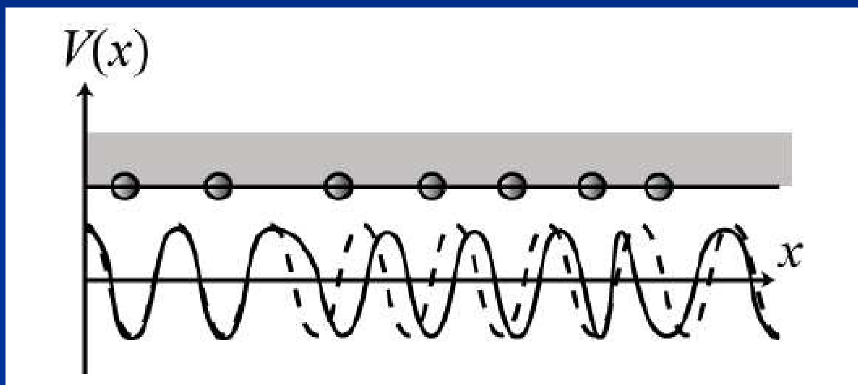


Random (smooth) chemical potential

No localization

Can break commensurate phases

Backward scattering (q » 2 π ρ_0)



$$\frac{dK}{dl} = -\frac{K^2}{2}\tilde{D}_{\mathrm{b}}$$

$$\frac{dD}{dl} = (3-2K)\tilde{D}_{\mathrm{b}}$$

Responsible for localization (Bose Glass)

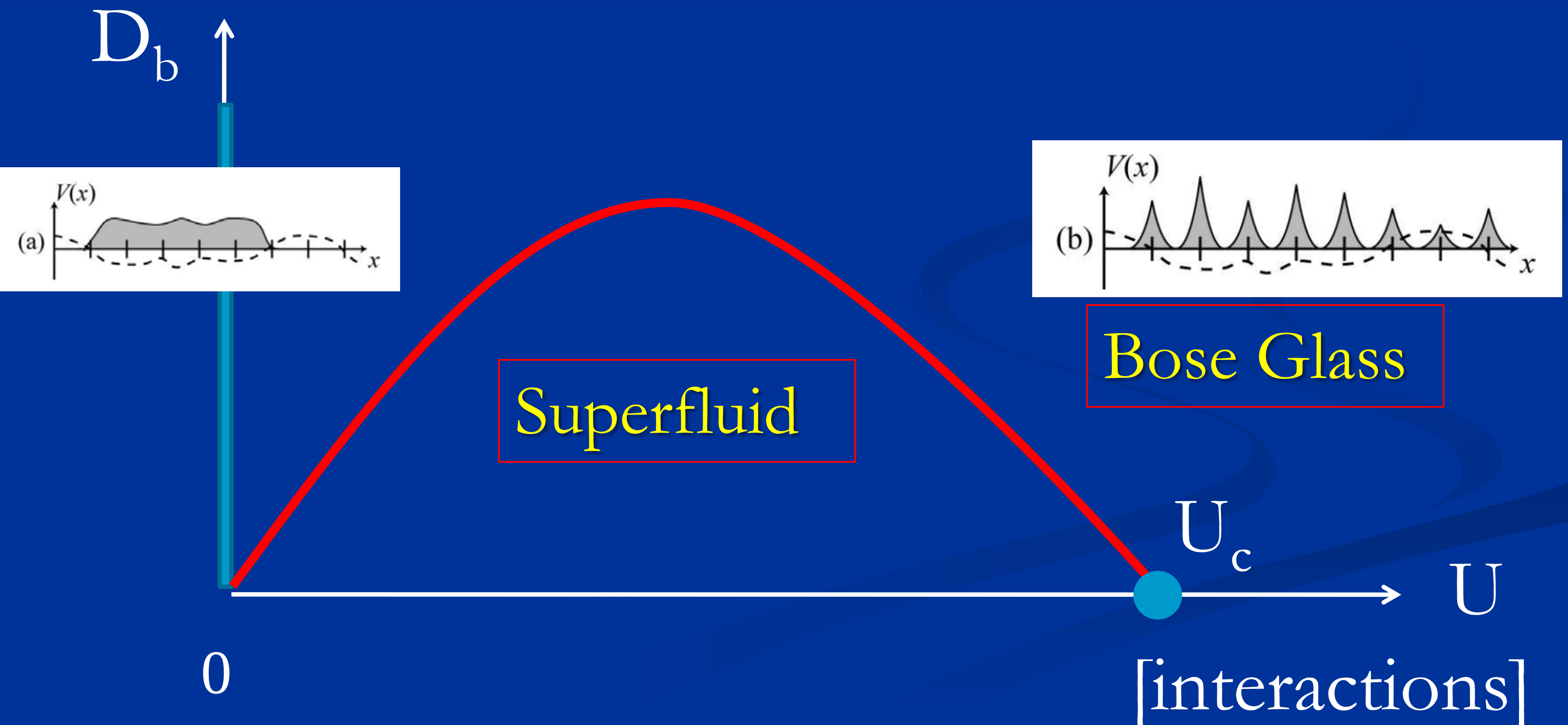
Pinning of a CDW of bosons

$$-D\sum_{ab}\int d\tau d\tau' dx \cos(2\phi_a(x\tau) - 2\phi_b(x\tau'))$$

Bose glass phase

TG + H. J. Schulz EPL 3 1287 (87); PRB 37 325 (1988);

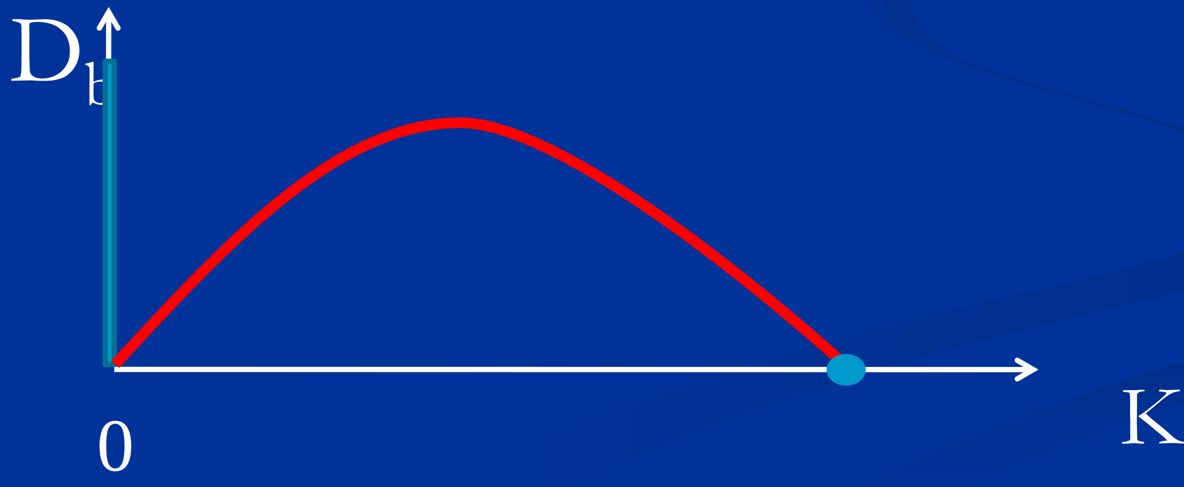
M.P.A. Fisher et al. PRB 40 546 (1989)



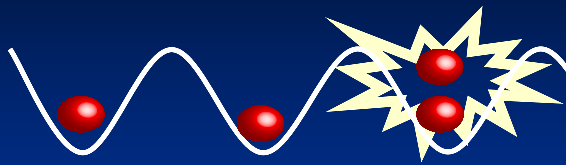
SU-BG transition in d=1

$$\langle \psi(r)\psi^\dagger(0)\rangle \sim \left(\frac{1}{r}\right)^{\frac{1}{2K}} \qquad K \to 3/2$$

Universal exponent at the SU-BG transition !

