Nucleosynthesis and Related Nuclear Data Needs

Lecture 3

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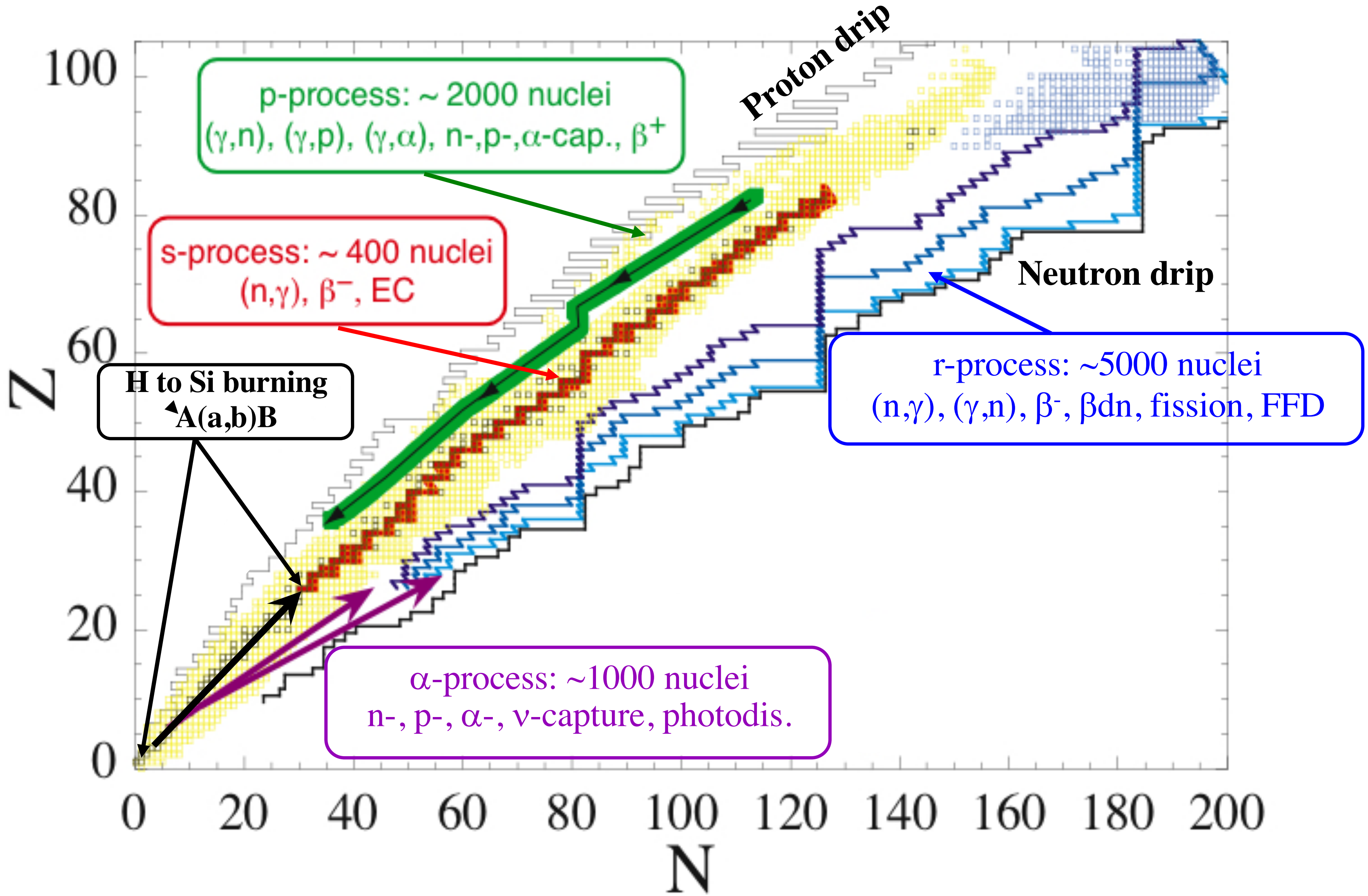
RELATED NUCLEAR DATA NEEDS

Lecture 3

- β-decay in a stellar plasma
- Nuclear reactions in an astrophysical plasma
- Nuclear needs for the various nucleosynthesis processes

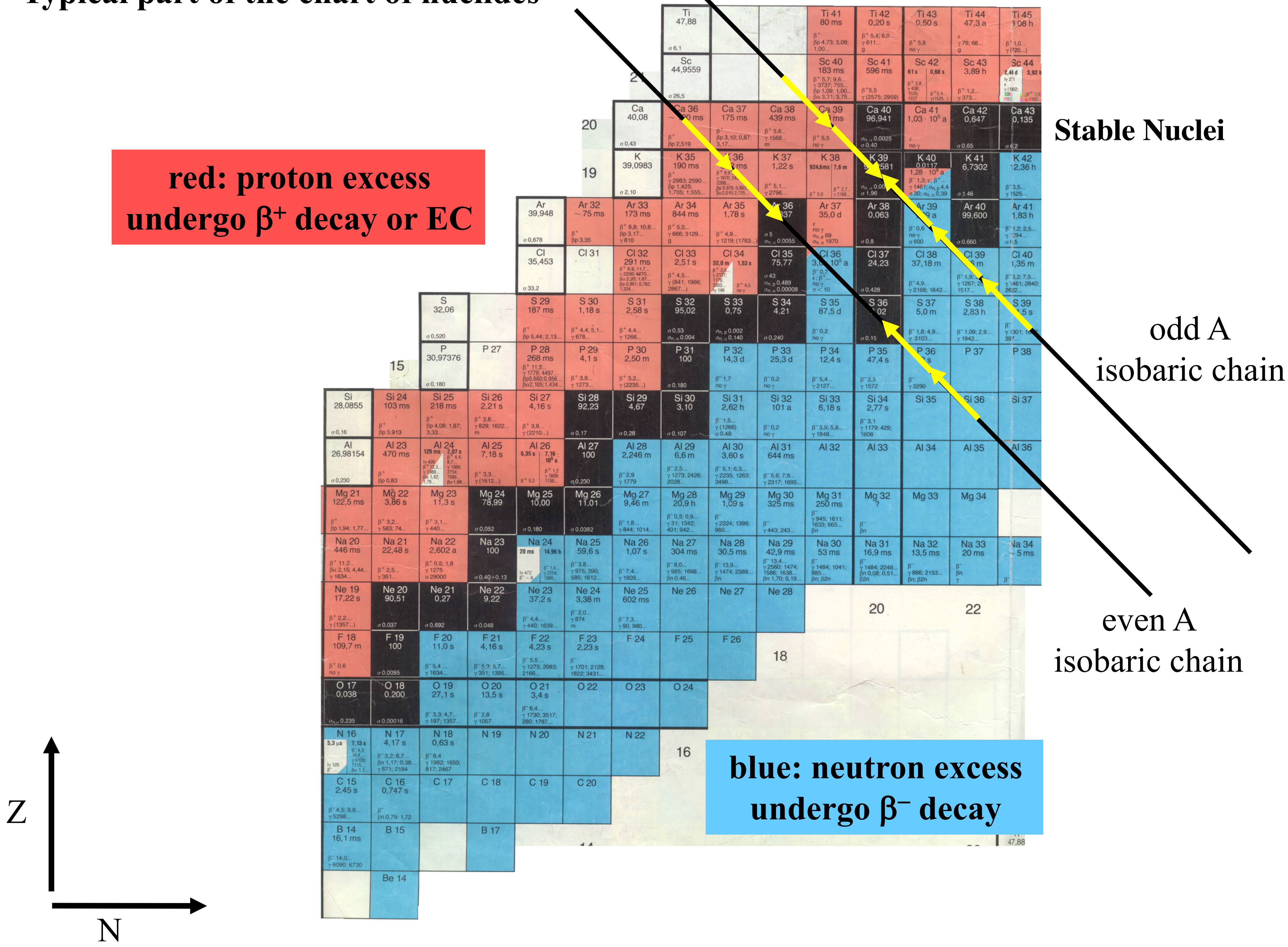
Many different nuclear needs for the different nucleosynthesis applications



Some specificities of the astrophysical plasma

- Energy region of "almost no event" for charged-particle reactions:
  - low cross sections unreachable experimentally
- Unstable species involved:
  - limited experimental data are available
- Exotic species involved:
  - n-rich and n-deficient nuclei out of reach from experiment
- Large number of properties and nuclei involved
  - thousands of nuclei involved, tens of properties
- High-T environments:
  - thermalization effects of excited states by electron and photon interaction. Impact on reaction rates, β-decays
  - ionization effects: Impact on β-decays (bound state β-decay, continuum-$e^{-/+}$ capture); e^-- screening effects on reaction rates
- High-ρ environments (supernovae, neutron stars):
  nuclear binding understood in terms of a nuclear Equation of State (energy density and pressure of a system of nucleons as a function of matter density ρ)

Typical part of the chart of nuclides



In stellar conditions, the probability for these 3 processes can be quite different, in particular due to

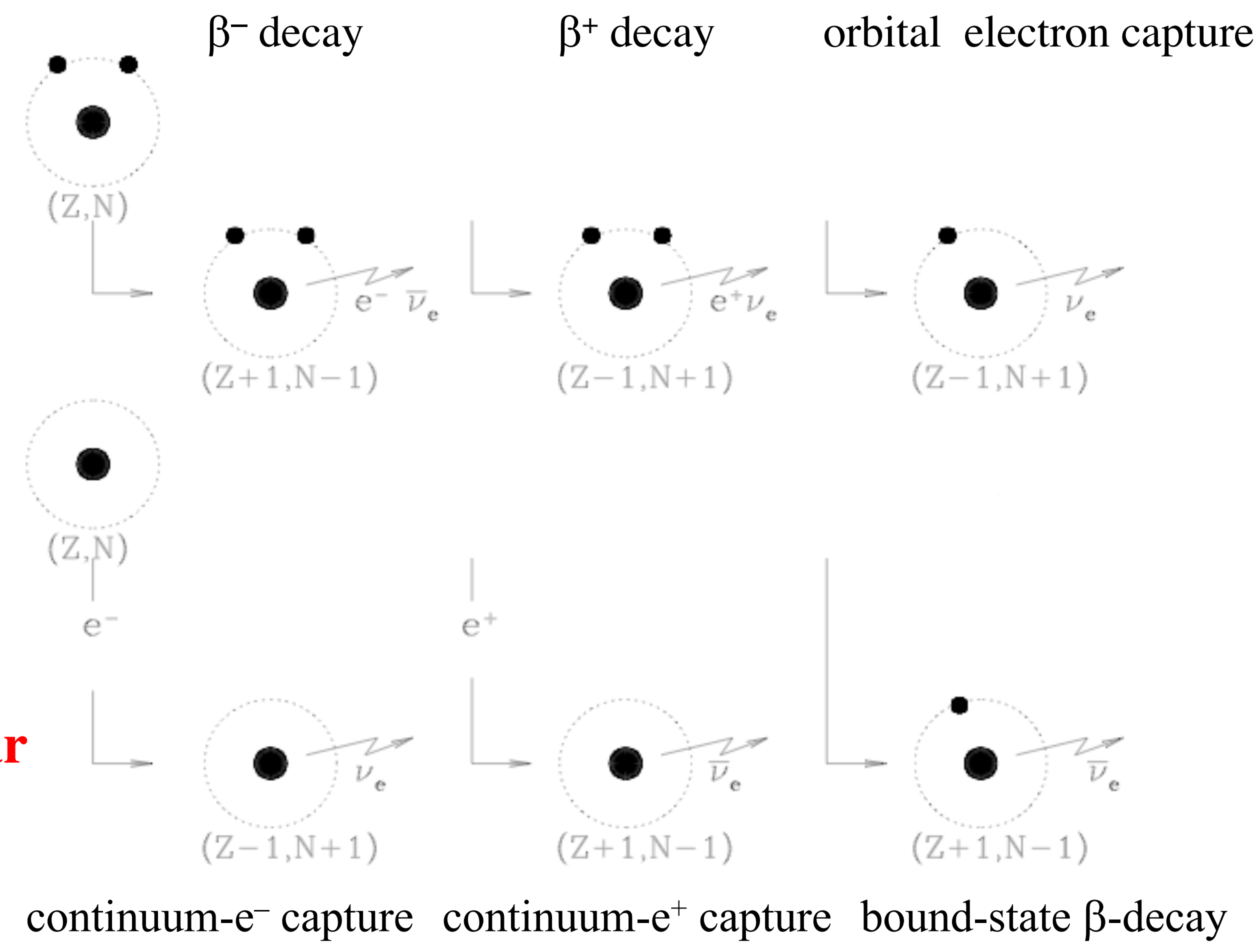
- the possible reduction of the phase space accessible to the emitted electron for the β^- decay. This reduction is due to the degeneracy of the Fermi-Dirac electron gas in many high-density astrophysical environments.
- the electron capture in degenerate conditions can make stable nuclei to become unstable if $\varepsilon_F > Q$ (e.g ^{14}N, ^{16}O, ^{20}Ne or ^{24}Mg in white dwarfs or protons in the core of exploding massive stars)
- the contribution of thermally populated excited states to the decay process
- the ionisation leading to large abundance of free electrons and their possible capture by nuclei
- the presence at high temperature ($T > 10^9$K) of $e^{\pm}$ pairs

  ($\gamma + \gamma \leftrightarrow e^+ + e^-$) and their possible capture

The different nuclear β-decay modes

Usual decay modes in the laboratory

Additional possible decay modes in stellar conditions

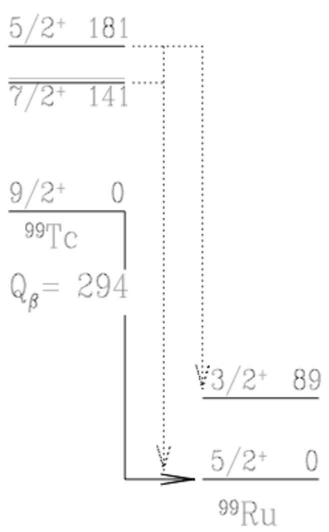


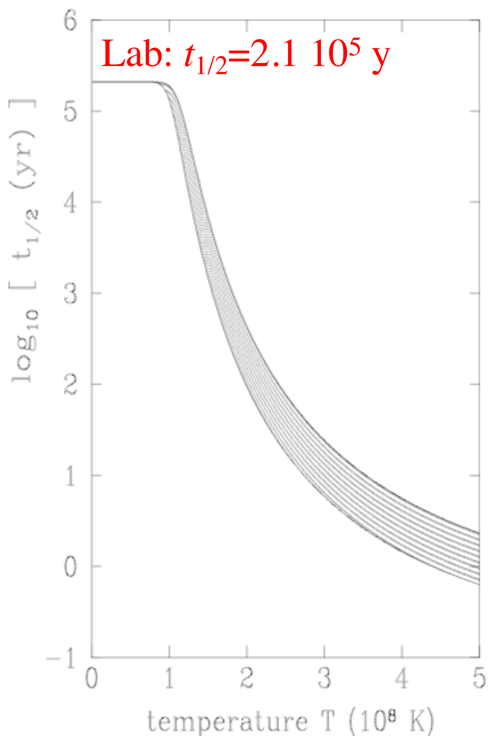
The contribution of thermally populated excited states to the decay process

At increasing temperatures, excited states are thermally populated and can contribute to the β-decay process and consequently modify (sometime very significantly) its half-life with respect to the laboratory value.

One example is ^{99}Tc, most probably the Tc isotope observed at the surface of AGB stars and produced by the s-process nucleosynthesis at temperatures of a few 10^8K. In such conditions, the 141 and 181 keV states can be sufficiently populated to contribute to the β-decay and reduce the half-life from the laboratory value of 2.1 10^5 y to about 10 years at T~3 10^8K.

$^{99}Tc \longrightarrow {}^{99}Ru + e^- + \bar{\nu}_e$





The decay of thermally populated states can play a significant role if

-the ground state β-decay is slow due to spin selection rules (large differences between spin and parities of initial and final states: forbidden transitions)

- the low-lying states can undergo faster β-decays than the ground state

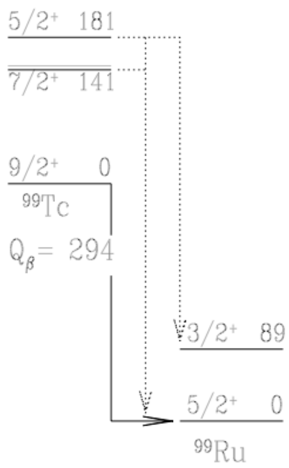
- temperatures are high enough to populate such excited states

→ **$\beta^{\pm}$-decay rates are *T*-dependent in stars**

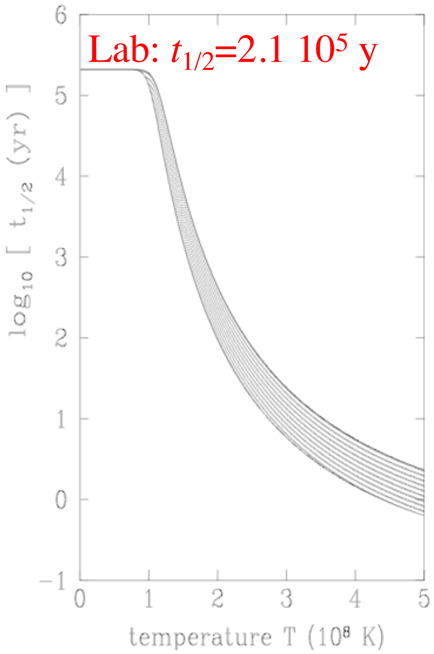
$$\lambda_\beta = \lambda_\beta(T)$$

The density dependence of the β-decays

$^{99}Tc \longrightarrow {}^{99}Ru + e^- + \bar{\nu}_e$



At low density
(non-degenerate e⁻ gas)

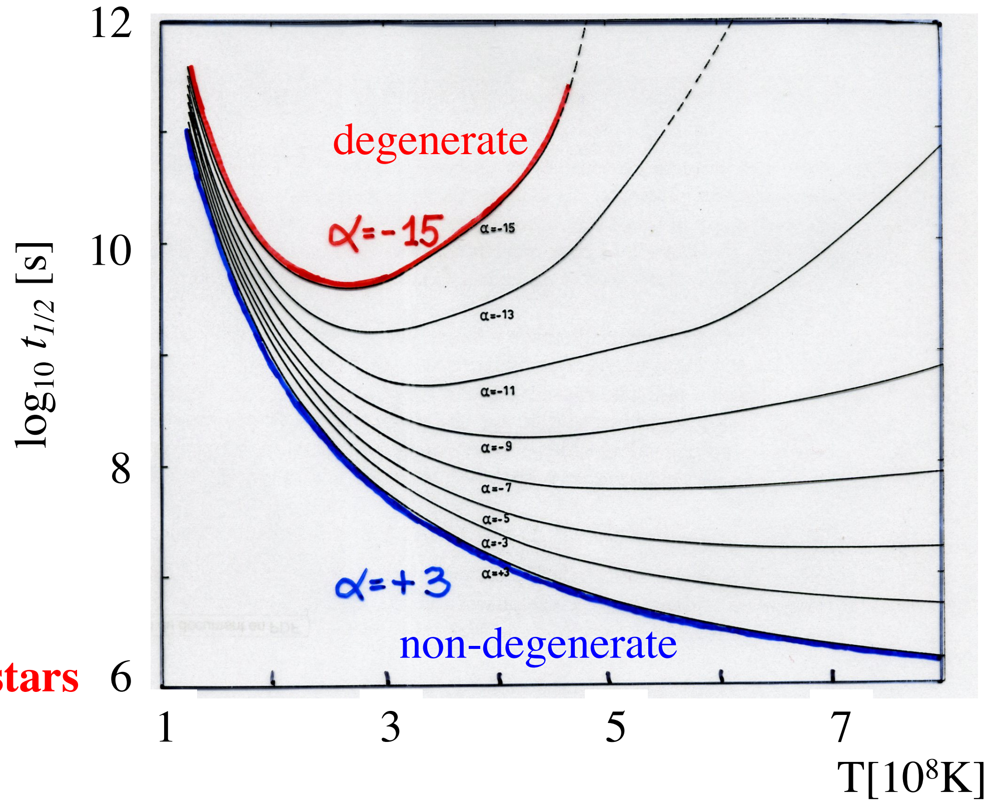


Fermi-Dirac distribution for the electron gas: probability for a continuum electron state at energy E to be unoccupied

$$f = 1 - \frac{1}{1 + \exp(\alpha + E/kT)}$$

degeneracy parameter $\alpha = -\eta = \alpha(T, \varepsilon_F) = \alpha(T, \rho)$

Lab: log $t_{1/2}$[s]~12.8



→ β-decay rates are *T*- and *ρ*-dependent in stars

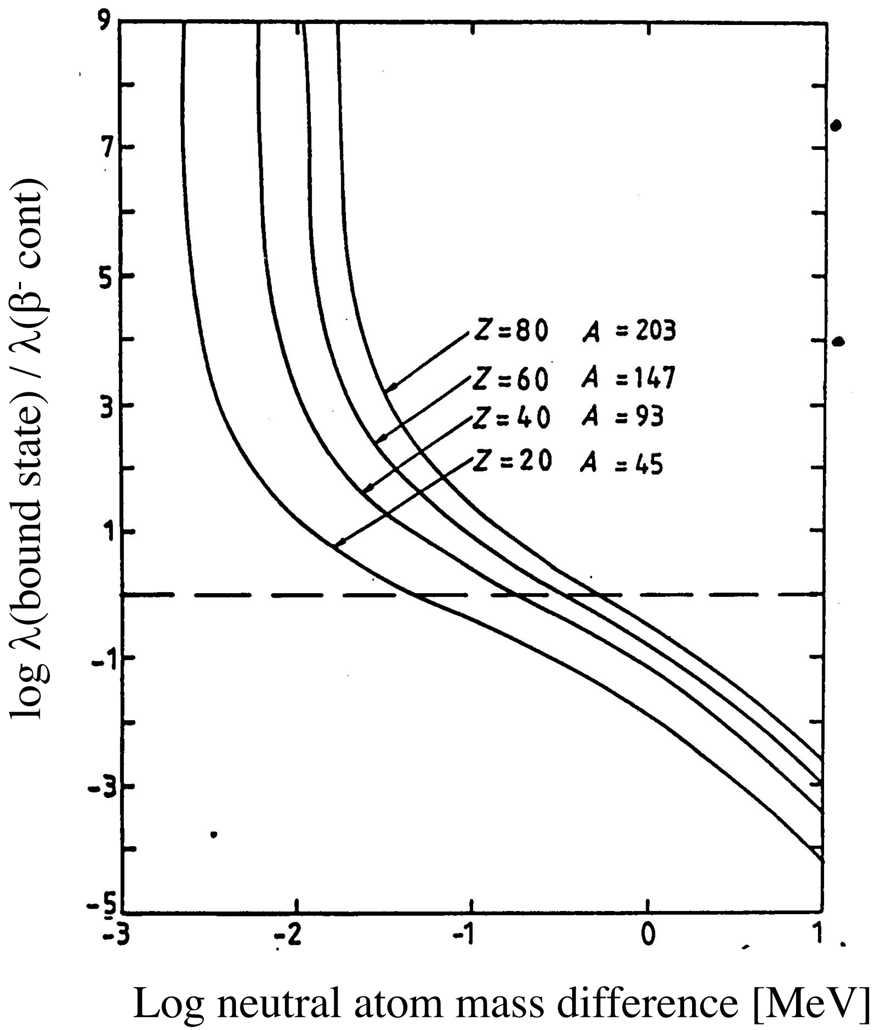
$$\lambda_\beta = \lambda_\beta(T, \rho)$$

The bound state β-decay process

$$(Z,N) \longrightarrow (Z+1,N-1) + e^- + \bar{\nu}_e$$

Continuum state:
"normal" β-decay

Vacant bound atomic state:
"bound state" β-decay



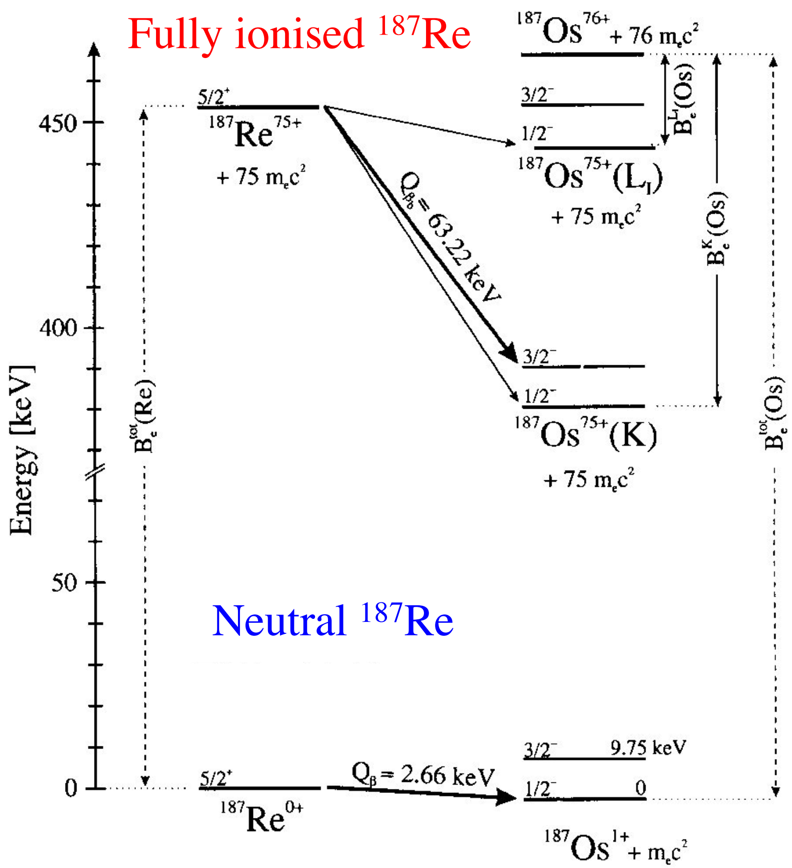
In neutral atoms, only weakly bound state are available for the decay electron, so that in terrestrial conditions, bound state β decay is marginal.

