



# *Nonlinear electrodynamics in Weyl semimetals: Floquet bands and photocurrent generation*

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**Theory**

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**Experiment**

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**Qiong Ma, Pablo Jarillo-Herrero (MIT)**

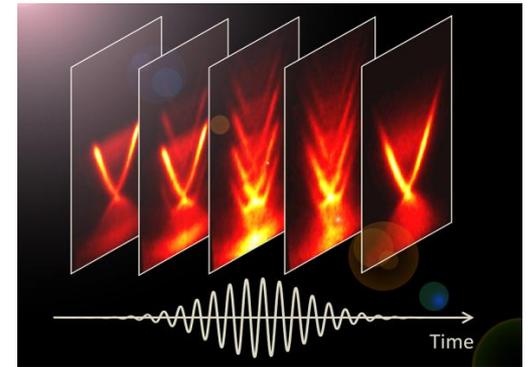


# Outline

*Nonequilibrium physics: light + topological matter + dynamics*

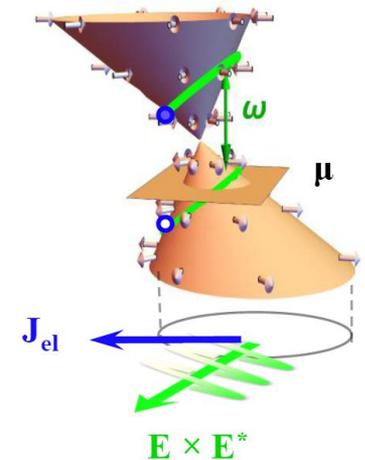
## ■ Floquet-Bloch bands in gapless topological materials

- Mahmood, CKC, et. al., Nature Physics, 2016
- CKC, Lee, et. al., PRL, 2016
- CKC, Oh, Han and Lee, PRB, 2016



## ■ Photocurrent in Weyl semimetals

- CKC, Lindner, Refael and Lee, PRB, 2017
- Ma, Xu, CKC, et. al. Nature Physics, 2017



## Example of driven system: Kapitza Pendulum

**New physics can emerge when physical systems are driven far away from equilibrium**



(<https://www.youtube.com/watch?v=rwGAzy0noU0>)

# Motivation – Nonequilibrium Floquet bands

## Equilibrium [H]

## Nonequilibrium [H(t)≠H(t+T)]

**Evolution:**

$$U(t) = e^{-iHt}$$

$$U(t, 0) = e^{-i \int_0^t dt' H(t')}$$

**Eigenenergies  
and  
eigenstates:**

$$U(t) |n\rangle = e^{-iE_n t} |n\rangle$$

$$U(T, 0) |n_F\rangle = e^{-iE_{F,n} T} |n_F\rangle$$

?

**State  
evolution:**

$$|\psi(t)\rangle = \sum_n \langle n | \psi(0) \rangle |n(t)\rangle$$

$$|\psi(t, 0)\rangle = \sum_n \langle n_F | \psi(0) \rangle |n_F(t)\rangle$$

**“Floquet-wave”:**  $|n_F(t)\rangle = e^{-iE_n t} |p_n(t)\rangle$

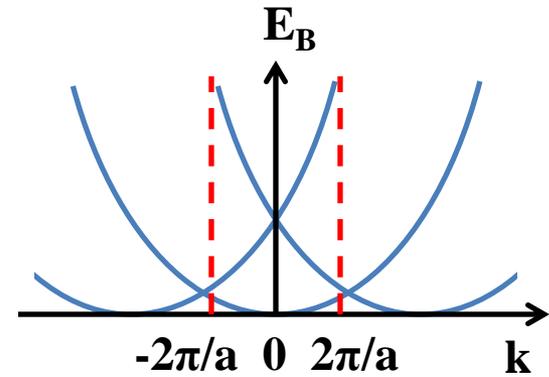
$$|p_n(t)\rangle = |p_n(t + T)\rangle$$

**Well-defined quasi-Hamiltonian in periodically driven systems**

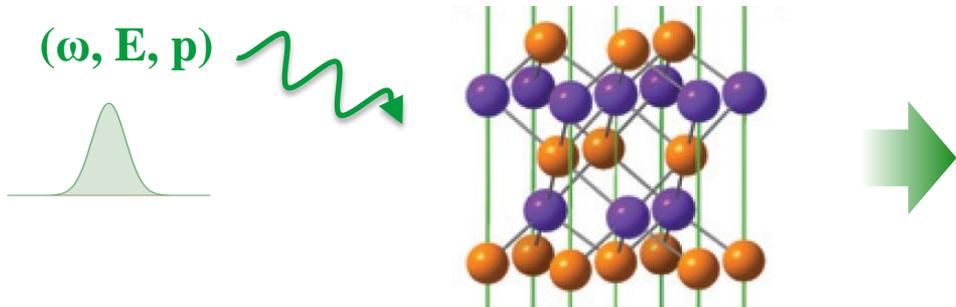
# Motivation – Floquet-Bloch bands

- Spatial periodicity in lattice  
→ Bloch bands

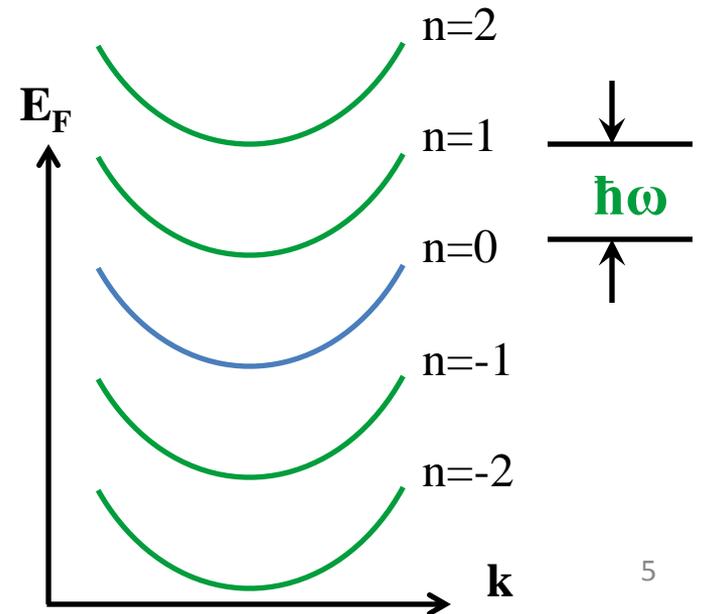
$$\psi_B(r) = e^{ikr} u(r) \quad u(r) = u(r + R)$$



- Temporal periodicity due to laser drives  
→ Floquet bands



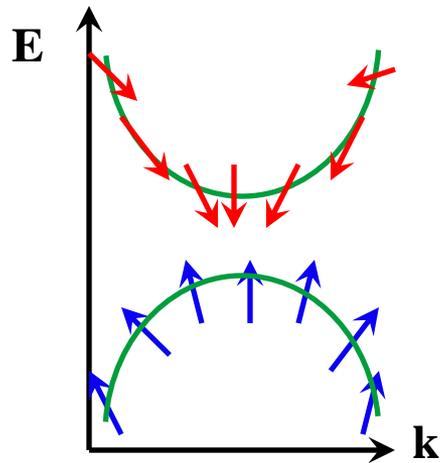
$$\psi_F(t) = e^{-iE_F t} p(t) \quad p(t) = p(t + T)$$



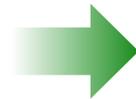
# Motivation – Floquet topological insulator

Light induced topological matter (Lindner, Refael and Galitski, 2011)

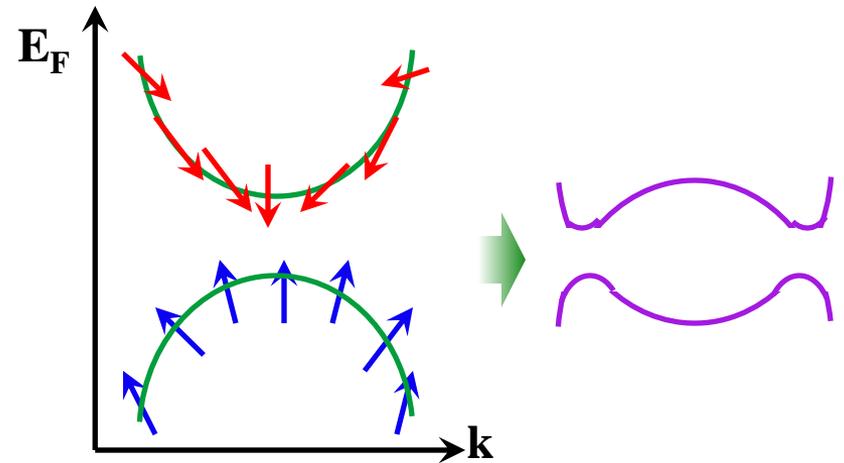
Ordinary insulator



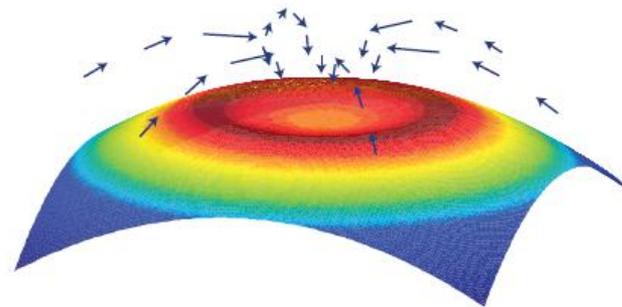
Laser drive



Photoinduced band inversion

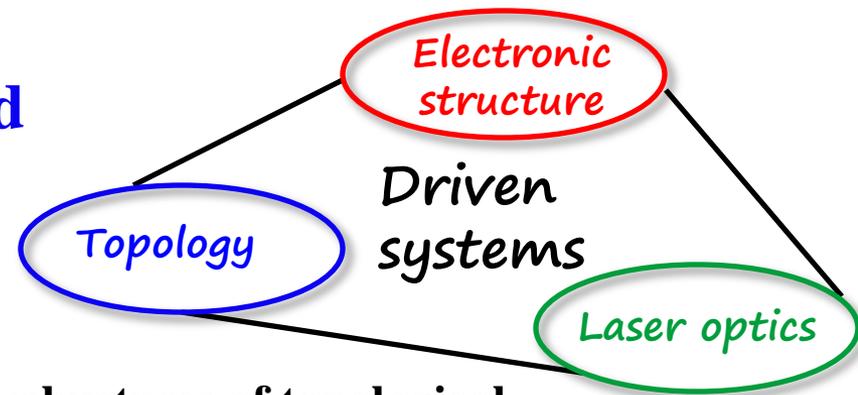


Nontrivial Floquet band



Tuning topology by light!

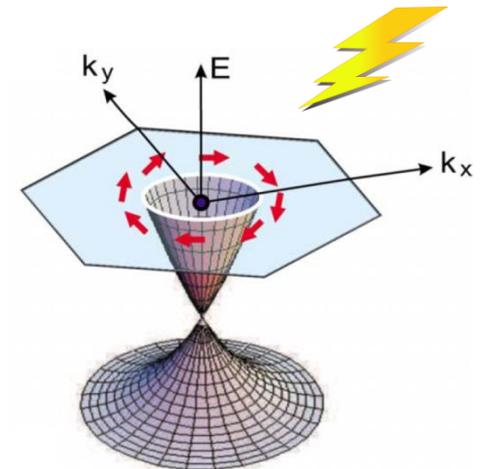
# Many new questions to be explored



- Floquet-band manipulation
- Interplay between intense laser drive and robustness of topological materials (e.g. 2D Dirac, 3D Dirac or Weyl semimetals)
- Roles of symmetry
- Topological phase transitions
- Experimental relevance: photoemission, photoinduced transport phenomena, optical responses, etc.
- And more:
  - Dynamics/evolution
  - Dissipation
  - Heating
  - Disorder
  - Strong correlation

