Topological Phases of Matter with Ultracold Atoms and Photons

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Advanced School and Workshop on Quantum Science and Quantum Technologies,
ICTP Trieste, September 2017

Overview

Lectures 1 & 2

Introduction to Topological Phases of Matter

Lecture 3

Topological Phases of Matter with Ultracold Atoms

Lecture 4

Topological Phases of Matter with Photons

Overview

Lectures 1 & 2

Introduction to Topological Phases of Matter

Lecture 3

Topological Phases of Matter with Ultracold Atoms

Some reviews/lecture notes for ultracold atoms on:

Rotating gases: Cooper, Adv. Phys. 57 539-616 (2008).

Artificial gauge fields: Dalibard et al. Rev. Mod. Phys. 83, 1523 (2011).

Spin-orbit coupling: Zhai, Int. J. Mod. Phys. B 26, 1230001 (2012)

Artificial gauge fields: Goldman et al., Rep. Prog. Phys. 77, 126401 (2014).

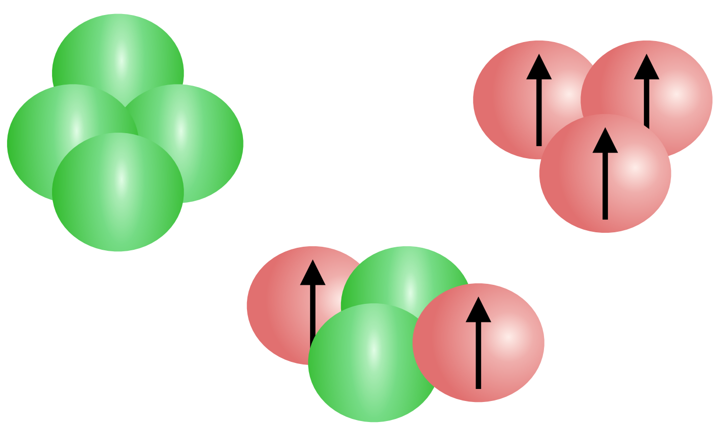
Great lecture notes on artificial gauge fields: Dalibard, arXiv:1504.05520

Chern bands: Goldman et al. arXiv:1507.07805 (in book "Universal Themes of Bose-Einstein Condensation")

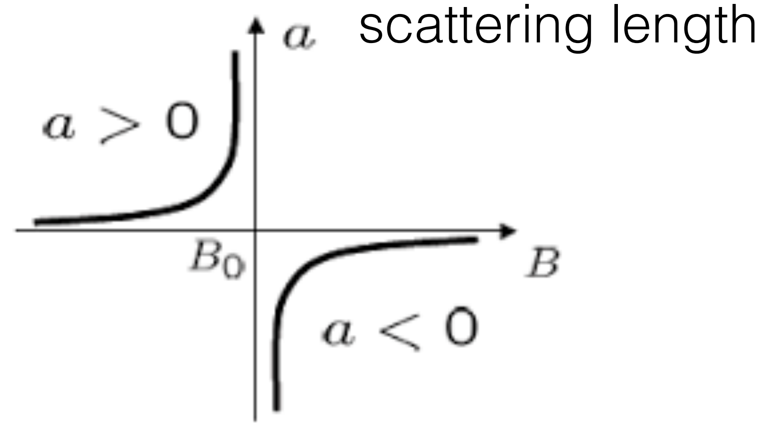
Topological physics with optical lattices: Goldman et al., Nature Physics 12, 639-645 (2016)

Ultracold atoms

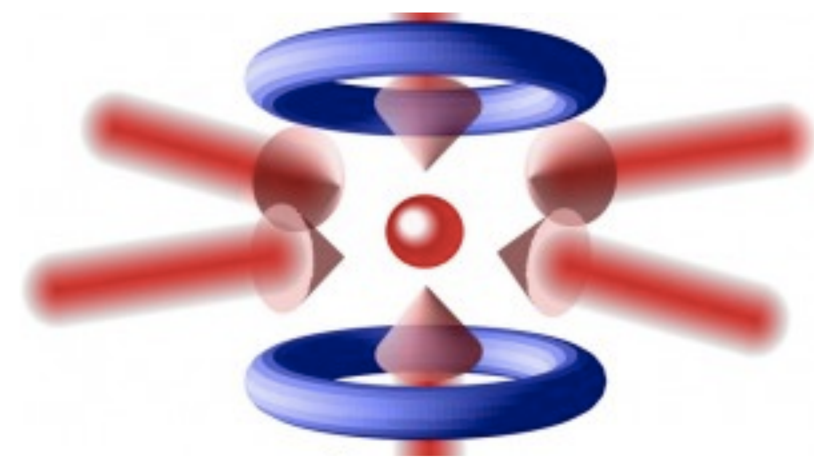
Different species: bosons, fermions, Bose-Fermi mixtures



Tuneable interactions (Feshbach resonances)

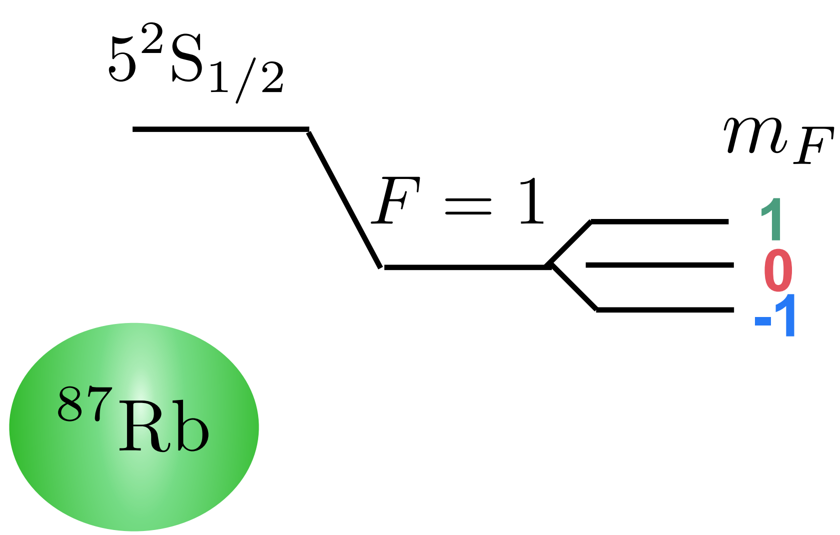


Controllable and tuneable…

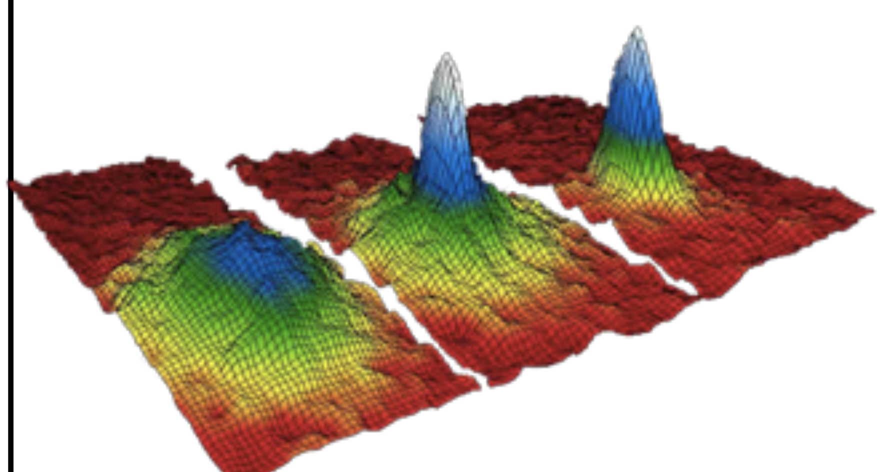


No disorder/impurities

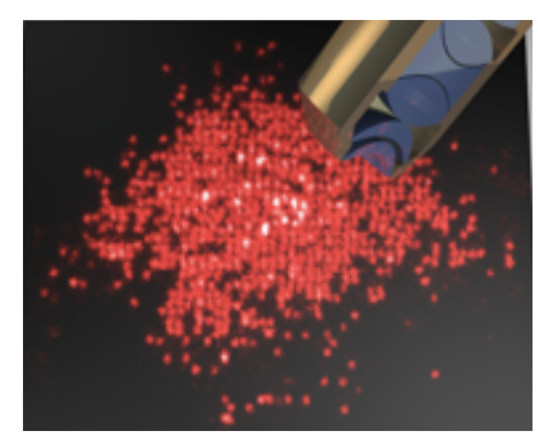
Addressable internal states: e.g. hyperfine states…



Access momentum distribution (time-of-flight imaging)



Access density (in-situ imaging)

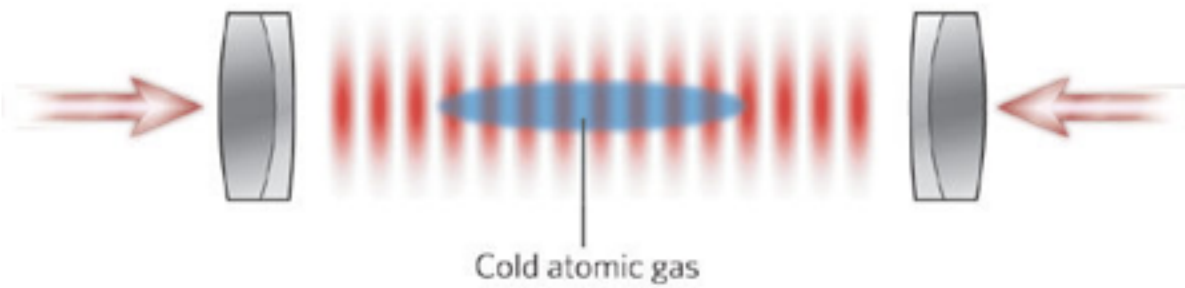


& many other observables….

Controlling atoms with light

Optical dipole potential $V(\mathbf{x}) = \alpha |E(\mathbf{x})|^2$

Interfere lasers to create optical lattices in 1D, 2D, 3D….



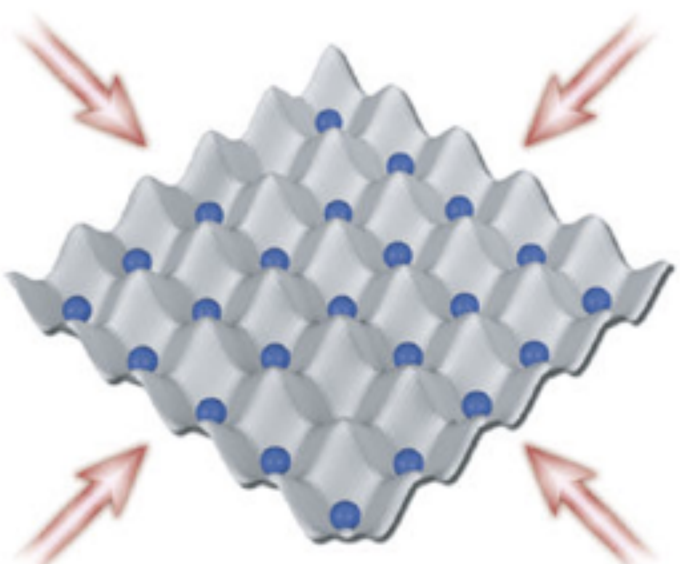
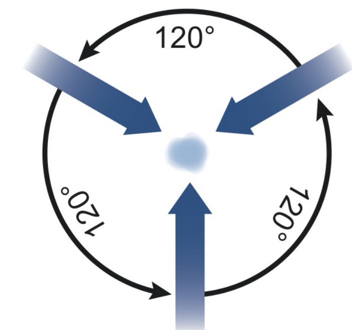
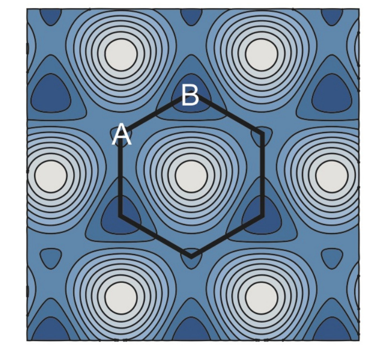


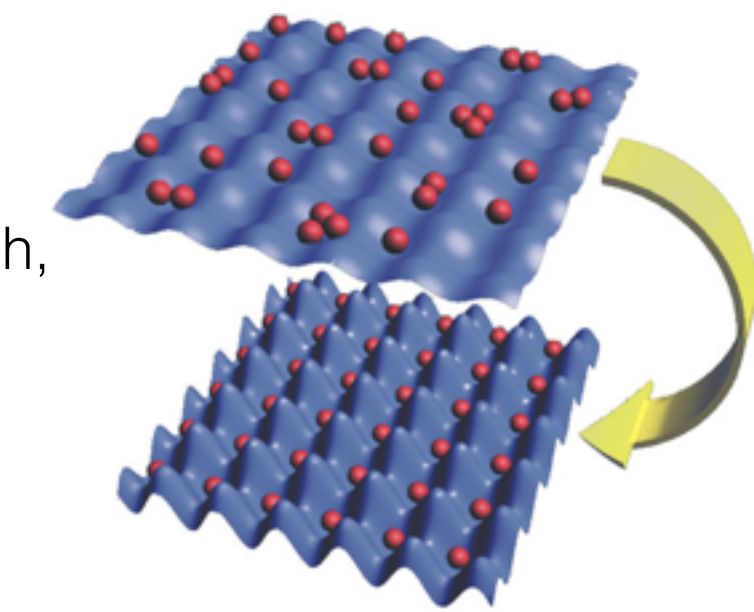
Figure from Bloch, Nature 453, 1016 (2008)

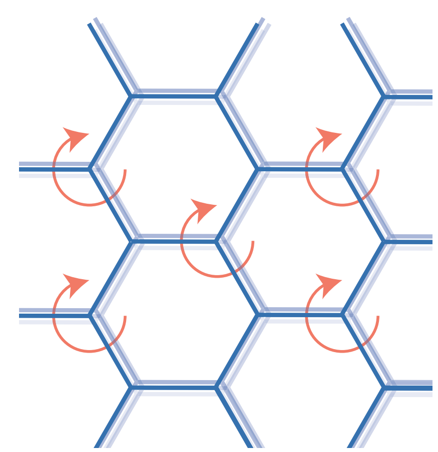
Arbitrary geometries, e.g. honeycomb:





Tuneable lattice depth, e.g. superfluid to Mott insulator





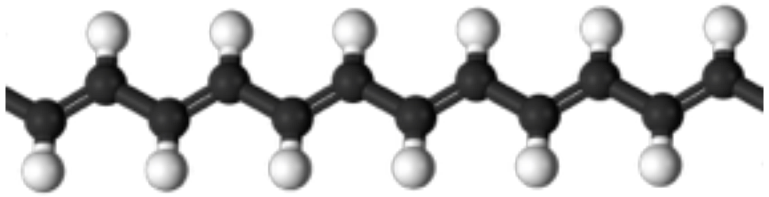
Dynamical lattices (e.g. lasers of different frequencies, piezo-electrics…)

Lecture 3

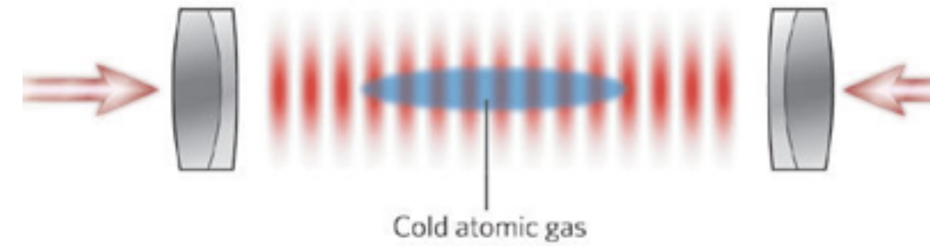
- How can we engineer topology for cold atoms?
  - SSH Model & Topological Pumps
  - Quantum Hall systems
  - Quantum spin Hall systems & topological superfluids
- How can we probe topology with cold atoms?
- Future perspectives

Lecture 3

- How can we engineer topology for cold atoms?
  - **SSH Model & Topological Pumps**
  - Quantum Hall systems
  - Quantum spin Hall systems & topological superfluids
- How can we probe topology with cold atoms?
- Future perspectives



SSH Model



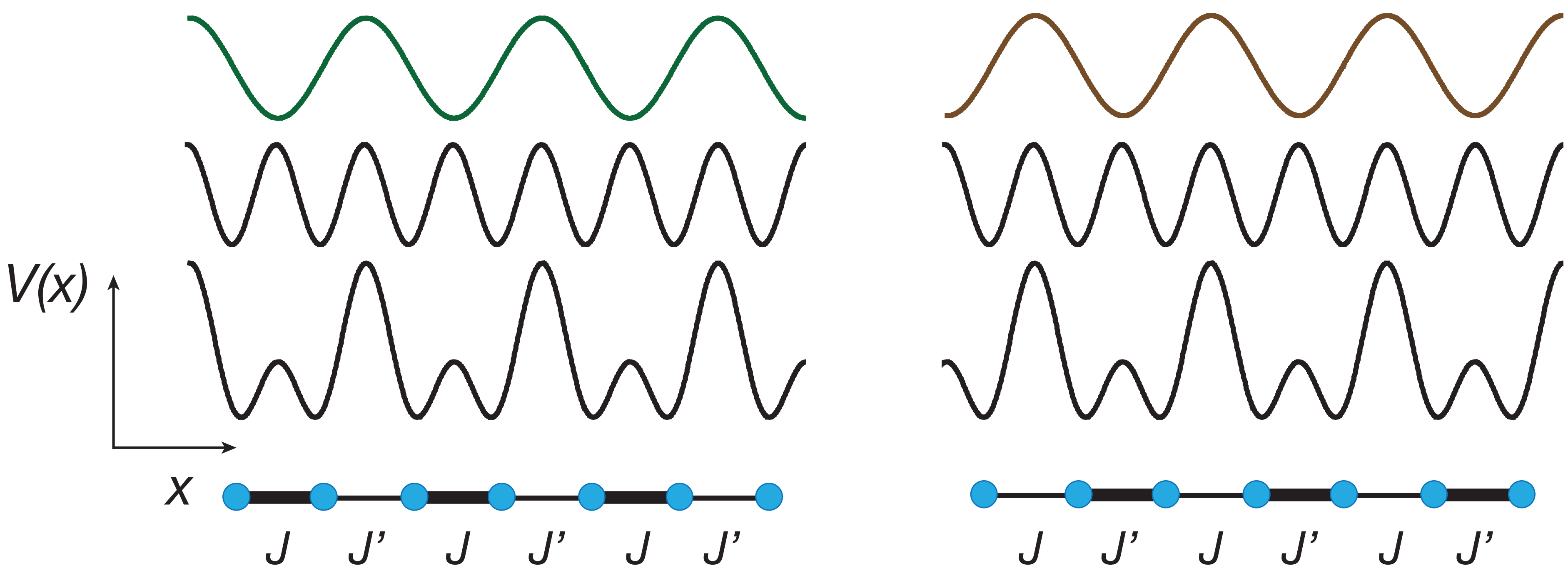
1D Superlattice: Superposition of a long and short optical lattice along one dimension

SSH model: 2-site superlattice $d_l = 2d_s$

Munich: Atala, M et al. Nat. Phys. 9, 795 (2013).

