Laboratoire Kastler Brossel

Collège de France, ENS, UPMC, CNRS

Introduction to Ultracold Atoms

An overview of experimental techniques

Fabrice Gerbier (*fabrice.gerbier@lkb.ens.fr*)

Advanced School on Quantum Science and Quantum Technologies, ICTP Trieste

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Atomic physics born with spectroscopy at the end of the 19th century.

Progressed hand-in-hand with quantum mechanics in the years 1900-1930.

AMO -Atomic, Molecular and Optical Physics : *dilute* gases (as opposed to *dense* liquids and solids).

Common view in the early 50's was that AMO physics was essentially understood, with little left to discover. Sixty years later, this view has been proven wrong.

AMO Physics underwent a serie of revolutions, each leading to the next one :

- the 1960's : the laser
- the 1970's : laser spectroscopy
- the 1980's : laser cooling and trapping of atoms and ions
- the 1990's : quantum degenerate atomic gases (Bose-Einstein condensates and Fermi gases)
- the 2000's : femtosecond frequency combs

The quantum mechanical description of an atom introduces several quantum numbers to describe its state :

- internal quantum numbers to describing the relative motion of electrons with respect to the nuclei,
- external quantum numbers, *e.g.* center of mass position $\hat{\boldsymbol{R}}$.

In spectroscopy, electromagnetic fields are used to probe the structure of internal states. Extensions of the same techniques developped for spectroscopy allow one to control the internal degrees of freedom coherently.

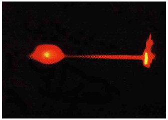
Laser cooling and trapping techniques allow one to do the same with the external degrees of freedom of the atom.

Milestones in ultracold atom physics

- first deflection of an atomic beam observed as early as 1933 (O. Frisch)
- revival of study of radiative forces in the lates 1970's; first proposals for laser cooling of neutral atoms (Hänsch – Dehmelt) and ions (Itano – Wineland) Why ? **Rise of the LASER**

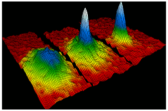
Laser cooling and trapping:

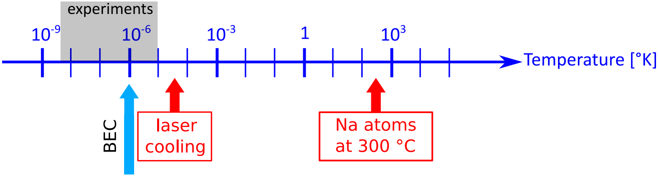
- 1980 : Slowing and bringing an atomic beam to rest
- 1985 : Optical molasses
- 1988 : magneto-optical traps , sub-Doppler cooling
- 1997 : Nobel Prize for Chu, Cohen-Tannoudji, Phillips



Quantum degenerate gases:

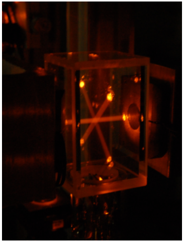
- Bose-Einstein condensation in 1995 [Cornell, Wieman, Ketterle : Nobel 2001]
- Degenerate Fermi gases in 2001 [JILA]



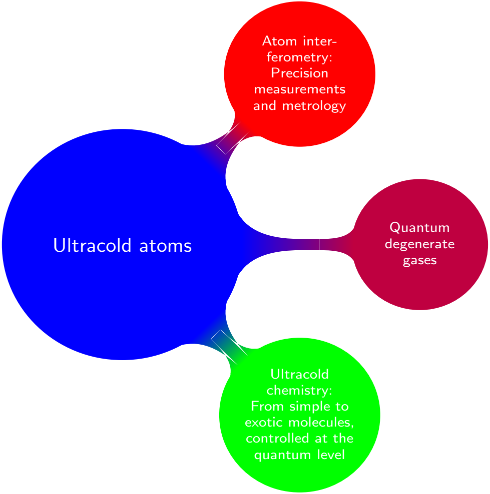


Why are ultracold atoms (and molecules) still interesting today ?

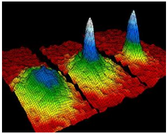
Laser cooling and trapping of neutral atoms



Nobel Prize 1997 :
S. Chu,
C. Cohen-Tannoudji,
W. D. Phillips



Quantum gases :
Bose-Einstein condensation (1995)



Degenerate Fermi gases (1999)

Nobel Prize 2001 :
E. Cornell,
W. Ketterle,
C. Wieman

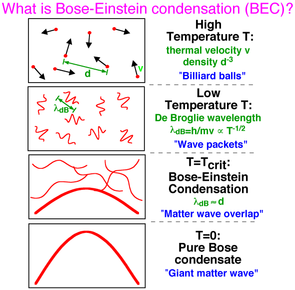
Ultracold atomic gases as many-body systems

