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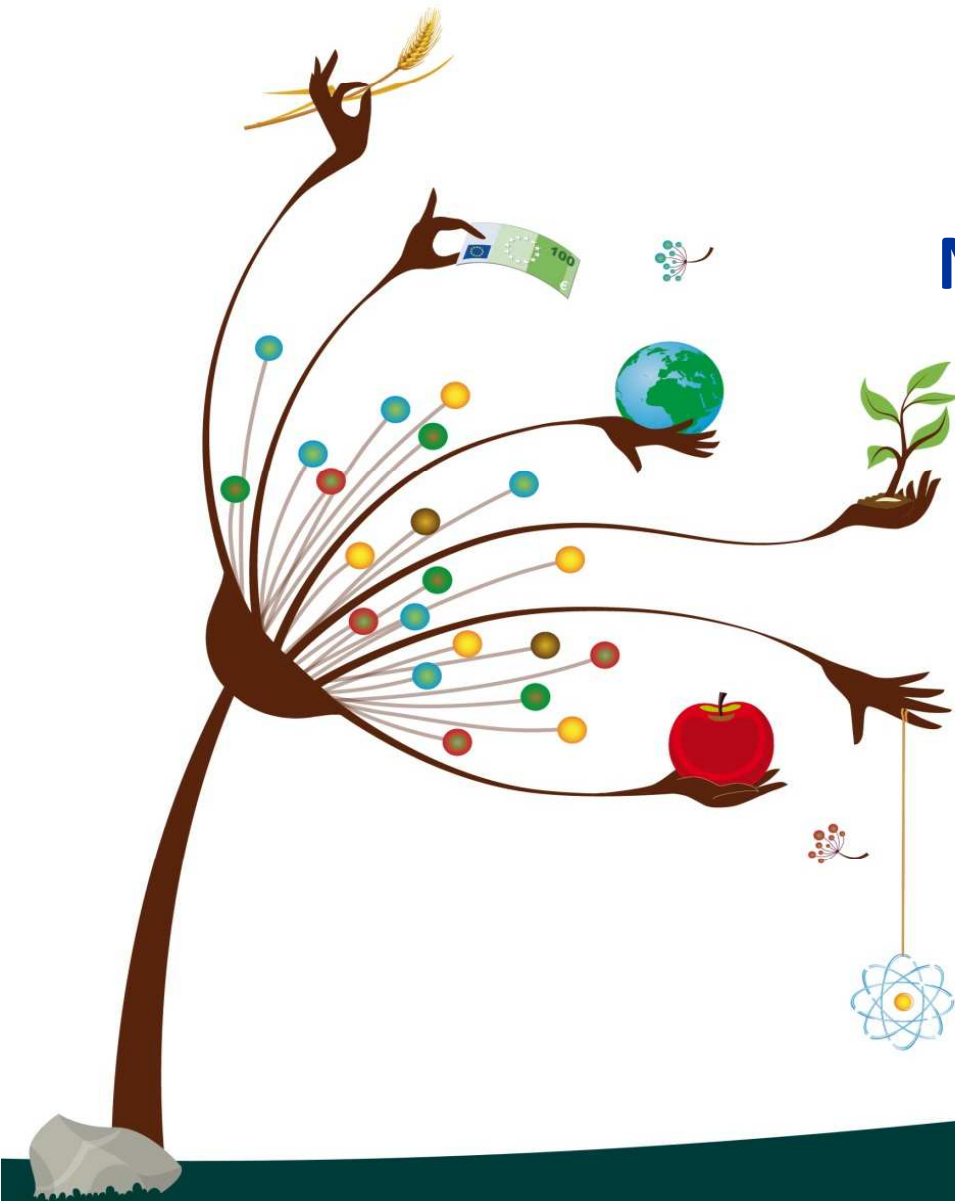
## Neutron time-of-flight measurements and transmission measurements

Peter Schillebeeckx

Workshop on the Evaluation of Nuclear Reaction Data for Applications

2 – 13 October 2017

ICTP, Trieste, Italy

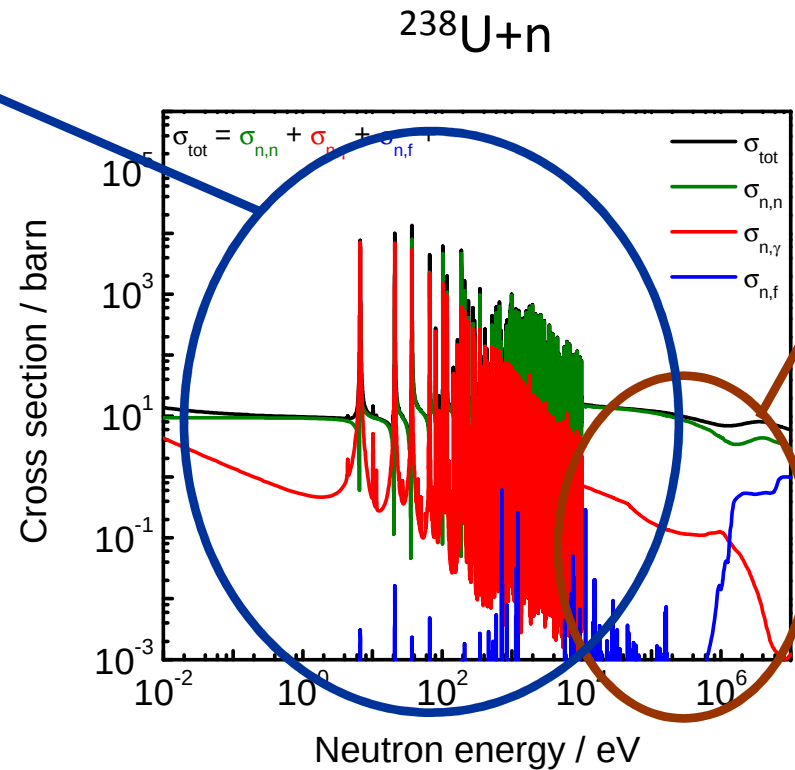


European  
Commission

# Neutron induced reaction cross sections

GELINA

Van de Graaff



White neutron source  
+  
Time-of-flight (TOF)

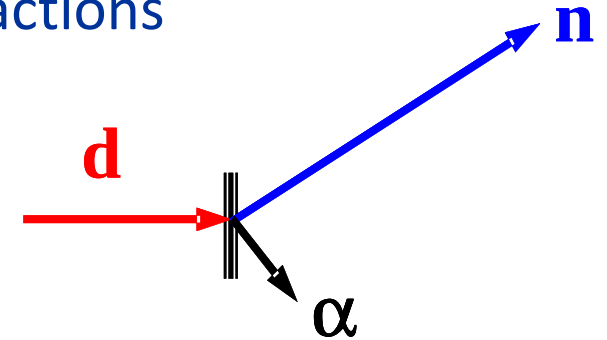
Mono-energetic neutrons  
(cp,n) reactions

# Mono-energetic neutron beams by (cp,n) reactions



quasi mono-energetic neutrons produced  
via nuclear reactions

e.g.  $T(d,n)^4He$



${}^7Li(p,n){}^7Be$

$E_n$ : 0 - 5.3 MeV

$T(p,n){}^3He$

$E_n$ : 0 - 6.2 MeV

$D(d,n){}^3He$

$E_n$ : 1.8 - 10.1 MeV

$T(d,n){}^4He$

$E_n$ : 12.1 - 24.1 MeV



# Mono-energetic neutron beams by (cp,n) reactions

- nuclear reaction,
- neutron emission angle,
- ion energy.

${}^7\text{Li}(p,n){}^7\text{Be}$   $E_n$ : 0 - 5.3 MeV

$\text{T}(p,n){}^3\text{He}$   $E_n$ : 0 - 6.2 MeV

$\text{D}(d,n){}^3\text{He}$   $E_n$ : 1.8 - 10.1 MeV

$\text{T}(d,n){}^4\text{He}$   $E_n$ : 12.1 - 24.1 MeV

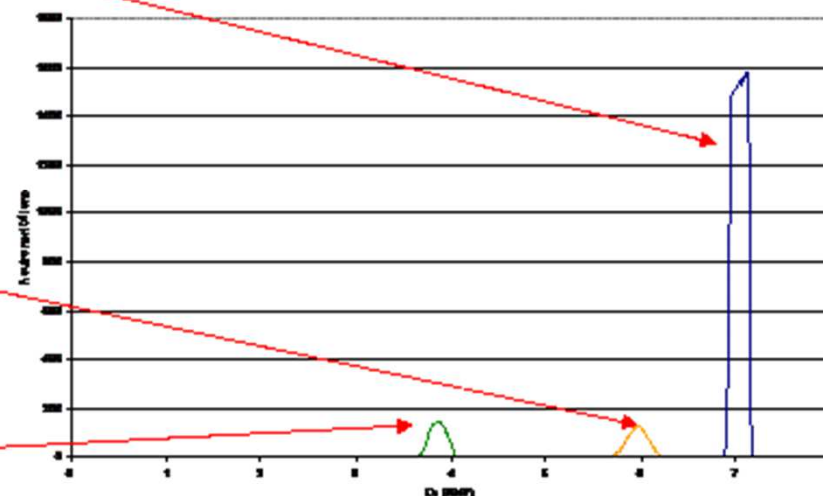
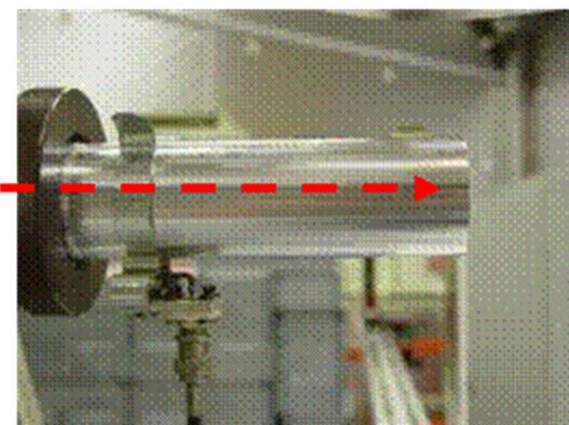
Ion beam

20 cm

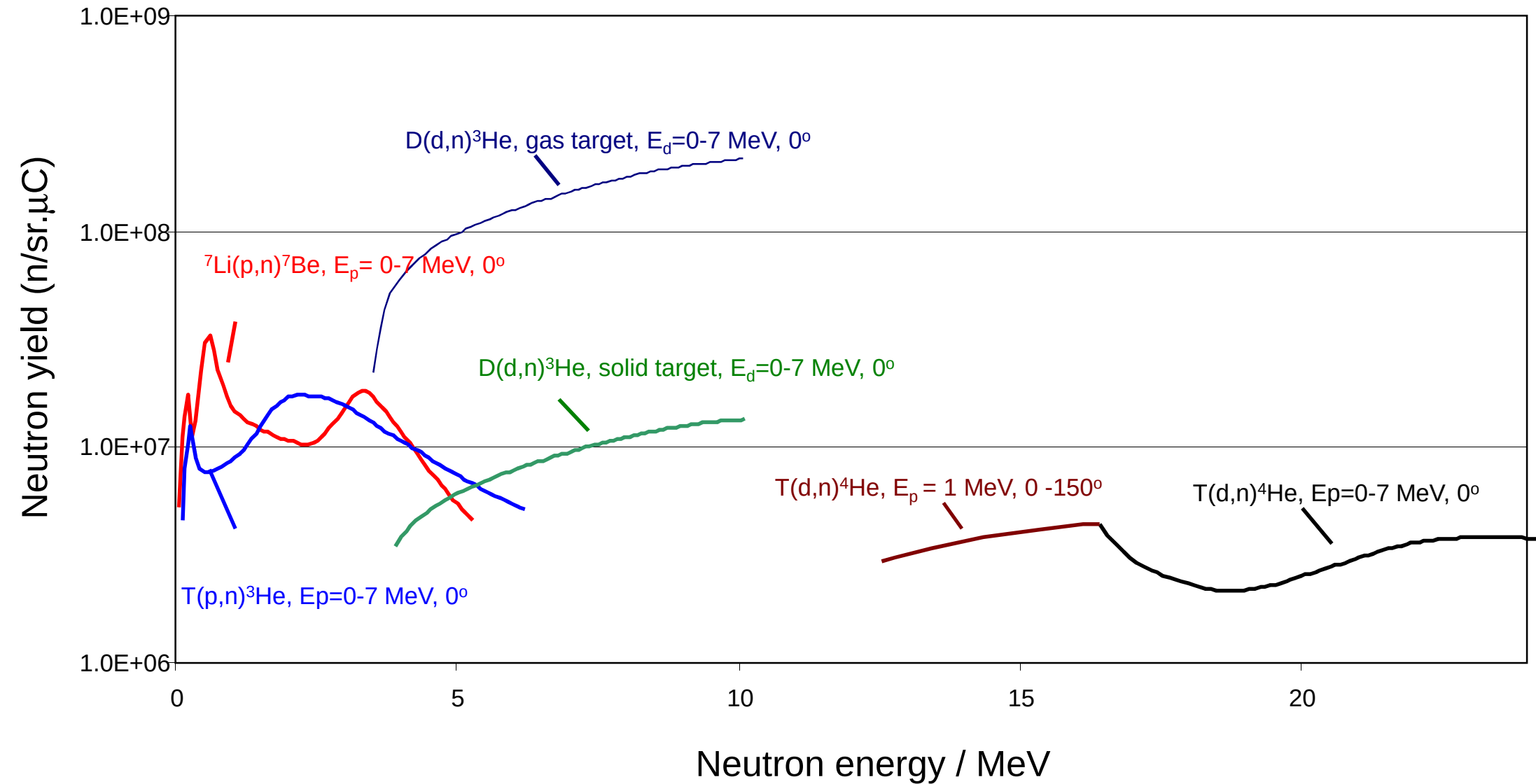
0 deg.

40 deg.

80 deg.



# Mono-energetic neutron beams by (cp,n) reactions



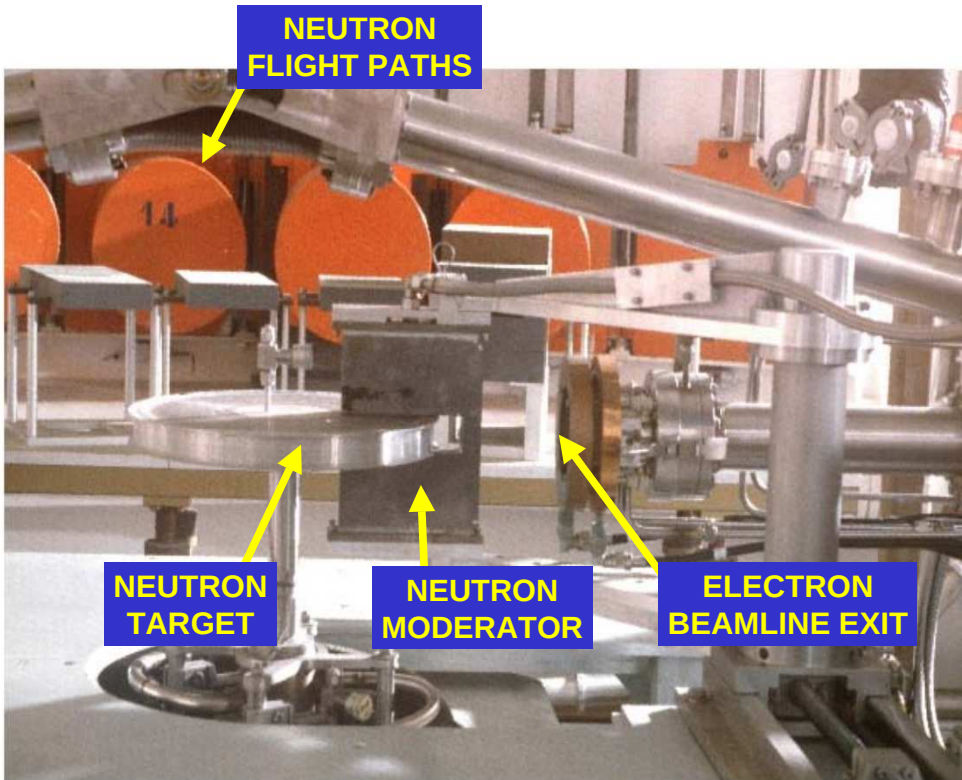
# Time-of flight facility GELINA



- Pulsed white neutron source  
( $10 \text{ meV} < E_n < 20 \text{ MeV}$ )
- Neutron energy : time – of – flight (TOF)
- Multi-user facility: 10 flight paths (10 m – 400 m)
- Measurement stations with special equipment:
  - Total cross section measurements
  - Reaction cross section measurements

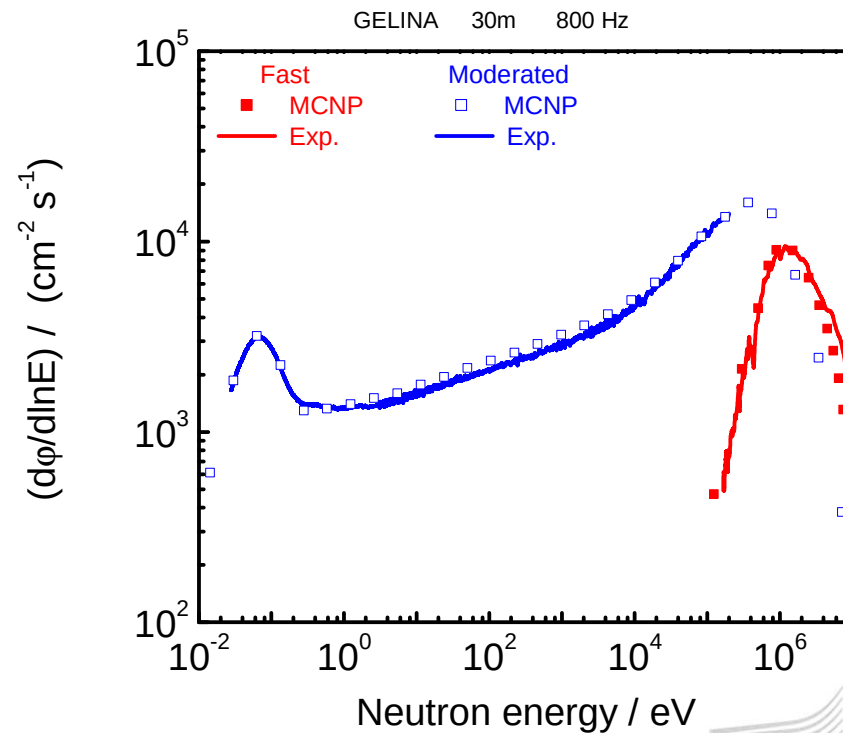


# GELINA : neutron production

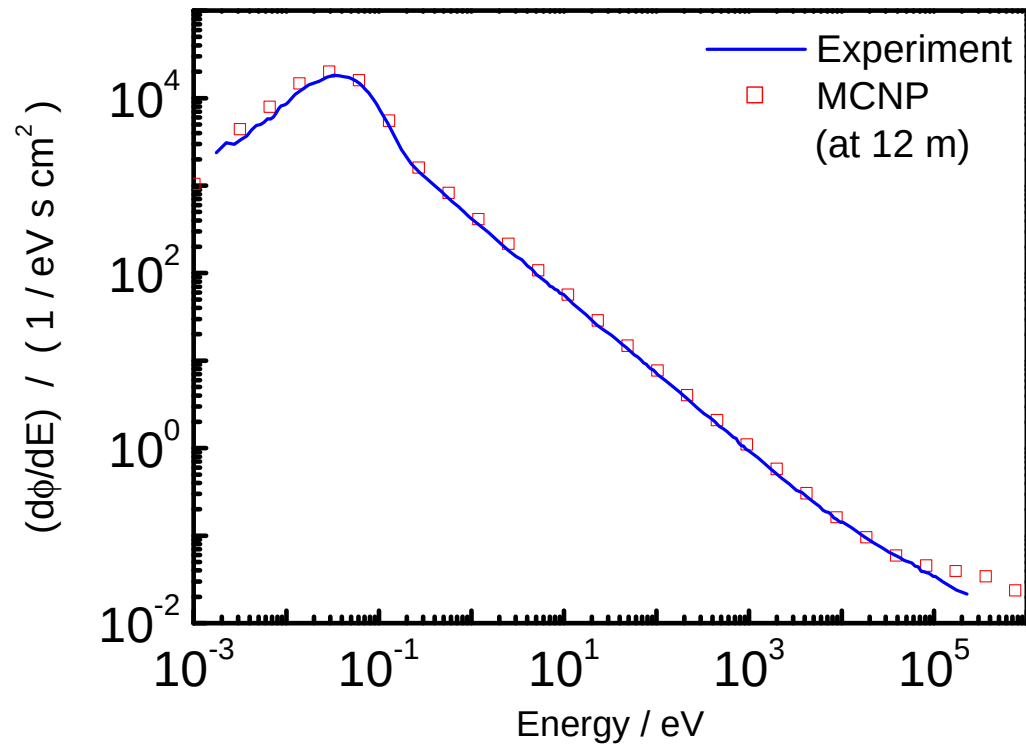


## Neutron production:

- Bremsstrahlung in U-target
- $(\gamma, n)$  and  $(\gamma, f)$  in U-target
- Low energy part enhanced by moderator (water in Be-canning)



# Characteristics of neutron source



- Isotropic emission

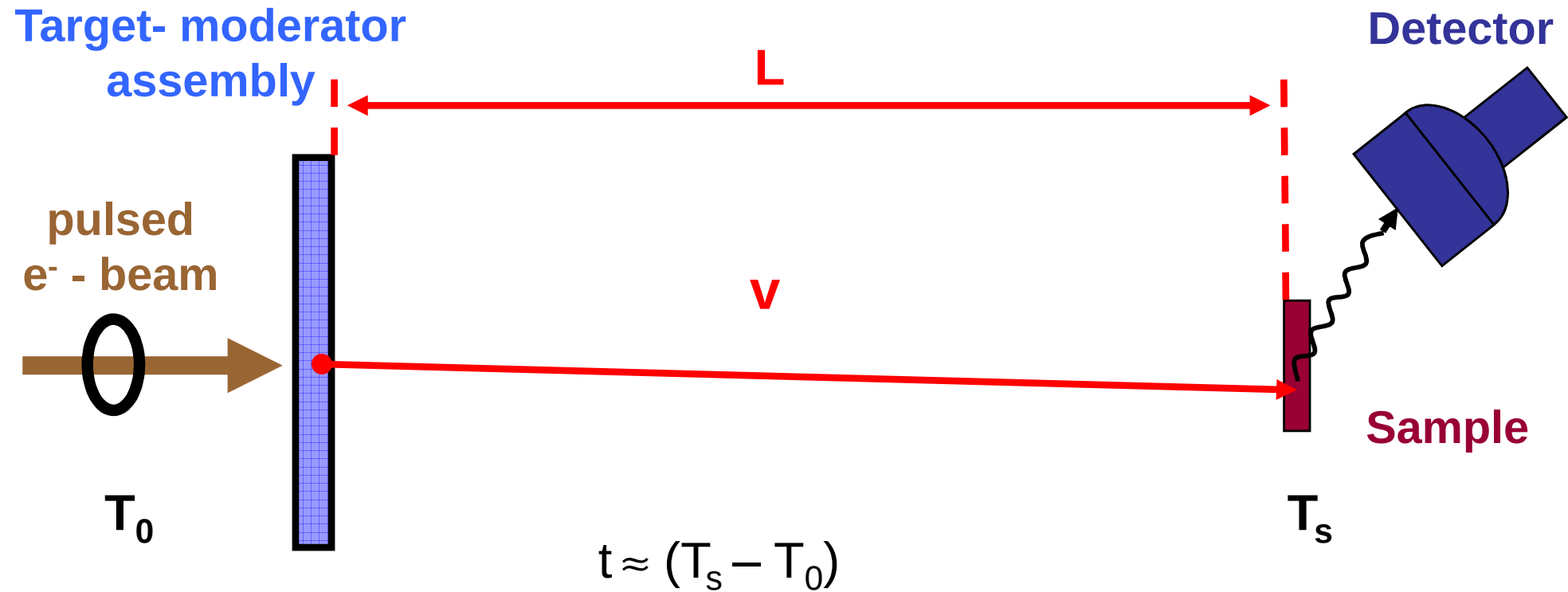
$$\phi_n(L) \propto \frac{1}{L^2}$$

- Moderated Spectrum

Maxwellian +  $1/E$



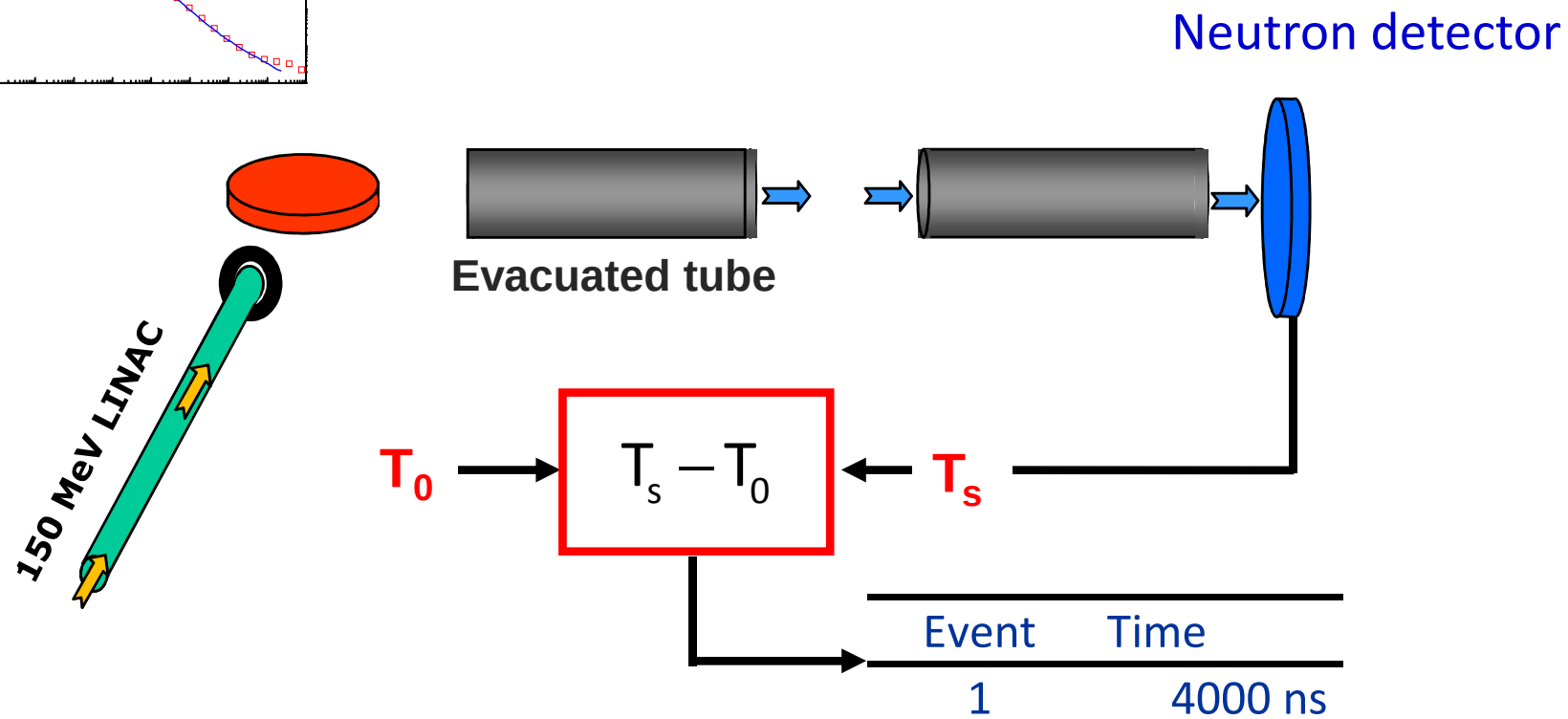
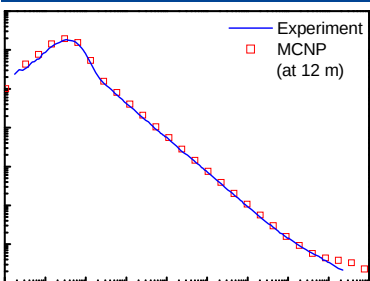
# TOF - measurements



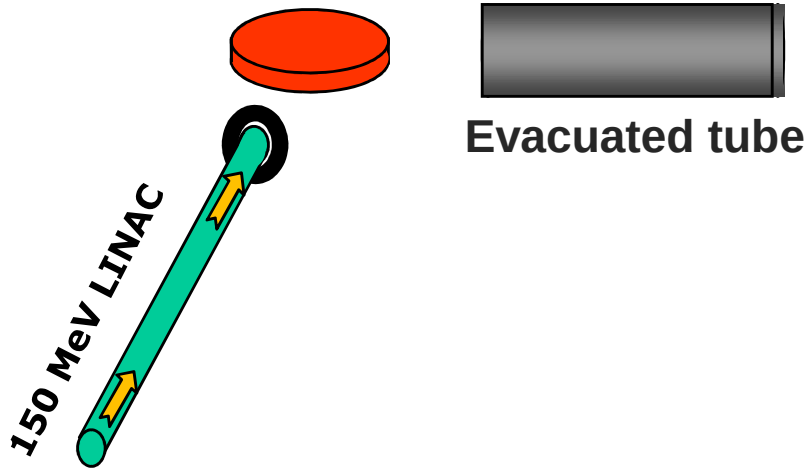
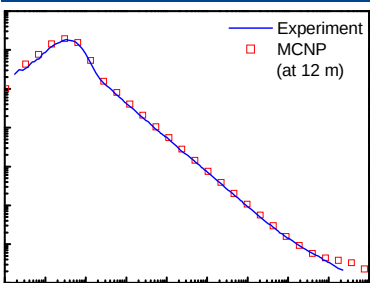
$$v = \frac{L}{t}$$

$$\Rightarrow E = mc^2(\gamma - 1) \cong \frac{1}{2}mv^2$$

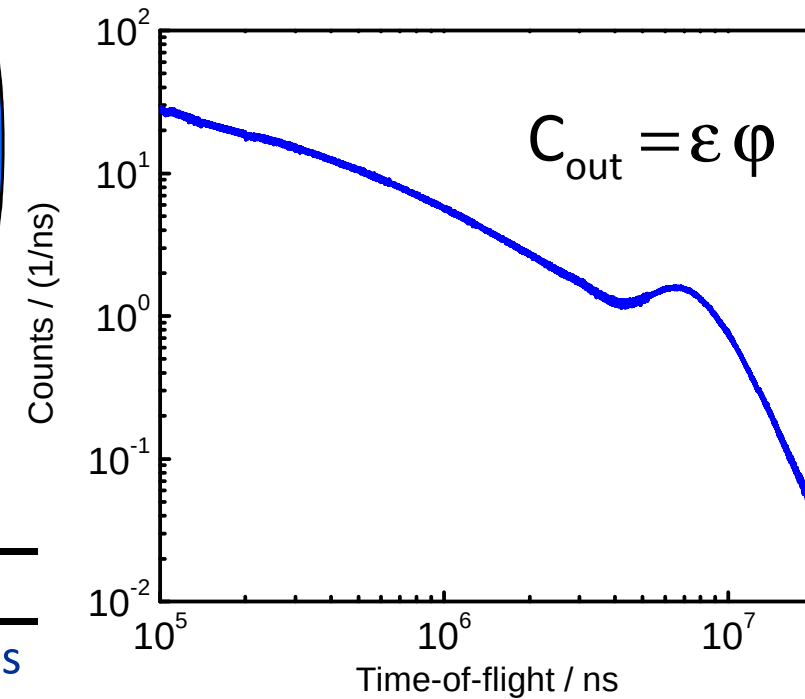
# TOF - measurements



# TOF - measurements



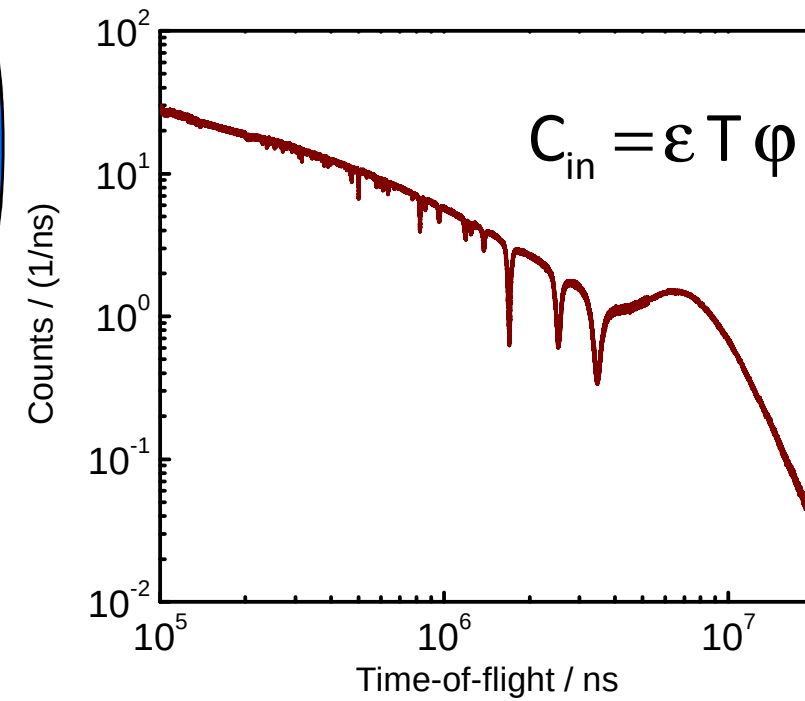
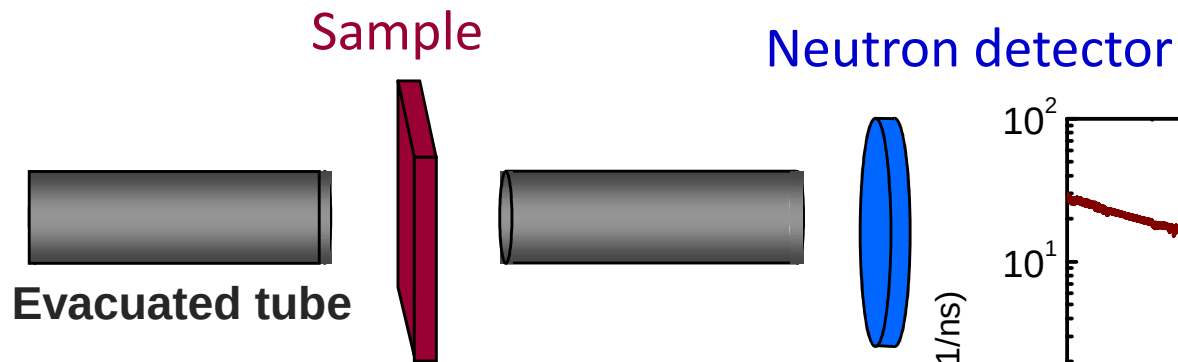
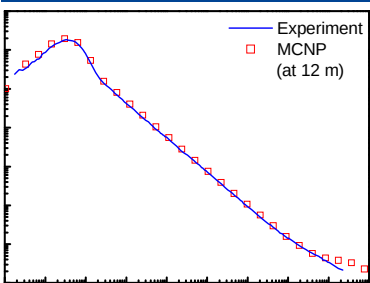
Neutron detector