First long lattice, $\phi = 0$

Second long lattice, $\phi' = \pi$



Long Lattices

Short Lattice

Superlattice potential

Dimerization D1

Dimerization D2

[Experiment: Interferometric measurement of bulk topological invariant (Zak phase) in the two dimerizations]

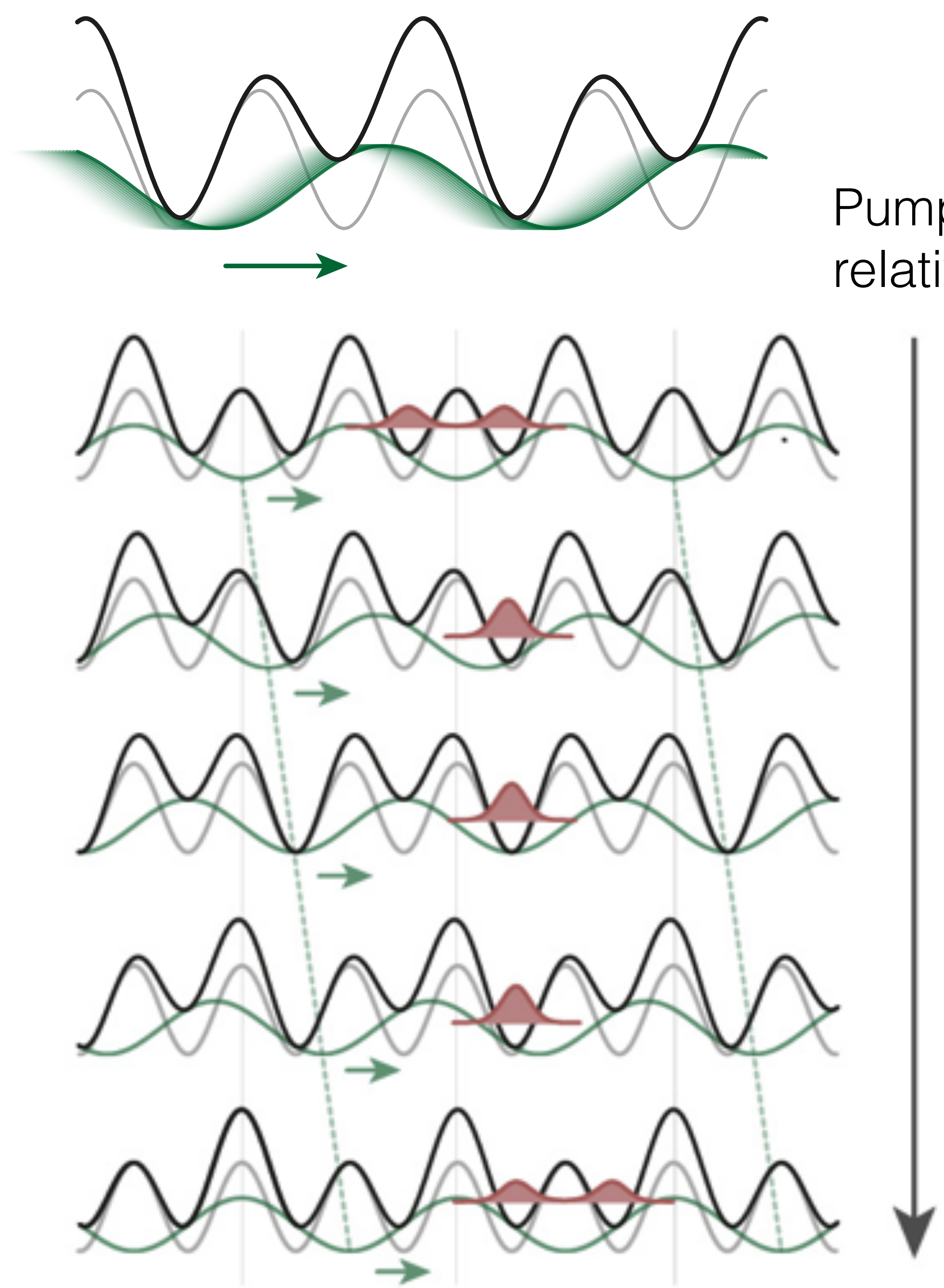
	Symmetry			d							
	Time-reversal	Particle-hole	Chiral	1	2	3	4	5	6	7	8
BDI	1	1	1	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$

Topological Pumps

And in photonics (Lecture 4)

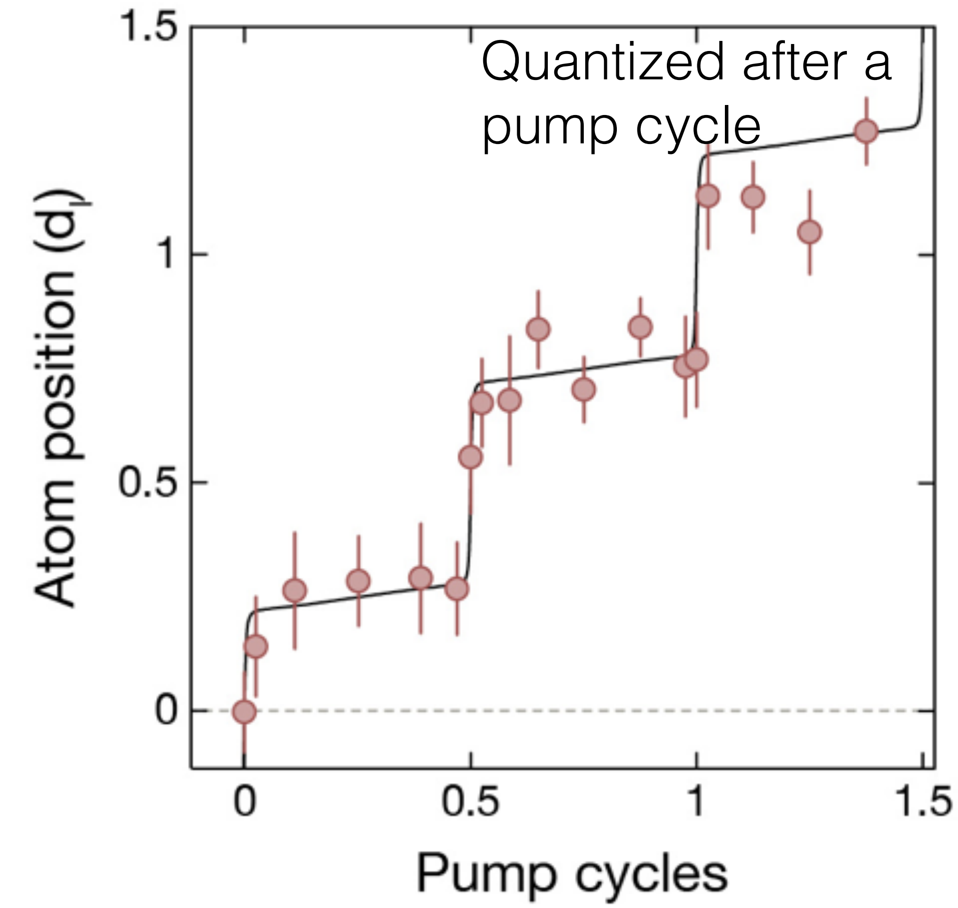
1D Pump —> Dynamical 2D QH Effect (*First Chern Number*)

Munich: Lohse, et al. Nat. Phys. 12, 350–354 (2016).
Kyoto: Nakajima,. et al. Nat. Phys. 12, 296–300 (2016).
Maryland: Lu et al. Phys. Rev. Lett. 116, 200402 (2016).



Pumping: Move long relative to short lattice

$x(T) = \nu_n$



Before we talked about pumping the Harper model:

$$\mathcal{H}_{1D} = -J\sum_m \left[a_m^\dagger a_{m+1} + \text{h.c.} + 2\cos\left(2\pi\alpha m + \varphi\right) a_m^\dagger a_m\right].$$

This is a slight variant of this model with bipartite NN hoppings ($J_1(t) \neq J_2(t)$) and $\alpha = 1/2$

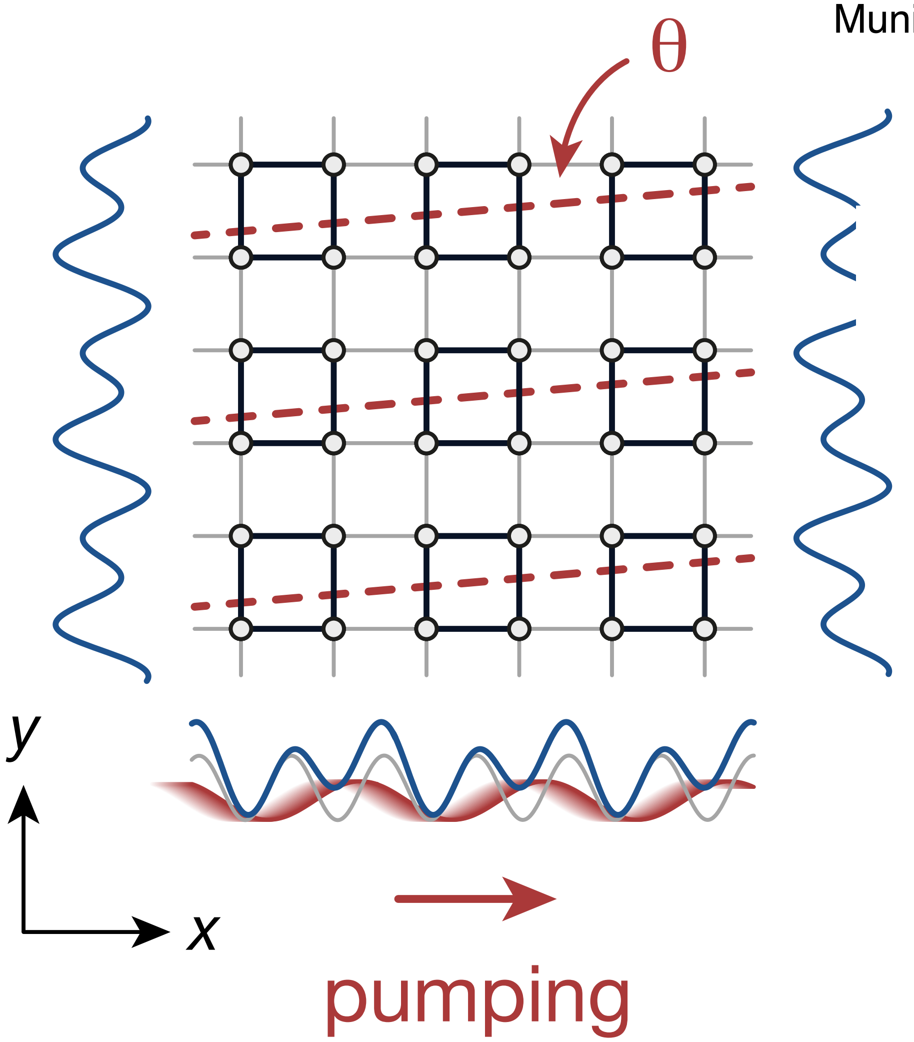
Topological Pumps

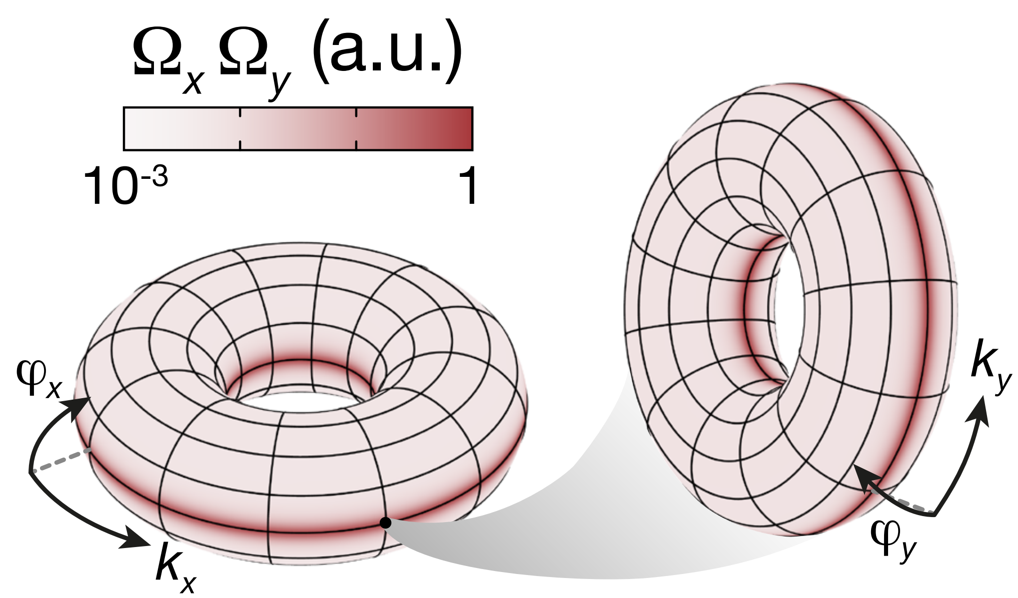
And in photonics
(Lecture 4)

2D Pump —> Dynamical 4D QH Effect (*Second Chern Number*)

Munich: Lohse, M et al. arXiv:1705.08371

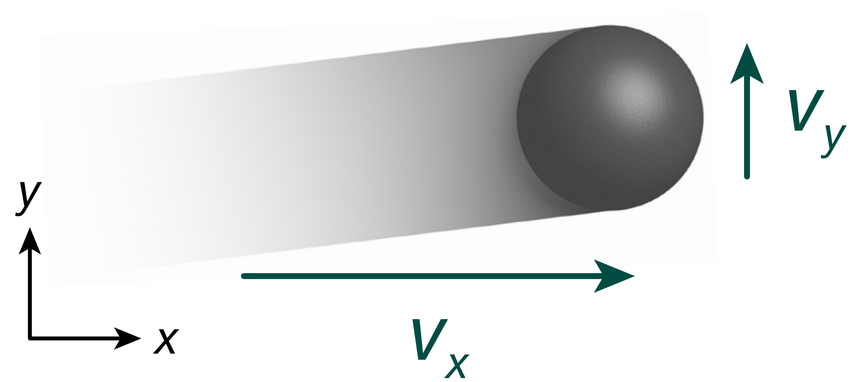
$$j_y = \frac{q^3}{h^2} E_x B_{wz} \nu_n^{(2)}$$

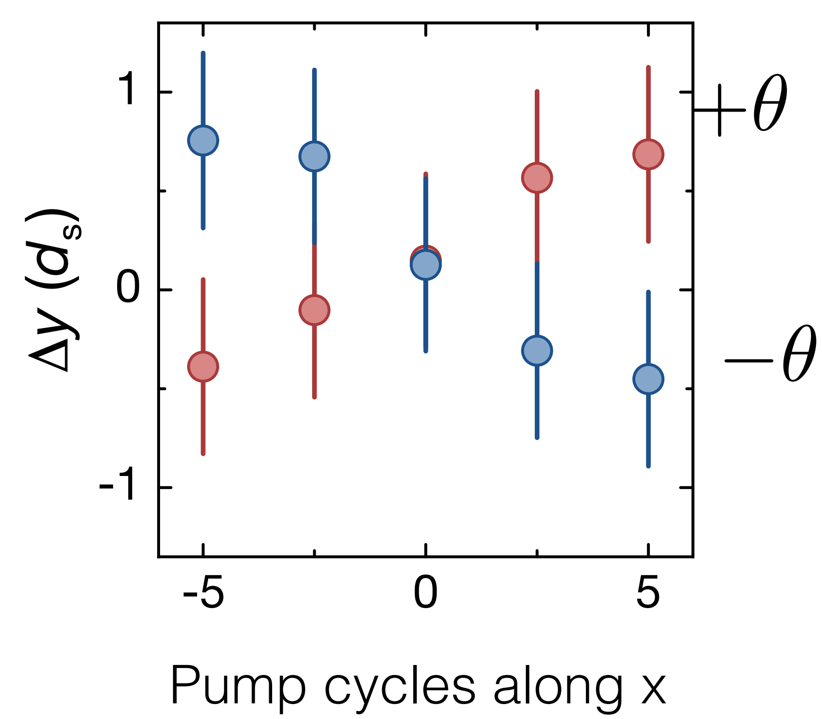




2D Superlattice: 2 x the 1D superlattices

2 physical directions + 2 phases = effective 4D parameter space





"Magnetic perturbation": Angled tilt of long lattice along y so that pumping x leads to response in y

Lecture 3

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  - SSH Model & Topological Pumps
  - **Quantum Hall systems**
  - Quantum spin Hall systems & topological superfluids

- How can we probe topology with cold atoms?