Quantum degeneracy : phase space density $n\lambda_{\rm dB}^3 > 1$

n: spatial density

$\lambda_{\rm dB} = \sqrt{\frac{2\pi\hbar^2}{mk_BT}}$: thermal De Broglie wavelength

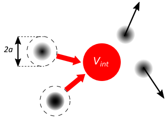
From W. Ketterle group website, http://www.cua.mit.edu/



Interacting atoms, but dilute gas: $na^3 \ll 1$

a : scattering length for $s-$wave interactions

$8\pi a^2$: scattering cross-section (bosons)



$$a \ll n^{-1/3} \ll \lambda_{\rm dB}$$

Typical values (BEC of ^{23}Na atoms) :

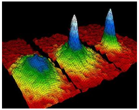
$$a \sim 2\,\text{nm}$$
$$n^{-1/3} \sim 100\,\text{nm}$$
$$\lambda_{\rm dB} \sim 1\,\mu\text{m at } T = 100\,\text{nK}$$

Ultracold atomic gases as model systems for many-body physics :

- dilute but *interacting* gases
- tunability (trapping potential, interactions, density, ...) and experimental flexibility
- microscopic properties well-characterized
- well-isolated from the external world

Bose-Einstein condensates :

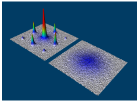
Superfluid gas
"Atom laser"



JILA, MIT, Rice (1995)

Optical lattices :

Superfluid-Mott insulator transition

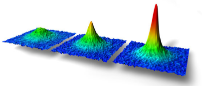


Munich 2002

superfluid → solid-like

BEC-BCS crossover :

fermions pairing up to form composite bosons

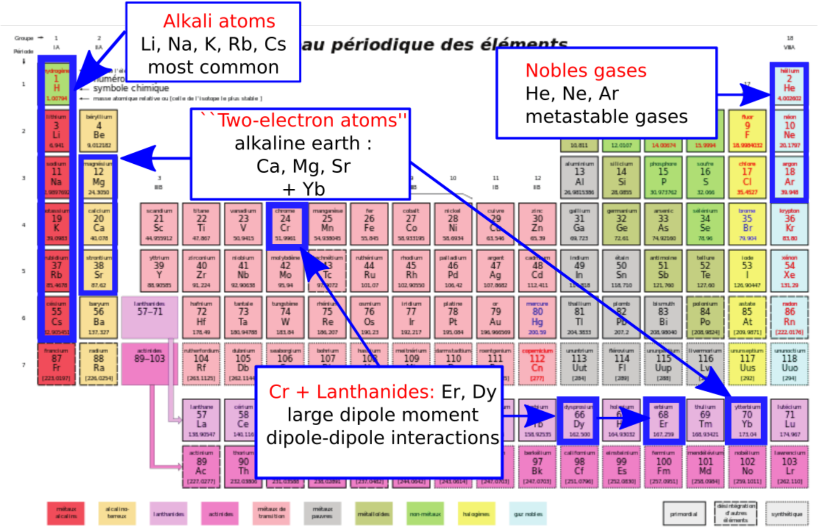


JILA, MIT, ENS (2003-2004)

Condensation of fermionic pairs

Many other examples :

- gas of impenetrable bosons in 1D,
- non-equilibrium many-body dynamics,
- disordered systems, ...



Alkali atoms
Li, Na, K, Rb, Cs
most common
Nobles gases
He, Ne, Ar
metastable gases
``Two-electron atoms''
alkaline earth :
Ca, Mg, Sr
+ Yb
Cr + Lanthanides: Er, Dy
large dipole moment
dipole-dipole interactions

Outline of the course

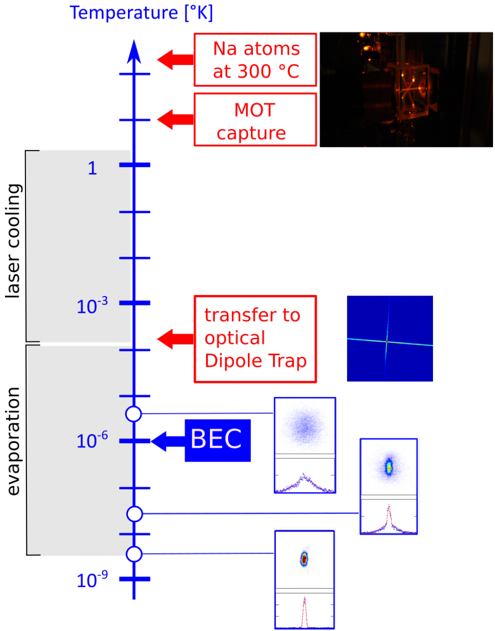
1. Introduction to quantum gases : an experimental point of view (Today)
    1. Introduction
    2. Overview of techniques for trapping and imaging
    3. Bose-Einstein condensation
2. Optical lattices (Today-Thursday)
3. Superfluid-Mott insulator transition (Friday)