In stellar conditions, this decay can become significant if

- **complete ionization** for the atomic state to be vacant
- **low *Q*-value transition** to be energetically favorable (allowed transition); capture takes place in the deeply bound orbits

A famous example: the bound state β-decay of ^{187}Re

The β decay of the ^{187}Re-^{187}Os pair play an important role in the study of the Re-Os clock ($t_{1/2}$(^{187}Re)~ 4.2 10^{10}y) to estimate the age of the Galaxy

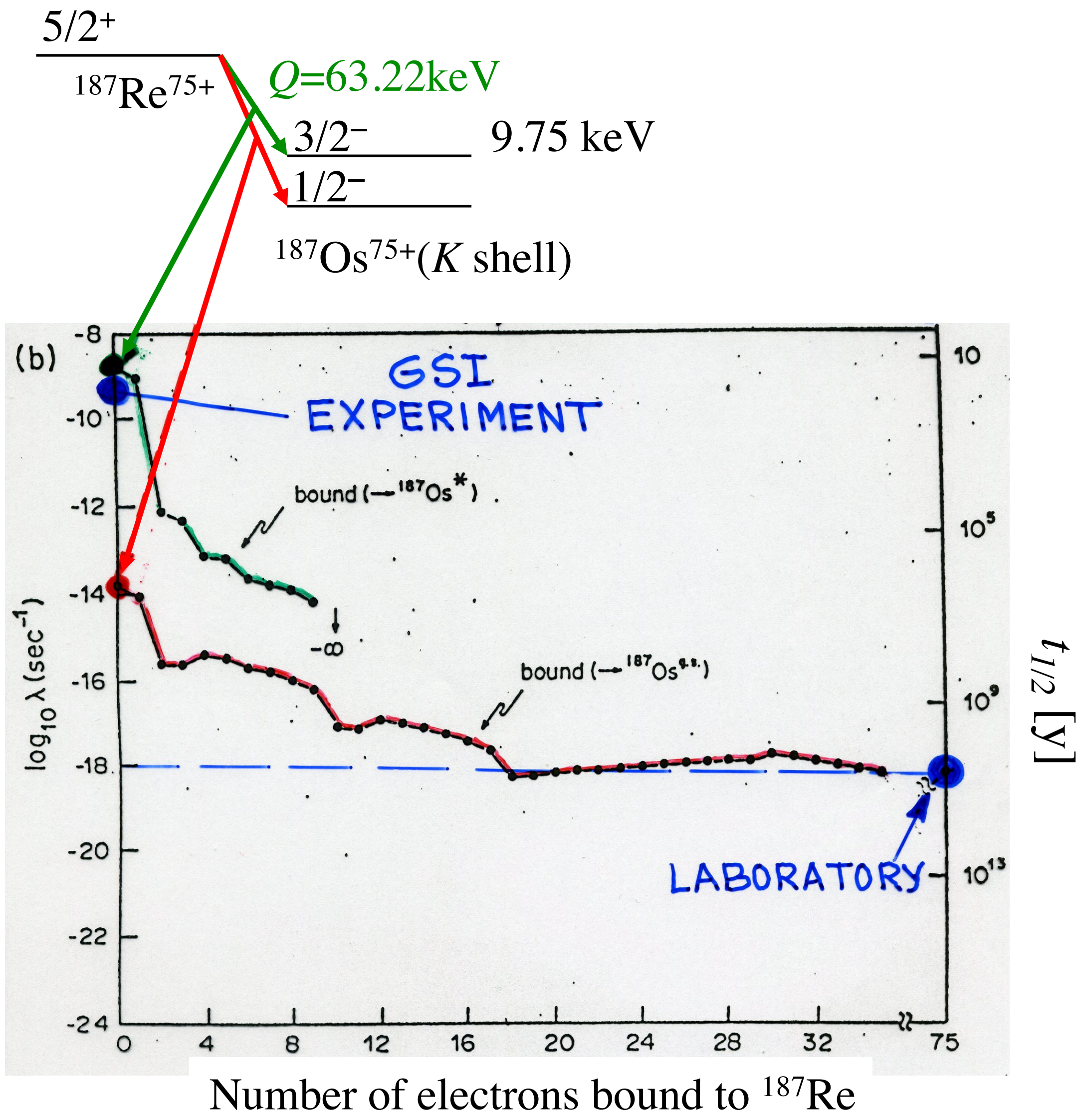


For fully ionised $^{187}Re^{75+}$, β decay to the continuum is forbidden because the electronic cloud in Os is stronger bound by $\Delta B_e^{tot}=B_e^{tot}(Os)-B_e^{tot}(Re)=15.31$keV than in Re. But

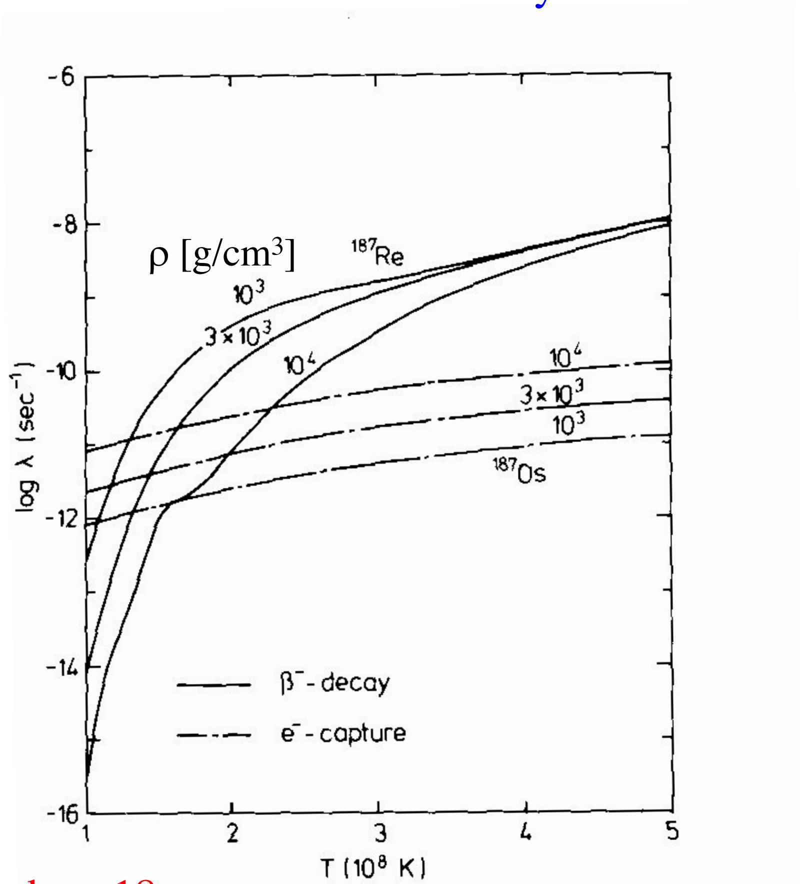
- $^{187}Os^{76+}$ can decay by capturing an electron from the continuum (^{187}Os is stable in the lab)
- $^{187}Re^{75+}$ is unstable against bound state β-decay with the electron bound in the *K* shell with the large Q=73keV value. The first ^{187}Os excited state at 9.75keV can be fed in a non-unique first forbidden transition (Q=63.22keV) with the half-life $t_{1/2}$=14yr, i.e 10^9 times shorter than for the neutral ^{187}Re

Terrestrial conditions: neutral ^{187}Re can only decay (normal β-decay with the electron in the continuum) through the unique, first forbidden transition to the ^{187}Os ground state: Q=2.66 keV and $t_{1/2}$~ 4.2 10^{10}y

Experimental confirmation of the bound state β-decay of fully stripped $^{187}Re^{75+}$



Final T- and ρ-dependence of ^{187}Os and ^{187}Re decays



Lab ~ -18

Application to cosmic rays

The cosmic-ray abundances of some radioactive nuclides can be used for estimating the age of these high-energy particles, or more precisely to provide some answers on some typical timescales that can shed some light in the complicated origin of cosmic rays.

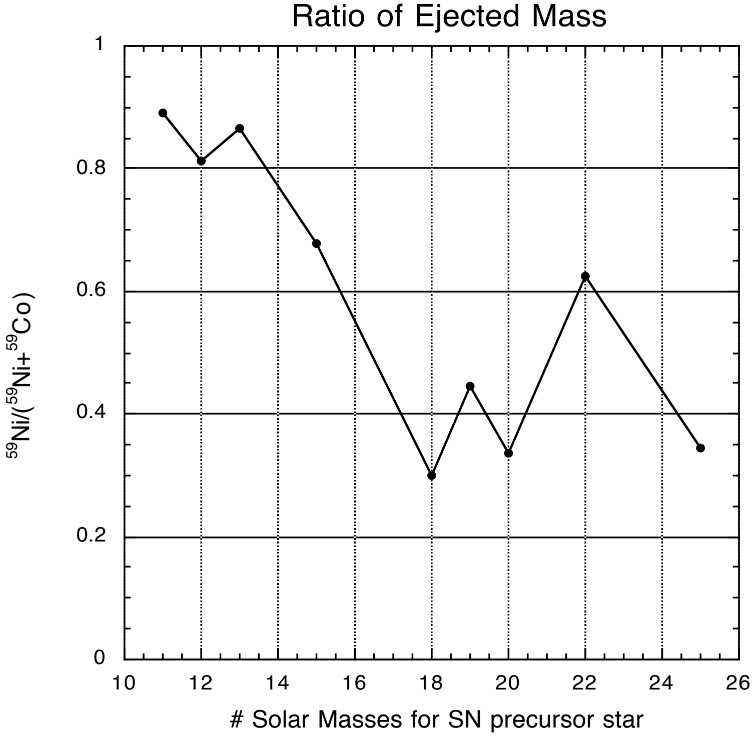
In particular, energy for cosmic rays comes from supernova explosions. **But does a supernova accelerate the nuclei that were synthesized in that supernova, or does it accelerate ambient material in the interstellar medium ?**

The ^{59}Ni electron capture can be used to answer this question.

$$^{59}\text{Ni} + e^- \rightarrow {}^{59}\text{Co with } t_{1/2} = 0.76 \text{ x } 10^5 \text{ yr}$$

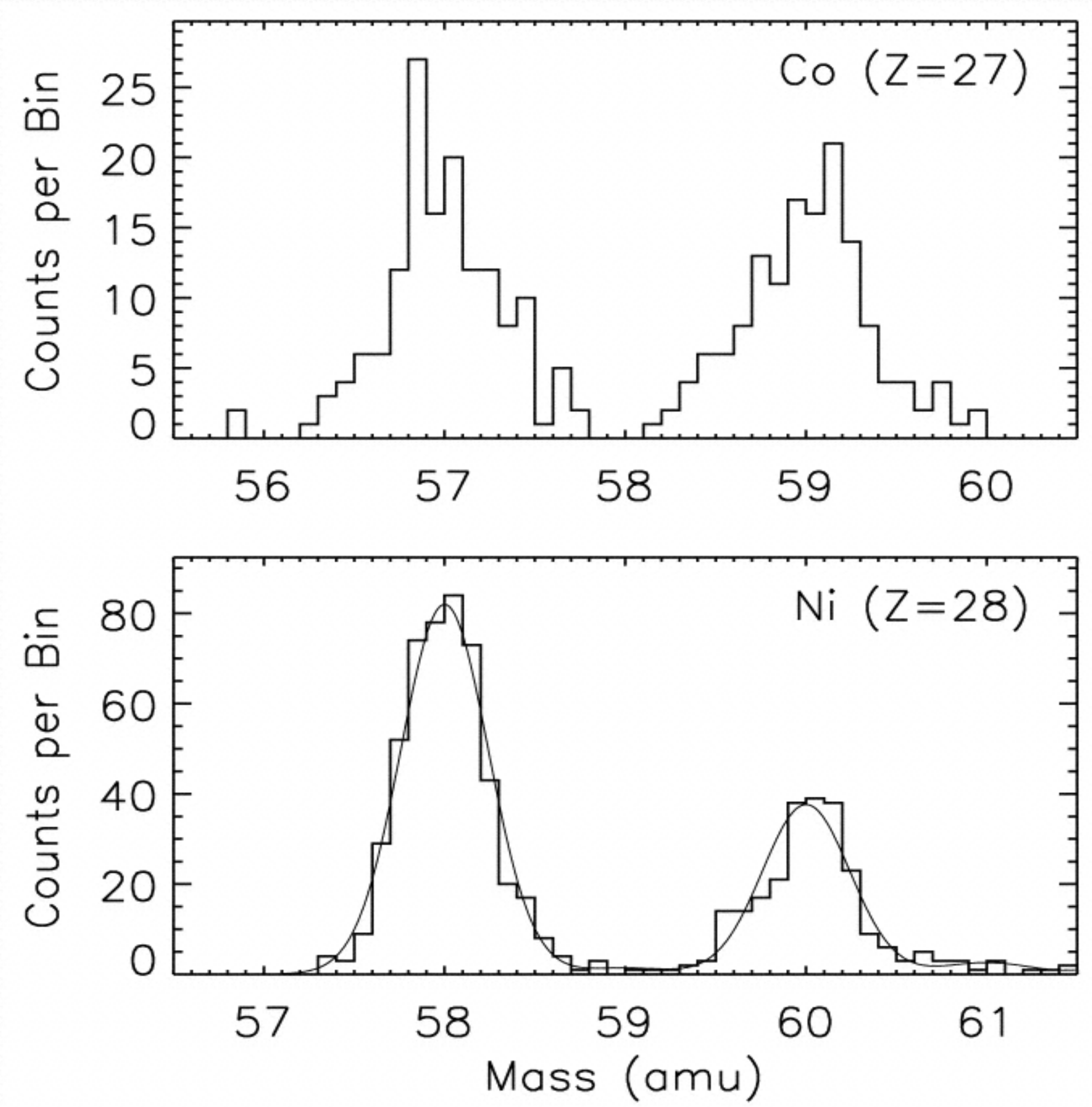
^{59}Ni β-decay in contrast is energetically forbidden, so bare ^{59}Ni nucleus is stable. In acceleration to cosmic-ray energies, nuclei are stripped of all their electrons. So, if supernovae accelerate freshly synthesized material, ^{59}Ni should survive, since ^{59}Ni is still produced in substantial amount by massive stars.

The fraction of mass 59 estimated to be synthesized as ^{59}Ni in type II supernovae



Observation: no detection of ^{59}Ni in the cosmic ray: The ^{59}Ni has all decayed to ^{59}Co.

→ **some 10^5 years between nucleosynthesis and acceleration of cosmic rays.**
(or the core part of the SN ejecta is not accelerated, only the wind and ISM material)



The continuum-e$^{\pm}$ capture

At relatively high temperature (typically T>10^{9}K), a significant amount of photons of the Planck distribution have an energy exceeding $2m_ec^2$=1.022MeV (where m_e is the electron rest mass). This is the lowest energy required for a electron-positron pair creation ($e^{\pm}$).
Since the probability to produce, but also annihilate, an $e^{\pm}$ pair is non-zero, in a system in thermodynamic equilibrium, an equilibrium $e^{\pm}$ pair density will be reached. At very high temperatures, (e.g in advanced burning stages of $M > 40M_{o}$ massive stars), the $e^{\pm}$ pairs are responsible for a non-negligible pressure that need to be taken into account in the equation of state. Similarly, their capture on stable or unstable nuclei can occur, especially in degenerate conditions (high-density environments).
The positrons like the electrons obey the Fermi-Dirac statistics if thermodynamic equilibrium is reached.

$$2\,\gamma \rightleftharpoons e^{+} + e^{-}$$

On the basis of the electroneutrality, the electron and positron densites (n_e) are related to the total density (ρ) by:

$$\sum_k Z_k Y_k = Y_{e^-} - Y_{e^+} \qquad \text{where} \qquad n_e = \rho N_{av} Y_e$$

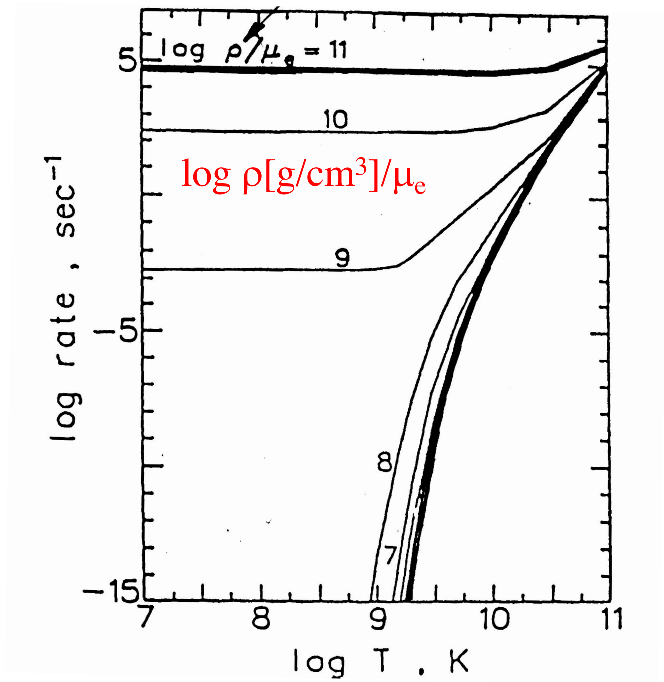
$$\rho/\mu_e = n_e/N_{av}$$

The positrons/electrons have a kinetic energy, so that exo- as well as endothermic transitions are possible

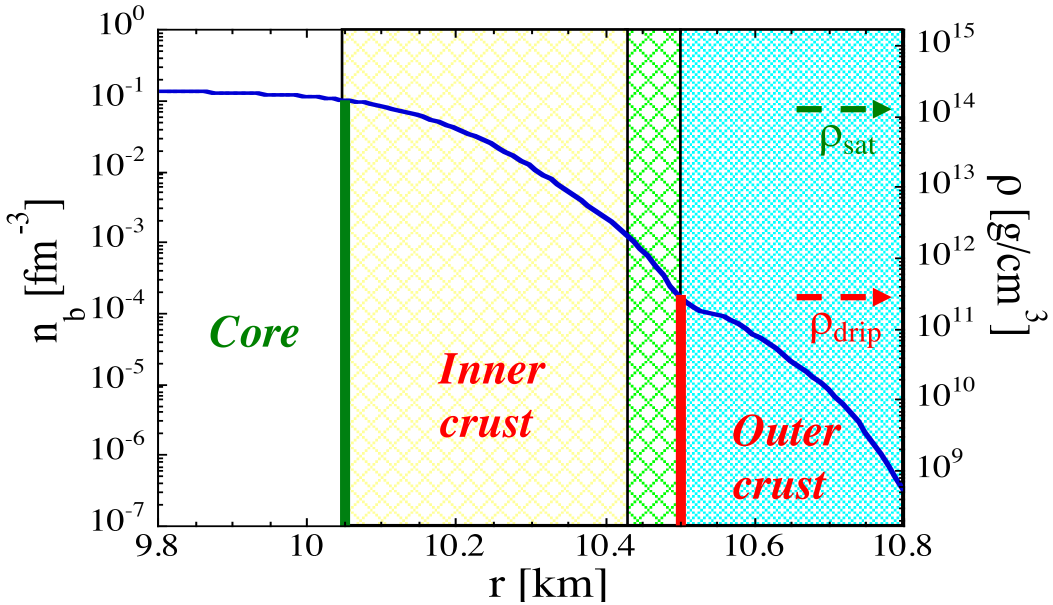
e.g. ^{56}Fe + e^- $\longrightarrow$ ^{56}Mn + ν_e (Q= -3.7MeV)

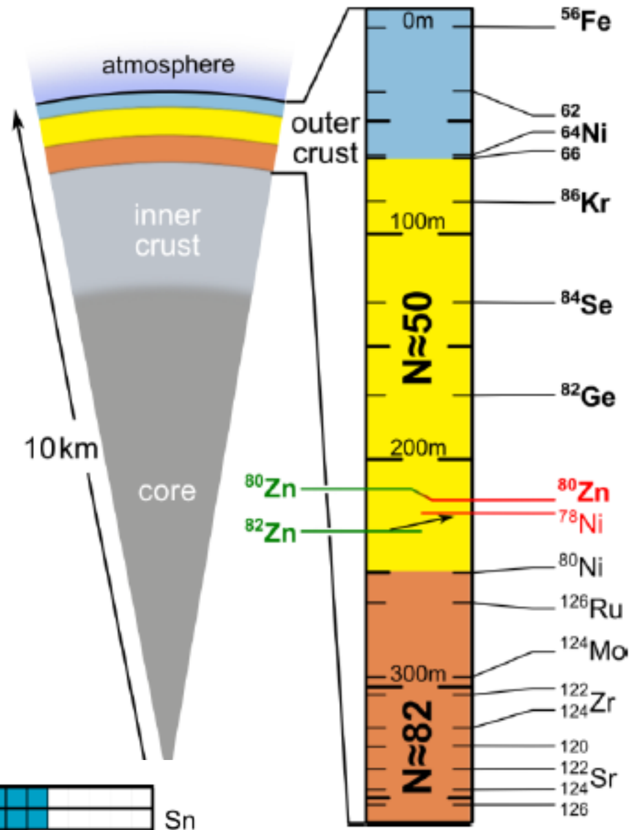
Neutronisation of the matter in high-density environments displacement of the "valley of β stability" (e.g neutron star)

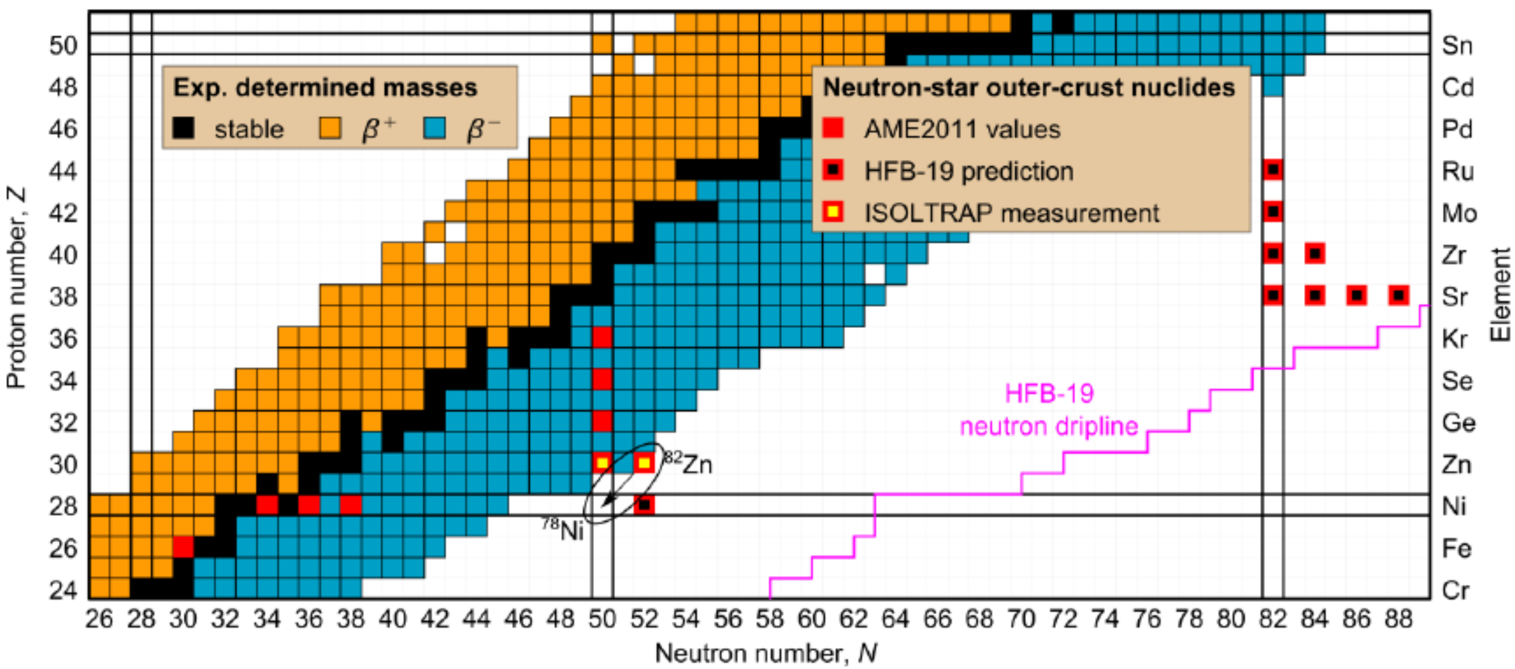
^{56}Fe + e^- $\longrightarrow$ ^{56}Mn + ν_e



Composition of the outer crust of neutron stars is dictated by the capture of relativistic highly degenerate electrons





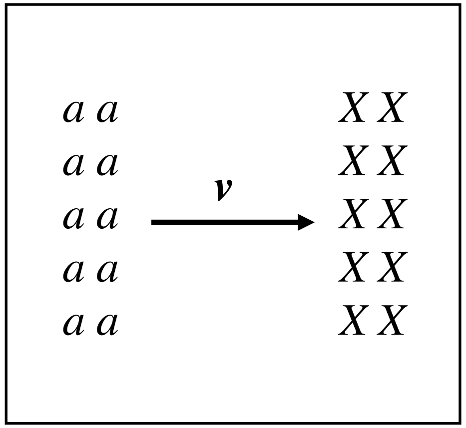


Nuclear reaction cross sections and reaction rates

The energy balance provides the energy produced (or absorbed) Q by each nuclear reaction. To estimate the number of reactions taking place per unit time and unit volume, we introduce the concept of *reaction cross section* which is a measure of the probability that a given reaction takes place.