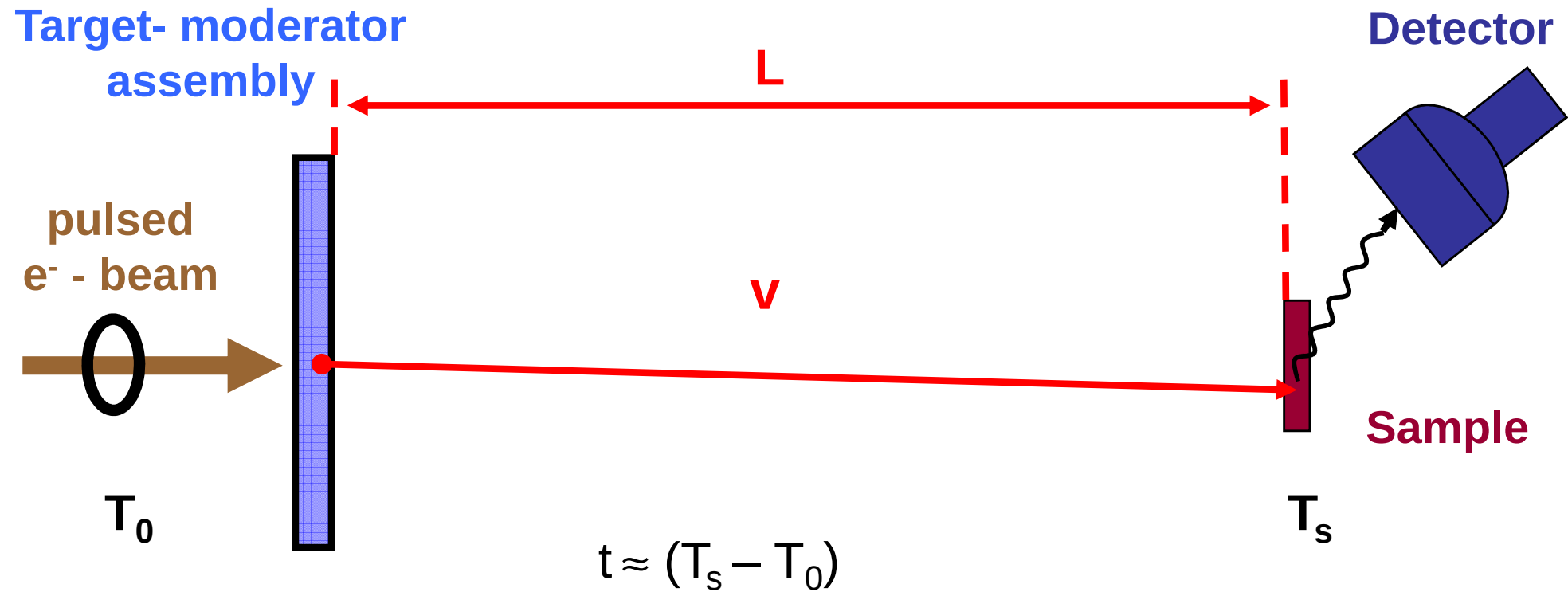
| Event | Time      |
|-------|-----------|
| 1     | 4000 ns   |
| 2     | 50000 ns  |
| 3     | 750000 ns |
| 4     | 500 ns    |
| .     | .         |
| .     | .         |



# TOF - measurements

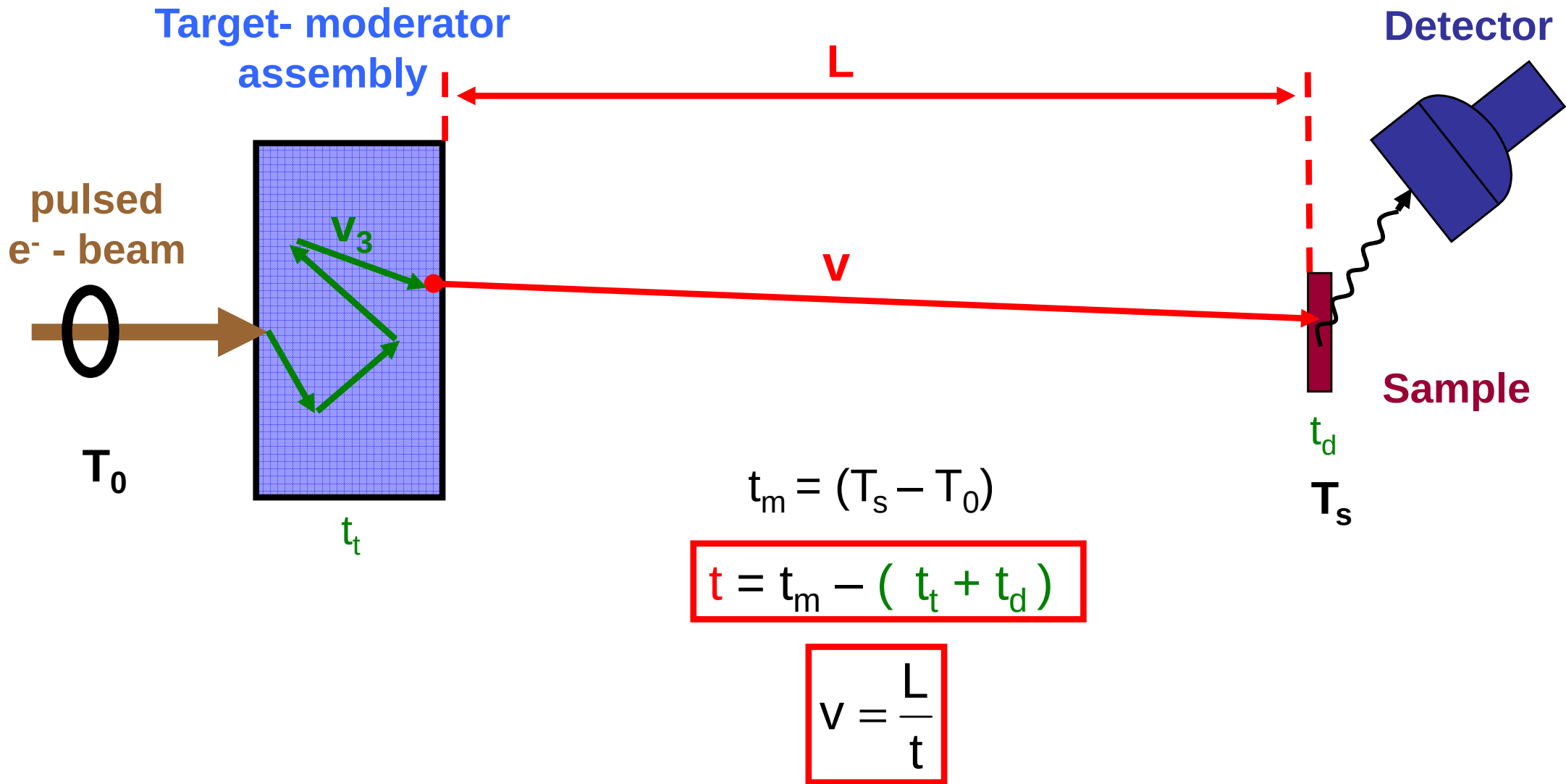


# TOF - measurements



$$v = \frac{L}{t} \quad \Rightarrow \quad E = mc^2(\gamma - 1) \cong \frac{1}{2}mv^2$$

# TOF - measurements



# TOF - measurements

$$v = \frac{L}{t}$$

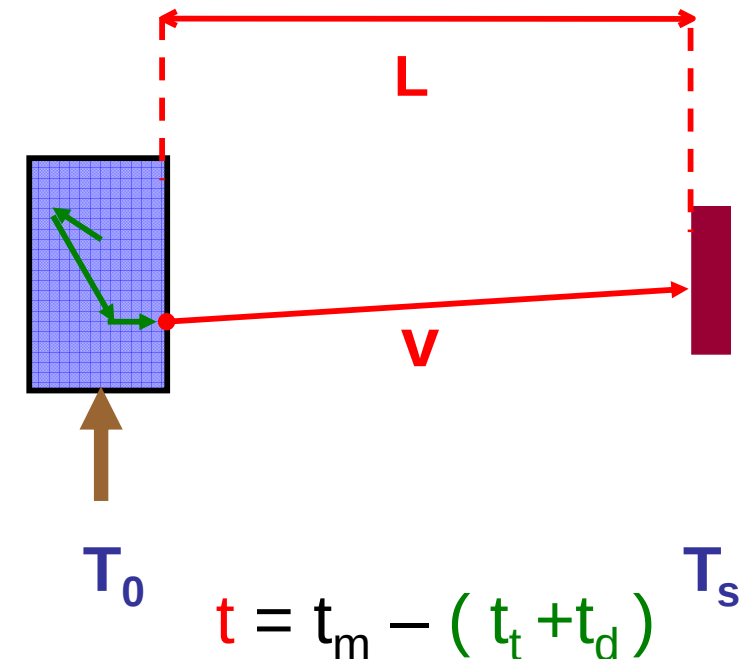
$$\frac{\Delta v}{v} = \sqrt{\frac{\Delta t^2}{t^2} + \frac{\Delta L^2}{L^2}}$$

$$E = mc^2(\gamma - 1)$$

$$\frac{\Delta E}{E} = (1 + \gamma)\gamma \frac{\Delta v}{v}$$

$$E \cong \frac{1}{2}mv^2$$

$$\frac{\Delta E}{E} \cong 2 \frac{\Delta v}{v}$$



$$E \cong \left( 72.298 \frac{L}{t} \right)^2$$

E : eV

t :  $\mu s$

L : m

$\Rightarrow$  1 eV neutron :  $v \cong 13.8 \times 10^3$  m/s

$\Rightarrow$   $\gamma$ -ray :  $c \cong 3 \times 10^8$  m/s  
 $\cong 300$  m/ $\mu s$



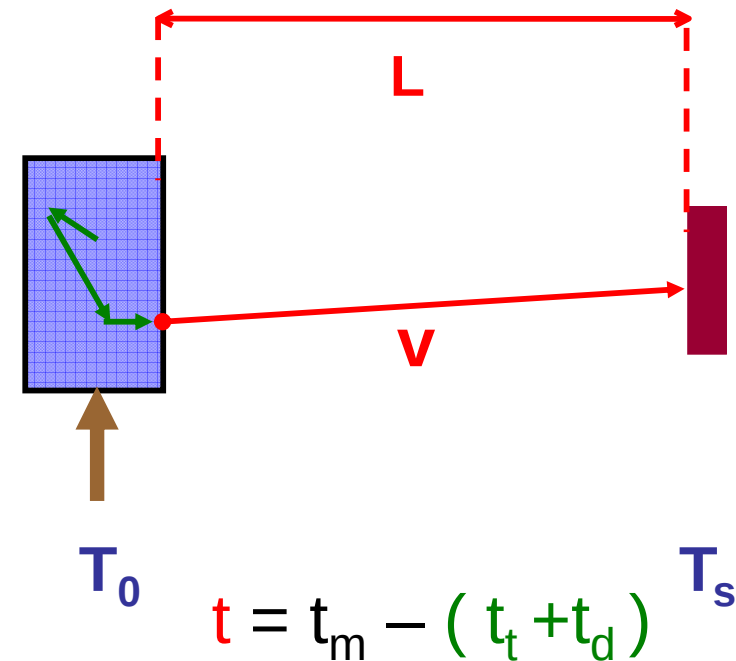
# Response of TOF-spectrometer

$$v = \frac{L}{t}$$

$$\frac{\Delta v}{v} = \sqrt{\frac{\Delta t^2}{t^2} + \frac{\Delta L^2}{L^2}}$$

$$\frac{\Delta v}{v} = \frac{1}{L} \sqrt{v^2 \Delta t^2 + \Delta L^2}$$

$$\frac{\Delta v}{v}$$



# Response of TOF-spectrometer

$$v = \frac{L}{t}$$

$$\frac{\Delta v}{v} = \sqrt{\frac{\Delta t^2}{t^2} + \frac{\Delta L^2}{L^2}}$$

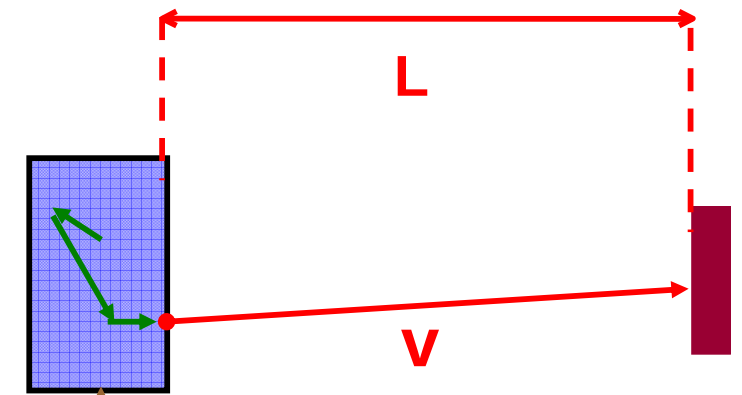
- $\Delta L$  (~ 1 mm)
- $\Delta t$ 
  - Initial burst width
  - Time jitter detector & electronics
  - Neutron transport in target - moderator
  - Neutron transport in detector

$\Delta T_0$

$\Delta T_s$

$\Delta t_t$

$\Delta t_d$



$$t = t_m - (t_t + t_d)$$

$$t_m = (T_s - T_0)$$

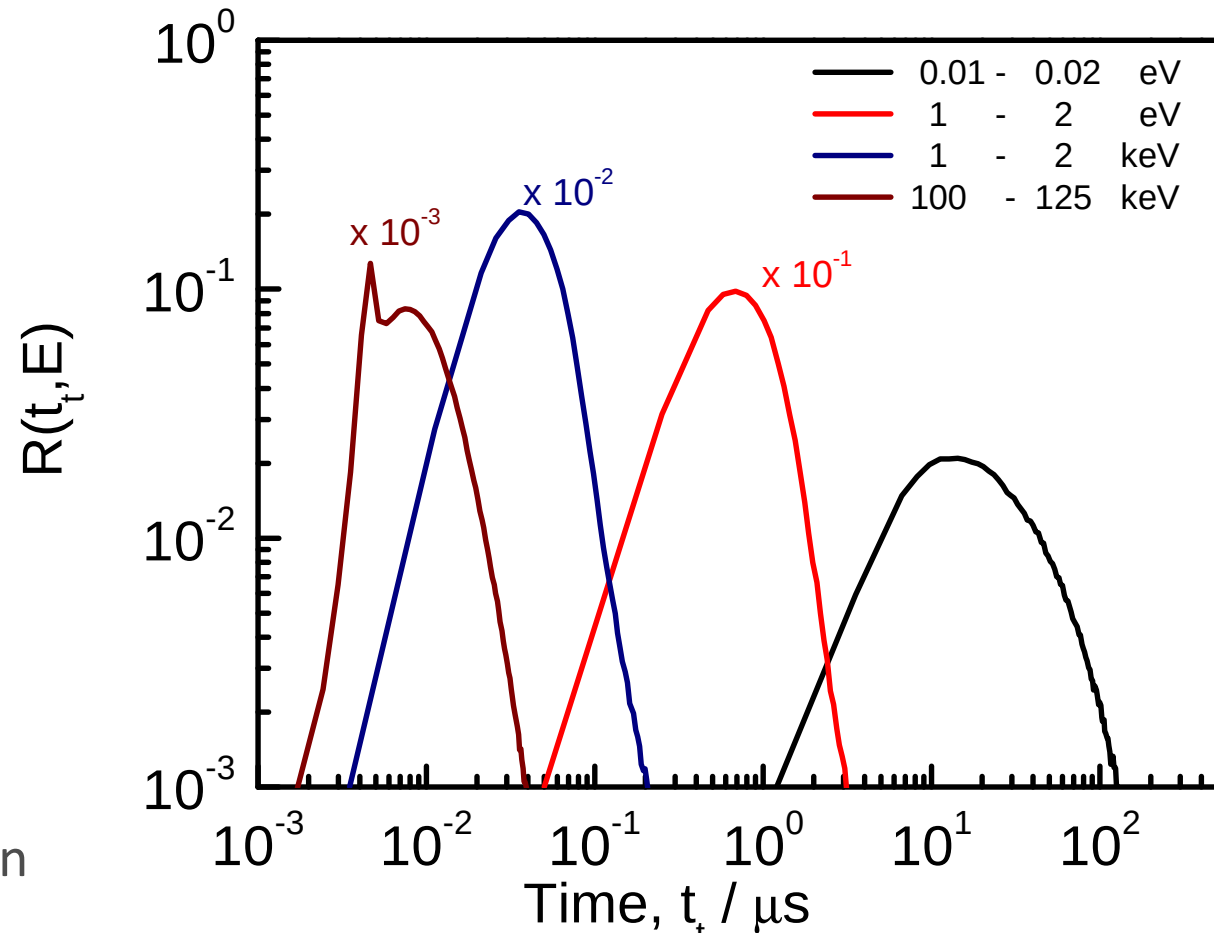
# Probability distribution of $t_t$ : GELINA

$$R(t_t, E) = R'(L(t_t), E) \left| \frac{dL}{dt} \right|$$

Transformation of variables :

$$L_t = v t_t$$

- $L_t$  : equivalent distance
- $v$  : neutron velocity when leaving the moderator
- $t_t$  : time difference between neutron creation and moment it leaves the moderator



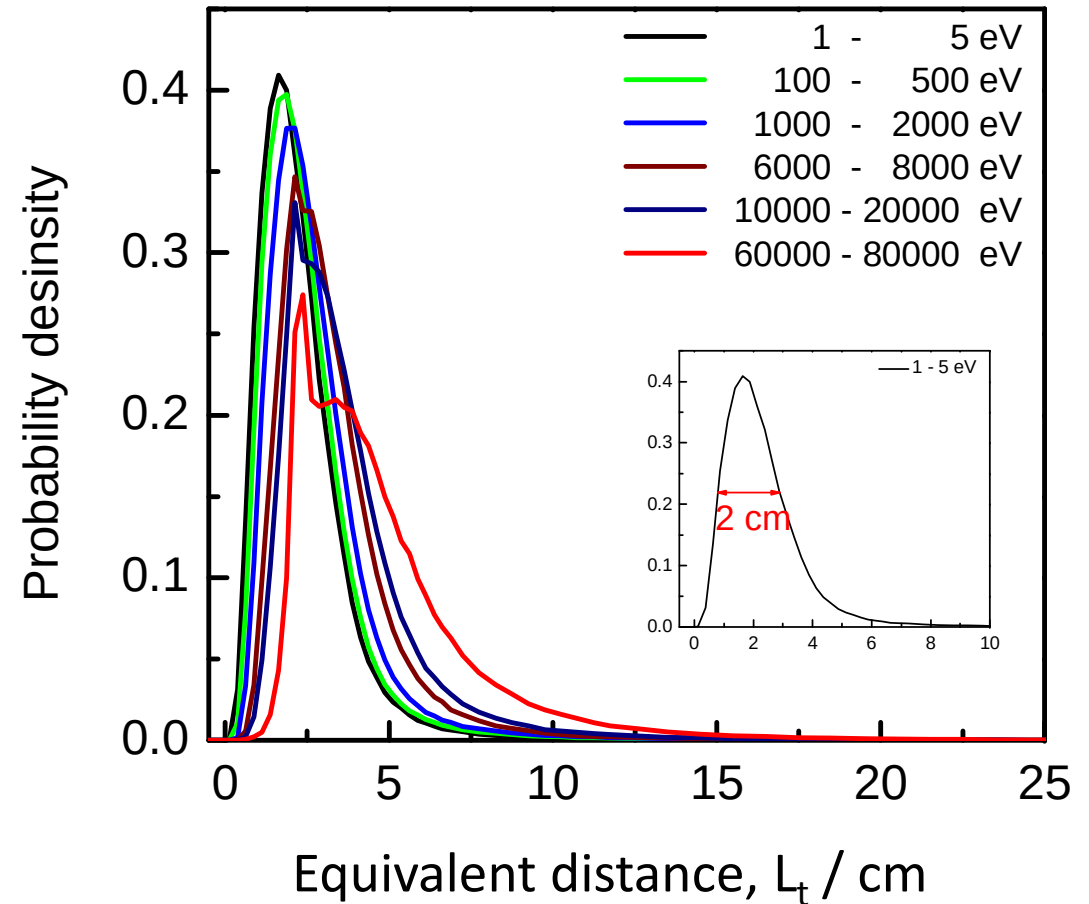
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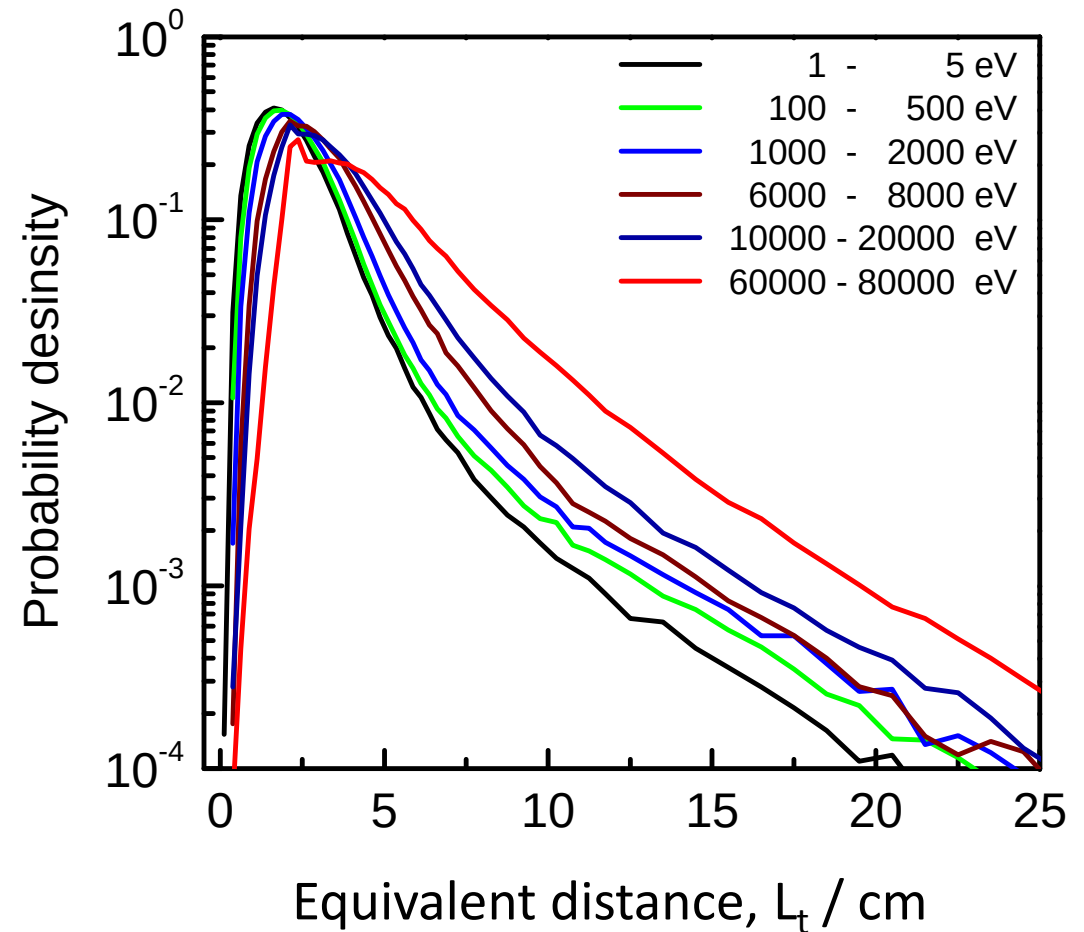
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- $v$  : neutron velocity when leaving the moderator
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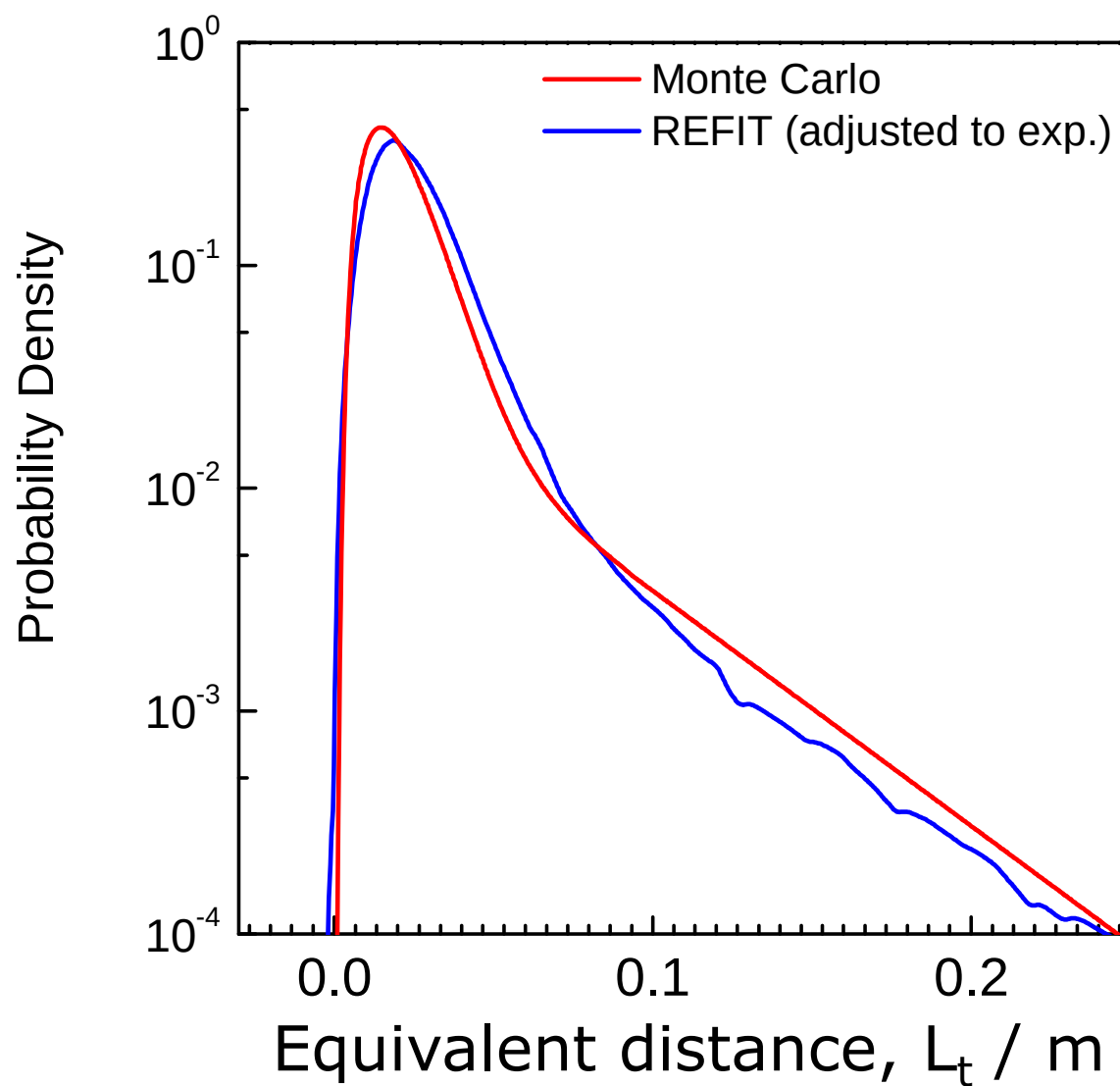
# Probability distribution of $L_t$ : GELINA

$$R(t_t, E) = R'(L(t_t), E) \left| \frac{dL}{dt} \right|$$

- Between 0.5 eV and 1000 eV:  $\chi^2$   
 $R'(L_t, E_n)$  dominated by  $\chi^2$  due to moderation process and almost independent of  $E_n$
- Below 0.5 eV :  $\chi^2$  + storage
- Above 1000 keV : more complex

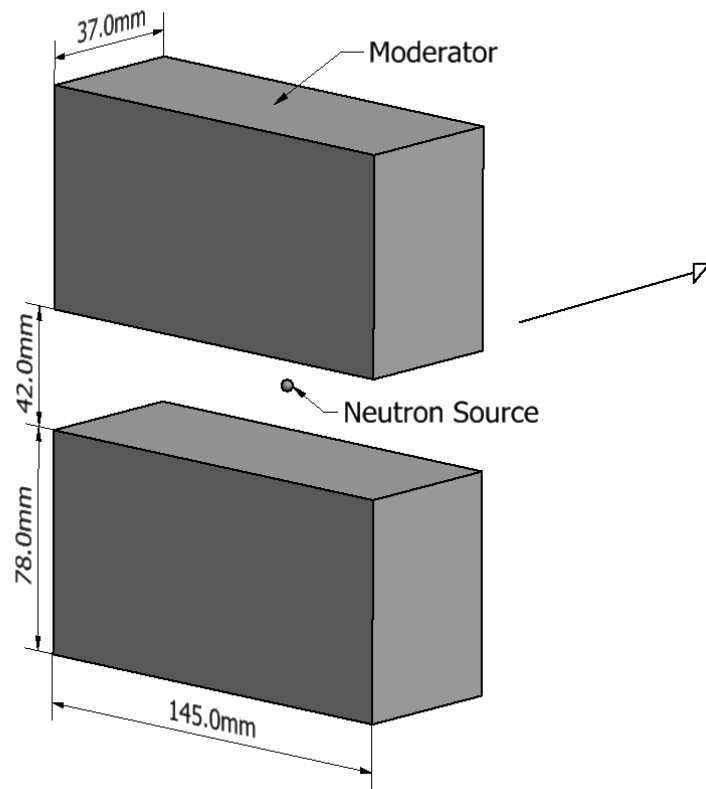


# Probability distribution of $L_t$ : GELINA



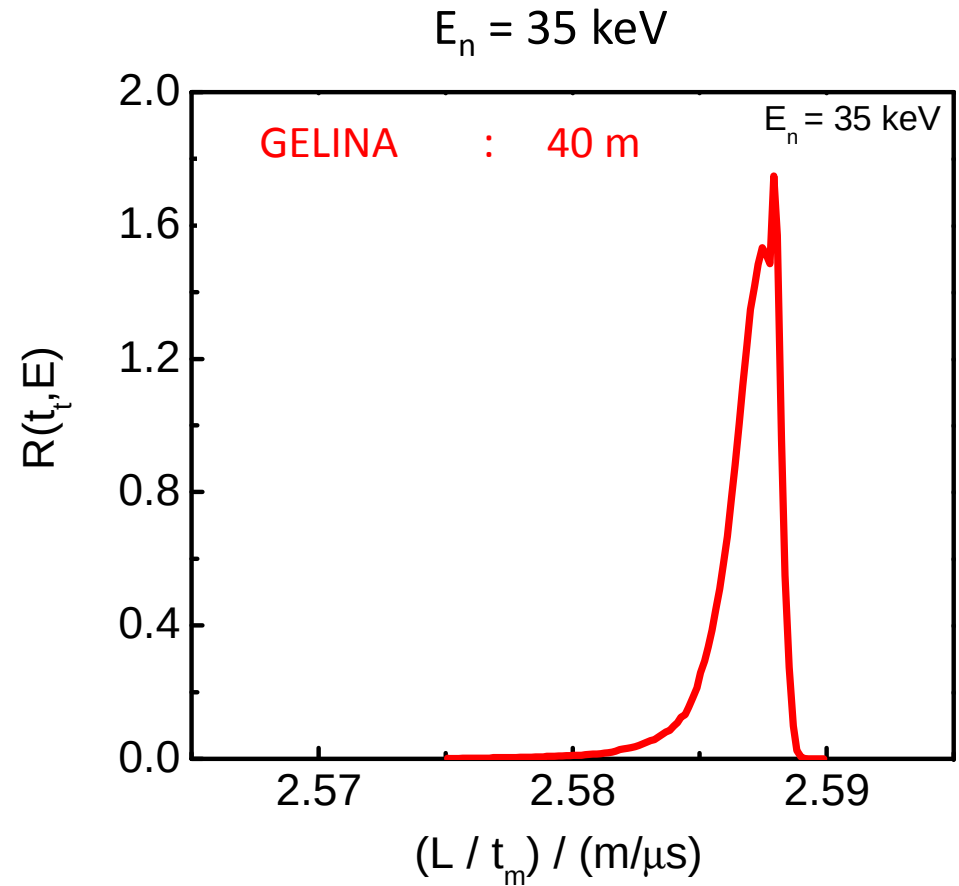


# $P(t_t, E)$ : photonuclear



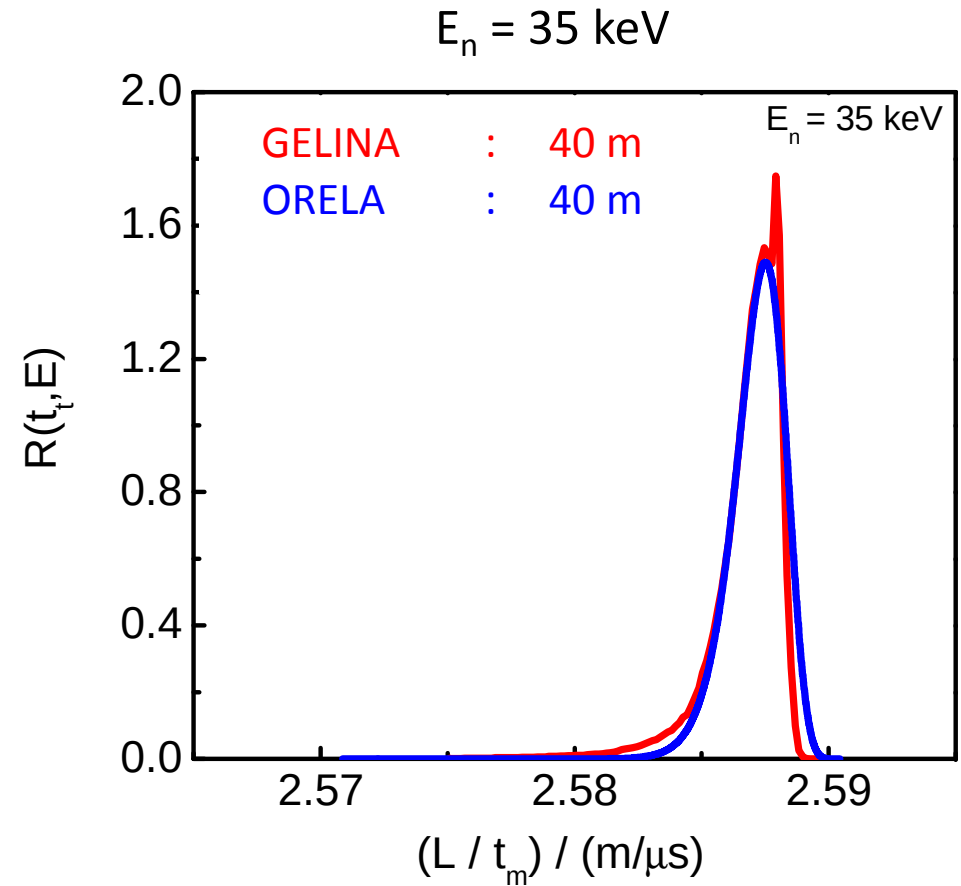
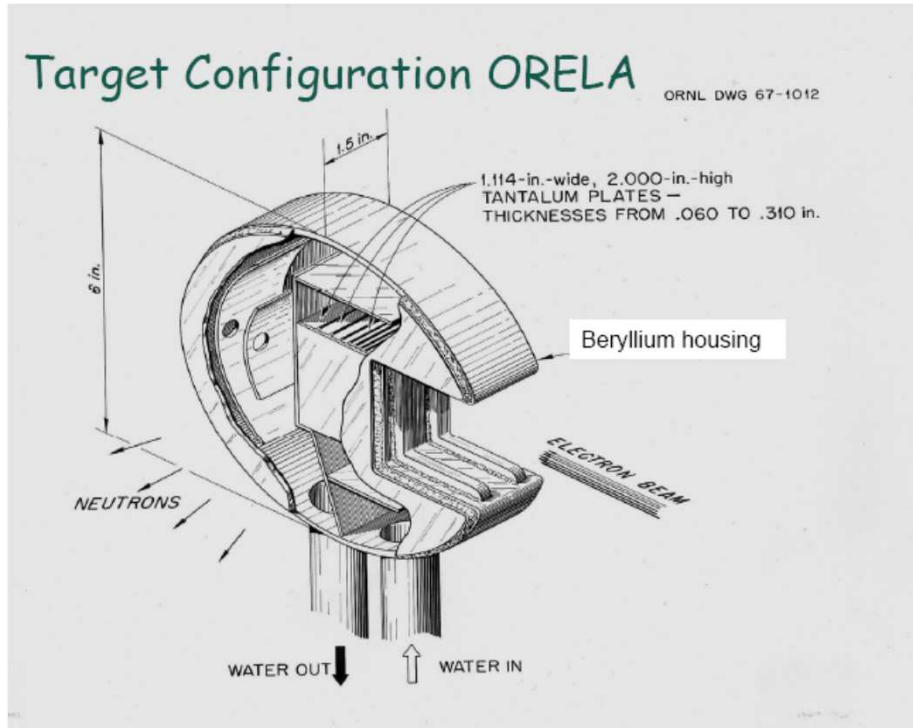
$$\Delta L \text{ (FWHM)} = 2 \text{ cm}$$