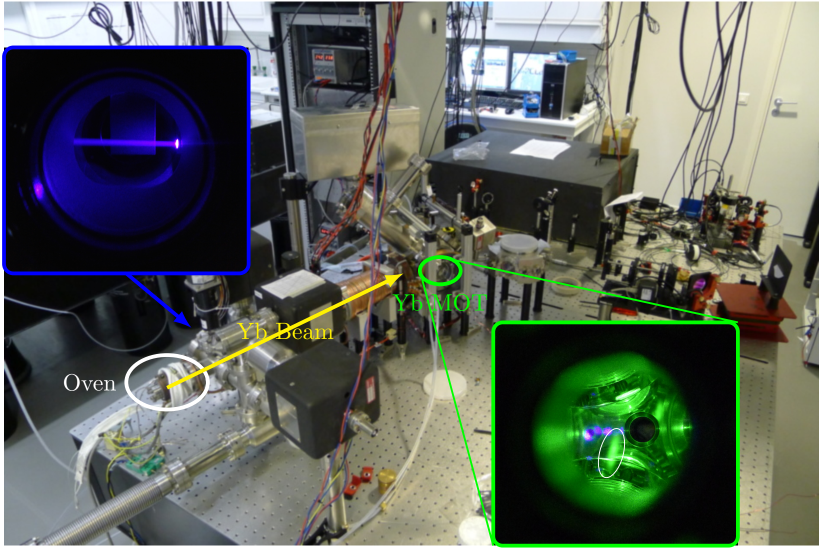
Typical experimental sequence :

- catch atoms in a magneto-optical trap
- laser cooling to $\sim 50\,\mu$K
- transfer to conservative trap (no resonant light): optical trap or magnetic trap
- evaporative cooling to BEC

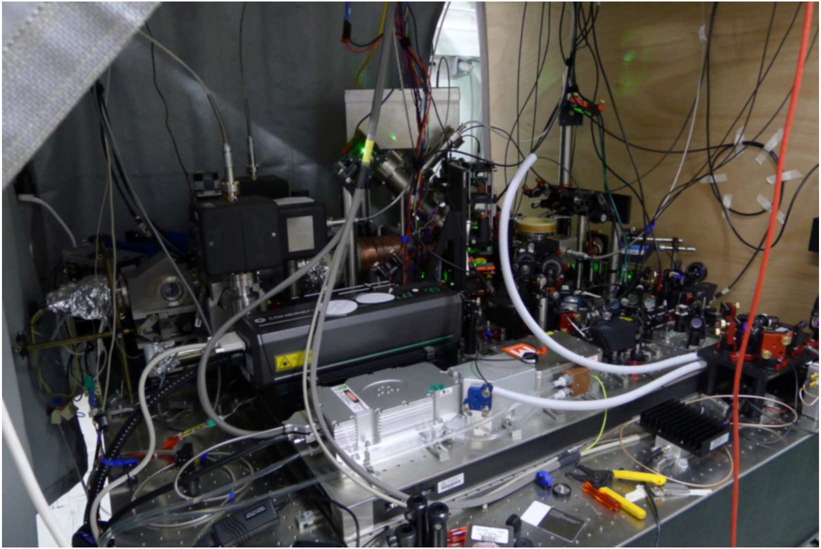
Take a picture of the cloud

Repeat




Yb MOT
Yb Beam
Oven

Ytterbium BEC experiment at LKB



Why is a two-step sequence necessary ?

Quantum gases : Phase-space density $\mathcal{D} = n\lambda_{\rm th}^3 \geq 1$

- Step 1: laser cooling in a magneto-optical trap
- Step 2 : evaporative cooling in a conservative trap

Laser cooling relies on the interaction between the atoms and a near-resonant laser. Spontaneous emission of photons is :

- essential to provide necessary dissipative mechanism to cool the motional degrees of freedom of the atoms,
- but also intrinsically random. This randomness prevents to cool the atoms below a certain limiting temperature !

Typical MOT of Yb :

$n \sim 10^{10} - 10^{11}$ at/cm^3, $T \sim 10\,\mu$K, $\lambda_{\rm dB} \sim 40\,$nm, $\mathcal{D} \sim 10^{-6} - 10^{-5}$

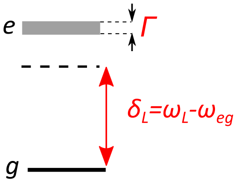
To overcome the limitations of laser cooling, all experiments (with one exception) follow the same path :

- trapping in a *conservative trap*: optical trap or magnetic trap,
- evaporative cooling to quantum degeneracy.