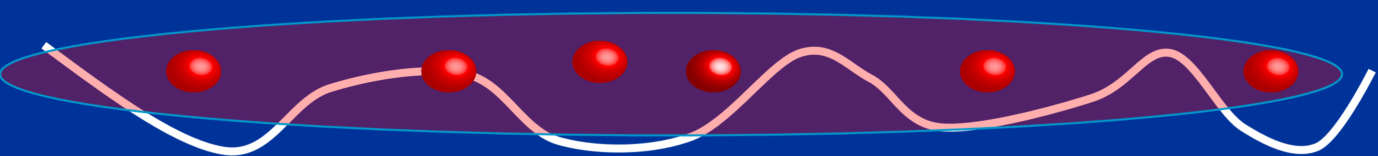


Phases on a lattice



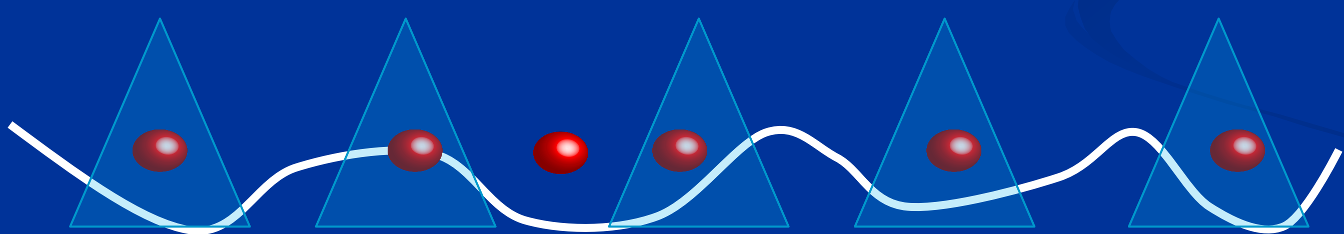
$$\frac{dN}{d\mu} = 0$$

- Mott insulator: incompressible; $\langle \psi \rangle = 0$



$$\frac{dN}{d\mu} \neq 0$$

- Superfluid: compressible; $\langle \psi \rangle \neq 0$



$$\frac{dN}{d\mu} \neq 0$$

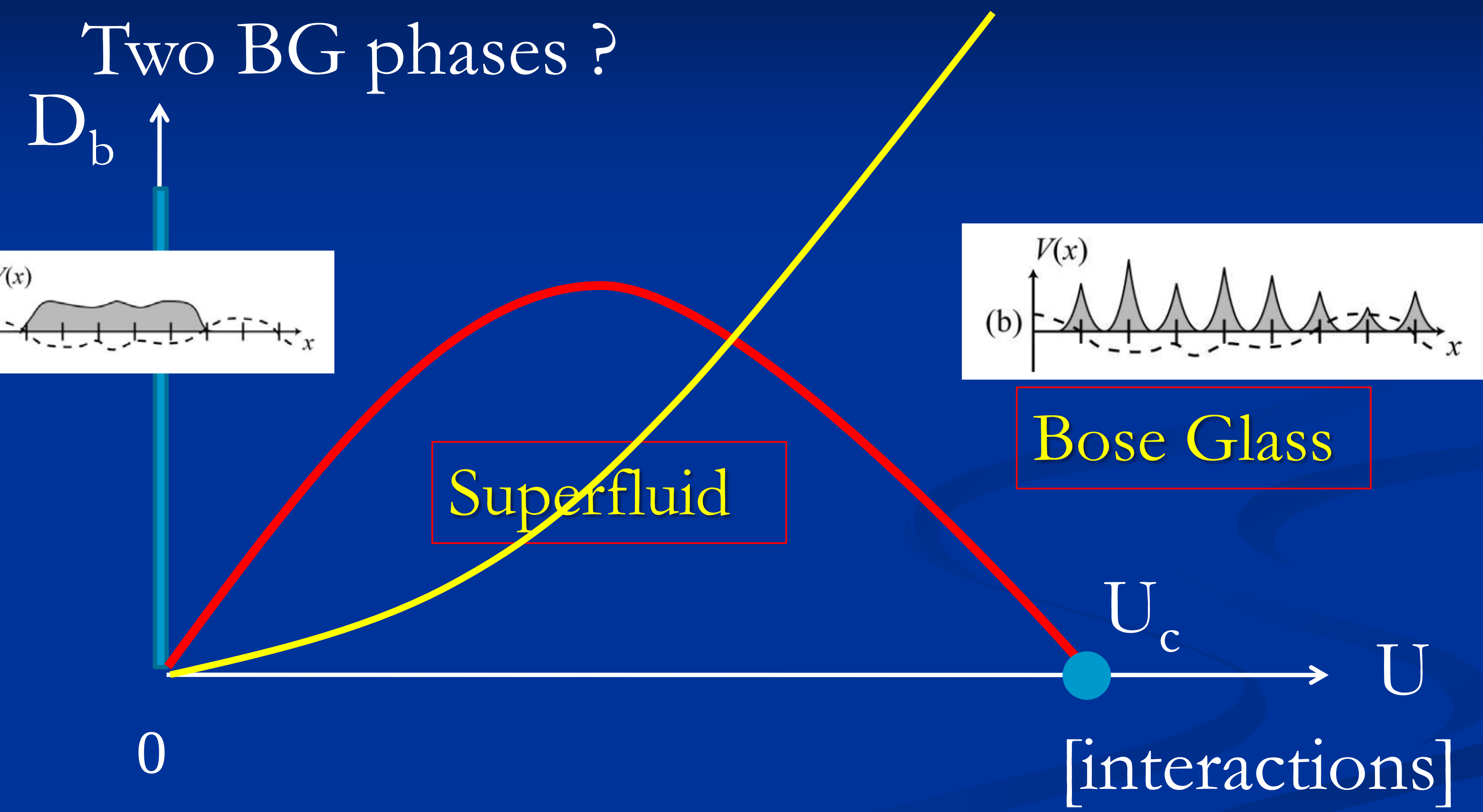
- Bose Glass : compressible; $\langle \psi \rangle = 0$

TG, P. le Doussal PRB 53 15206 (96); T. Nattermann et al. PRL 91 056603 (03); E. Altman et al PRB 81 174528 (10),…….

Questions on the phase diagram

E. Altman et al. PRB 81 174528 (2010)
Z. Ristivojevic, A. Petkovic, P. Le Doussal, TG PRL (2012)
L. Pollet et al. PRB 87 144203 (2013)
Z. Yao et al New J. Phys. 18, 045018 (2016)
E.V.H Doggen et al. arxiv/1704.02257

Phase diagram



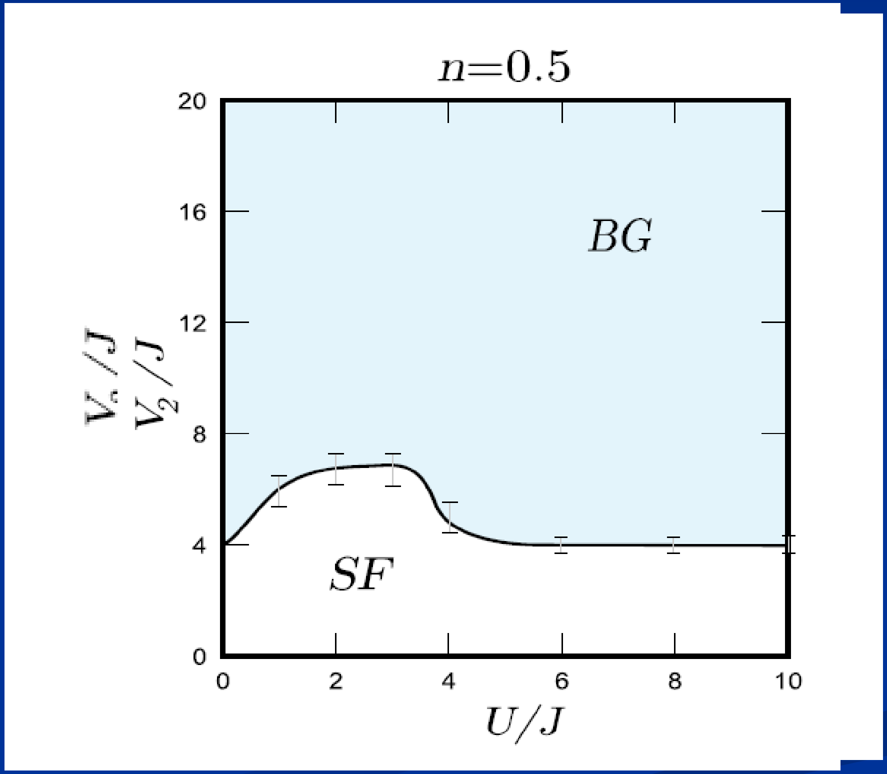
Order parameter ? Moments of distribution ?

Other potentials: Biperiodics

$$V(x) = V_0 \cos(Q_0 x) + V_1 \cos(Q_1 x)$$

- U= 0
  Aubry-André model
- Localization transition

Effect of interactions?
Same as ``true" disorder ?