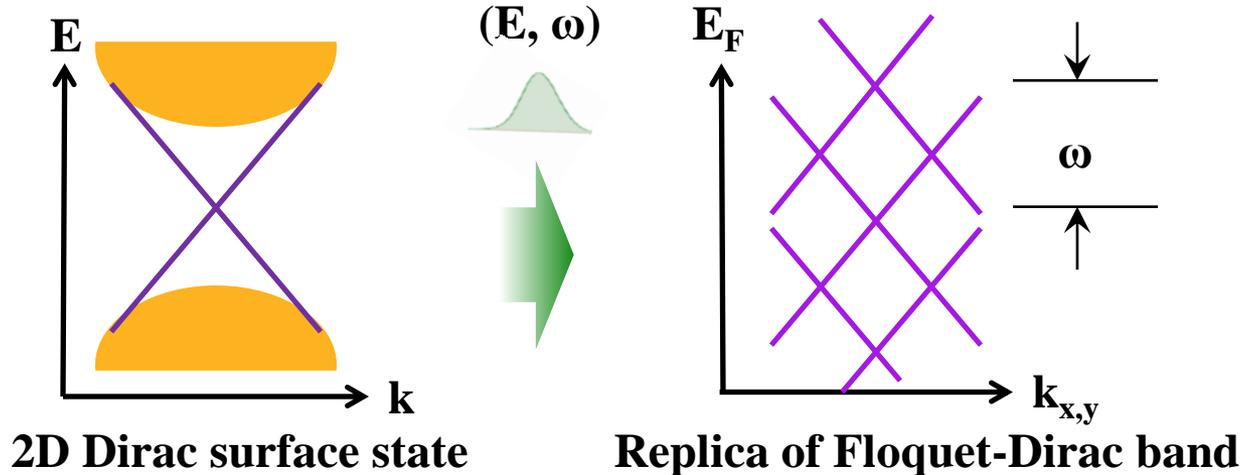
# *Driven 2D Massless Dirac Fermions*

- **2D Floquet-Bloch bands**
- **Time-resolved ARPES**
- **New experiment+theory findings**

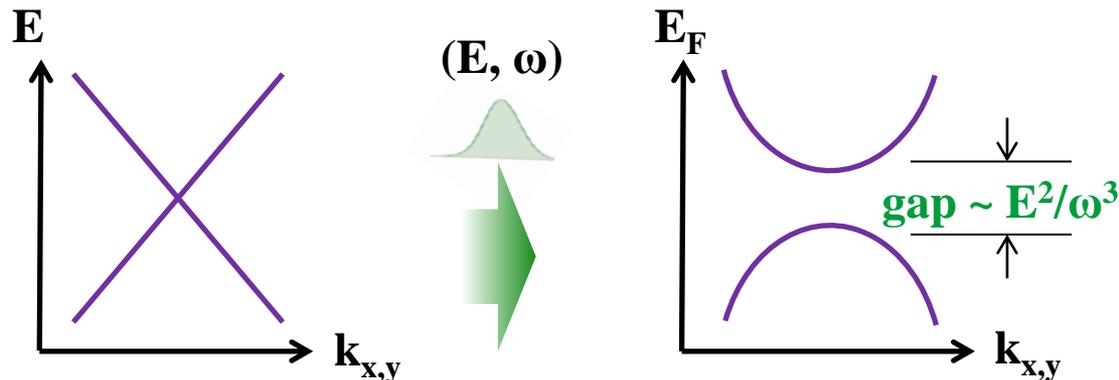


# Driving the surface of 3D topological insulator

## Linearly polarized drive:



## Circularly polarized drive:



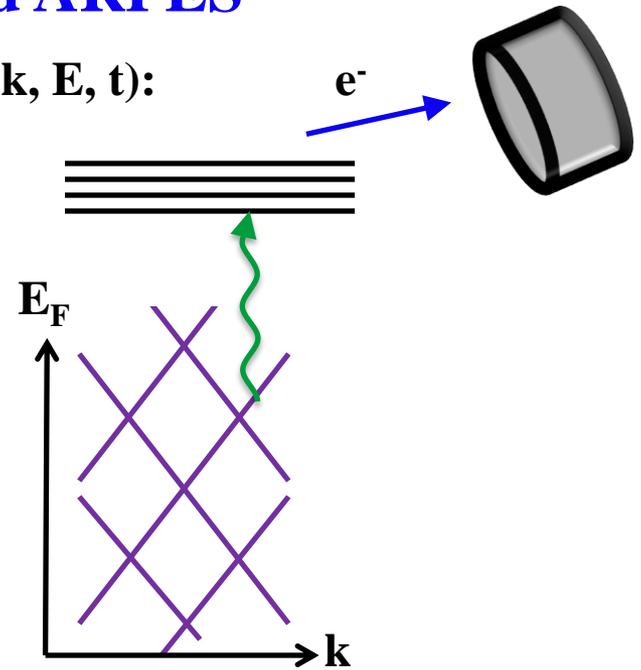
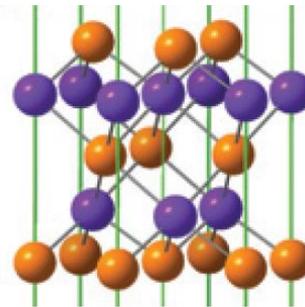
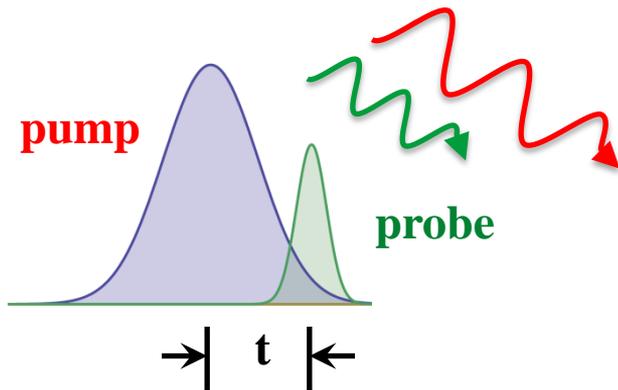
**Light induced band gap due to broken time-reversal symmetry**

**Magnus expansion:**

$$H_F = H_0 + \frac{1}{\omega} \sum_n \frac{1}{n} [H_{-n}, H_n] + O\left[\frac{1}{\omega^2}\right]$$

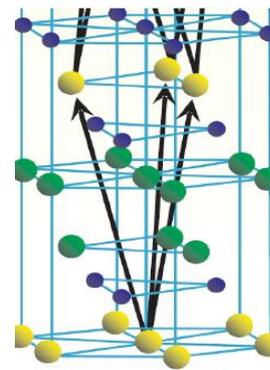
# Experimental advance in Time-Resolved ARPES

Pump-probe measurement of photoexcited electron ( $k, E, t$ ):

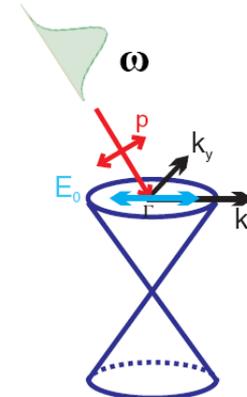


Experiment in Gedik's group  
@ MIT:

Sub-pico second laser pulse  
driving 3D TI  $\text{Bi}_2\text{Se}_3$



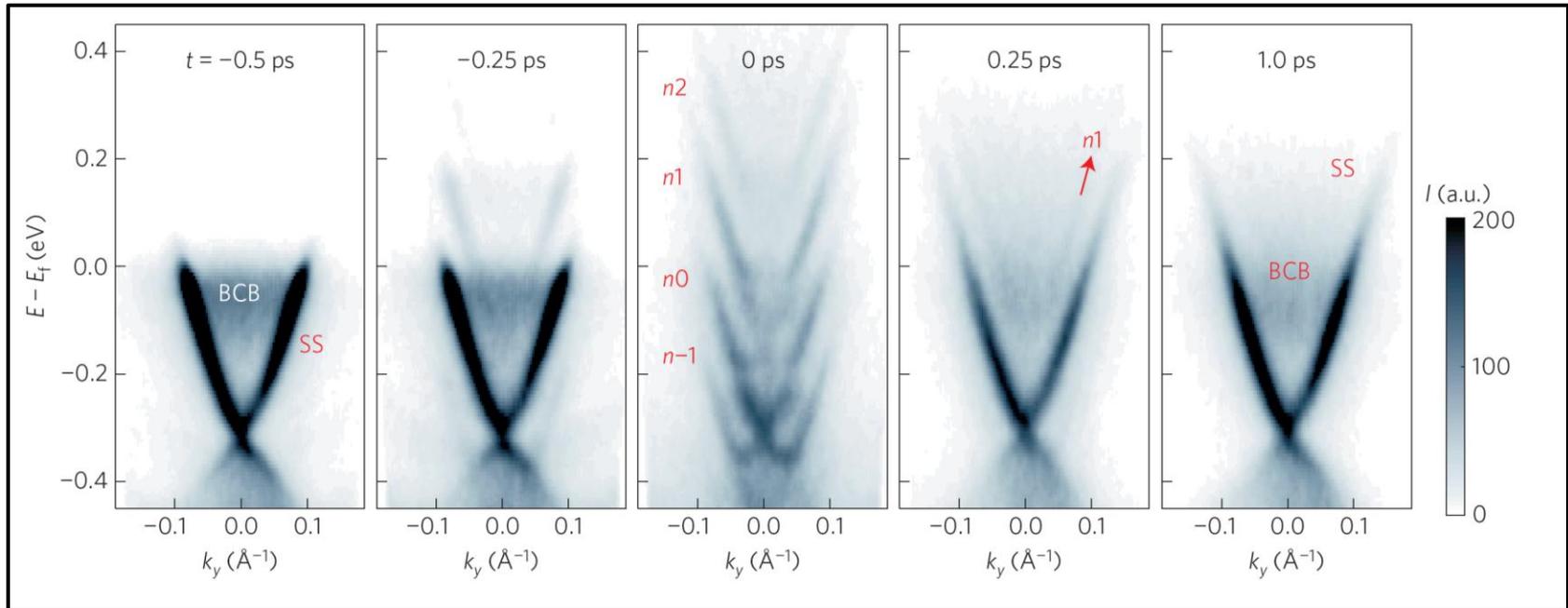
- Bi
- Se1
- Se2



# Floquet-Bloch band on the surface of topological insulator

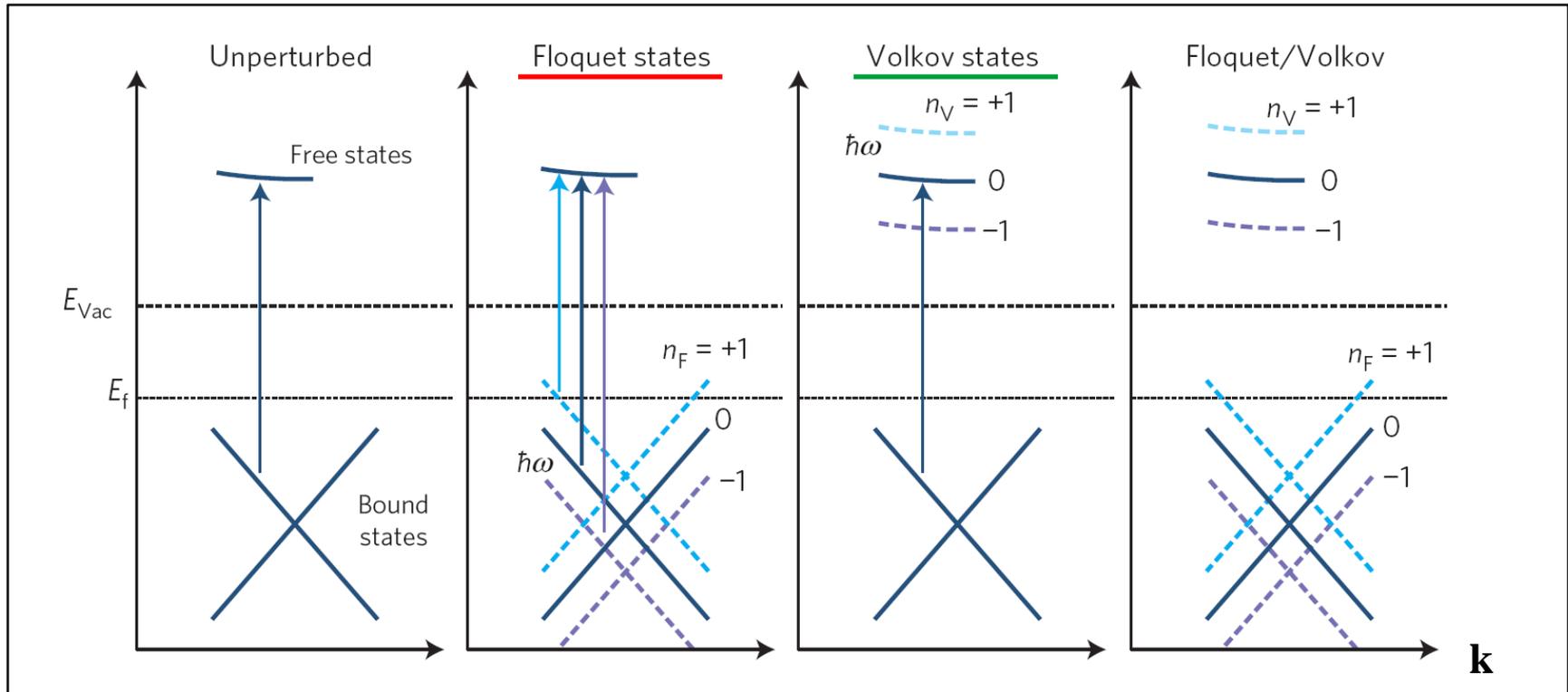
## Floquet-Bloch bands by driving the surface of $\text{Bi}_2\text{Se}_3$