The MOT remains a mandatory first step. The *trap depth* ($k_B \times$mK) requires laser-cooled atoms for efficient loading and subsequent evaporative cooling.

Two-level atom interacting with a monochromatic laser field:

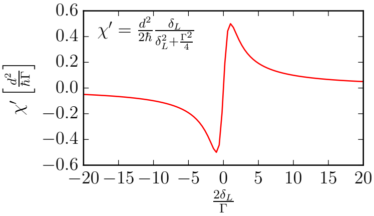
- Electric field : $\boldsymbol{E} = \frac{1}{2}\boldsymbol{\mathcal{E}}(\boldsymbol{r})e^{-i\omega_L t + i\phi} + \mathrm{c.c.}$
- Γ: transition linewidth
- d: electric dipole matrix element
- $\delta_L = \omega_L - \omega_{\mathrm{eg}}$: detuning from atomic resonance

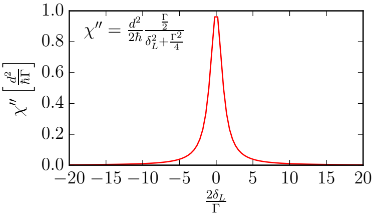


Low light intensity : Linear response of the atomic electric dipole driven by laser light

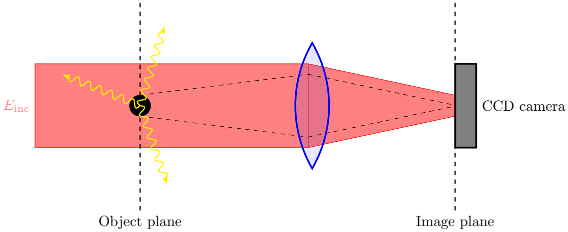
Complex susceptibility : $\chi = \chi' + i\chi''$ such that $\langle \hat{d} \rangle = \left(\chi E_L e^{i(\omega_L t + \phi_L)} + \mathrm{h.c.}\right)$

$$\chi' = \frac{d^2}{2\hbar}\frac{\delta_L}{\delta_L^2 + \frac{\Gamma^2}{4}}, \qquad \chi'' = \frac{d^2}{2\hbar}\frac{\frac{\Gamma}{2}}{\delta_L^2 + \frac{\Gamma^2}{4}}$$





Basic setup :



Propagation in a dielectric medium :

$$\nabla^2 \boldsymbol{E} - \frac{1}{c^2}\frac{\partial^2 \boldsymbol{E}}{\partial t^2} = \mu_0 \frac{\partial^2 \boldsymbol{P}}{\partial t^2}$$

$$\boldsymbol{P} = \epsilon_0 \chi \boldsymbol{E} = n_{\text{at}} \langle \hat{\boldsymbol{d}} \rangle$$

χ: dipole susceptibility
$n_{\text{at}}(\boldsymbol{r})$: atomic density

Writing $E(\boldsymbol{r}, t) = \mathcal{E} e^{i\varphi_L} e^{i(k_L z - \omega_L t)}$, and invoking a slowly-varying envelope approximation for $\mathcal{E}$ and ϕ_L (terms $\propto \Delta\mathcal{E}, \Delta\varphi_L$ neglected) :

$$\frac{d\mathcal{E}}{dz} = -\frac{k_L n_{\text{at}}}{2} \chi'' \mathcal{E}(z), \ : \text{Beer-Lambert law}$$

$$\frac{d\varphi_L}{dz} = \frac{k_L n_{\text{at}}}{2} \chi' \varphi_L(z) \ : \text{dephasing}$$

NB : the atomic density must also vary smoothly *along* z on the scale λ_L.

Absorption imaging

We rewrite the equation for $\mathcal{E}$ in terms of the laser intensity: $I = \frac{\epsilon_0 c}{2}|\mathcal{E}|^2$

$$\frac{dI}{dz} = -\kappa I(z) : \text{Beer-Lambert law}, \qquad \kappa = \frac{k_L n_{\rm at}}{2}\chi'' = \frac{3\lambda_L^2}{2\pi}\frac{n_{\rm at}}{\left(\frac{2\delta_L}{\Gamma}\right)^2 + 1}$$

Interpretation in terms of scattered photons :

$d(\text{Photon flux}) = -\,(n_{\rm at} dz dA)\,\sigma \times (\text{Photon flux})$, with a scattering cross-section

$$\sigma = \frac{\sigma_0}{\left(\frac{2\delta_L}{\Gamma}\right)^2 + 1}, \quad \sigma_0 = \frac{3\lambda_L^2}{2\pi}, \quad \kappa = \sigma n_{\rm at}$$

Maximum on resonance ($\sigma = \sigma_0$ for $\delta_L \approx 0$).