$\Rightarrow$  about half of the moderator thickness



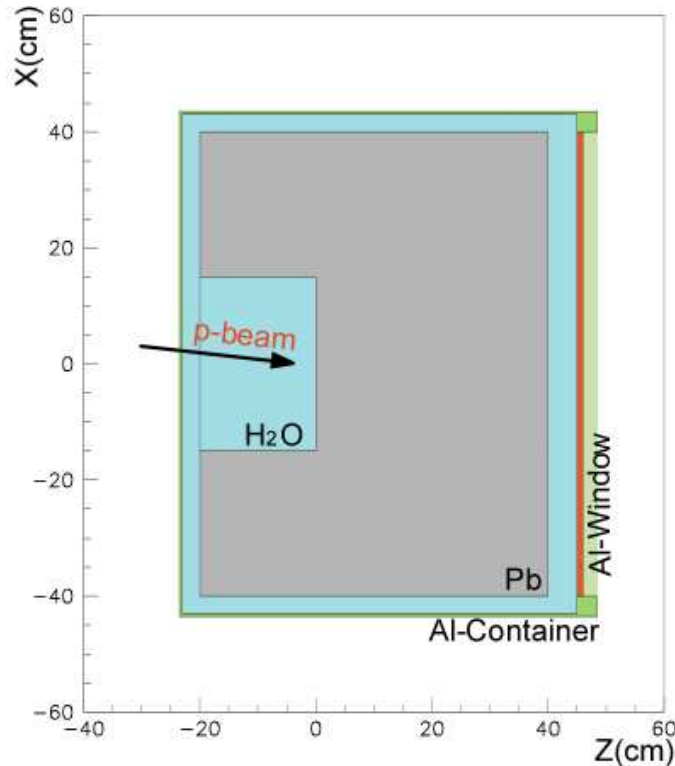
|            |   |                           |
|------------|---|---------------------------|
| Resolution | : | $\Delta L \text{ (FWHM)}$ |
| GELINA     | : | 2 cm                      |

# $P(t_t, E)$ : photonuclear

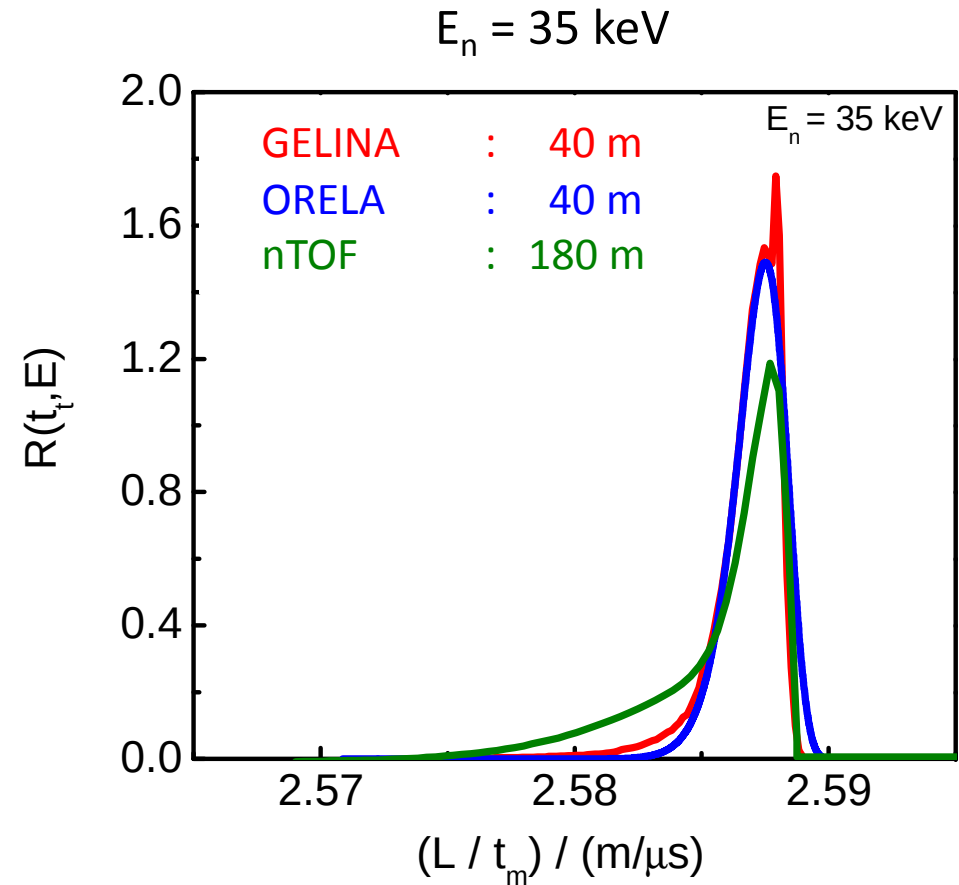


|            |   |                   |
|------------|---|-------------------|
| Resolution | : | $\Delta L$ (FWHM) |
| GELINA     | : | 2 cm              |
| ORELA      | : | 2 cm              |

# $P(t_t, E)$ : photonuclear $\Leftrightarrow$ spallation reactions

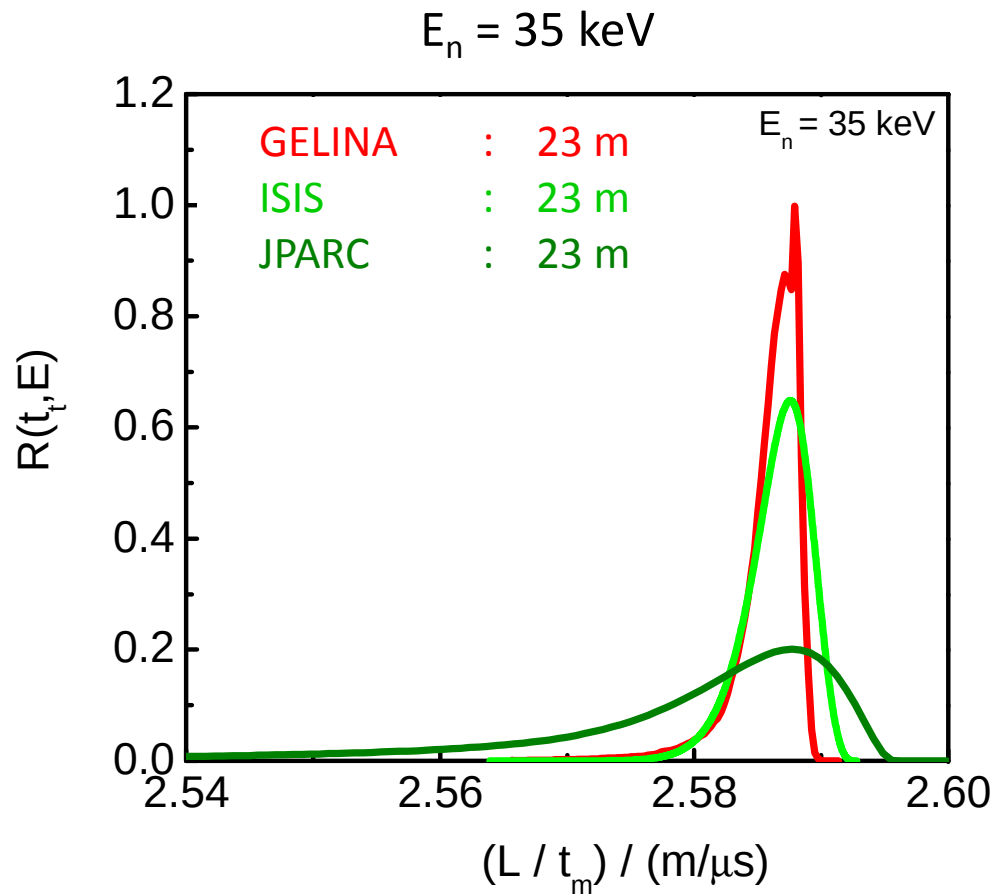


|                            |                                         |
|----------------------------|-----------------------------------------|
| Dimensions                 | : $80 \times 80 \times 60 \text{ cm}^3$ |
| Pure Lead                  | : 4 t                                   |
| H <sub>2</sub> O moderator | : 5 cm                                  |
| Al-window                  | : 1.6 mm                                |
| Al-container               | : 140 l                                 |

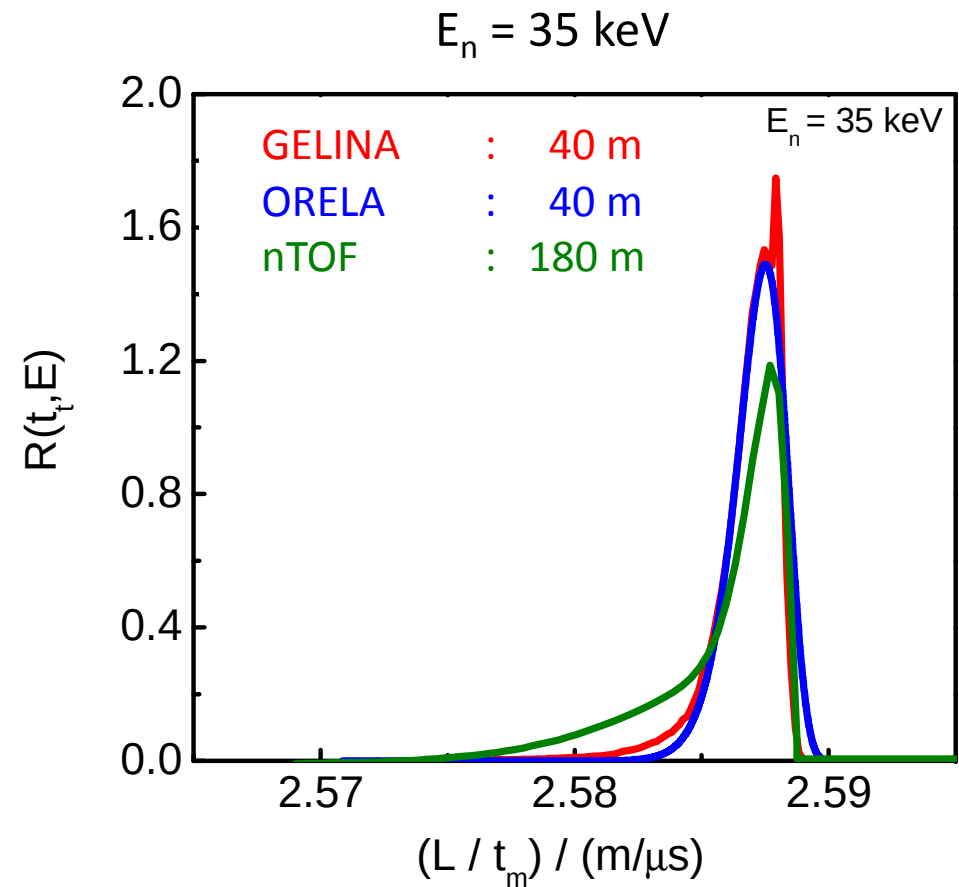


|            |                     |
|------------|---------------------|
| Resolution | : $\Delta L$ (FWHM) |
| GELINA     | : 2 cm              |
| ORELA      | : 2 cm              |
| nTOF       | : 10 cm             |

# $P(t_t, E)$ : photonuclear $\Leftrightarrow$ spallation reactions



Resolution :  $\Delta L$  (FWHM)  
 ISIS (INES) : 5 cm  
 J-PARC (MLF/ANNRI) : 13 cm  
 Strongly depend on target/moderator  
 configuration (coupled/decoupled)



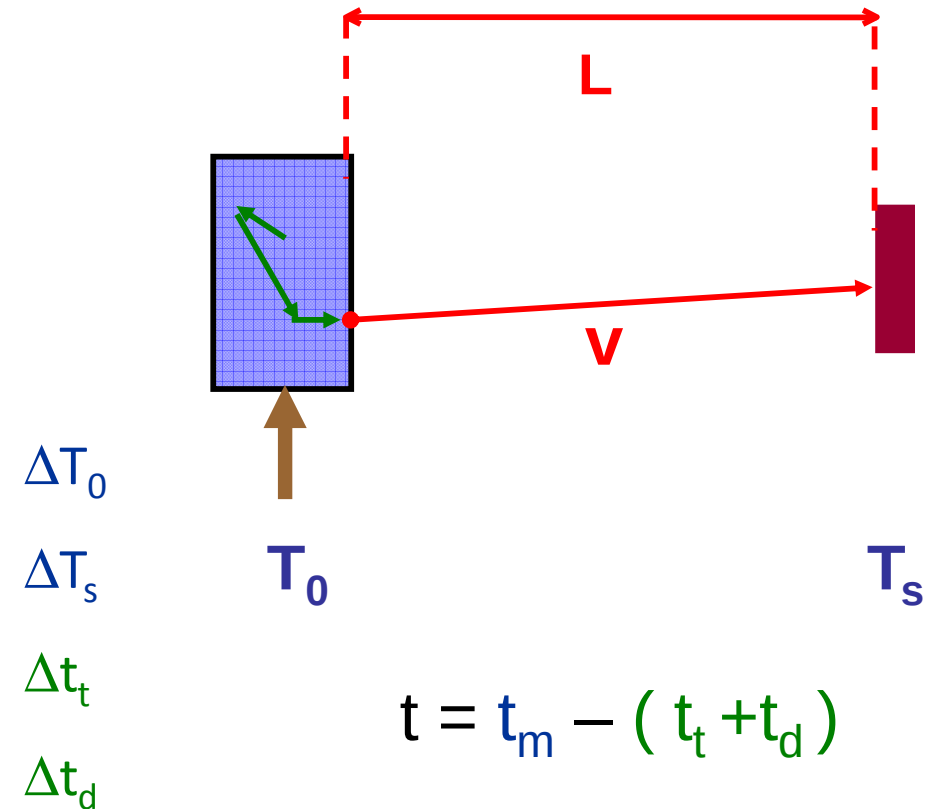
Resolution :  $\Delta L$  (FWHM)  
 GELINA : 2 cm  
 ORELA : 2 cm  
 nTOF : 10 cm

# Response of TOF-spectrometer

$$v = \frac{L}{t}$$

$$\frac{\Delta v}{v} = \sqrt{\frac{\Delta t^2}{t^2} + \frac{\Delta L^2}{L^2}}$$

- $\Delta L$  (~ 1 mm)
- $\Delta t$ 
  - Initial burst width
  - Time jitter detector & electronics
  - Neutron transport in target - moderator
  - Neutron transport in detector



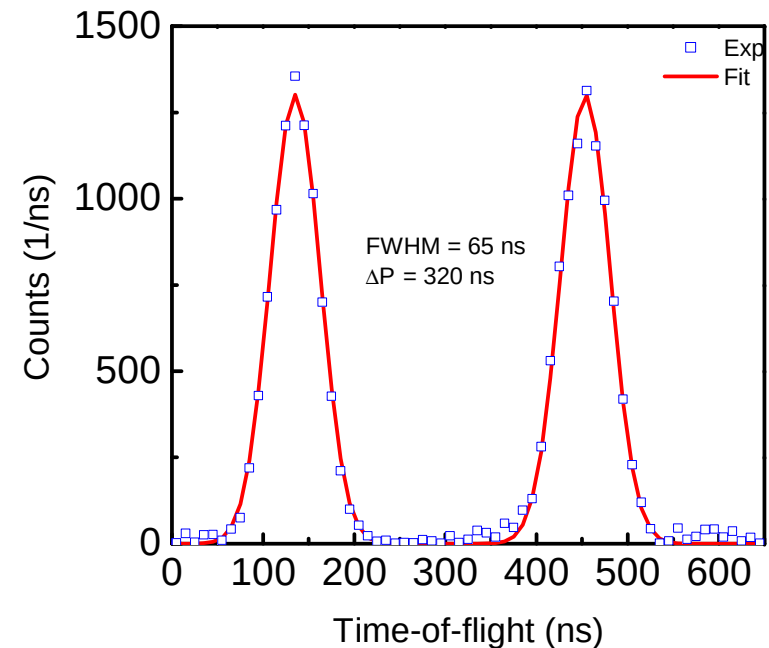
# Initial burst ( $T_0$ )

- Single burst : mostly normal (Gaussian) distribution

- GELINA :  $\Delta T_0 = 1$  ns (FWHM)
- ORELA :  $\Delta T_0 = 4$  ns (FWHM)
- nTOF :  $\Delta T_0 = 10$  ns (FWHM)

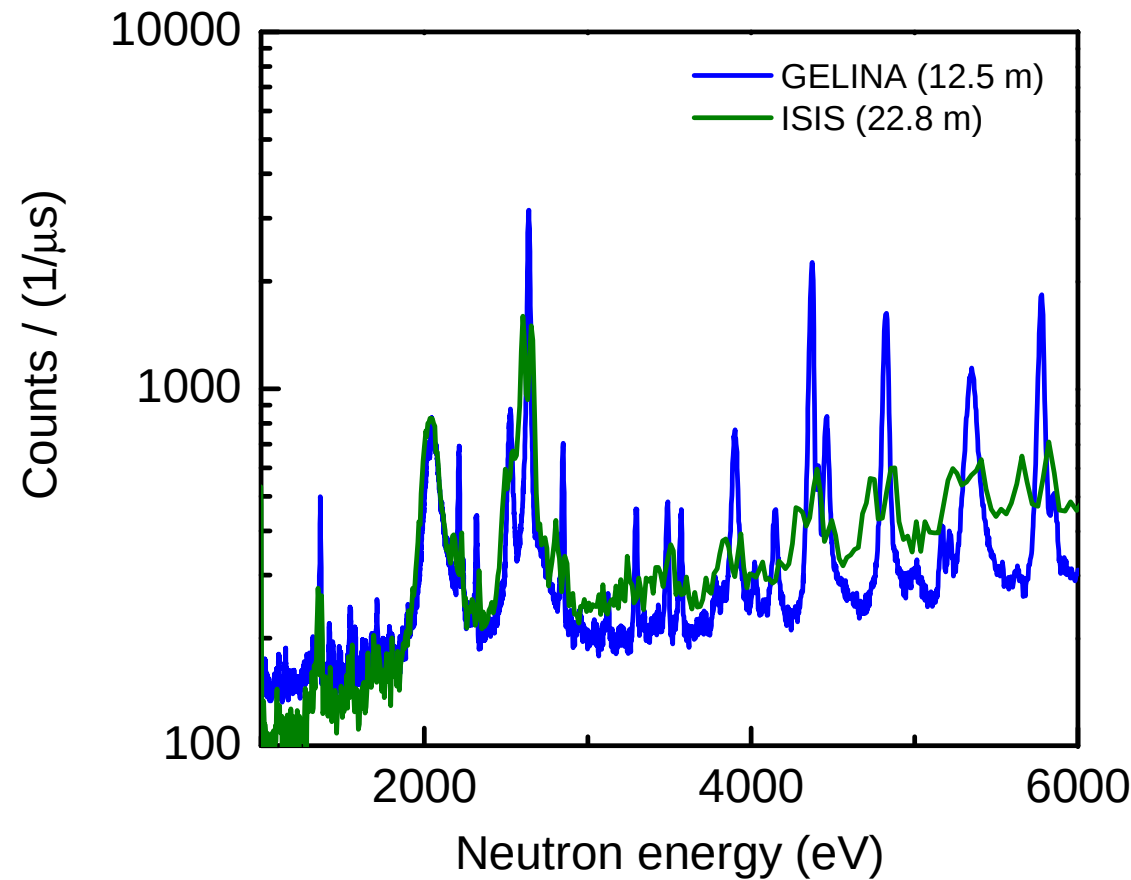
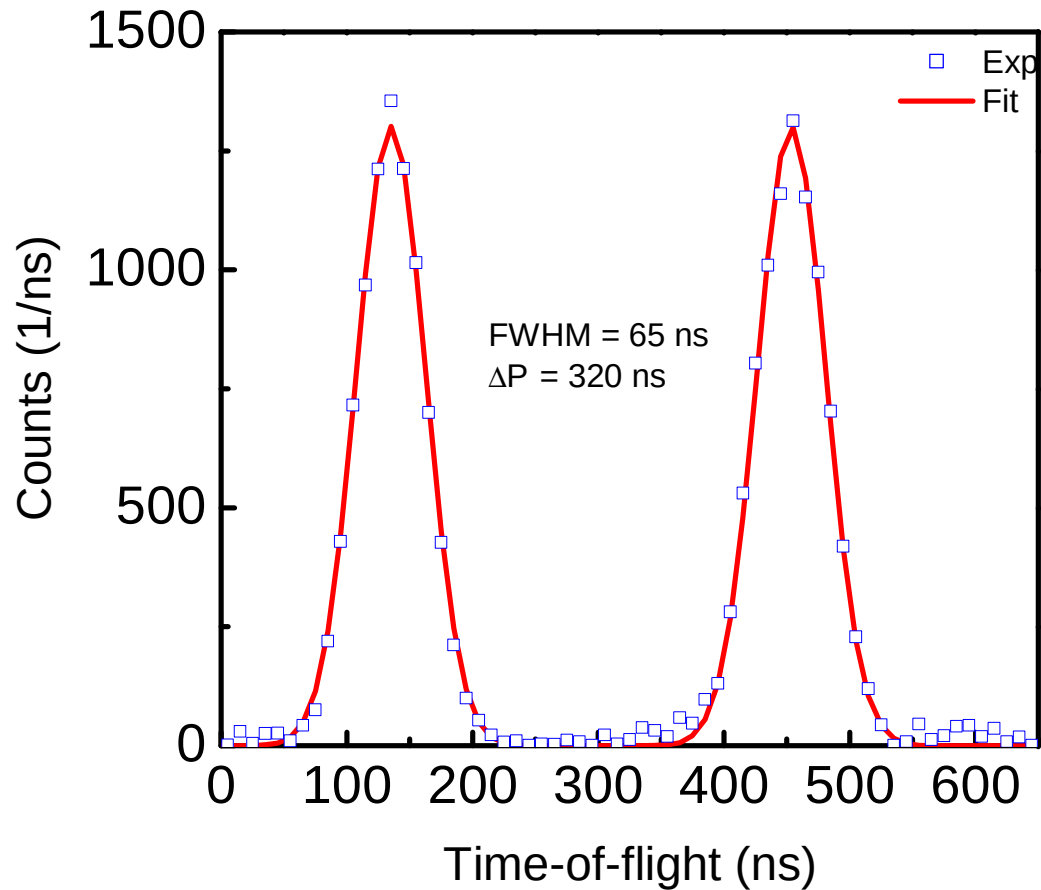
- Double pulse structure :

- ISIS
- J-PARC



# Response GELINA (12.5 m) < - > ISIS (22.8 m)

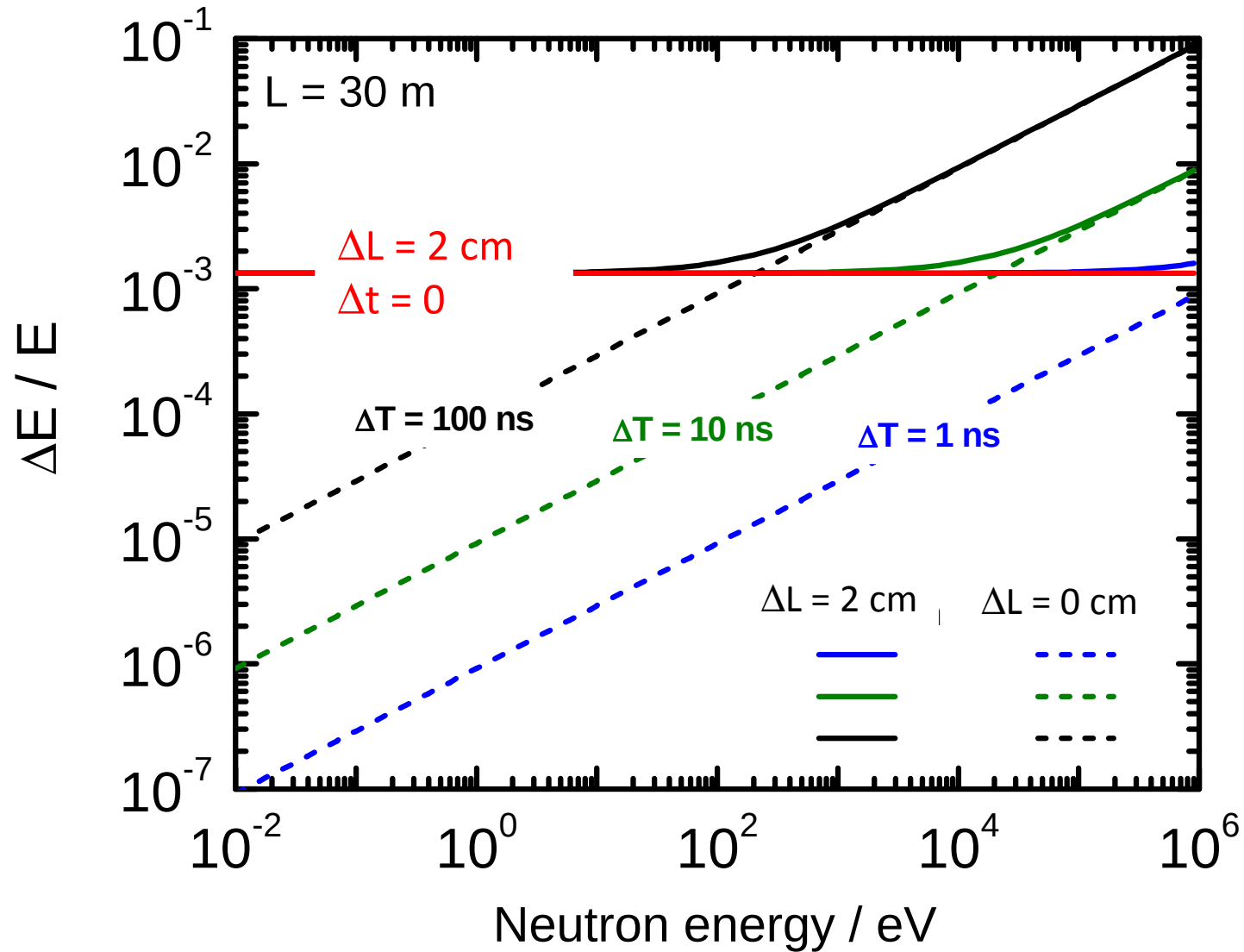
Double pulsed proton beam



Double pulsed proton beam also at J-PARC



# TOF : resolution components



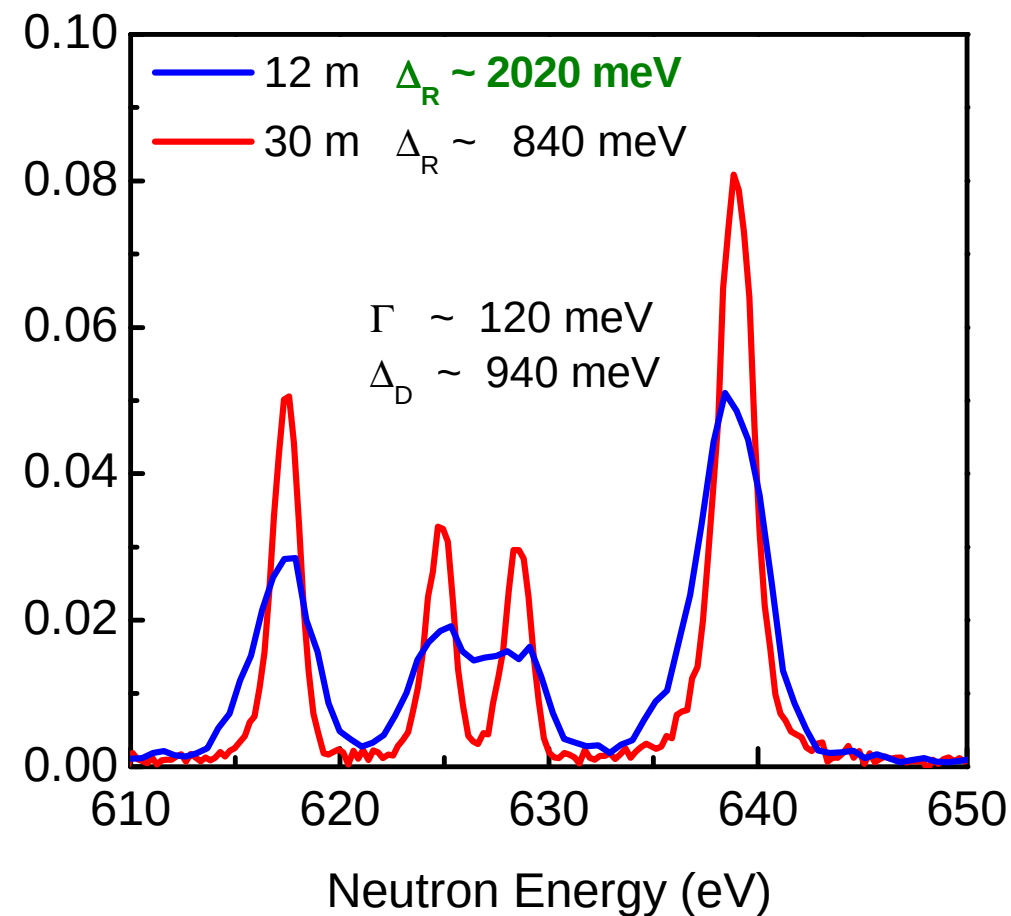
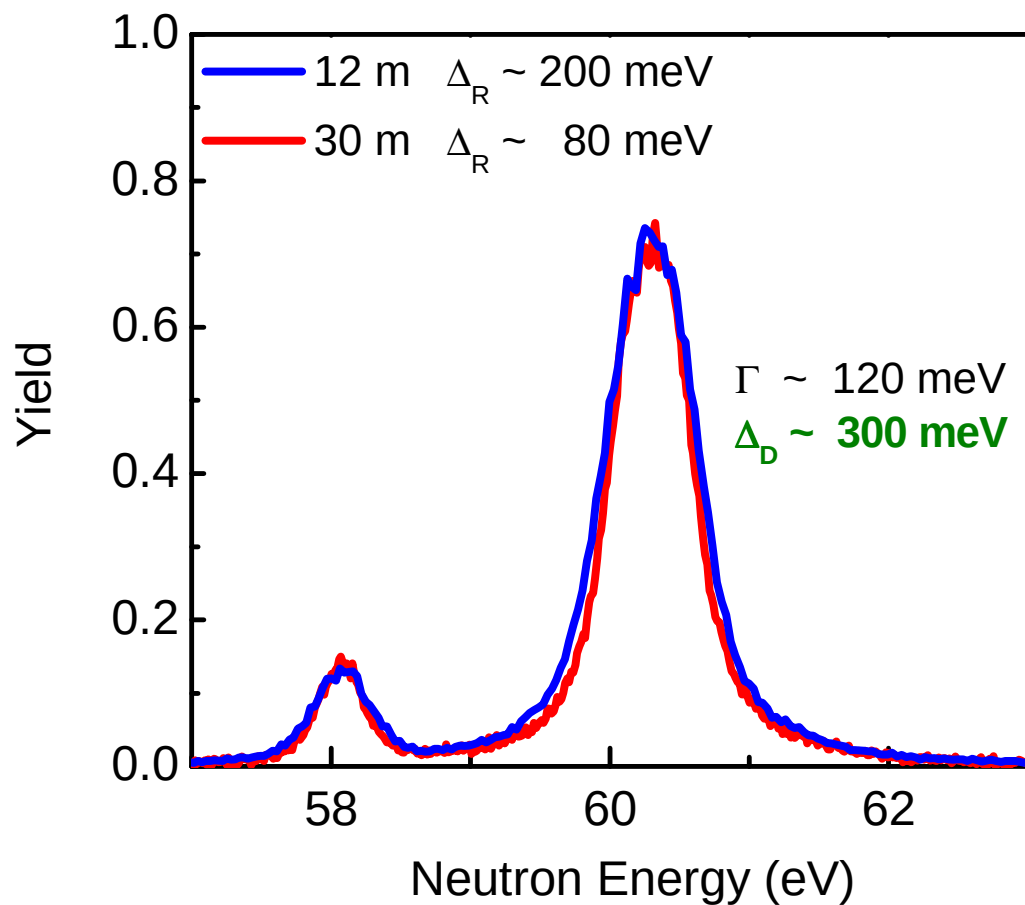
$$\frac{\Delta v}{v} = \frac{1}{L} \sqrt{v^2 \Delta t^2 + \Delta L^2}$$

$$\frac{\Delta E}{E} \cong 2 \frac{\Delta v}{v}$$

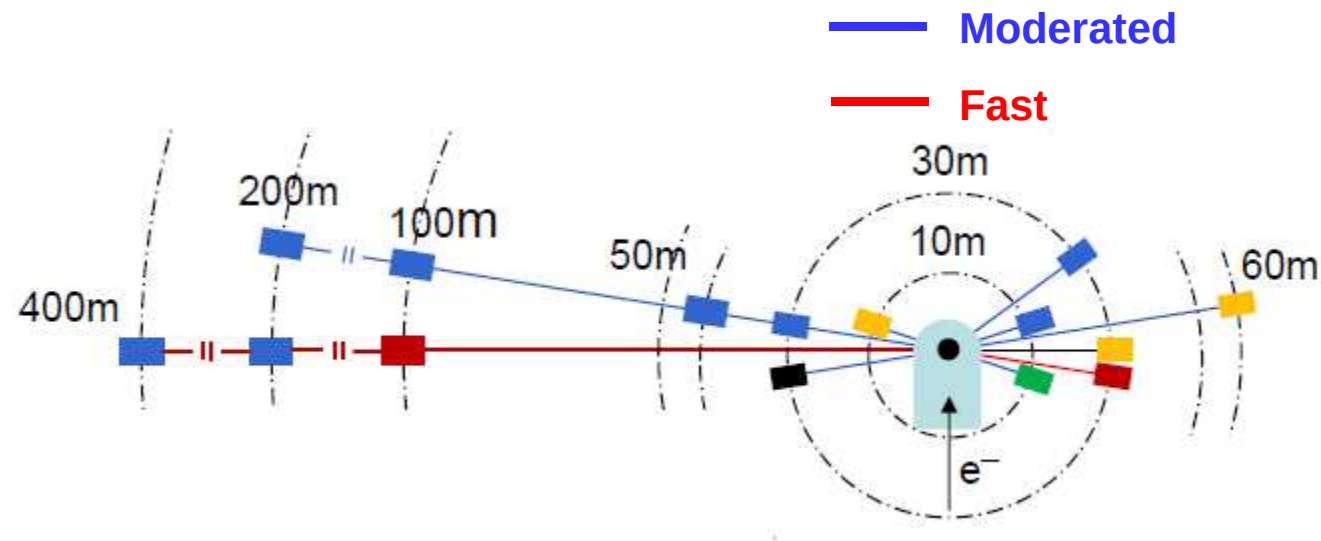
# $^{197}\text{Au}(n,\gamma)$ at $L = 12$ m and $30$ m

$$\frac{\Delta E}{E} \cong 2 \sqrt{\left(\frac{\Delta L}{L}\right)^2 + \left(\frac{\Delta t}{t}\right)^2}$$

$$\Delta_D = \sqrt{\frac{4 E k_B T}{m_x / m_n}} \quad \text{FWHM} = 2 \sqrt{\ln 2} \Delta_D$$