Let us consider X particles bombarded by a uniform flux of projectiles a. We define the cross section as

$$\sigma[\mathrm{cm}^2] = \frac{\text{nbr of reaction per nucleus } X \text{ and per unit time}}{\text{nbr of incident particles } a \text{ per unit surface and per unit time}}$$



σ can be interpreted as the surface of X seen by the particle a and leading to a reaction, even if this picture is physically not correct !
$\sigma=\sigma(v)$ is a function of the relative velocity between X and a and σ is therefore symmetrical for both types of particles.

Let N_X be the uniform density of the target nuclei and N_a the one of the projectiles. The flux of the incident particles is $N_a v$ [cm^{-3} x cm/s=$\mathrm{cm}^{-2}\ \mathrm{s}^{-1}$].
The reaction rate, defined as the number of reactions taking place per time unit and volume unit is therefore given by

$$r_{aX} = \frac{1}{1+\delta_{aX}} \sigma(v)\, v\, N_a N_X$$

If $a=X$ the number of particle pairs per volume unit is $N^2/2$ and not N^2 --> $\delta_{aX}=1$

For incident charged-particles

$$\sigma(E) = \pi \lambdabar^2 P(E) = \pi \frac{\hbar^2}{p^2} P(E) = \pi \frac{\hbar^2}{2mE} P(E)$$

The Coulomb penetration factor can be estimated in the case of a zero centrifugal barrier (l=0)

$$P(E) \propto \exp\left(-\frac{2\pi Z_X Z_a e^2}{\hbar v} \right) = \exp(-2\pi\eta)$$ where η is the Sommerfeld parameter

$$\boxed{\sigma(E) \equiv \frac{S(E)}{E} \exp\left(-bE^{-1/2}\right)}$$ where $b = 2\pi\eta E^{1/2} = 31.28\ Z_X Z_a \mu^{1/2}\ \text{keV}^{1/2}$

where $S(E)$ is called the *astrophysical S-factor* which presents the advantage of not being very much energy dependent (except for resonant reactions) since the geometrical ($1/E$) and the Coulomb repulsion [$\exp(-bE^{-1/2})$] factors have been explicitly extracted. $S(E)$ represents the intrinsic nuclear part of the reaction probability.

- $S(E)$ does not vary significantly in the energy region of astrophysics interest, and is therefore often expressed as $S(E)=S_0+S'_0\,(E-E_0)+\ldots$
- $S(E)$ may vary strongly on small energy scales due to the resonance phenomenon

For incident neutrons

$$\sigma(E) \approx \pi \lambda\!\!\!^{-2} P(E) \propto \frac{\sqrt{E}}{E} \propto \frac{1}{\sqrt{E}} \propto \frac{1}{v}$$

This relation is known as the **1/*v*-law of neutron capture**

Due to the absence of the Coulomb barrier, reactions involving neutrons are extremely fast (by far faster than reactions with charged particles), so that the neutron half-life in a stellar plasma is extremely small (of the order of the micro- or milli-second). Neutrons can therefore not be found in traditional stellar medium, since as soon as they are produced, they are absorbed. They play a negligible role in the nuclear energy production. In contrast, they play a role of first importance in the nucleosynthesis of elements heavier than iron. Neutron captures are responsible for almost all nuclei heavier than iron (i.e about 2/3 of stable nuclei in the Universe). The identification of the astrophysical sites in which neutrons can be produced in sizeable amount remains one of the major mysteries of modern astrophysics.

The stellar plasmas in which nuclear reactions take place are in most cases *non-degenerate* and *non-relativistic*. In that case, the particle velocities obey a Maxwell-Boltzmann normalised distribution. Let $\phi(v)$ be the normalised distribution of relative velocities, i.e. $\phi(v)dv$ is the probability that the relative velocity between both particles ranges between v and $v+dv$. The reaction rate for a given velocity v can therefore be generalised to the distribution $\phi(v)$ through

$$r_{aX} = \frac{1}{1+\delta_{aX}} N_a N_X \int_0^\infty \sigma(v) v \, \phi(v) dv = \frac{1}{1+\delta_{aX}} N_a N_X \langle \sigma v \rangle \qquad \text{with} \quad \int_0^\infty \phi(v) dv = 1$$

From the reaction rate, we can now derive the differential equations describing

- the destruction of species X: $$\frac{\partial N_X}{\partial t} = -(1+\delta_{aX}) r_{aX} = -N_a N_X \langle \sigma v \rangle = -\frac{N_X}{\tau_a(X)}$$ independent of δ_{aX}

  where $\tau_a(X)$ is the destruction timescale of X against reaction with a, i.e.

$$\tau_a(X) = \frac{1}{N_a \langle \sigma v \rangle}$$

- the production of species Y: $$\frac{\partial N_Y}{\partial t} = +r_{aX} = \frac{1}{1+\delta_{aX}} N_a N_X \langle \sigma v \rangle$$ if $<\sigma v>$ is the $X(a,b)Y$ reaction rate

The energy production rate by the *X(a,b)Y* reaction per unit time

- and per unit volume $\varepsilon_{X(a,b)Y} = r_{X(a,b)Y} \, Q_{X(a,b)Y}$ [MeV/cm^3/s]
- and per unit mass $\varepsilon_{X(a,b)Y} = r_{X(a,b)Y} \, Q_{X(a,b)Y} / \rho$ [MeV/g/s]

The total energy ε_{nuc} per volume unit or per mass unit is obtained directly by summing up the contribution of each nuclear reaction taking place in the stellar plasma.

Normalised distribution of relative velocities

In a non-degenerate non-relativistic plasma, the velocity distributions obey the Maxwell-Boltzmann law, but what law do the relative velocities follow ?

For the X particles, the Maxwell-Boltzmann distribution of absolute velocities can be written as

$$N_X\,\phi(v_X)\,d^3v_X = N_X\left(\frac{m_X}{2\pi\,k_BT}\right)^{3/2}\exp\left(-\frac{m_x v_X^2}{2k_BT}\right)d^3v_X \qquad \text{such that} \qquad \int\phi(v_X)\,d^3v_X = 1$$

The reaction rate is therefore expressed as a double integral on

$$N_X\,\phi(v_X)\,d^3v_X\;N_a\,\phi(v_a)\,d^3v_a = N_XN_a\frac{(m_Xm_a)^{3/2}}{(2\pi\,k_BT)^3}\exp\left(-\frac{m_x v_X^2 + m_a v_a^2}{2k_BT}\right)d^3v_X\;d^3v_a$$

In terms of the centre-of-mass and relative velocities, we can now write

$$\begin{cases}\vec{v}_x = \vec{V} + \dfrac{m_a}{m_X + m_a}\vec{v}\\[2ex] \vec{v}_a = \vec{V} - \dfrac{m_X}{m_X + m_a}\vec{v}\end{cases} \qquad N_X\,\phi(v_X)\,d^3v_X\;N_a\,\phi(v_a)\,d^3v_a = N_XN_a\frac{(m_Xm_a)^{3/2}}{(2\pi\,k_BT)^3}\exp\left(-\frac{(m_x + m_a)V^2}{2k_BT} - \frac{\mu v^2}{2k_BT}\right)d^3V\,d^3v$$

$$r = \int\sigma(v)v\,N_X\,\phi(v_X)\,d^3v_X\;N_a\,\phi(v_a)\,d^3v_a = N_XN_a\underbrace{\int\left(\frac{m_X + m_a}{2\pi\,k_BT}\right)^{\frac{3}{2}}\exp\left(-\frac{(m_x + m_a)V^2}{2k_BT}\right)d^3V}_{=1}\;\int\sigma(v)v\left(\frac{\mu}{2\pi\,k_BT}\right)^{\frac{3}{2}}\exp\left(-\frac{\mu v^2}{2k_BT}\right)d^3v$$

$$\Rightarrow \quad \boxed{r = N_XN_a\int\sigma(v)v\left(\frac{\mu}{2\pi\,k_BT}\right)^{\frac{3}{2}}\exp\left(-\frac{\mu v^2}{2k_BT}\right)4\pi v^2dv}$$

$$\mu = \frac{m_Xm_a}{m_X + m_a}$$

$$d^3v = 4\pi v^2dv$$

The relative velocity distribution is therefore also a Maxwell-Boltzmann distribution based on the reduced mass μ

$$\mu = \frac{m_X m_a}{m_X + m_a} \longrightarrow \phi(v)dv = \left(\frac{\mu}{2\pi k_B T}\right)^{\frac{3}{2}} \exp\left(-\frac{\mu v^2}{2k_B T}\right) 4\pi v^2 dv$$

Usually, the cross section is estimated as a function of the relative kinetic energy E, rather than as a function of the relative velocity v. In the non-relativistic case,

$$E = \frac{1}{2}\mu v^2$$

In this case

$$\psi(E)dE = \phi(v)dv = \frac{2}{\sqrt{\pi}} \frac{E}{k_B T} \exp\left(-\frac{E}{k_B T}\right) \frac{dE}{\sqrt{k_B T E}} \qquad dv = \sqrt{\frac{2}{\mu}} \frac{1}{2\sqrt{E}} dE$$

$$\langle \sigma v \rangle = \int_0^\infty \sigma(E) v(E) \psi(E) dE = \frac{2}{\sqrt{\pi}} \int_0^\infty \sigma(E) \sqrt{\frac{2E}{\mu}} \frac{E}{k_B T} \exp\left(-\frac{E}{k_B T}\right) \frac{dE}{\sqrt{k_B T E}}$$

$$\boxed{r = N_X N_a \langle \sigma v \rangle = N_X N_a \frac{8\pi}{\sqrt{\mu}} \left(\frac{1}{2\pi k_B T}\right)^{\frac{3}{2}} \int_0^\infty \sigma(E) E \exp\left(-\frac{E}{k_B T}\right) dE}$$

The major difficulty in the estimate of the nuclear reactions of astrophysics interest is therefore to determine the cross section $\sigma(E)$ and its energy dependence in the energy range of relevance.

The nuclear reaction rate will be important if the abundance of the target and the projectile is high and if the interaction probability (reaction cross section) is high too.

Reaction rate of astrophysical interest for a target in its GS

$$\langle \sigma v \rangle = \frac{8\pi}{\sqrt{\mu}} \left(\frac{1}{2\pi k_B T} \right)^{\frac{3}{2}} \int_0^{\infty} \sigma(E) E \exp\left(-\frac{E}{k_B T} \right) dE$$

Traditionally expressed in terms of

- $N_A\langle\sigma v\rangle$ in $mol^{-1}\ cm^3\ s^{-1}$ (N_A = Avogadro nbr)
- $\langle\sigma\rangle = \langle\sigma v\rangle / v_T$ in mb for neutrons (MACS)

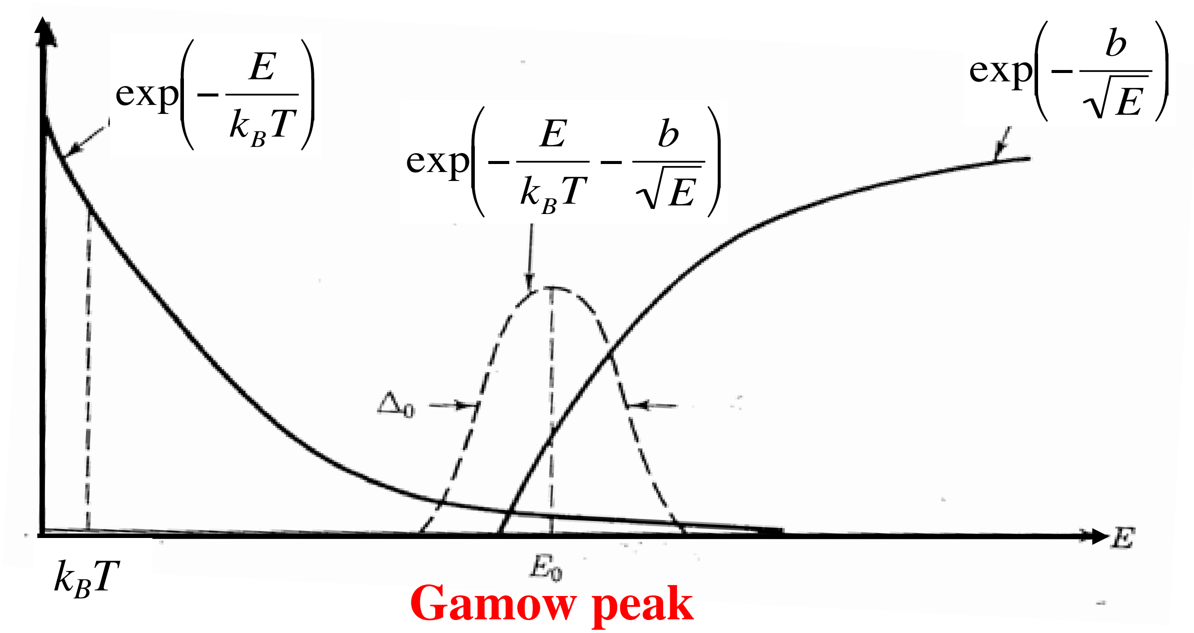
  where v_T is the thermal velocity $v_T = \sqrt{\frac{2 k_B T}{\mu}}$

Nuclear reaction rates in stellar plasmas

On the basis of the energy-dependent expression of the cross section *for charged-particle induced reactions*

$$\sigma(E) \equiv \frac{S(E)}{E}\exp\left(-bE^{-1/2}\right) \qquad \text{where } b = 2\pi\eta E^{1/2} = 31.28\, Z_X Z_a \mu^{1/2}\ \text{keV}^{1/2}$$

$$\Rightarrow \quad \langle\sigma v\rangle = \frac{8\pi}{\sqrt{\mu}}\left(\frac{1}{2\pi k_B T}\right)^{\frac{3}{2}}\int_0^\infty \sigma(E) E \exp\left(-\frac{E}{k_B T}\right) dE = \sqrt{\frac{8}{\pi\mu}}\frac{1}{(k_B T)^{3/2}}\int_0^\infty S(E)\exp\left(-\frac{E}{k_B T}-\frac{b}{\sqrt{E}}\right) dE$$



The integrant is a strongly peaked function around the so-called Gamow energy E_0 defined from

$$\frac{d}{dE}\left(\frac{E}{k_B T} + bE^{-1/2}\right)_{E=E_0} = \frac{1}{k_B T} - \frac{1}{2}bE_0^{-3/2} = 0 \quad \Rightarrow \quad E_0 = \left(\frac{bk_B T}{2}\right)^{2/3}$$

Or numerically $E_0=1.220(Z_X^2 Z_a^2\,\mu\, T_6^2)^{1/3}$ keV where T_6 is the temperature in million of degrees

Generally, $E_0 \gg k_B T = 0.086 T_6$ keV indicating that the barrier penetration factor favours highly energetic particles in the tail of the Maxwell-Boltzmann distribution.
It is also possible to estimate in a first approximation the width of the Gamow peak by replacing the integrant by

$$\exp\left(-\frac{E}{k_B T} - bE^{-1/2}\right) \approx I_{\max} \exp\left[-\left(\frac{E-E_0}{\Delta/2}\right)^2\right]$$

$$\longrightarrow \quad I_{\max} = \exp\left(-\frac{E_0}{k_B T} - bE_0^{-1/2}\right) = \exp\left(-\frac{3E_0}{k_B T}\right) \qquad \frac{E_0}{k_B T} = \frac{1}{2} b E_0^{-1/2}$$

Or numerically: $I_{\max}=\exp[-42.46\,(Z_X^2 Z_a^2 \mu / T_6)^{1/3}]$

And requesting both second derivatives to be equal at $E=E_0$ (same curvature):

d/dE

$$\left[-\frac{1}{k_B T} + \frac{b}{2} E^{-3/2}\right] \exp\left(-\frac{E}{k_B T} - bE^{-1/2}\right) \approx -2 I_{\max} \frac{E-E_0}{(\Delta/2)^2} \exp\left[-\left(\frac{E-E_0}{\Delta/2}\right)^2\right]$$

d^2/dE^2

$$\left[\left(-\frac{1}{k_B T} + \frac{b}{2} E^{-3/2}\right)^2 - \frac{b}{2}\frac{3}{2} E^{-5/2}\right] \exp\left(-\frac{E}{k_B T} - bE^{-1/2}\right) \approx -2 I_{\max} \frac{1}{(\Delta/2)^2} \left[-2\frac{(E-E_0)^2}{(\Delta/2)^2} + 1\right] \exp\left[-\left(\frac{E-E_0}{\Delta/2}\right)^2\right]$$

at $E=E_0$

$$\begin{cases} \dfrac{2}{(\Delta/2)^2} = \dfrac{3b}{4} E_0^{-5/2} \\[2ex] \dfrac{1}{k_B T} = \dfrac{1}{2} b E_0^{-3/2} \end{cases} \qquad \frac{(\Delta/2)^2}{2} = \frac{4}{3} k_B T E_0 \quad \Rightarrow \quad \Delta = \frac{4}{\sqrt{3}} \sqrt{k_B T E_0}$$

Δ is approximatively twice the geometrical mean of $k_B T$ and E_0. Hence, $\Delta < E_0$

Or numerically $\Delta=0.75(Z_X^2 Z_a^2\,\mu\, T_6^5)^{1/6}$ keV

For example for the $^{12}C(p,\gamma)^{13}N$ reaction, at T=30 10^6 K (twice the central temperature in the sun) we find: E_0=38 keV and Δ=23 keV, while $k_B T$=2.6 keV and B_{coul}~3MeV.

The energy of astrophysics interest, i.e. the energy at the Gamow peak is extremely weak (by far smaller than the height of the Coulomb barrier). To estimate the reaction rate experimentally or theoretically, we need to estimate the astrophysical *S*-factor in the vicinity of the Gamow energy E_0 in an energy range of the order of Δ.

At such energies (~keV), the interaction probability is extremely small, this explains why the characteristic timescales for nuclear burning in stars is very long (10 billion years for the sun to burn its hydrogen).

At such low energies, it remains almost impossible these days to determine the cross sections experimentally (of the order of the picobarn 10^{-12} barn; 1 barn=10^{-24} cm^2=10^{-28} m^2). Indirect methods need to be developed to provide information that will enable us to extrapolate more safely the *S*-factor in the energy range of astrophysics interest.

For *radiative neutron captures* (with l=0), the cross section obeys the $1/v$-law, so that

$$\langle \sigma v \rangle \propto \sqrt{\frac{8}{\pi\mu}} \frac{1}{(k_B T)^{3/2}} \int_0^\infty \sqrt{E} \exp\left(-\frac{E}{k_B T}\right) dE$$

And the integrant is maximum for

$$\frac{d}{dE}\left(\sqrt{E}\exp\left(-\frac{E}{k_B T}\right)\right)_{E=E_0} = \frac{1}{2\sqrt{E_0}} - \frac{\sqrt{E_0}}{k_B T} = 0 \quad \Rightarrow \quad \boxed{E_0 = \frac{k_B T}{2}}$$

so that $$\langle \sigma v \rangle \propto \sqrt{\frac{8}{\pi\mu}} \frac{1}{(k_B T)^{3/2}} \int_0^\infty \sqrt{E} \exp\left(-\frac{E}{k_B T}\right) dE = \sqrt{\frac{8}{\pi\mu}} \frac{\sqrt{\pi}}{2} = \sqrt{\frac{2}{\mu}} = \text{Constant}$$

$$\int_0^\infty \sqrt{x}\, e^{-x} dx = 2\int_0^\infty y^2 e^{-y^2} dy = \int_0^\infty e^{-y^2} dy = \frac{\sqrt{\pi}}{2}$$

In such specific conditions, the neutron capture reaction *rate* is temperature and density independent !

Rem: It should be kept in mind that the results obtained here are based on a gross estimate of the energy dependence of the reaction cross section ($S(E)$ constant for charged particles, and the $1/v$-law for neutrons). Such a dependence can be drastically different in the neighbourhood of a resonance, leading to a very strong variation of σ on a very small energy range. These resonances, and most particularly those located in the Gamow energy range $E_0 \pm \Delta/2$ of astrophysical interest play a fundamental role and can strongly affect the nuclear reactions in a stellar plasma.

Contribution from thermally populated excited states

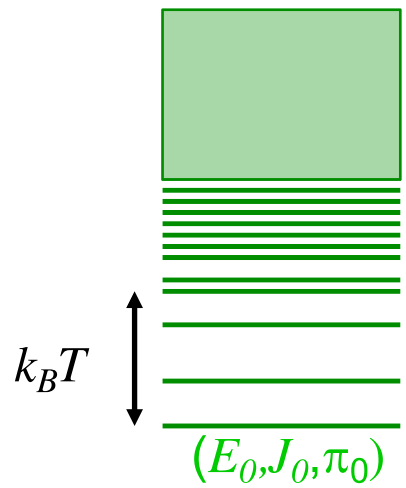
In hot astrophysical plasmas, *a target nucleus exists in its ground as well as excited states*.
In a thermodynamic equilibrium situation, the relative populations of the various levels of nucleus I^{μ} with excitation energies $\varepsilon^{\mu}{}_{I}$ obey a Maxwell-Boltzmann distribution.
The effective stellar rate of per pair of particles in the entrance channel at temperature T taking due account of the contributions of the various target excited states μ is thus expressed in a classical notation (in cm^3 s^{-1} mole^{-1}) as

$$N_{\mathrm{A}}\langle\sigma v\rangle^{*}_{jl}(T) = \left(\frac{8}{\pi m}\right)^{1/2}\frac{N_{\mathrm{A}}}{(kT)^{3/2}\;G_I(T)}\int_0^{\infty}\sum_{\mu}\frac{(2J_I^{\mu}+1)}{(2J_I^{0}+1)}\sigma^{\mu}_{jl}(E)E\exp\left(-\frac{E+\varepsilon_I^{\mu}}{kT}\right)dE$$

where k is the Boltzmann constant, m the reduced mass of the $I^0 + j$ system, N_{A} the Avogadro number, and $G(T)$ the temperature-dependent normalised partition function given by

$$G_I(T) = \sum_{\mu}\frac{2J_I^{\mu}+1}{2J_I^{0}+1}\exp\left(-\frac{\varepsilon_I^{\mu}}{kT}\right)$$

$k_B T$ [keV] = 86.2 T [10^9K]



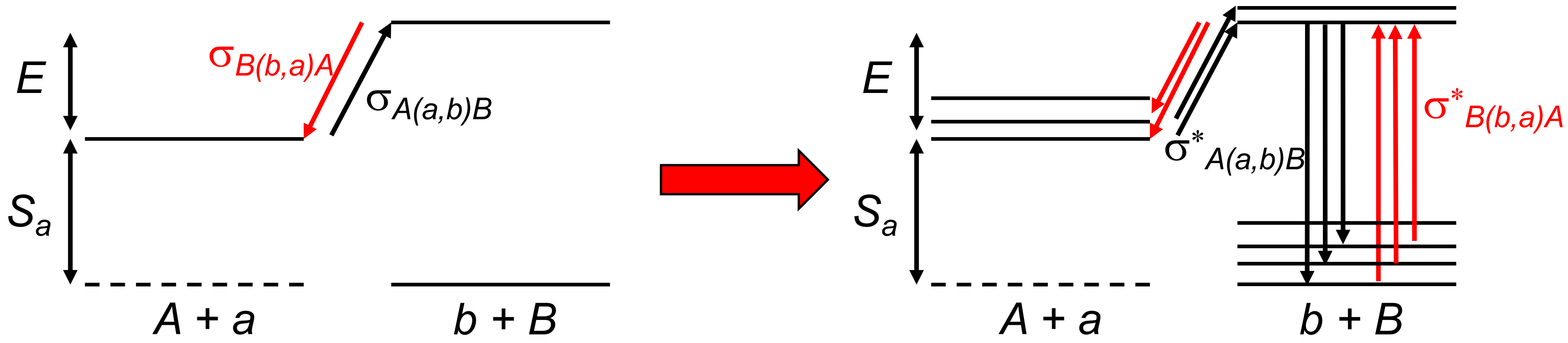
Detailed balance and reverse reactions in stellar conditions

Reverse reactions can be estimated with the use of the reciprocity theorem. In particular, the stellar photo-dissociation rates (in s^{-1}) are classically derived from the reverse radiative capture rates by

$$\lambda^*_{(\gamma,j)}(T) = \frac{(2J_I^0+1)(2J_j+1)}{(2J_L^0+1)} \frac{G_I(T)}{G_L(T)} \left(\frac{mkT}{2\pi\hbar^2}\right)^{3/2} \langle\sigma v\rangle^*_{(j,\gamma)} \; e^{-Q_{j\gamma}/kT}$$

where $Q_{j\gamma}$ is the Q-value of the $I^0(j,\gamma)L^0$ capture reaction.