J. Vidal, D. Mouhanna, TG PRL 83 3908 (1999); PRB 65 014201 (2001)

G. Roux et al. PRA 78 023628 (2008);
T. Roscilde, Phys. Rev. A 77, 063605 2008;
X. Deng et al PRA 78, 013625 (2008);

Beyond RG

Average over disorder

$$H = \tfrac{c}{2}\int (\nabla u)^2 d^d x + \rho_0 \sum_K \int e^{iK(x-u(x))} V(x)$$

- Using replica method

$$S = \sum_a c \int (\nabla u_a)^2 d^{d+1} x$$

$$-\rho_0 \Delta \sum_{a,b} \sum_K \int \cos(K(u_a(x,\tau) - u_b(x,\tau'))) d^d x d\tau d\tau'$$

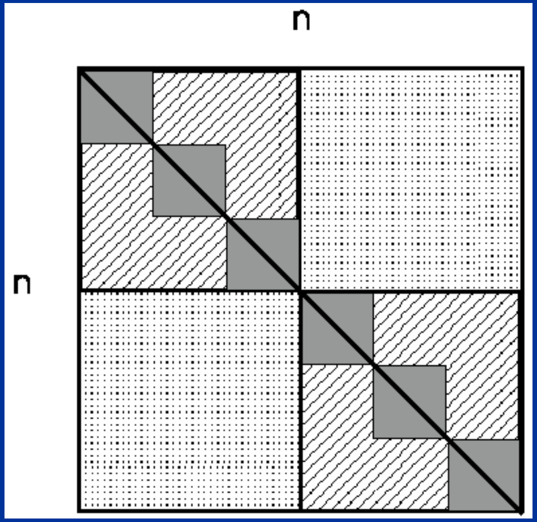
- Need to take the limit n -> 0

Variational Method

Find the best quadratic Hamiltonian

$$S_0 = \sum_{ab} \sum_{i\omega_n} \int G^{-1}_{ab}(i\omega_n, q) \Phi^a_{i\omega_n, q} \Phi^b_{-i\omega_n, -q} d^d q$$

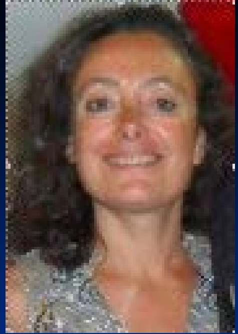
Minimize $$F_{\mathrm{var}} = F_0 + \left\langle (S - S_0) \right\rangle_{S_0}$$



TG, P. Le doussal PRB 52 1242 (1995); PRB 53 15206 (1996)

Replica symmetry breaking

Glassy physics ??



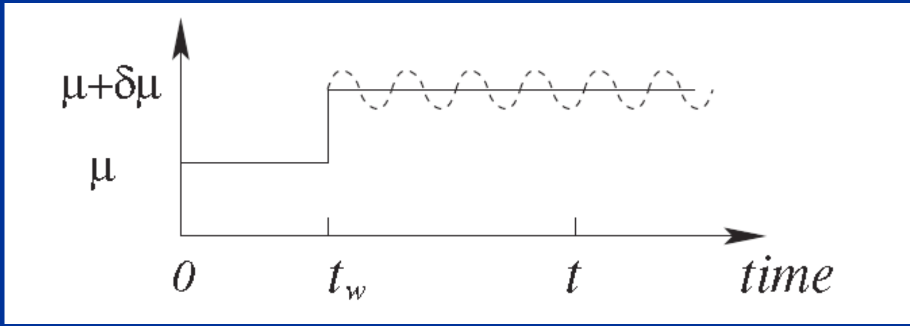
Aging

PRL 96, 217203 (2006) PHYSICAL REVIEW LETTERS week ending 2 JUNE 2006

Dynamic Compressibility and Aging in Wigner Crystals and Quantum Glasses

Leticia F. Cugliandolo,[1,2] Thierry Giamarchi,[3] and Pierre Le Doussal[2]

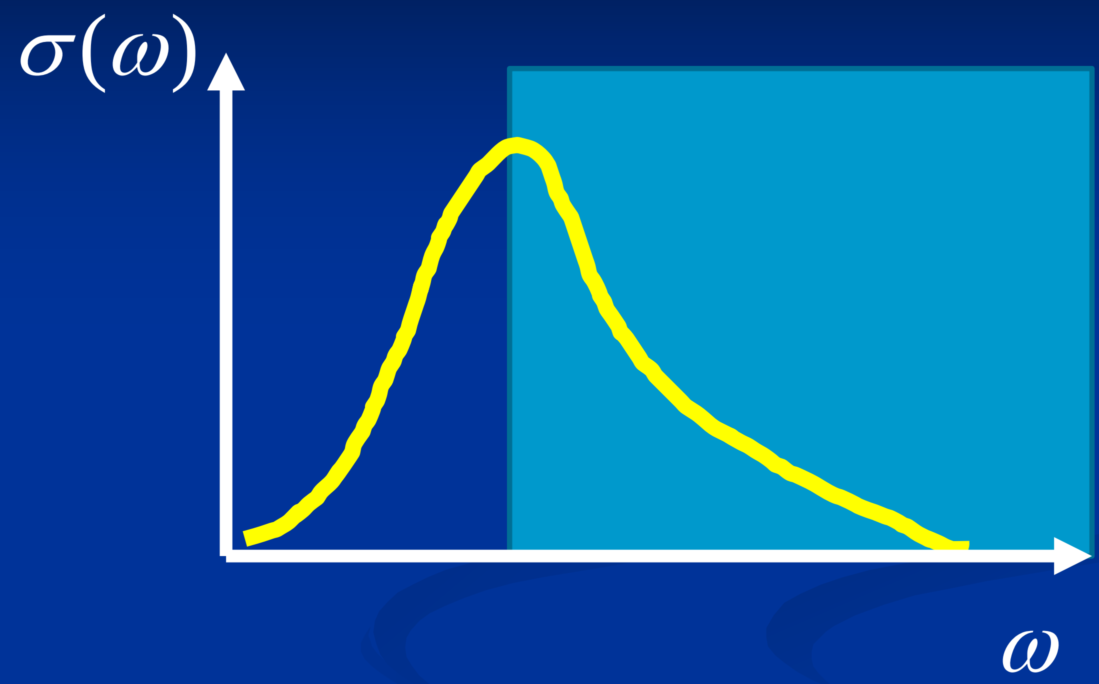
$$H = \int_x \left\{ \frac{\hbar^2 \Pi^2(x)}{2m\rho_0} + \frac{c}{2}[\nabla u(x)]^2 \right\} - \int_x U(x)\rho_0 \cos\{Q[x - u(x)]\},$$



Transport

a.c. and d.c transport (hight T)

