Non-equilibrium thermodynamics of quantum processes: two

or an invitation to study stochastic thermodynamics of quantum processes

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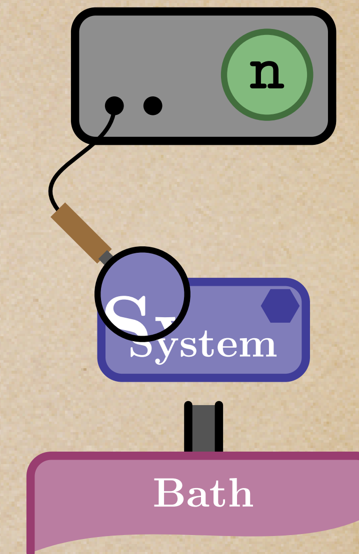
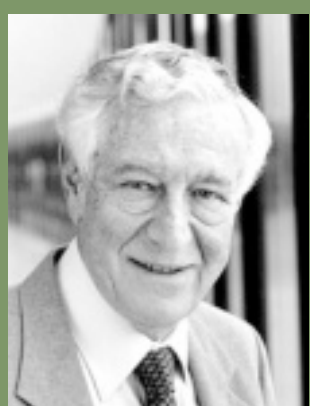
(ICTP, Trieste, 4 September 2017)

Content & structure

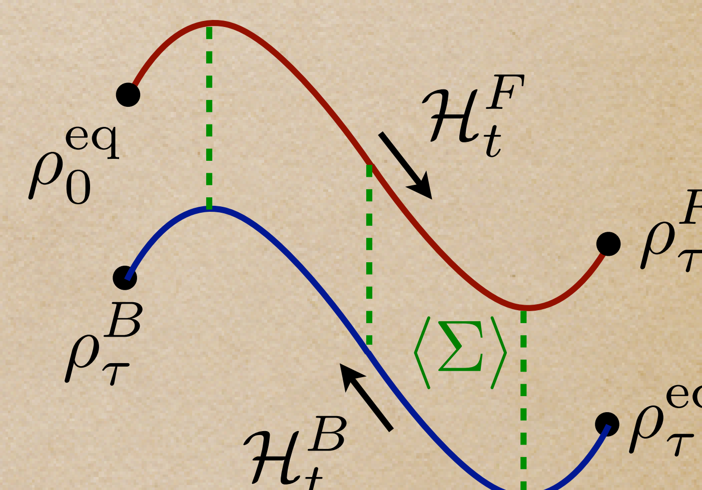


Non-equilibrium definition of thermodynamic work: fluctuation theorems



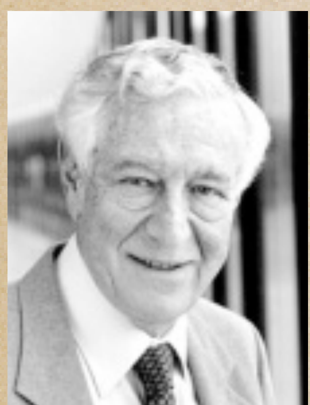
Landauer principle & quantum (open-system) dynamics

Irreversibility & entropy production in closed q-systems



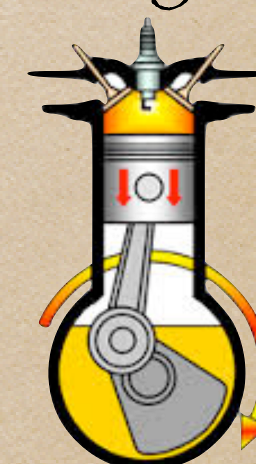
Quantum correlations, coherences and thermodynamics