Note that, in stellar conditions, the reaction rates for targets in thermal equilibrium obey reciprocity since the forward and reverse channels are symmetrical, in contrast to the situation which would be encountered for targets in their ground states only.



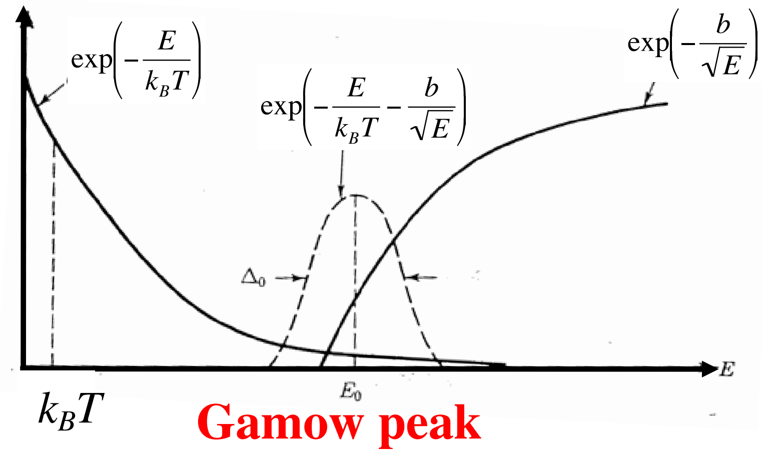
In Summary:

for all reacting species in the stellar plasma, we need to estimate experimentally or theoretically the astrophysical reaction rate at the temperature T of the plasma:

$$N_A \langle \sigma v \rangle^*_{jl}(T) = \left(\frac{8}{\pi m}\right)^{1/2} \frac{N_A}{(kT)^{3/2}\, G_I(T)} \int_0^\infty \sum_\mu \frac{(2J_I^\mu + 1)}{(2J_I^0 + 1)} \sigma^\mu_{jl}(E) E \exp\left(-\frac{E + \varepsilon_I^\mu}{kT}\right) dE$$

- For charged particles: around the Gamow energy E_0 (**far below the Coulomb barrier**)

$$\sigma(E) \equiv \frac{S(E)}{E} \exp\left(-bE^{-1/2}\right)$$

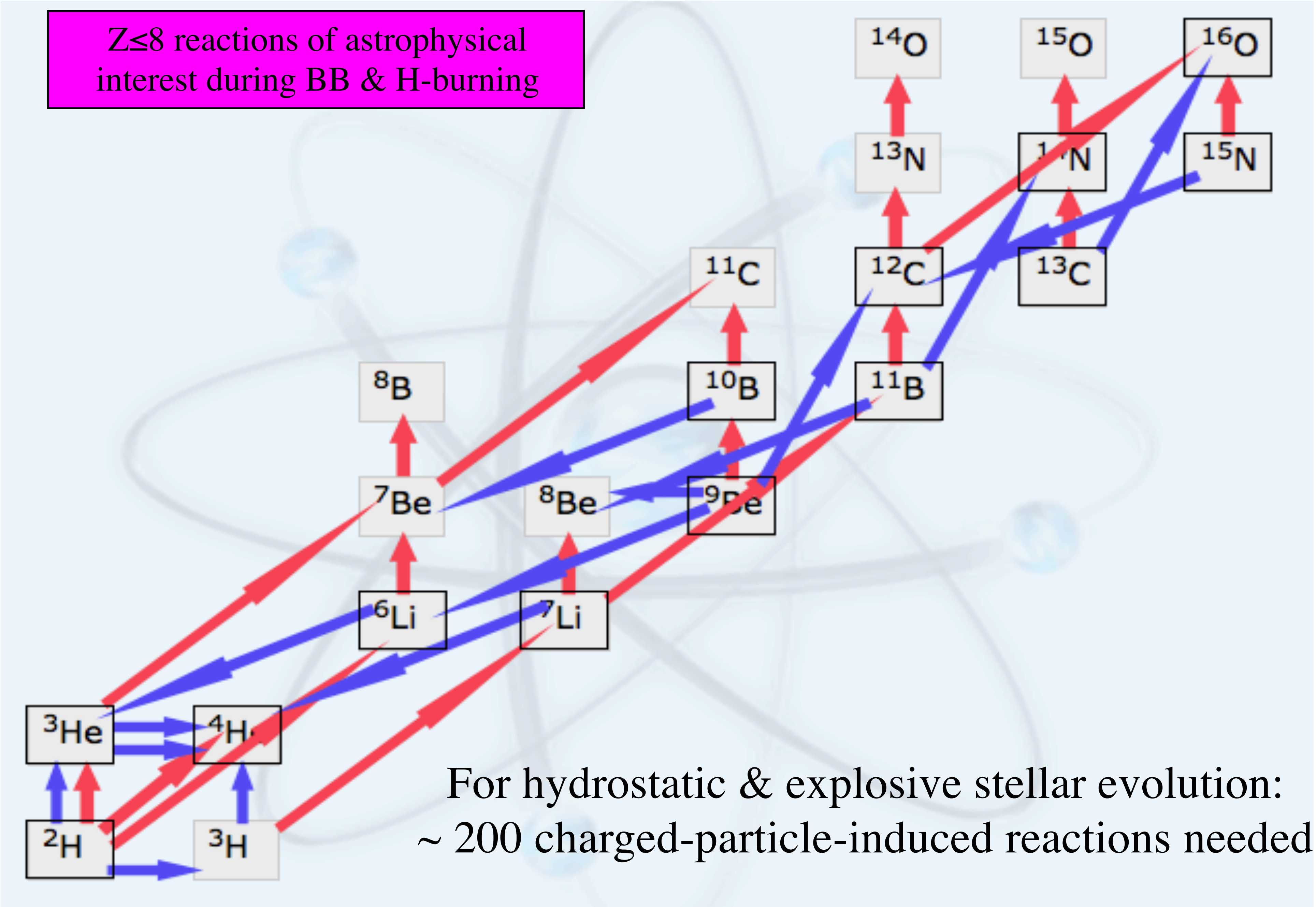


- For neutrons: around $k_B T$ (10-100 keV for s- or r-process nucleosynthesis)

$$\sigma(E) \approx \pi \lambda\!\!\!^{-2} P(E) \propto \frac{\sqrt{E}}{E} \propto \frac{1}{\sqrt{E}} \propto \frac{1}{v} \qquad (1/v\text{-law})$$

- Contribution from thermally excited states in target nuclei must be taken into account

Z≤8 reactions of astrophysical interest during BB & H-burning



For hydrostatic & explosive stellar evolution:
~ 200 charged-particle-induced reactions needed

Compilation/Evaluation/Dissemination of reaction cross sections

Compilation/**Evaluation**/Dissemination of available nuclear data (including error bars) for ~ 200 charged-particle-induced reactions from H to ^{56}Fe involved in Big Bang, H-, He-, C-, O-, Si-burning

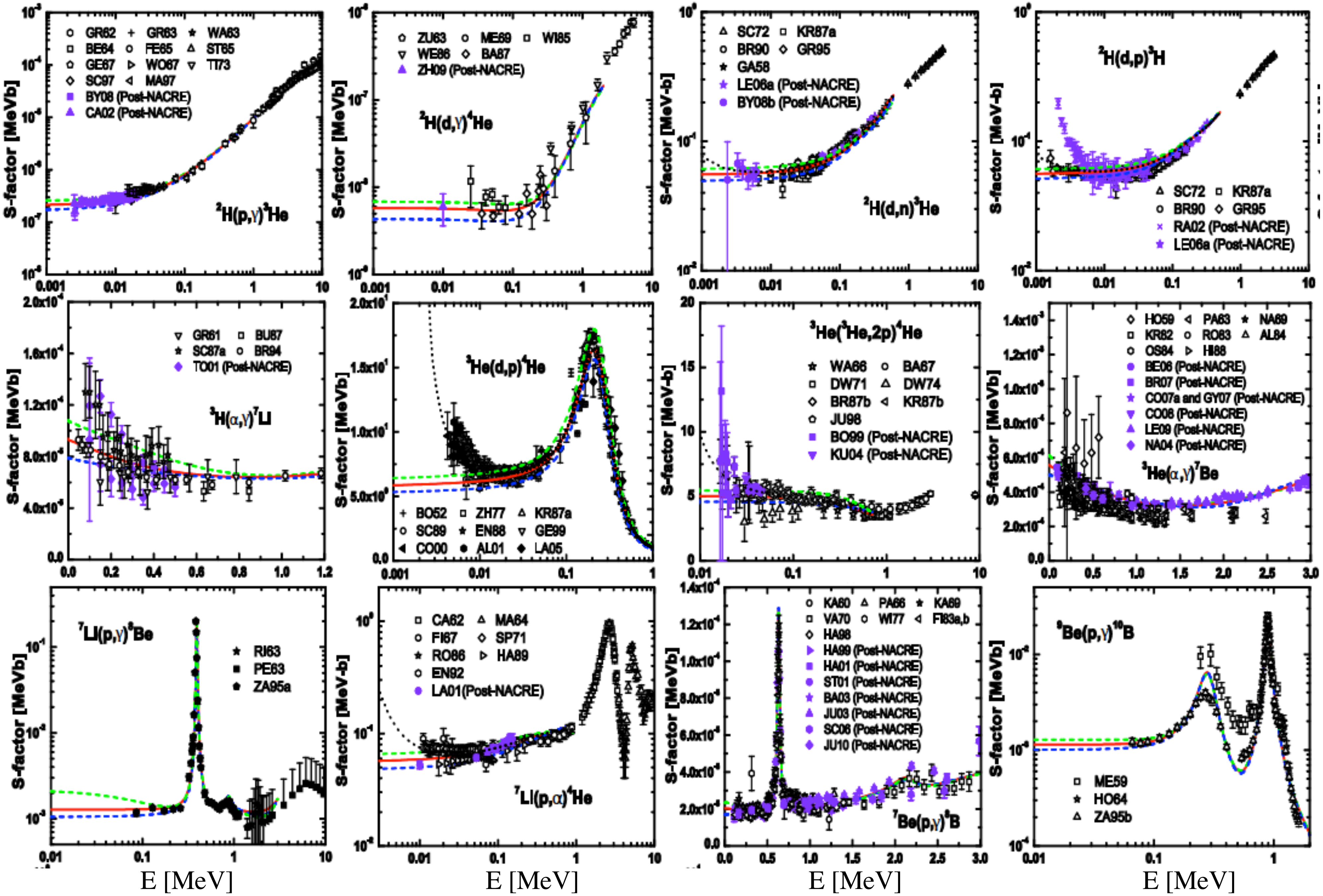
major recent evaluation efforts (with error bar estimates)

- ☞ NACRE II: Xu et al. Nucl. Phys. A 918, 61 (2011) : ***A*****=1-15**
- ☞ Iliadis et al. Nucl. Phys. A 841, 1 (2010): ***A*****=16-40**
- ☞ Descouvemont et al: At. Data Nucl. Data Tables 88 (2004) 203 : ***A*****=1-7**
- ☞ NACRE: Angulo et al. Nucl. Phys. 656, 3 (1999): ***A*****=1-28**

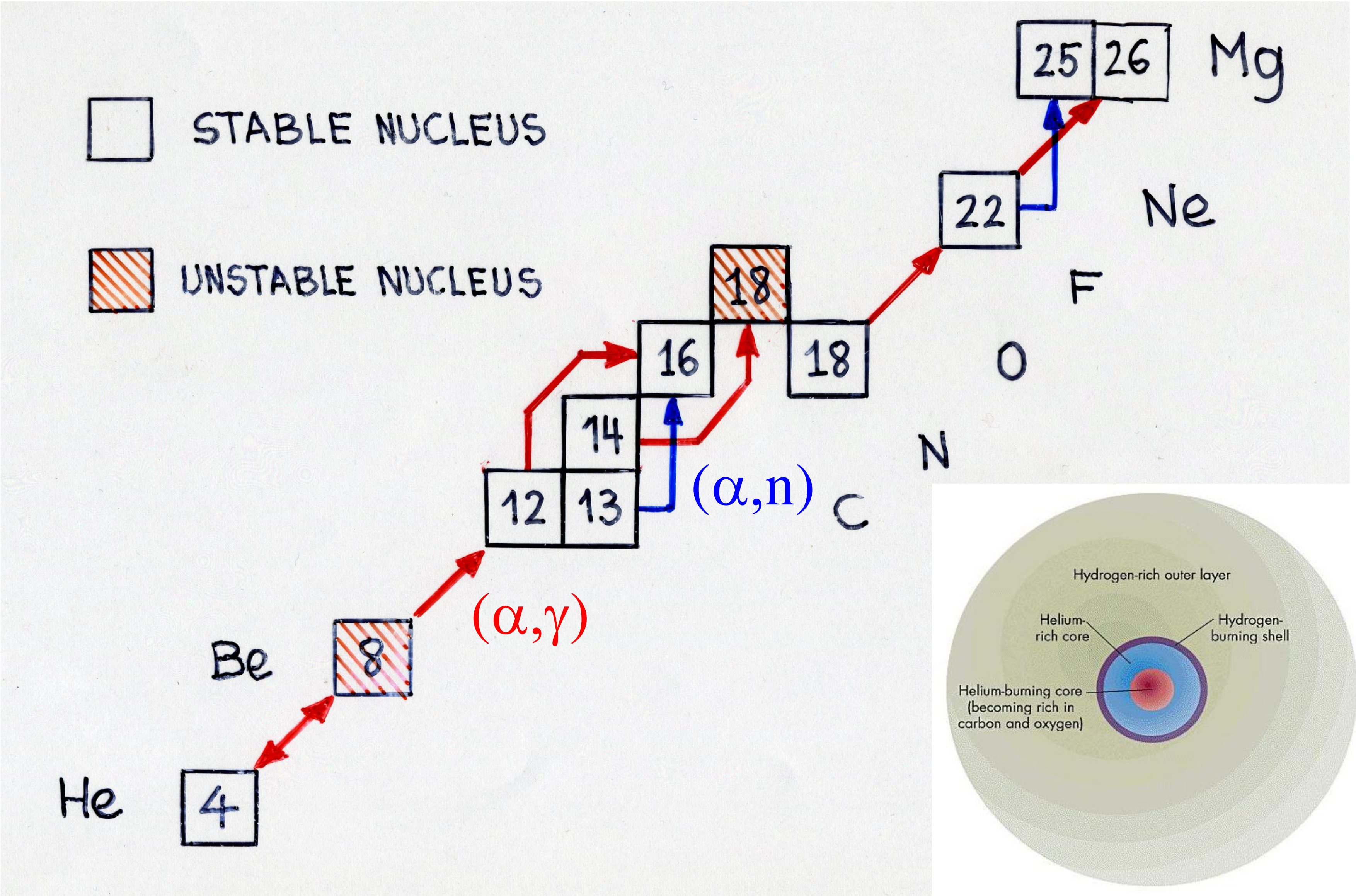
→ Tabulation of reaction rates in a Temperature grid

Uncertainties on nuclear reaction rates may impact nucleosynthesis predictions. Robustness of predictions needs to be checked with respect to *nuclear uncertainties*.

Evaluation of reaction cross sections (S-factors)



Non-explosive He burning



Non-explosive He burning

He-burning involves principally the reactions **$3\alpha \rightarrow {}^{12}C$ and ${}^{12}C(\alpha,\gamma){}^{16}O$** and the destruction of ${}^{13}C$ and ${}^{14}N$ to produce ${}^{22}Ne$ and ${}^{25,26}Mg$

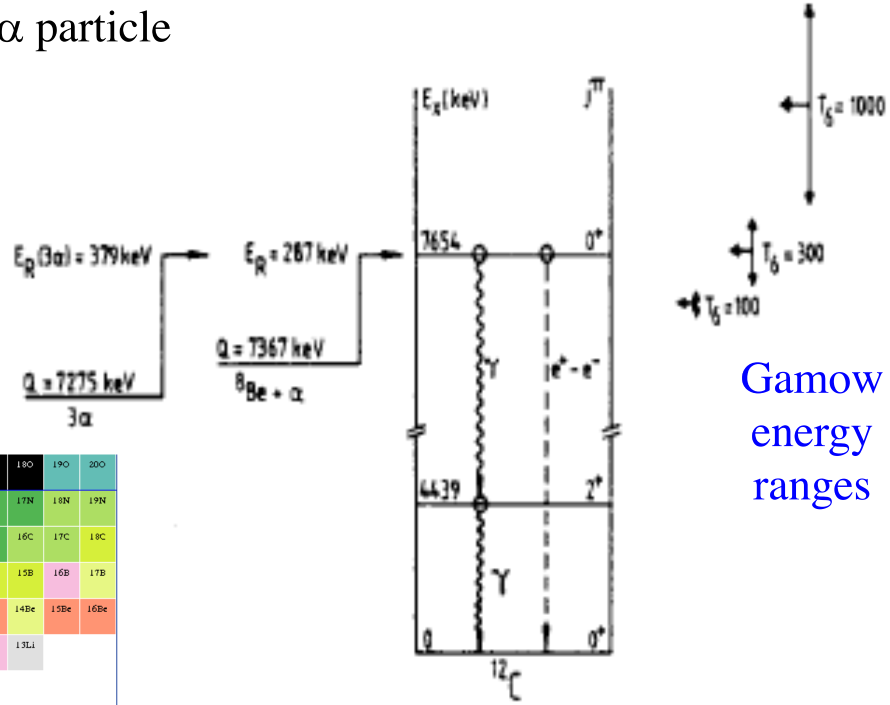
$3\alpha \rightarrow {}^{12}C$ has a positive Q-value but is highly unlikely if it involves a 3-body interaction. In reality, it corresponds to a two-steps reaction, namely

1st step: $\alpha + \alpha \rightarrow {}^{8}Be$ (Q=–92.1keV) but ${}^{8}Be$ is unstable ($\tau_{1/2} \sim 10^{-16}$s) and is dissociated into 2α

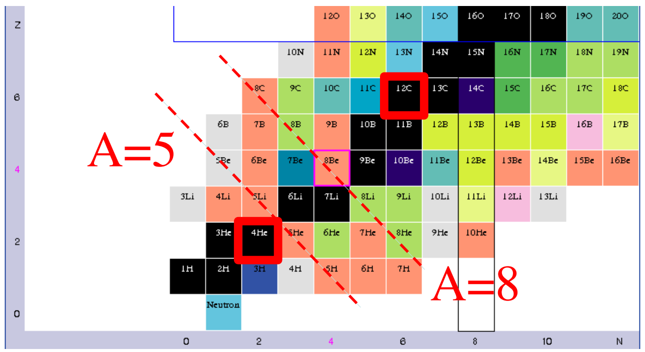
Under hydrostatic He-burning conditions: $T_8 \sim 3$ and $\rho \sim 10^5$g/cm^3 $\longrightarrow \dfrac{N({}^{8}\mathrm{Be})}{N({}^{4}\mathrm{He})} = 10^{-10}$

enabling the equilibrated ${}^{8}Be$ to recapture an α particle

2d step: $\alpha + {}^{8}Be \rightarrow {}^{12}C$ (Q=7.37MeV)

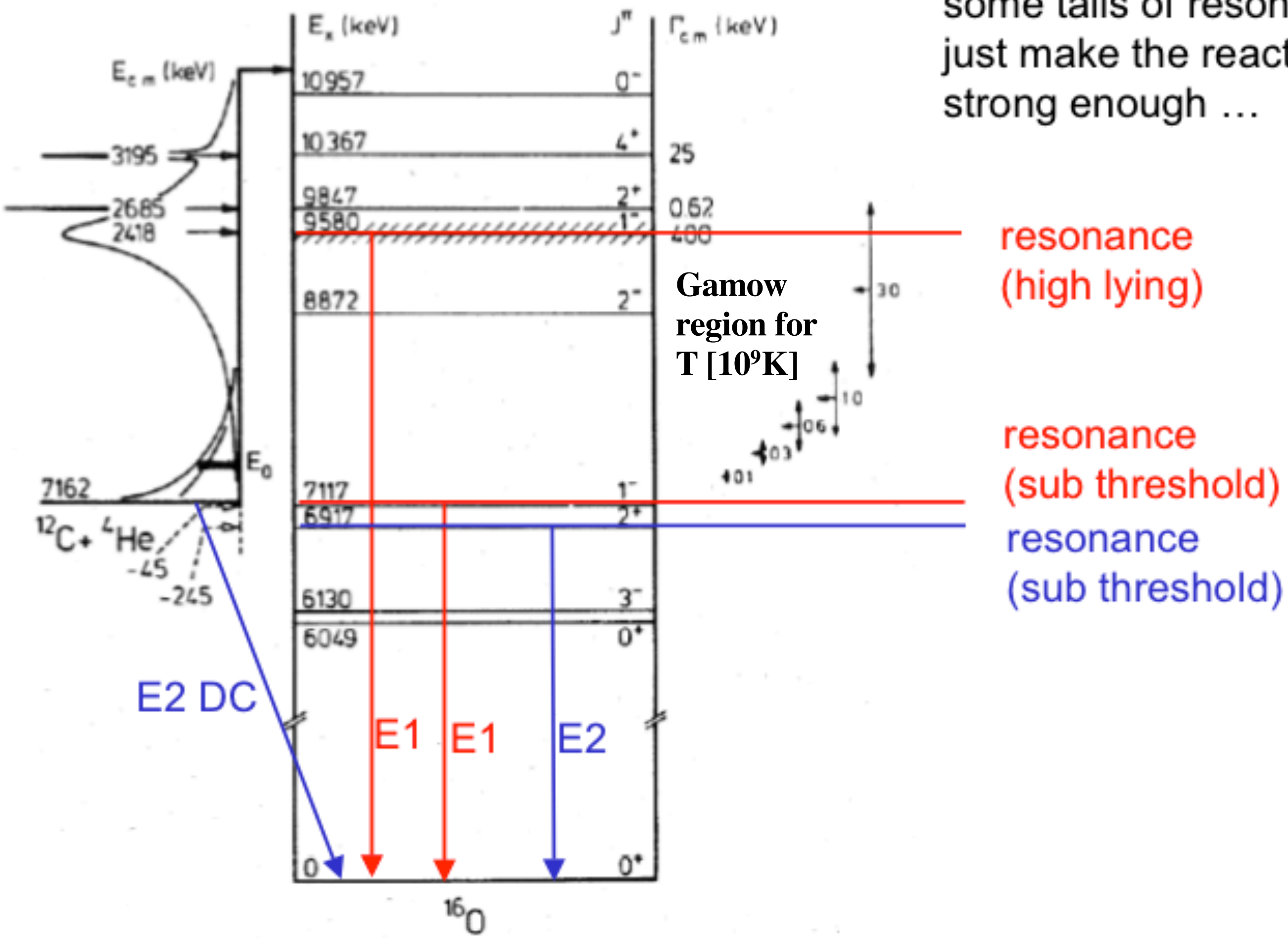


To explain the presence of ${}^{12}C$ in the Universe, Hoyle stipulated in 1954 the existence of a resonance in ${}^{12}C$ corresponding to the $J^{\pi}=0^+$ level at $E \sim 7.7$MeV (this state is now known as the Hoyle state)



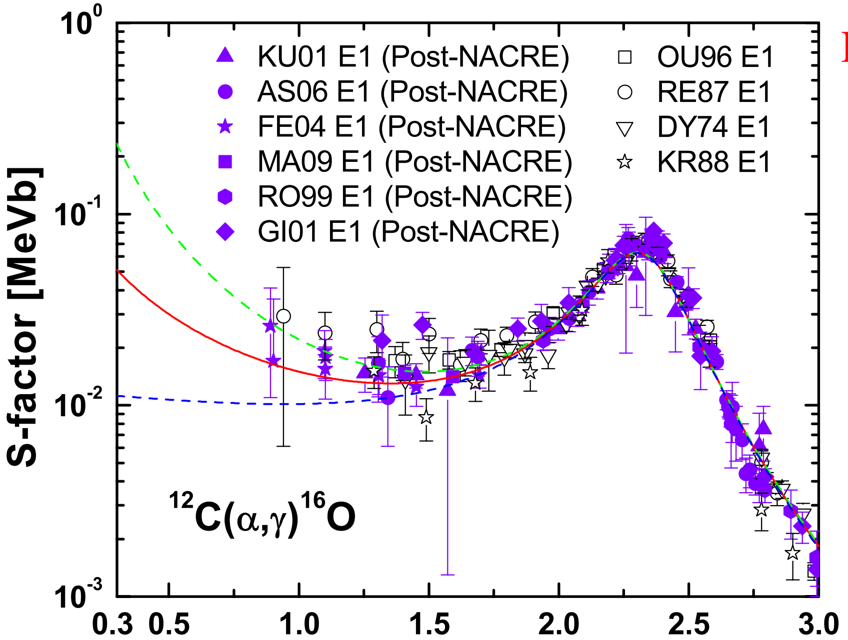
The special case of $^{12}C(\alpha,\gamma)^{16}O$

$^{12}C(\alpha,\gamma)^{16}O$ reaction rate defines the equilibrium $^{12}C/^{16}O$ ratio at the end of He-burning