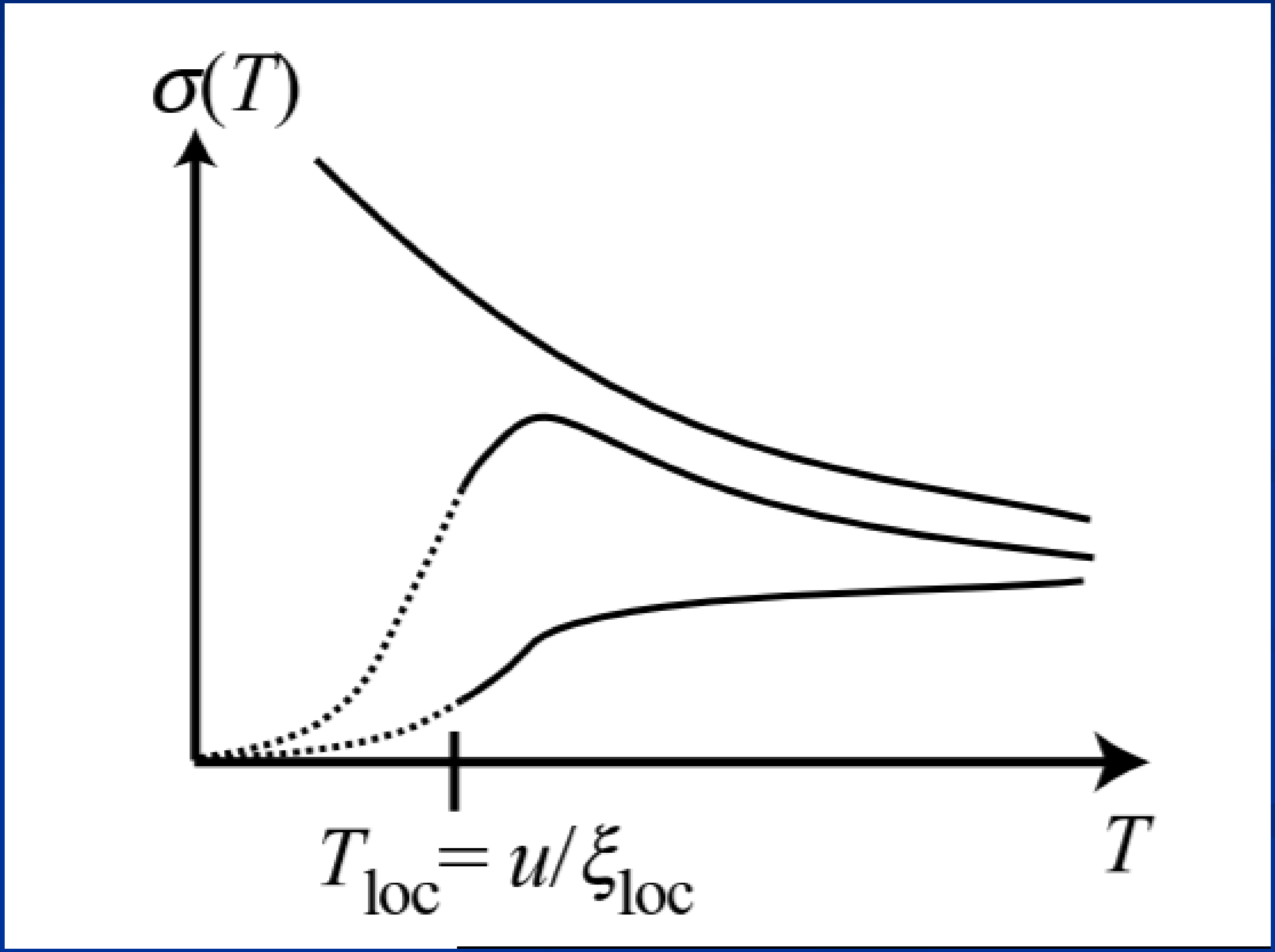
$$\xi_{\rm loc} \sim \alpha \left(\frac{1}{K^2 \tilde{D}_{\rm b}}\right)^{\frac{1}{3-2K}}$$



$\omega > \omega_{loc}$ $\quad \sigma(\omega) \propto \omega^{-\nu}$

$\omega < \omega_{loc}$ $\quad \sigma(\omega) \propto \omega^{2}$

d.c. transport (high T)



Finite T transport (low T)

With a thermal bath:

Variable range hopping

T. Nattermann, TG, P. Le doussal, PRL 91, 056603 (2003)
arXiv:cond-mat/0403487

Without a thermal bath:
Many Body Localization -> See B. Altshuler's lectures

$$\frac{S}{\hbar} = \int_0^L dx \int_0^{\beta\hbar u} dy \frac{1}{2\pi K}[(\partial_y \phi)^2 + (\partial_x \phi)^2],$$

$$S_{\text{dis}}/\hbar = -\frac{1}{2}\int \frac{dxdyA(x)}{2\pi K\alpha^2} e^{i[\phi(x,y)-\zeta(x)]} + \text{H.c.},$$

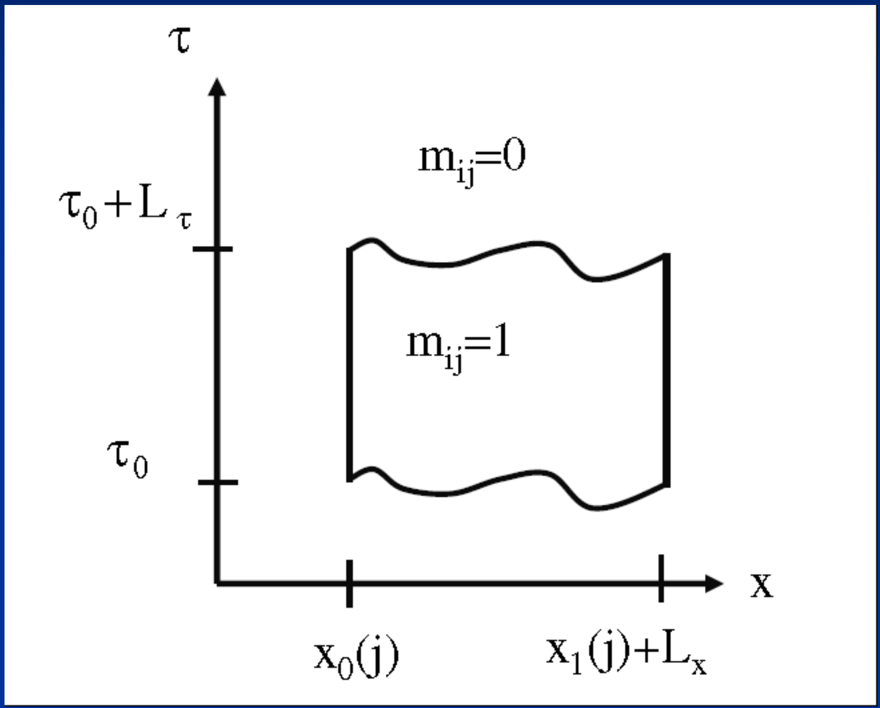
$$S_E/\hbar = \int dxdy\tilde{E}\phi(x,y),$$

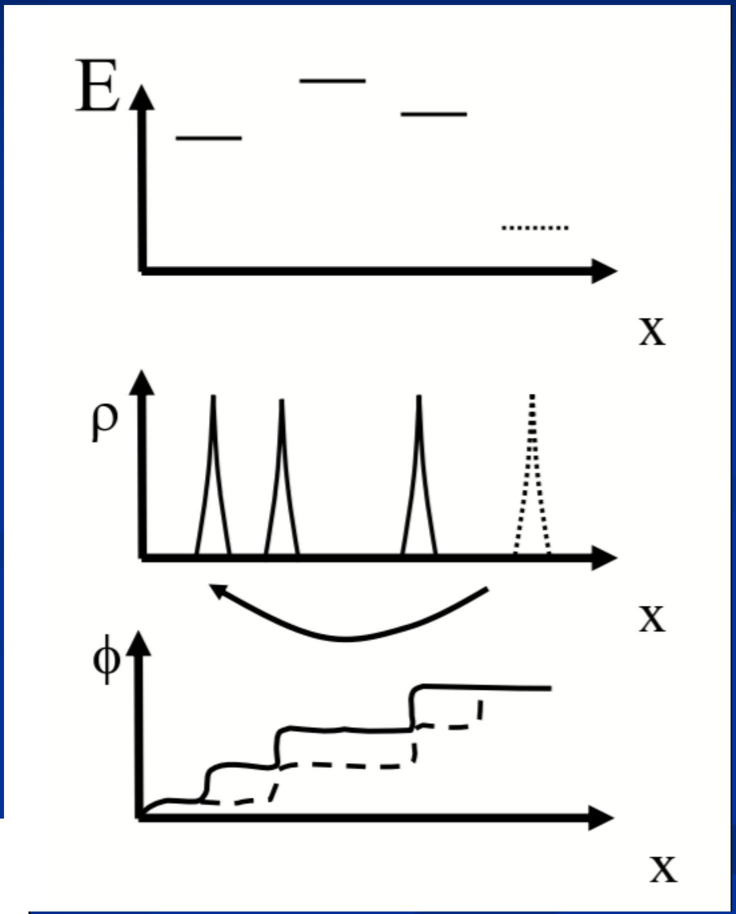
$$\frac{H}{u^*\hbar} = \frac{1}{2\pi K^*\alpha}\sum_{i=1}^{N}[(\phi_{i+1} - \phi_i)^2 - A^* \cos(\phi_i - \zeta_i)],$$

$$\frac{H}{u^*\hbar} = \frac{2\pi}{K^*\alpha}\sum_{i=1}^{N}(n_{i+1} - n_i - f_i)^2 - \frac{N}{2\pi K^*\alpha},$$