# TOF – measurements at GELINA



- $(n, \gamma)$
- $(n, \text{tot})$
- $(n, f)$  and  $(n, cp)$
- $(n, n' \gamma)$
- $(n, n)$



Velocity from TOF

$$v = \frac{L}{t}$$

- Neutron fluence rate  $\Rightarrow L \searrow$

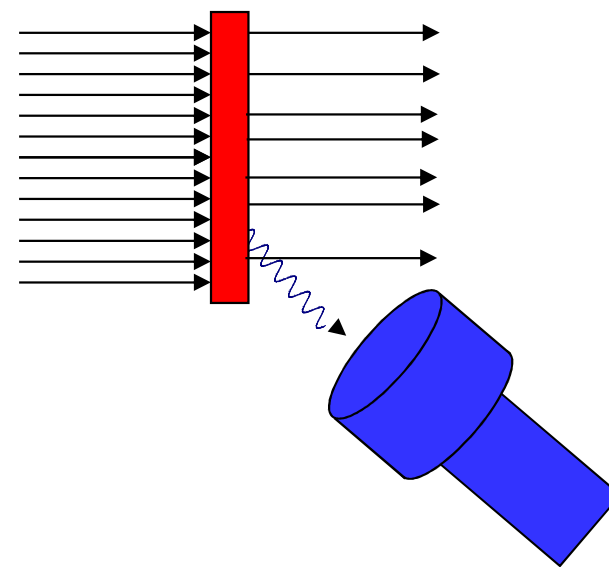
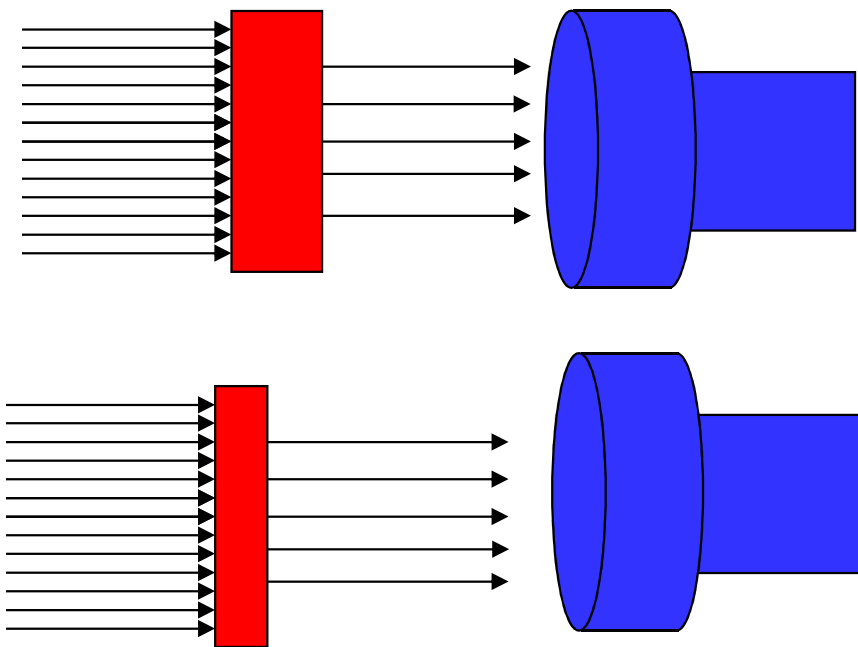
$$\phi(L) \propto \frac{1}{L^2}$$

- Resolution  $\Rightarrow L \nearrow$

$$\frac{\Delta v}{v} = \frac{1}{L} \sqrt{v^2 \Delta t^2 + \Delta L^2}$$

# Resonance parameters

- Determination of resonance parameters is a complex process and requires a set of complementary data
- Combine reaction cross section measurements on thin samples with transmission measurements on samples with different thicknesses



# Transmission measurements

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- Basic principles
- Background measurements (black resonance filters)
- Resolved resonance region
- Unresolved resonance region

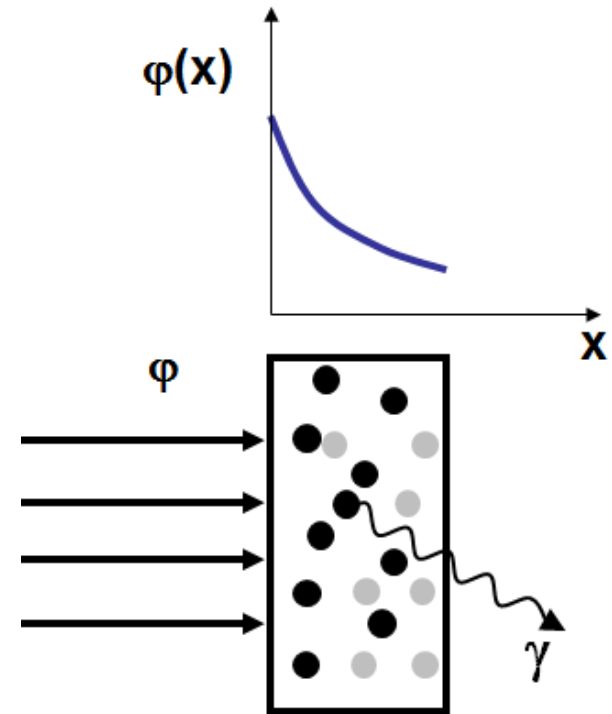
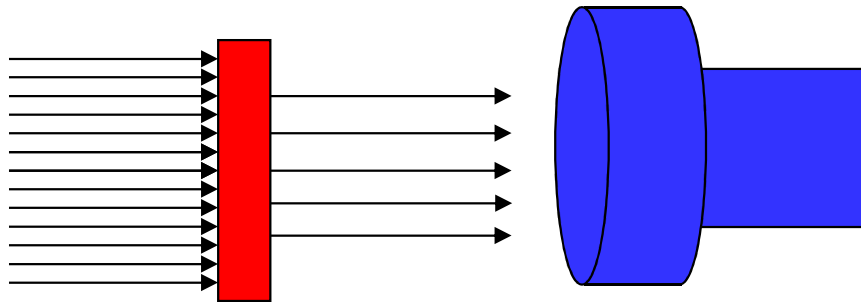
# Cross section measurements

## Transmission

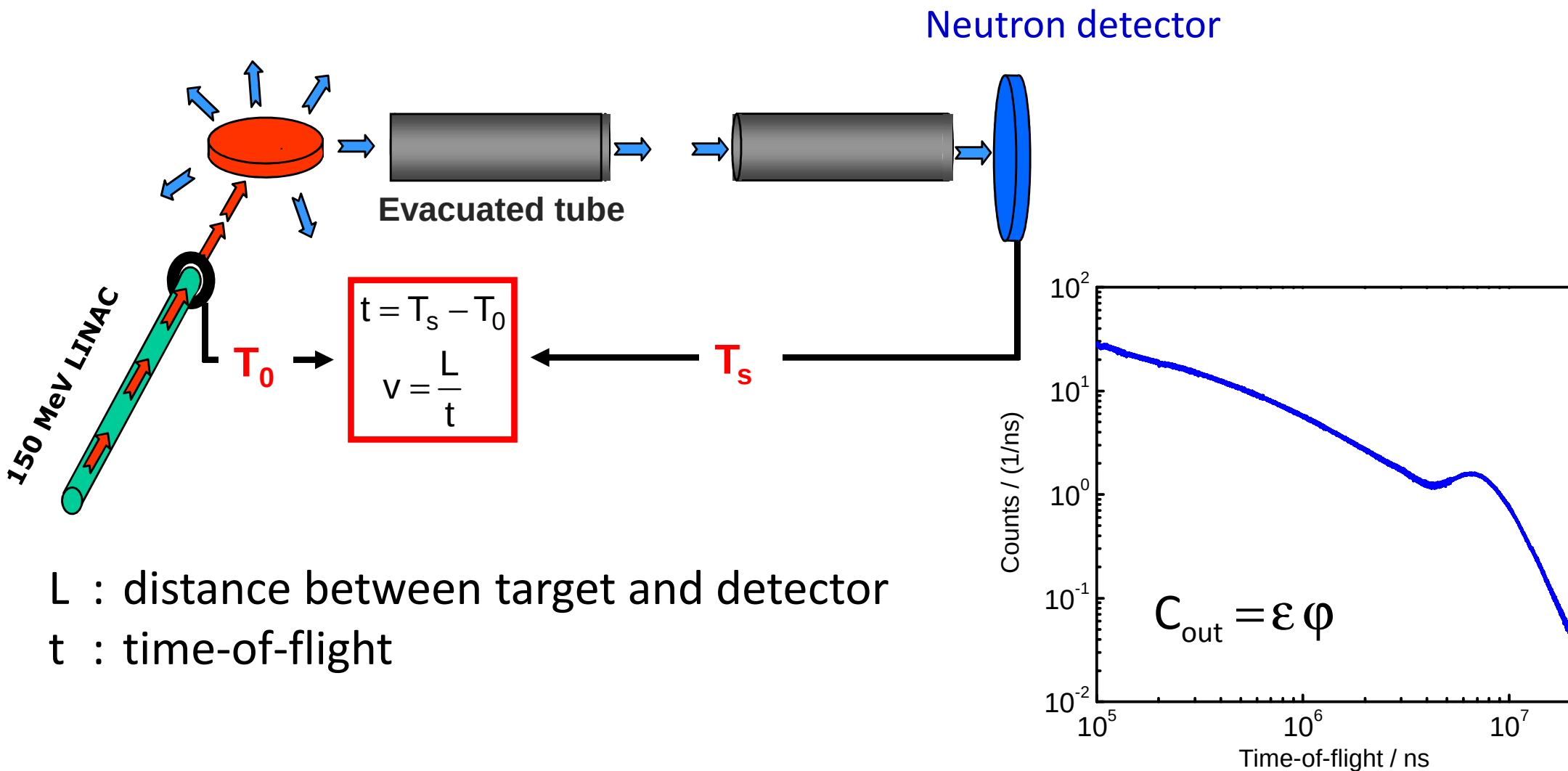
$$T \cong e^{-n \bar{\sigma}_{\text{tot}} x}$$

T : transmission

Fraction of the neutron beam traversing the sample without any interaction

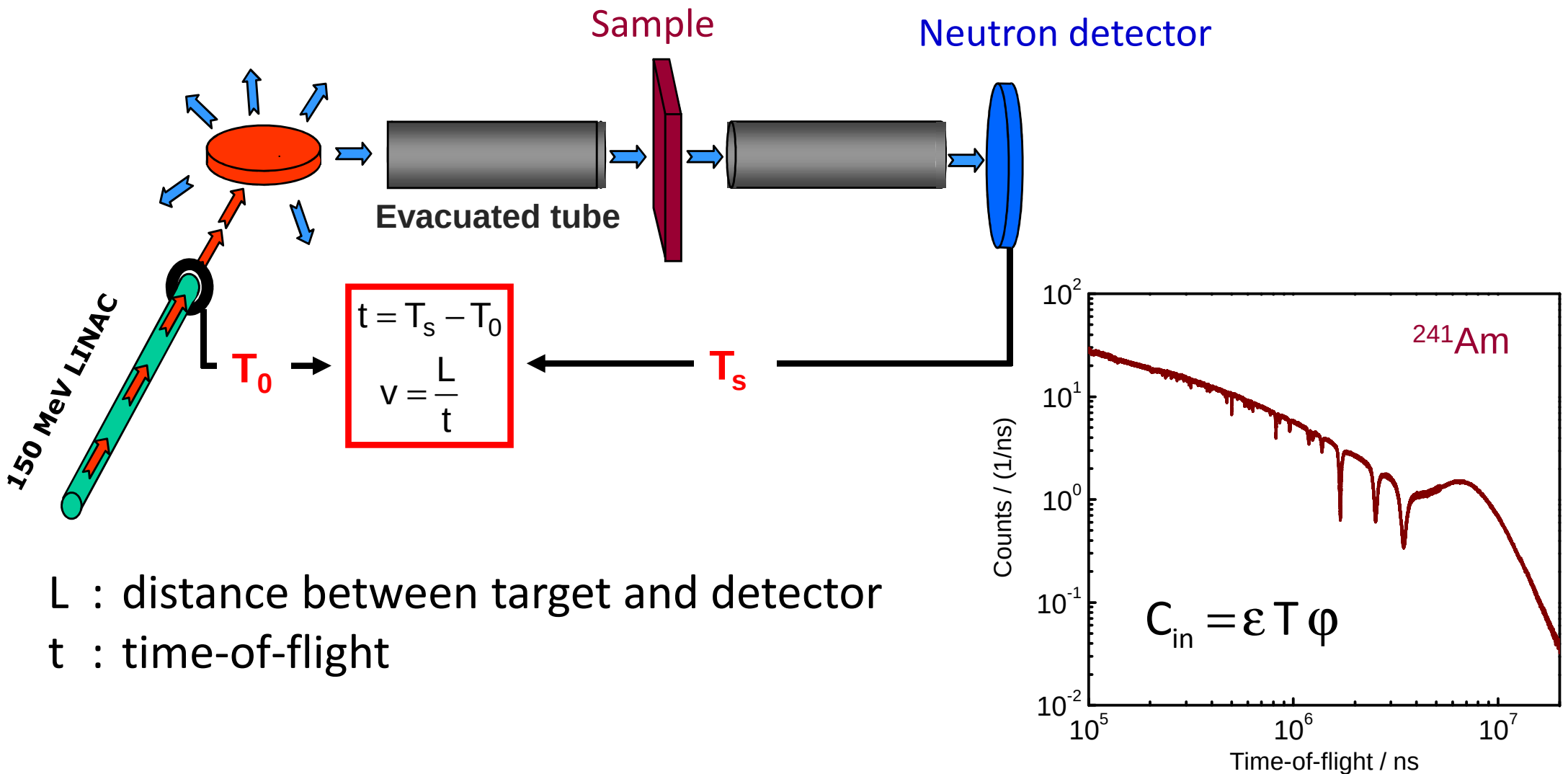


# TOF-cross section measurements



$L$  : distance between target and detector  
 $t$  : time-of-flight

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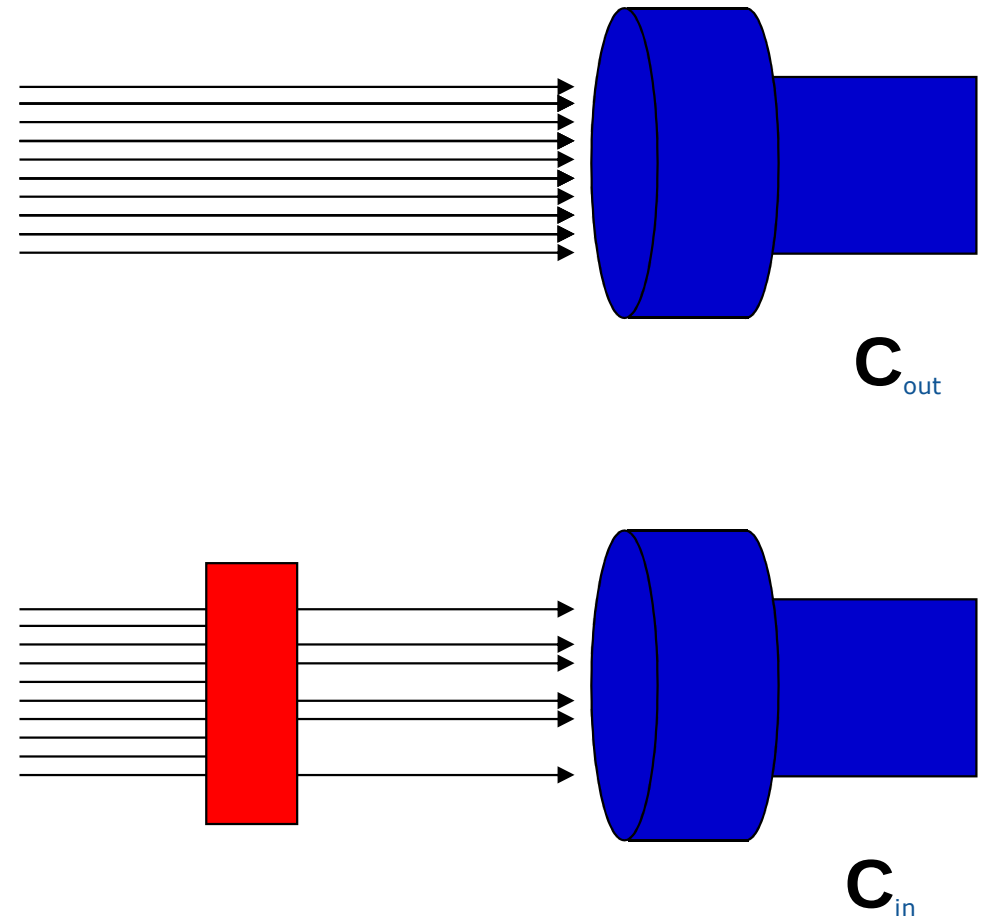


# Transmission measurements

## Transmission

$$T_{\text{exp}} = \frac{C_{\text{in}}}{C_{\text{out}}}$$

- Incoming neutron flux cancels
- Detection efficiency cancels



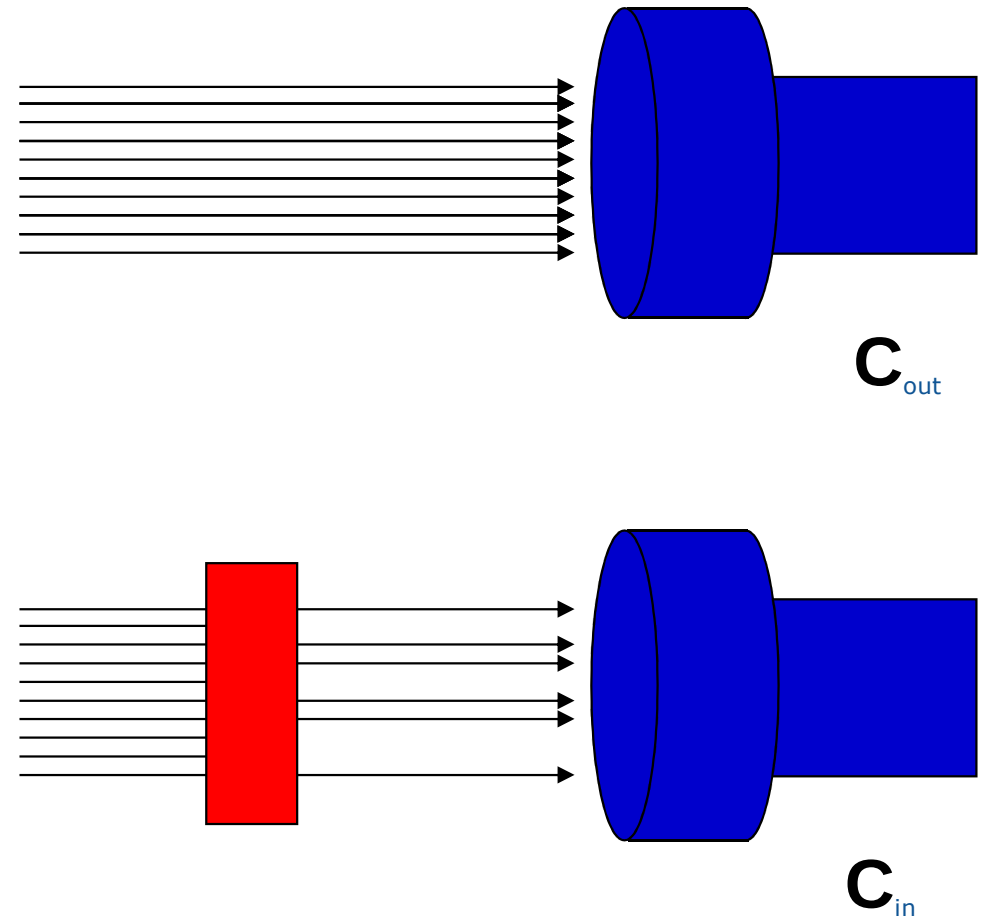
# Transmission measurements

## Transmission

$$T_{\text{exp}} = \frac{C_{\text{in}}}{C_{\text{out}}} \propto e^{-n\bar{\sigma}_{\text{tot}}}$$

- Incoming neutron flux cancels
- Detection efficiency cancels

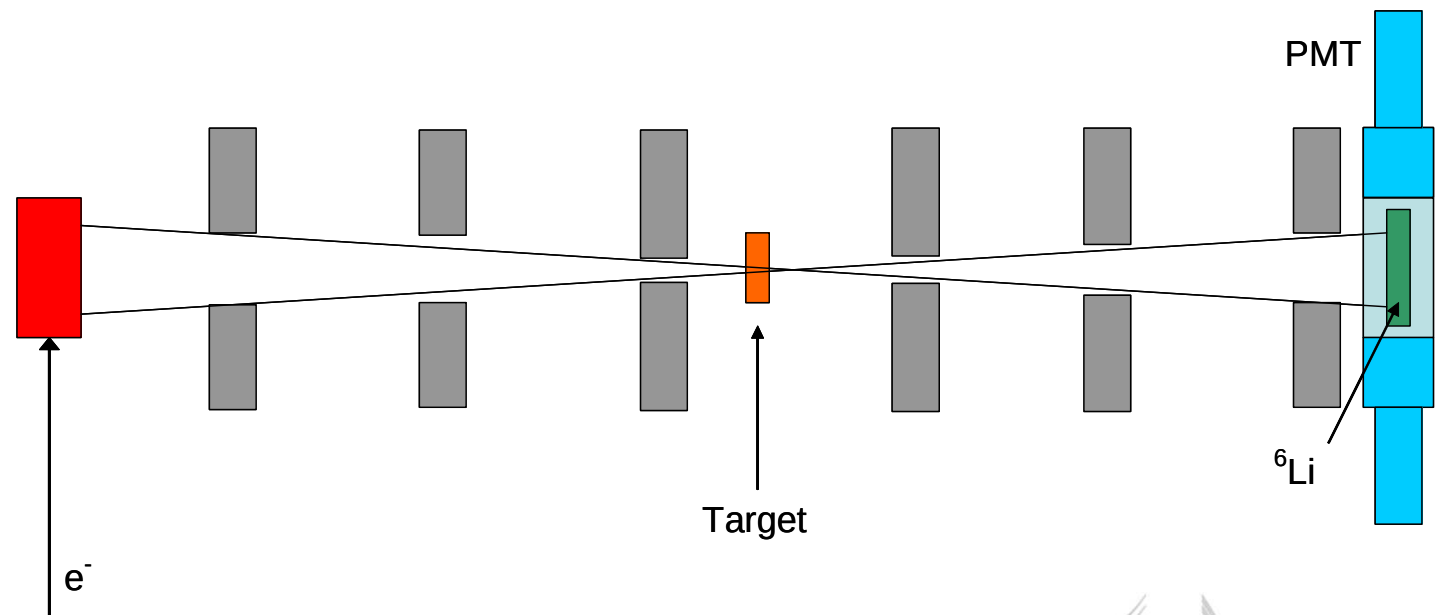
⇒ Direct relation between  $T_{\text{exp}}$  and  $\sigma_{\text{tot}}$



# Transmission : principle

$$T_{\text{exp}} = \frac{C_{\text{in}}}{C_{\text{out}}} \propto e^{-n\bar{\sigma}_{\text{tot}}}$$

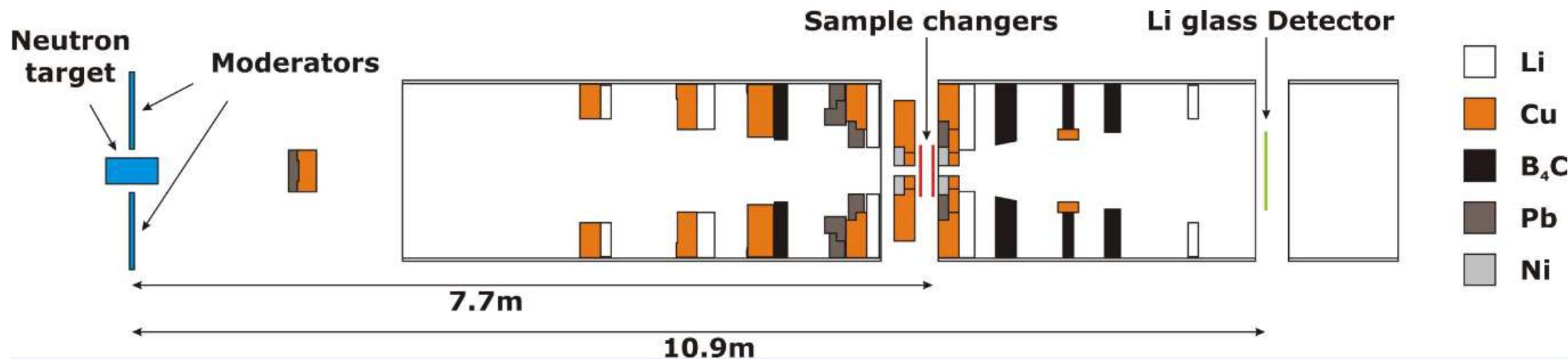
- (1) All detected neutrons passed through the sample
- (2) Neutrons scattered in the target do not reach detector
- (3) Sample perpendicular to parallel neutron beam  
 $\Rightarrow$  Good transmission geometry (collimation)
- (4) Homogeneous target (no spatial distribution of  $n$ )



# Transmission measurements

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# Transmission station at GELINA

**$^6\text{Li}$  detector**



**Castle**



**Neutron target +  
moderators**

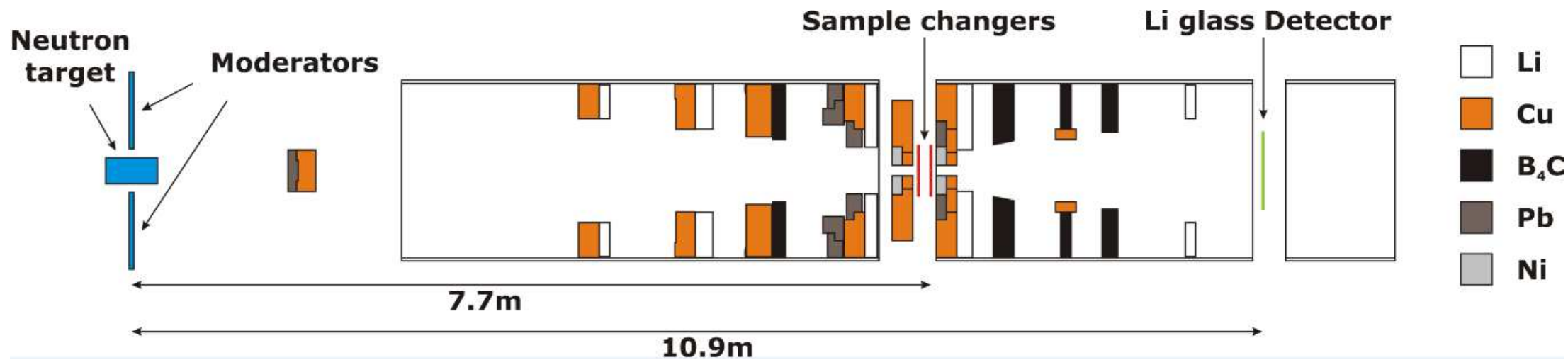
# Transmission : principle

$$T_{\text{exp}} = \frac{C_{\text{in}}}{C_{\text{out}}} \propto e^{-n\bar{\sigma}_{\text{tot}}}$$

## Detectors

Low energy :  ${}^6\text{Li}(n,t)\alpha$       Li-glass

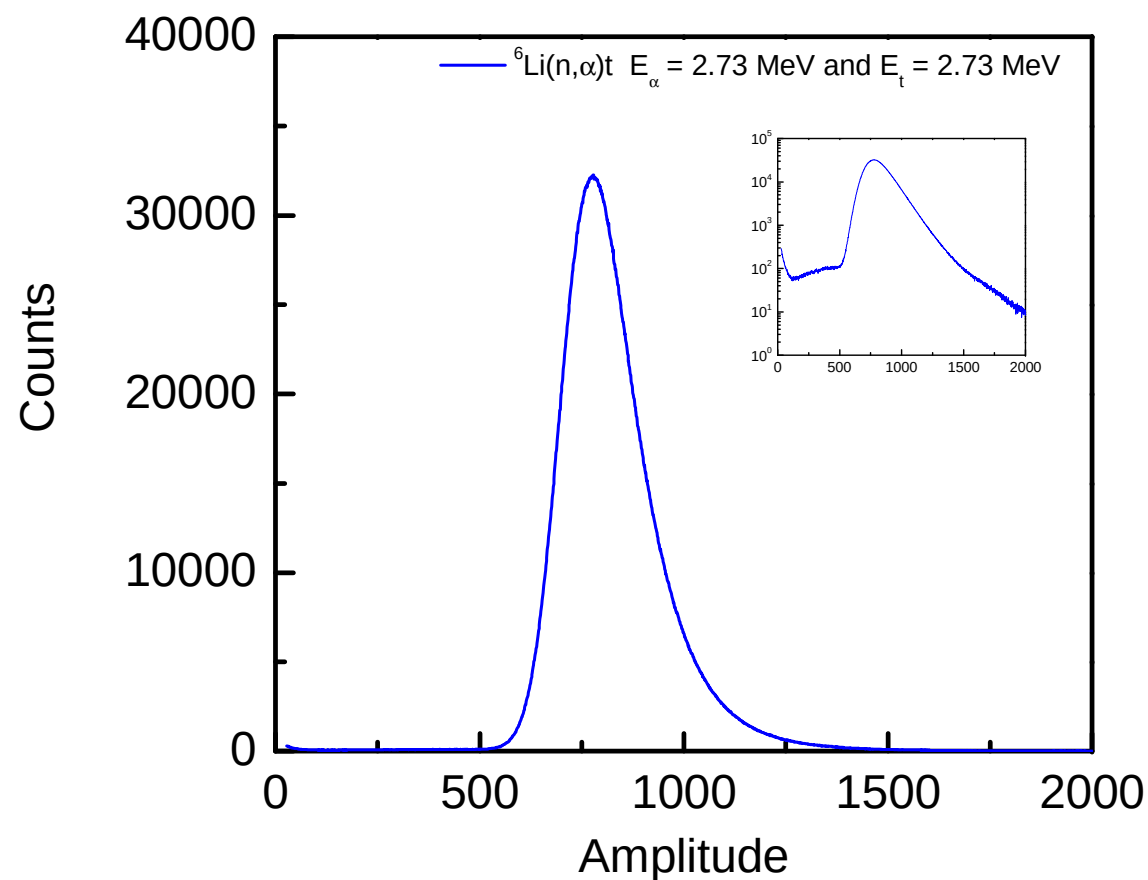
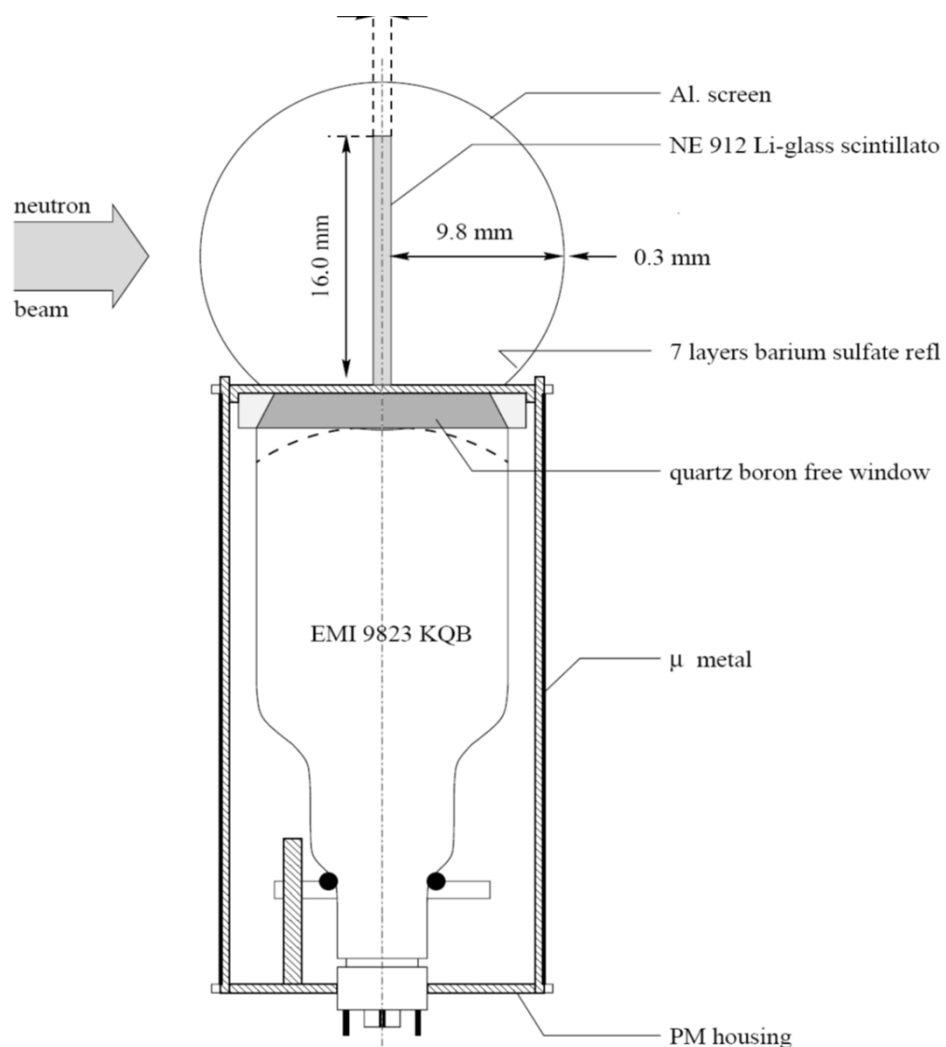
High energy :  $\text{H}(n,n)\text{H}$       Plastic scintillator