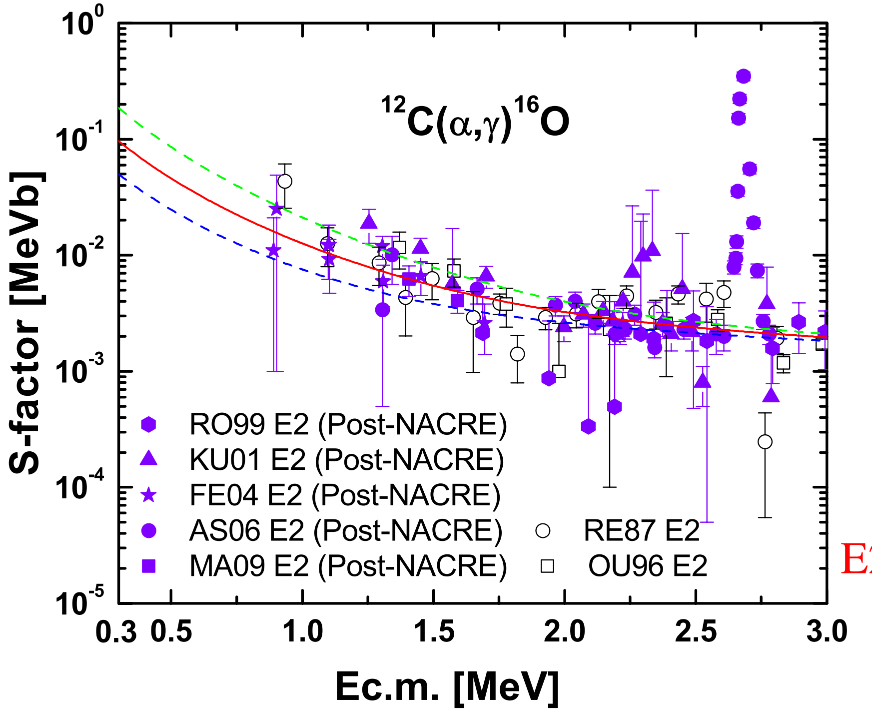


complications:
- very low cross section makes direct measurement impossible
- subthreshold resonances cannot be measured at resonance energy
- Interference between the E1 and the E2 components

Reaction contribution in $^{12}C(\alpha,\gamma)^{16}O$

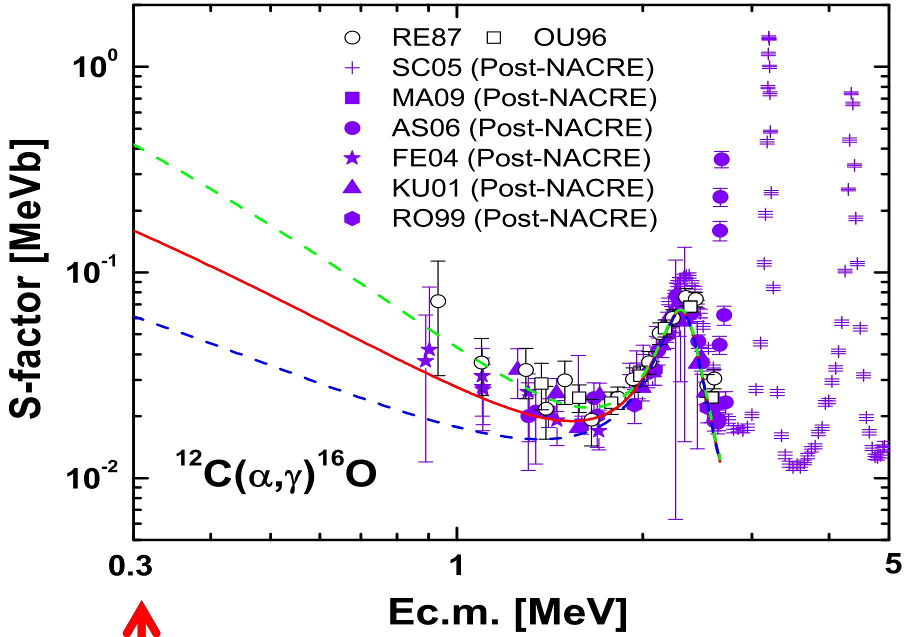


E1 component



E2 component

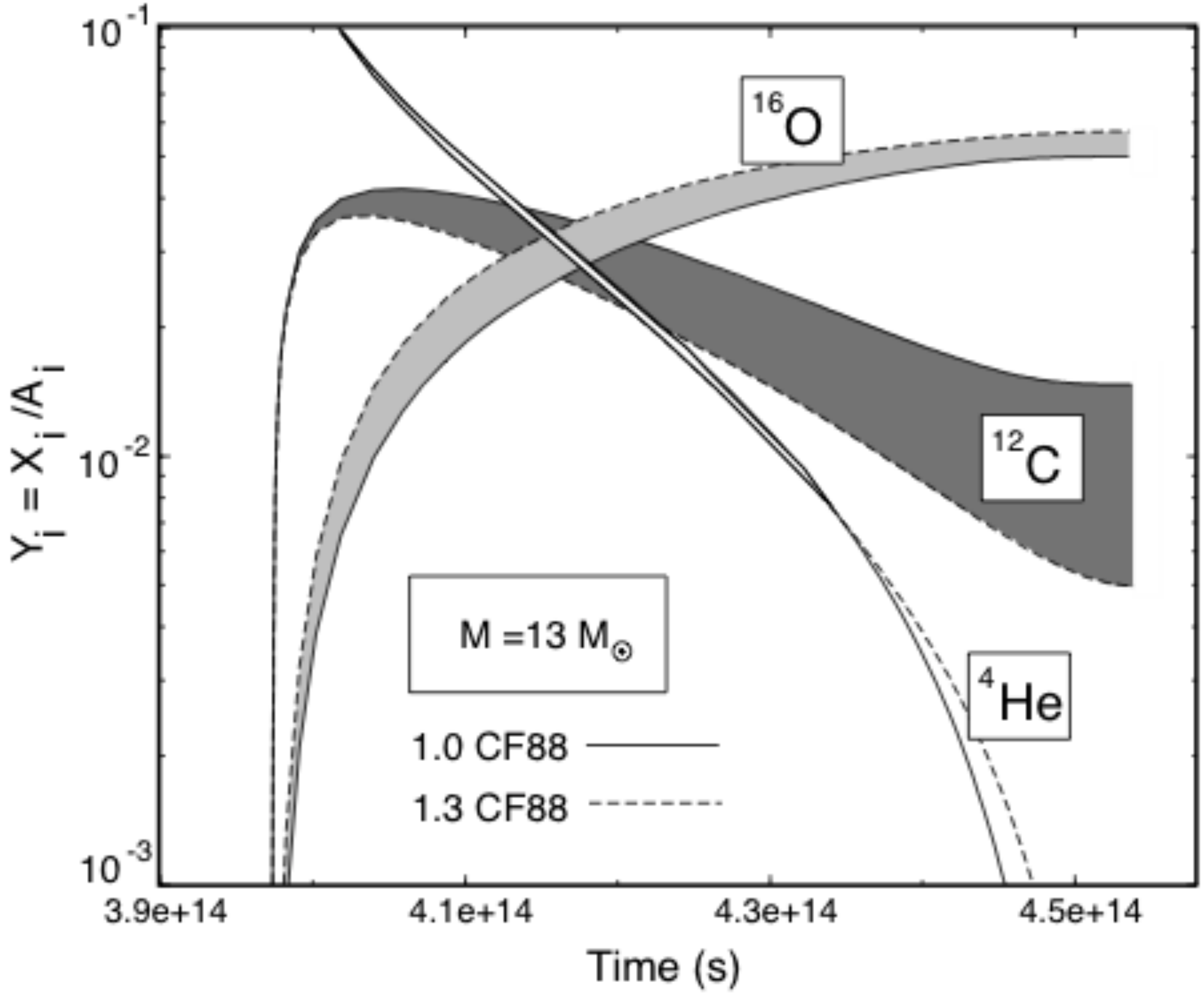
The low-energy extrapolation of the $^{12}C(\alpha,\gamma)^{16}O$ reaction rate



$E_G \sim 0.3$MeV at T_6=200

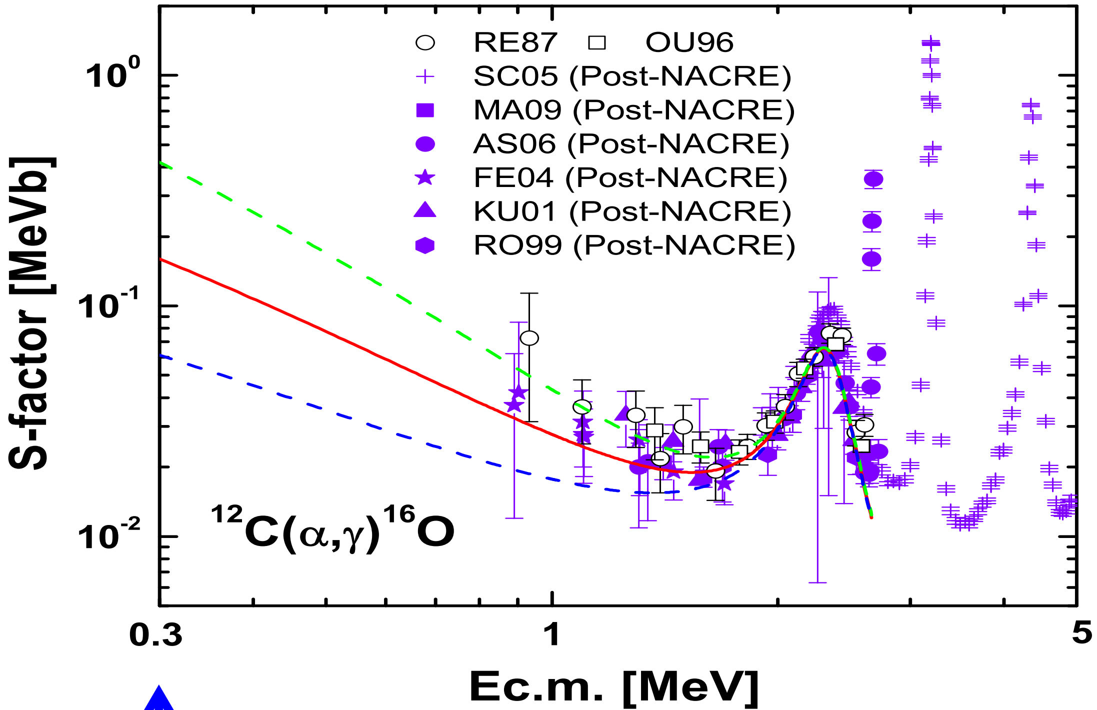
$$E_G = 1.220 (Z_X^2 Z_a^2 \mu T_6^2)^{1/3} \text{ keV}$$

Evolution of the ^{12}C and ^{16}O abundance during He-burning of a $13M_\odot$ star for 2 different estimates of the $^{12}C(\alpha,\gamma)^{16}O$ reaction rate



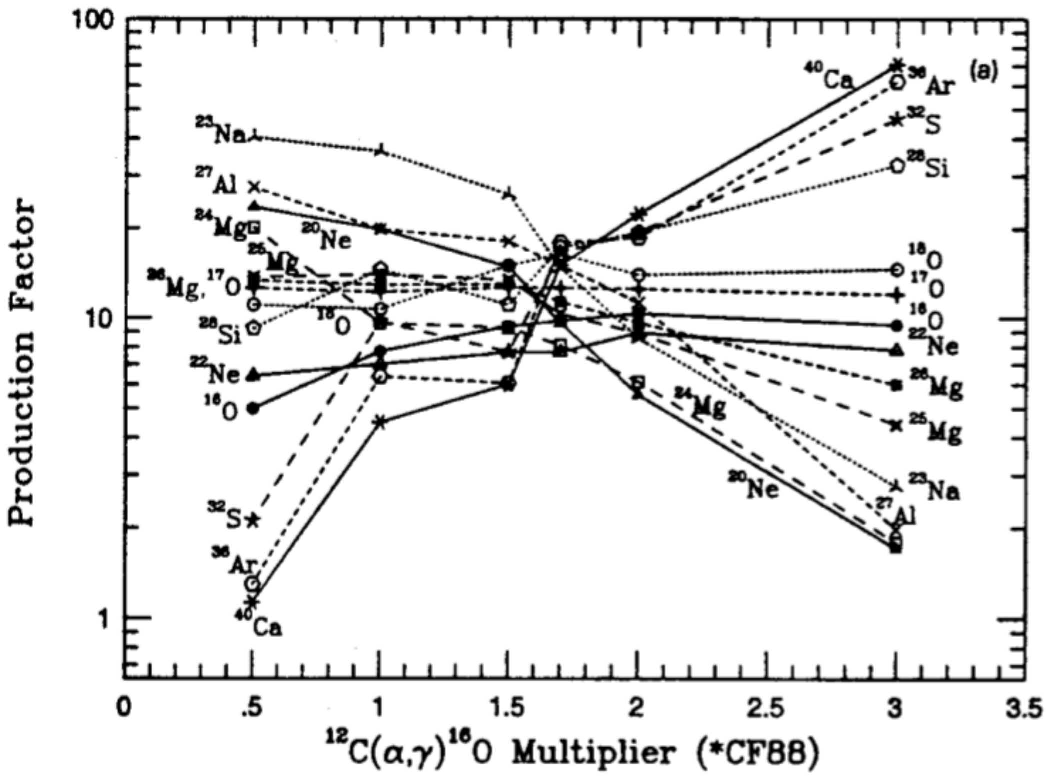
Influence of the $^{12}C(\alpha,\gamma)^{16}O$ reaction rate uncertainties on astrophysical observables

The low-energy extrapolation of the $^{12}C(\alpha,\gamma)^{16}O$ reaction rate



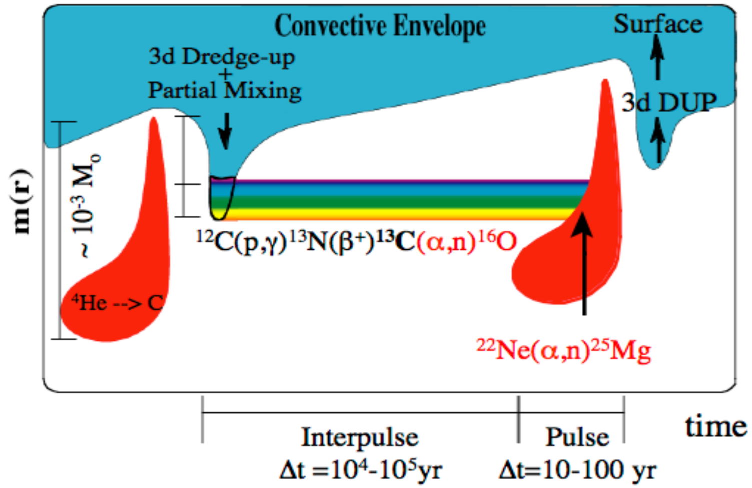
$E_G \sim 0.3$MeV at T_6=200

Na to Ca overproduction in massive stars



But an astrophysics simulation is NOT a nuclear physics experiment

The s-process nucleosynthesis



n-source: $^{13}C(\alpha,n)^{16}O$; $^{22}Ne(\alpha,n)^{25}Mg$

n-captures: ~ 80% known experimentally (E_n~10-30 keV), more being measured

- at nToF: $^{90,91}Zr$, ^{139}La, ^{151}Sm, $^{204,206,207}Pb$, ^{209}Bi
- by activation: ^{58}Fe, ^{59}Co, ^{64}Ni, $^{63,65}Cu$, $^{79,81}Br$, $^{85,87}Rb$ (Heil 08), $^{74,76}Ge$, ^{75}As, $^{184,186}W$ (Marganiec 09)
- by activation + AMS: ^{40}Ca (Dillman 09), ^{62}Ni (Nasser 05)
- on long-lived nuclei: $^{60}Fe(n,\gamma)$ (Uberseder 09), ^{182}Hf (Vockenhuber, 07), ^{14}C (Reifarth 08)

~ 20% still to be determined theoretically (unstable nuclei)
+ thermalisation effects & non-thermalisation of given isomers

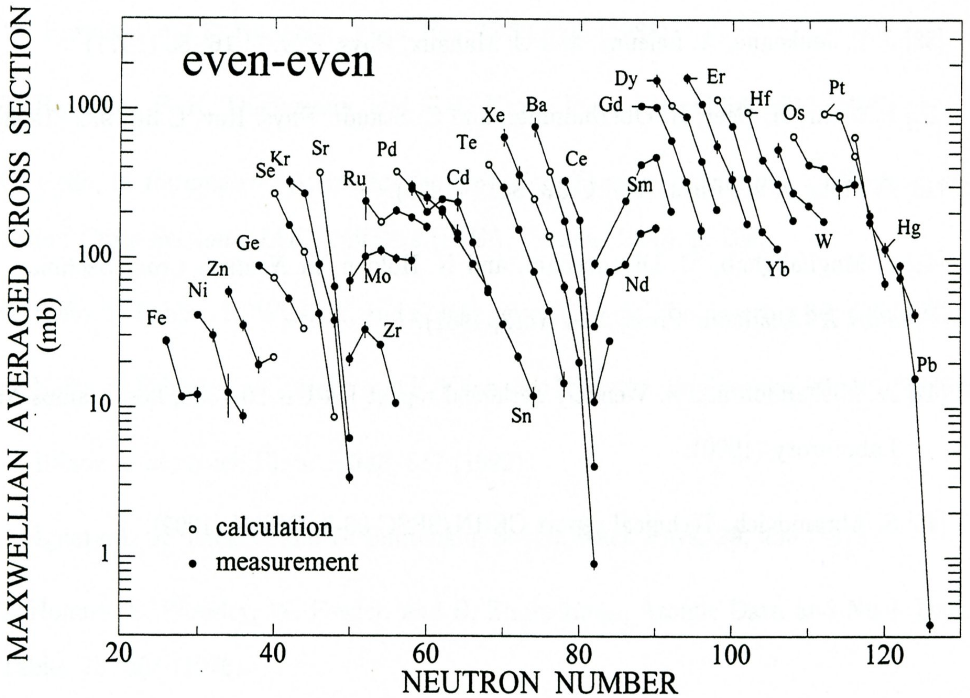
Beta-decays: T- and ρ-dependence of the stellar rate (Takahashi & Yokoi 1987)

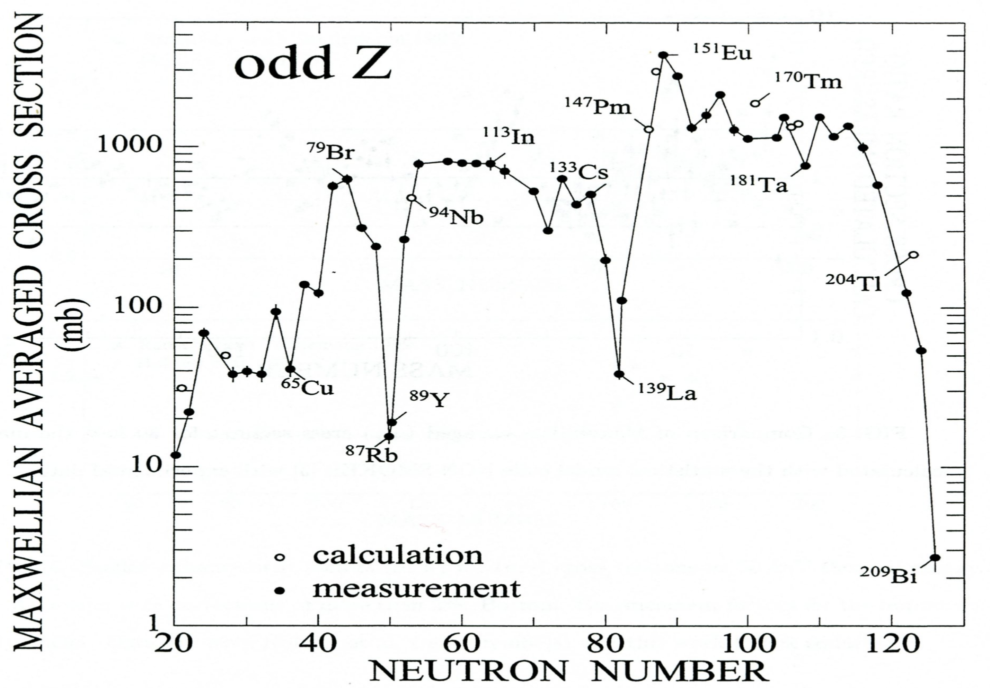
Still many branching points affected by ~ factor of 3 due to unknown *log ft*

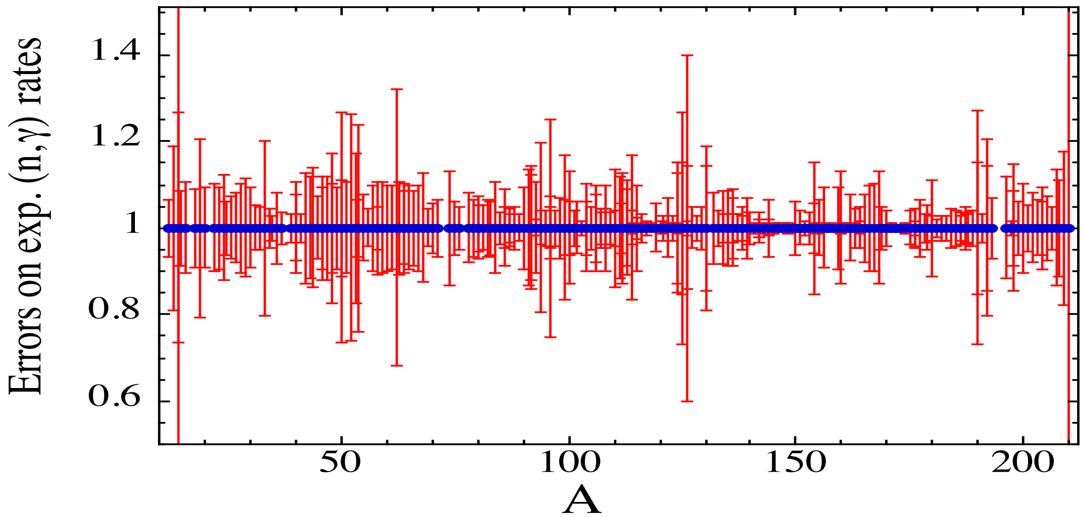
Experimental (n,γ) rates

About 80% of the radiative neutron capture rates of relevance for the s-process are known experimentally

30 keV Maxwellian-averaged cross sections (T~3.5 10^8K)







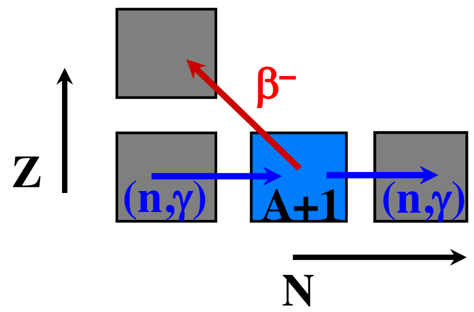
$$\langle \sigma \rangle = \frac{\langle \sigma v \rangle}{v_T}$$

Uncertainties affecting *T*- and *ρ*-dependent β-decay rates of s-process branching points (Takahashi & Yokoi 1987)