Plan of the discussion



Landauer principle & quantum (open-system) dynamics

Finite-time Thermodynamics

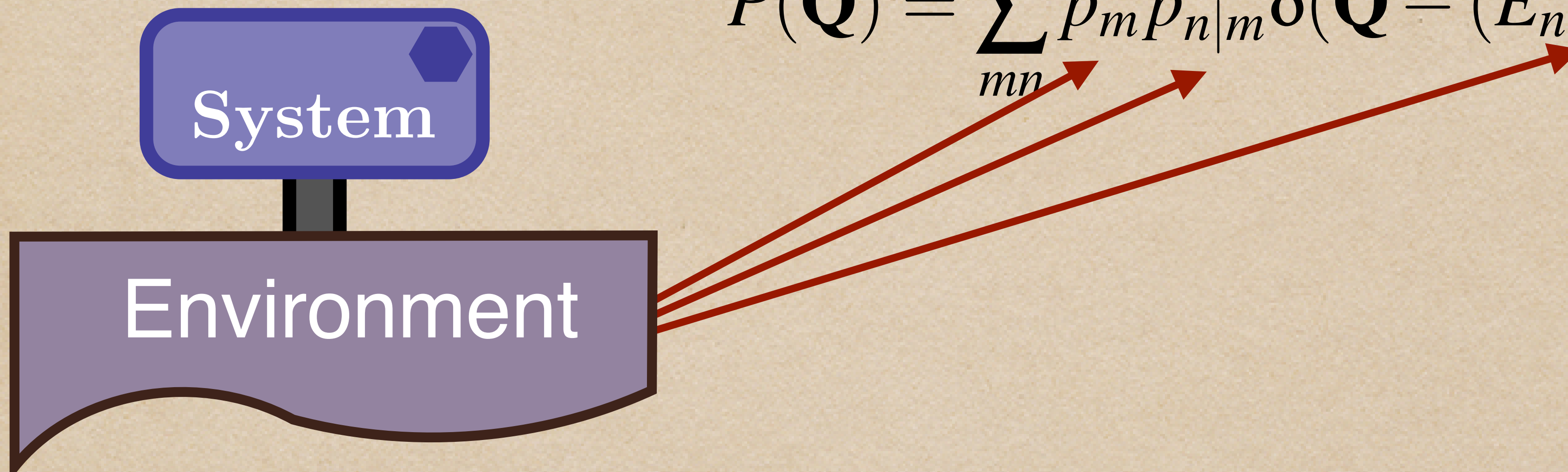


0) Non-equilibrium interpretation of non-unitality

1) Non-equilibrium version of Landauer principle: A tighter bound than the original one?

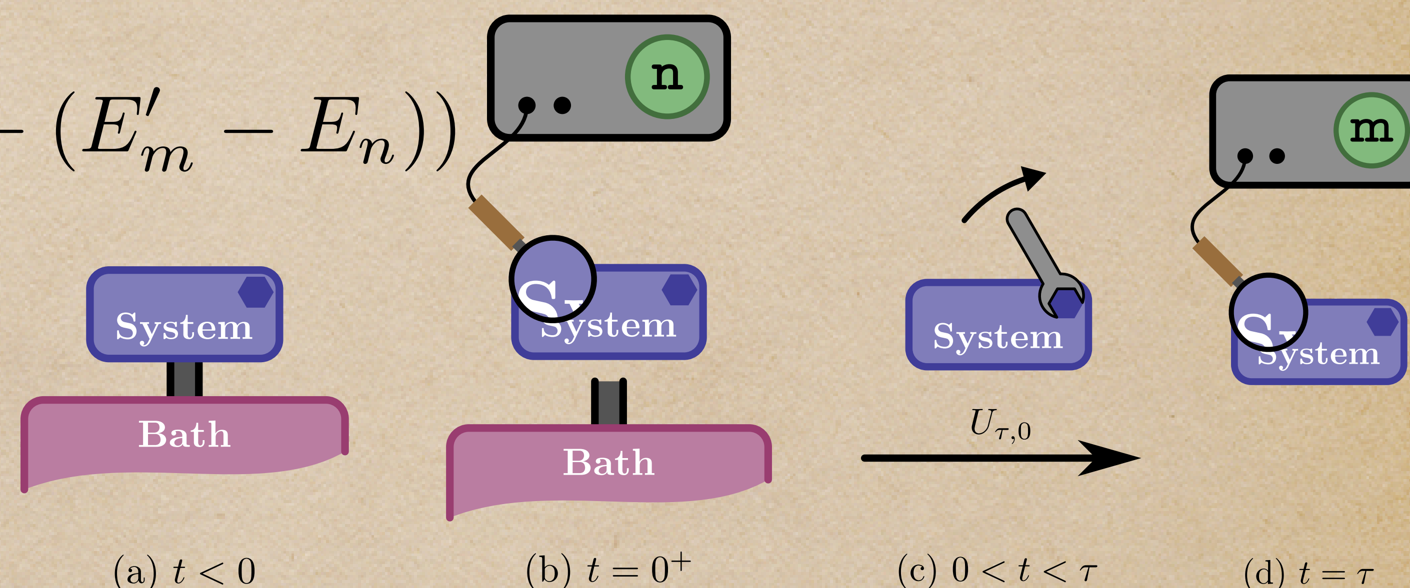
How about heat?