$$n_i^0 = m_0 + \sum_{j<i}[f_i],$$

VRH: solitonic excitations

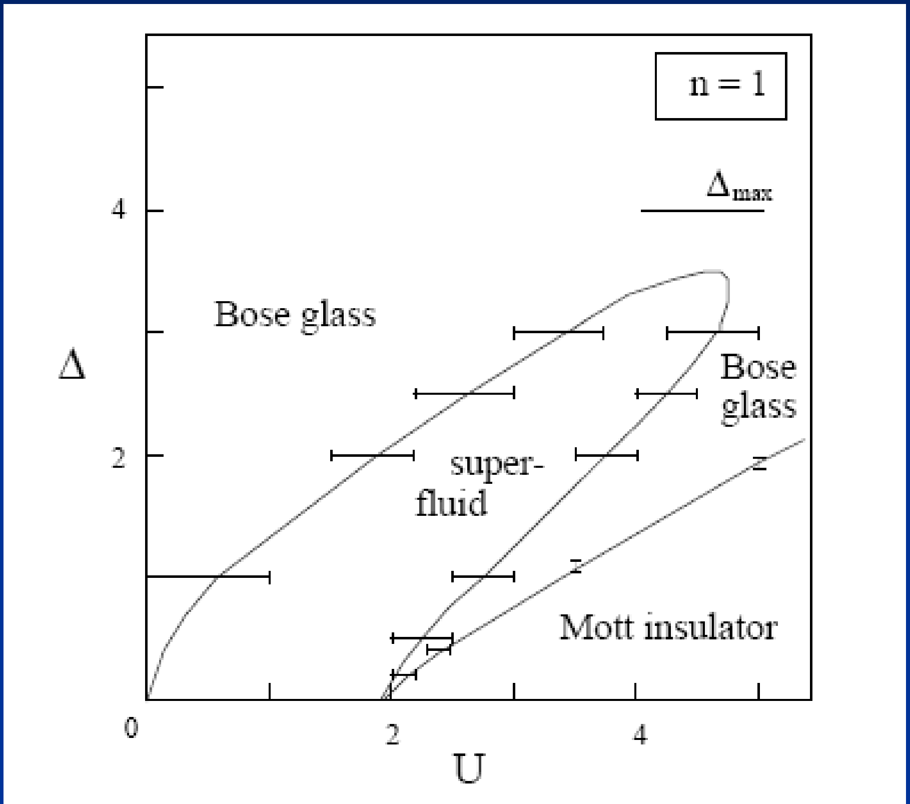




$$\sigma(T) \propto e^{-(S^*/\hbar)} = \exp\left[-\frac{4\pi}{K^*}\sqrt{2\beta\Delta}\right].$$

The hunt for the Bose glass

Numerics (disorder)



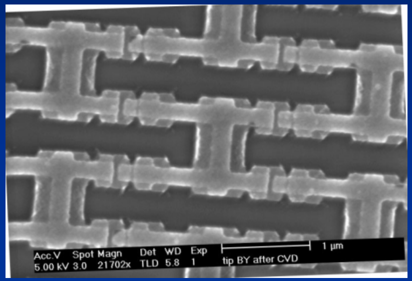
S. Rapsch, U. Schollwoeck, W. Zwerger EPL 46 559 (1999);

G. Batrouni et al. PRL 65 1765 (90);
N. V. Prokof'ev and B. V. Svistunov, PRL 80 4355 (96);
N. Prokofev et al. PRL 92 015703 (04);
O. Nohadani et al. PRL 95, 227201 (05)
K. G. Balabanyan et al. PRL 95, 055701 (05);
L. Pollet et al. PRL 103, 140402 (2009)
…..

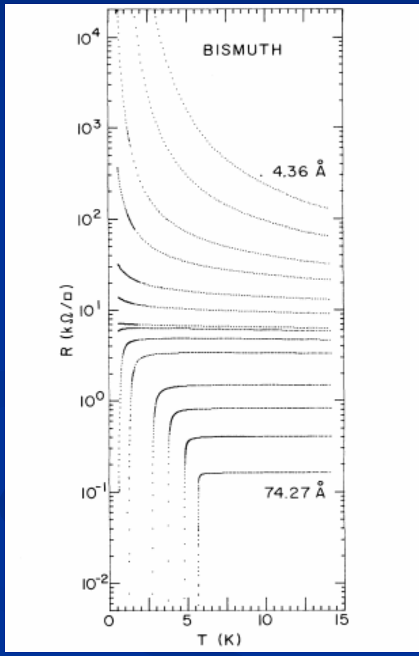
Experiments

First generation of experiments

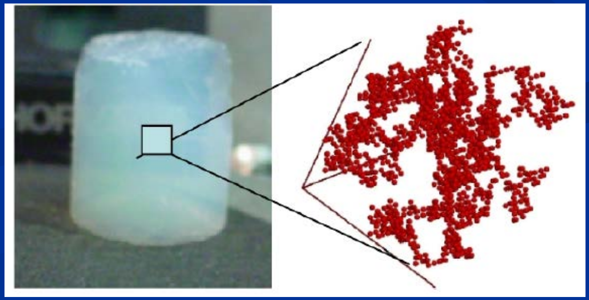
Josephson junction arrays



Disordered superconducting films
(D.B. Haviland et al, PRL 62 2180 (89))

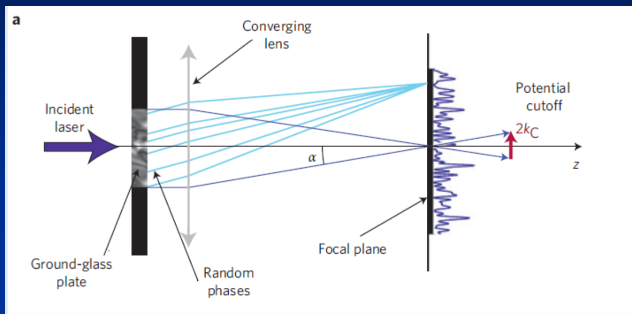


Helium in porous media

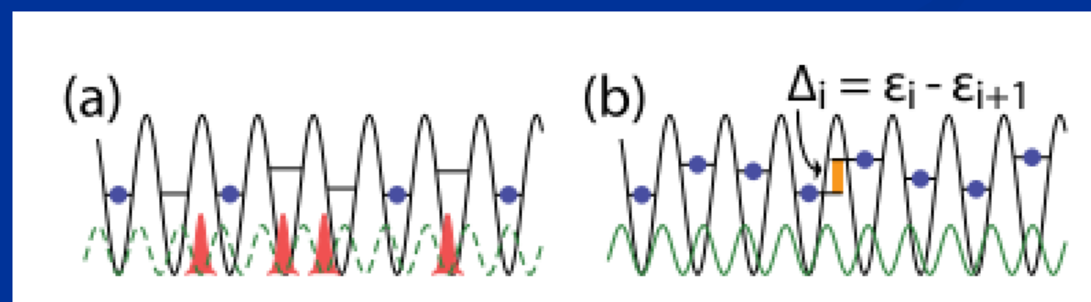


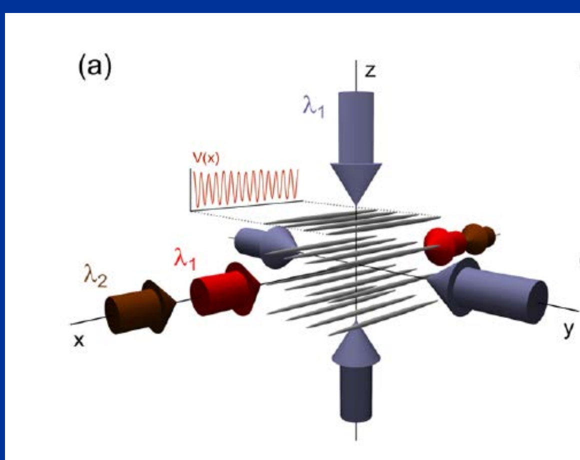
Cold atoms: Control of Interactions and Disorder (quasi-periodic)

Cold atomic gases



Speckle (Palaiseau, Florence, Urbana)





Mixtures (Stony Brook)

Biperiodic lattices (Florence)

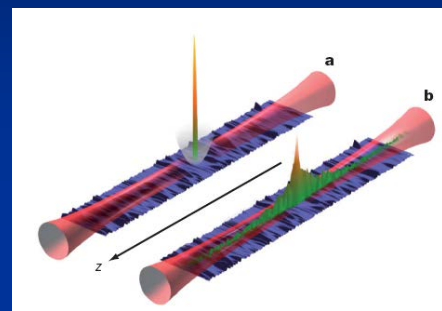
Anderson localization

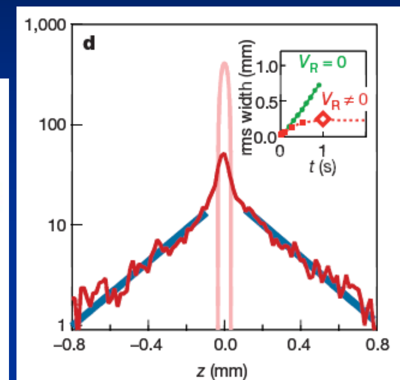
Vol 453|12 June 2008|doi:10.1038/nature07000 nature