(F. Mahmood, CKC, et. al., Nature Physics, 2016)



- $\text{CO}_2$  laser:  $\hbar\omega \sim 120$  meV
- Gap  $\sim 60$  meV, match well with theory
- Spectral weight discrepancy

# Interference between Floquet and Volkov effects



$$I(\vec{k}, E) \sim \int_{t_i}^{t_f} dt \int_0^{t_f-t} d\tau s(t)s(t+\tau) \sum_{\sigma_1, \sigma_2, \sigma_f} \underbrace{M_k^*(\sigma_f, \sigma_1)M_k(\sigma_f, \sigma_2)}_{\text{spin-probe effect}}$$

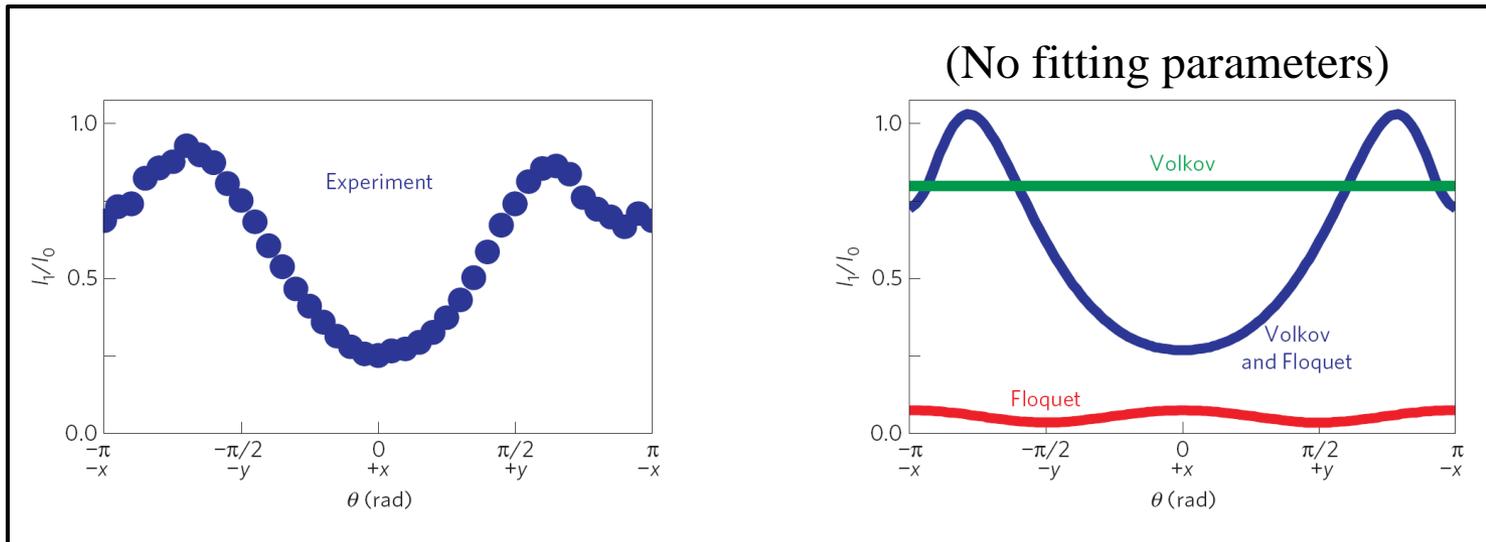
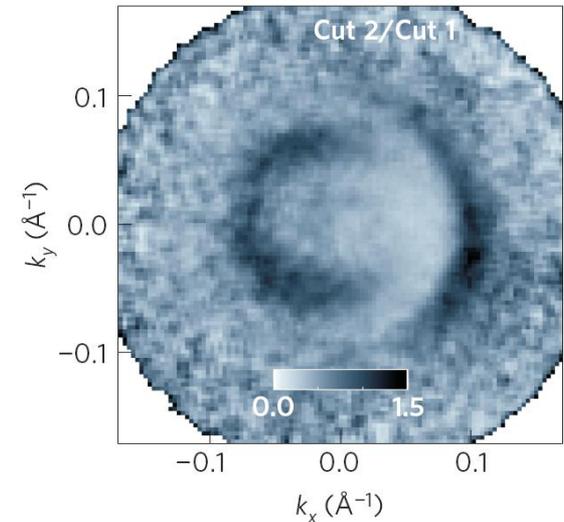
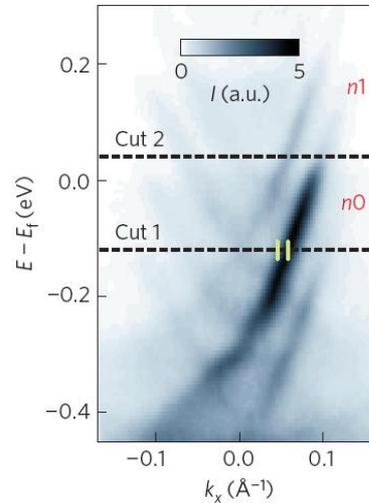
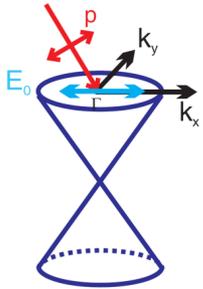
$$\times 2 \operatorname{Re} \left[ \underbrace{G_{k, \sigma_1, \sigma_2}^<(t, \tau)}_{\text{Floquet}} e^{-iE\tau/\hbar} \right] \underbrace{e^{-i \int_t^{t+\tau} dt' H_{\text{LAPE}}(t')/\hbar}}_{\text{Volkov}}$$

**Floquet**

**Volkov**

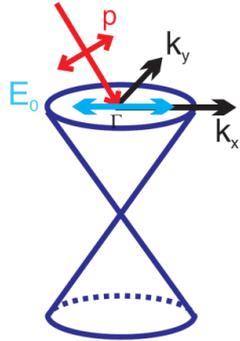
# Spectral weights analysis

**P-polarized pump:**

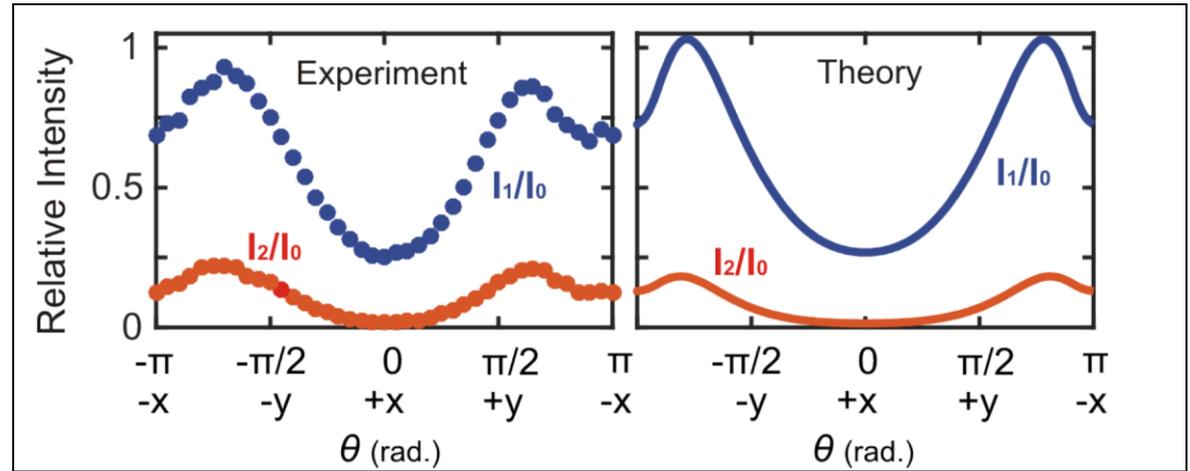


# More spectral weight analysis

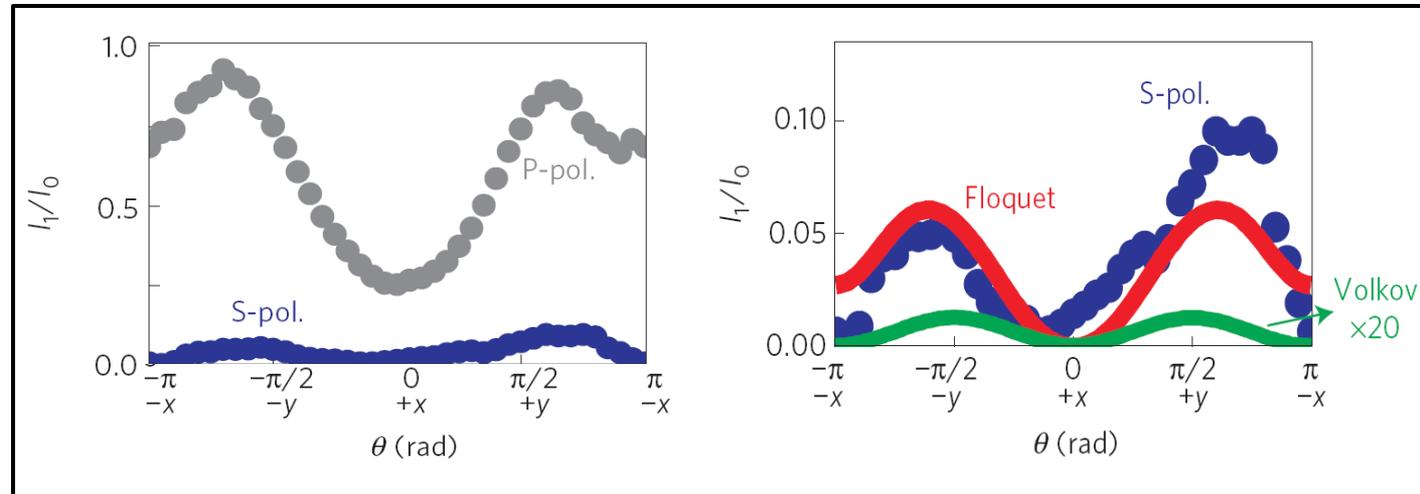
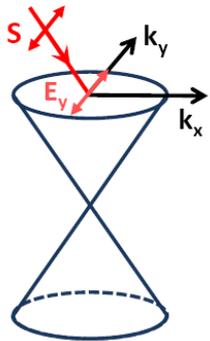
**P-polarized pump:**



**Higher order Floquet bands**

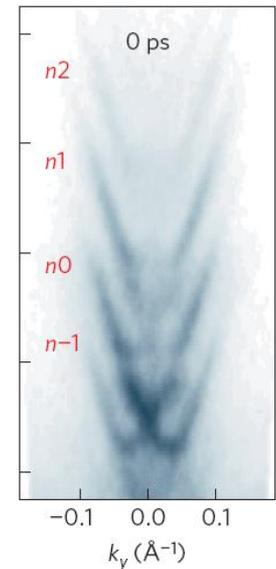


**Purely intrinsic Floquet band using S-polarized pump**



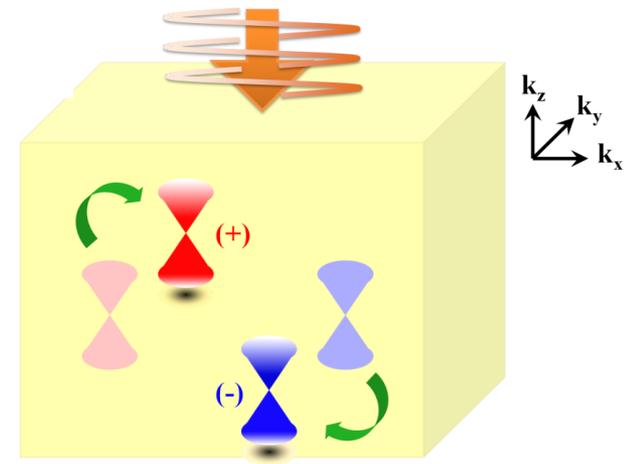
## Summary (what we learnt...)