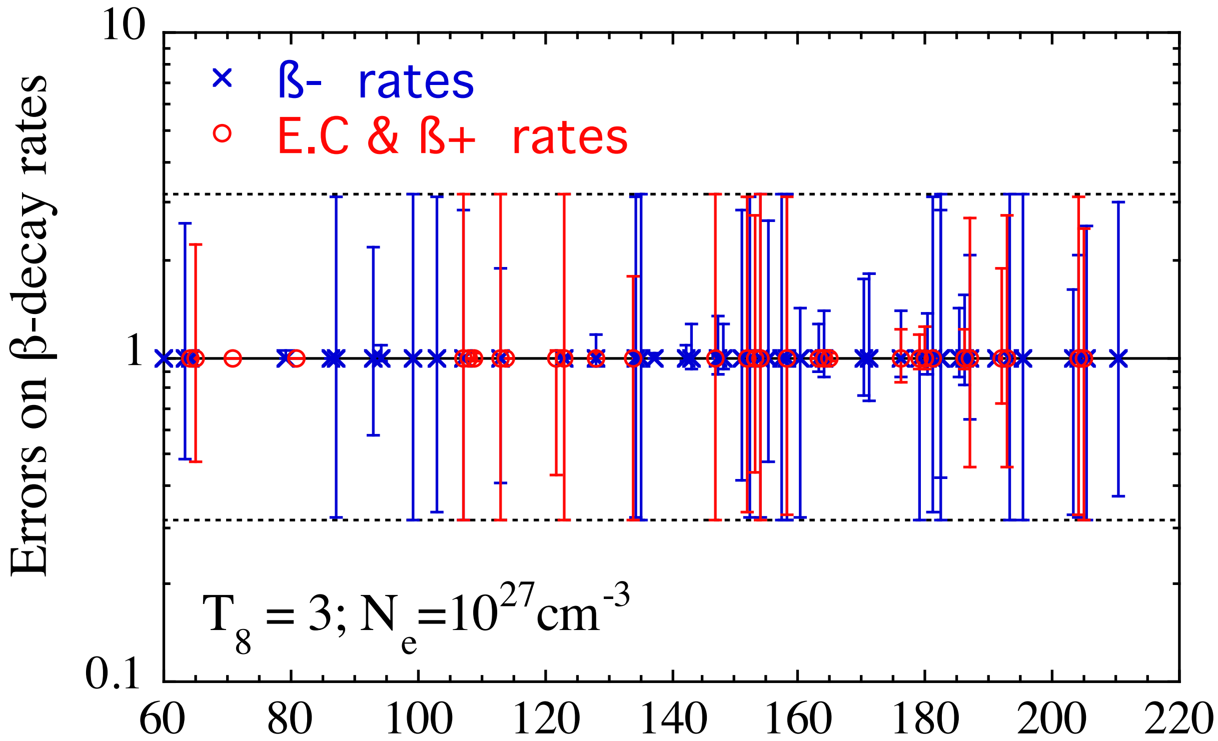
$$f_\beta = \frac{\lambda_\beta}{\lambda_\beta + \lambda_n} = \frac{\langle\sigma\rangle_A N_s(A)}{\langle\sigma\rangle_{A+1} N_s(A+1)}$$

where $\lambda_n = N_n \langle\sigma\rangle_A v_T$

$\lambda_\beta = \lambda_\beta(T,\rho)$



Still many branching points affected by factor of ~3 due to unknown *log ft* (± 0.5)



e.g. β^- ^{63}Ni

^{99}Tc

$^{134\text{-}135}Cs$

^{151}Sm

$^{152\text{-}154}Eu$

^{152}Eu

β^+ ^{107}Ag

^{123}Te

^{204}Tl

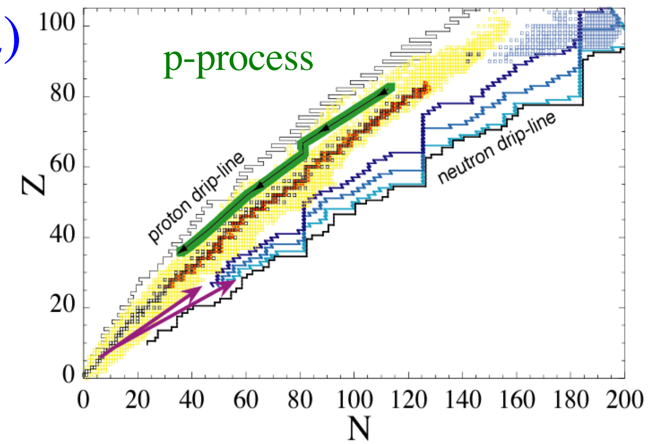
^{205}Pb

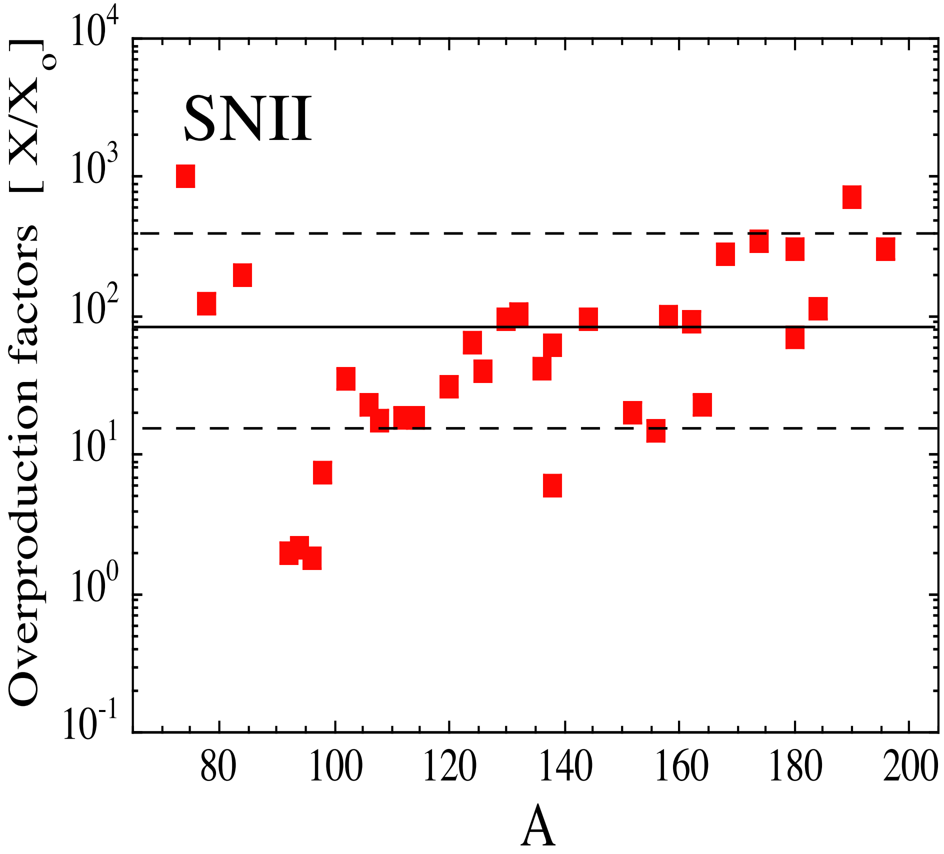
The p-process nucleosynthesis

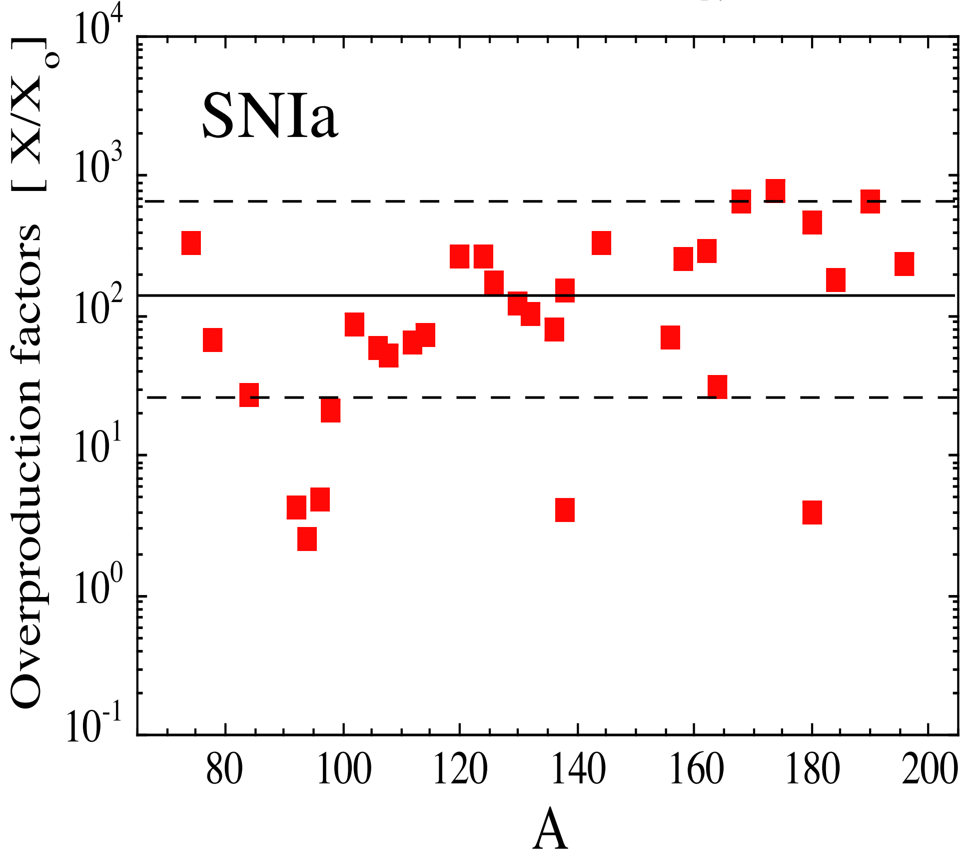
Nuclear needs:
- Photodisintegration rates (γ,n), (γ,p), (γ,α)
- Neutron-, proton, alpha-capture rates
- β⁺-decay rates
- ν-nucleus interaction rates

For about 2000 neutron-deficient nuclei

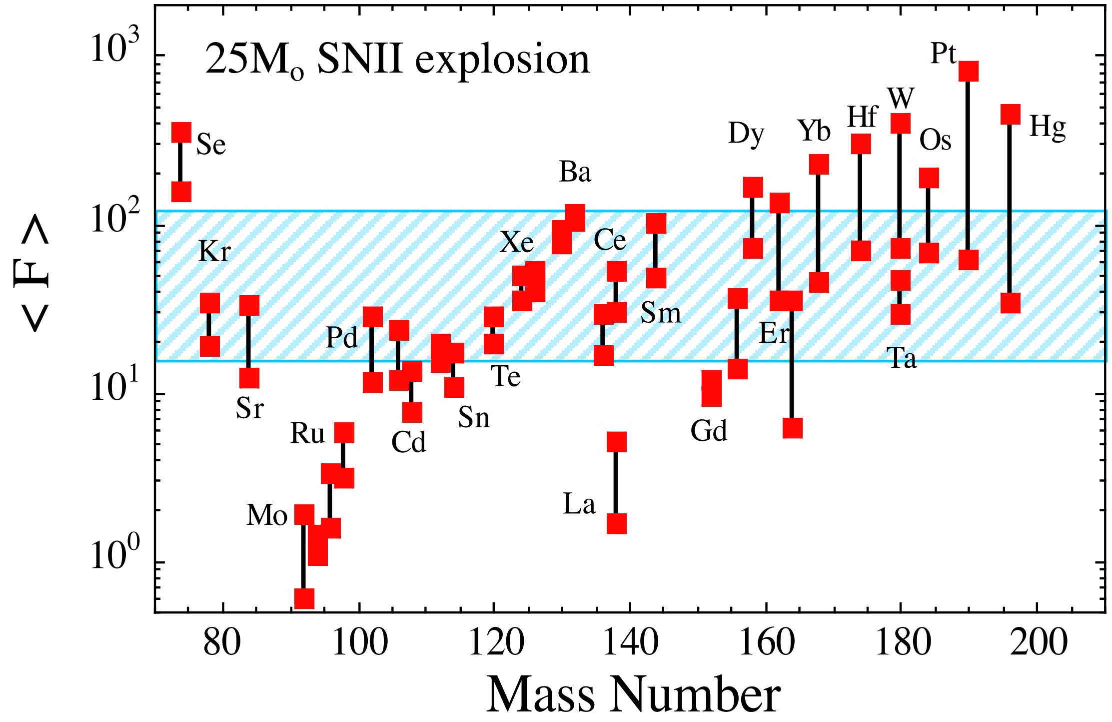
→ essentially based on theoretical HF predictions







Impact of the nuclear uncertainties on the p-nuclide production



Major nuclear uncertainties from

- GLOBAL alpha-nucleus optical potentials (heavy A>150 p-nuclides)
- GLOBAL nucleon-nucleus potential, NLD, γ-strength (light <90 p-nuclides)

(The ^{92,94}Mo, ^{96,98}Ru discrepancies are most probably not related to nuclear issues)

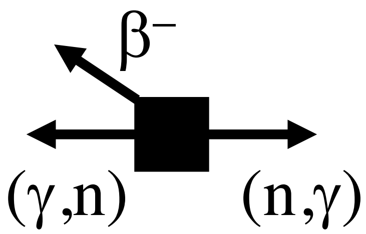
- p- and α-captures: new measurements (Demokritos, Debrecen, Kalrsruhe, Achen,…) but still not enough contraints on global potential - more theoretical work welcome too
- γ-ray strengths: new experimental information (Konan, Oslo, Duke, GSI, Dresden, …), but still open debate on the low-energy tail and PDR - more theoretical work welcome

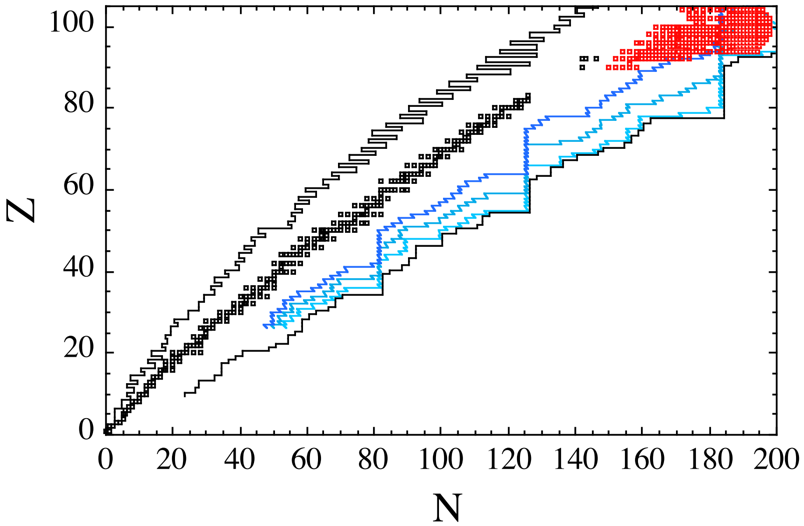
Nuclear phyics input for the r-process nucleosynthesis

(n,γ) – (γ,n) – β competition & Fission recycling

Main needs

- β-decay
- (n,γ) and (γ,n) rates
- Fission (nif, sf, βdf) rates
- Fission Fragments Distributions





Nucleosynthesis requires RATES for some 5000 nuclei !

(and not only masses or β-decay along the oversimplified so-called "r-process path")

→ simulations rely almost entirely on theoretical predictions

In turn, theoretical models are tuned on available experimental data

Challenge in theoretical nuclear physics (essential for r-process applications)

PHENOMENOLOGICAL DESCRIPTION

↓

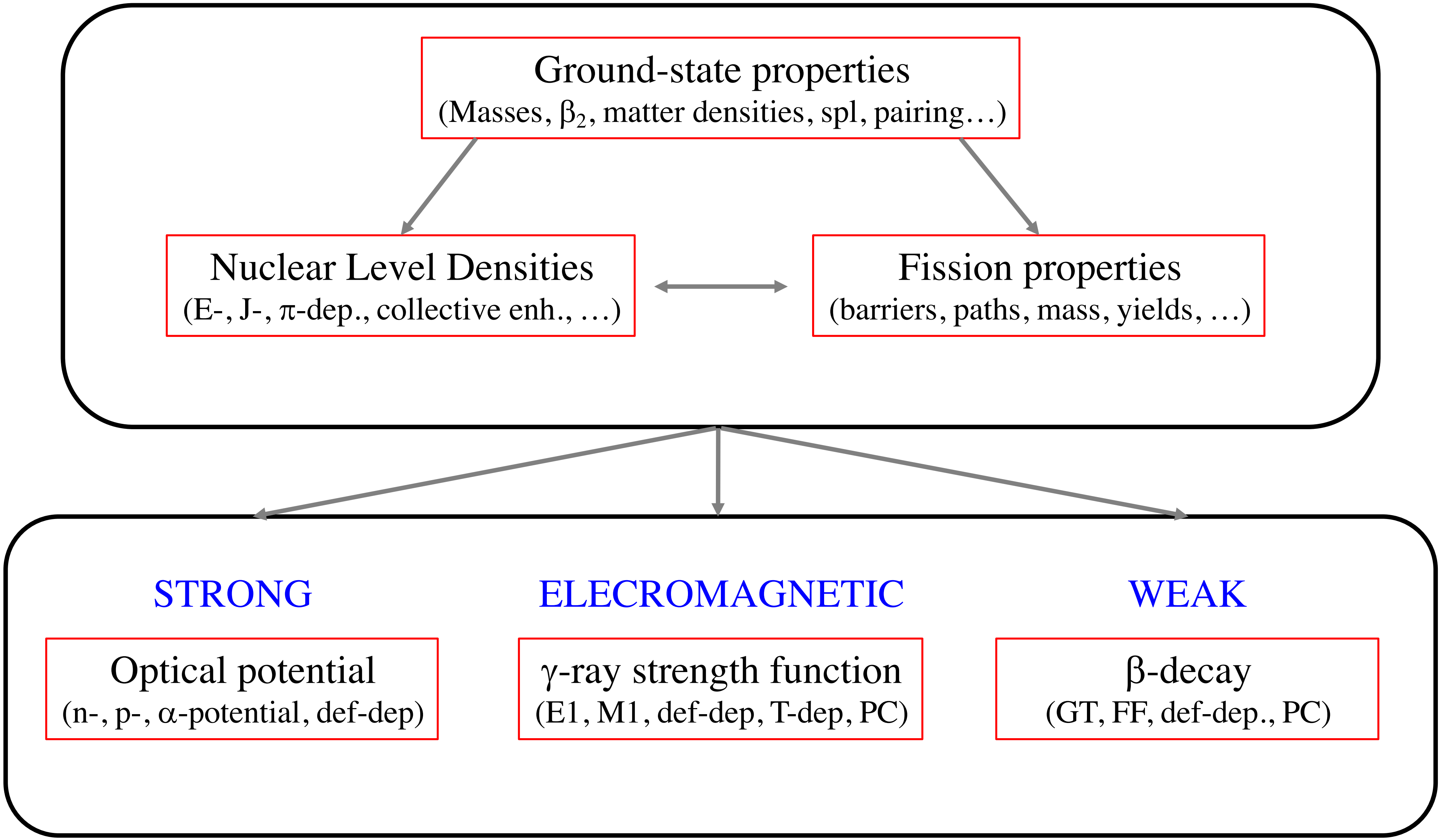
UNIVERSAL GLOBAL MICROSCOPIC DESCRIPTION

UNIVERSAL:	capable of predicting *all properties* of relevance
GLOBAL:	capable of predicting the properties of *all nuclei*
MICROSCOPIC:	for more *reliable extrapolations* from valley of stability to drip lines

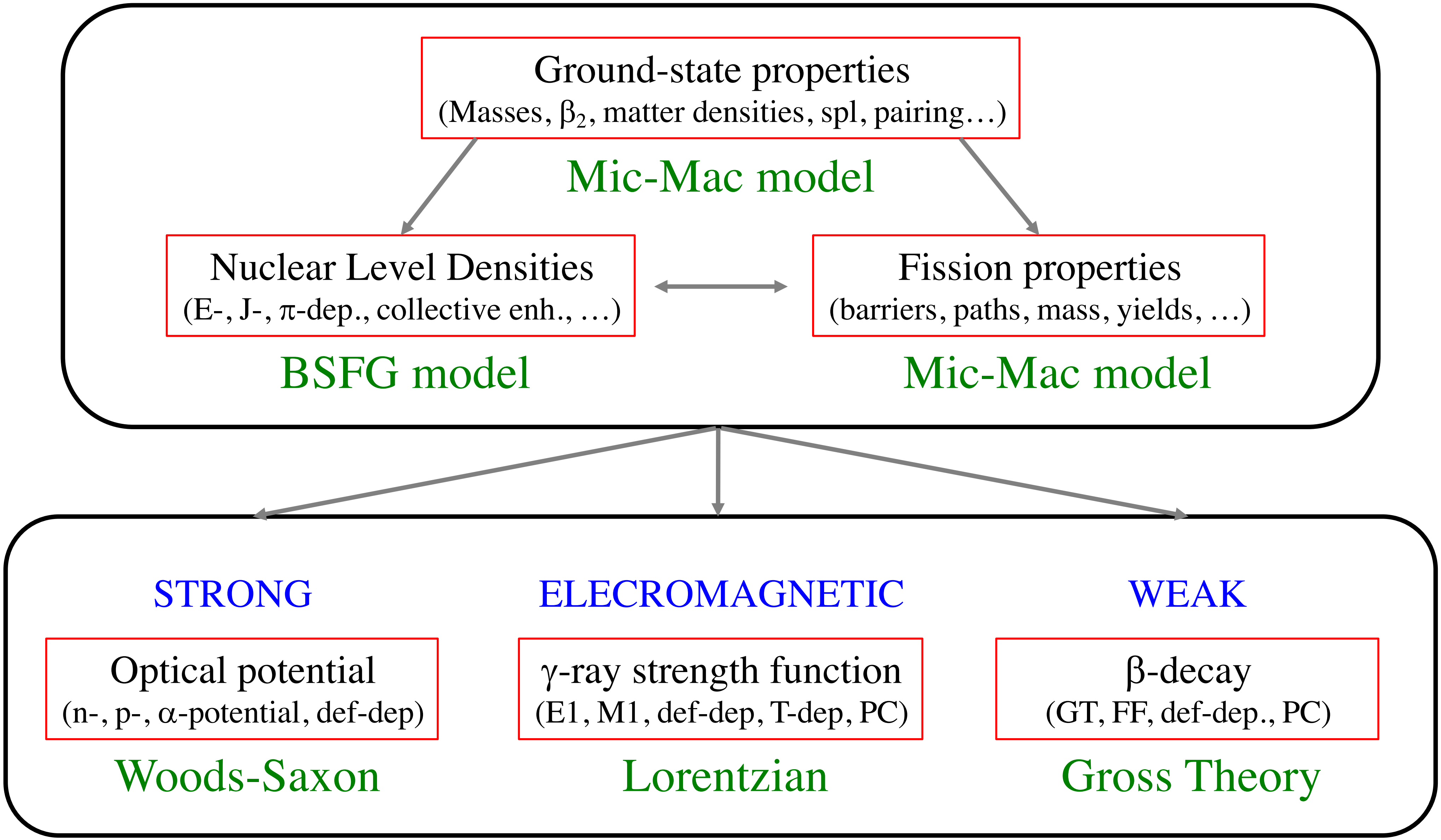
A necessary condition for a true predictive power

a challenge that will require a continued experimental & theoretical effort

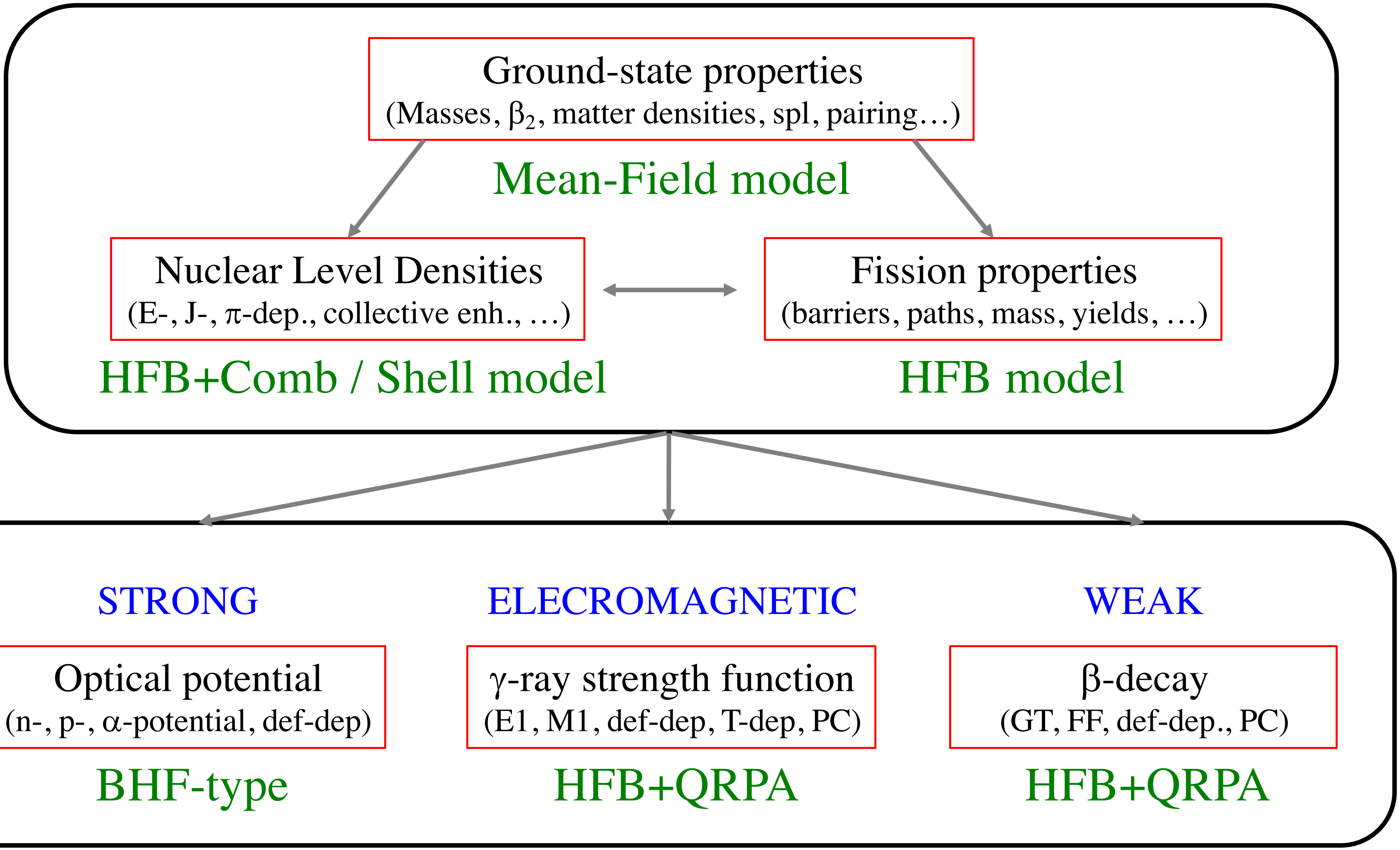
Nuclear inputs to nuclear reaction codes (e.g TALYS)