LETTERS

Direct observation of Anderson localization of matter waves in a controlled disorder

Juliette Billy[1], Vincent Josse[1], Zhanchun Zuo[1], Alain Bernard[1], Ben Hambrecht[1], Pierre Lugan[1], David Clément[1], Laurent Sanchez-Palencia[1], Philippe Bouyer[1] & Alain Aspect[1]



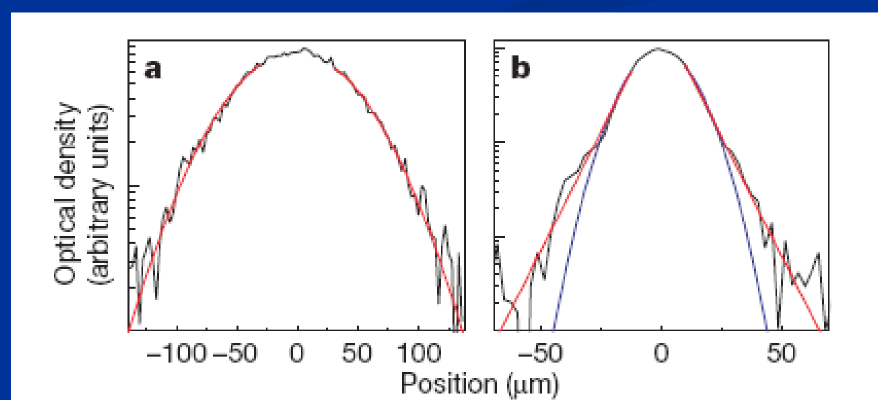


Vol 453|12 June 2008|doi:10.1038/nature07071 nature

LETTERS

Anderson localization of a non-interacting Bose–Einstein condensate

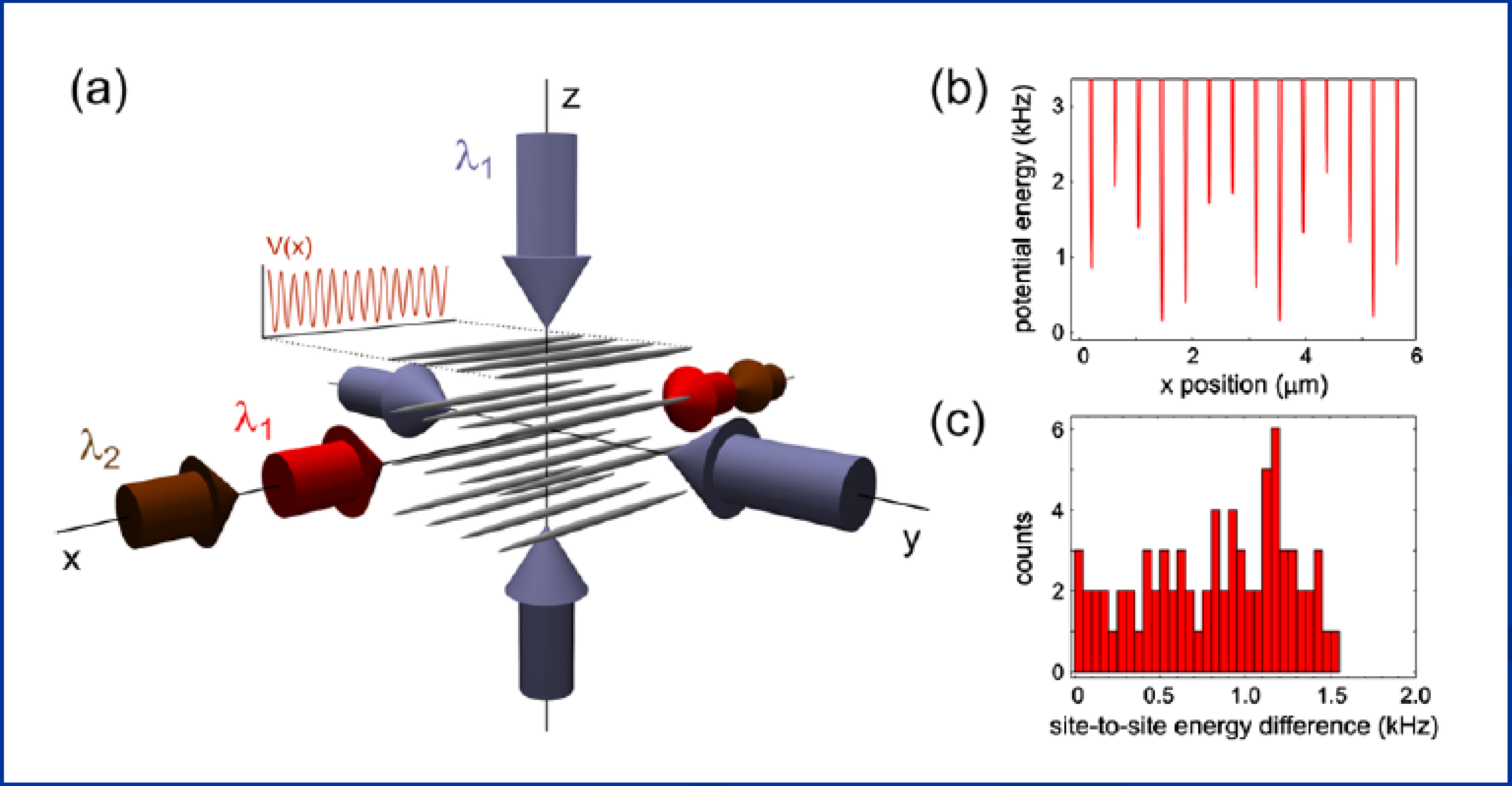
Giacomo Roati[1,2], Chiara D'Errico[1,2], Leonardo Fallani[1,2], Marco Fattori[1,2,3], Chiara Fort[1,2], Matteo Zaccanti[1,2], Giovanni Modugno[1,2], Michele Modugno[1,4,5] & Massimo Inguscio[1,2]



Aubry-Andre Model (Ann. Isr. Phys. Soc. **3, 133 1980**)

Experiments with interactions !

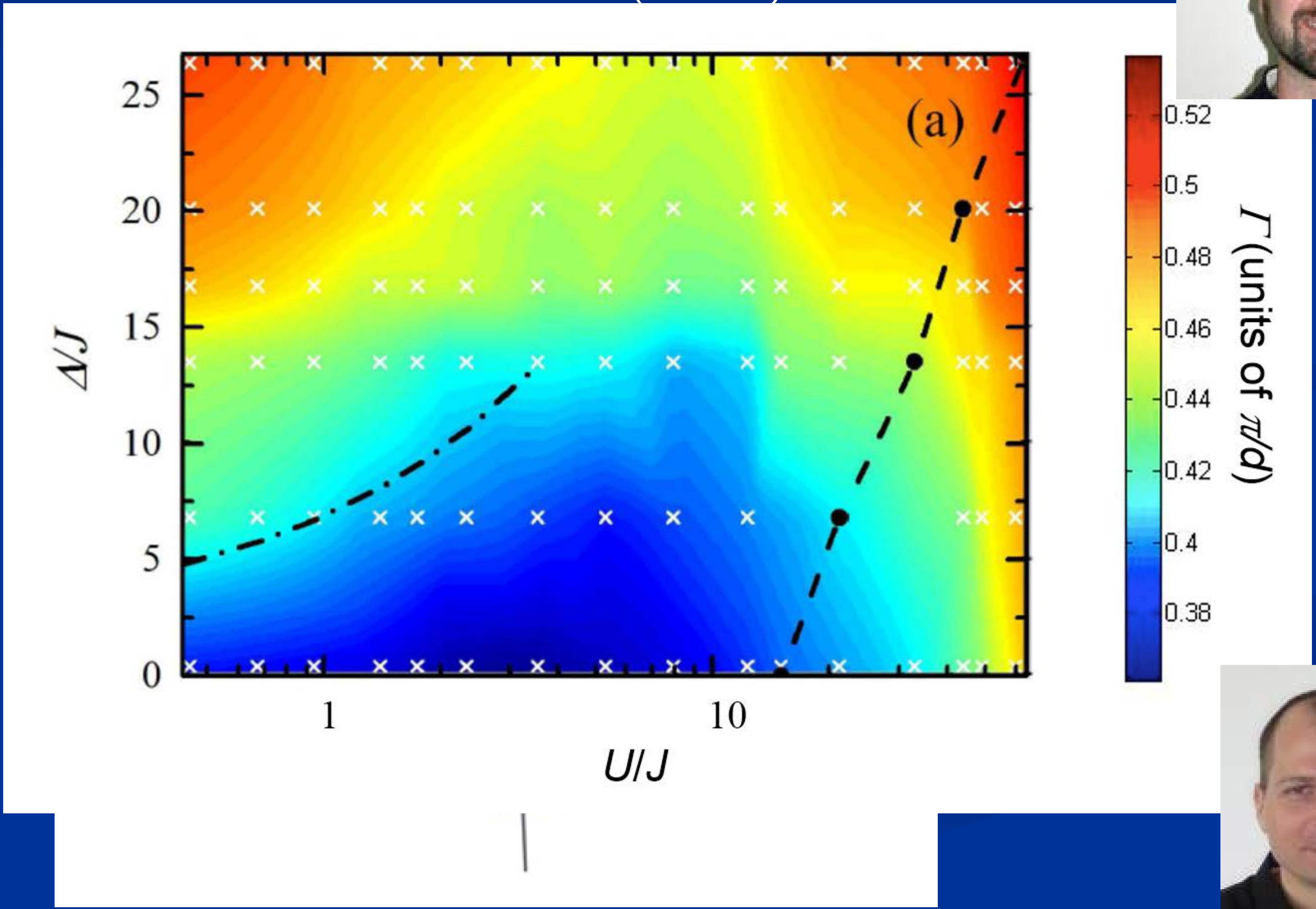
L. Fallani et al. PRL 98, 130404 (2007)