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Quantum Hall systems

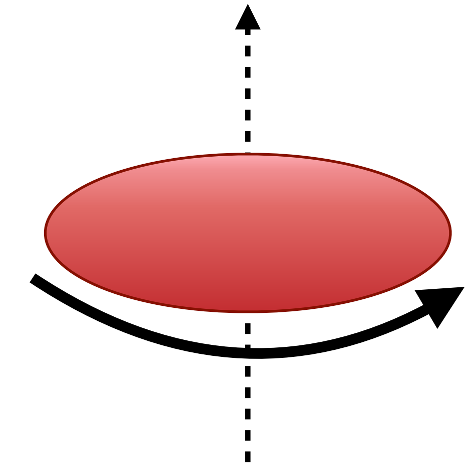
Ultracold atoms are neutral so can't use the coupling of the charge with external electromagnetic fields, e.g. Lorentz force $F_{\text{Lorentz}} = q\mathbf{v} \times \mathbf{B}$

Different mechanisms:

1. **Rotation**
2. **Dressed states**
3. **Tight-binding schemes**
4. **Laser-assisted tunnelling (internal states)**
5. **Synthetic dimensions**

	Symmetry			d							
	Time-reversal	Particle-hole	Chiral	1	2	3	4	5	6	7	8
A	0	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$

$$\mathbf{\Omega}_R = \Omega_R \hat{z}$$



1. Rotation

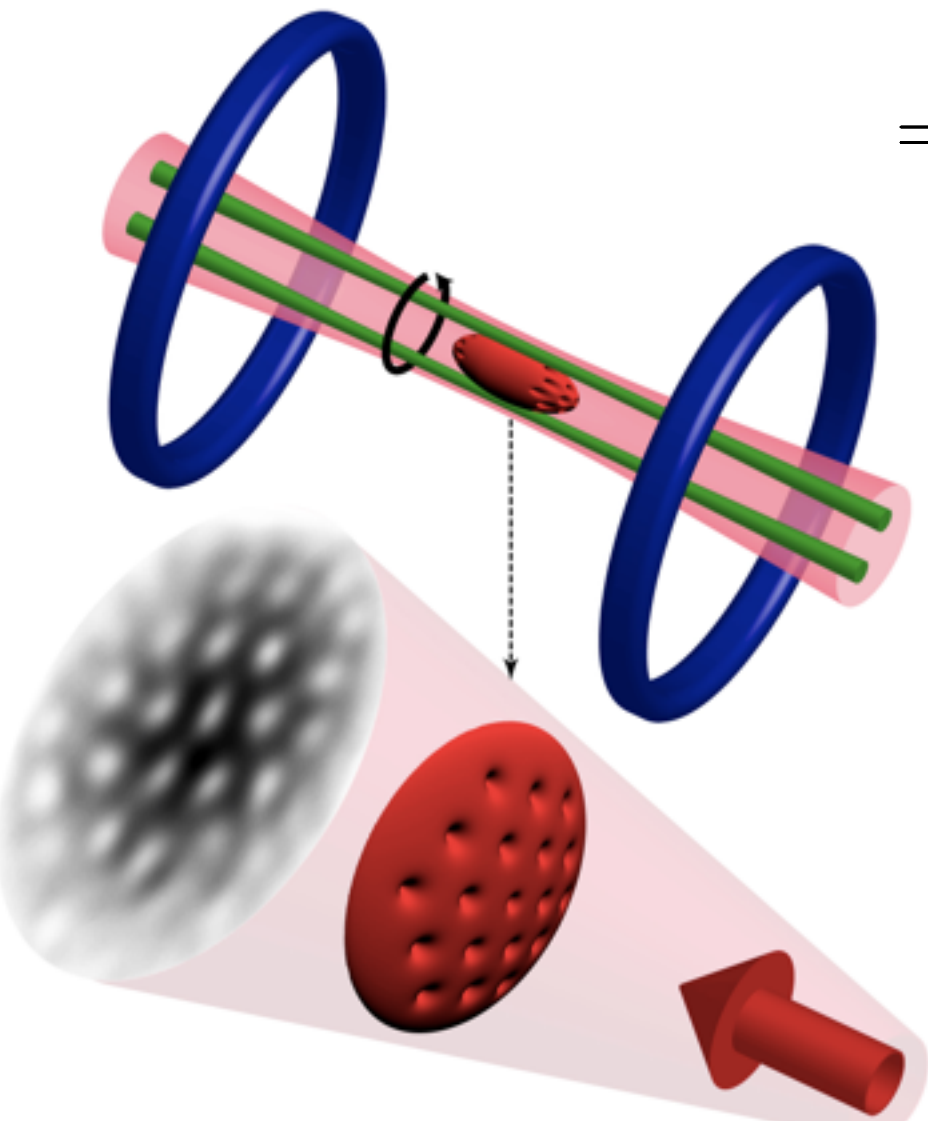
$F_{\mathrm{Lorentz}} = q\mathbf{v} \times \mathbf{B}$ mimicked in the rotating frame by $F_{\mathrm{Coriolis}} = 2M\mathbf{v} \times \mathbf{\Omega}_R$

A 2D gas in the rotating frame:

Harmonic trap — Rotation

$$\mathcal{H}_R = \frac{\mathbf{p}^2}{2M} + \frac{1}{2}M\omega_\perp^2(x^2+y^2) - \mathbf{\Omega}_R \cdot \mathbf{r} \times \mathbf{p}$$

$$= \frac{(\mathbf{p} - M\mathbf{\Omega}_R \times \mathbf{r})^2}{2M} + \frac{1}{2}M(\omega_\perp^2 - \Omega_R^2)(x^2+y^2)$$



$$q\mathbf{B}_{\mathrm{eff}} = 2M\mathbf{\Omega}_R$$

c.f.

$$\mathcal{H}_B = \frac{(\mathbf{p} - q\mathbf{A})^2}{2M}$$

"Centrifugal limit"

Landau level-like if $\Omega_R = \omega_\perp$

but atoms fly away if $\Omega_R > \omega_\perp$

$\Omega_R = 0.993\omega_\perp$ (Boulder)

Drawbacks:

- Artificial magnetic field *in rotating frame* so non-axisymmetric perturbations can lead to heating
- Hard to reach strongly-correlated regime $n_\phi = \dfrac{qB_{\mathrm{eff}}}{h} \ll n$

Figure from: http://cua.mit.edu/ketterle_group/experimental_setup/BEC_I.htm

Cooper, Adv. Phys. 57 539-616 (2008).

2. Dressed states

Remember what we talked about in Lecture 1

$$\gamma_n = \oint_{\mathcal{C}} \mathrm{d}\mathbf{R} \cdot \mathcal{A}_n(\mathbf{R}) = \int_{\mathcal{S}} \mathrm{d}\mathbf{S} \cdot \Omega_n(\mathbf{R})$$

Berry phase	**Berry connection**	**Berry curvature**
$\gamma_n = i \oint_{\mathcal{C}} \langle n(\mathbf{R})\vert \frac{\partial}{\partial \mathbf{R}} \vert n(\mathbf{R})\rangle \mathrm{d}\mathbf{R}$	$\mathcal{A}_n(\mathbf{R}) = i\langle n(\mathbf{R})\vert \frac{\partial}{\partial \mathbf{R}} \vert n(\mathbf{R})\rangle$	$\Omega_n(\mathbf{R}) = \nabla \times \mathcal{A}_n(\mathbf{R})$
Analogous to magnetic flux $\Phi = \int_{\mathcal{S}} \mathrm{d}\mathbf{S} \cdot \mathbf{B}(\mathbf{r})$	Analogous to a magnetic vector potential $\mathbf{A}(\mathbf{r})$	Analogous to a magnetic field $\mathbf{B}(\mathbf{r}) = \nabla \times \mathbf{A}(\mathbf{r})$

2. Dressed states

Concept: Engineer a real-space Berry curvature *to mimic a magnetic field*

General recipe:

1. Take an atom with N internal states (N>2)

2. Couple states with space-dependent fields (e.g. lasers)

atom-light coupling (in rotating-wave approx.)

$$H_{\mathrm{dress}}(\mathbf{r})|n(\mathbf{r})\rangle = E_n(\mathbf{r})|n(\mathbf{r})\rangle$$

N “dressed states”

3. Prepare the atom in a given dressed state $|l(\mathbf{r})\rangle$ and then let it move adiabatically

$$\Psi(\mathbf{r},t) = \sum_n \phi_n(\mathbf{r},t)|n(\mathbf{r})\rangle \qquad i\hbar\frac{\partial\Psi(\mathbf{r},t)}{\partial t} = \left(\frac{p^2}{2M} + H_{\mathrm{dress}}\right)\Psi(\mathbf{r},t)$$

After algebra (& using adiabaticity)

$$i\hbar\frac{\partial\phi_l(\mathbf{r},t)}{\partial t} = \left(\boxed{\frac{(p-\mathcal{A}_l(\mathbf{r}))^2}{2M}} + E_l(\mathbf{r}) + W(\mathbf{r})\right)\phi_l(\mathbf{r},t)$$

[“geometric scalar potential”]

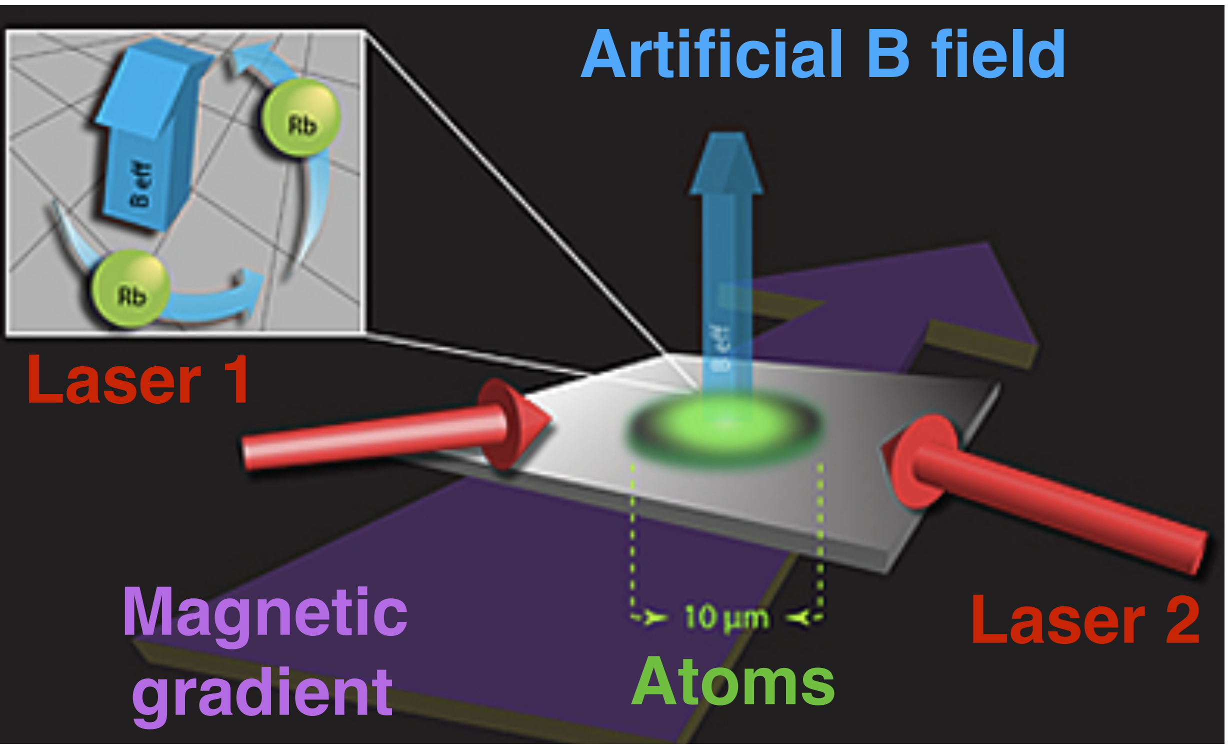
$$\mathcal{A}_l(\mathbf{r}) = i\langle l(\mathbf{r})|\frac{\partial}{\partial\mathbf{r}}|l(\mathbf{r})\rangle$$

Challenges: Atomic species? Adiabaticity? Lifetime? Heating? …. many schemes!

See e.g. review: Dalibard et al. Rev. Mod. Phys. 83, 1523 (2011).

2. Dressed states

Experiment: Spielman group (Maryland) Lin et al., Nature, 462, 628 (2009)



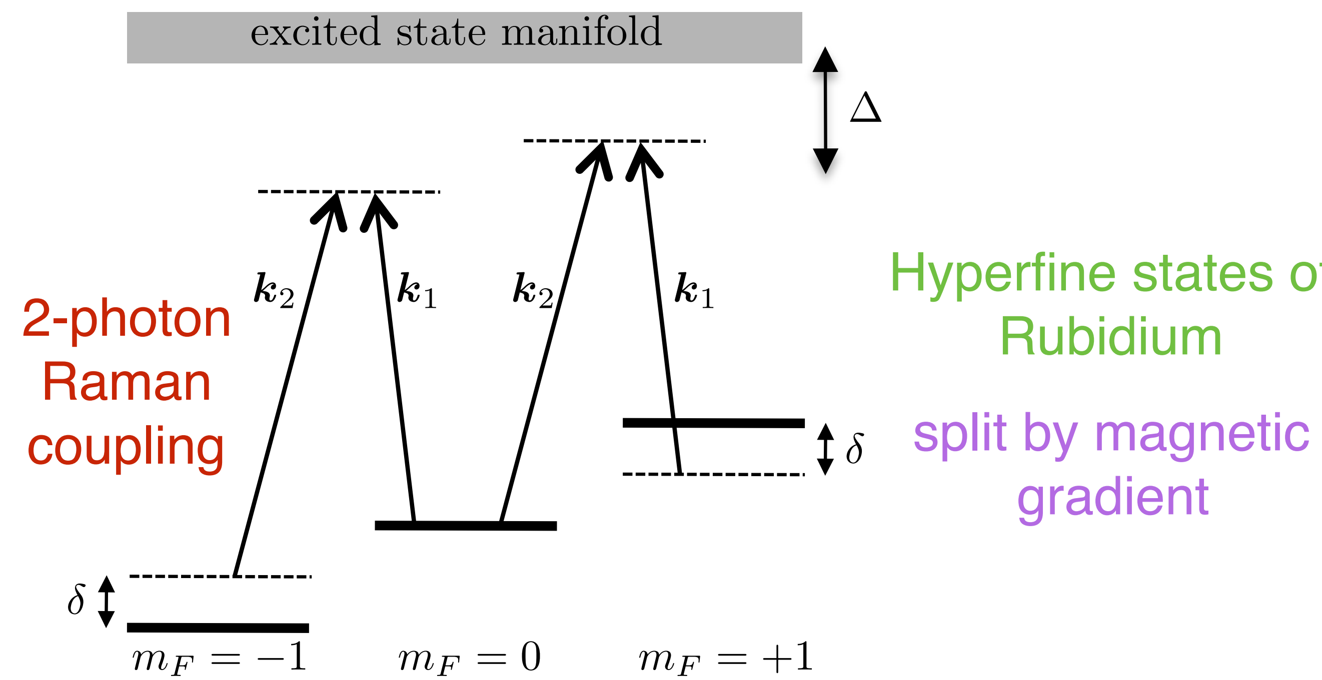
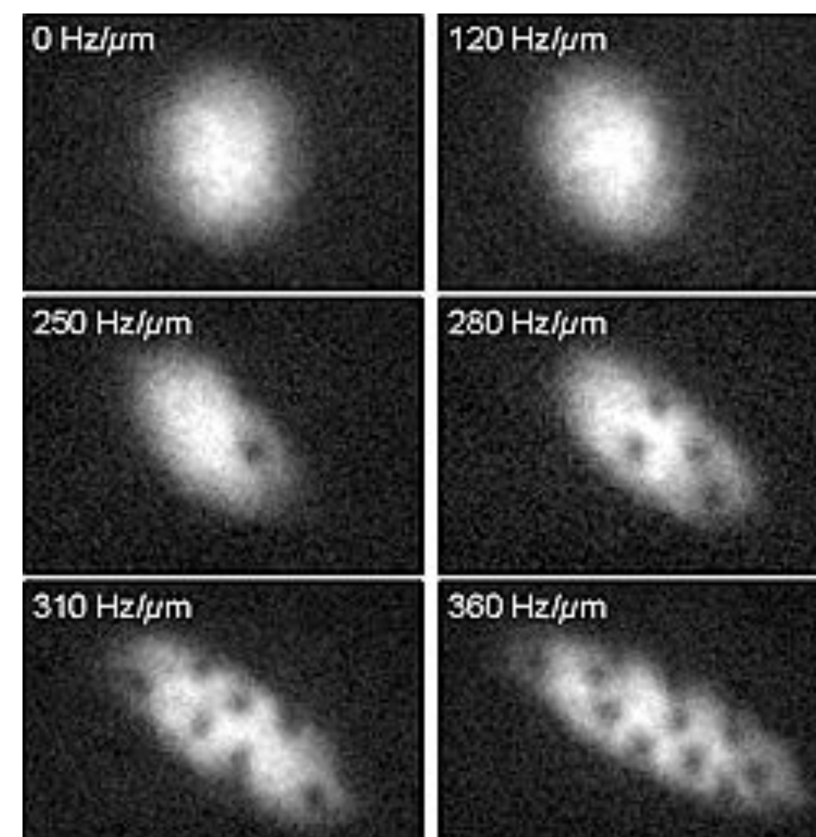


Figure from: https://www.nist.gov/news-events/news/2009/12/jqi-researchers-create-synthetic-magnetic-fields-neutral-atoms



- Limited by heating from photon scattering
- Hard to get high enough artificial magnetic flux to reach strongly-correlated regime