The intensity on the camera (assuming that the focal depth of the imaging system is $\gg$ cloud size) gives a magnified version of the transmitted intensity,

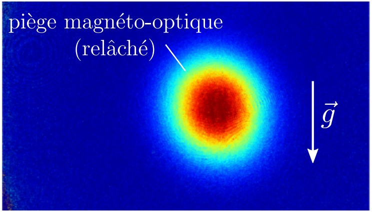
$$I_t(x,y) = I_{\rm inc}(x,y) e^{-\sigma \int n_{\rm at}(x,y,z)dz}$$

One calls $\tilde{n}(x,y) = \int n_{\rm at}(x,y,z)dz$ the *column density* and $OD(x,y) = \sigma\tilde{n}(x,y)$ the *optical depth*.

Absorption signal :

$$\tilde{n}(x,y) = \frac{1}{\sigma}\ln\left(\frac{I_{\rm inc}(x,y)}{I_t(x,y)}\right)$$

Absorption image of a MOT of Yb atoms after a time of flight of $t \sim 10\,\text{ms}$, during which atom are released from the trap and expand freely.



Why do we need to take pictures after a time of flight ?

- To reduce absorption. Often the optical depth is too large: $OD \sim 5 - 10$ for a MOT, $OD \sim 100$ or more for a BEC. Images are “pitch black” and dominated by noise.
- To avoid photon reabsorption and multiple scattering, which makes the above description invalid and the interpretation of images difficult.

In-situ imaging are also possible, though easier with dispersive techniques than with absorption imaging.

Time-of-flight (t.o.f.) experiment :

- suddenly switch off the trap potential at $t = 0$,
- let the cloud expand for a time t.

The spatial distribution for long times is proportional to the initial *momentum distribution* $\mathcal{P}_0$ evaluated at $\boldsymbol{p} = \frac{M\boldsymbol{r}}{t}$:

$$\boxed{n_{\mathrm{at}}(\boldsymbol{r}, t) \longrightarrow_{t\to\infty} \left(\frac{M}{t}\right)^3 \mathcal{P}_0\left(\boldsymbol{p} = \frac{M\boldsymbol{r}}{t}\right)}$$

This results holds for a classical or a quantum gase, indistinctively, *provided one can neglect the role of interactions during the expansion*.

For non-degenerate gases, temperature can be inferred from the cloud size after t.o.f.

For a Boltzmann gas,

$$\frac{\langle p_x^2 \rangle}{2M} = \frac{k_B T}{2}$$

and suitable generalizations for Bose or Fermi gases.

For large detuning $\delta_L \gg \Gamma$, the atom-laser interaction can be pictured using the Lorentz model of an elastically bound electron.

The electric field induces an oscillating electric dipole moment $\boldsymbol{d} \propto \boldsymbol{\mathcal{E}}$.

The (time-averaged) potential energy of the induced dipole is

$$V(\boldsymbol{r}) = -\frac{1}{2}\langle \boldsymbol{d} \cdot \boldsymbol{E} \rangle = \frac{d^2 |\boldsymbol{\mathcal{E}}(\boldsymbol{r})|^2}{4\hbar\delta_L}$$

- $\delta_L < 0$ (red detuning) : $V < 0$, atoms attracted to intensity maxima
- $\delta_L > 0$ (blue detuning) : $V > 0$, atoms repelled from intensity maxima

Radiated energy : $\dot{W} = \langle \dot{\boldsymbol{d}} \cdot \boldsymbol{E} \rangle = \hbar\omega_L \Gamma_{\rm sp}$: energy absorbed by the dipole from the field, and reradiated by spontaneous emission at a rate $\Gamma_{\rm sp}$.

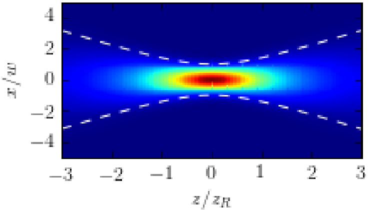
Usually $\Gamma_{\rm sp} \approx \frac{\Gamma \Omega_L(\boldsymbol{r})^2}{8\delta_L^2} \ll 1\,s^{-1}$ for far-off resonance dipole traps and optical lattices.

Some numbers for ^{87}Rb atoms and a relatively weak trap :

- $\Gamma/2\pi \approx 6\,$MHz,
- $\lambda_{eg} \approx 780\,$nm [$\omega_L/2\pi \approx 4 \times 10^{14}$ Hz]
- $\lambda_0 \approx 1064\,$nm, $\delta_L/2\pi \approx -3 \times 10^{13}$ Hz,
- Laser parameters : power $P = 200\,$mW, beam size $100\,\mu$m

One finds a potential depth $|V| \sim h \times 20\,$kHz $\sim k_B \times 1\,\mu$K.