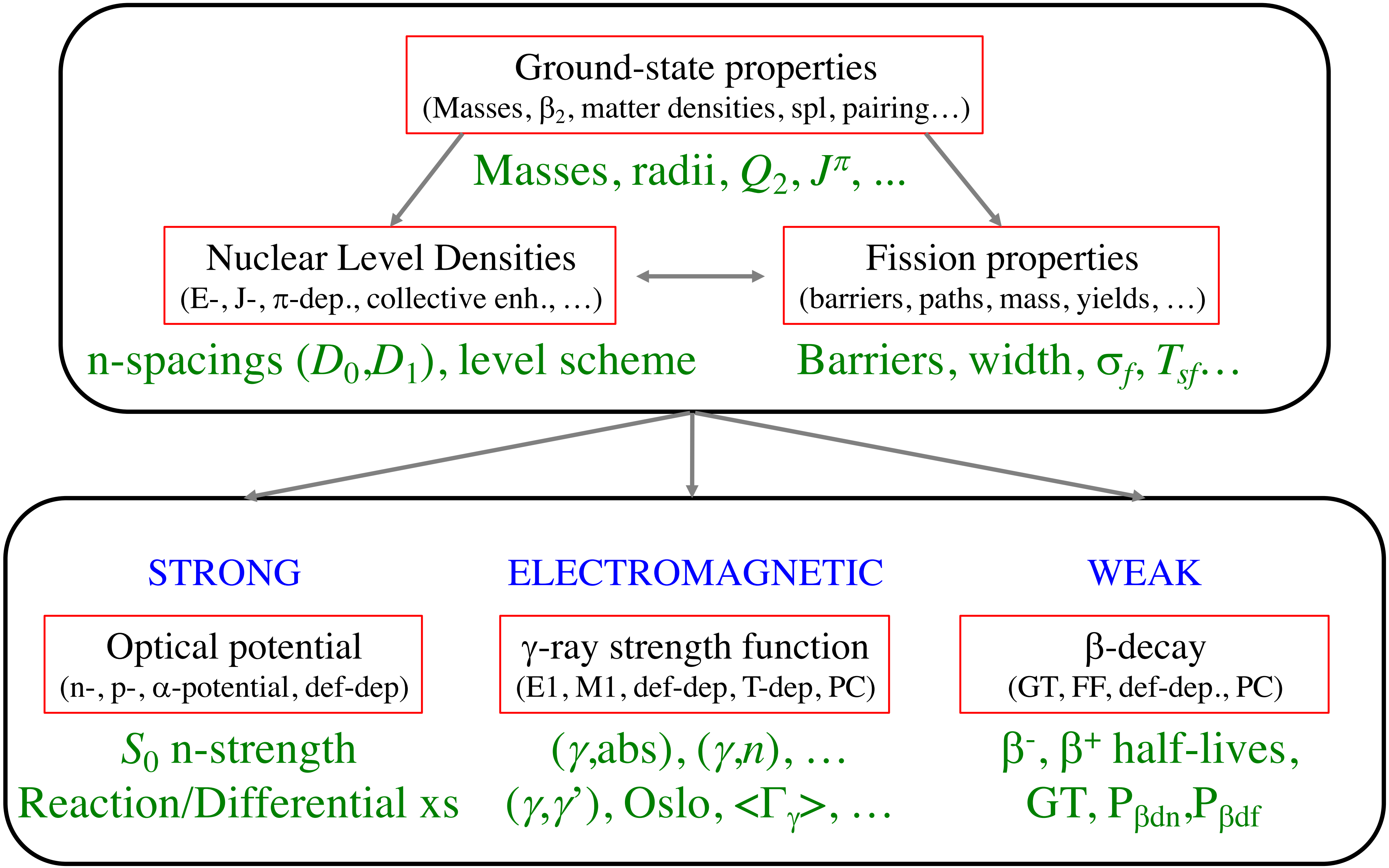
Nuclear inputs to nuclear reaction codes (e.g TALYS)



Nuclear inputs to nuclear reaction codes (e.g TALYS)



Constraints on theoretical models from measurements



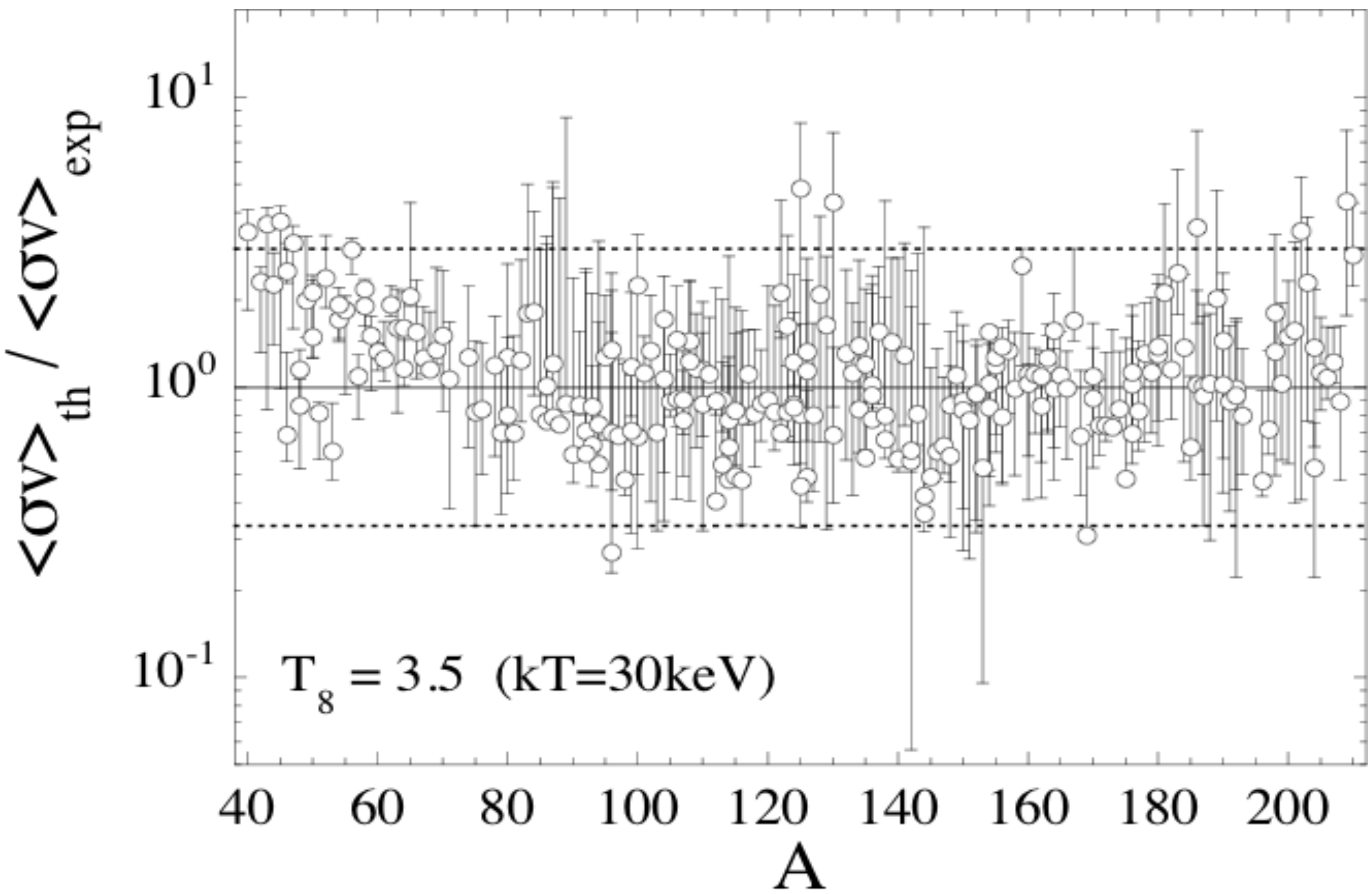
STILL MANY OPEN QUESTIONS IN REACTION THEORY

- **The reaction model**
  - **CN vs Direct capture for low-S_n & Isolated Resonance Regime**
- **Nuclear inputs to the reaction model** (almost no exp. data !)
  - **GS properties: masses (correlations - GCM, odd-nuclei)**
  - **E1-strength function: GDR tail, PR, ε_γ=0 limit, *T*-dep, PC**
  - **Nuclear level Densities (at low *E*): *J*- and π-description, pairing, shell and collective effects & damping**
  - **Optical potential: the low-*E* isovector imaginary component**
  - **Fission: fission paths, NLD at the saddle points, FFD**
- **The β-decay rates**
  - **Forbidden transitions, deformation effects, odd-nuclei, PC**

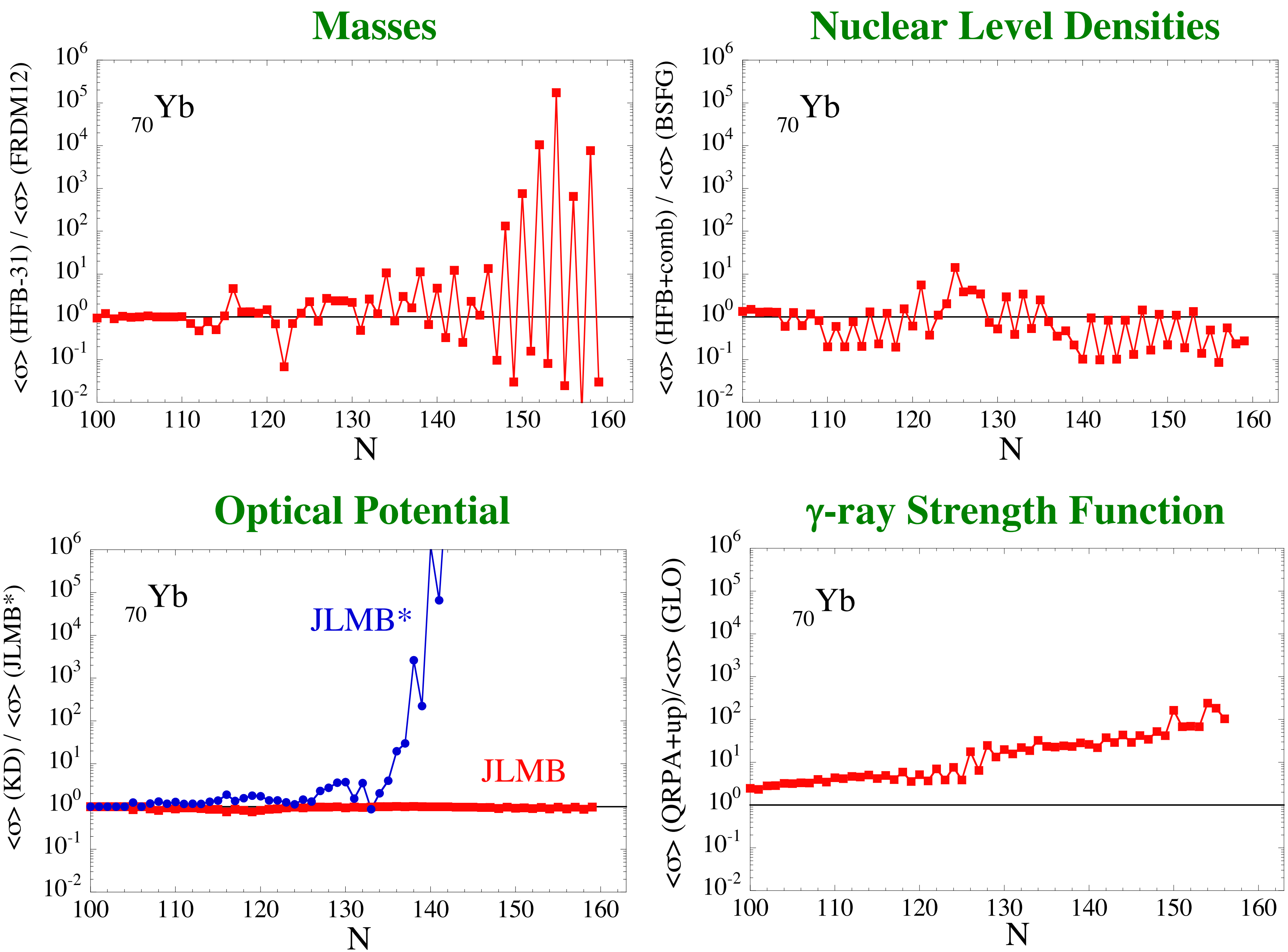
We are still far from being capable of estimating ***reliably*** the radiative neutron capture and β-decay of exotic n-rich nuclei
(and fission properties even for known nuclei)

Models exist, but corresponding uncertainties are usually not estimated

Comparison of known (n,γ) reaction rates with the statistical model estimates

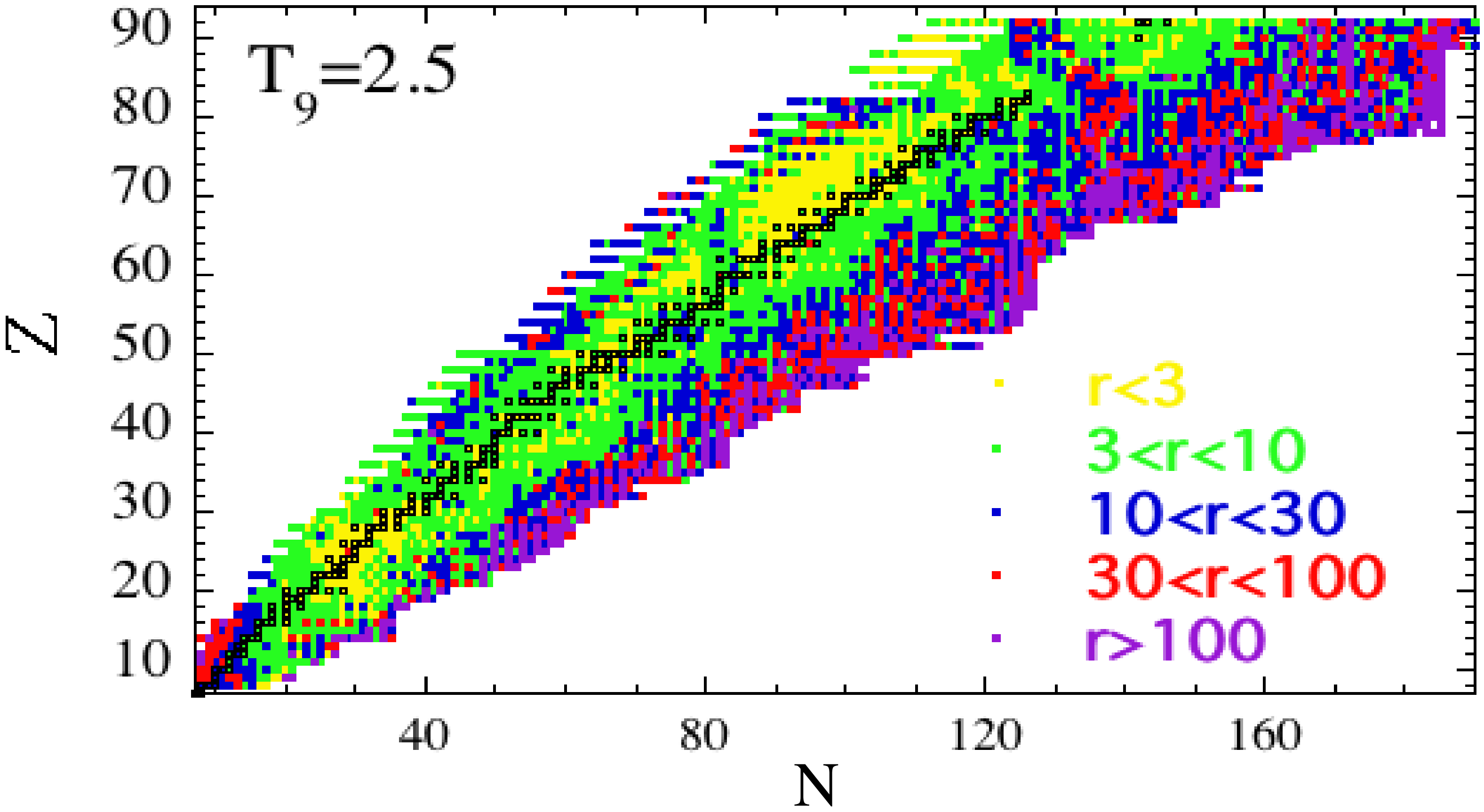


Impact of the various ingredients on the radiative neutron capture (E_n~100keV)



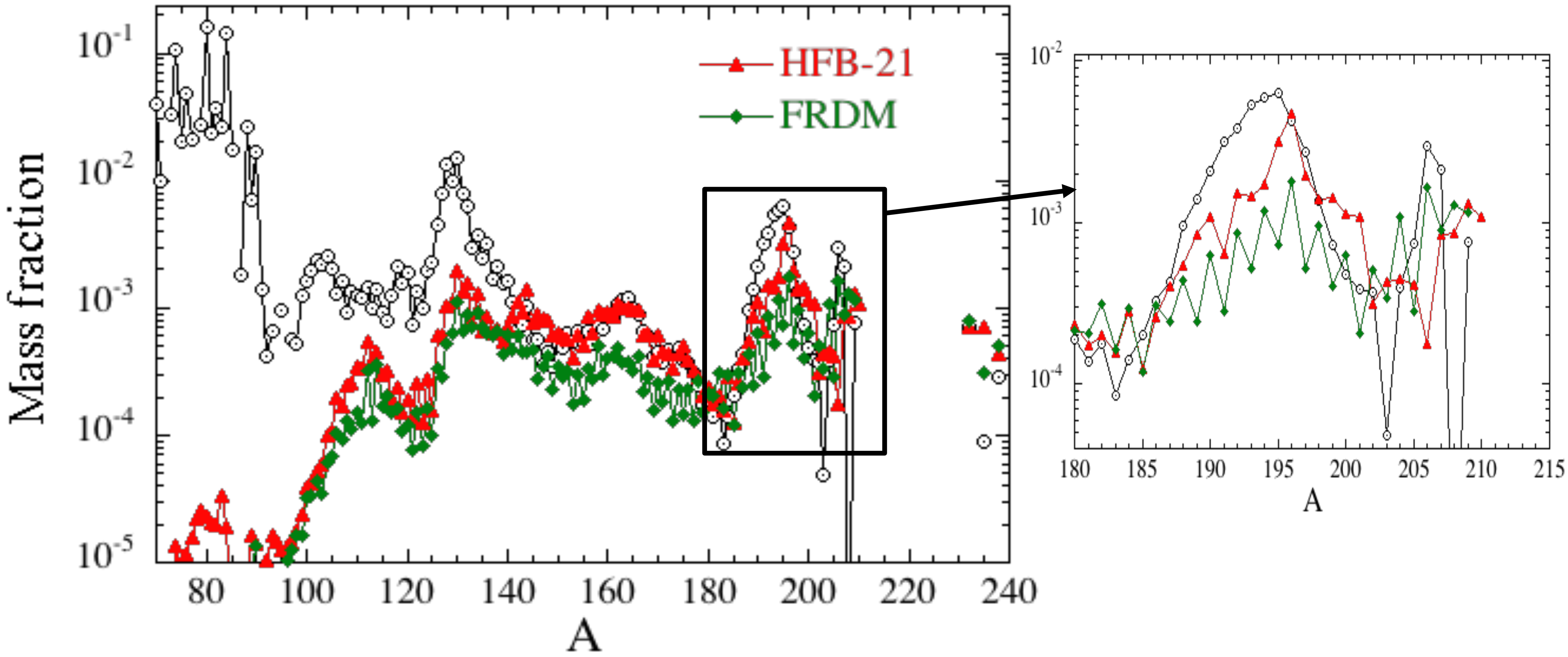
Uncertainties in the prediction of the radiative neutron capture rates

Maximum deviations for different input parameters



Impact of masses on the r-process nucleosynthesis in NS mergers

- GT2 β-decay rates with consistently estimated Q_β
- n-capture rates estimated within the HF+PE+DC model



TALYS and the calculation of astrophysical rates

```
projectile n
element    u
mass       278
energy 1.
astro  y
astrogs n
ldmodel   5
fismodel 5
alphaomp 5
massmodel 2
strength 4
```

Calculate astrophysical rates $N_A<\sigma v>^*$
and MACS for (n,γ)
(→ files astrorate.tot, astrorate.g, …)
on a defined T-grid (30 T: 1e5K → 1e10K)

astrogs = y → assume targets in GS only
estimate $N_A<\sigma v>$ (not $N_A<\sigma v>^*$)
(by default astrogs=n)

Microscopic inputs based on the
Skyrme HFB models

TALYS and the calculation of astrophysical rates

```
projectile n
element    u
mass       278
energy 1.
astro  y
astrogs n
ldmodel  6
fismodel 5
alphaomp 5
massmodel 3
strength 8
strengthM1 8
```

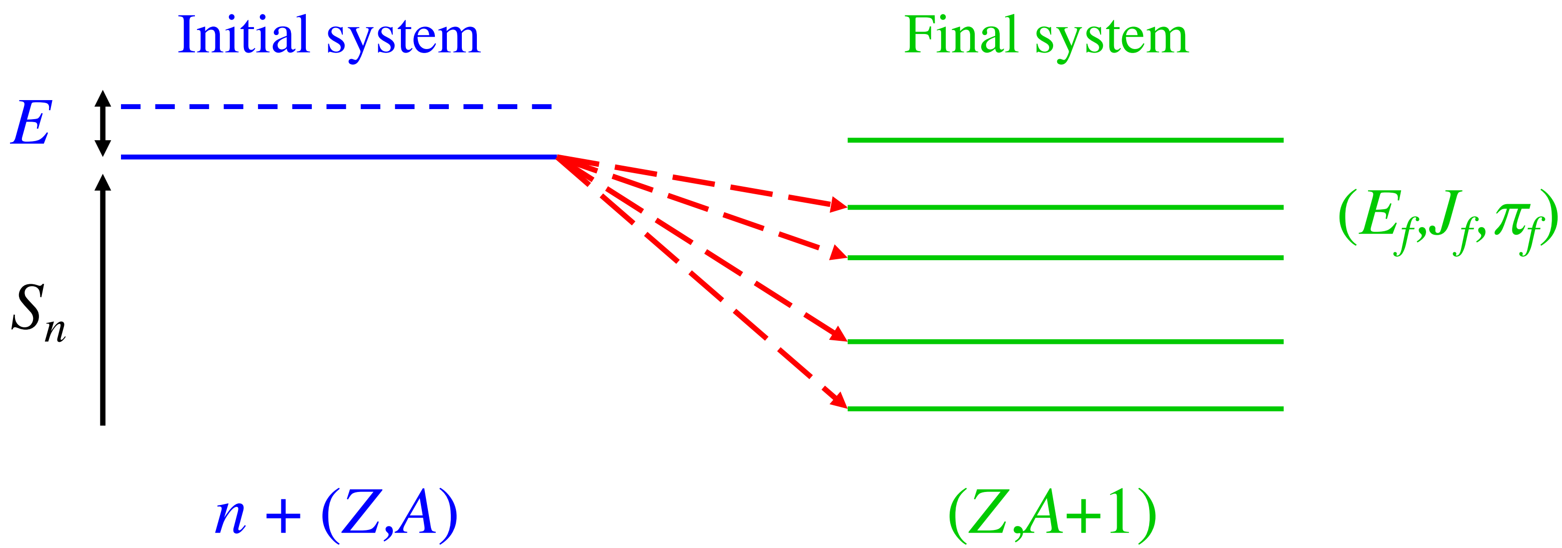
Calculate astrophysical rates $N_A\langle\sigma v\rangle^*$
and MACS for (n,γ)
(→ files astrorate.tot, astrorate.g, …)
on a defined *T*-grid (30 T: 1e5K → 1e10K)

astrogs = y → assume targets in GS only
estimate $N_A\langle\sigma v\rangle$ (not $N_A\langle\sigma v\rangle^*$)
(by default astrogs=n)

Microscopic inputs based on the
Gogny-HFB models

Direct captures

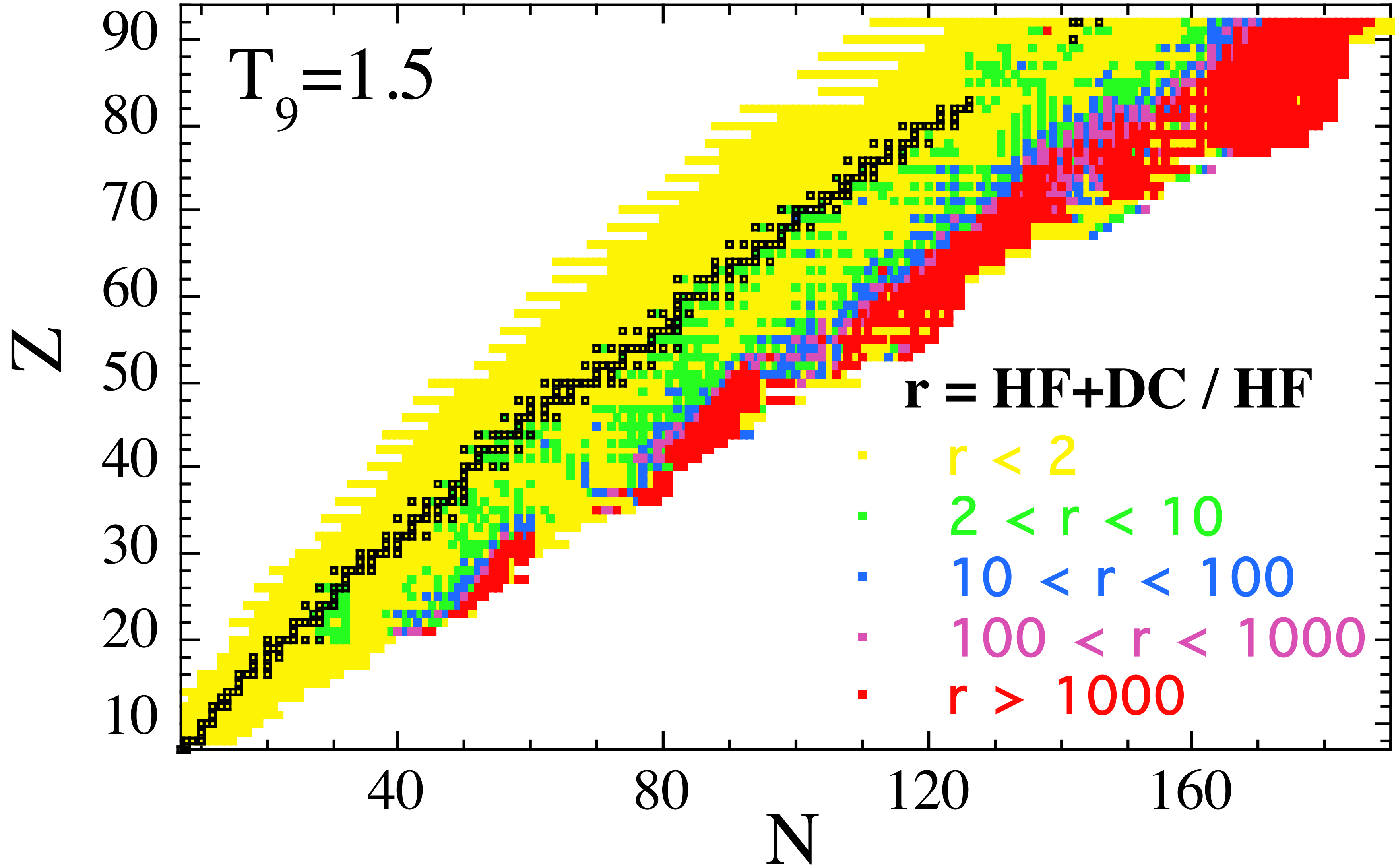
Direct scatter of incoming neutrons into a bound state without formation of a Compound Nucleus (particularly important for light and low-S_n n-rich nuclei)



Different models exist (in particular the so-called potential model) but
Requires a detailed knowledge of

- detailed spectroscopy of low-excited states (E_f, J_f,π_f), including the spectroscopic factor of each excited state, describing the overlap between the antisymmetrized wave function of the initial system $(Z,N)+n$ and the final state f in $(Z,N+1)$
- n-nucleus interaction potential

Comparison of the Statistical and Direct Captures



TALYS and the calculation of astrophysical rates

```
projectile n
element    u
mass       278
energy 1.
astro  y
astrogs n
ldmodel   6
fisgmodel 5
alphaomp 5
massmodel 3
strength 8
strengthM1 8

racap y
```

astro y: Calculate astrophysical rates $N_A\langle\sigma v\rangle^*$ and MACS for (n,γ) (→ files astrorate.tot, astrorate.g, …) on a defined T-grid (30 T: 1e5K → 1e10K)

astrogs = y → assume targets in GS only
estimate $N_A\langle\sigma v\rangle$ (not $N_A\langle\sigma v\rangle^*$)
(by default astrogs=n)

ldmodel, fismodel, alphaomp, massmodel, strength, strengthM1: Microscopic inputs based on the Gogny-HFB models

racap y: Direct capture contribution consistently added to the resonance contribution

The fundamental role of β-decay rates

(including βdn & βdf)

Gross Theory :

the β-strength function is estimated by folding one-particle strength function via a simple pairing scheme taking into account the correspondin sum rules and even-odd effects.