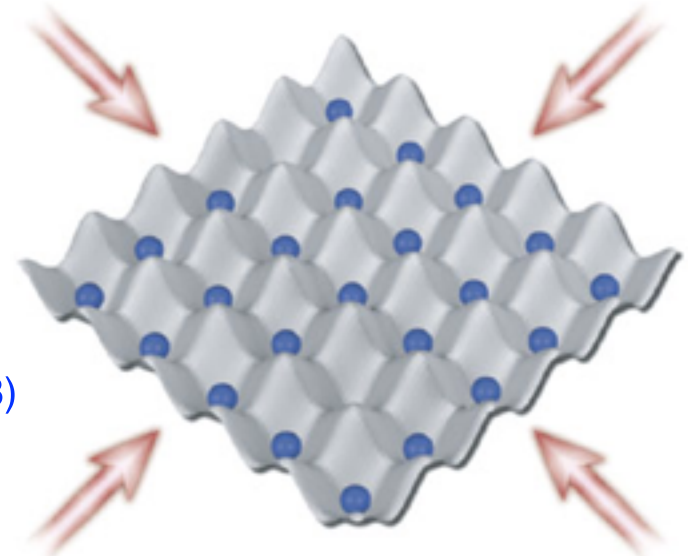
↓

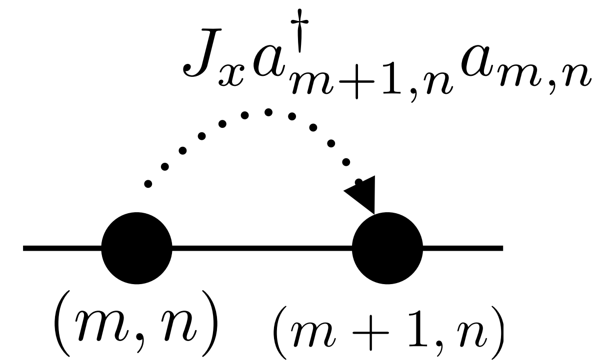
Could be overcome with the optical flux lattice schemes Cooper, PRL, 106, 175301 (2011) and following works…

Tight-binding lattice schemes

Deep optical lattices

Figure from
Bloch, Nature
453, 1016 (2008)





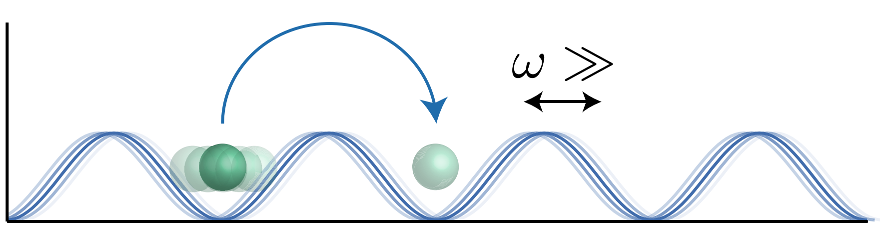
+ Peierls substitution

$$J_x \to J_x e^{i\theta^x_{m,n}}, \qquad \theta^x_{m,n} = -\frac{e}{\hbar}\int_{\mathbf{r}_{m,n}}^{\mathbf{r}_{m+1,n}} \mathbf{A}\cdot d\mathbf{x}$$

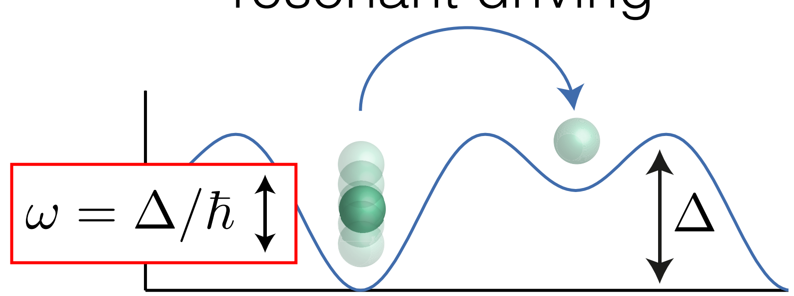
e.g.
Harper-Hofstadter model,
Haldane model…

How to engineer the right spatially-dependent Peierls phases?

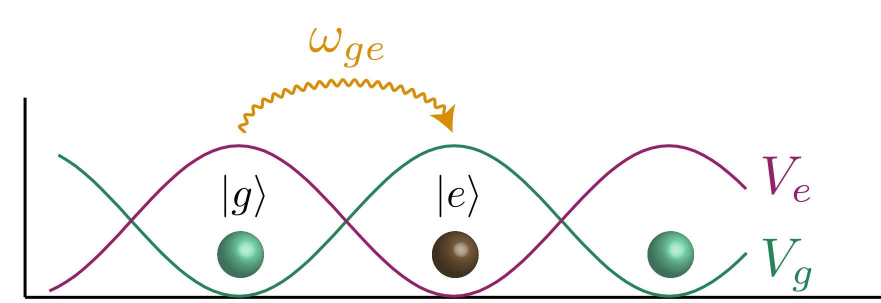
Shaking



Superlattices +
resonant driving



4. Laser-assisted tunnelling (internal states)



Figures from Goldman et al. arXiv:1507.07805

3. Floquet engineering

Very(!) brief intro to Floquet theory:

System modulated periodically in time $H = H_0 + V(t)$ (static: H_0; periodic driving: $V(t)$)

$V(t+T) = V(t)$

$T = 2\pi/\omega$

$$U(T) = \mathcal{T}\exp\left(-i\int_0^T dt H(t)\right)$$

Stroboscopic evolution captured by time-independent effective Hamiltonian:

$$U(T) = \exp\left(-iTH_{\text{eff}}\right)$$

H_0 and H_{eff} can be in **different** topological classes

Typically assume high-frequency driving ($\omega \gg$ all other frequencies) and then calculate effective Hamiltonian perturbatively, e.g. at lowest order:

$$H^{\text{eff}} = \frac{1}{T}\int_0^T H(t)\mathrm{d}t$$

Concept: Design driving to engineer an artificial magnetic field in the effective Hamiltonian

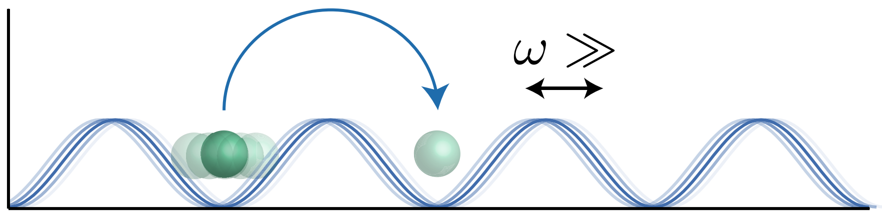
[N.B. Outside of high-frequency limit can have topology with no static analogue: *"anomalous Floquet topological systems"*]

For lots more about Floquet theory, see e.g:

M. Bukov et al. Advances in Physics, 64, 139, (2015)

N. Goldman et al., arXiv:1507.07805

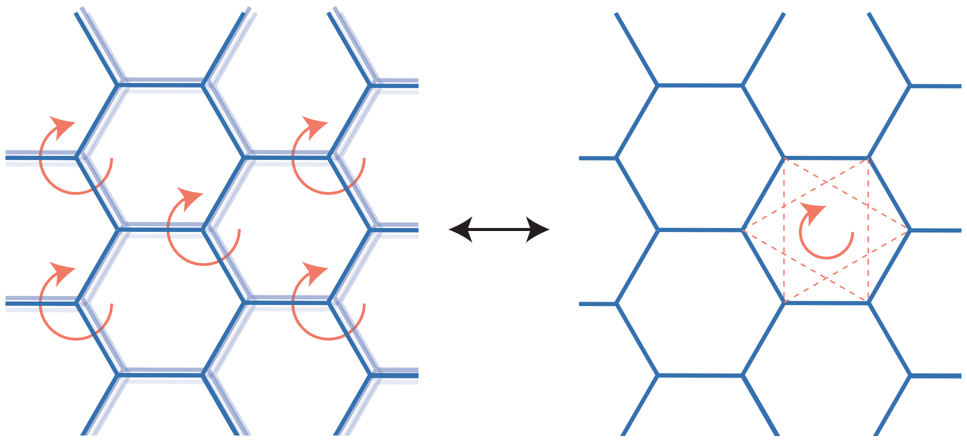
3. (A): Shaking

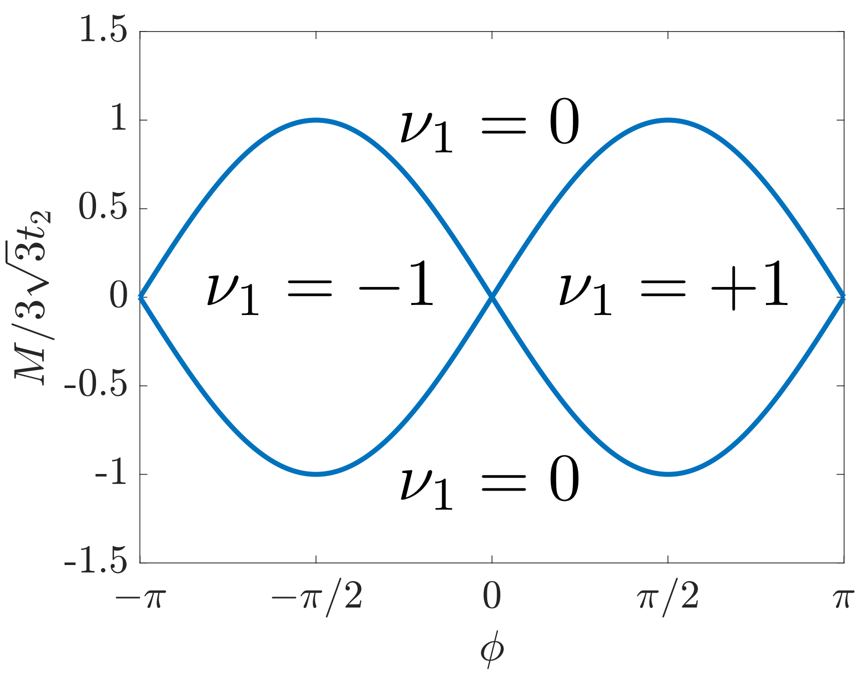


Shaking the lattice off-resonantly with high frequency.

The shaking renormalises the tight-binding hopping amplitudes in the effective Hamiltonian.

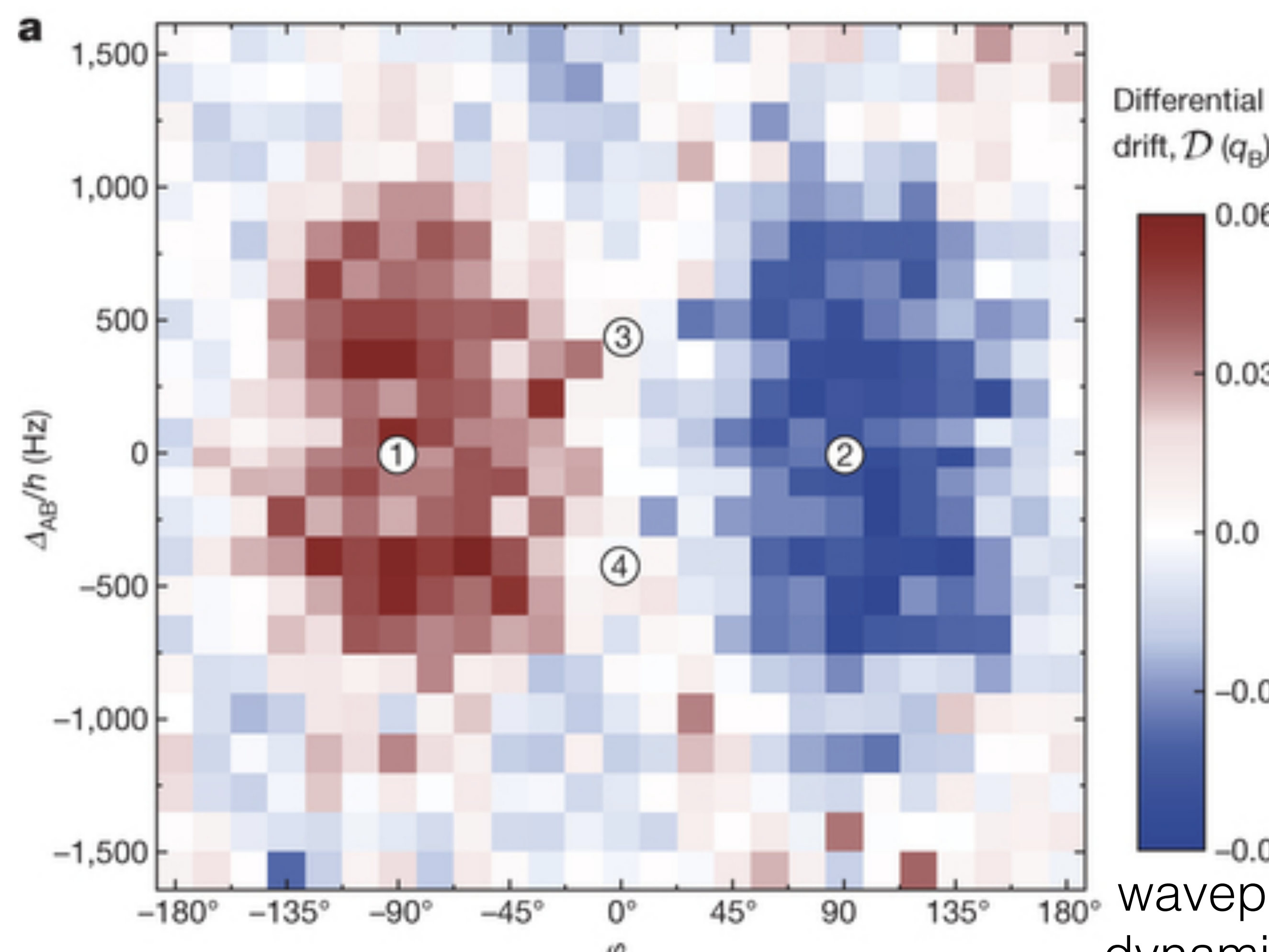
Example: circularly shaken hexagonal lattice —> Haldane model





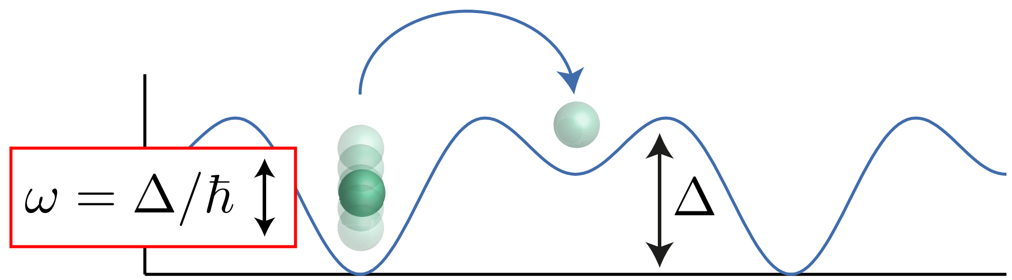
Topological phase diagram of Haldane model
(Zurich 2014)

Jotzu et al., Nature 515, 237 (2014)



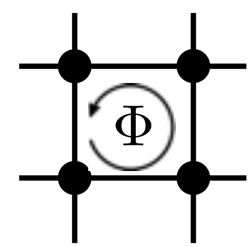
wavepacket dynamics

3. (B): Superlattices + resonant driving

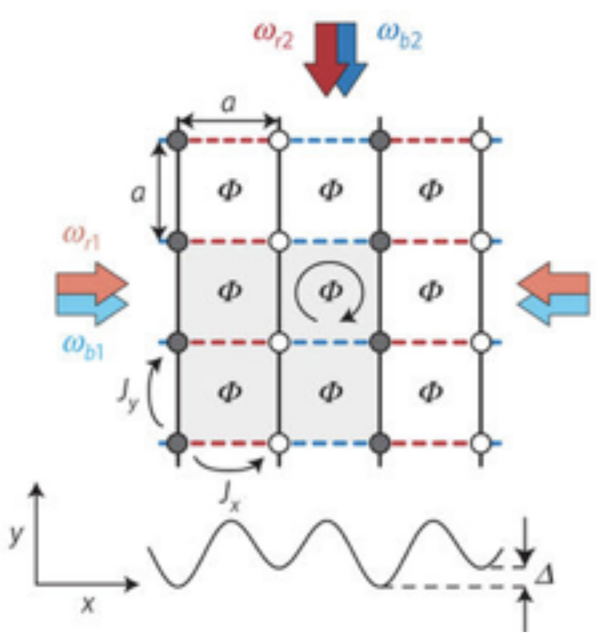


- Superlattice inhibits normal hopping processes.
- Resonant lasers turn back on tunnelling and control the hopping amplitudes in the effective Hamiltonian.