Trapping atoms in such a potential requires sub-μK temperatures, or equivalently degenerate (or almost degenerate) gases.



Actual laser beams have Gaussian profile :
Dipole potential of the form

$$U(\boldsymbol{r}) = -U_0 \frac{e^{-2\frac{x^2+y^2}{w^2}}}{1+\left(\frac{z}{z_R}\right)^2}$$

Harmonic trap near the bottom of the potential (particles with energy $\ll U_0$):

$$U(\boldsymbol{r}) = -U_0 + \sum_{x_\alpha = x,y,z} \frac{1}{2} m\omega_\alpha^2 x_\alpha^2$$

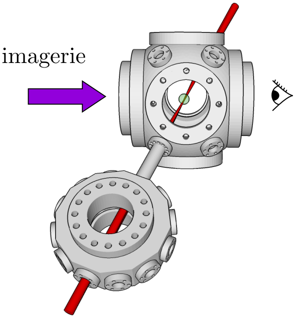
$$\omega_x = \omega_y = \sqrt{\frac{4U_0}{Mw^2}}, \qquad \omega_z = \sqrt{\frac{2U_0}{Mz_R^2}} = \frac{w}{\sqrt{2}z_R}\omega_x$$

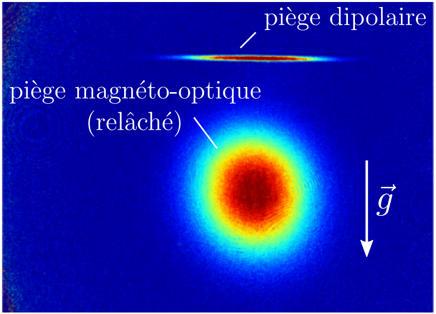
Typical example for Ytterbium atoms and a strong trap :

- $\lambda_{eg} = 399\, nm$
- $\lambda_L = 532\, nm$
- $\delta_L = -2 \cdot 10^{14}$ Hz
- $P = 10$ W, $w = 50\,\mu$m
- $s \approx 10^{-6}$
- $I = 2$ MW/m^2 at focus
- $U_0 \approx k_B \times 700\,\mu$K
- $\omega_x/(2\pi) \approx 1.2$ kHz
- $\omega_z/(2\pi) \approx 3$ Hz

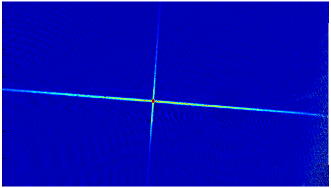
Loading a dipole trap

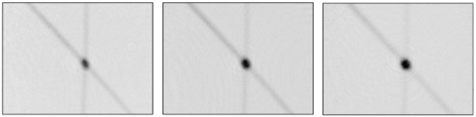
Single beam dipole trap (side view) :





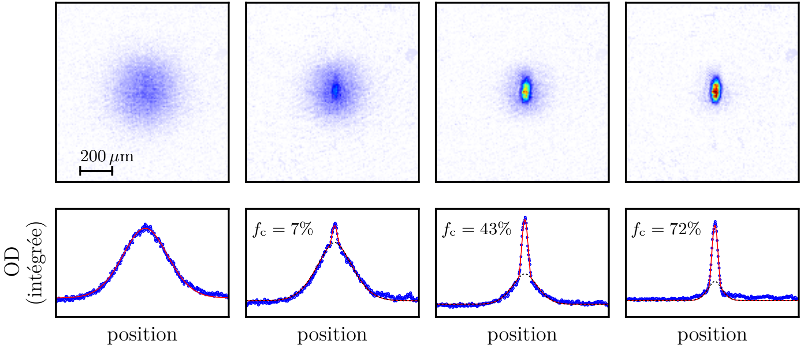
Crossed dipole trap (viewed from above):





t=100 ms t=300 ms t=600 ms

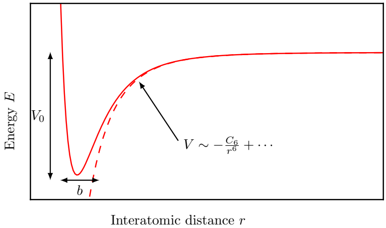
Ytterbium BEC, LKB :



[figure from A. Dareau's PhD thesis]

Emergence of a narrow and dense peak on top of the broad and dilute background of thermal atoms.
Hallmark of Bose-Einstein condensation.
Qualitatively what is expected from non-interacting atoms, but all quantitative comparison fail: Including atomic interactions is essential to understand the properties of quantum gases.