- **Driving 2D Dirac generates Floquet bands and tunable gaps controlled by laser polarization, frequency and intensity through TR breaking**
- **Spectral weights are quantitatively understood in terms of intrinsic and extrinsic Floquet effects**
- **An excellent moment for more exotic ideas!**



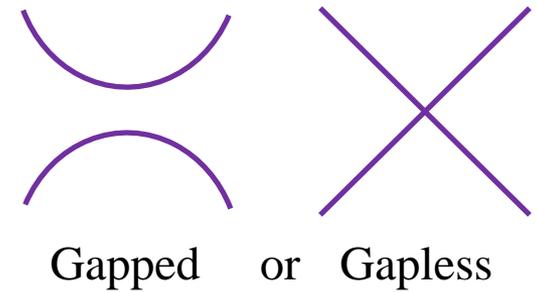
## *Driven 3D Weyl Semimetals*

- **Role of chirality**
- **Photoinduced anomalous Hall effect**
- **Semimetal transitions by light**



# Weyl fermion – 3D band touching points

## Why is 3D special?



**Any 2-band Hamiltonian:** 
$$H = f_0 I_2 + \sum_{i=x,y,z} f_i(\vec{k}) \sigma_i = \begin{pmatrix} f_0 + f_z & f_x - i f_y \\ f_x + i f_y & f_0 + f_z \end{pmatrix}$$

**Band touching (points) iff** 
$$f_x(\vec{k}_0) = f_y(\vec{k}_0) = f_z(\vec{k}_0) = 0$$

**In 3D, conditions generically satisfied *without fine tuning***  
→ **robust against perturbation**

**In 2D, additional symmetries required to force, say  $f_z(\mathbf{k}) = 0$  (e.g. graphene)**  
→ **not robust** if one of those symmetries is removed

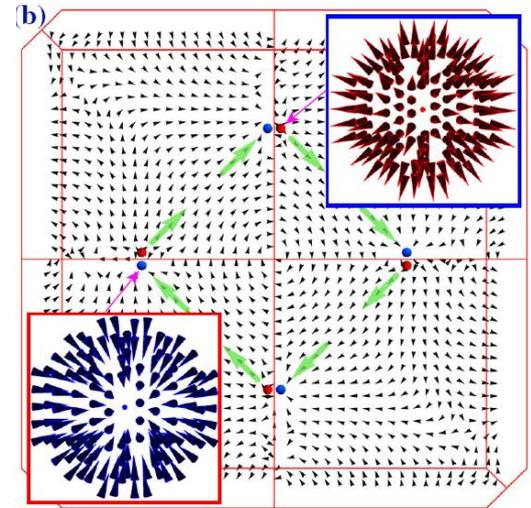
# Weyl semimetals: 3D Chiral fermion

## Features:

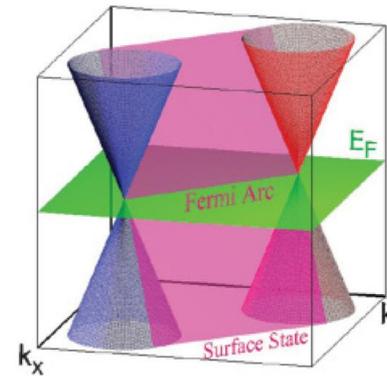
- 3D linearly band touching points
- Come in a pair of opposite chirality (Nielsen-Nyomiya theorem)
- Monopole and anti-monopole of Berry curvature in momentum space
- Fermi arc surface states
- Chiral anomaly
- Can be created by breaking TR or I symmetry of 3D Dirac semimetals



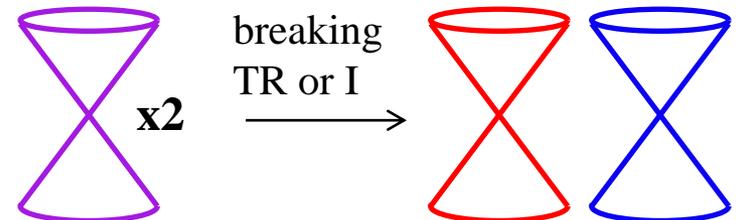
## Berry curvature of TaAs



(H. Weng, et. al., PRX, 2015)



(X. Wan, et. al., PRB, 2011)

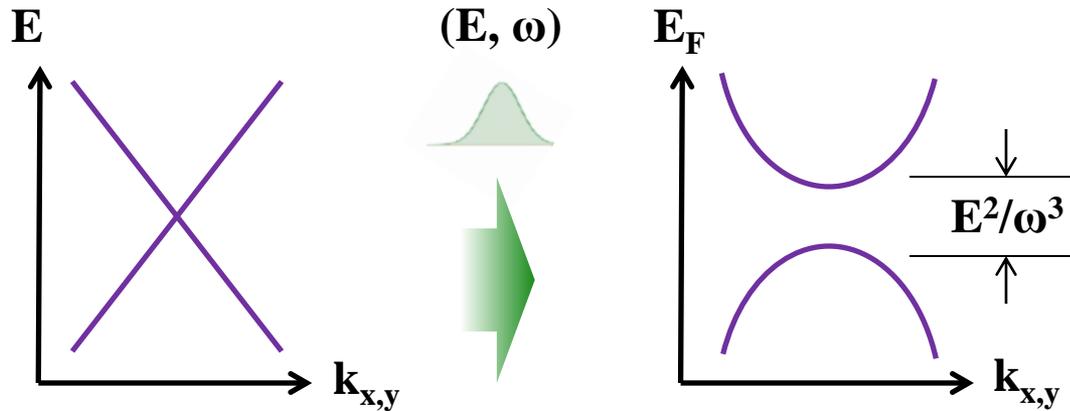


3D Dirac with both TR and I

3D Weyl

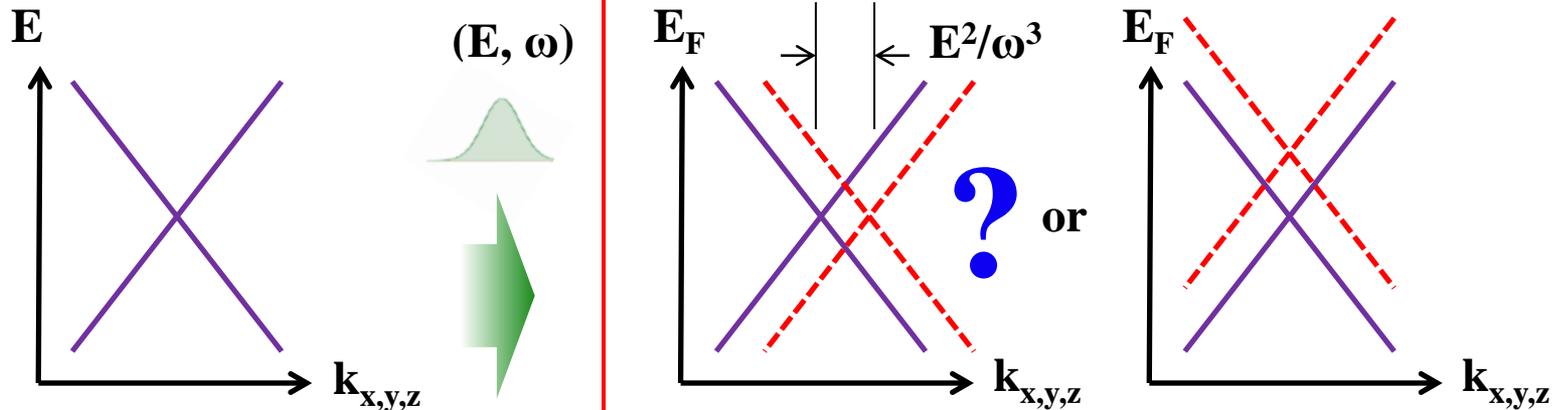
# Effects of chiral photons on Dirac and Weyl fermions

## 2D Dirac (TR required):



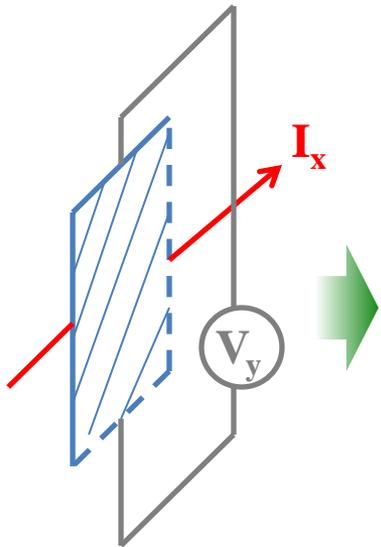
**Anomalous Hall Effect!**

## 3D Weyl (TR not required):

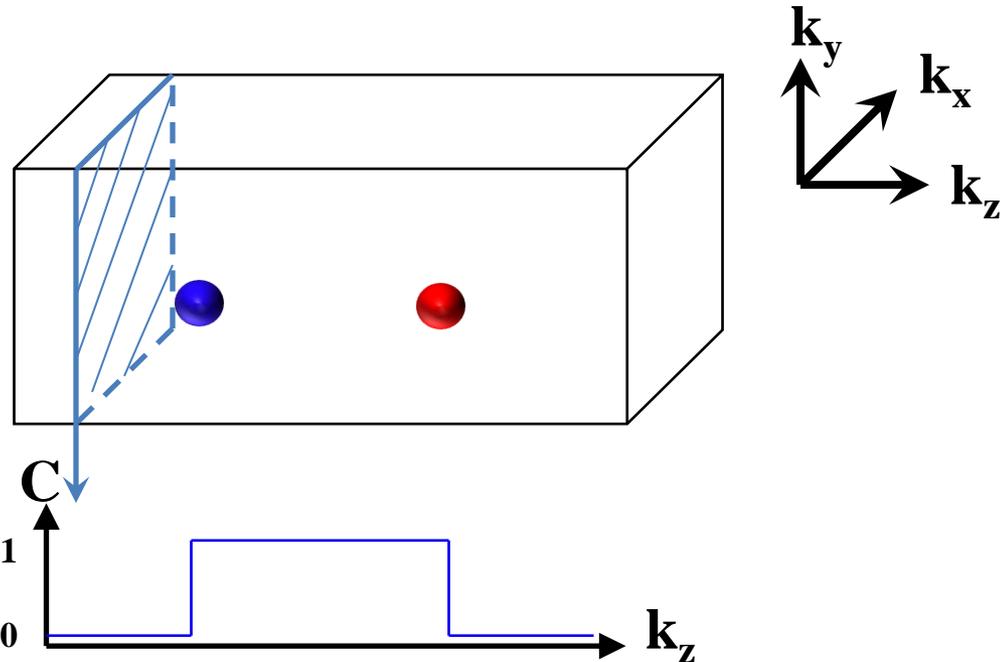


# Anomalous Hall Effect in Weyl semimetals

View as a stack of 2D layers with well-defined topological invariant and  $\sigma_{xy}$



$$\sigma_{xy} = C e^2/h$$



$$\sigma_{xy} = \frac{e^2}{2\pi h} \Delta K_z$$

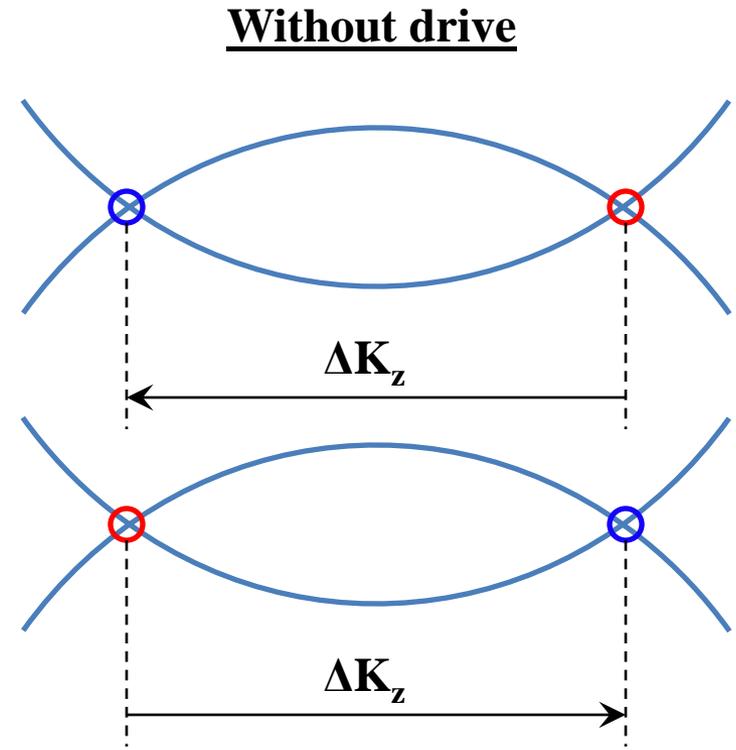
(Yang, Lu and Ran, PRB, 2011)