Example: Harper-Hofstadter Model



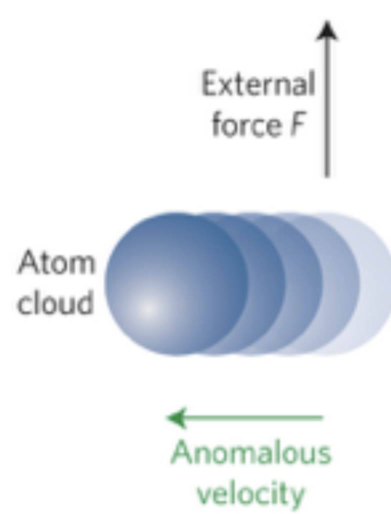
Munich: Aidelsburger et al., PRL, 111, 185301 (2013)
MIT: Miyake et al, PRL, 111, 185302 (2013)

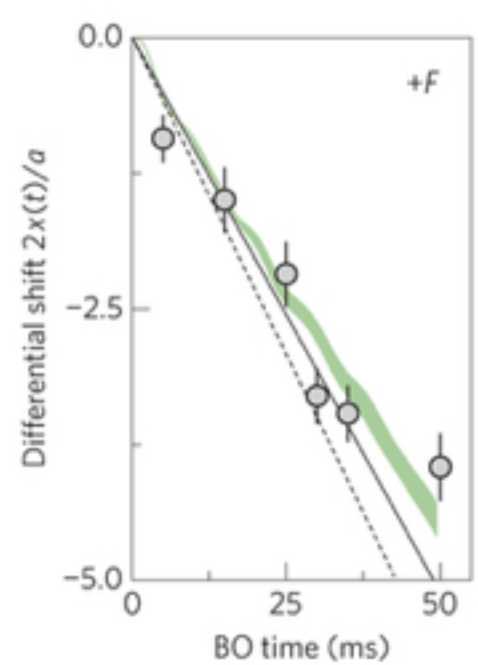


Peierls phase inherited from spatially-dependent driving phases

Measurement of First Chern Number from dynamics (Munich, 2015)

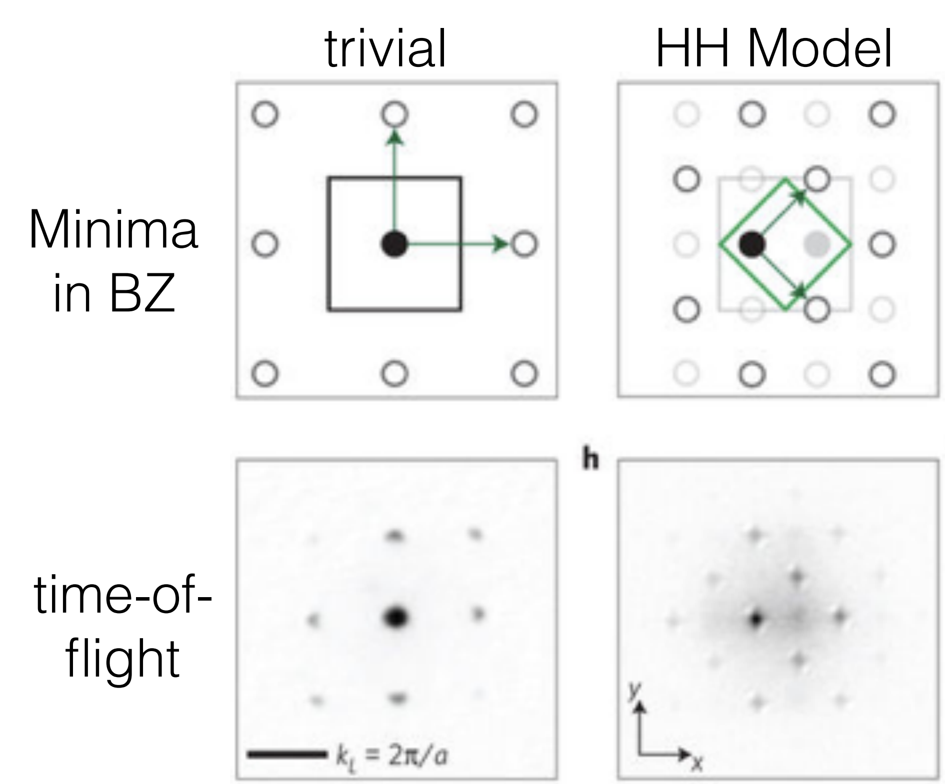
Aidelsburger et al., Nat. Phys, 11,162 (2015)





Bose-Einstein condensate in the HH model (MIT, 2015)

Kennedy et al., Nat. Phys, 11, 859 (2015)



Also chiral currents in HH ladders: (Munich, 2014) Atala et al. Nat. Phys., 10, 558, (2014)

3. Strong correlations?

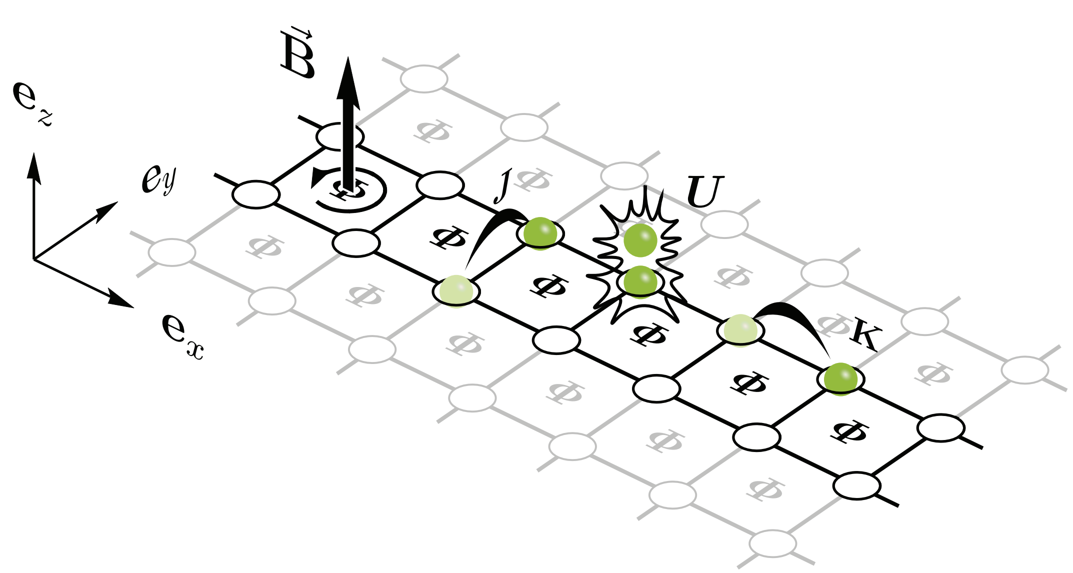
High magnetic flux densities: ✓
~ a flux quantum per plaquette

Low particle densities: ✓
~ a particle per plaquette (e.g. Mott insulator)

But still big challenges:

- How to reduce excitations due to driving?
- How to reduce temperature? e.g. typical topological band-gap $\Delta \sim 10\mathrm{n}K$
- How to adiabatically-prepare a strongly-correlated topological state starting from the initial topologically-trivial system (before turning on driving)?

Recent first step?

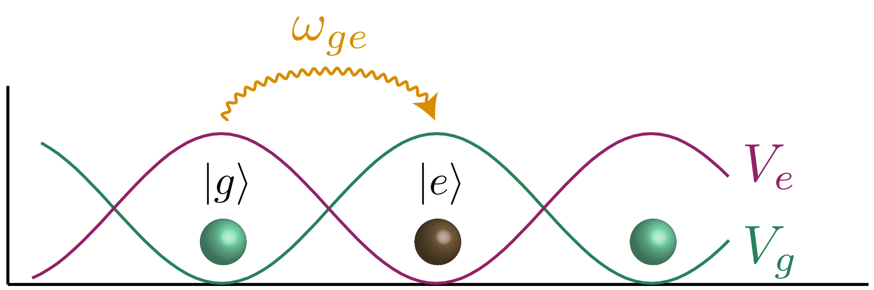


Harvard: Tai et al., arXiv:1612.05631

Strong interactions in the few-body limit of a Harper-Hofstadter ladder

Combined high resolution imaging with complex Peierls phases realised through superlattice + resonant driving approach

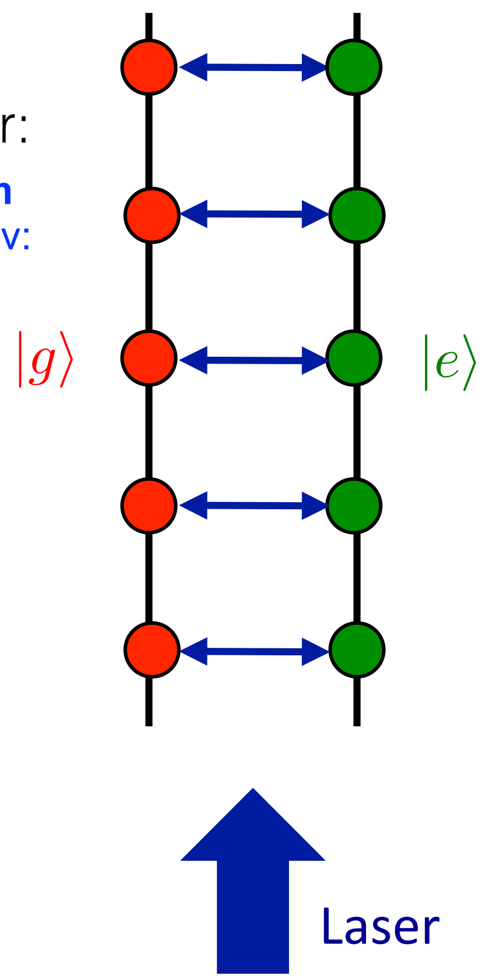
4. Laser-assisted tunnelling (internal states)



- Different internal states so no NN hopping processes.
- Lasers restore tunnelling by resonantly coupling the states. [c.f. 3b]

Top-view of a ladder:

Figure from Dalibard, arXiv: 1504.05520



Many schemes for different configurations / internal states etc….

Good reviews: Dalibard et al. Rev. Mod. Phys. 83, 1523 (2011), Goldman et al., Rep. Prog. Phys. 77, 126401 (2014), Dalibard, arXiv:1504.05520…

e.g.
laser phase $\phi = ky$
gets "printed on" the effective tunnelling amplitudes
[NB in a 2D lattice, need additional tricks to rectify the flux]

An important realisation:

Different internal states don't need to be on different lattice sites

instead can view states themselves as a "synthetic dimension"

Boada et al., PRL, 108, 133001 (2012), Celi et al., PRL, 112, 043001 (2014)

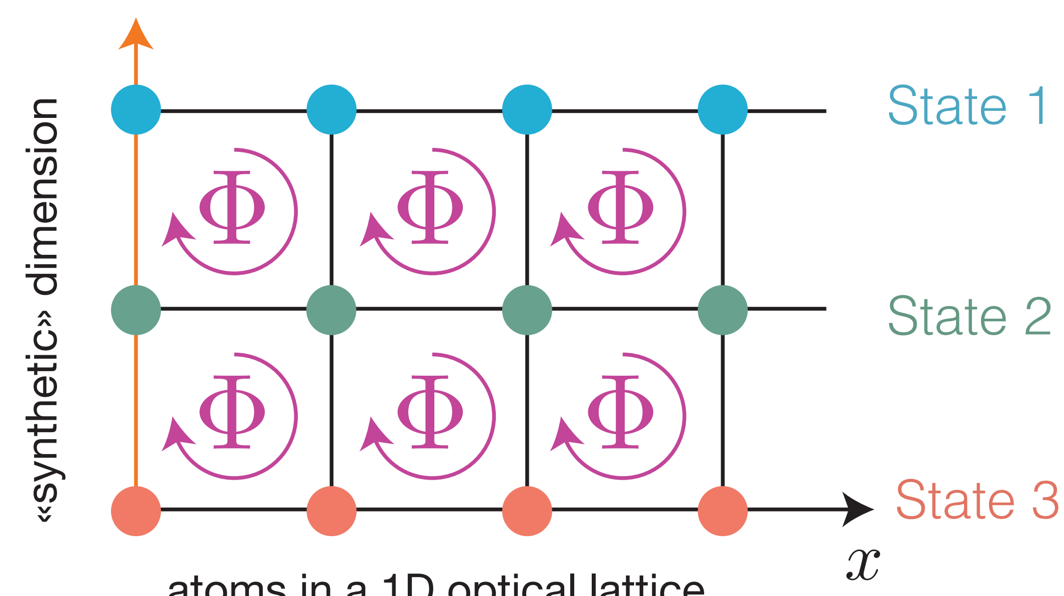
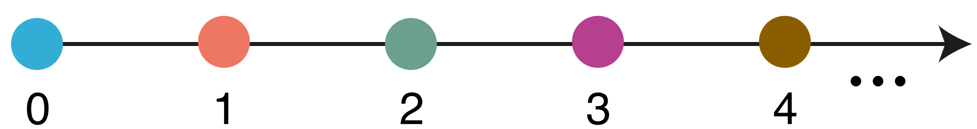


Figure from Goldman et al., Nat. Phys. 12, 639 (2016)

5. Synthetic dimensions

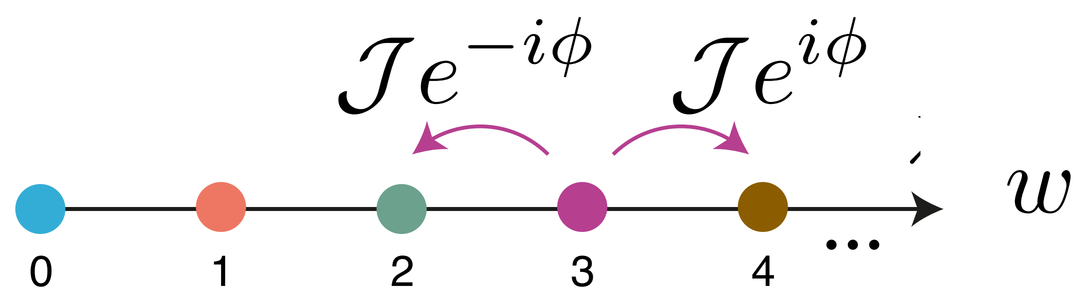
Concept:

1. Identify a set of states and reinterpret as sites in a synthetic dimension

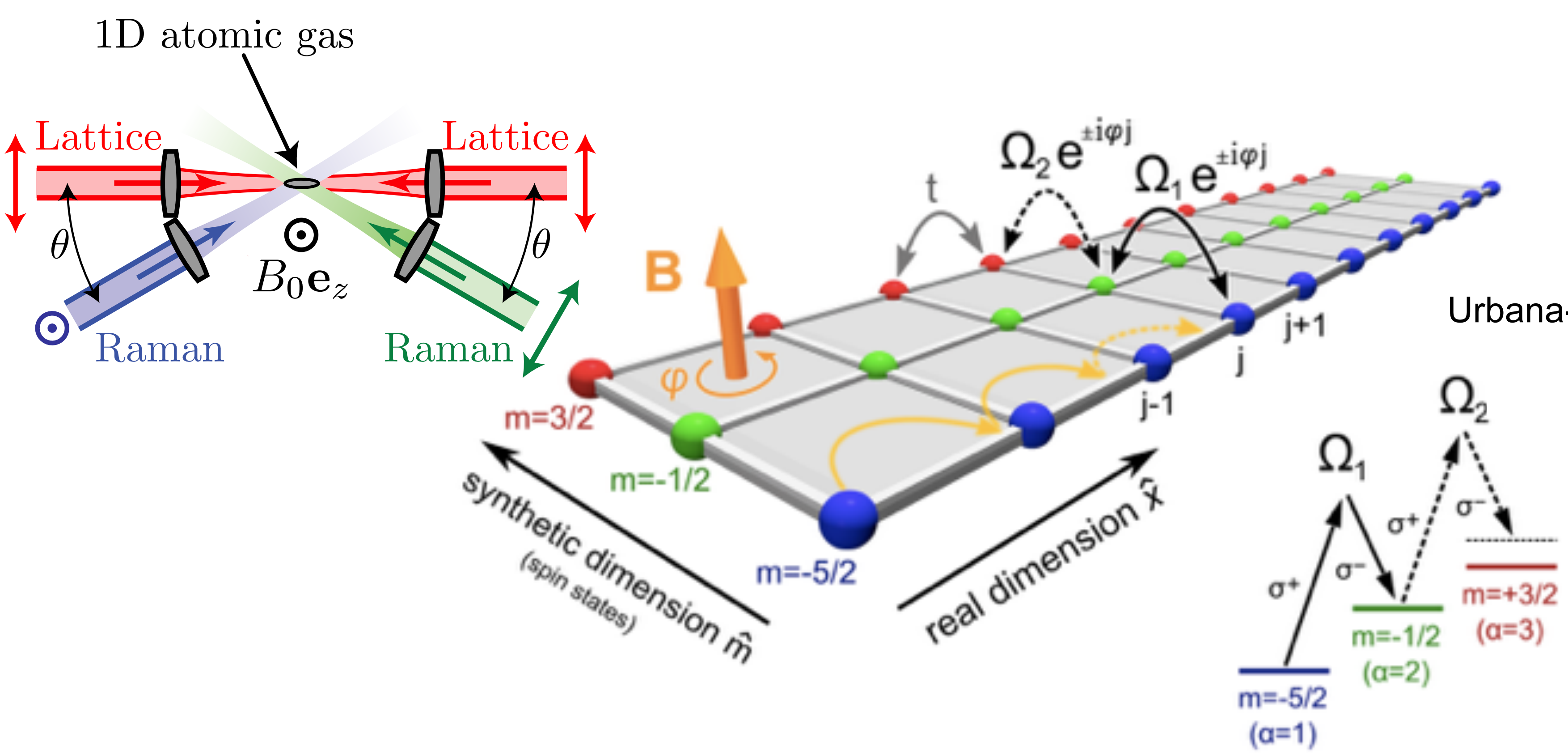


First proposed by:
Boada et al., PRL, 108, 133001 (2012),
Celi et al., PRL, 112, 043001 (2014)

2. Couple these modes to simulate a tight-binding “hopping”



3. Combine with real spatial dimensions or more synthetic dimensions as desired



Using internal atomic states:
Florence: Mancini et al, Science, 349, 1510 (2015)
Livi et al, Phys. Rev. Lett. 117, 220401 (2016)
Maryland: Stuhl et al. Science, 349, 1514 (2015)
Boulder: Kolkowitz et al, Nature, 542, 66 (2017)

Using discrete momentum states:
Urbana-Champaign: Alex et al., Sci. Adv. 3, e1602685 (2017)

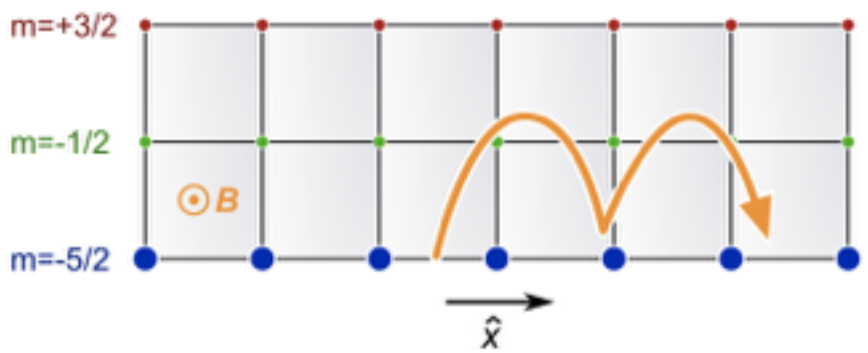
Using harmonic trap states:
Theory: Price et al, PRA 95, 023607 (2017)

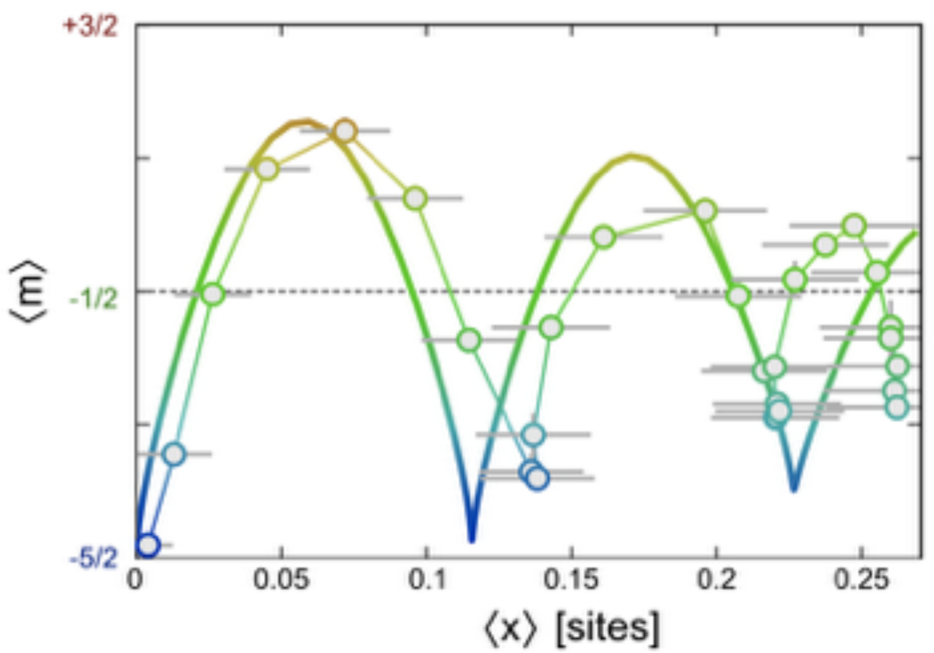
And in photonics
(Lecture 4)

5. Synthetic dimensions

Why is this an interesting approach?