$$P(\mathbf{Q}) = \sum_{mn} p_m p_{n|m} \delta(\mathbf{Q} - (E_n - E_m))$$



$$P_F(W) = \sum_{n,m} p_n^0 p_{m|n}^{\tau} \delta\left(W - (E'_m - E_n)\right)$$

Work Distribution

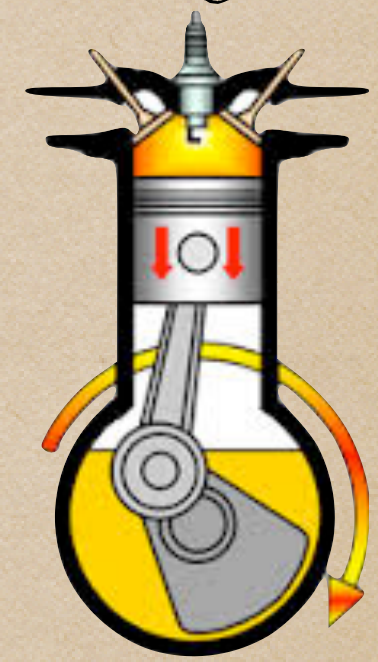


Talkner, Lutz, and Haenggi, Phys. Rev. E 75, 050102 (2007)

Goold, Poschinger, and Modi, Phys. Rev. E 90, 020101 (2014)

Link to fluctuation theorems

Finite-time Thermodynamics



Statements linking the stochastic properties of thermodynamically relevant quantities to equilibrium features

Jarzinsky equality $\langle e^{-\beta W} \rangle = e^{-\beta \Delta F}$

free-energy change

Jarzynski, PRL 78 2690 (1997)

Standard fluctuation relations hold unchanged for <u>unital</u> open channels

A. E. Rastegin, J. Stat. Mech. P06016

Mr. Landauer's principle

The principle

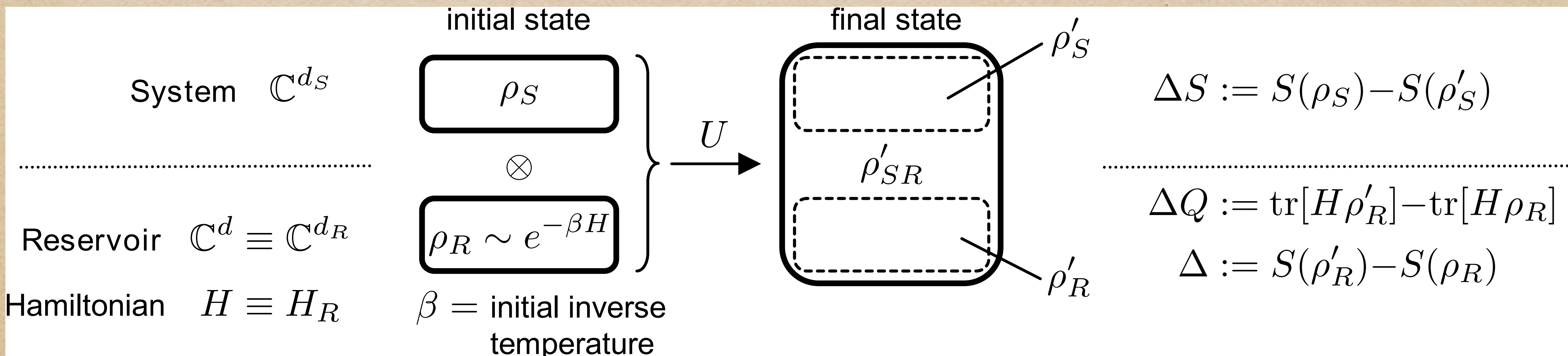
"Any logically irreversible manipulation of information... must be accompanied by a corresponding entropy increase in non-information bearing degrees of freedom of the information processing apparatus or its environment"

$$\beta\langle Q\rangle \geq \Delta S$$

Rolf Landauer

Can we let Landauer's principle emerge from microscopic equations?

(Im)proving Landauer: Reeb & Wolf



1st result $$\beta\Delta Q \;=\; \Delta S + I(S':R') + D(\rho'_R\|\rho_R) \;\geq\; \Delta S$$

2nd result $$\beta\Delta Q \;\geq\; \Delta S + \frac{2(\Delta S)^2}{\log^2(d-1)+4}$$