In general,  $\delta\sigma_{ij} = \frac{e^2}{2\pi h} \epsilon_{ijk} \delta\nu_k$  with Chern vector  $\delta\nu_k = \sum_I \chi_W^{(I)} \cdot \delta q_k^{(I)}$

# AHE in TR Weyl semimetals

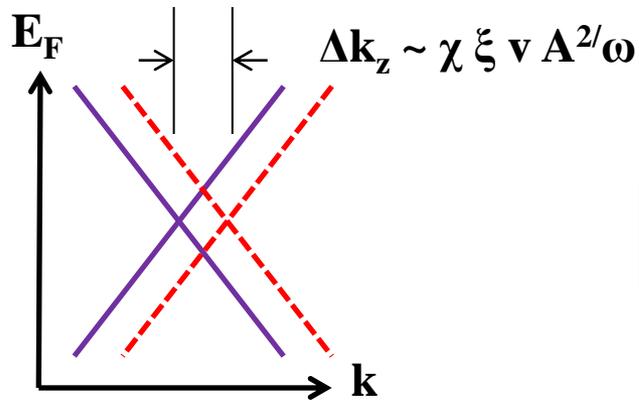
With TR,  $\sigma_{xy}$  from TR Weyl pairs cancel each other

**No AHE in TR Weyl semimetal!**



$$\sigma_{xy} / \sigma_0 = (\Delta K_z) + (-\Delta K_z) = 0$$

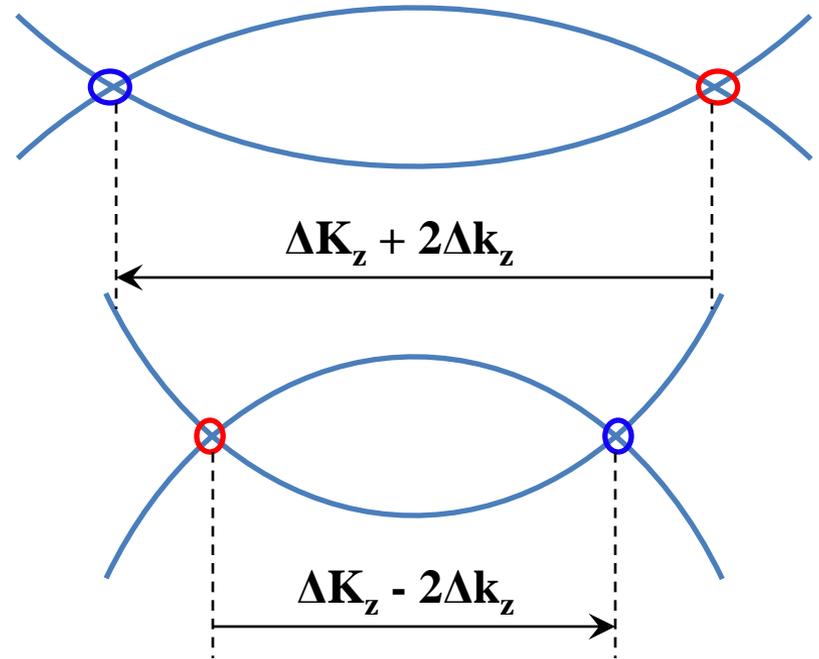
# AHE in driven TR Weyl semimetals



Photoinduced Weyl nodes shift in a chirality ( $\chi$ ) and polarization ( $\xi$ ) dependent manner

Lead to **photoinduced AHE**

Driven



$$\sigma_{xy} / \sigma_0 \sim 4 \xi v A^2 / \omega$$

$$\delta\nu_z = \sum_I \chi_W^{(I)} \cdot \delta q_z^{(I)} \approx (\text{no. of nodes}) \xi v A^2 / \omega$$

(CKC, et. al., PRL, 2016)

# Chirality-dependent Weyl node shift

**Low-energy Weyl Hamiltonian coupled to AC drive propagating along z:**

$$H_w = v \vec{k}_i \cdot \sigma_i \quad \vec{k} \rightarrow \vec{k} + A(\cos \omega t, \xi \sin \omega t, 0)$$

**Effective Floquet contribution:**

$$\Delta H_F \approx \frac{1}{\omega} [H_{-1}, H_1] = - \left( \frac{\xi v^2 A^2}{\omega} \right) \sigma_z$$

$$\boxed{k_z \rightarrow k_z - \frac{\xi v A^2}{\omega} \quad \text{or} \quad k_z - \frac{\chi \xi |v| A^2}{\omega}} \quad \text{chirality: } \chi = \frac{v}{|v|}$$

**Anisotropy:**

$$H_W(\vec{q}) = q_i \alpha_i \sigma_0 + q_i \beta_{ij} \sigma_j$$

**Coupling to higher bands:**

$$H_{\text{lin}}(\vec{q}) = \begin{pmatrix} H_W(\vec{q}) & q_i C_i \\ q_i C_i^\dagger & D_0 + q_i D_i \end{pmatrix}$$

# Lattice model study

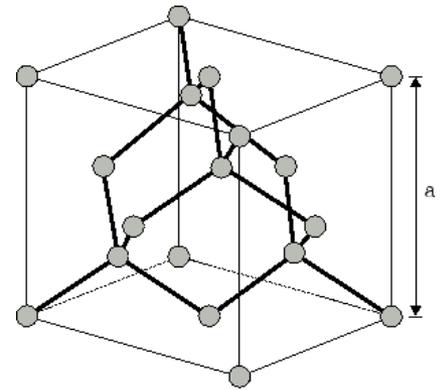
**Hopping model on diamond lattice that breaks inversion symmetry**

$$H = - \sum_{\langle i,j \rangle} (t c_i^\dagger c_j + \text{H.c.}) + \sum_i E_i c_i^\dagger c_i \\ + i\lambda \sum_{\langle\langle i,j \rangle\rangle} (c_i^\dagger \mathbf{e}_{ij} \cdot \mathbf{s} c_j - \text{H.c.}).$$

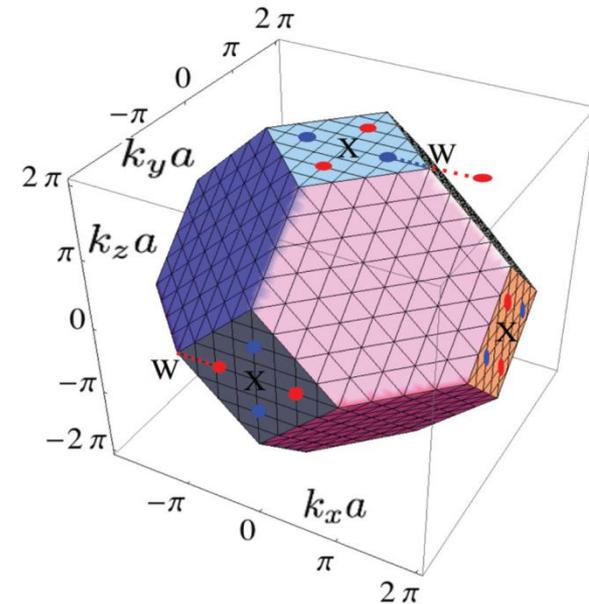
**Supports 12 Weyl nodes (6 +ve and 6 -ve)**

$$k_0 = 2 \sin^{-1} \left( \frac{\epsilon}{4\lambda} \right) < \pi$$

$\vec{k}_{W_-}^{(1,2)} = (\pm k_0, 0, 2\pi),$	$\vec{k}_{W_+}^{(3,4)} = (0, \pm k_0, 2\pi),$
$\vec{k}_{W_+}^{(5,6)} = (\pm k_0, 2\pi, 0),$	$\vec{k}_{W_-}^{(7,8)} = (2\pi, \pm k_0, 0),$
$\vec{k}_{W_-}^{(9,10)} = (0, 2\pi, \pm k_0),$	$\vec{k}_{W_+}^{(11,12)} = (2\pi, 0, \pm k_0),$



Lattice structure



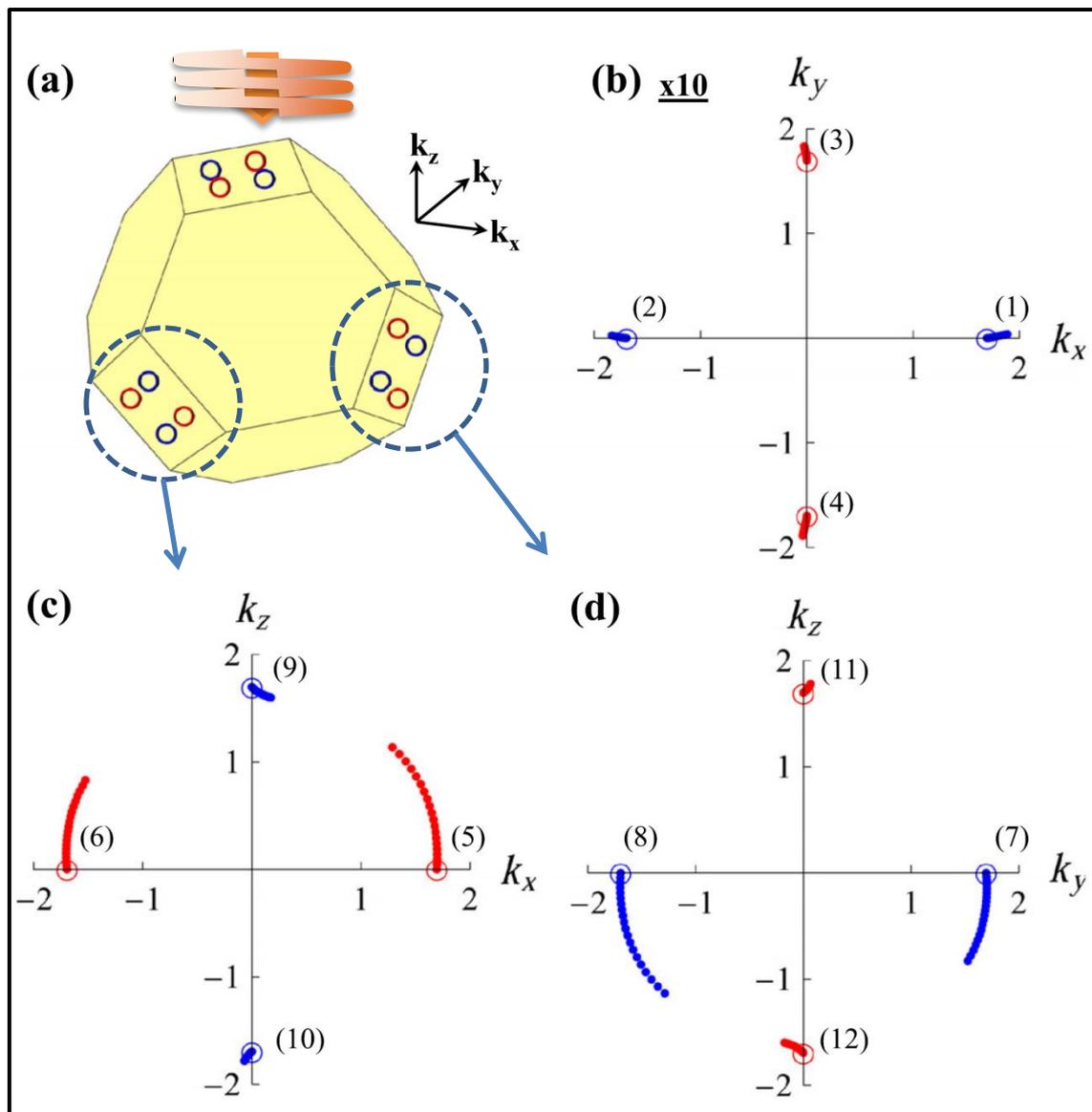
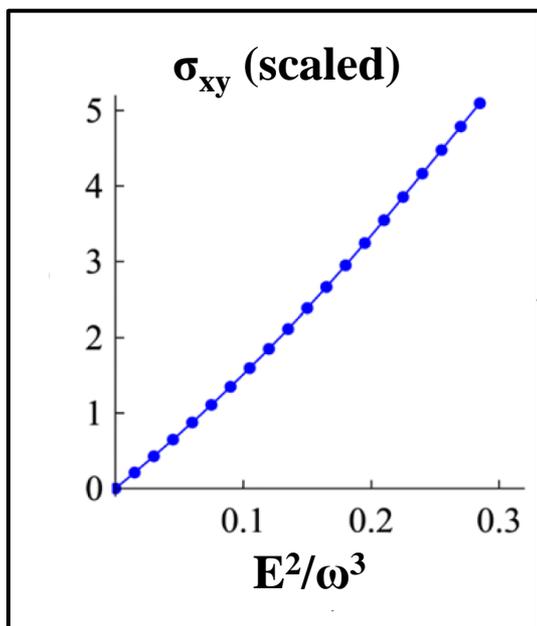
(Ojanen, PRB, 2013)