- Probing edge physics



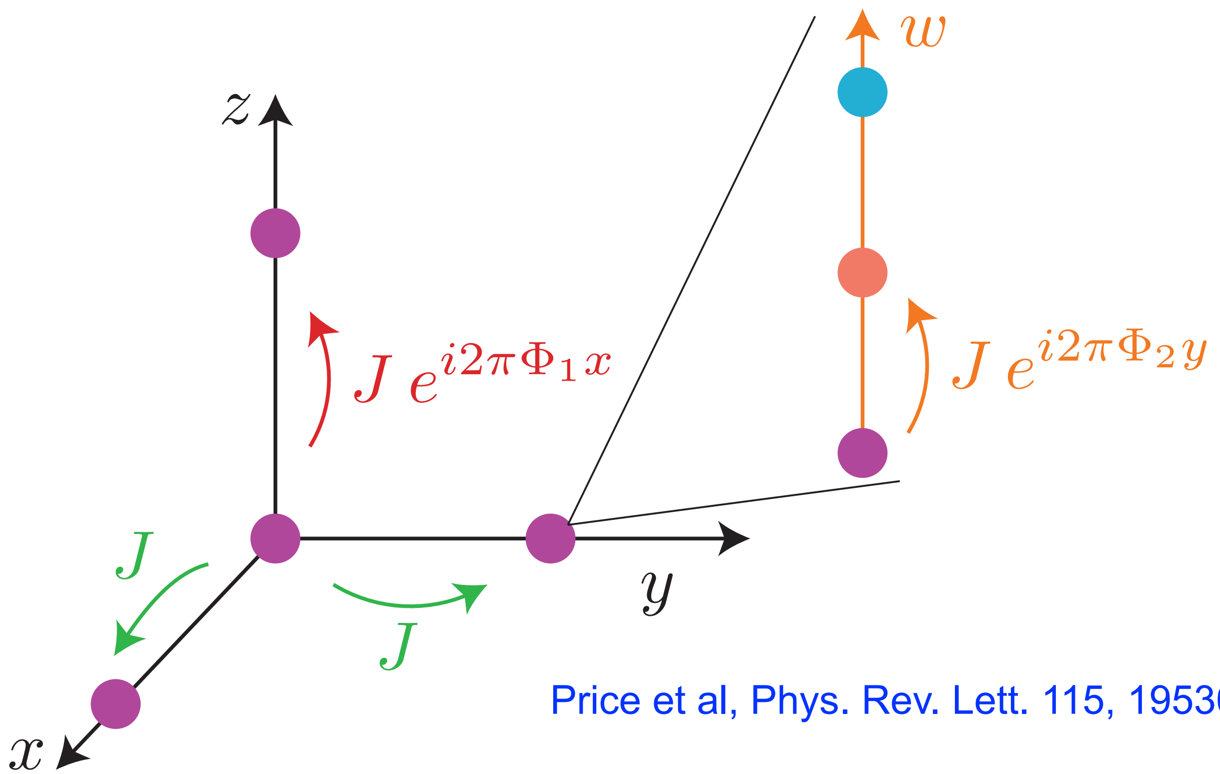


Using internal atomic states:
Florence: Mancini et al, Science, 349, 1510 (2015)
Maryland: Stuhl et al. Science, 349, 1514 (2015)

N.B. Chiral currents in real-space HH ladders
Munich: Atala et al. Nat. Phys., 10, 558, (2014)

- Unusual interactions — What are the interactions like in terms of the synthetic dimension? FQH possible?

- Higher dimensions!
  e.g. 4D quantum Hall effect



Price et al, Phys. Rev. Lett. 115, 195303 (2015)

$$j_x = \frac{q^3}{h^2} E_y B_{zw} \sum_{n \in occ.} \nu_n^{(2)}$$

[c.f. Lecture 1]

Lecture 3

- How can we engineer topology for cold atoms?
    - SSH Model & Topological Pumps
    - Quantum Hall systems
    - **Quantum spin Hall systems & topological superfluids**

- How can we probe topology with cold atoms?

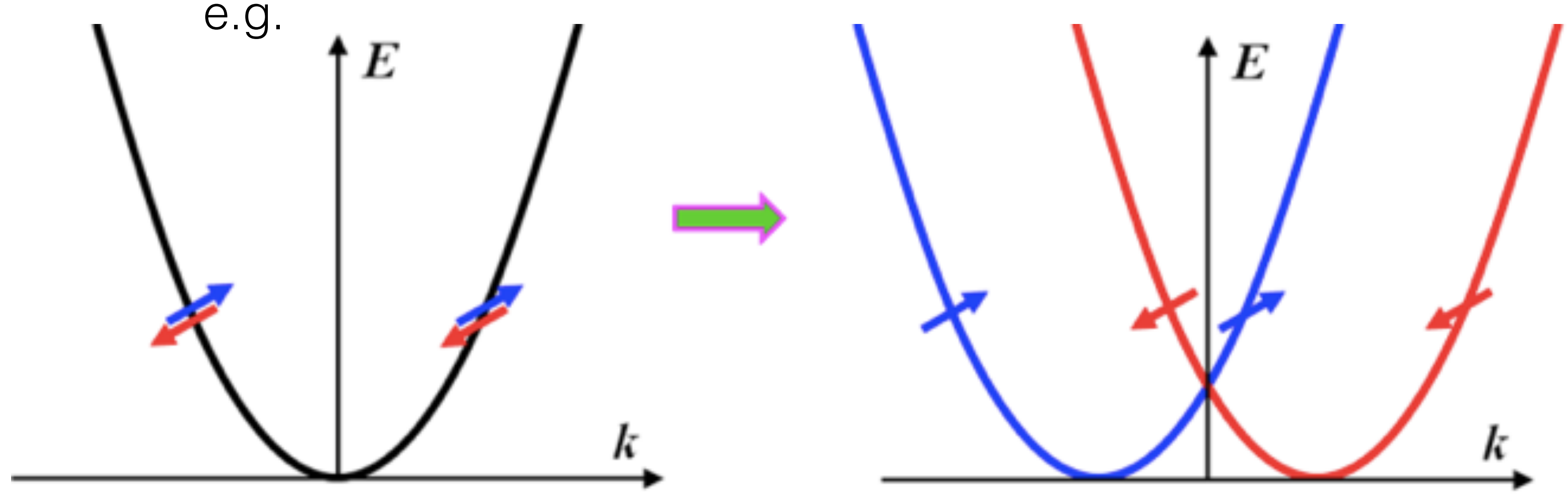
- Future perspectives

Spin-orbit coupling

$$\hat{H} = \frac{\left(\hat{\boldsymbol{p}} - \hat{\boldsymbol{A}}(\hat{\boldsymbol{r}})\right)^2}{2M} + \dots$$

now gauge field is ensemble of Pauli matrices, not just a vector

e.g.



In ultracold atoms:

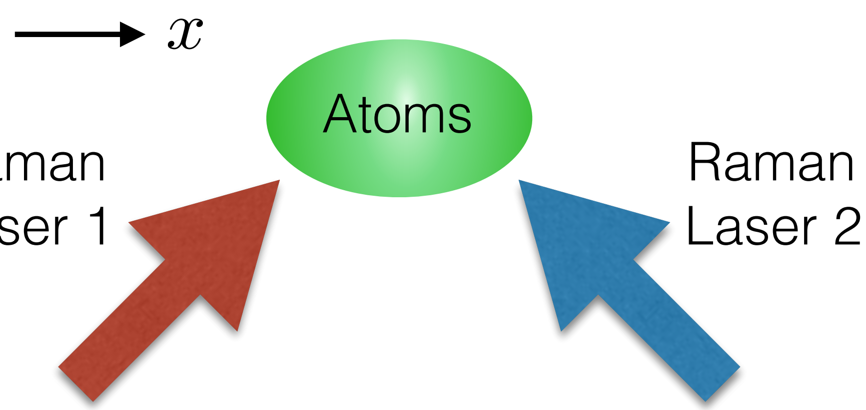
1. Choose two internal atomic states to act like “spin-up” and “spin-down”

2. Couple motional states to the internal states of the atom

[NB these artificial non-Abelian gauge fields can be richer than usual solid-state SO-coupling]

See e.g. :
Dalibard et al. Rev. Mod. Phys. 83, 1523 (2011).
Zhai, Int. J. Mod. Phys. B 26, 1230001 (2012)
Goldman et al., Rep. Prog. Phys. 77, 126401 (2014)

1D Spin-Orbit Coupling



Raman transition gives atoms a momentum kick along x

$$\hat{H}_0 = \begin{pmatrix} \frac{k_x^2}{2m} + \frac{\delta}{2} & \frac{\Omega}{2} e^{2ik_0x} \\ \frac{\Omega}{2} e^{-2ik_0x} & \frac{k_x^2}{2m} - \frac{\delta}{2} \end{pmatrix},$$

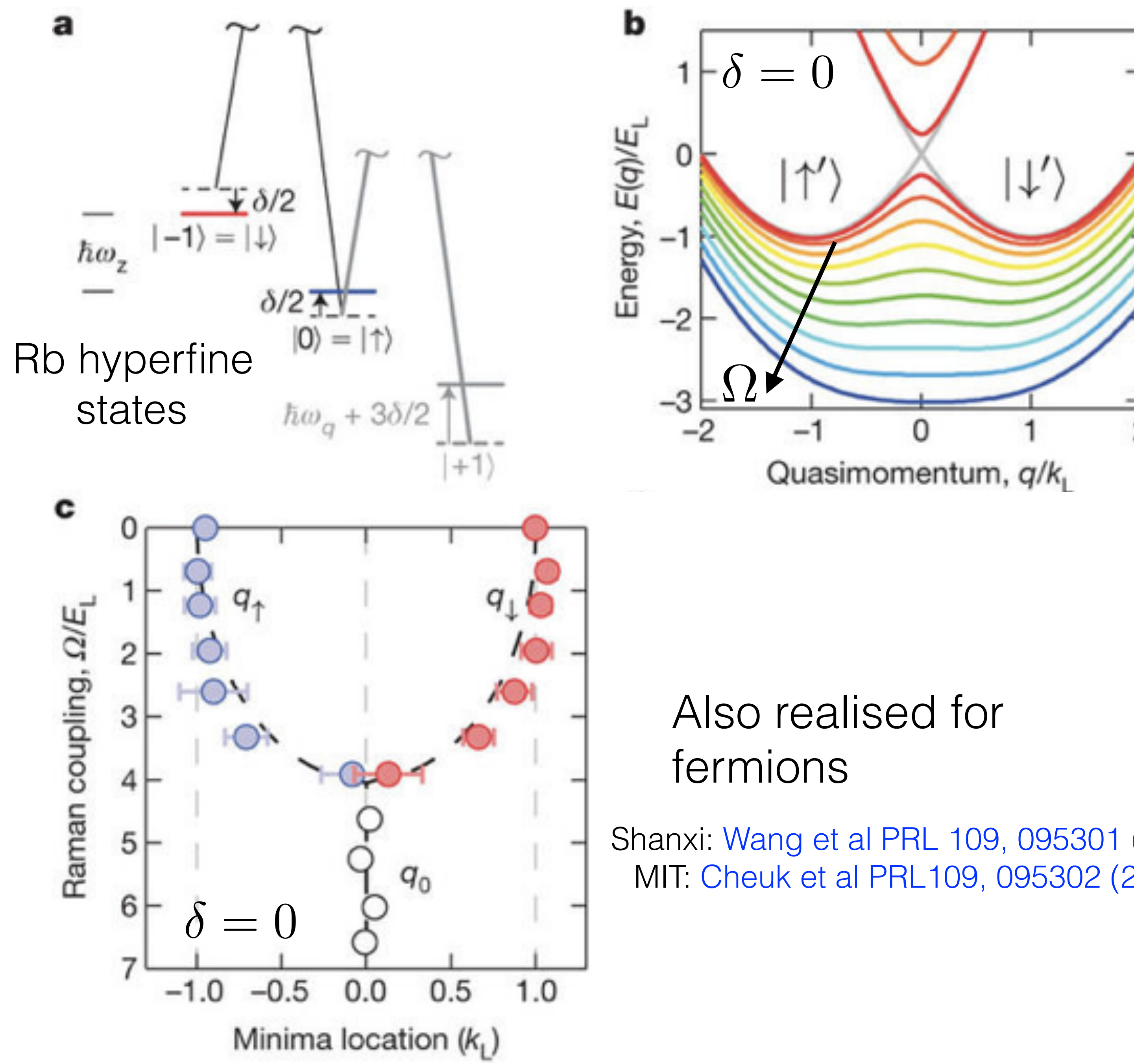
Resulting Hamiltonian (after some tricks):

$$\hat{H}_0 = \frac{(k_x - k_0\sigma_x)^2}{2m} - \frac{\delta}{2}\sigma_x + \frac{\Omega}{2}\sigma_z,$$

Effective Zeeman terms

Equal mixture of “Rashba” and “Dresselhaus” SO-coupling

Pioneered in Maryland experiment: Lin, et al. Nature 471, 83 (2011).



Also realised for fermions

Shanxi: Wang et al PRL 109, 095301 (2012),
MIT: Cheuk et al PRL109, 095302 (2012).