# Lithium-glass scintillator : energy deposition

${}^6\text{Li}(n,t)\alpha$

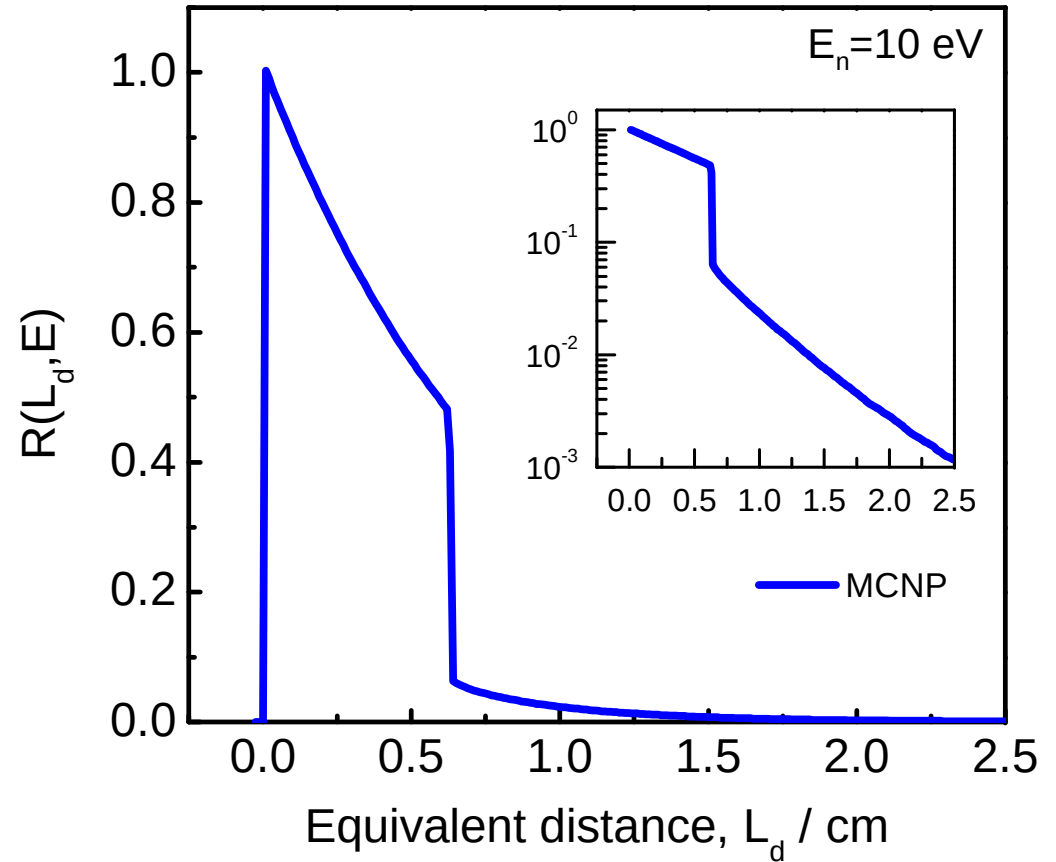
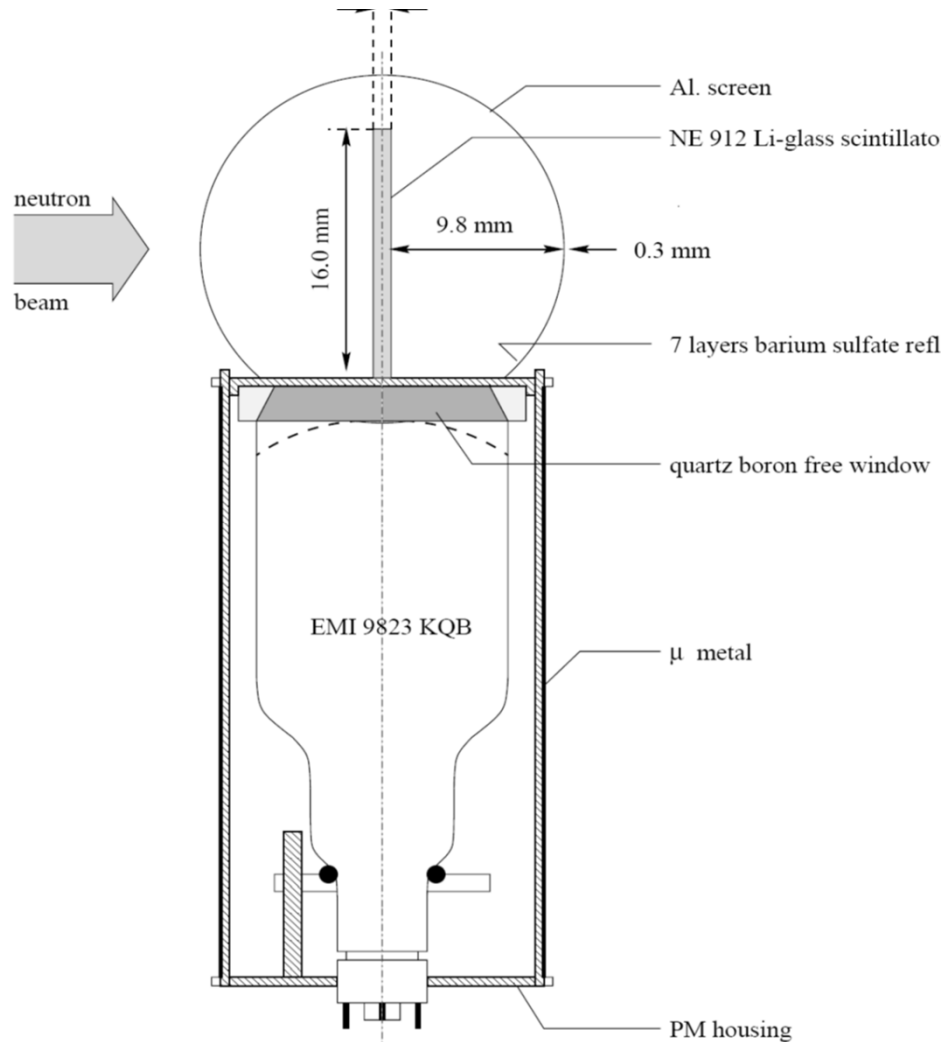
Scintillator + PMT



# Lithium-glass scintillator : resolution

${}^6\text{Li}(n,t)\alpha$

Scintillator + PMT

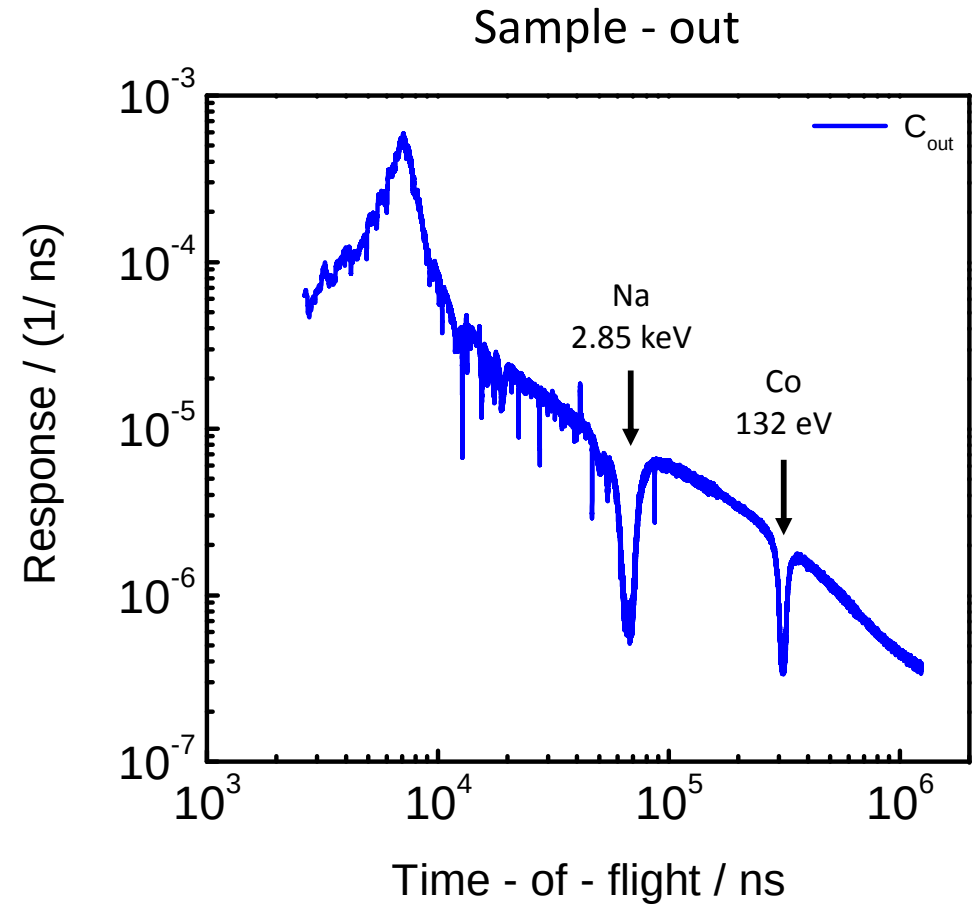
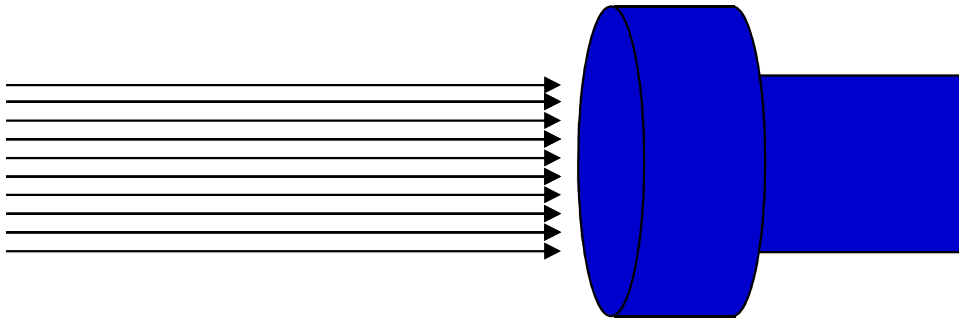




# Transmission : sample-out

- Detector

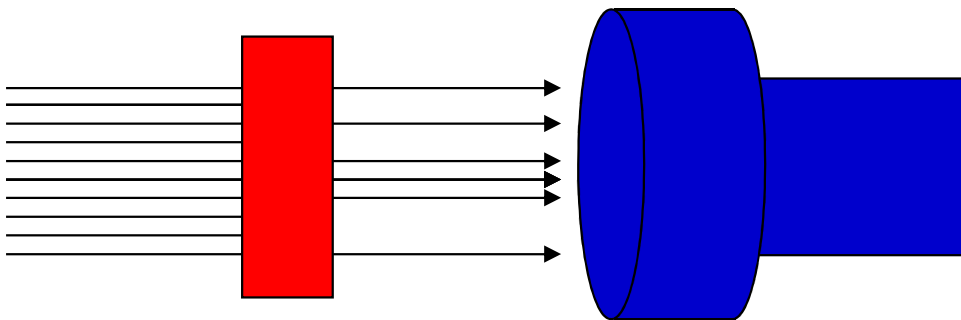
- NE912 Li-glass scintillator, 95% enriched in  $^6\text{Li}$   
diameter : 101.1 mm  
thickness : 6.35 mm
- at 49.34 m from neutron source



# Transmission : sample-in

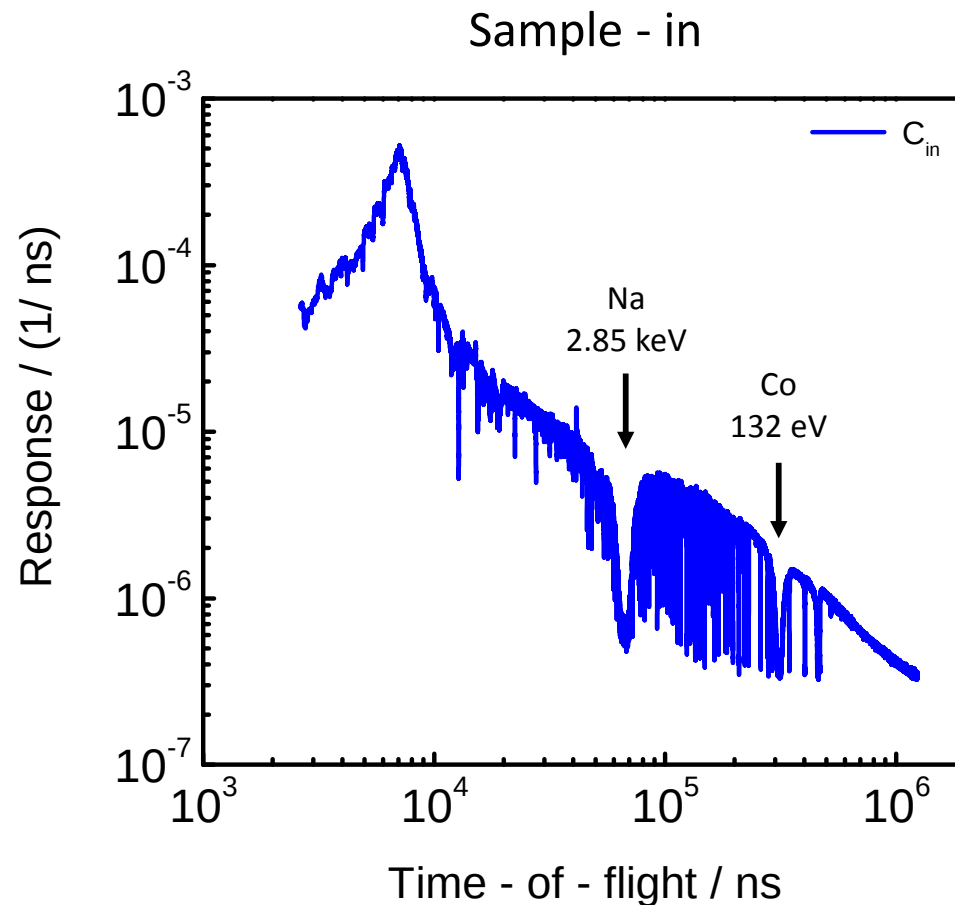
- **Detector**

- NE912 Li-glass scintillator, 95% enriched in  $^6\text{Li}$   
diameter : 101.1 mm  
thickness : 6.35 mm
- at 49.34 m from neutron source



- **Sample**

- Au metal foil  
50 mm x 50 mm x 3 mm  
1.757 (0.004)  $10^{-2}$  at/b
- at 23.78 m from neutron source



# Background: black resonance technique

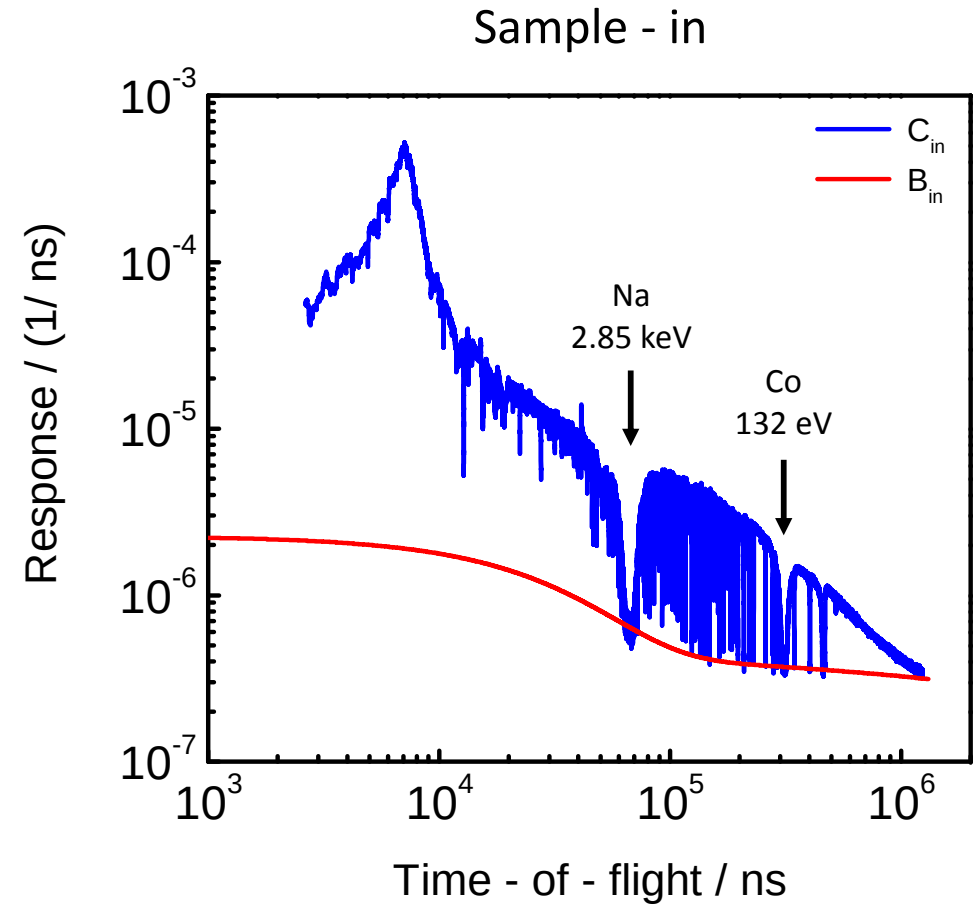
## Black resonance filter

- strong resonance at  $E_r$

$$T = e^{-n\bar{\sigma}_{\text{tot}}} \approx 0$$

- removes all neutrons at TOF corresponding to  $E_r$

⇒ Remaining counts are due to background

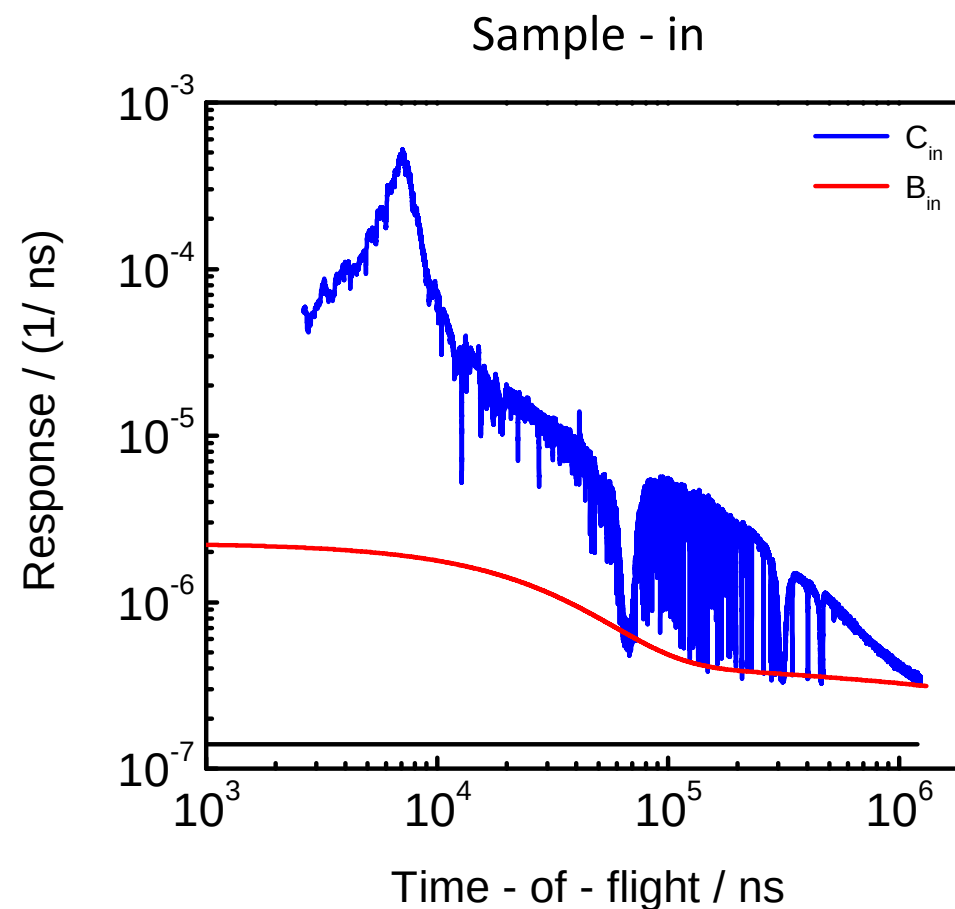


# Background: black resonance technique

$$B(t) = B_0$$

- $B_0$  time independent

From measurement with no beam

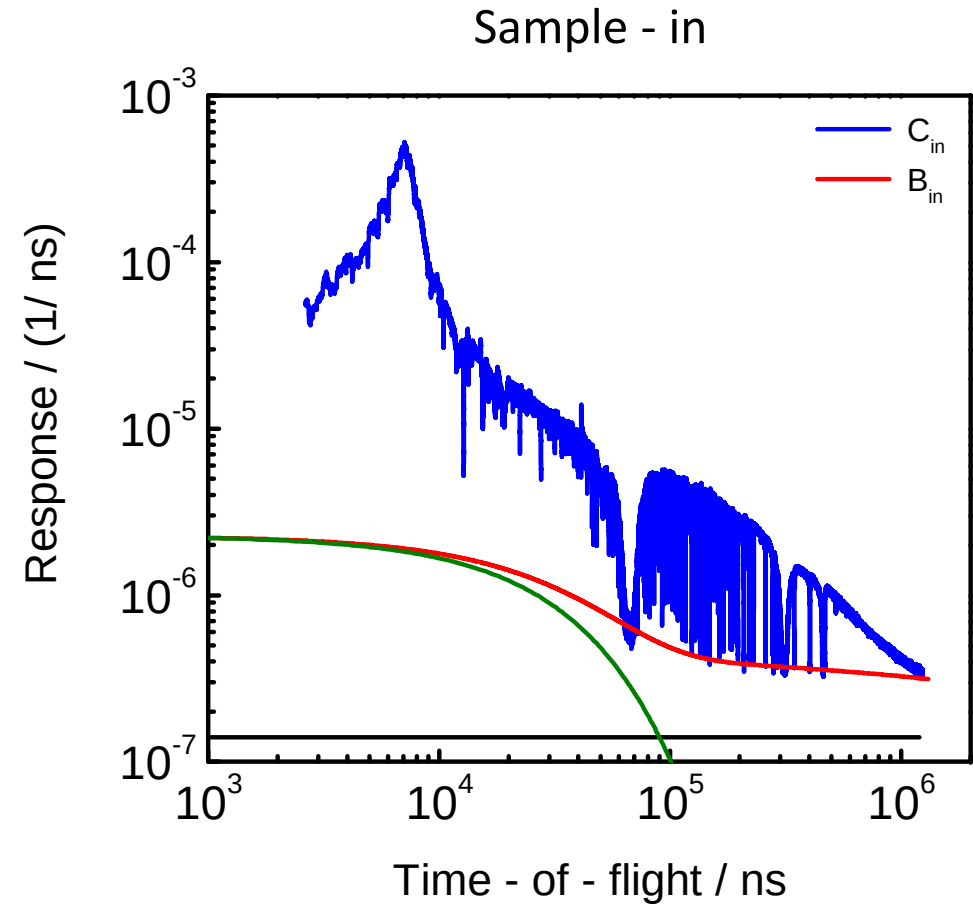


# Background: black resonance technique

$$B(t) = B_0 + B_\gamma(t)$$

- $B_0$  time independent
- $B_\gamma(t)$   ${}^1\text{H}(n, \gamma)$   $E_\gamma = 2.2 \text{ MeV}$   
 $b_1 e^{-\lambda_1 t}$

Shape from measurement with thick poly-ethylene filter in the beam to remove neutrons

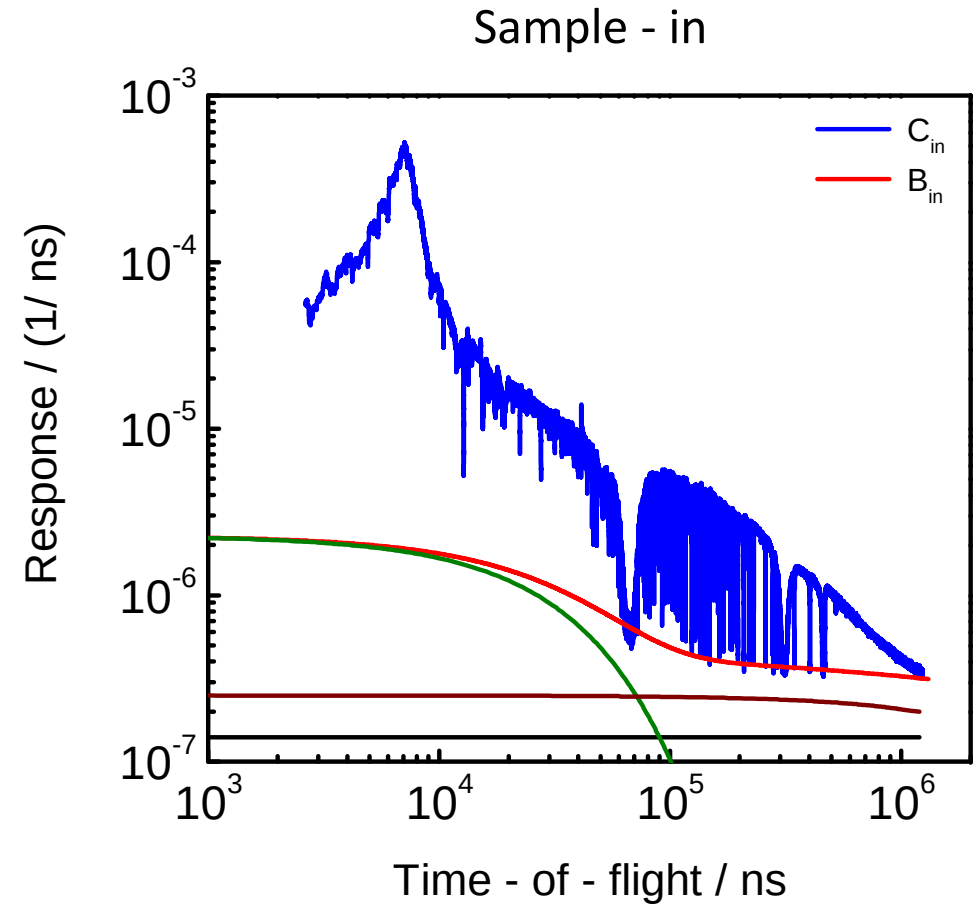


# Background: black resonance technique

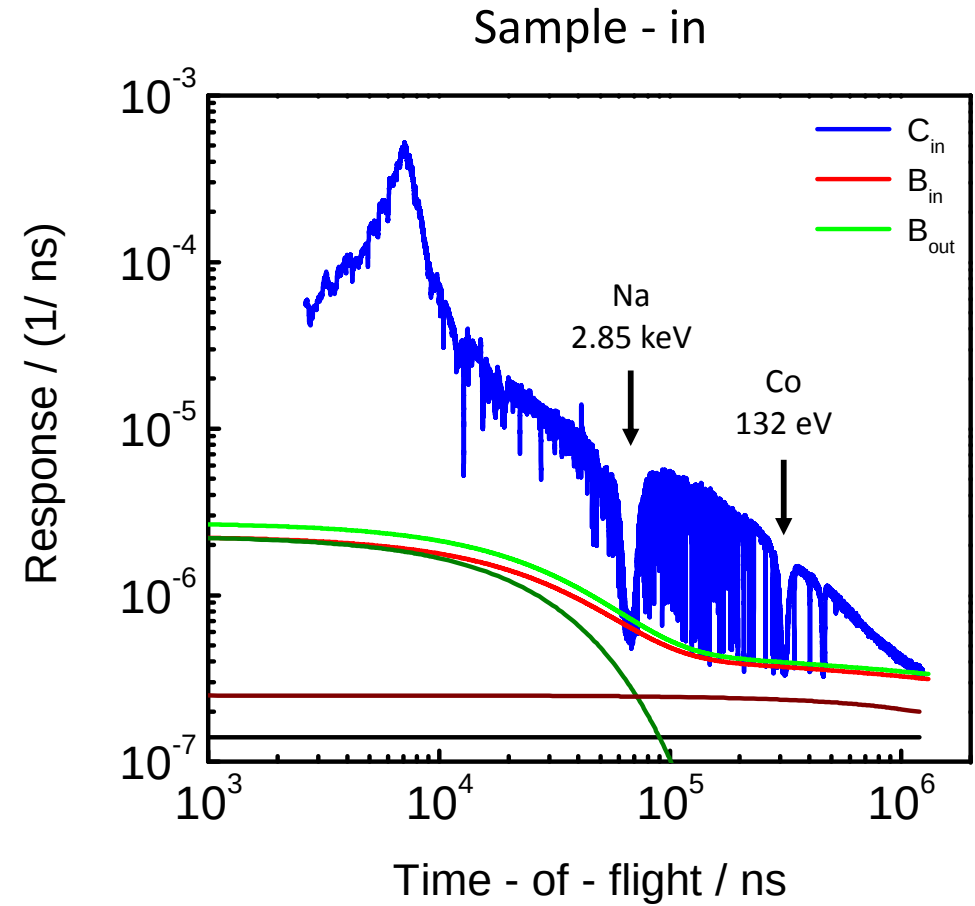
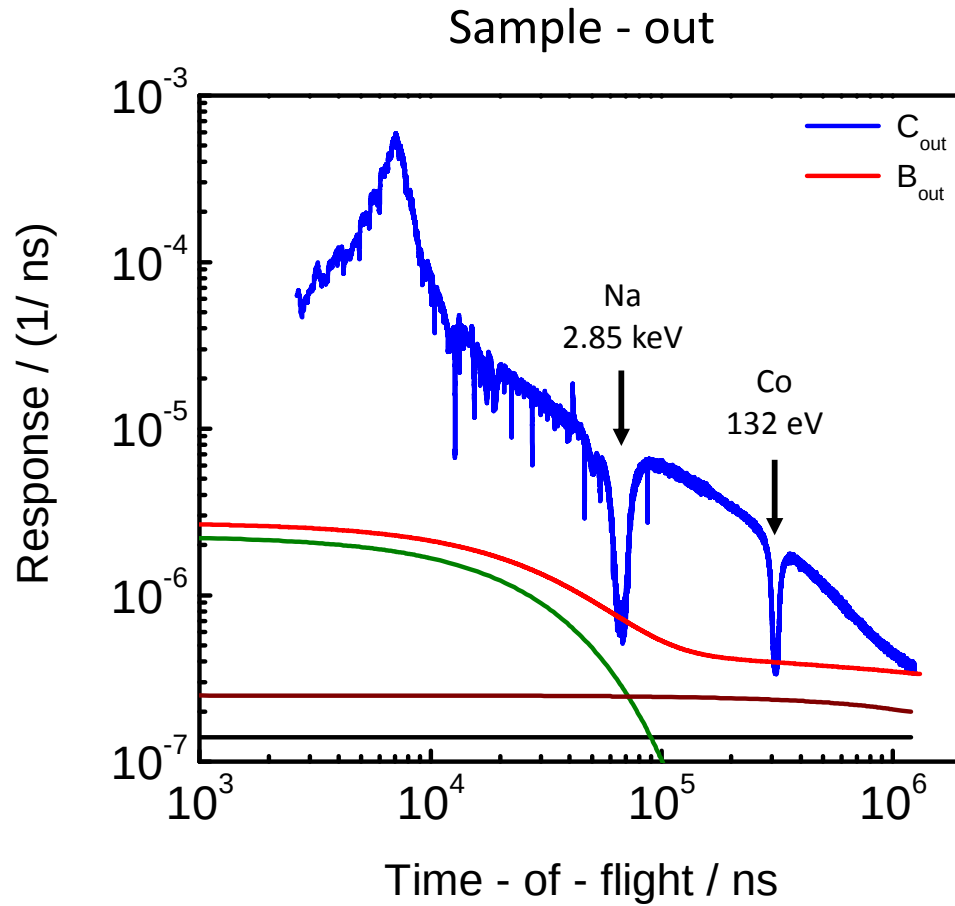
$$B(t) = B_0 + B_\gamma(t) + B_n(t)$$

- $B_0$  time independent
- $B_\gamma(t)$   $^1\text{H}(n, \gamma)$   $E_\gamma = 2.2 \text{ MeV}$   
 $b_1 e^{-\lambda_1 t}$
- $B_n(t)$  scattered neutrons  
 $b_2 e^{-\lambda_2 t}$

Shape from measurement with  
Pb-filter + black resonance filters



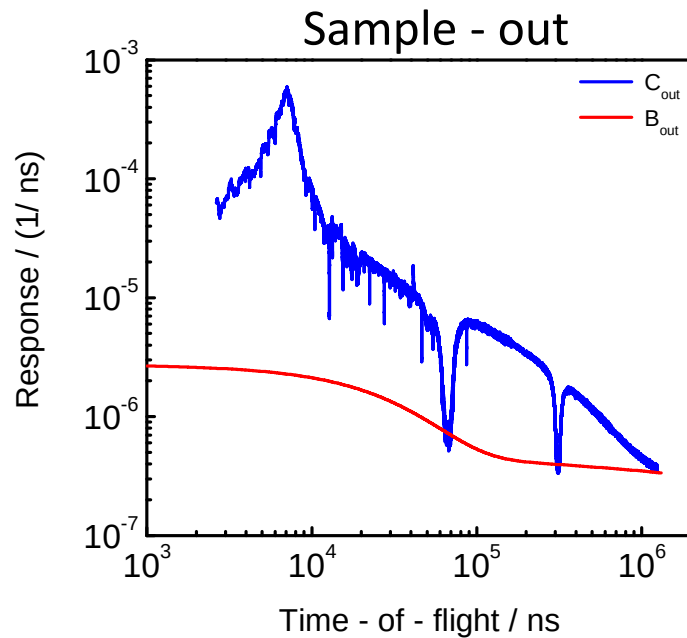
# Background: black resonance technique



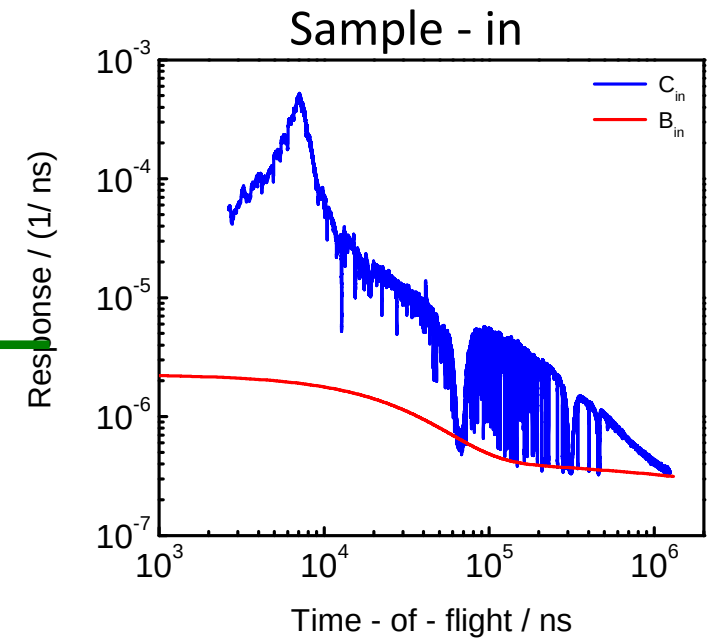
Background influenced by sample  
⇒ use of fixed background filters to adjust  $b_1$  and  $b_2$

$$B = b_0 + \boxed{b_1} e^{-\lambda_1 t} + \boxed{b_2} e^{-\lambda_2 t}$$

# TOF-spectra $\Rightarrow T_{\text{exp}}$

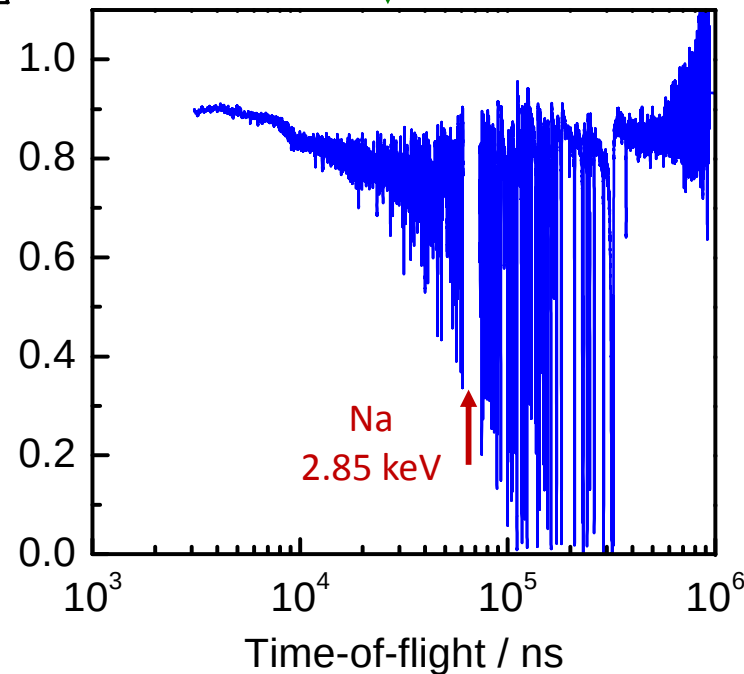


$$T_{\text{exp}} = \frac{C_{\text{in}} - K B_{\text{in}}}{C_{\text{out}} - K B_{\text{out}}}$$