# Lattice model study

Model Hamiltonian

 (periodic drive)

Floquet Hamiltonian



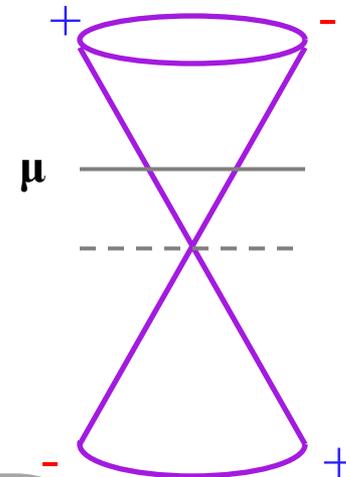
(CKC, et. al., PRL, 2016)

# Effects of doping

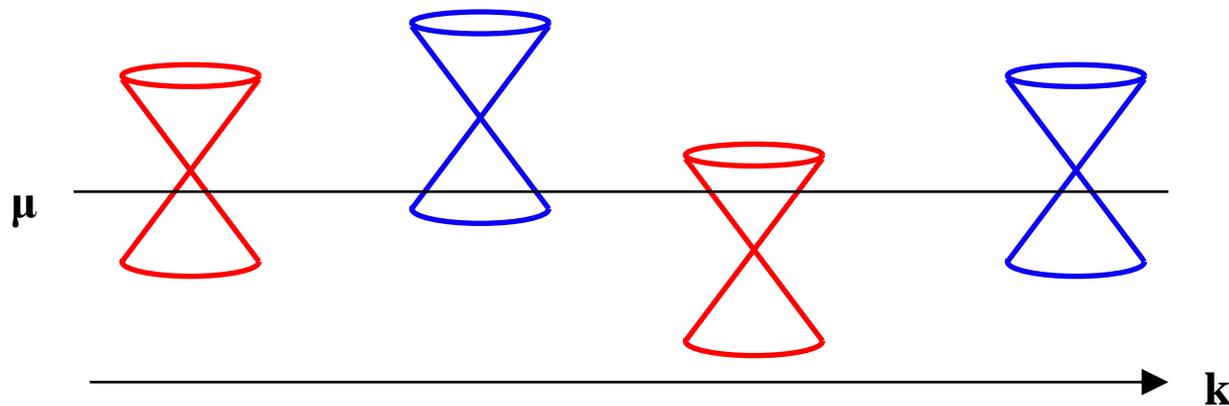
**Doping only leads to negligible correction**  
 $\sim O(\mu^2)$

$$\delta\sigma_{xy}(\mu) \sim \frac{e^2}{4\pi^2\hbar} \int_{0 \leq k \leq k_F} d^3k \Omega(\vec{k}) \cdot \hat{k}_z \sim O(\mu^2)$$

$$\Omega_{\pm}(\vec{k}) \sim \mp \frac{\vec{k}}{2k^3}$$

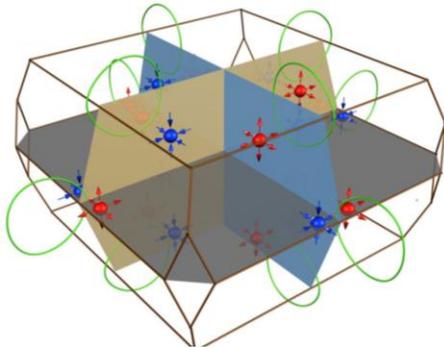


**In sensitive to node positions:**

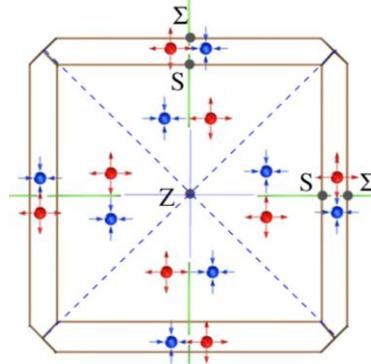


# Experimental estimation on TaAs family

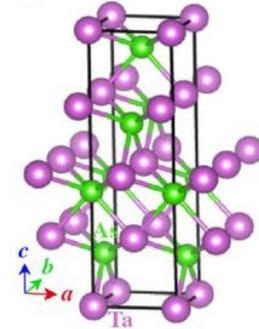
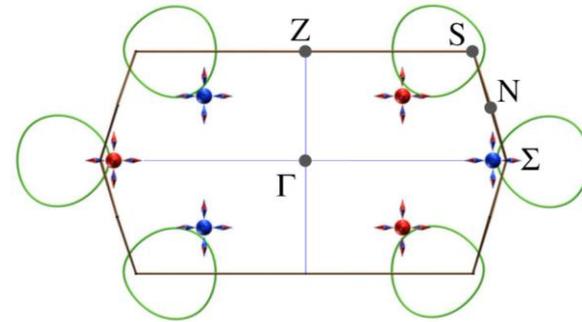
Weyl family of nonmagnetic material: TaAs, TaP, NbAs and NbP



24 Weyl nodes



Mirror and TR symmetry



CW drive { Sample size: 100 $\mu$ m x 100 $\mu$ m x 100 nm  
CO<sub>2</sub>-laser:  $\hbar\omega = 120\text{meV}$ , P = 1W  
Average Fermi velocity: 2 eV $\text{\AA}$   
Hall current: 1A

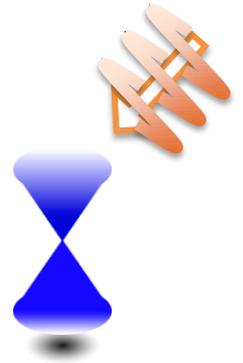


$V_H \sim 130\text{ nV}$

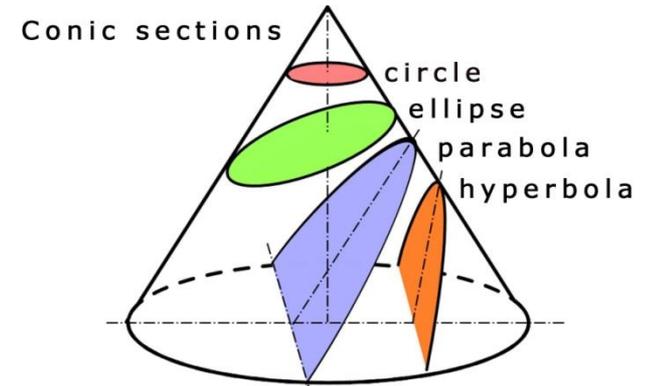
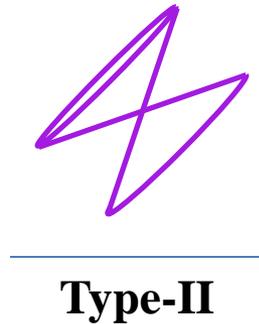
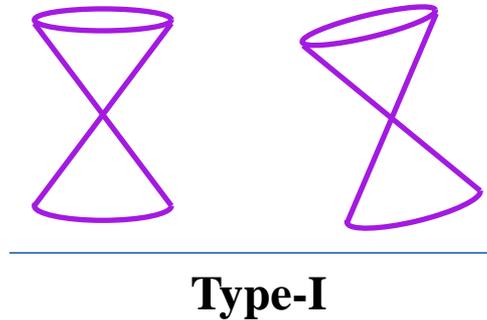
Pulsed drive Faraday angle:  $\theta_F \sim \frac{2\alpha}{n_0 + n_s} \frac{\sigma_{xy}(0)}{\sigma_0} \sim 200\text{ mrad (Weyl semimetal)}$

compared to :  $\sim 7\text{ mrad (graphene)}$

*Can we do more?*



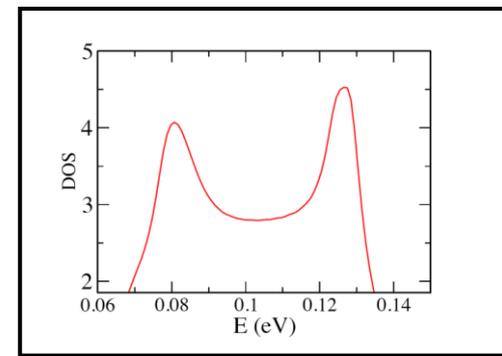
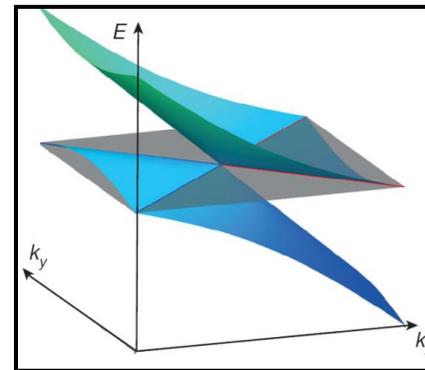
# Two types of Weyl cones



**Conic section  
Fermi surfaces**

## Type-II Weyl features:

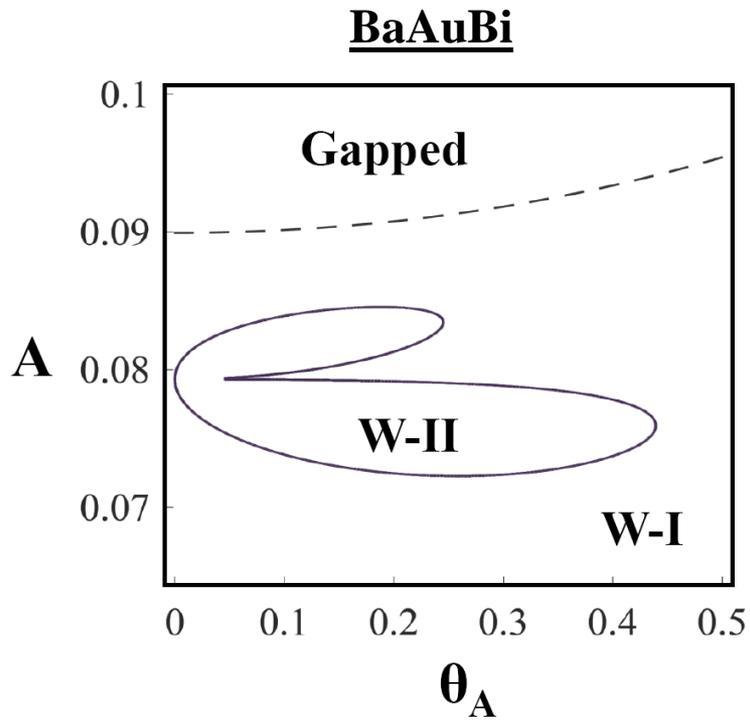
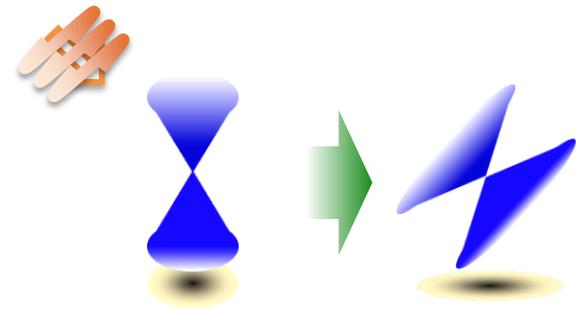
- Open Fermi surfaces
- Finite electronic DOS
- Fermi arc surfaces states
- Anisotropic chiral anomaly



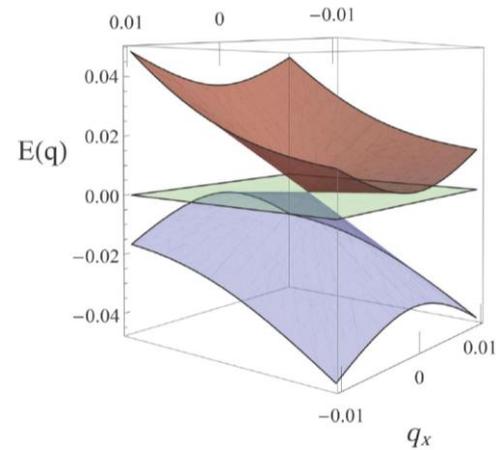
(A. A. Soluyanov, et. al., Nature, 2015)