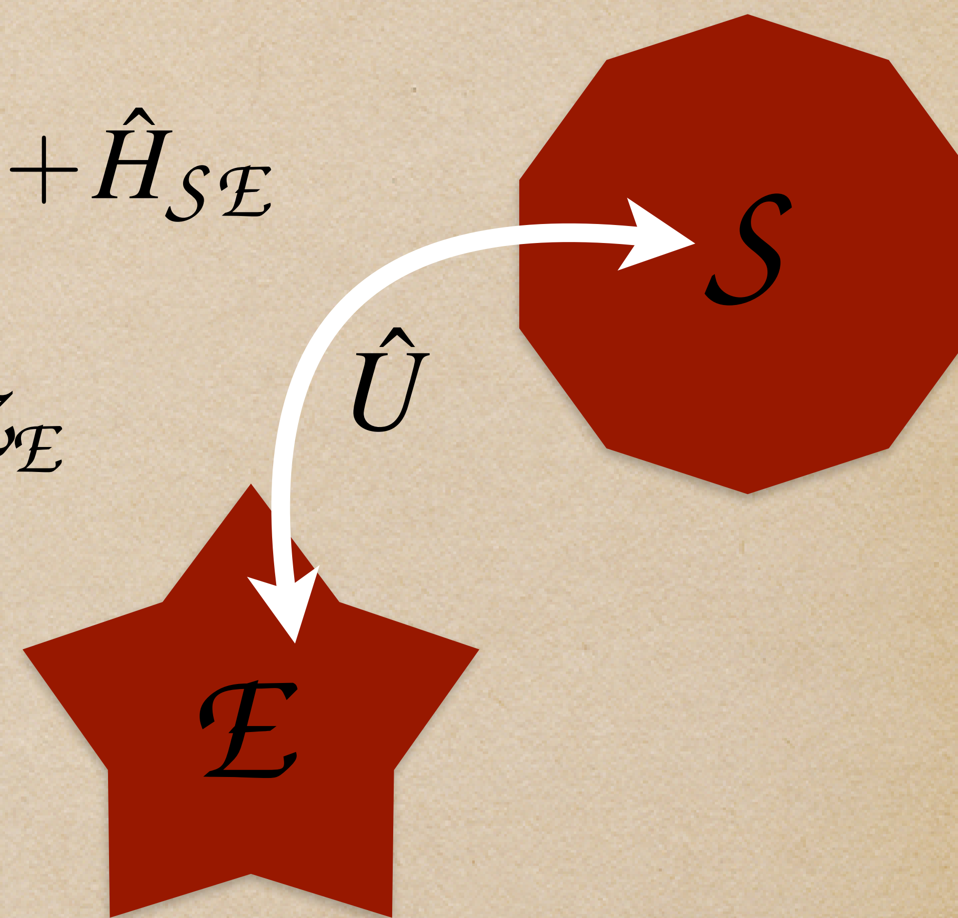
D. Reeb, and M. Wolf, NJP 16, 103011 (2014)

(Im)proving Landauer: our way

$$\hat{H} = \hat{H}_{\mathcal{S}} + \hat{H}_{\mathcal{E}} + \hat{H}_{\mathcal{S}\mathcal{E}}$$

$$\hat{\rho}_{\mathcal{S}\mathcal{E}} = \hat{\rho}_{\mathcal{S}} \otimes e^{-\beta \hat{H}_{\mathcal{E}}} / Z_{\mathcal{E}}$$

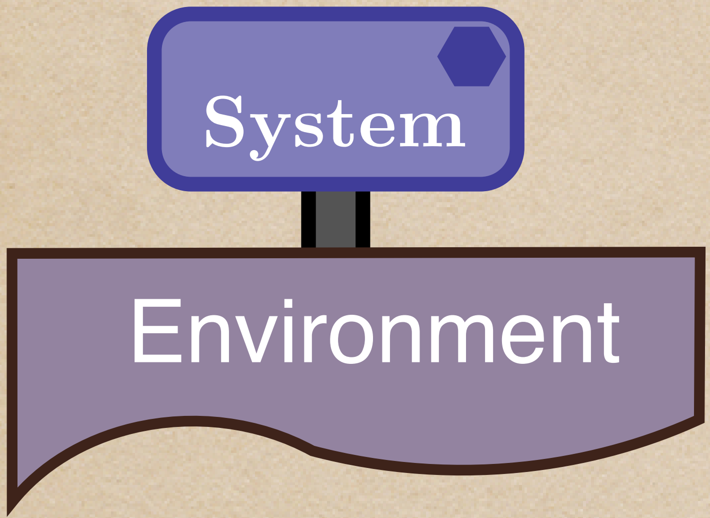
$$\hat{\rho}'_{\mathcal{E}} = \mathrm{tr}_{\mathcal{S}}[\hat{U}(\hat{\rho}_{\mathcal{S}} \otimes \hat{\rho}_{\mathcal{E}})\hat{U}^{\dagger}] = \sum_{l} \hat{A}_l \hat{\rho}_{\mathcal{E}} \hat{A}_l^{\dagger}$$



$$\hat{A}_{l=jk} = \sqrt{\lambda_j} \langle s_k | \hat{U} | s_j \rangle \qquad \sum_l \hat{A}_l^{\dagger} \hat{A}_l = \mathbb{1}_{\mathcal{E}}$$