Factor K introduces a correlated uncertainty due to the background model

Transmission

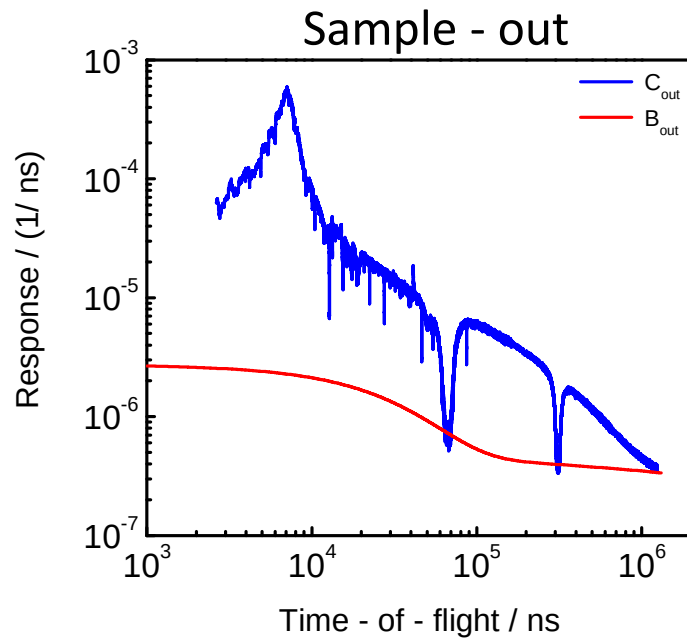


Sample - in

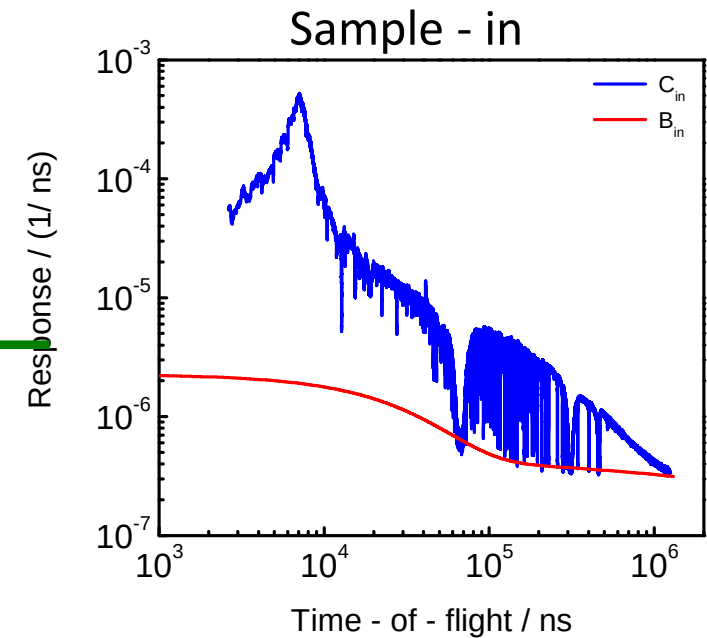
$$K = 1.00 \pm 0.03$$



# TOF-spectra $\Rightarrow T_{\text{exp}}$

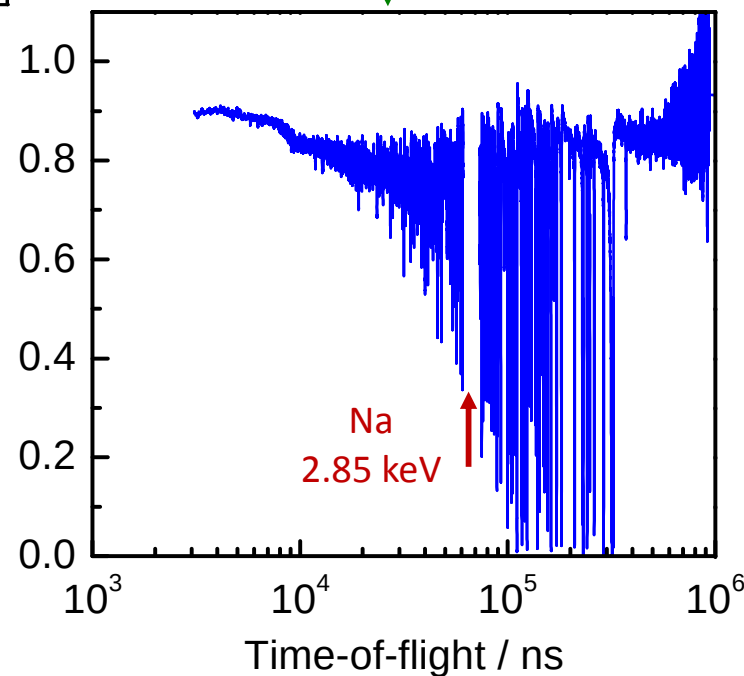


$$T_{\text{exp}} = N \frac{C_{\text{in}} - K B_{\text{in}}}{C_{\text{out}} - K B_{\text{out}}}$$



Factor N introduces a correlated uncertainty due to the normalisation

Transmission

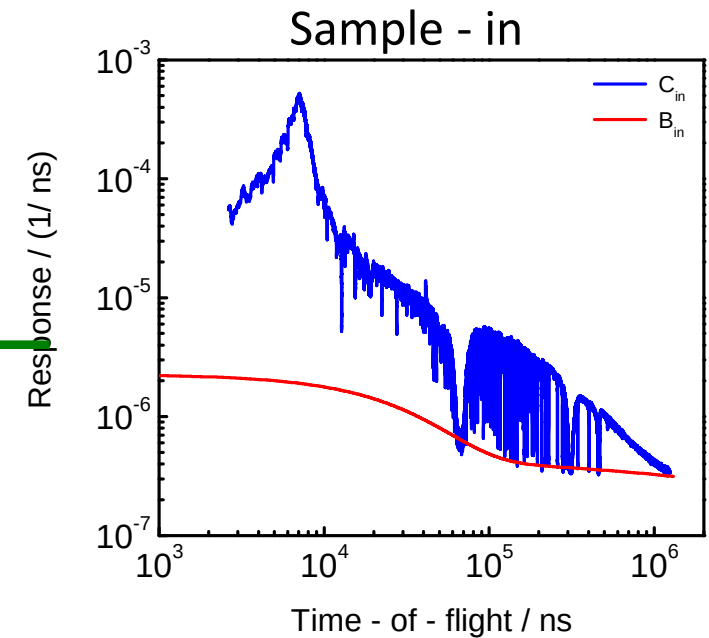
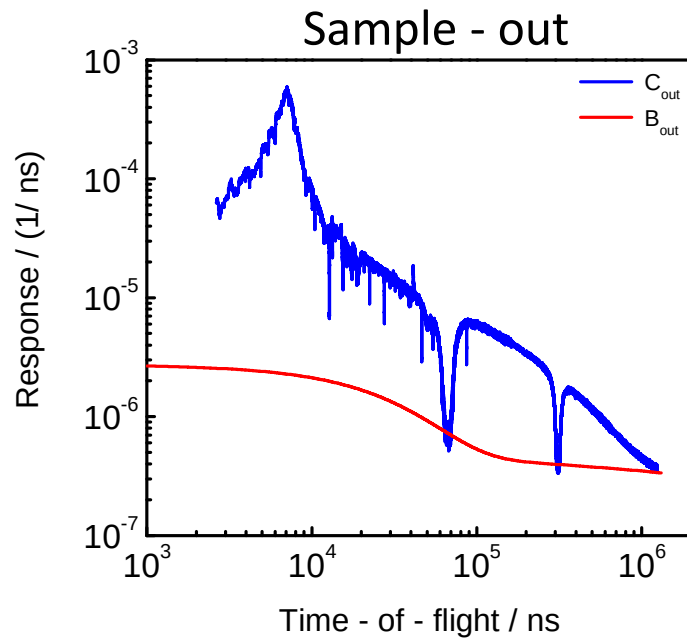


Sample - in

$$K = 1.00 \pm 0.03$$

$$N = 1.0000 \pm 0.0025$$

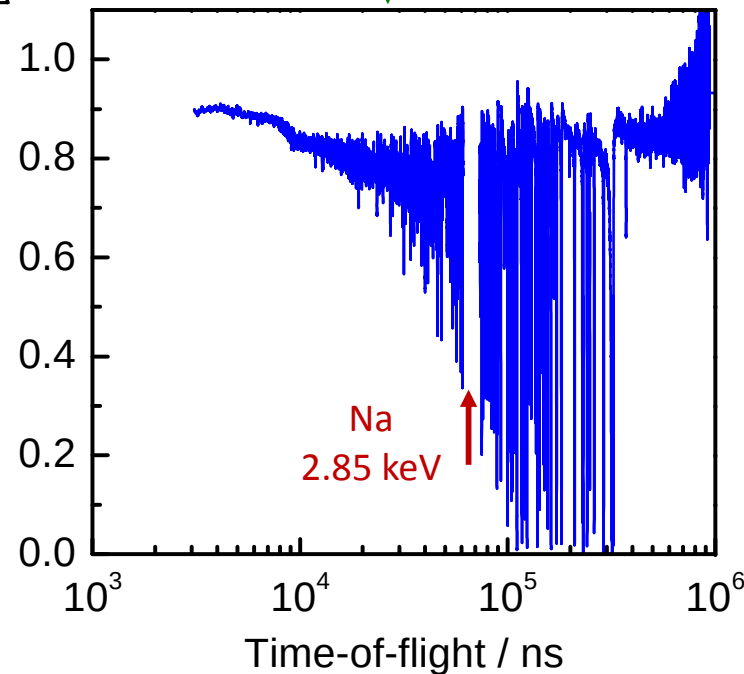
# TOF-spectra $\Rightarrow T_{\text{exp}}$



$$T_{\text{exp}} = N \frac{C_{\text{in}} - K B_{\text{in}}}{C_{\text{out}} - K B_{\text{out}}}$$

Repeated measurements of sample-in and sample-out cycles to avoid impact of:

- Electronic drifts
- Variations in beam intensity

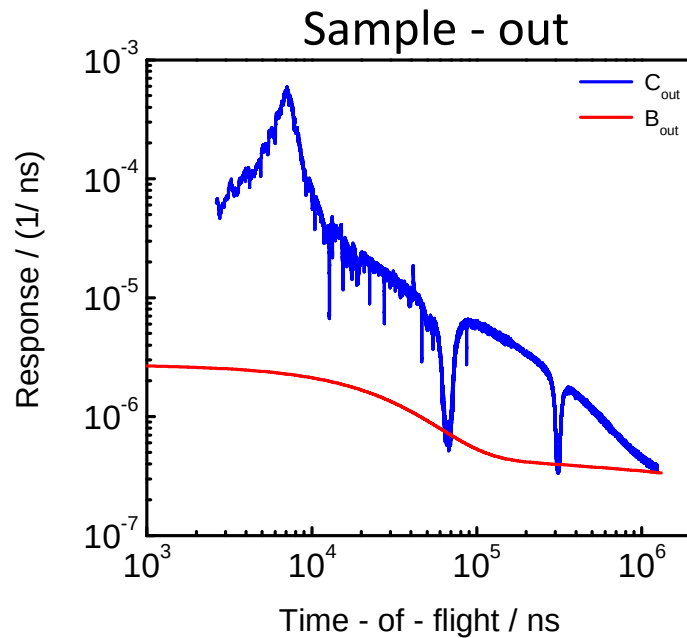


Sample - in

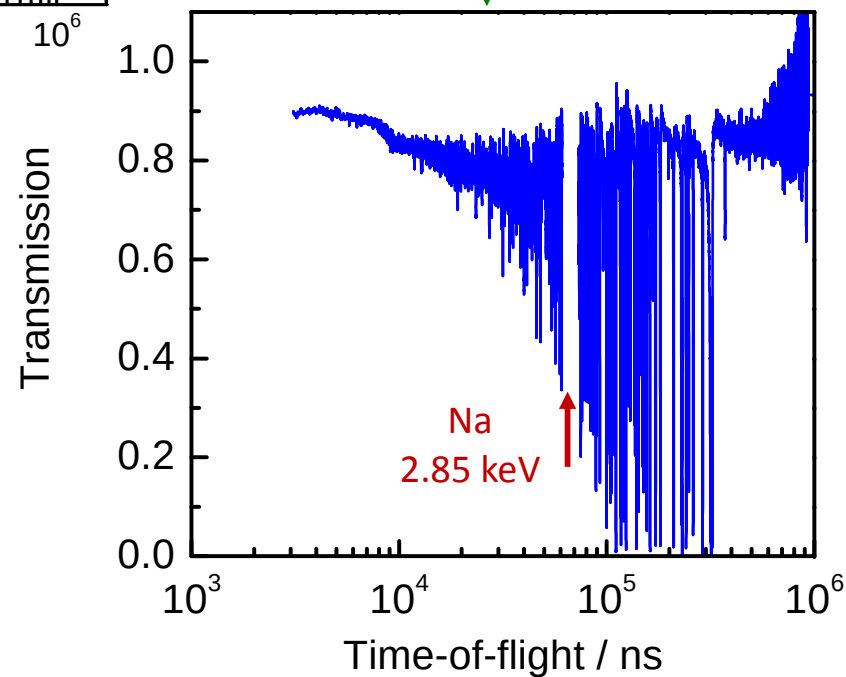
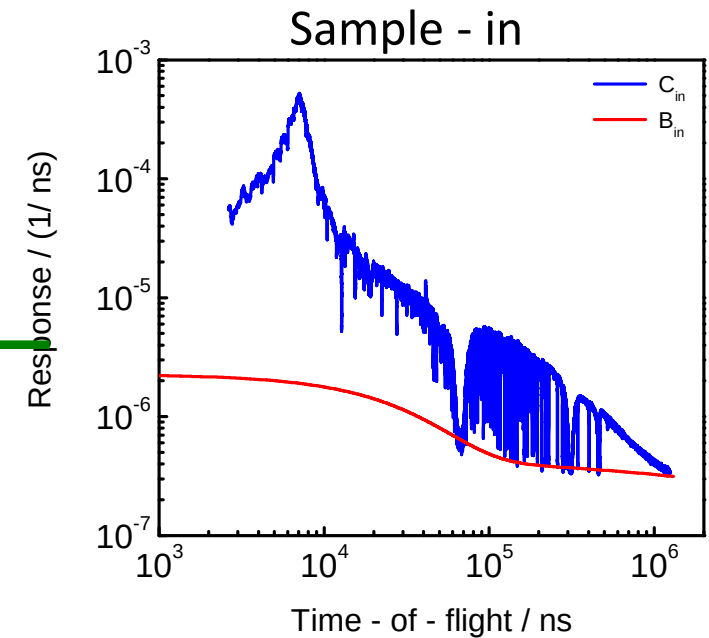
$$K = 1.00 \pm 0.03$$

$$N = 1.0000 \pm 0.0025$$

# TOF-spectra $\Rightarrow T_{\text{exp}}$



$$T_{\text{exp}} = N \frac{C_{\text{in}} - K B_{\text{in}}}{C_{\text{out}} - K B_{\text{out}}}$$



Sample - in

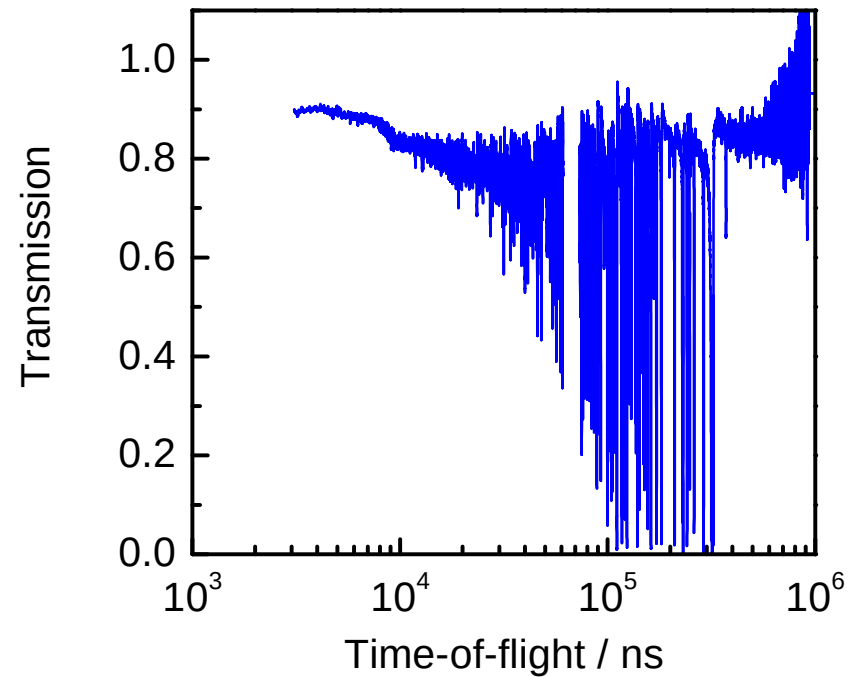
$$\frac{u_K}{K} \approx 3\%$$

$$\frac{u_N}{N} \approx 0.25\%$$

# Resonance region

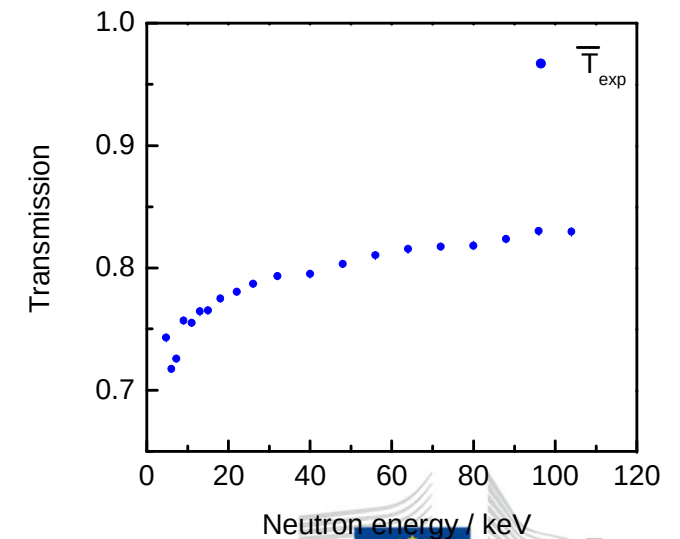
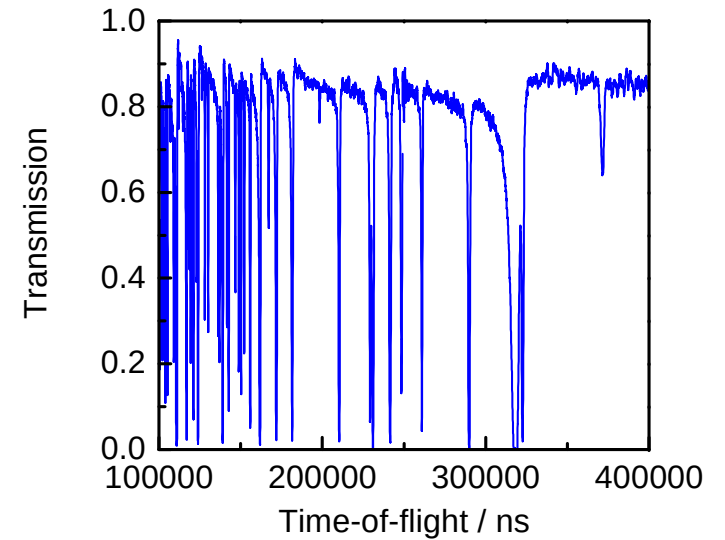
## Resonance Region $D > \Gamma$

- Resolved Resonance Region  $\Delta_R < D$
- Unresolved Resonance Region  $\Delta_R > D$



RRR  
Resonance shape analysis

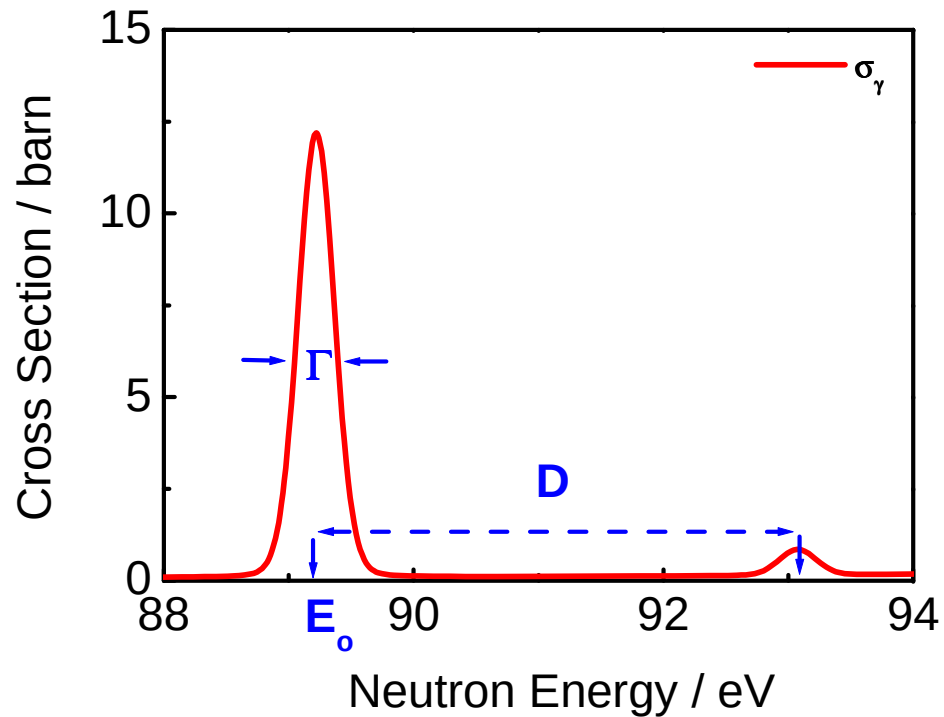
URR  
Average parameters



# Data analysis

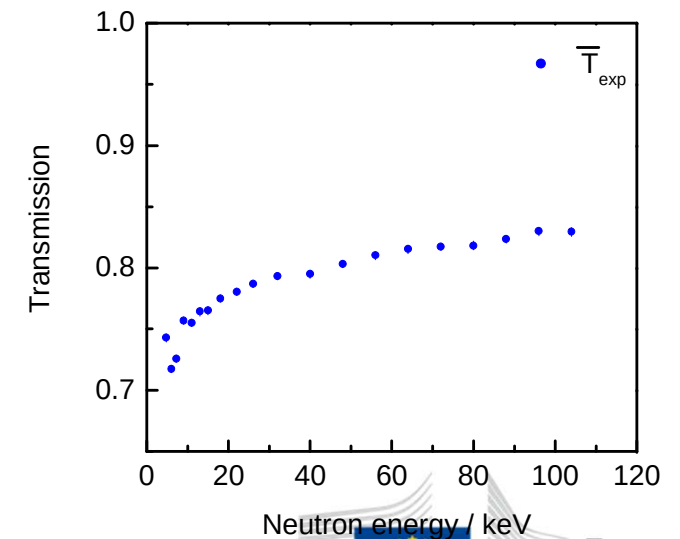
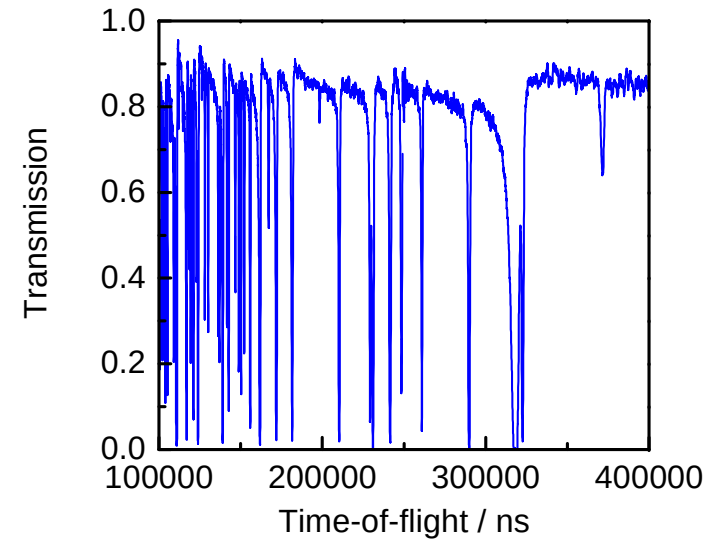
## Resonance Region $D > \Gamma$

- Resolved Resonance Region  $\Delta_R < D$
- Unresolved Resonance Region  $\Delta_R > D$



RRR  
Resonance shape analysis

URR  
Average parameters



# Transmission data : $^{241}\text{Am} + n$

Determine resonance parameters from a GLSQ fit to the experimental data

$$T_{\text{exp}} = \frac{C_{\text{in}} - B_{\text{in}}}{C_{\text{out}} - B_{\text{out}}} \quad \frac{u_{T_{\text{exp}}}}{T_{\text{exp}}} < 0.25\%$$

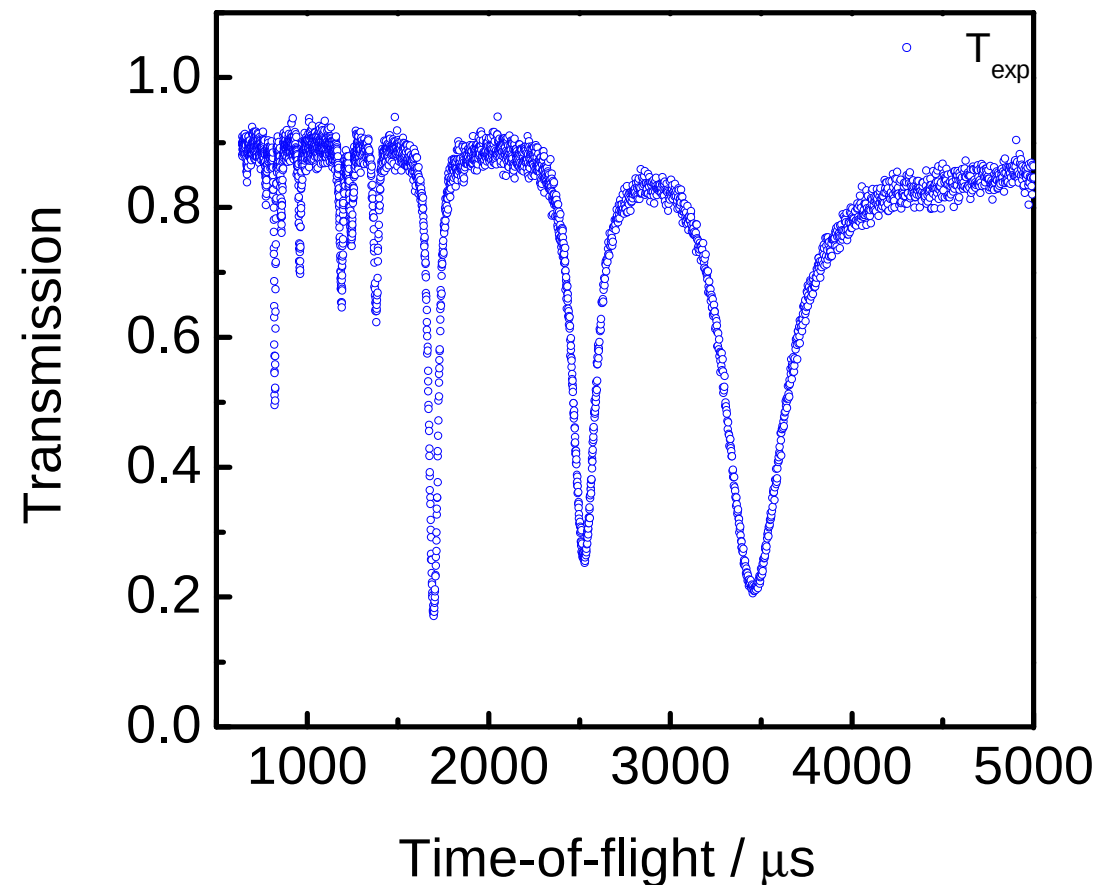
$$T_M(t) = \int R(t, E) e^{-n \bar{\sigma}_{\text{tot}}(E)} dE$$

$R(t_m, E)$  : response of TOF-spectrometer

$\sigma_{\text{tot}}$  : total cross section

$n$  : areal number density  
total number of atoms per unit area

$$\chi^2(\text{RP}) = (T_{\text{exp}} - T_M)^T V_{T_{\text{exp}}}^{-1} (T_{\text{exp}} - T_M)$$



# Transmission data : $^{241}\text{Am} + n$

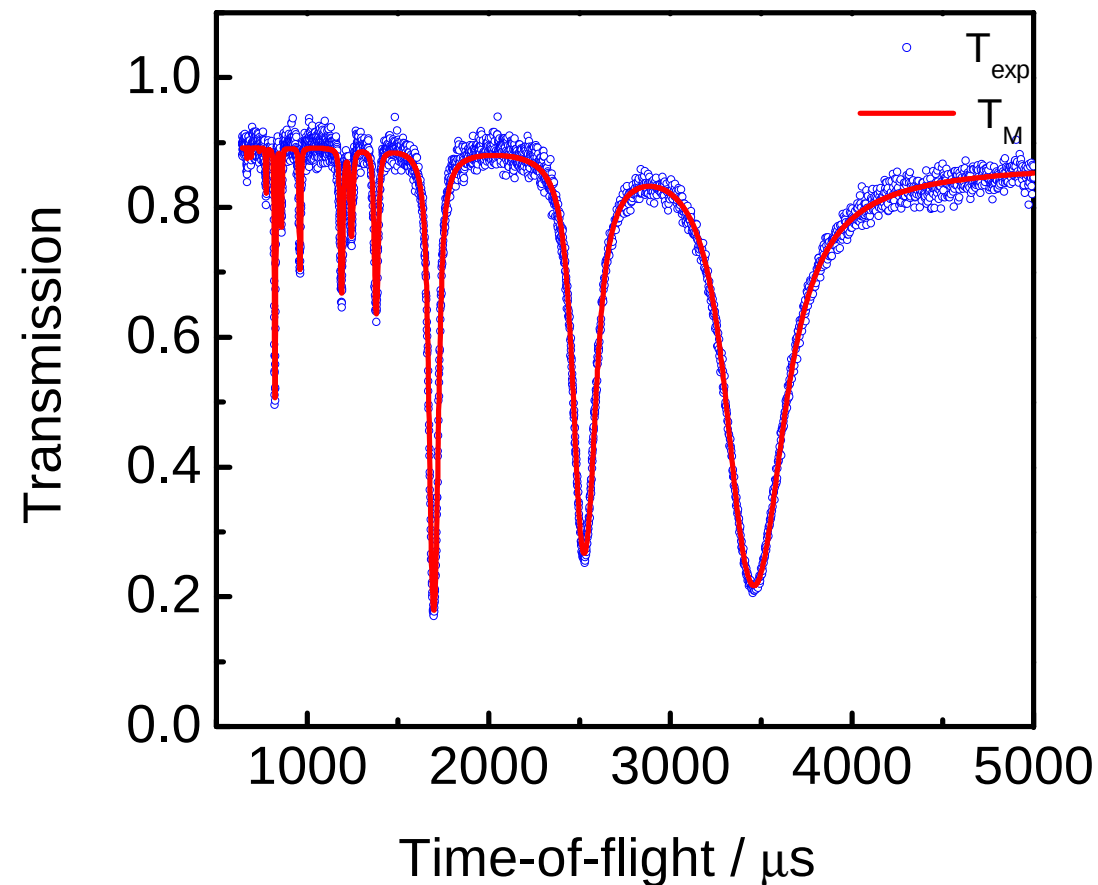
Determine resonance parameters from a GLSQ fit to the experimental data  
Resonance shape analysis (RSA) with **REFIT**

$$T_{\text{exp}} = \frac{C_{\text{in}} - B_{\text{in}}}{C_{\text{out}} - B_{\text{out}}} \quad \frac{u_{T_{\text{exp}}}}{T_{\text{exp}}} < 0.25\%$$

$$T_M(t) = \int R(t, E) e^{-n \bar{\sigma}_{\text{tot}}(E)} dE$$

$R(t_m, E)$  : response of TOF-spectrometer  
 $\sigma_{\text{tot}}$  : total cross section  
 $n$  : areal number density  
total number of atoms per unit area

$$\chi^2(\text{RP}) = (T_{\text{exp}} - T_M)^T V_{T_{\text{exp}}}^{-1} (T_{\text{exp}} - T_M)$$



# Transmission data : $^{241}\text{Am} + n$

Determine resonance parameters from a GLSQ fit to the experimental data  
Resonance shape analysis (RSA) with **REFIT**