Quasi-periodics and interactions

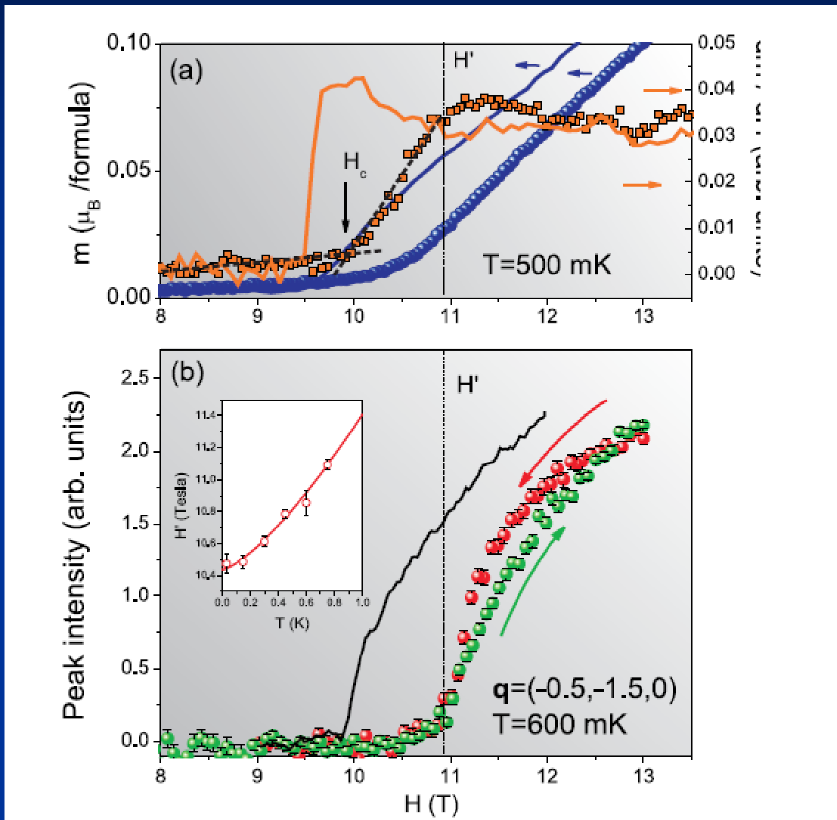
C. D'Errico, E. Lucioni et al. PRL (2014);

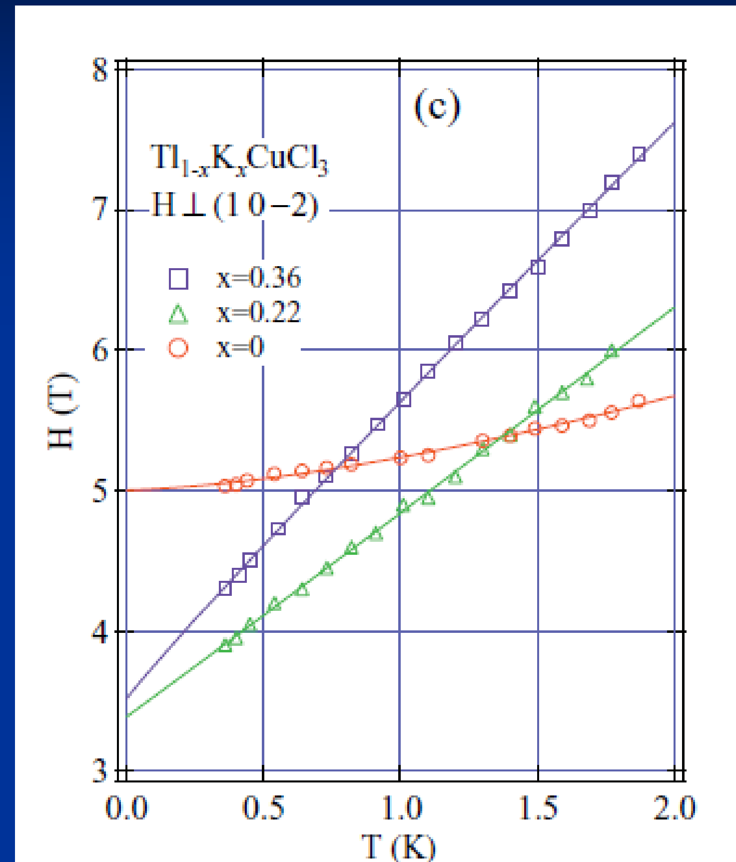
L. Gori et al PRA 93 033650 (2016)



Magnetic insulators

Bose glass in dimer systems





IPA-Cu(Cl0.95Br0.05)$_3$

T. Hong et al.

Phys. Rev. B **81**, 060410 (2010)