- short-range repulsion (Coulomb interaction)
- long range attractive tail (van der Vaals interaction)
- potential depth $\sim 100 - 1000\,\mathrm{K}$
- range $b \sim$ a few Å
- many (10-100s) bound molecular states



For low enough collision energies, the scattering amplitude is characterized by a single length a (the scattering length) : the complicated details of the potential are "washed out" on scales $\gg a$.

The idea of the pseudopotential method : replace the true interaction potential by a fictitious one, tuned to give the same scattering length as the exact one.

$$V(\boldsymbol{r}_1 - \boldsymbol{r}_2) \to V_{\mathrm{pseudo}}(\boldsymbol{r}_1 - \boldsymbol{r}_2) = g\delta(\boldsymbol{r}_1 - \boldsymbol{r}_2), \qquad g = \frac{4\pi\hbar^2 a}{M}$$

g is chosen to reproduce the same scattering cross-section as the true potential $\sigma = 8\pi a^2$ *when* V_{pseudo} *is treated in the Born approximation.*

Mean-field theory of interacting bosons at $T = 0$

$$\hat{H} = \underbrace{\sum_{i=1}^{N} \frac{\hat{p}_i^2}{2M}}_{E_{\text{kin}}} + \underbrace{\sum_{i=1}^{N} U(\hat{r}_i)}_{E_{\text{pot}}} + \underbrace{\sum_{i\neq j} V_{\text{int}}(\hat{r}_i - \hat{r}_j)}_{E_{\text{int}}}$$

- assume that the many-body wavefunction keeps the same form as for non-interacting bosons,

$$\Psi(\boldsymbol{r}_1, \cdots, \boldsymbol{r}_N) = \phi(\boldsymbol{r}_1) \times \cdots \times \phi(\boldsymbol{r}_N)$$

  Macroscopic occupation of a single-particle orbital ϕ (*not* the ground state of the trap potential U)

- replace V_{int} by a pseudopotential $V_{\text{pseudo}} = g\delta(\boldsymbol{r})$, with $g = \frac{4\pi\hbar^2 a}{M}$ and a the scattering length.

Mean-field energy functional :

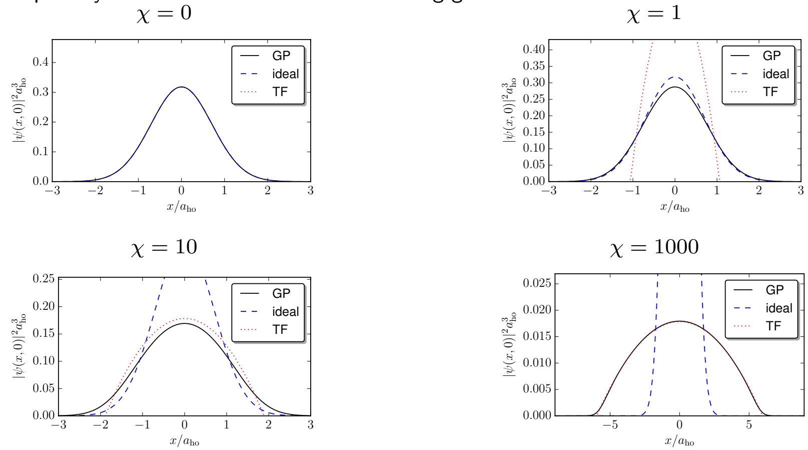
$$E_N(\phi, \phi^*) = \int d^3\boldsymbol{r} \left(-\frac{\hbar^2}{2M} N\phi^* \Delta\phi + U(\boldsymbol{r}) N|\phi|^2 + \frac{g}{2} N(N-1)|\phi|^4 \right)$$

Minimization of E_N under the constraint $\langle \phi | \phi \rangle = 1$:

$$\text{Gross-Pitaevskii equation}: \ \mu\psi = -\frac{\hbar^2}{2M}\Delta\psi + U(\boldsymbol{r})\psi + g|\psi|^2\psi$$

condensate wavefunction $\psi = \sqrt{N}\phi$ (normalized to N)

We introduce a parameter $b = \frac{R}{a_{\rm ho}}$, with $a_{\rm ho} = \sqrt{\frac{\hbar}{M\omega}}$ the harmonic oscillator length, to quantify deviations from the non-interacting ground state.