$^{241}\text{Am}$  :  $I^\pi = 5/2^-$   $\ell = 0$  from shape

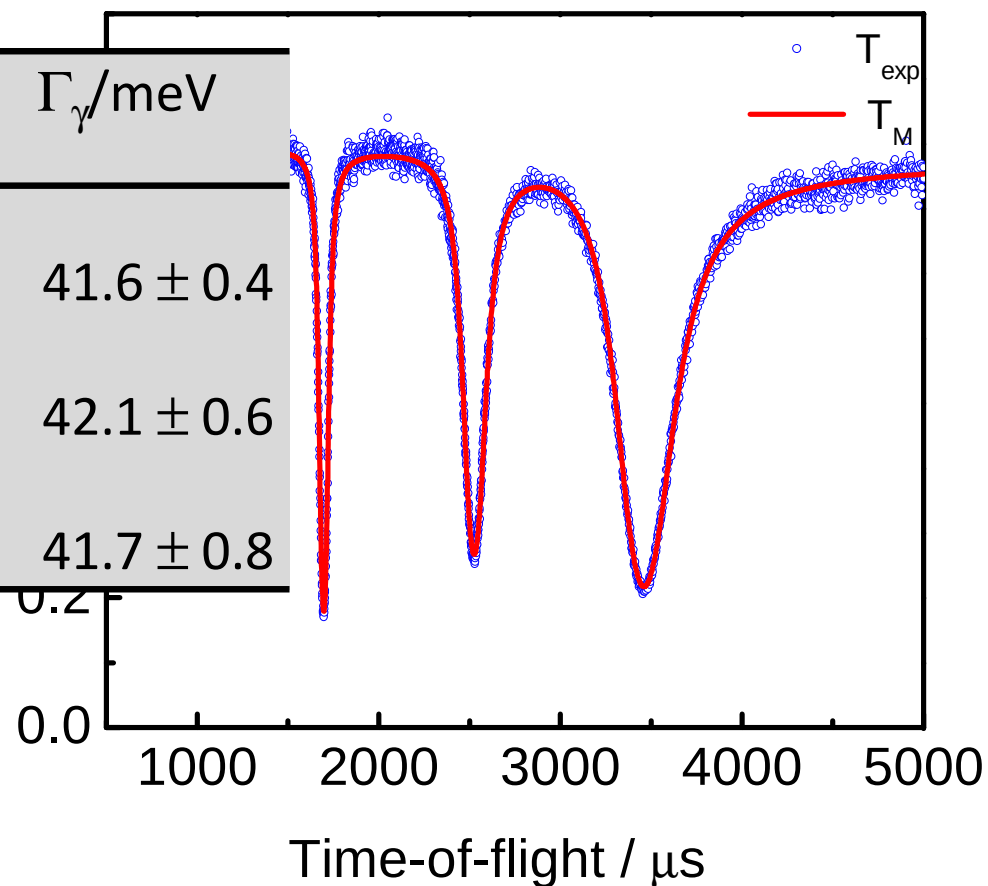
$\Delta_D \sim 25$  meV and  $\Delta_R \sim 2.5$  meV ( $L = 25$  m)

| Energy /eV | J | g    | $g\Gamma_n/\text{meV}$ | $\Gamma_\gamma/\text{meV}$ |
|------------|---|------|------------------------|----------------------------|
| 0.306      | 3 | 7/12 | $0.0373 \pm 0.0002$    | $41.6 \pm 0.4$             |
| 0.574      | 2 | 5/12 | $0.0629 \pm 0.0004$    | $42.1 \pm 0.6$             |
| 1.271      | 3 | 7/12 | $0.2176 \pm 0.0023$    | $41.7 \pm 0.8$             |

Uncertainties only due to counting statistics

⇒ **absolute measurement**

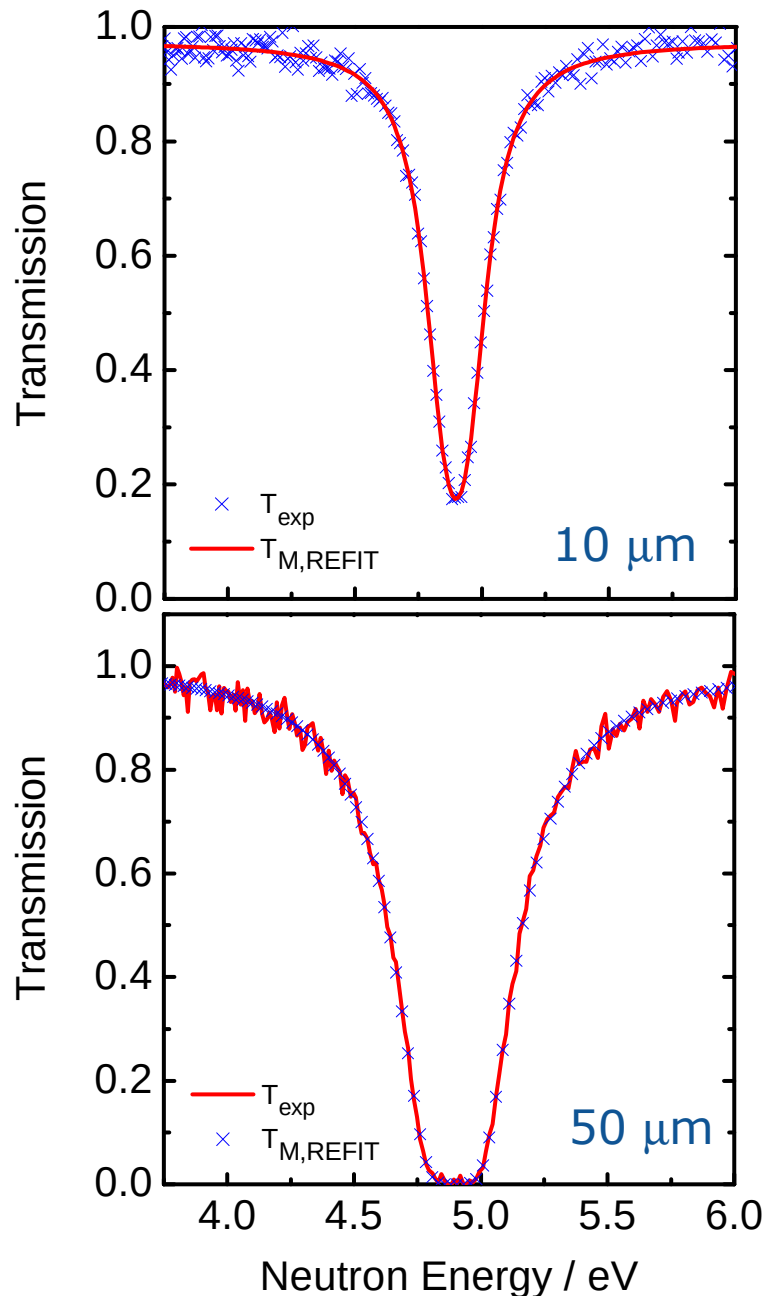
⇒ **no reference to any other cross section**





# 4.9 eV resonance for $^{197}\text{Au}+n$

$^{197}\text{Au} : I^\pi = 3/2^+$



## Resonance shape analysis with REFIT

$\ell = 0$  from shape, *spin  $J = 2$  from fit with capture data*

$\Delta_D \sim 80$  meV and  $\Delta_R \sim 5$  meV ( $L = 50$  m)

$$\Gamma_n = (15.06 \pm 0.08) \text{ meV}$$

$\Rightarrow$

$$\Gamma_\gamma = (121.7 \pm 1.3) \text{ meV}$$

$$\Gamma_n = (14.66 \pm 0.30) \text{ meV}$$

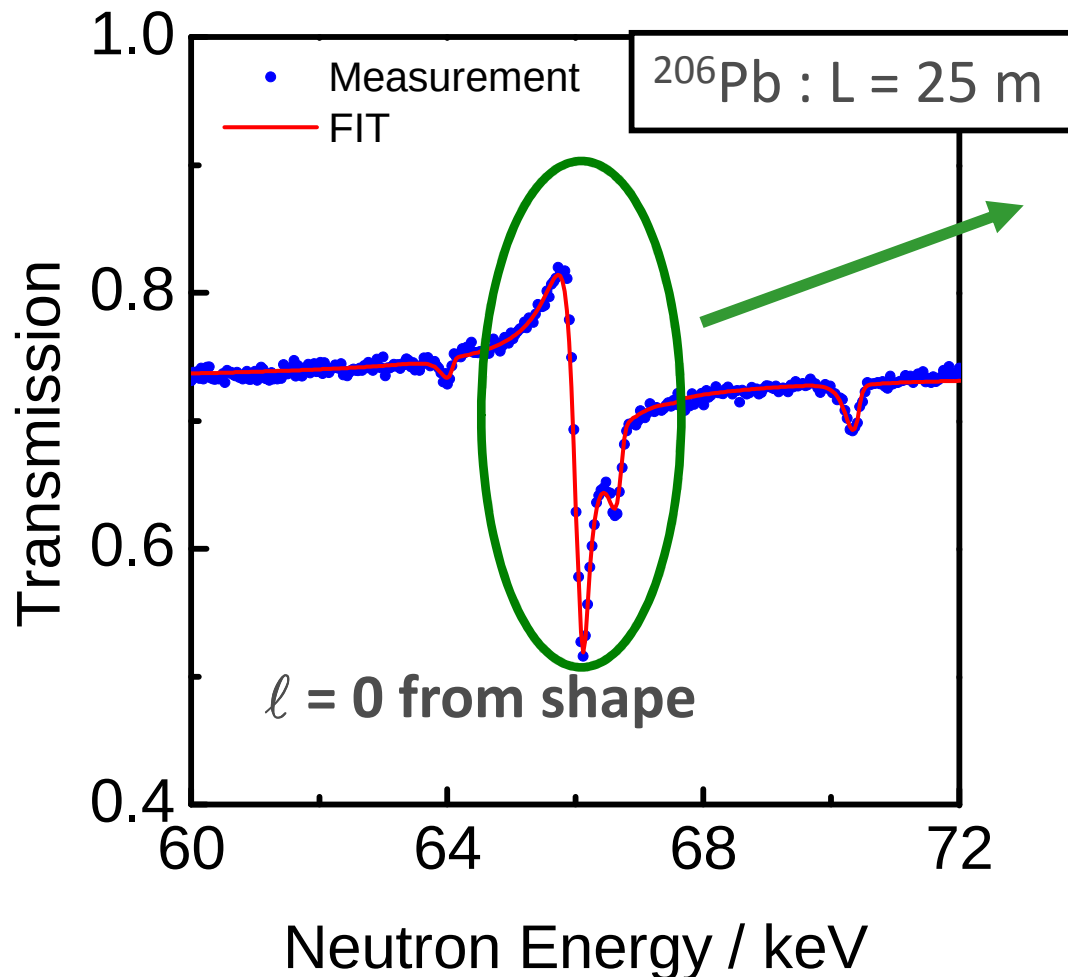
$\Rightarrow$

$$\Gamma_\gamma = (124.8 \pm 3.7) \text{ meV}$$

Uncertainties only due to counting statistics

# Determination of scattering radius

$$\sigma_{\text{tot}}(E_n) = g \frac{\pi}{k_n^2} \frac{\Gamma_n \Gamma}{(E_n - E_R)^2 + (\Gamma/2)^2} + g \frac{4\pi}{k_n} \frac{\Gamma_n (E_n - E_R) R}{(E_n - E_R)^2 + (\Gamma/2)^2} + g 4\pi R^2$$



Interference

**$R = 9.55 (0.02) \text{ fm}$**

Borella et al., Phys. Rev. C 76 (2007) 014605

# Determination of statistical factor

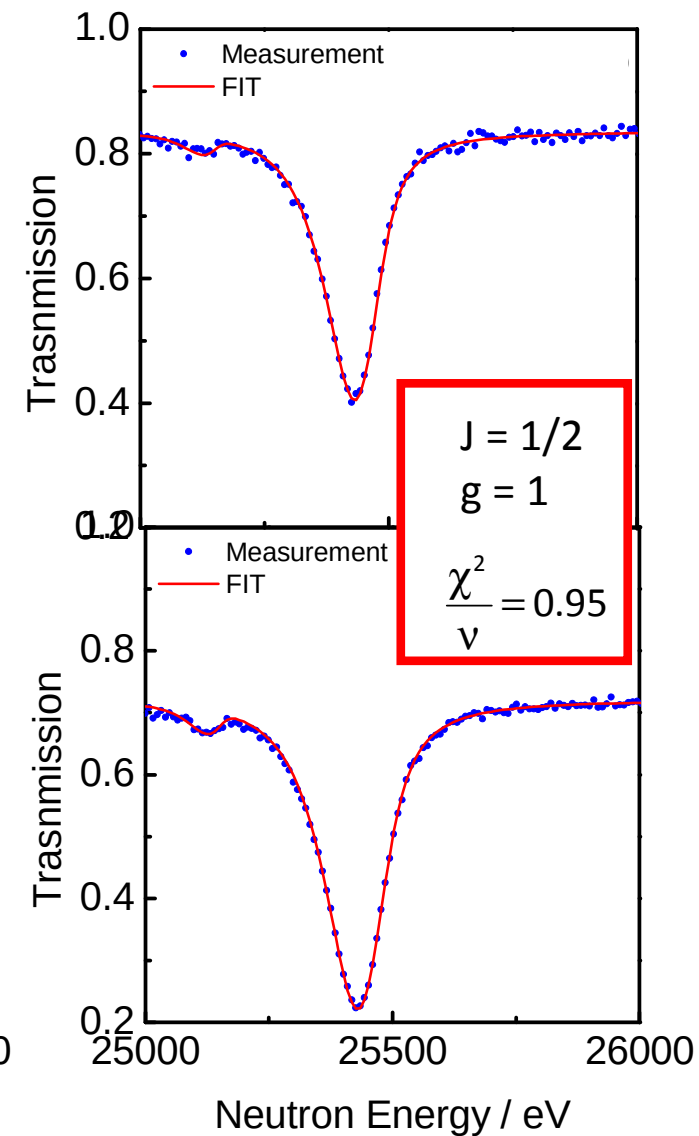
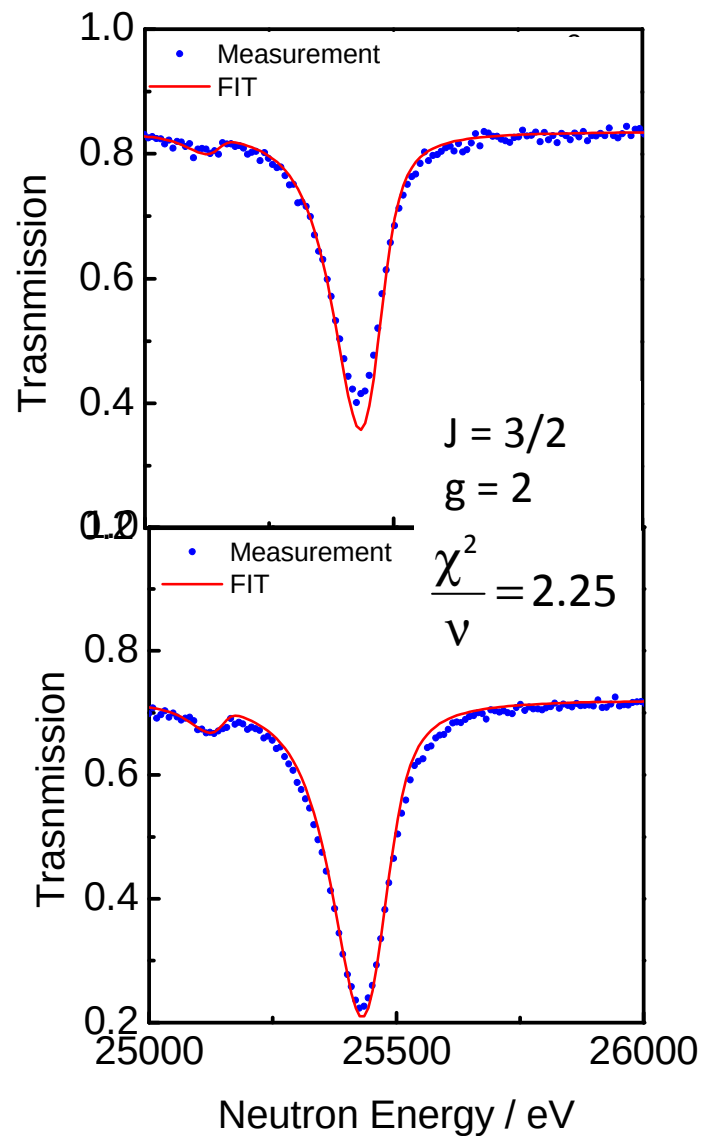
$^{206}\text{Pb} (I^\pi = 0^+) + n$

$\ell = 1$  from shape

$$g = \frac{2J+1}{2(2I+1)}$$

$\Rightarrow J = 1/2$

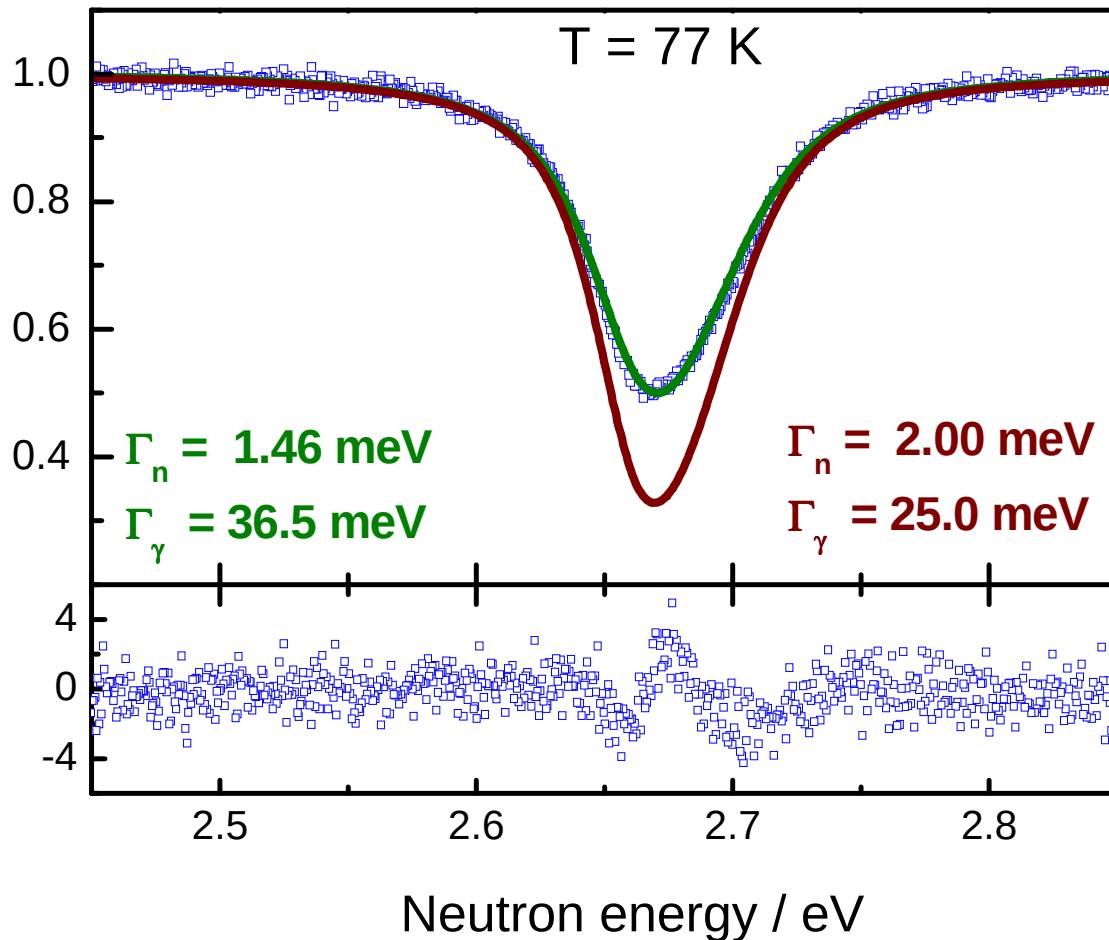
$\Rightarrow g = 1$



Borella et al., Phys. Rev. C 76 (2007) 014605

# REFIT : homogeneous sample

PuO<sub>2</sub> powder mixed with carbon powder



Homogeneous sample

$$T_M(t) = \int R(t, E) T(E, \bar{\sigma}_{\text{tot}}, n) dE$$

$$T(E, n, \bar{\sigma}_{\text{tot}}) = e^{-n \bar{\sigma}_{\text{tot}}(E)}$$

Peak cross section underestimated

Width overestimated

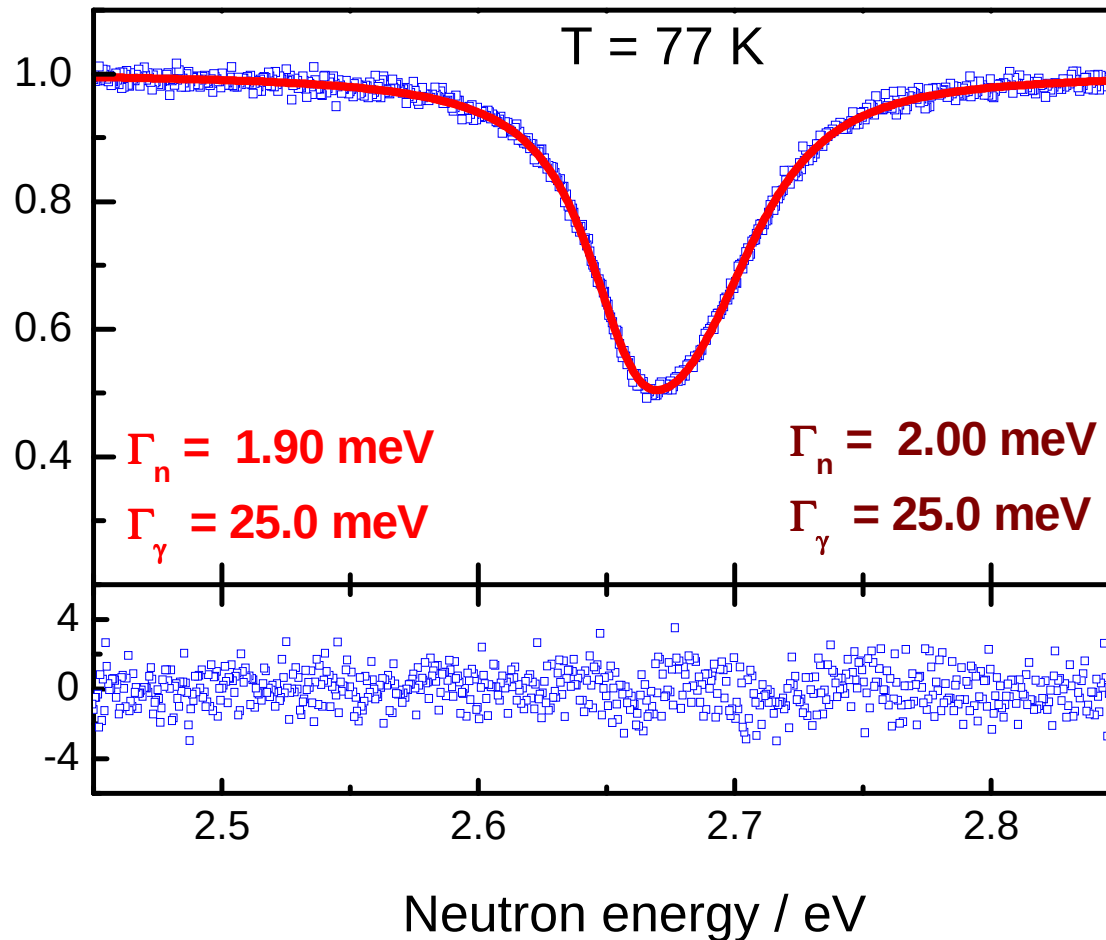
<sup>242</sup>Pu

$\Gamma_n$  underestimated

$\Gamma_\gamma$  overestimated

# REFIT : sample inhomogeneities

PuO<sub>2</sub> powder mixed with carbon powder



Account for particle size distribution

$$T_M(t) = \int R(t, E) T(E, \bar{\sigma}_{\text{tot}}, n, \dots) dE$$

$$T(E, \bar{\sigma}_{\text{tot}}, n, \dots) = (1 - f_h) \int e^{-\sum_k x n'_k \bar{\sigma}_{\text{tot},k}} p(x) dx + f_h$$

$$p(x) = \frac{1}{x \sqrt{2\pi s^2}} e^{-\frac{(\ln x + s^2/2)^2}{2s^2}}$$

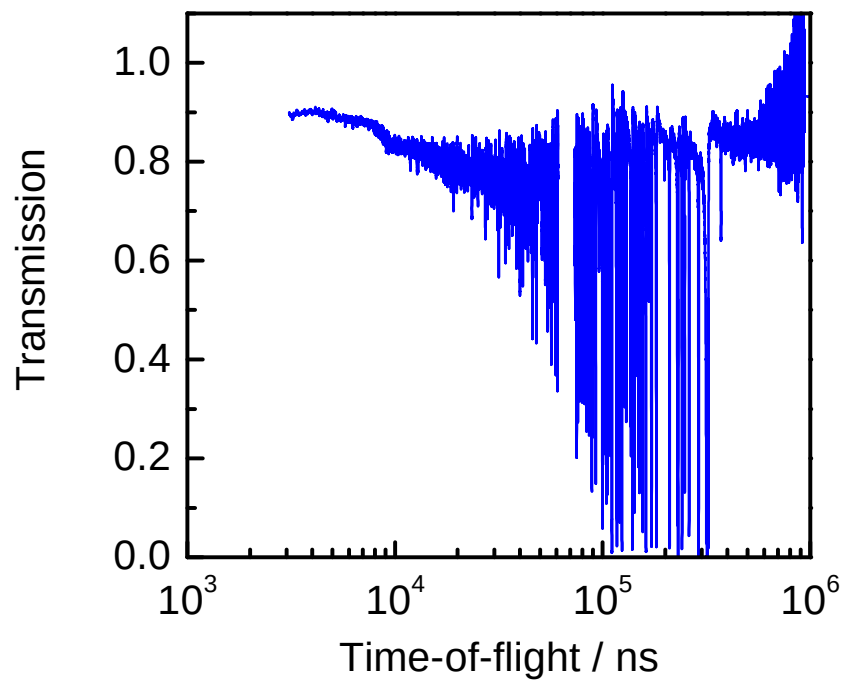
$$n'_k = \frac{n_k}{f_h}$$

See Becker et al.

# Resonance region

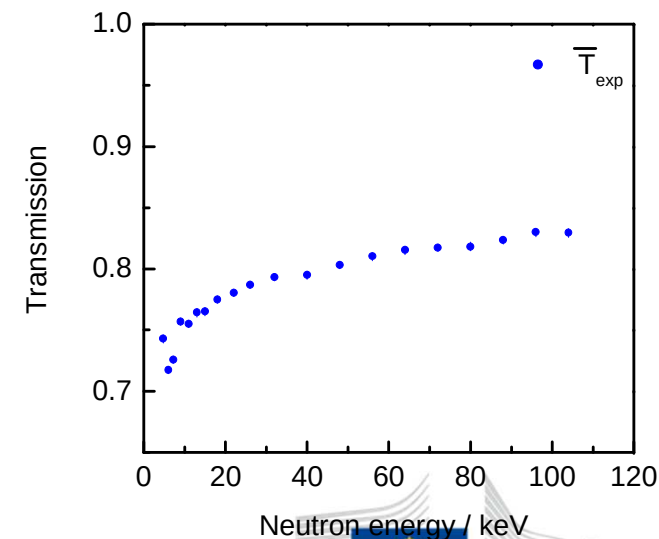
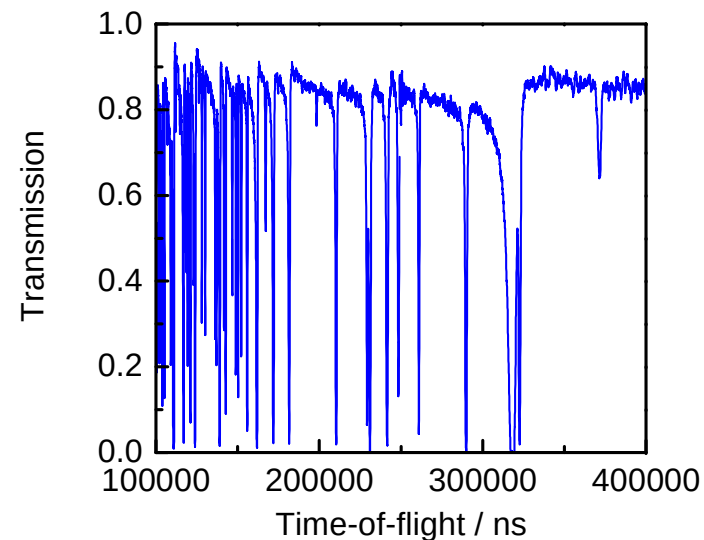
## Resonance Region $D > \Gamma$

- Resolved Resonance Region  $\Delta_R < D$
- Unresolved Resonance Region  $\Delta_R > D$

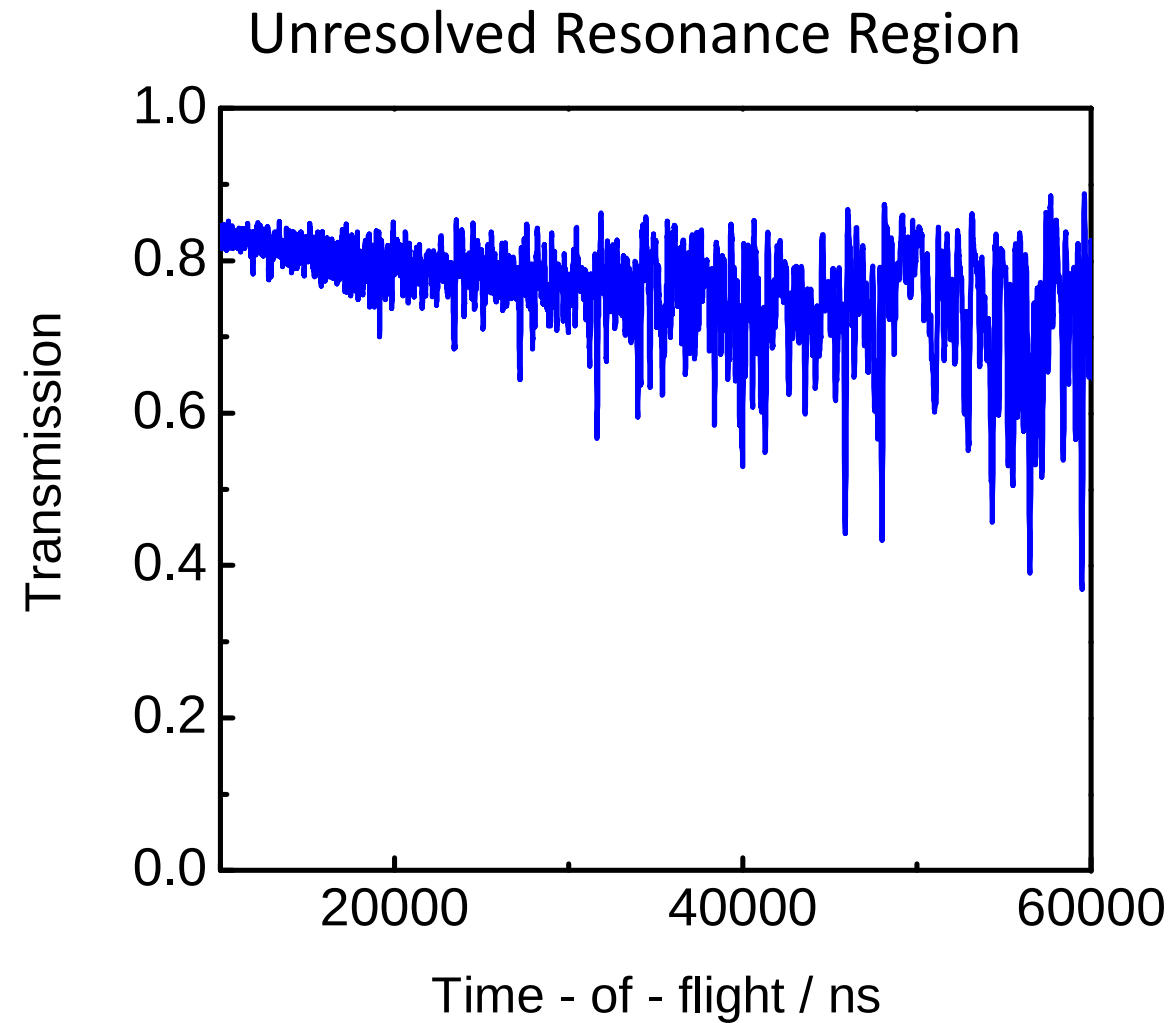
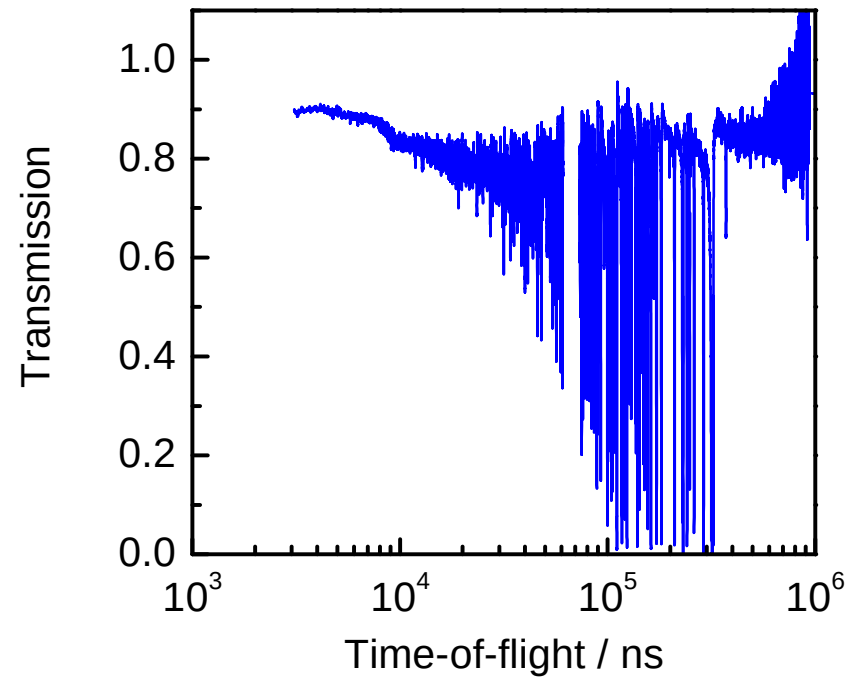