QRPA approach (Skyrme, Gogny, RMF) with some level of approximations: TDA, separable interactions, inconsistency between Ground & Excited states, spherical approximation, GT only, …

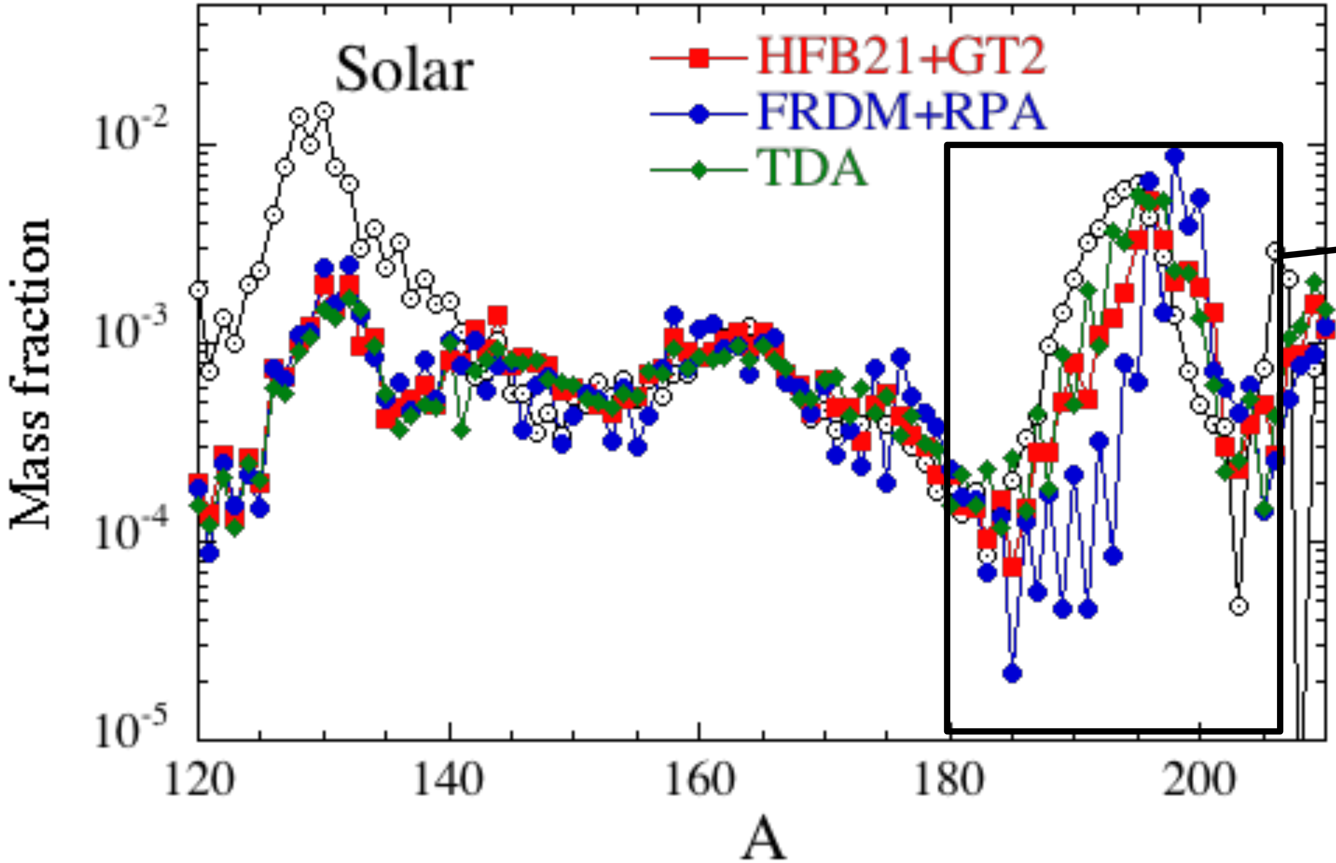
Recent work within

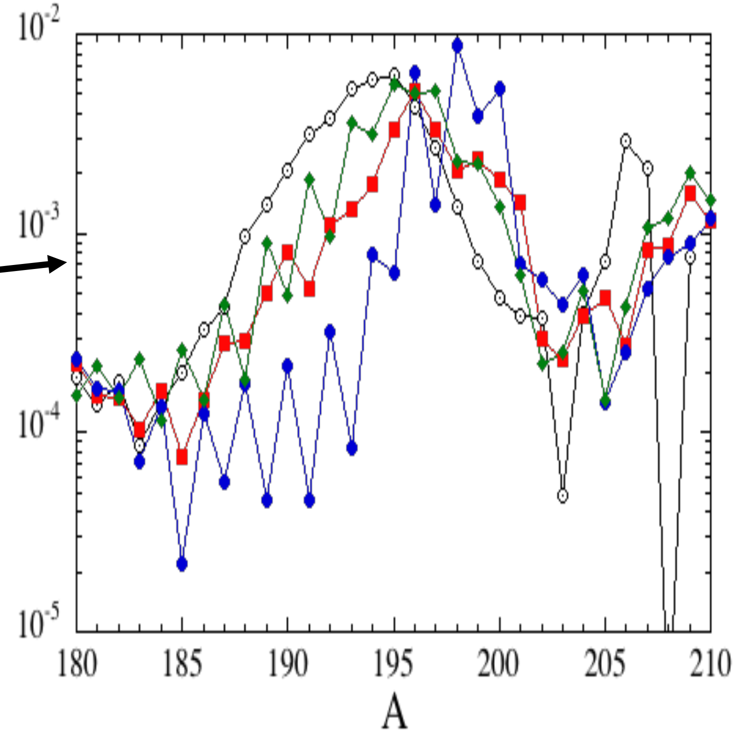
- EDF+Fermi Liquid Theory (Borzov 2010): spherical, FF incl.
- RHB+QRPA (Marketin et al. 2014): spherical, FF incl.
- Gogny HFB+QRPA (Martini & Péru 2014): def, GT, no FF (yet)

In practice, only a few complete tables (publicly) available

- Tachibana et al. (1990): HFB + Gross Theory 2 (GT + FF)
- Klapdor et al. (1984): Tamm-Dancoff approximation
- Möller et al. (2003): FRDM + QRPA & gross theory for FF

Impact of β-decay rates on the r-process nucleosynthesis in NS mergers





Large impact of the β-decay rate – set the synthesis timescales

(βdn also influences the location of the peak with the late capture of neutrons released)

→ Need at least deformed "microscopic" calculation (HFB+QRPA) including GT+FF transitions, odd nuclei, PC, ….

Fission probabilities and fragment distribution

Fission processes (**spontaneous, β-delayed, neutron-induced, photo**) and fission fragment distributions of relevance for estimating the (in particular in sites like NSM)

- termination point of the r-process or production of SH
- production of light species (A~110-160) by fission recycling
- heating of the matter (affecting the light curve)
- production of radiocosmochronometers (U, Th)

Complicate nuclear physics associated with

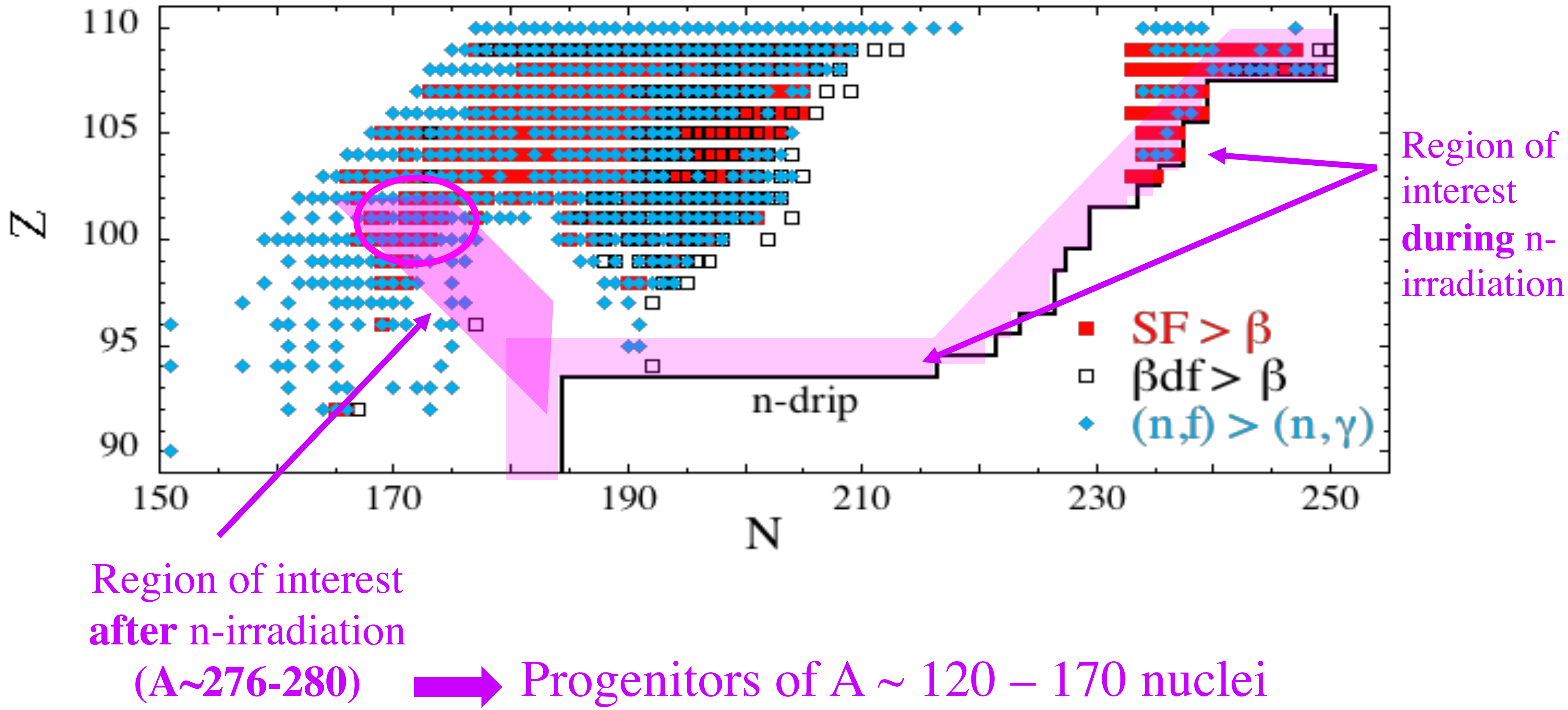
- Full Potential Energy Surfaces (fission barriers/paths, collective mass, …)
- NLD at the saddle points (transition states) & in isomeric well (class-II states)
- Fission fragment distributions

+ coupling with competitive n-, γ-, β-channels

for some 2000 heavy exotic n-rich nuclei with $90 \leq Z \leq 110$

→ Real effort needed to improve *predictions* of fission properties (Still far from being achieved, even for U and Th !)

HFB-14 prediction of fission probabilities

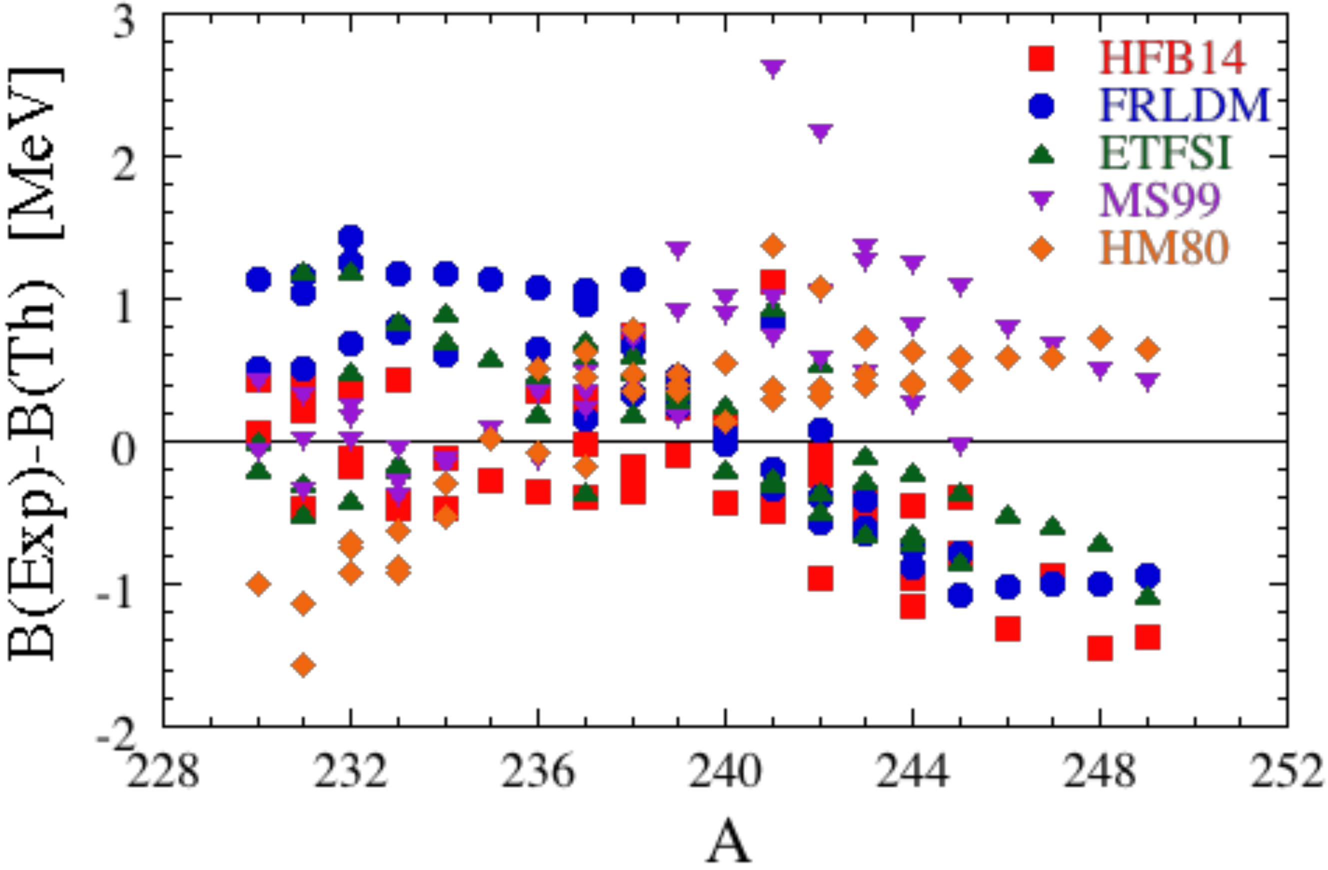


Progenitors of A ~ 120 – 170 nuclei

Obviously, still many uncertainties affecting the prediction of the input physics necessary to estimate the (sf, βdf, nif) rates & fission fragment distribution

Description of the primary fission barriers by global models

45 « empirical » primary barriers 88 ≤ Z ≤ 96 (RIPL-3)



Model	σ [MeV]
HFB-14	0.60
FRLDM	0.81
ETFSI	0.57
MS99	0.82
HM80	0.66

Urgent need to improve the global prediction of barriers within « microscopic » models

e.g. mean-field model including l-r asymmetry & triaxial shape & long-range correlations

Fission Fragment Distribution

Fission fragment distribution plays a ***fundamental*** role, especially in scenarios where fission recycling is very efficient (NSM)

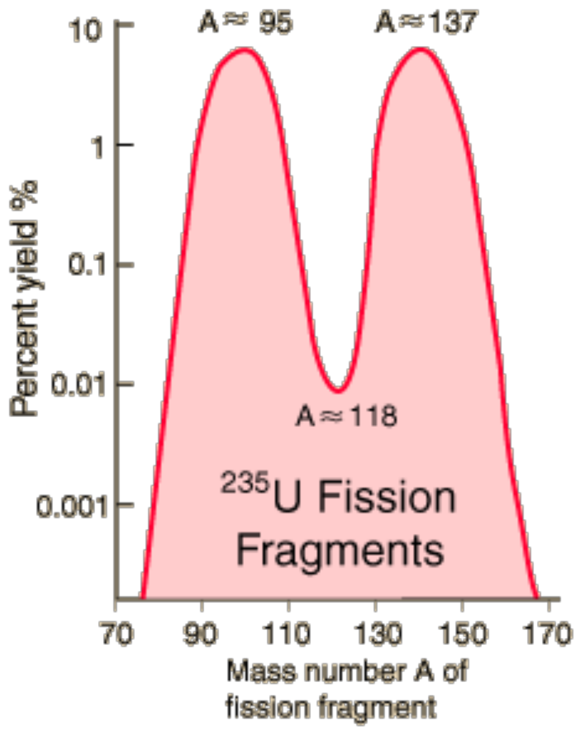
- Final r-abundance distribution (110 ≤ A ≤ 170) shaped by the FFD
- Emission of prompt neutrons that will be at late times

Many different phenomenological approaches exist, based on systematics, i.e highly-parametrized multi-Gaussian-type fits, with adjustement on available experimental FFD

→ Almost all kinds of FFD can be extrapolated for exotic nuclei !

→ Need for « serious » microscopic description of collective dynamics

(e.g time-dep. Schrödinger eq.)



Sensitivity to the fission fragment distribution

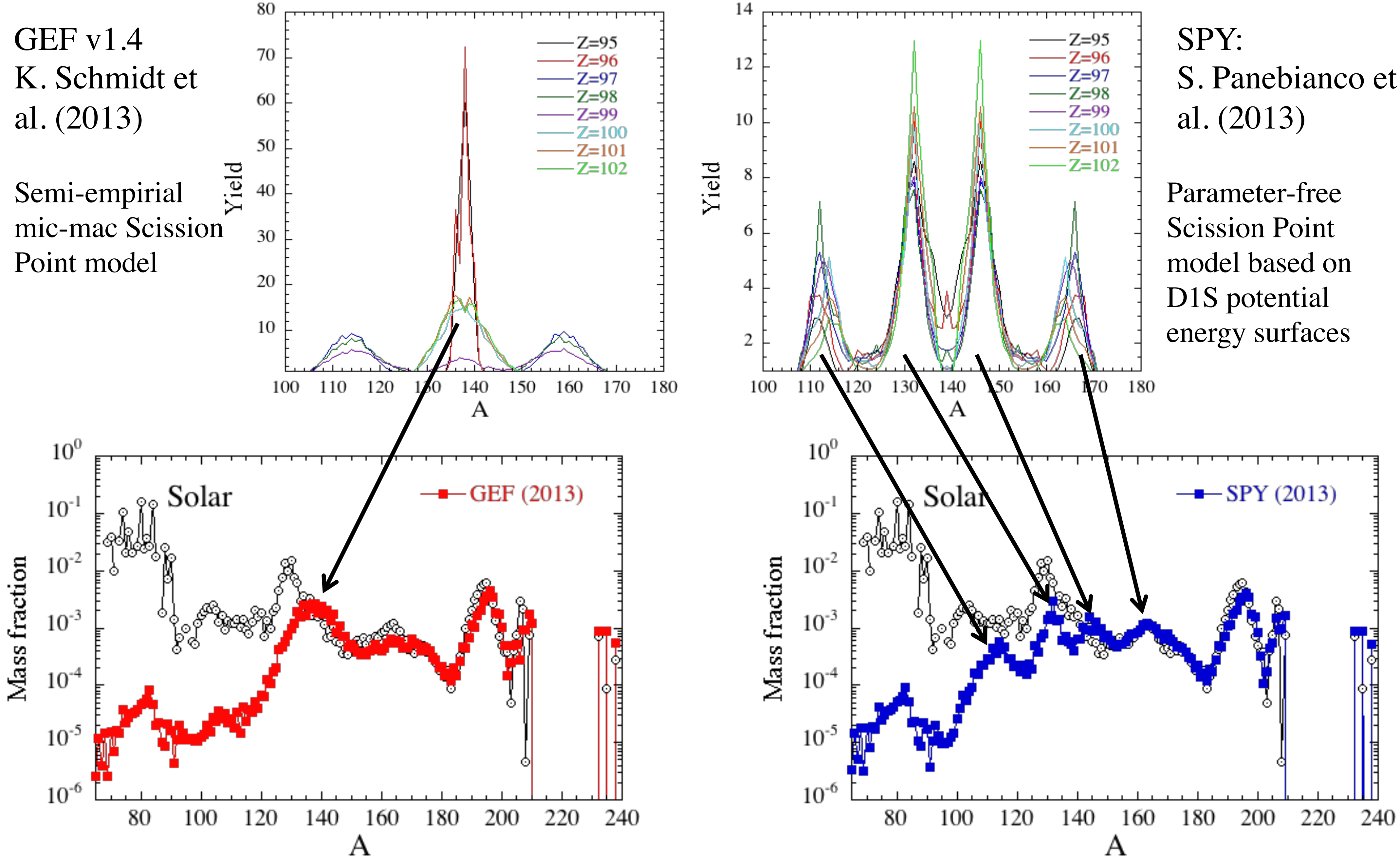
along the A=278 isobar (from the N=184 closed shell)

GEF v1.4
K. Schmidt et al. (2013)

Semi-empirial mic-mac Scission Point model

SPY:
S. Panebianco et al. (2013)

Parameter-free Scission Point model based on D1S potential energy surfaces



Conclusions

	ASTRO	NUCLEAR	OBS
BIG-BANG	+	+	+
A<56 SYNTHESIS	+	+ –	+
S-PROCESS	–	+ –	+ –
P-PROCESS	–	–	–
R-PROCESS	– –	– –	–

Conclusions

Nuclear physics is a necessary but a not sufficient condition for Nuclear Astrophysics

Still many open nuclear physics questions

The exact role of nuclear physics in Astrophysics will remain unclear as long as the astrophysics sites and the exact nuclear mechanisms of relevance are not fully under control

Reference textbooks and review papers

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  M. Arnould, K. Takahashi, Rep. Prog. Phys. 62 (1999) 395-464
- "S-process nucleosynthesis: nuclear physics and the classical model"

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  M. Arnould, S. Goriely, K. Takahashi, Phys. Rep. 450 (2007) 97
- "The p-process of stellar nucleosynthesis: astrophysics and nuclear physics status"

  M. Arnould, S. Goriely, Phys. Rep. 384(2003) 1