# Photoinduced type-II Weyl transition - 1

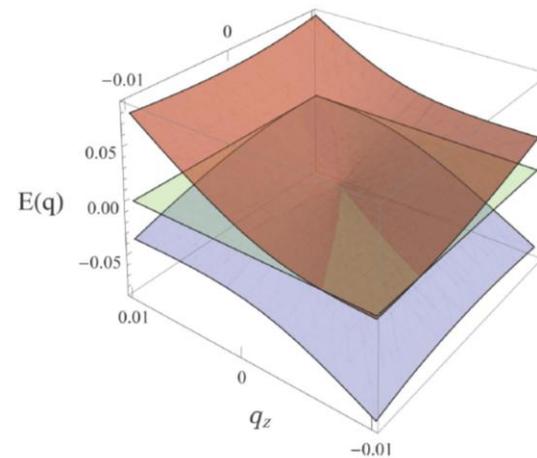
Floquet phase diagram as a function of drive amplitude ( $A$ ) and angle ( $\theta_A$ )



(CKC, Oh, Han and Lee, PRB, 2016)



**W-I**

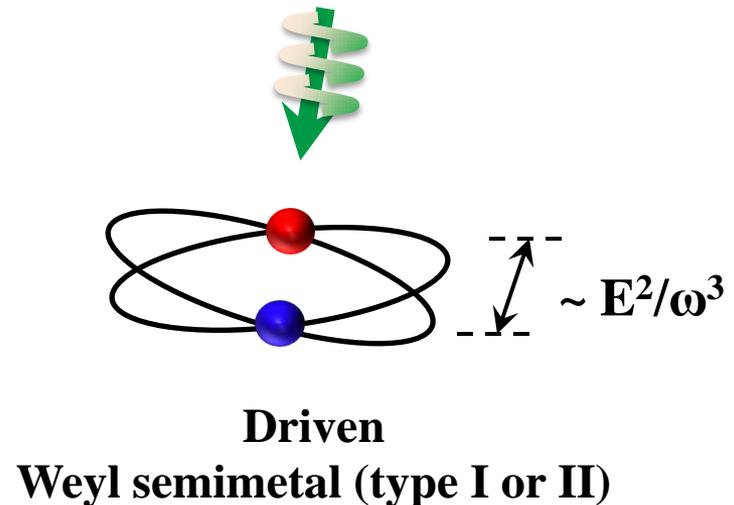
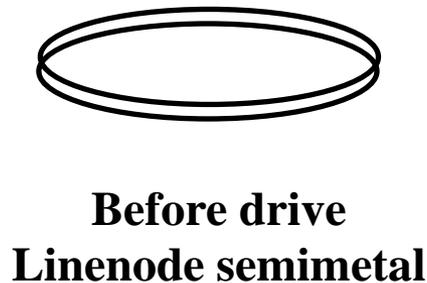


**W-II**

# Photoinduced type-II Weyl transition - 2

## Linenode semimetal:

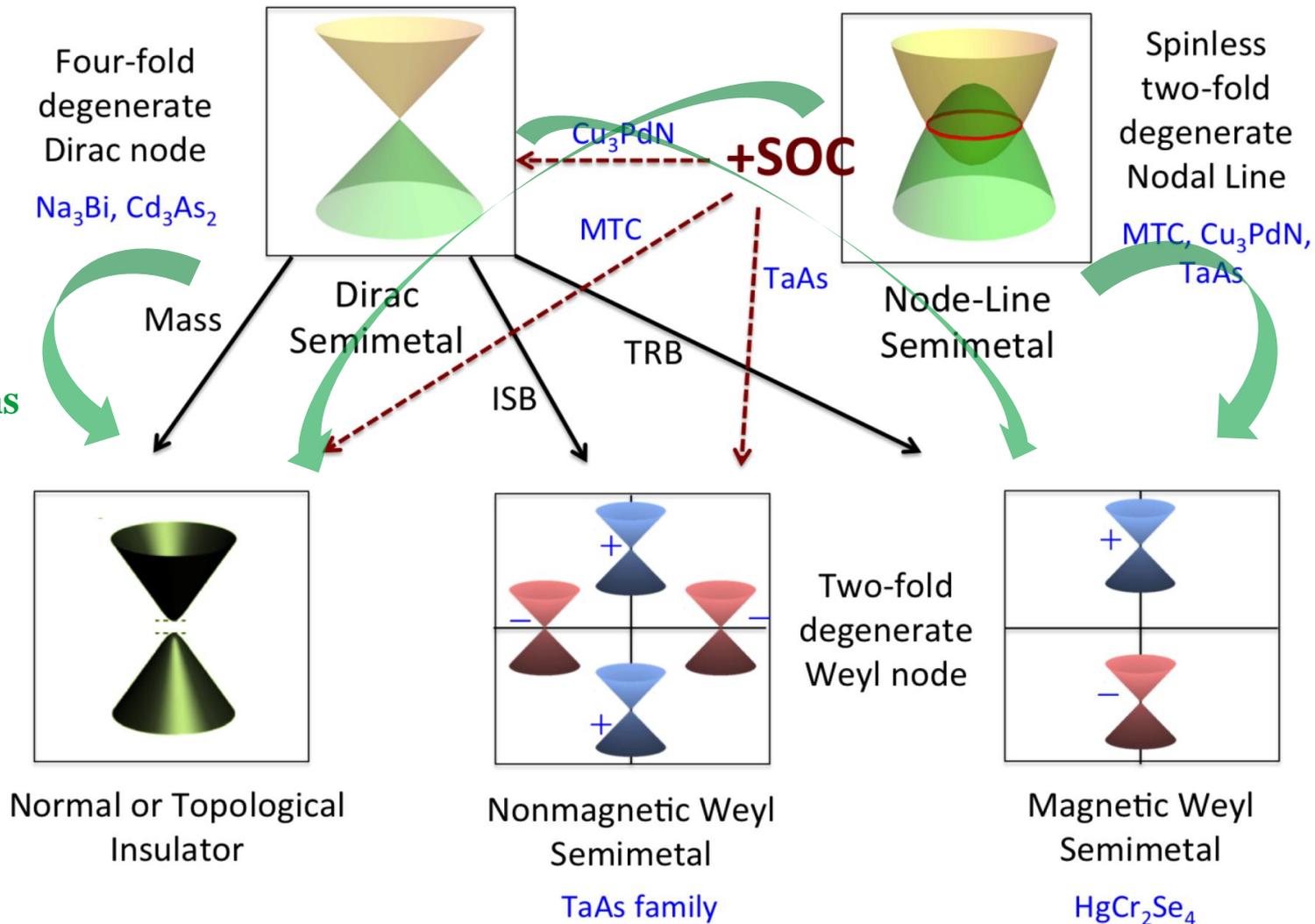
- 3D linearly band touching ring
- nearly flat drum-like surface state
- interesting Berry phase features



(CKC, Oh, Han and Lee, PRB, 2016)

# Topological semimetal transitions *by light!*

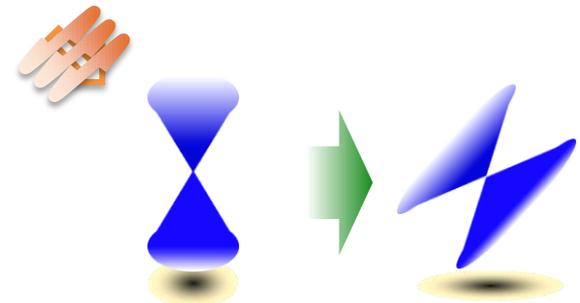
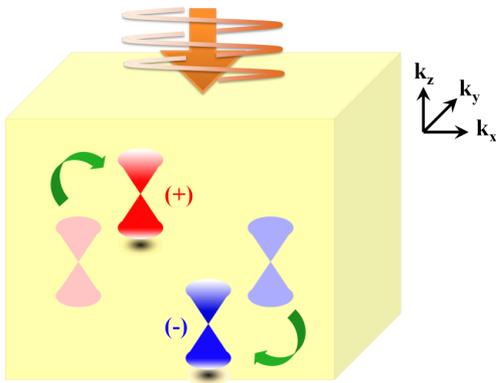
Light induced transitions



(Weng, Dai, Fang, JPCM, 2016)

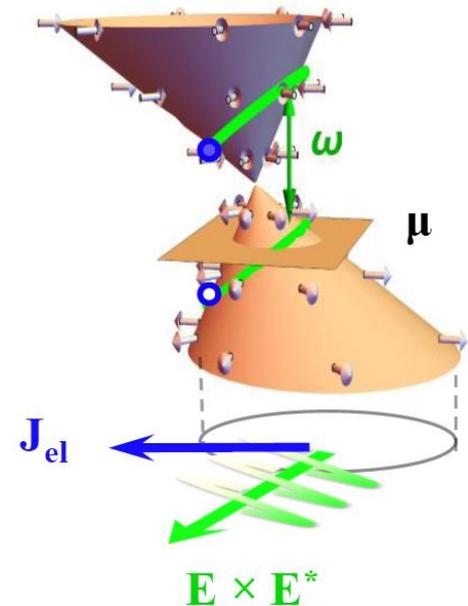
# Summary

- Driving Weyl semimetals **photoinduce anomalous Hall effect** (large effect, measurable by optical and transport experiments)
- Various ways to **photoinduce Weyl transitions** (changes of Fermi surfaces, surface states, transport properties...)



# *Photocurrents in Weyl semimetals*

- **Circular photogalvanic effect (CPGE)**
- **Weyl semimetals as infrared detector**



# Growing interests in nonlinear photovoltaic effects

## Intraband effects

- Gyrotropic magnetic:** Moore and Orenstein, PRL (2010); Zhong, Orenstein and Moore, PRL (2015)
- Quantum nonlinear Hall:** Sodemann and Fu, PRL (2015)
- Photovoltaic chiral magnetic:** Taguchi, et. al, PRB (2016)
- Emergent electromagnetic induction:** Ishizuka, et. al, PRL (2016)
- Photoinduced anomalous Hall:** Chan, et. al, PRL (2016)

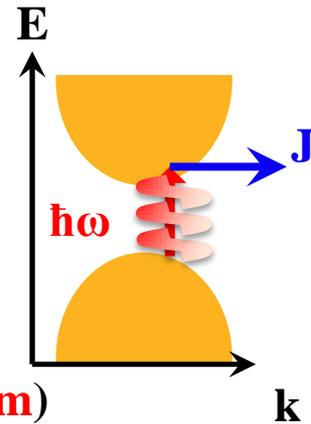
## Interband Circular Photogalvanic effect (CPGE)

- **Quantum wells:** Ganichev, et. al, Physica E (2001)
- **Nanotubes:** Ivchenko and Spivak, PRB (2003)
- **Noncentrosymmetric media:** Deyo, et. al, arXiv:0904.1917 (2009)
- **Weyl semimetals:**
  - Konig, et.al, PRB (2017)
  - Golub, et. al, JETP (2017)
  - de Juan, et. al, Nature Comm (2017)

# Infrared photodetection in various systems

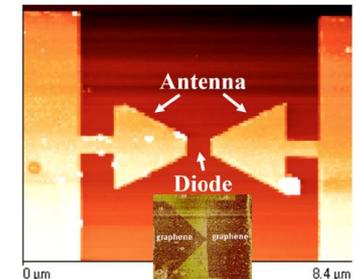
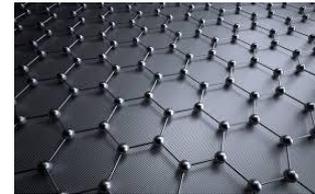
## Conventional semiconductors:

- High efficiency
- But, frequency range is limited by electronic bandgap ( **$\sim 300\text{meV}$  or  $4\mu\text{m}$** )
- \* Blackbody object at 300K has radiation peak  **$\sim 73\text{meV}$  or  $17\mu\text{m}$**



## Graphene:

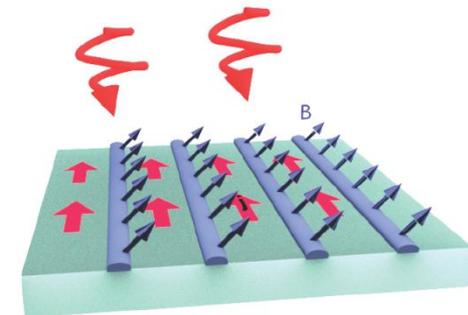
- No frequency limitation (in theory)
- Very low efficiency  
as low as  $\sim 0.00001$  for infrared detection



(Zhu, et al, IEEE J Quant. Electron, 2014)