RRR  
Resonance shape analysis

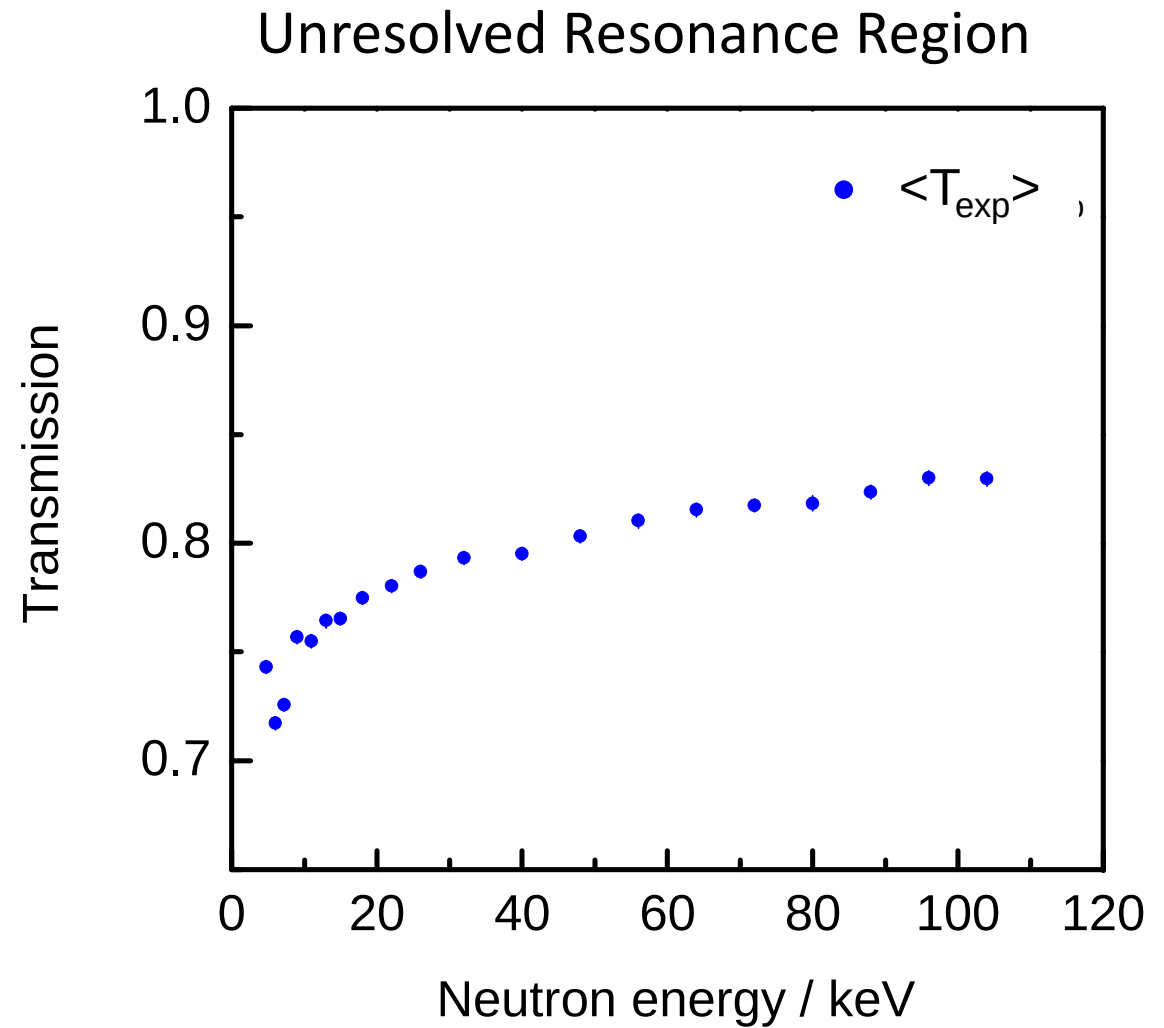
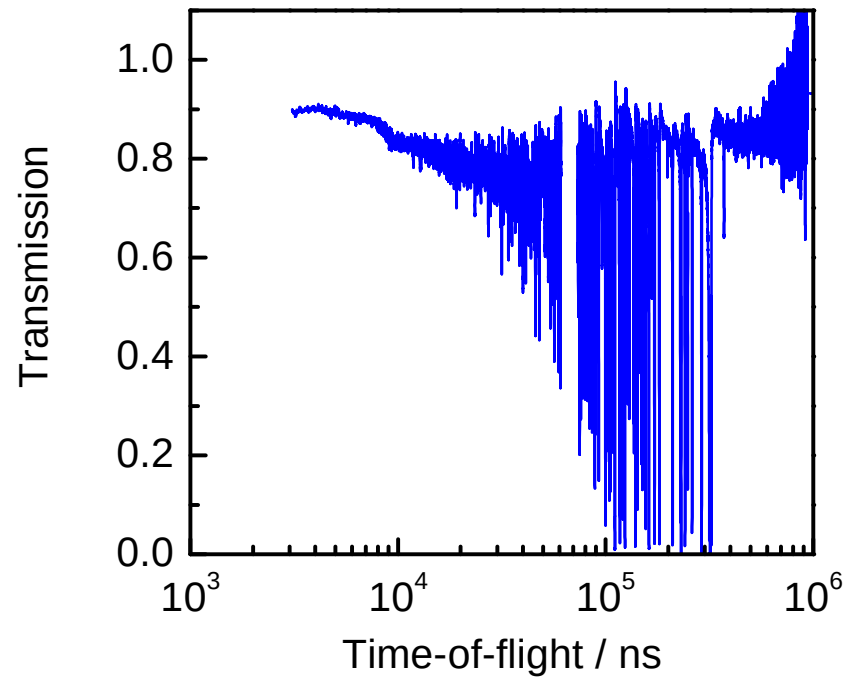
URR  
Average parameters



# Unresolved resonance region



# From average $T_{\text{exp}}$ to average $\sigma_{\text{tot}}$





## From average $T_{\text{exp}}$ to average $\sigma_{\text{tot}}$

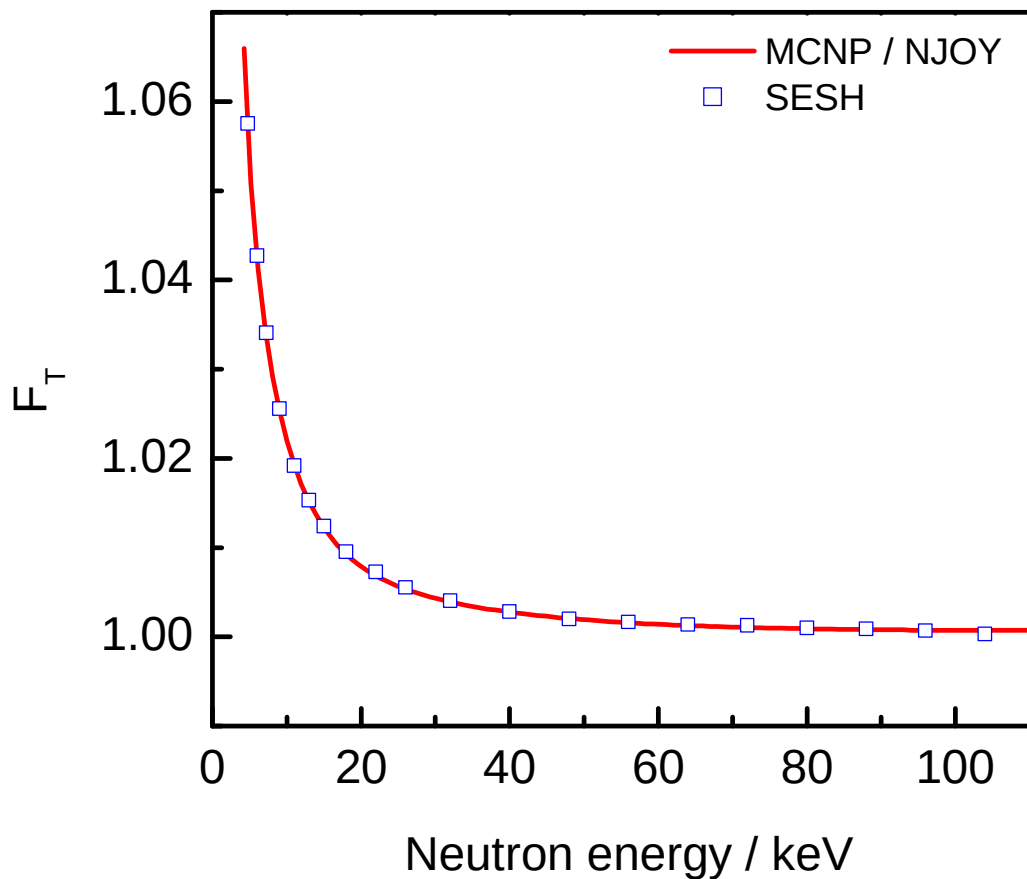
$$T = e^{-n \sigma_{\text{tot}}}$$

$$\langle T \rangle = \langle e^{-n \sigma_{\text{tot}}} \rangle = e^{-n \langle \sigma_{\text{tot}} \rangle} \left( 1 + \frac{n^2}{2} \text{var}(\sigma_{\text{tot}}) + \dots \right)$$

$$\langle T \rangle \neq e^{-n \langle \sigma_{\text{tot}} \rangle}$$

$$\langle \sigma_{\text{tot}} \rangle \neq \frac{-1}{n} \ln \langle T_{\text{exp}} \rangle$$

# From average $T_{\text{exp}}$ to average $\sigma_{\text{tot}}$



$$T = e^{-n\sigma_{\text{tot}}}$$

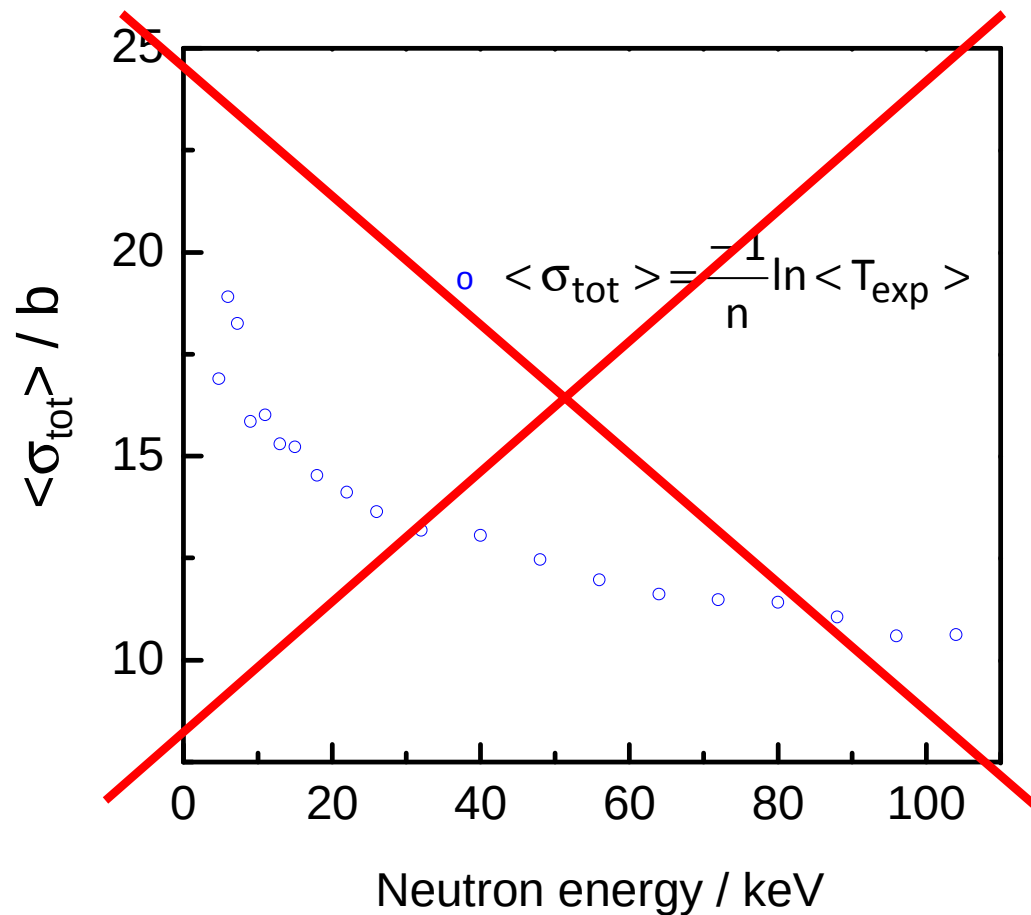
$$\langle T \rangle = \langle e^{-n\sigma_{\text{tot}}} \rangle = e^{-n\langle\sigma_{\text{tot}}\rangle} \left( 1 + \frac{n^2}{2} \text{var}(\sigma_{\text{tot}}) + \dots \right)$$

$$\langle T \rangle = \langle e^{-n\sigma_{\text{tot}}} \rangle \neq e^{-n\langle\sigma_{\text{tot}}\rangle}$$

$$F_T = \frac{\langle e^{-n\sigma_{\text{tot}}} \rangle}{e^{-n\langle\sigma_{\text{tot}}\rangle}} = 1 + \frac{n^2}{2} \text{var}(\sigma_{\text{tot}}) + \dots$$

$$\langle \sigma_{\text{tot}} \rangle = \frac{-1}{n} \ln \frac{\langle T_{\text{exp}} \rangle}{F_T}$$

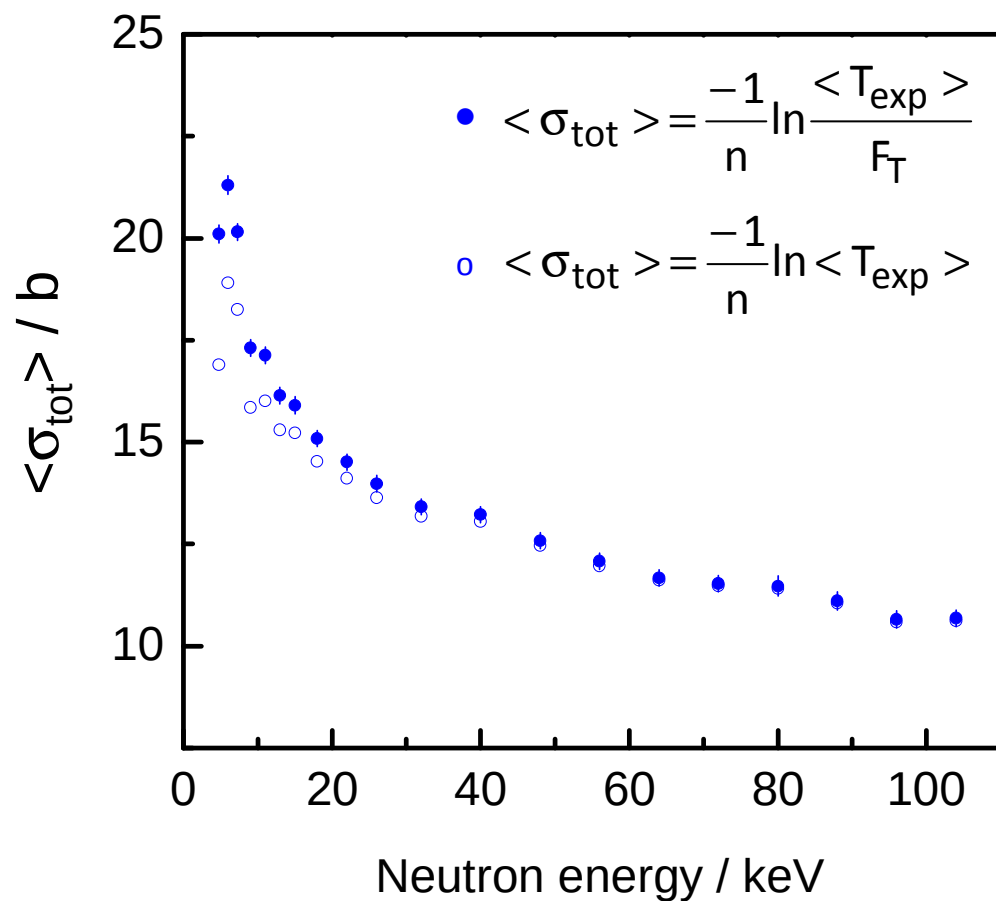
# From average $T_{\text{exp}}$ to average $\sigma_{\text{tot}}$



$$F_T = \frac{\langle e^{-n\sigma_{\text{tot}}} \rangle}{e^{-n\langle \sigma_{\text{tot}} \rangle}} = 1 + \frac{n^2}{2} \text{var}(\sigma_{\text{tot}}) + \dots$$

$$\langle \sigma_{\text{tot}} \rangle = \frac{-1}{n} \ln \frac{\langle T_{\text{exp}} \rangle}{F_T}$$

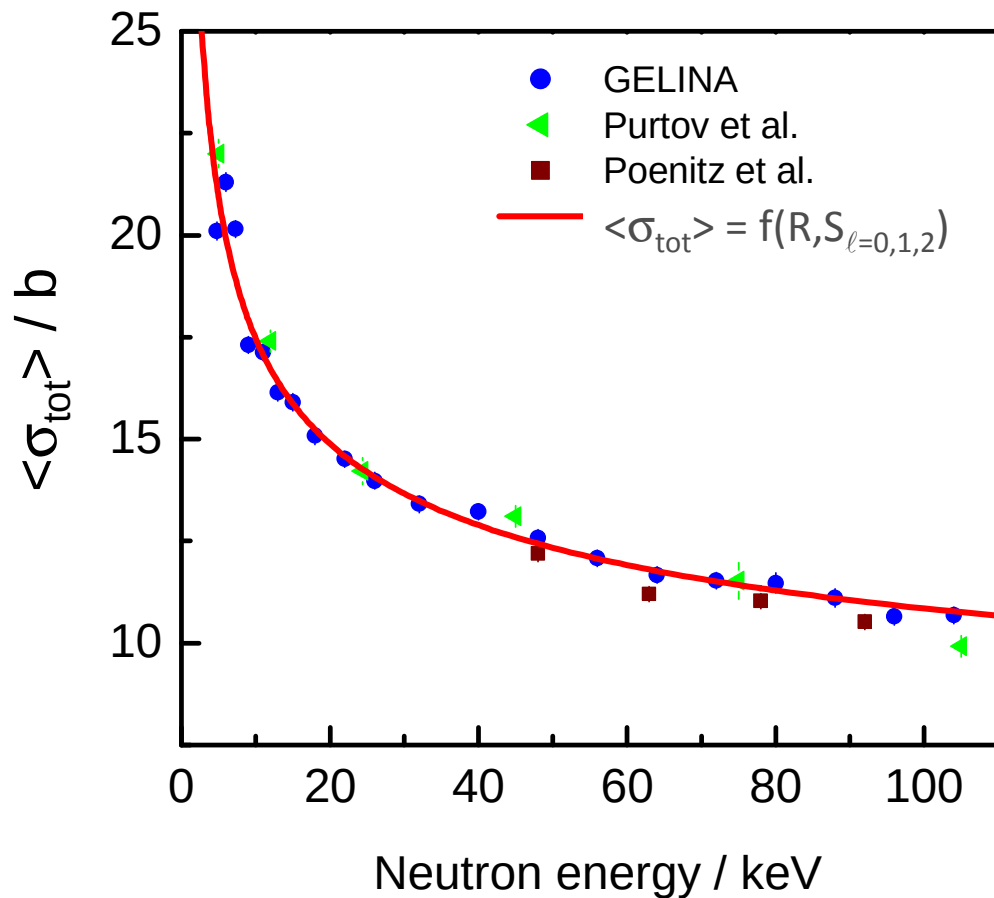
# From average $T_{\text{exp}}$ to average $\sigma_{\text{tot}}$



$$F_T = \frac{\langle e^{-n\sigma_{\text{tot}}} \rangle}{e^{-n\langle \sigma_{\text{tot}} \rangle}} = 1 + \frac{n^2}{2} \text{var}(\sigma_{\text{tot}}) + \dots$$

$$\langle \sigma_{\text{tot}} \rangle = \frac{-1}{n} \ln \frac{\langle T_{\text{exp}} \rangle}{F_T}$$

# Parameterisation by average parameters



$$\langle \sigma_{\text{tot}} \rangle = f(R, S_{\ell=0,1,2})$$

$$S_{\ell} = \frac{\sum_{j=1}^N (g \Gamma_n^{\ell})_j}{(2\ell + 1) \Delta E}$$

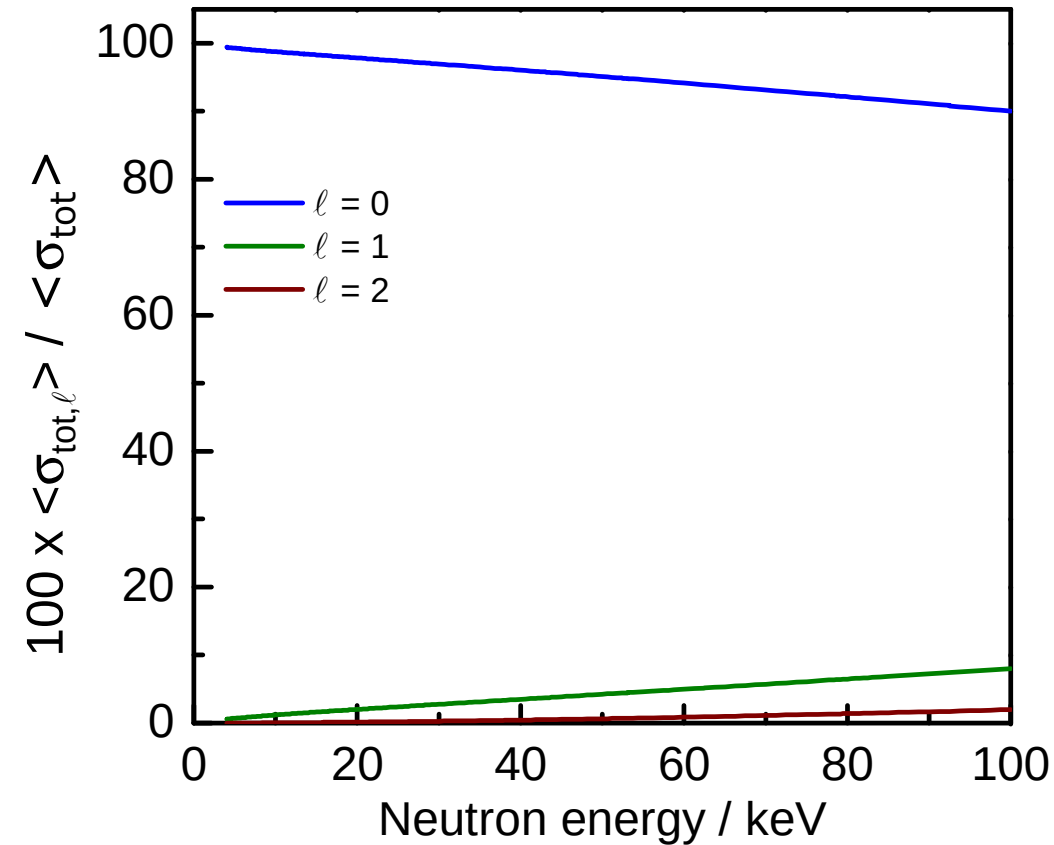
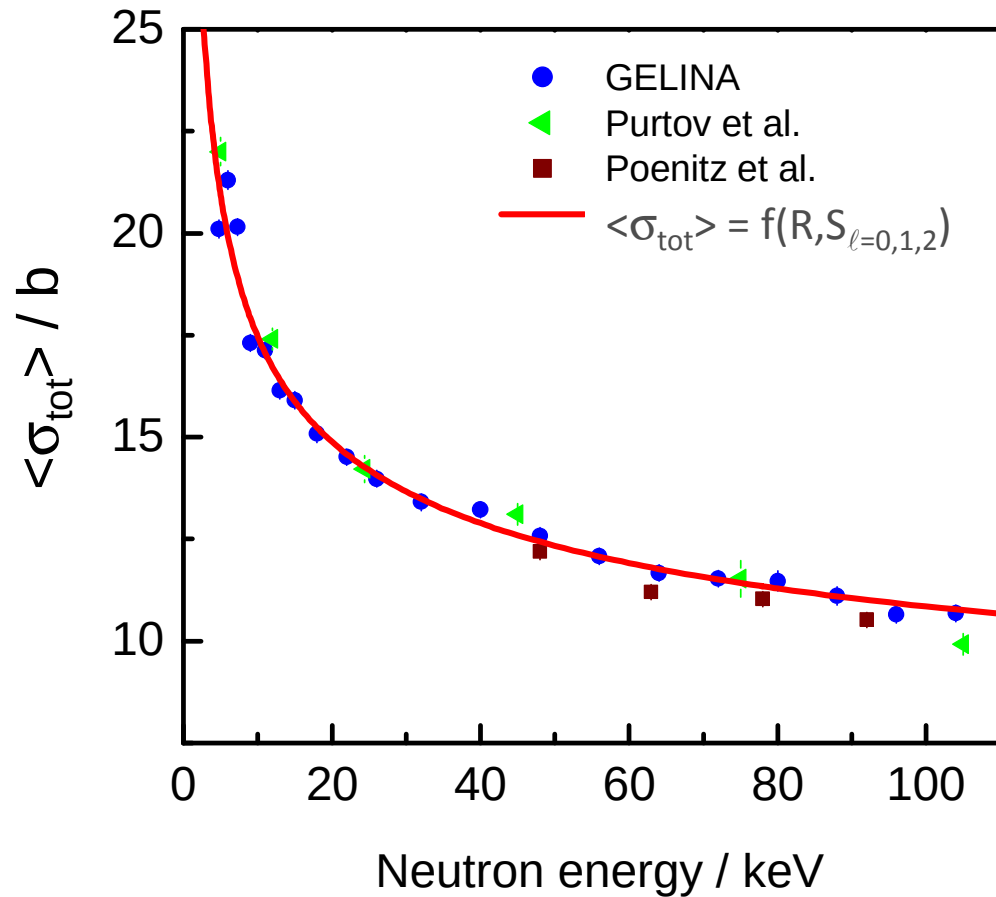
$$\langle \sigma_c \rangle = \frac{2\pi}{k_c^2} g_c \left[ 1 - \sqrt{1 - T_c} \cos(2\psi_c) \right]$$

$$T_c(E) = \frac{2\pi\sqrt{E}P_cS_c}{k_c a_c} \times \left[ \left( 1 + \frac{\pi\sqrt{E}P_cS_c}{2k_c a_c} - F_c R_c^{\infty} \right)^2 + \left( P_c R_c^{\infty} + \frac{\pi\sqrt{E}F_c S_c}{2k_c a_c} \right)^2 \right]^{-1}, \quad (7)$$

$$\psi_c(E) = \phi_c(k_c a_c) - \frac{1}{2} \left[ \tan^{-1} \frac{P_c R_c^{\infty} - \pi\sqrt{E}F_c S_c / 2k_c a_c}{1 - \pi\sqrt{E}P_c S_c / 2k_c a_c - F_c R_c^{\infty}} + \tan^{-1} \frac{P_c R_c^{\infty} + \pi\sqrt{E}F_c S_c / 2k_c a_c}{1 + \pi\sqrt{E}P_c S_c / 2k_c a_c - F_c R_c^{\infty}} \right], \quad (8)$$

$$\Gamma_n^{\ell} = \frac{1}{\sqrt{E_n / 1\text{eV}}} \frac{k_c a_c}{P_{\ell}} \Gamma_{n,\ell}$$

# Parameterisation by average parameters

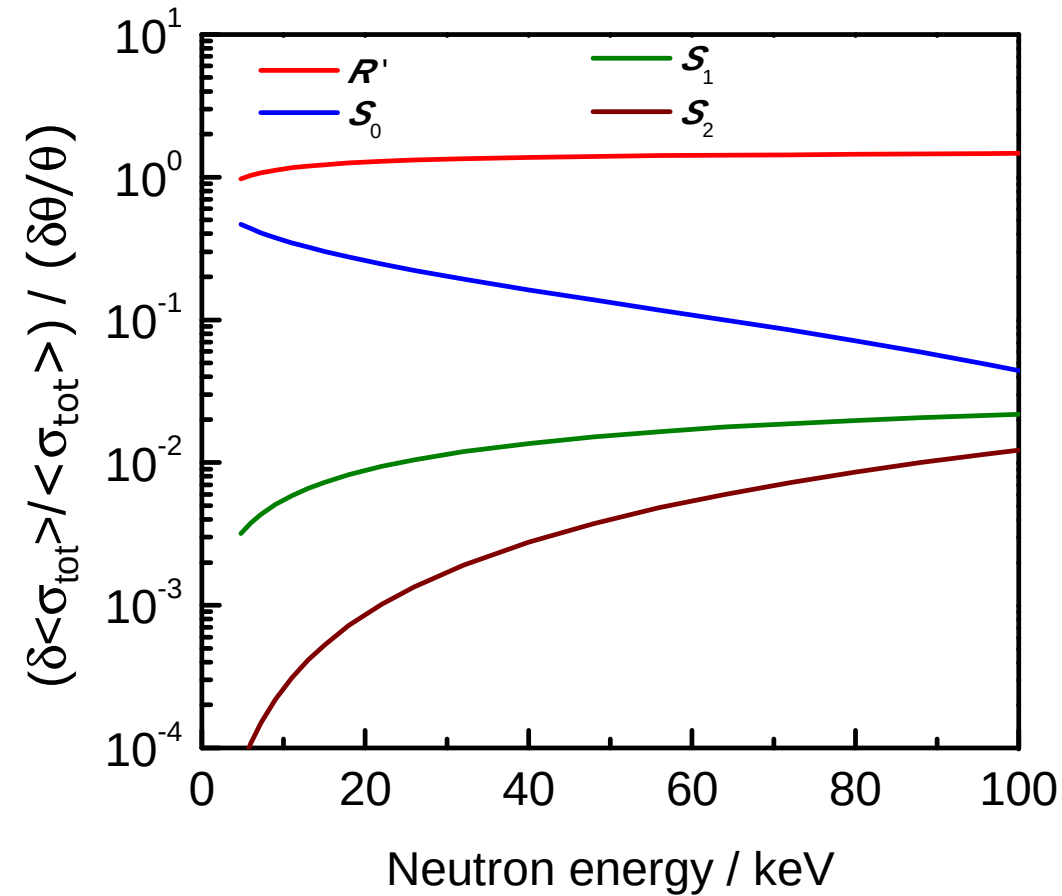
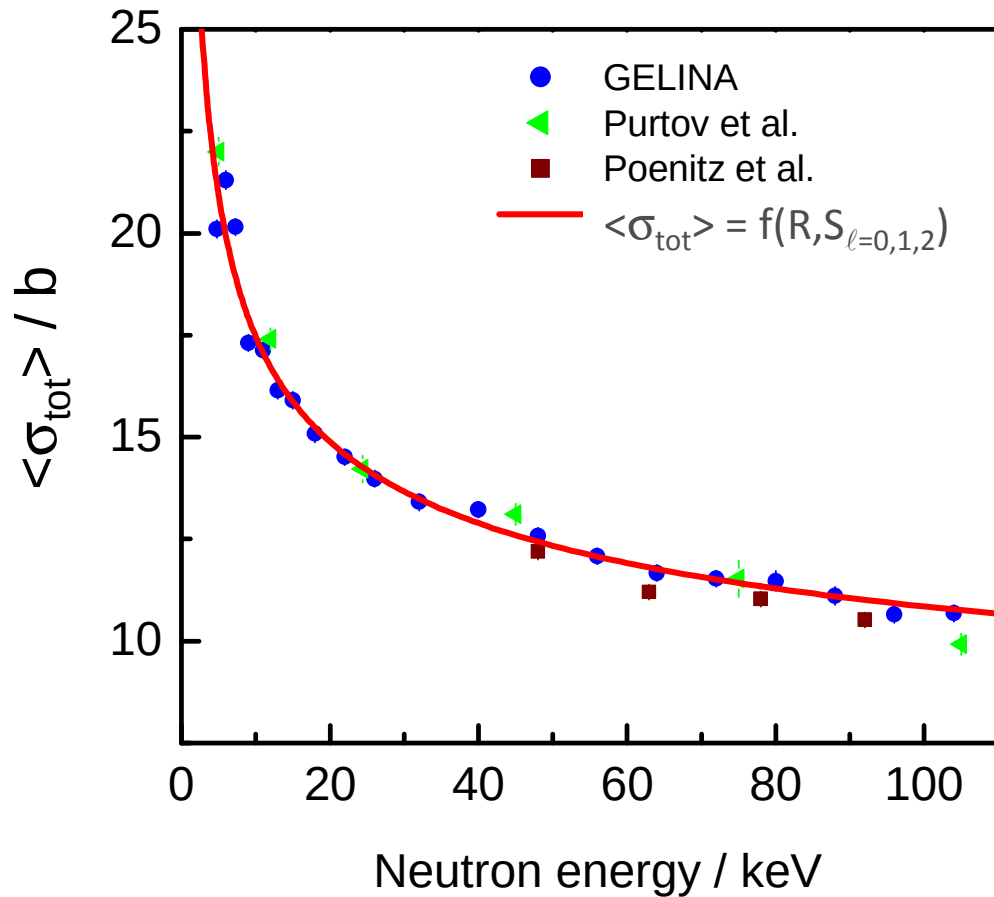


$$\langle \sigma_{\text{tot}} \rangle = f(R, S_{\ell=0,1,2})$$

$$S_{\ell} = \frac{\sum_{j=1}^N (g \Gamma_n^{\ell})_j}{(2\ell + 1) \Delta E}$$

$$\Gamma_n^{\ell} = \frac{1}{\sqrt{E_n / 1\text{eV}}} \frac{k_c a_c}{P_{\ell}} \Gamma_{n,\ell}$$

# Parameterisation by average parameters

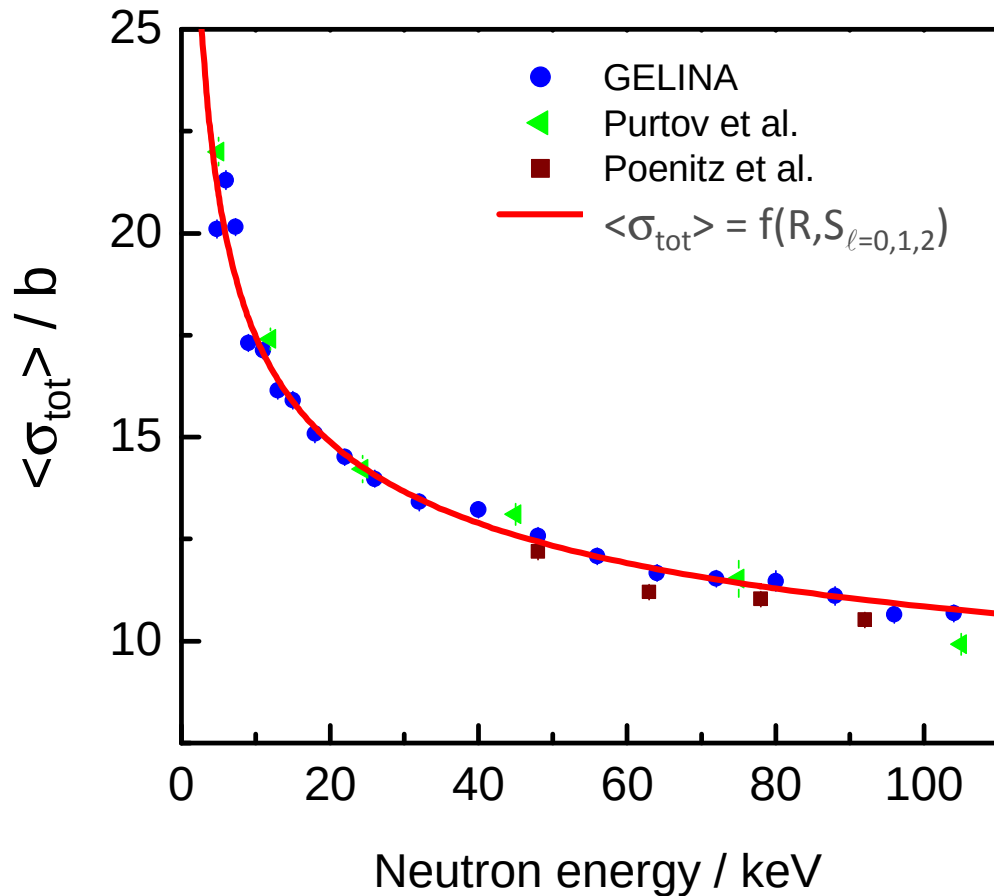


$$\langle \sigma_{\text{tot}} \rangle = f(R, S_{\ell=0,1,2})$$

$$S_{\ell} = \frac{\sum_{j=1}^N (g \Gamma_n^{\ell})_j}{(2\ell + 1) \Delta E}$$

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# Parameterisation by average parameters



$$\langle \sigma_{\text{tot}} \rangle = f(R, S_{\ell=0,1,2})$$

$$S_{\ell} = \frac{\sum_{j=1}^N (g \Gamma_n^{\ell})_j}{(2\ell + 1) \Delta E}$$

| Correlation matrix |                 |         |       |
|--------------------|-----------------|---------|-------|
| R / fm             | $9.12 \pm 0.09$ | 1       | -0.43 |
| $S_0 / 10^{-4}$    | $1.93 \pm 0.03$ | -0.43   | 1     |
| $S_1 / 10^{-5}$    | 5.64            | from OM |       |
| $S_2 / 10^{-4}$    | 3.48            |         |       |

Average parameters are essential  
for self-shielding calculations  
ENDF file in URR:

- Average resonance parameters : required
- Average point wise cross section : optional

$$\Gamma_n^{\ell} = \frac{1}{\sqrt{E_n / 1\text{eV}}} \frac{k_c a_c}{P_{\ell}} \Gamma_{n,\ell}$$



# Literature

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