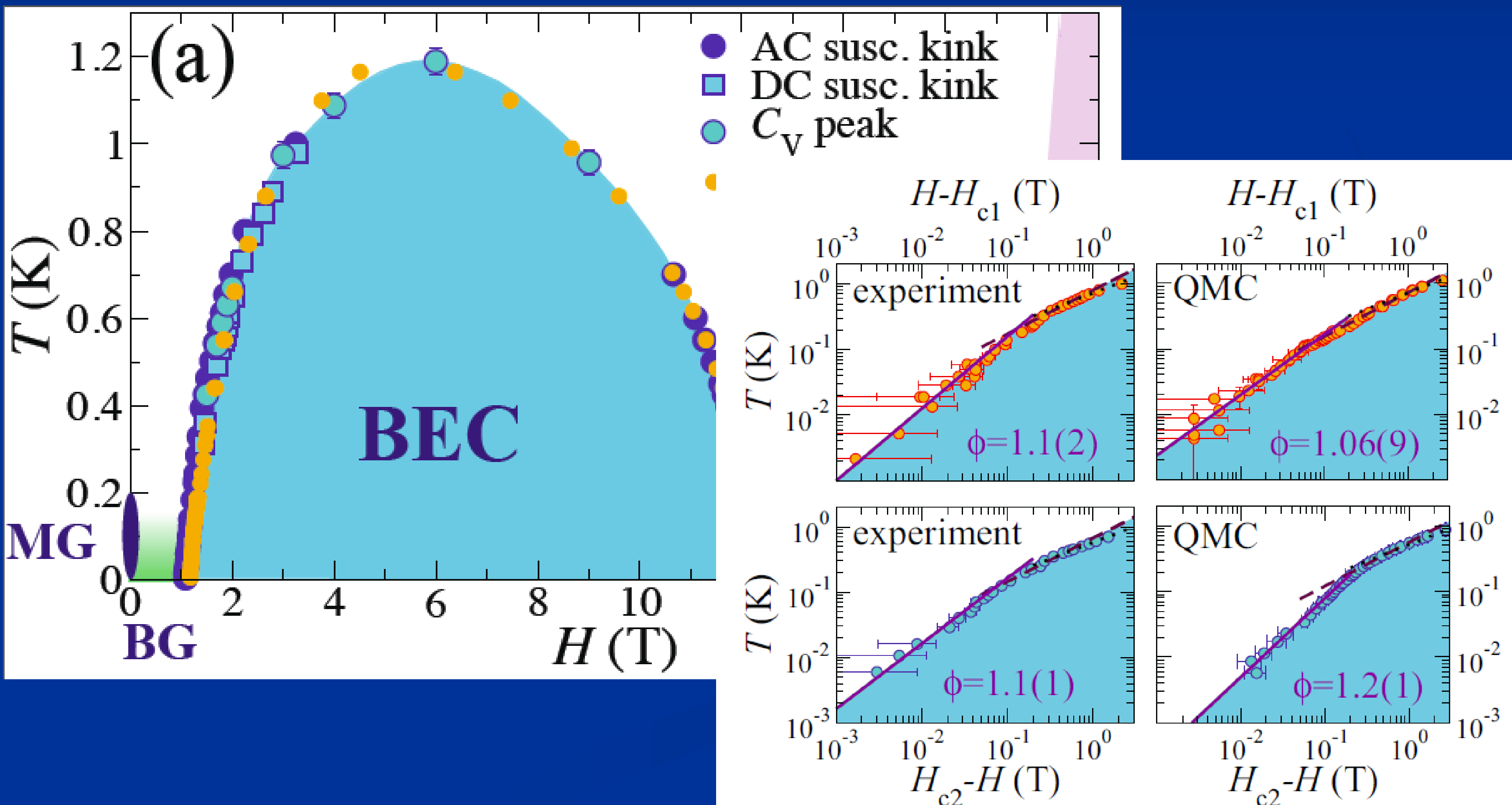
Tl1-xKxCuCl$_3$

F. Yamada et al.

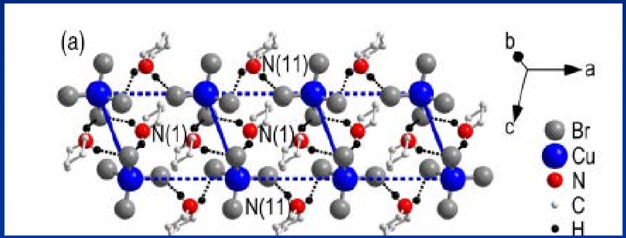
Phys. Rev. B **83**, 020409 (2011)

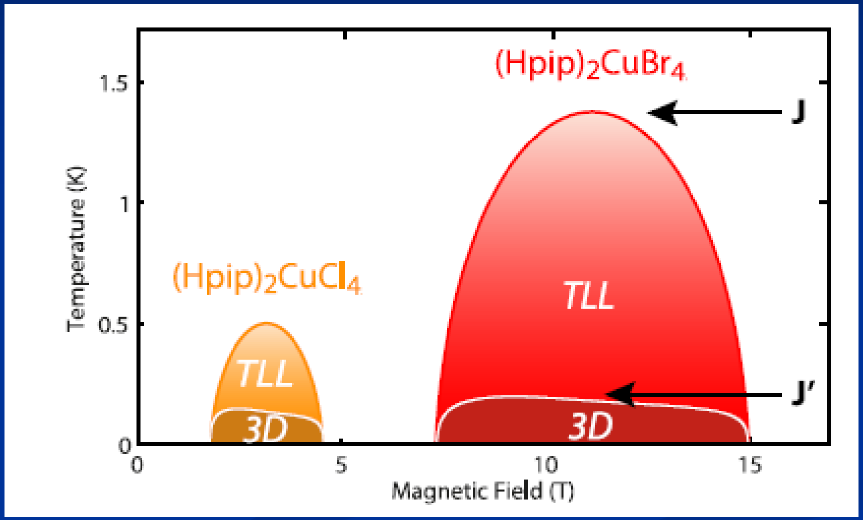
DTN Br

Rong Yu et al. Nature 489 379 (2013)



Disorder





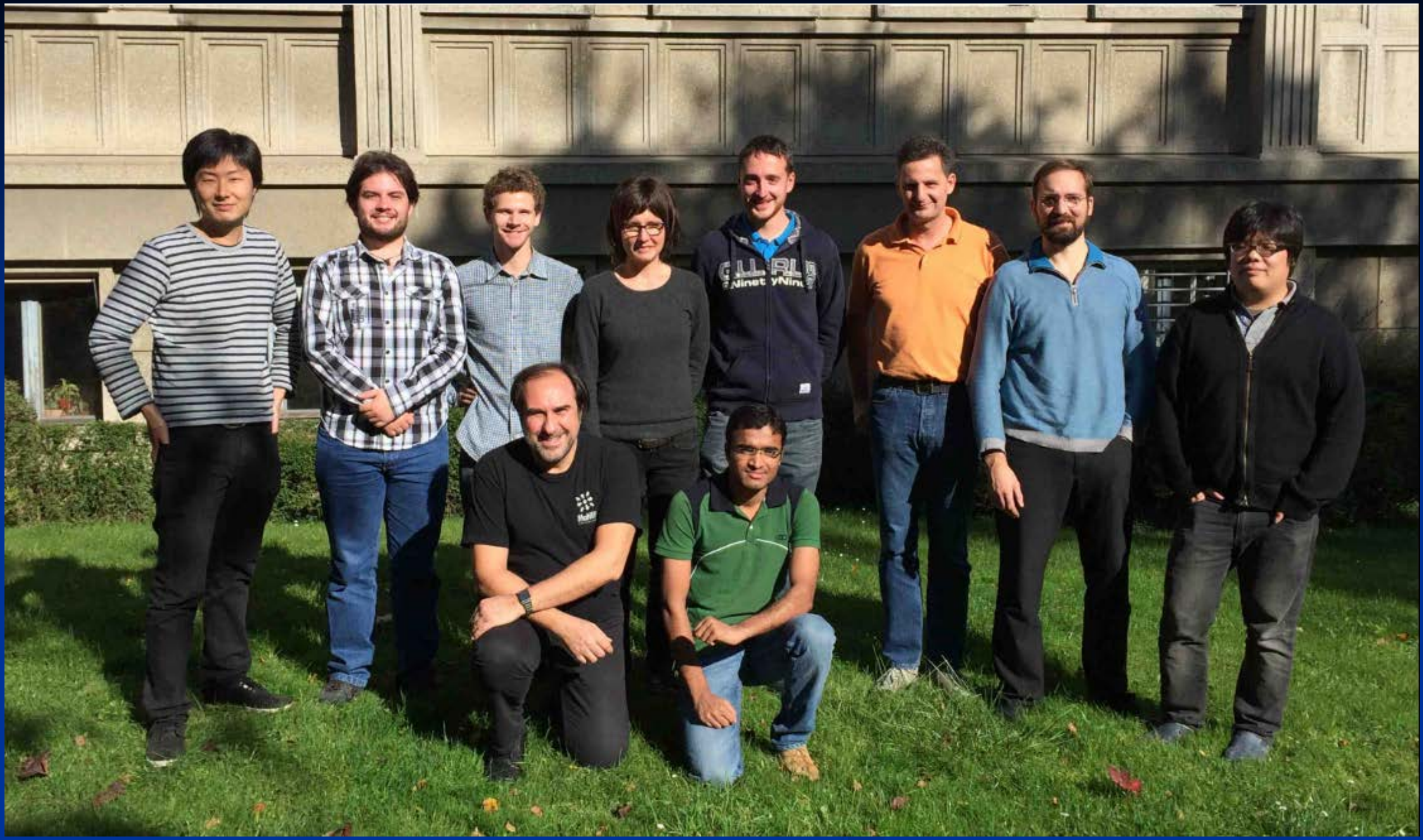
BPCB: B. C. Watson et al., PRL 86 5168 (2001)

M. Klanjsek et al., PRL 101 137207 (2008)

S. Ward et al. J. Phys C 25 014004 (2013)

Take home message

- Good theoretical methods to deal with the case of a ``simple” equilibrium 1d systems (analytics and numerics)
- Stepping stone to go beyond: many exciting questions and problems (out of equilibrium, disorder, many chains, etc.)
- Controled experimental realizations in condensed matter and cold atoms



(S. Uchino, S. Tapias, N. Kestin, L. Foini, N. Kamar, E. Coira, C. Berthod, A. Kantian, S. Furuya)

Charles Bardyn

Michele Filippone

Pjotrs Grisins

Shintaro Takayoshi

Anne-Maria Visuri