Goold, Paternostro, and Modi, PRL 114, 060602 (2015)

How about heat?



$$P(\mathbf{Q}) = \sum_{mn} p_m p_{n|m} \delta(\mathbf{Q} - (E_n - E_m))$$

Heat probability distribution!

Talkner, Lutz, and Haenggi, Phys. Rev. E 75, 050102 (2007)

(Im)proving Landauer: our way

$$P(Q) = \sum_{l,m,n} \langle r_n|\hat{A}_l|r_m\rangle(\hat{\rho}_{\mathcal{E}})_{mm}\langle r_m|\hat{A}_l^\dagger|r_n\rangle\delta(Q - E_{nm})$$

$$\langle e^{-\beta Q}\rangle = \int e^{-\beta Q}\, dQ\, P(Q) = \sum_l \mathrm{tr}[\hat{A}_l^\dagger \hat{\rho}_{\mathcal{E}} \hat{A}_l] = \mathrm{tr}[\hat{\mathbf{A}}\,\hat{\rho}_{\mathcal{E}}]$$

$$\hat{\mathbf{A}} = \sum_l \hat{A}_l \hat{A}_l^\dagger$$

Thermodynamic interpretation of the 'degree' of non-unitality of a process

Goold, Paternostro, and Modi, PRL 114, 060602 (2015)

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(Im)proving Landauer: our way

$$P(Q) = \sum_{l,m,n} \langle r_n|\hat{A}_l|r_m\rangle(\hat{\rho}_{\mathcal{E}})_{mm}\langle r_m|\hat{A}_l^\dagger|r_n\rangle\delta(Q - E_{nm})$$

$$\langle e^{-\beta Q}\rangle = \int e^{-\beta Q}\, dQ\, P(Q) = \sum_l \mathrm{tr}[\hat{A}_l^\dagger \hat{\rho}_{\mathcal{E}} \hat{A}_l] = \mathrm{tr}[\hat{\mathbf{A}}\,\hat{\rho}_{\mathcal{E}}]$$

$$\hat{\mathbf{A}} = \sum_l \hat{A}_l \hat{A}_l^\dagger$$

Jarzinsky equality $\langle e^{-\beta W}\rangle = e^{-\beta \Delta F}$

Not a proper fluctuation theorem

Talkner, Campisi, Haenggi, J Stat Mech P02025 (2009)
Goold, Paternostro, and Modi, PRL 114, 060602 (2015)

(Im)proving Landauer: our way

$$P(Q) = \sum_{l,m,n} \langle r_n|\hat{A}_l|r_m\rangle(\hat{\rho}_{\mathcal{E}})_{mm}\langle r_m|\hat{A}_l^\dagger|r_n\rangle\delta(Q - E_{nm})$$

$$\langle e^{-\beta Q}\rangle = \int e^{-\beta Q}\, dQ\, P(Q) = \sum_l \mathrm{tr}[\hat{A}_l^\dagger \hat{\rho}_{\mathcal{E}} \hat{A}_l] = \mathrm{tr}[\hat{\mathbf{A}}\,\hat{\rho}_{\mathcal{E}}]$$

$$= \mathrm{tr}[\hat{\rho}_{\mathcal{S}} \otimes \mathbb{1}_{\mathcal{E}} \hat{U}^\dagger\, \mathbb{1}_{\mathcal{S}} \otimes \hat{\rho}_{\mathcal{E}} \hat{U}] = \mathrm{tr}[\hat{\mathbf{M}}\,\hat{\rho}_{\mathcal{S}}]$$

Assume unitality $\langle Q\rangle \geq 0$

$$\beta\langle Q\rangle \geq \mathcal{B}_Q = -\ln(\mathrm{tr}[\hat{\mathbf{A}}\,\hat{\rho}_{\mathcal{E}}]) = -\ln(\mathrm{tr}[\hat{\mathbf{M}}\,\hat{\rho}_{\mathcal{S}}])$$

A tighter bound than Reeb and Wolf's ? Yes!

Goold, Paternostro, and Modi, PRL 114, 060602 (2015)