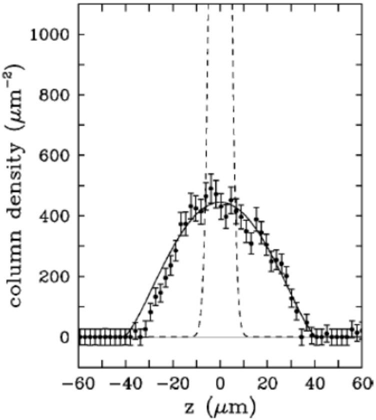


For small interactions (or $N \to 0$), the condensate forms in the trap ground state with $R = a_{\rm ho} = \sqrt{\frac{\hbar}{M\omega}}$.

Increasing g from 0 to its actual value, the condensate wavefunction changes from the Gaussian ground state of the harmonic oscillator to a flatter profile.

Density profile (*in-situ* dispersive imaging) of a Sodium BEC:

Dalfovo *et al.*, RMP 1998; Hau *et al.*, PRA 1998



Typical numbers (Na BEC):

$N \sim 10^6$,

$\overline{\omega}/2\pi = 15\,$Hz (after decompression),

Chemical potential :
$\mu \approx h \times 300\,Hz\approx k_B \times 30\,$nK

Peak density : $n \sim 2 \cdot 10^{13}$ at/cm^3,

Typical sizes : $R_{\mathrm{TF}} \sim 37\,\mu$m, $\zeta \sim 800\,$nm

Common numbers : $\mu/h \sim$kHz, $n \sim 10^{14}$ at/cm^3, $R_{\mathrm{TF}} \sim 10\,\mu$m, $\zeta \sim 100\,$nm
These number can change by factors up to 10 depending on atomic species and experimental details.

NB $1\,$kHz$\leftrightarrow$ $50\,$nK

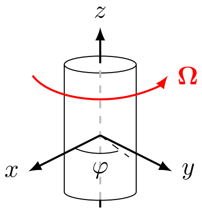
The rotating bucket experiment

BEC is a superfluid : dissipationless flow .
Interpretation of the phase θ of the condensate wavefunction :

$$\boldsymbol{v}_s = \frac{\hbar}{M}\boldsymbol{\nabla}\theta : \text{superfluid velocity}$$

Rotating bucket with cylindrical symmetry :

- a classical fluid will be "dragged along" and rotate with the container.
- velocity field of a "solid body" $\boldsymbol{v} = \boldsymbol{\Omega} \times \boldsymbol{r}$.



What happens if we have a superfluid inside the bucket ?

$\boldsymbol{v}_s = \frac{\hbar}{M}\boldsymbol{\nabla}\theta \implies$ Irrotational flow : $\boldsymbol{\nabla} \times \boldsymbol{v}_s = 0$

Superfluid stays at rest !

There is however another possibility : singularity of the phase θ (and of $\boldsymbol{v}_s$), which can only occur at a point where the density vanishes $n_0 = 0$.

In classical hydrodynamics, this corresponds to a vortex line:

$$\text{Velocity field}: \ \boldsymbol{v} = v_0 \boldsymbol{e}_\phi$$

$$\text{Circulation}: \ \oint \boldsymbol{v} \cdot \boldsymbol{dl} = 2\pi\rho v_0 = \mathcal{C}$$

$$\boldsymbol{\nabla} \times \boldsymbol{v}_s = \mathcal{C}\delta^{(2)}(\boldsymbol{\rho})\boldsymbol{e}_z$$

Quantization of circulation for a quantum fluid :

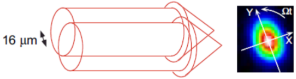
$$\mathcal{C} = \oint \boldsymbol{v}_s \cdot \boldsymbol{dl} = \frac{\hbar}{M} \oint \boldsymbol{\nabla}\theta \cdot \boldsymbol{dl} = \frac{\hbar}{M} \underbrace{[\theta(2\pi) - \theta(0)]}_{=2\pi \times \text{integer}} = s\frac{h}{M}, \qquad s \in \mathbb{Z}$$

Circulation must be quantized to keep the condensate wavefunctions single-valued.
s **: vortex charge.**

Velocity field far from the quantized vortex core : $\boldsymbol{v}_s = \frac{\hbar s}{M\rho}\boldsymbol{e}_\phi$

Experiments with an "optical spoon" stirring the condensate into rotation :

- spoon = fast rotating anisotropy on top of an otherwise rotationally symmetric potential
- BEC formed in equilibrium, then set in rotation
- Turbulent relaxation to an equilibrium state (in the rotating frame) after $\sim 1\,$s
- vortex nucleation above a critical rotation speed Ω_c.
- large BEC and high rotation : many vortices (up to 100 observed)
- vortices organize into a regular, triangular array (Abrikosov lattice)
- also present in type-II superconductors



$$\delta U(\vec{r}) = \frac{1}{2} m\omega_\perp^2 \left(\varepsilon_X X^2 + \varepsilon_Y Y^2\right)$$