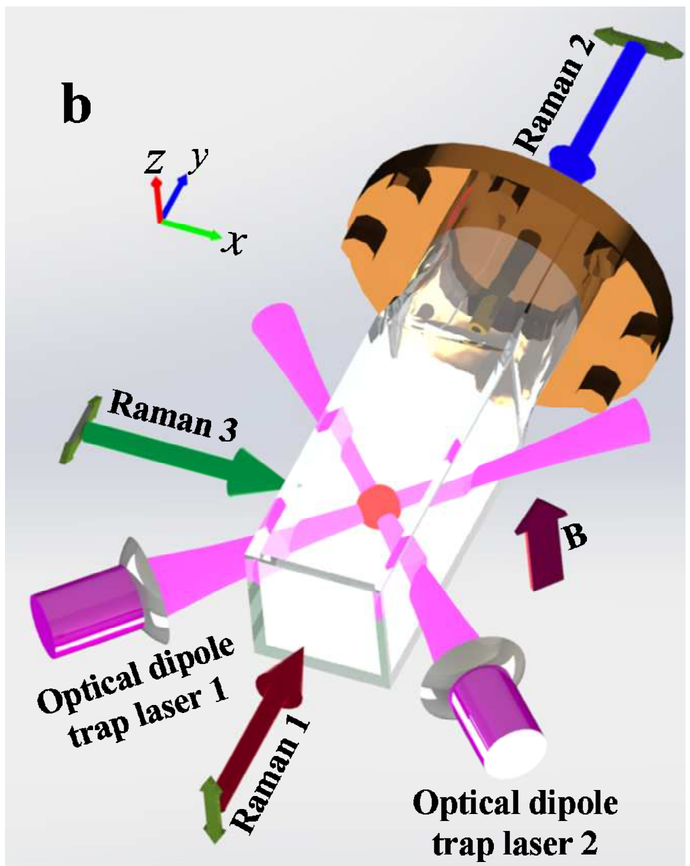
[N.B. Essentially same scheme as synthetic dimension with 2 internal states]

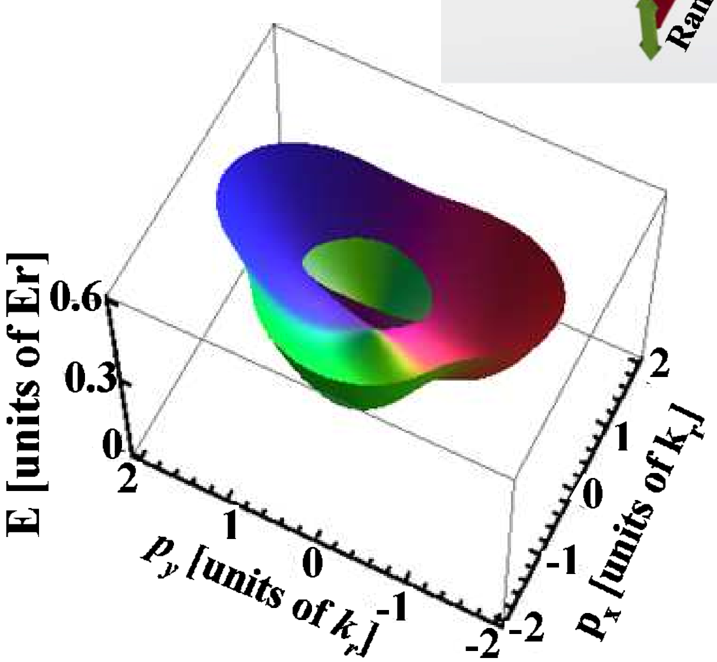
2D Spin-Orbit Coupling

First realization for fermions:

Shanxi: Huang, et al. Nat. Phys. 12, 540 (2016)

3 Raman lasers coupling 3 internal states



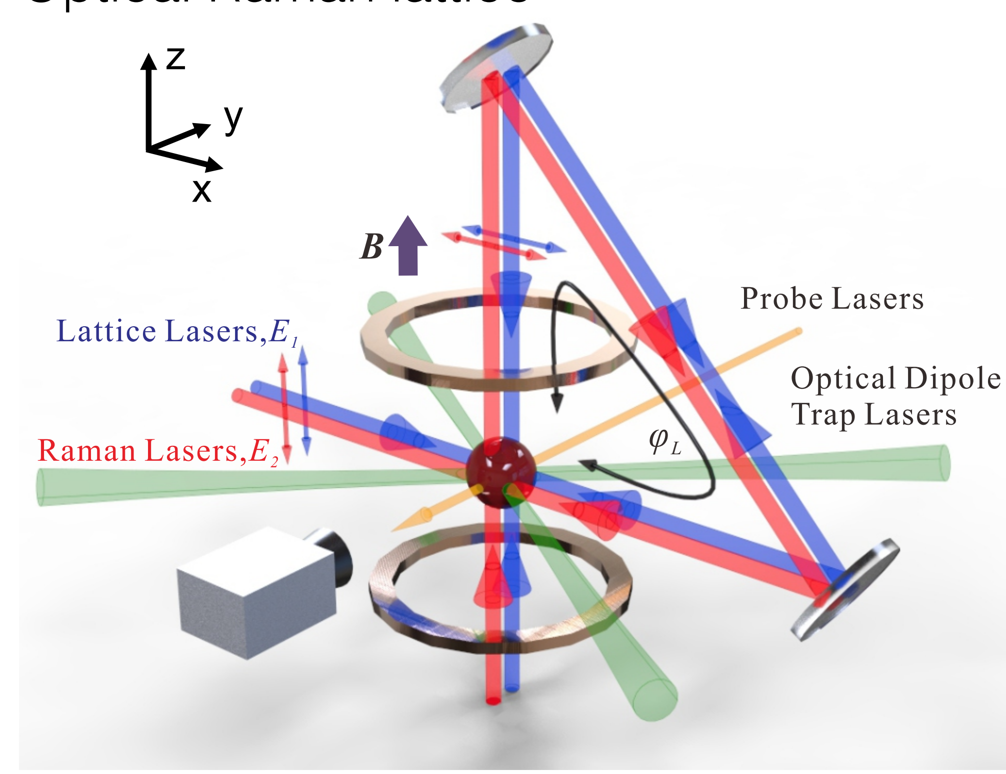


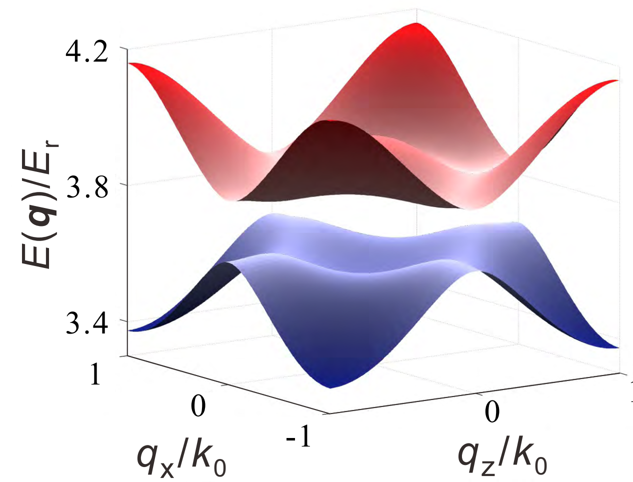
SO coupling for lowest two dressed states

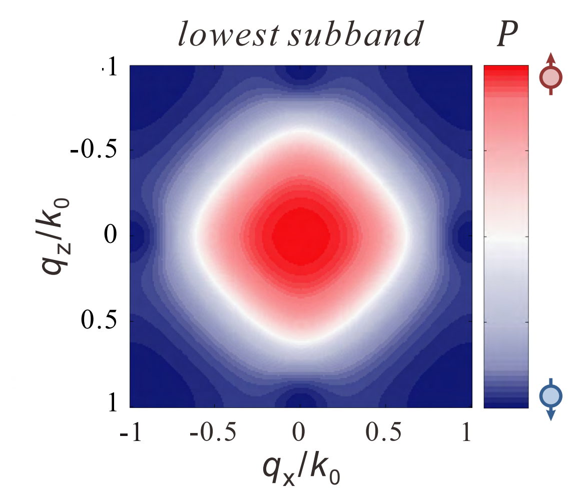
First realization for bosons:

Shanghai: Wu, et al. Science 354, 6308 (2016).

Optical Raman lattice







[NB breaks TRS]

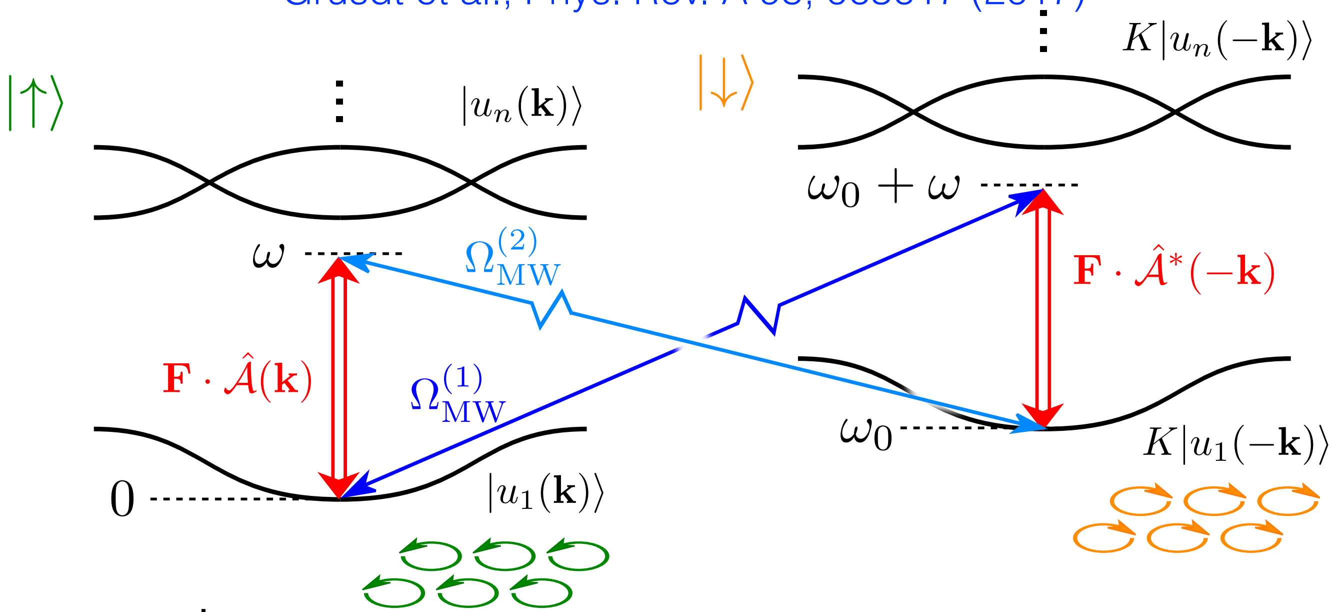
Towards topological insulators

Ingredients:

- Opposite effective magnetic fields for opposite spins
- Spin-orbit coupling that doesn't break TRS

still experimentally challenging!

Grusdt et al., Phys. Rev. A 95, 063617 (2017)



Spin-flipping terms : microwave pulse
+ near-resonant lattice shaking
(Floquet engineering)

Proposal:

identical time-reversed band structures
with opposite Chern numbers

Other proposals e.g.:

- Atomic chip

  Goldman et al., Phys. Rev. Lett. 105, 255302 (2010)

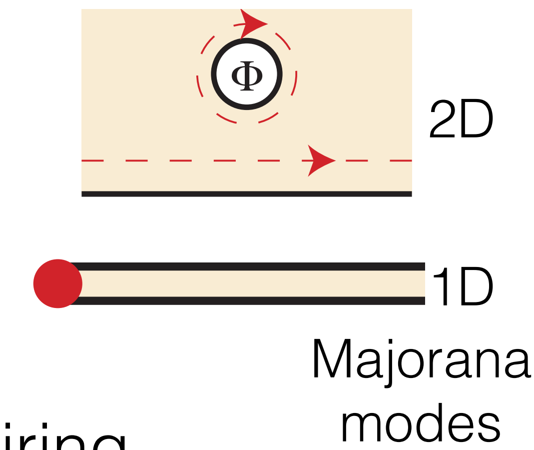
- Optical flux lattices

  Béri et al, Phys. Rev. Lett. 107, 145301 (2011)

	Symmetry			*d*							
	Time-reversal	Particle-hole	Chiral	1	2	3	4	5	6	7	8
AII	-1	0	0	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$

Towards topological superfluids

Symmetry				d							
	Time-reversal	Particle-hole	Chiral	1	2	3	4	5	6	7	8
D	0	1	0	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$

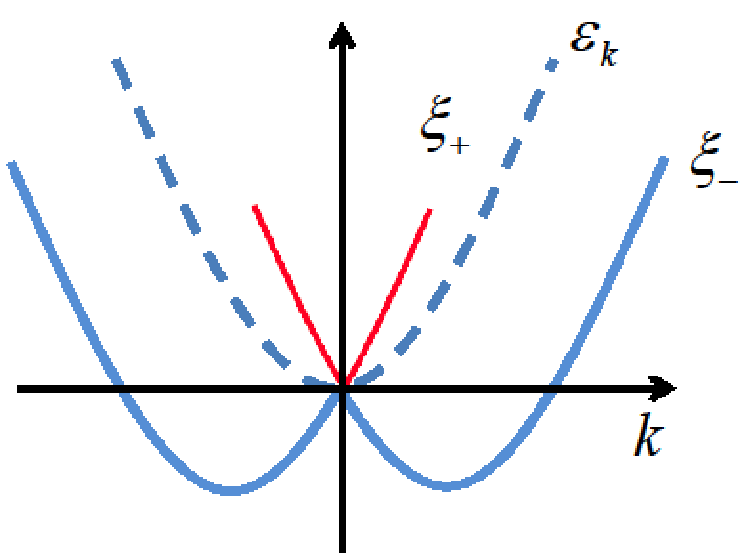


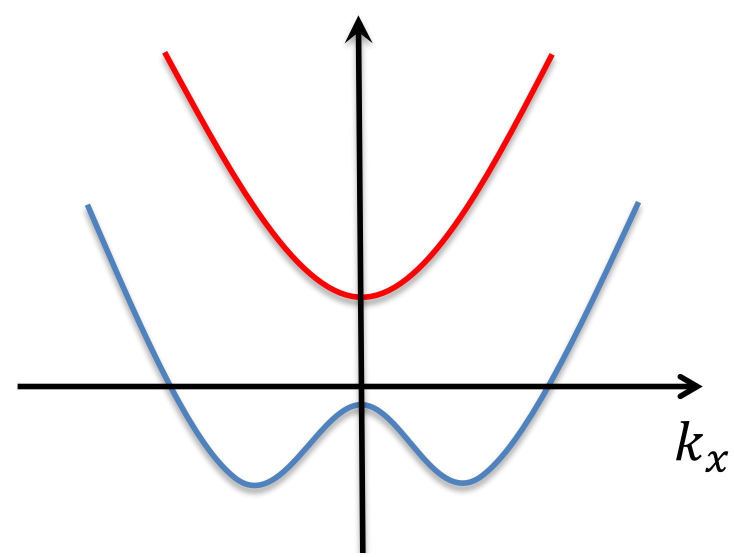
Simple models: fermionic superconductor/superfluid with p-wave pairing

Various proposals for how to get p-wave superfluids in cold atoms, including:

Zhang et al., PRL. 101, 160401 (2008), Jiang et al. PRL 106, 220402 (2011)…

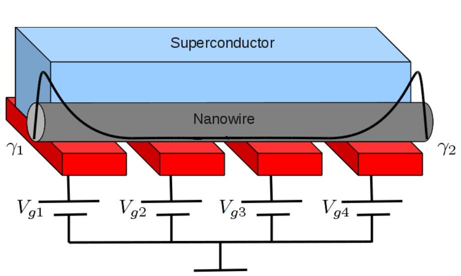
Spin-orbit coupling + Zeeman field + s-wave interactions





↓

SO-coupling can turn s-wave pairing to p-wave pairing



+ Zeeman field
$\Delta\sigma_z$
(Breaks TRS)

Put interacting fermions into Shanghai experiment?
Wu, et al. Science 354, 6308 (2016).

Lecture 3

- How can we engineer topology for cold atoms?
  - SSH Model & Topological Pumps
  - Quantum Hall systems
  - Quantum spin Hall systems & topological superfluids

- **How can we probe topology with cold atoms?**

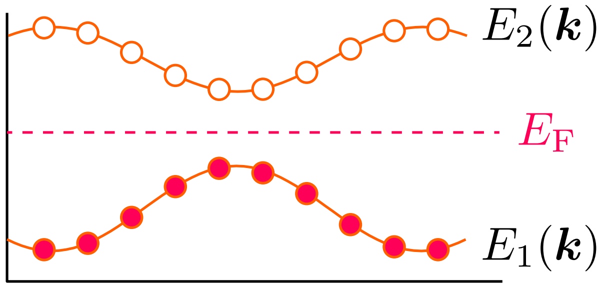
- Future perspectives

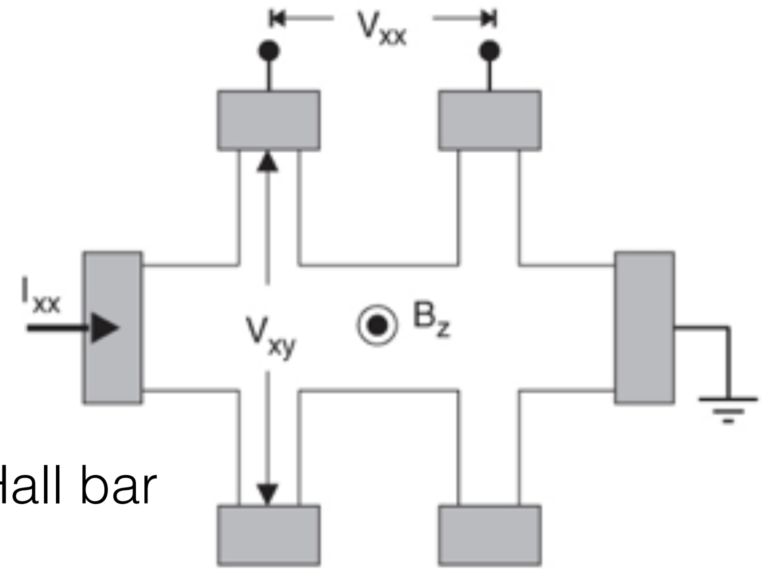
Probing Topology with Cold Atoms

1. Measuring topological bulk invariants through transport

in solid state:

electrons fill bands up to Fermi level





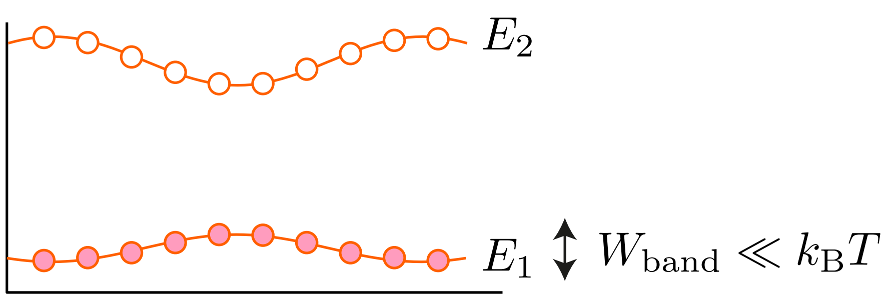
Hall bar

measurements of *currents* and *voltages*

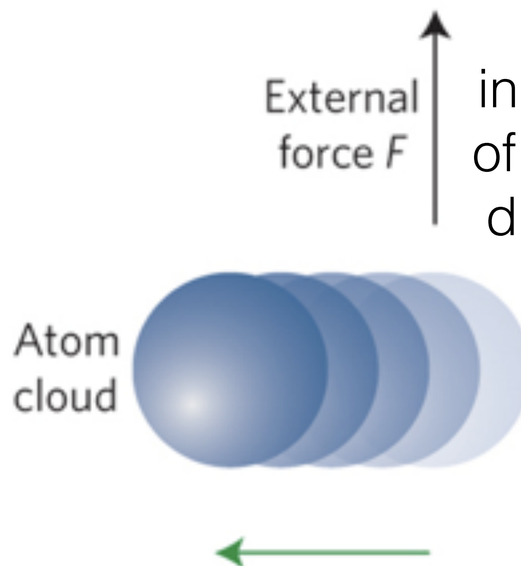
$$\sigma_{xy} = -\frac{e^2}{h} \sum_{n \in \text{occupied}} \nu_n$$

in cold atoms:

- fermions fill bands up to Fermi level
- bosons uniformly populate bands, e.g. due to temperature

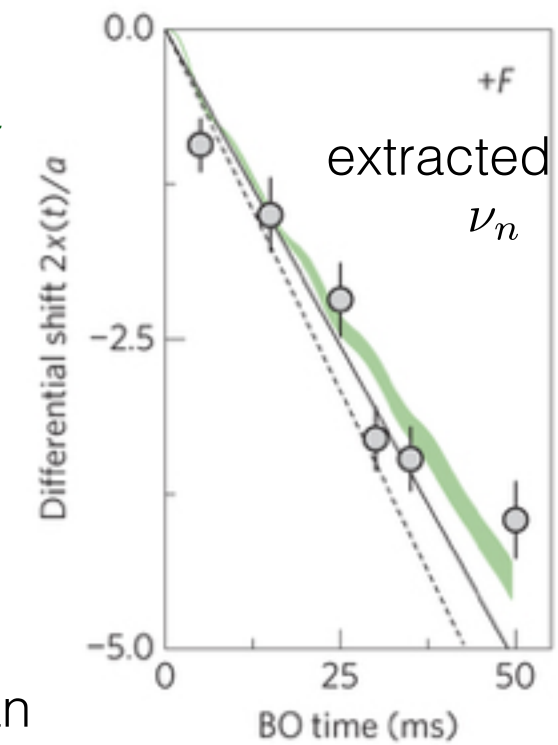


- semiclassical wavepacket : probes local geometrical Berry curvature



in-situ measurements of *center-of-mass drift* due to *external force*

Munich: Aidelsburger et al., Nat. Phys, 11,162 (2015)



$$\mathbf{v}_\mathrm{COM} = \frac{\mathbf{j}}{n}$$

N.B. particle density can depend on topology too

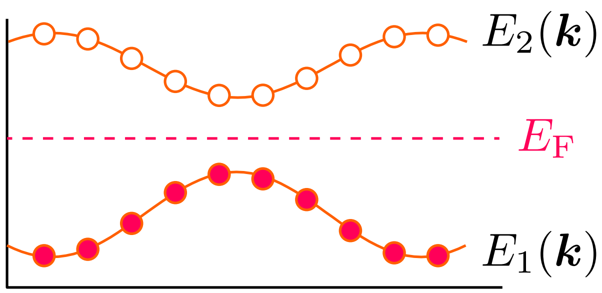
Price et al., Phys. Rev. B 93, 245113 (2016)

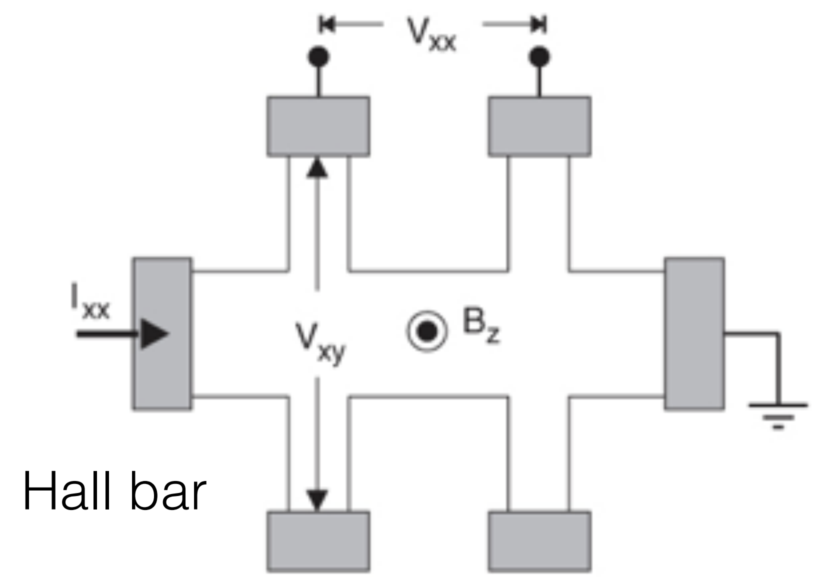
Probing Topology with Cold Atoms

1. Measuring topological bulk invariants through transport

in solid state:

electrons fill bands up to Fermi level



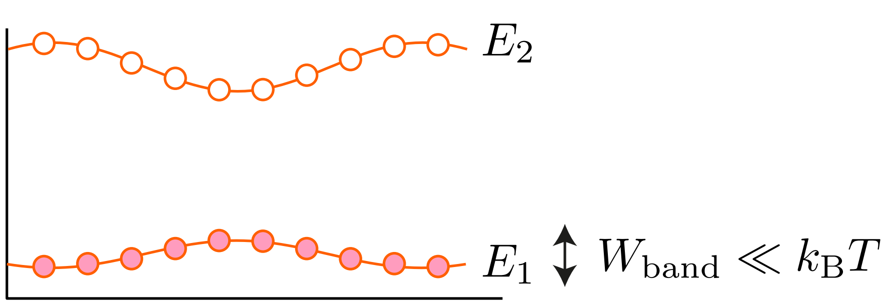


measurements of *currents* and *voltages*

$$\sigma_{xy} = -\frac{e^2}{h} \sum_{n \in \mathrm{occupied}} \nu_n$$

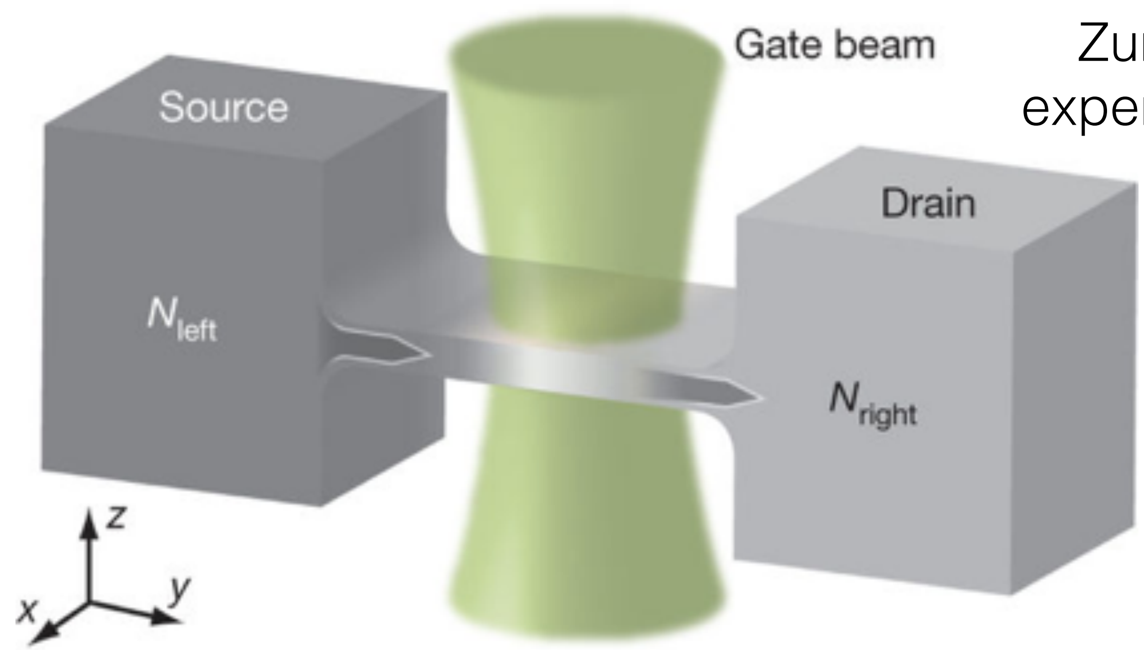
in cold atoms:

- fermions fill bands up to Fermi level
- bosons uniformly populate bands, e.g. due to temperature



- semiclassical wavepacket : probes local geometrical Berry curvature

Future perspective: measure currents for a topological system between two atomic reservoirs?



Zurich experiment

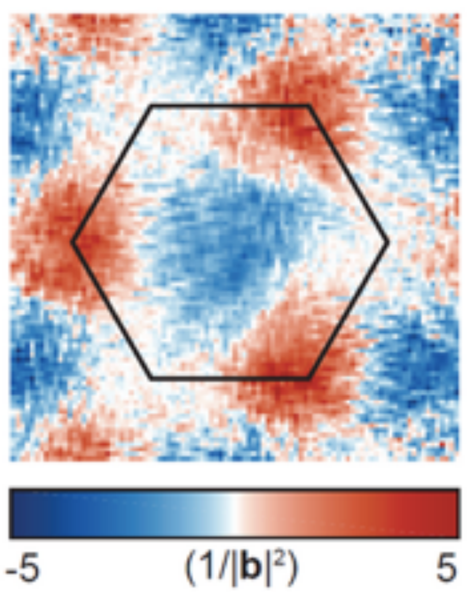
Figure from: Stadler et al., Nature 491, 736, (2012)

Probing Topology with Cold Atoms

2. Measuring topological bulk invariants in new ways (c.f. solid-state systems)

Time-of-flight measurements

Hamburg experiment

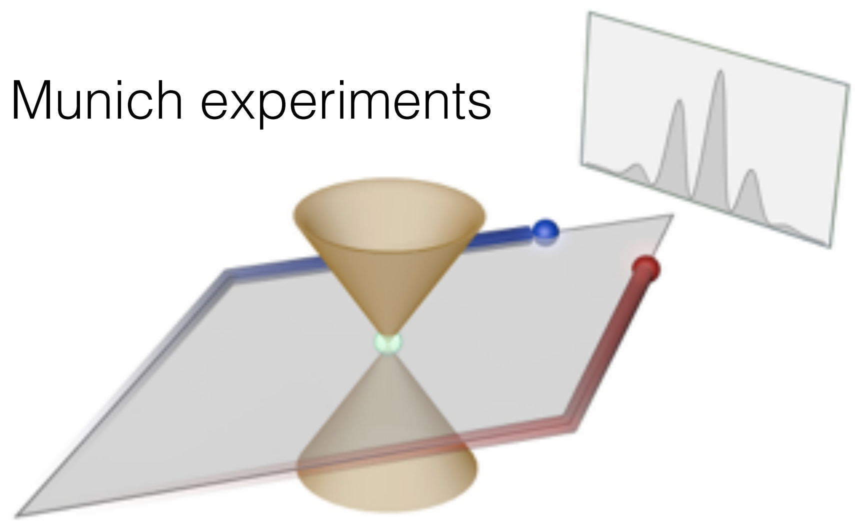


Use time-of-flight to determine eigenstates — from these can reconstruct geometrical and topological properties

Flaschner et al., Science 352, 1091 (2016)

Interferometric measurements

Munich experiments



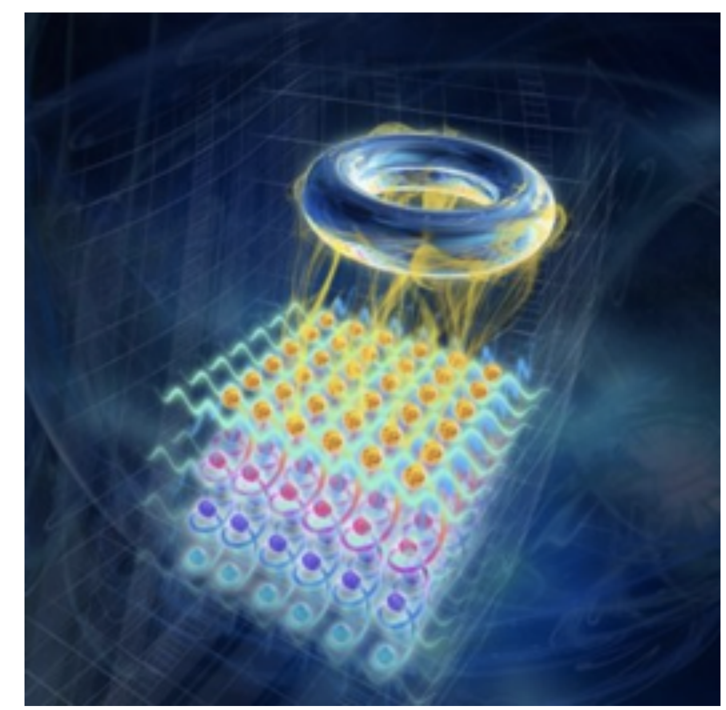
https://www.quantum-munich.de/media/aharonov-bohm-interferometer/

Interference due to Berry flux enclosed in momentum-space (analogous to Aharanov-Bohm)

Duca et al. Science 347, 288–292 (2015)

Also:
Zak phase: Atala et al, Nat. Phys. 9, 795 (2013)
Wilson loops: Li et al., Science 352, 1094 (2016)

Recent proposal: **Heating rates**



Credit: IQOQI Innsbruck / Harald Ritsch

Heating rate (i.e. rate of transfer to excited bands) due to shaking can be related to topological band invariants

Tran et al., Sci. Adv. 3, 8, e1701207 (2017)

Probing Topology with Cold Atoms

3. Probing topological edge states

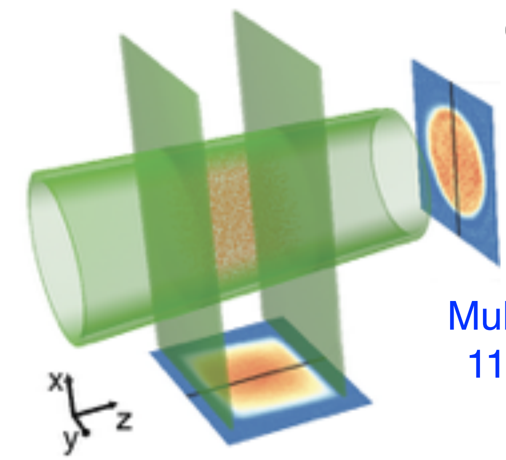
Challenges:

- Ultracold gases are typically in a harmonic trap —> "soft" confinement affects edge states
- Many more atoms in bulk than in edge states : how to isolate the edge signal?

Use synthetic dimensions [see earlier in the lecture]

Maybe use a box trap?

Cambridge experiment

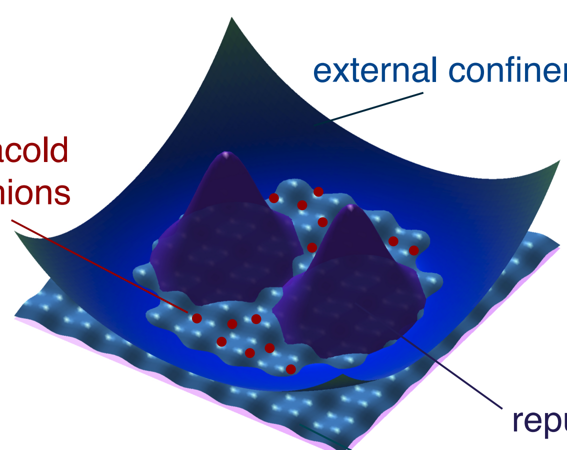


Mukherjee et al., PRL 118, 123401 (2017)

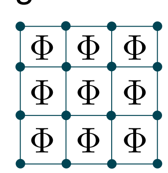
Make a topological interface

Goldman et al., Phys. Rev. A 94, 043611 (2016)

Density dynamics after a quench

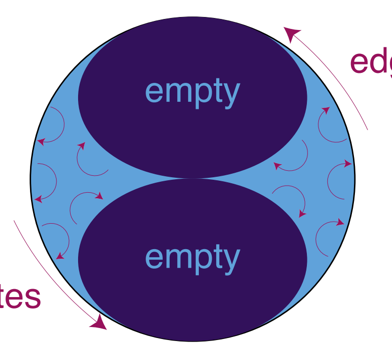


+ synthetic uniform magnetic flux

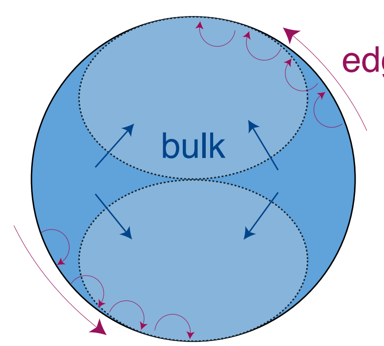


Goldman et al., PNAS 110(17) 6736-6741 (2013)

Equilibrium at t=0 (with walls)



After walls removal



Other proposals, e.g.:

- Spectroscopic imaging

  e.g. Goldman et al.PRL, 108, 255303, (2012).

- Dynamical instability of edge states

  Galilo et al., PRL 115, 245302 (2015)

Lecture 3

- How can we engineer topology for cold atoms?
    - SSH Model & Topological Pumps
    - Quantum Hall systems
    - Quantum spin Hall systems & topological superfluids
- How can we probe topology with cold atoms?
- **Future perspectives**

Future Perspectives

- What about using dissipation to engineer topology?

$$\mathrm{d}\hat{\rho}/\mathrm{d}t = i\left[\hat{\rho}, \hat{H}\right] + \sum_j \left(\hat{L}_j \hat{\rho} \hat{L}_j^\dagger - \frac{1}{2}\left\{\hat{L}_j^\dagger \hat{L}_j, \hat{\rho}\right\}\right)$$

Review: Goldman et al., Nat. Phys. 12, 639 (2016)

- What about dynamical gauge fields? (building connections to QED & QCD)

See e.g. reviews:
Goldman et al., Rep. Prog. Phys. 77, 126401 (2014).
U.-J. Wiese, Annalen der Physik 525, 777 (2013).

- Can we engineer new topological phases of matter, e.g. in higher dimensions?

- Strongly-correlated topological states of cold atoms? Majoranas?

Lecture 3

- How can we engineer topology for cold atoms?
  - SSH Model & Topological Pumps
  - Quantum Hall systems
  - Quantum spin Hall systems & topological superfluids

- How can we probe topology with cold atoms?

- Future perspectives

How does this compare/contrast with what we can do in photonics?
Lecture 4