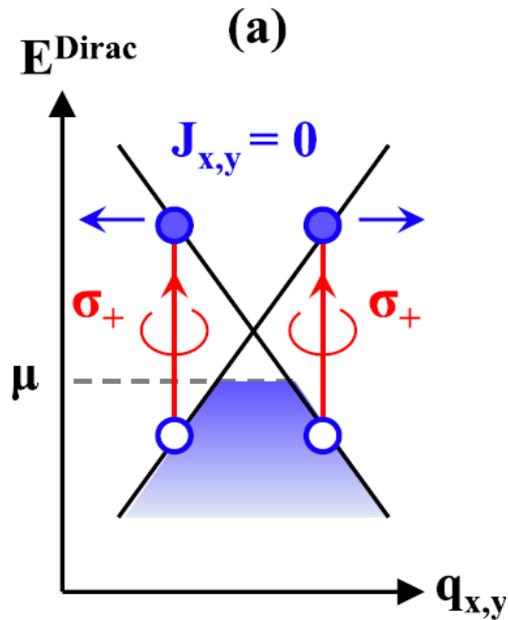
## 3D TI (surface state) + magnetic superlattice:

- Improved efficiency
- Require external coupling



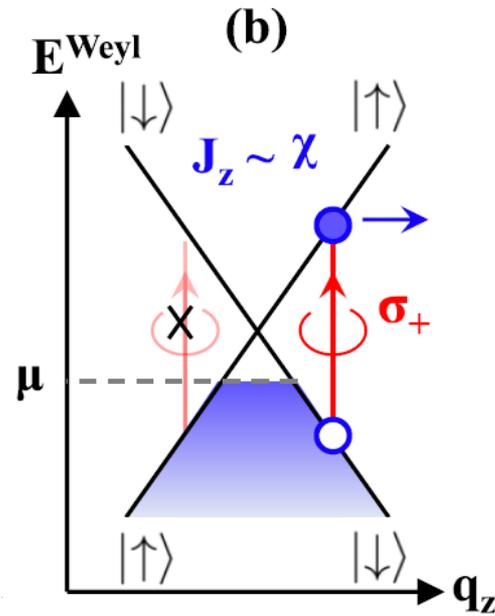
(Lindner, et. al, arXiv: 1403.0010)

# Circular photovoltaic effects in Dirac and Weyl systems



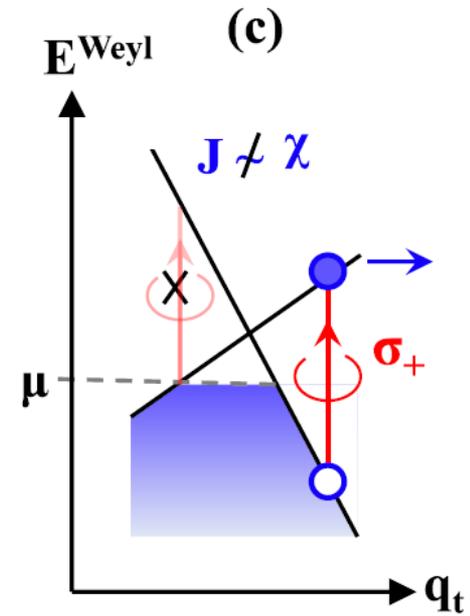
2D Dirac system

- Symmetric photoexcitation leads to zero current
- Inversion symmetry **forbids current**



3D Weyl system

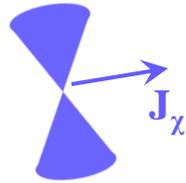
- Asymmetric photoexcitation
- Current direction governed by chirality
- **No net current ?**



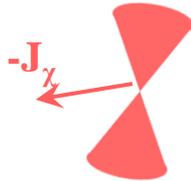
3D Weyl system (with tilt)

- Asymmetric excitation by Pauli blockade
- Current direction can be arbitrary
- **Net current in general**

# Centrosymmetry vs Non-centrosymmetry

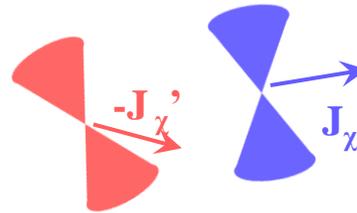


• IC

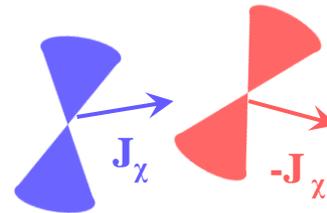


Centrosymmetric  
Weyl semimetal

Currents from  
positive and negative  
Weyl nodes cancel



• TRIM

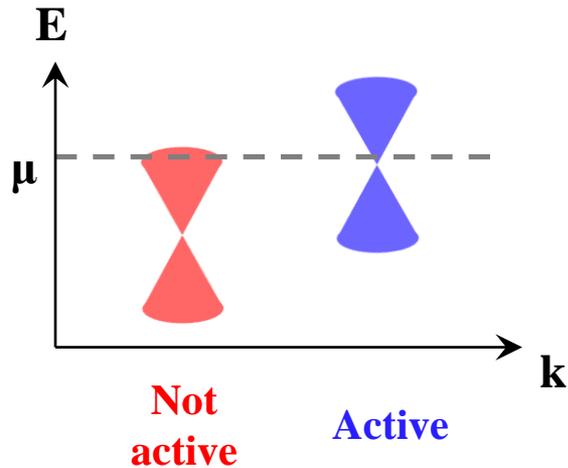


Non-centrosymmetric  
Weyl semimetal

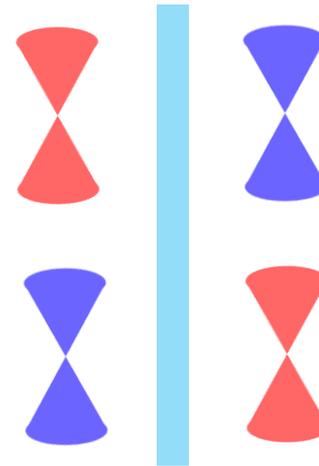
Positive and negative  
Weyl nodes are not  
symmetry related.  
No current cancellation  
in general.

**Necessary condition - 1: Break inversion symmetry**

# Role of mirror symmetry



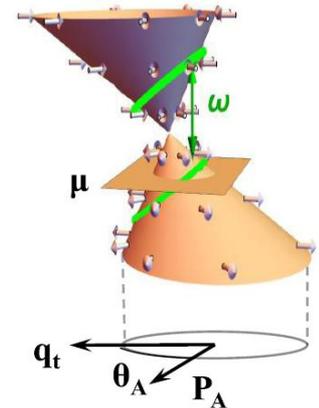
With  $\mu$  imbalance, expect to see a net photocurrent.



In many realistic materials (e.g. TaAs), the presence of mirror symmetry aligns the crossing points.

Still have a non-zero photocurrent?

Necessary condition - 2: Finite tilts of Weyl spectra



# Single Weyl node consideration

Single Weyl Hamiltonian: **tilt velocity** **Fermi velocity**

$$H_W(\vec{q}) = \hbar v_t \underline{q_t} \sigma_0 + \hbar v_F \underline{\hat{v}_{i,j}} q_i \sigma_j$$

**tilt direction**

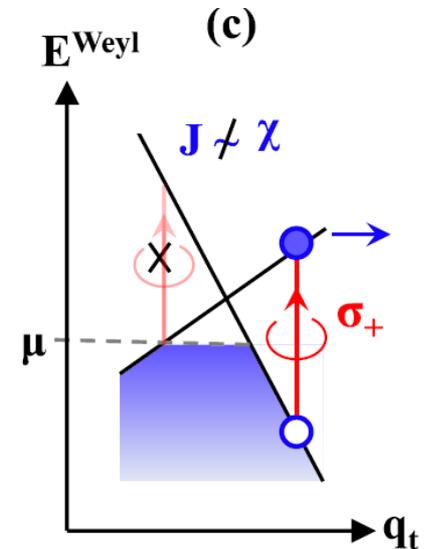
**Chirality  $\chi = \text{Sgn}\{\text{Det}[\mathbf{v}_F \mathbf{v}_{ij}]\}$**

Consider  $v_t \rightarrow -v_t$  and  $\hat{v}_{i,j} \rightarrow -\hat{v}_{i,j}$  ( $= \vec{q} \rightarrow -\vec{q}$ ),

$\Rightarrow J_i \rightarrow -J_i$   $\iff$  ( $\mathbf{J} = \mathbf{0}$  in inversion symmetric system)

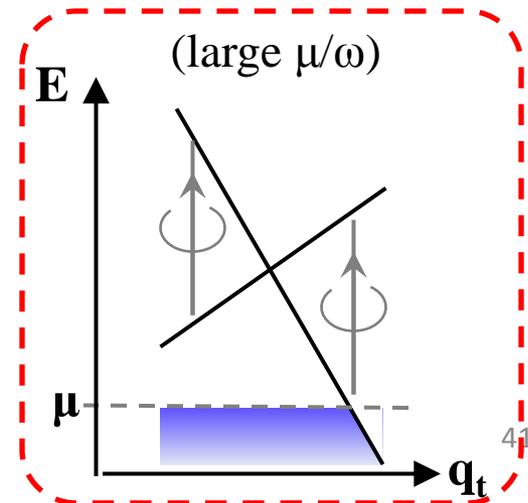
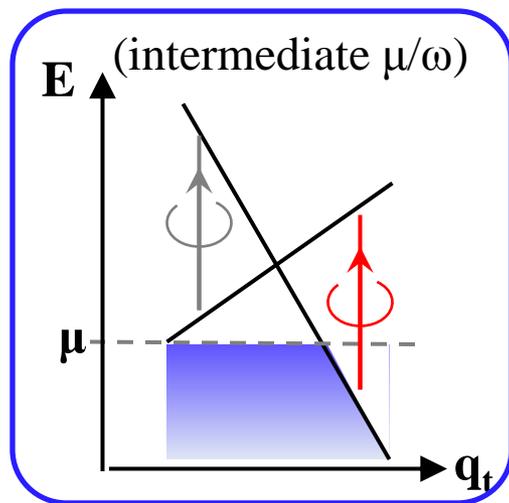
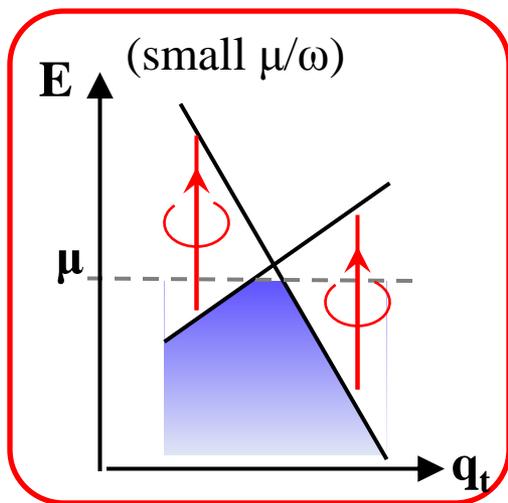
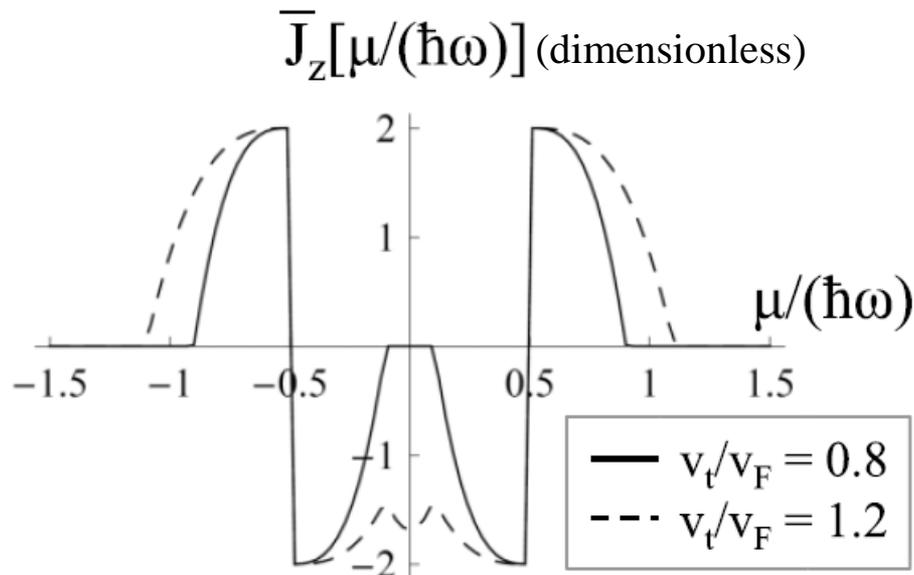
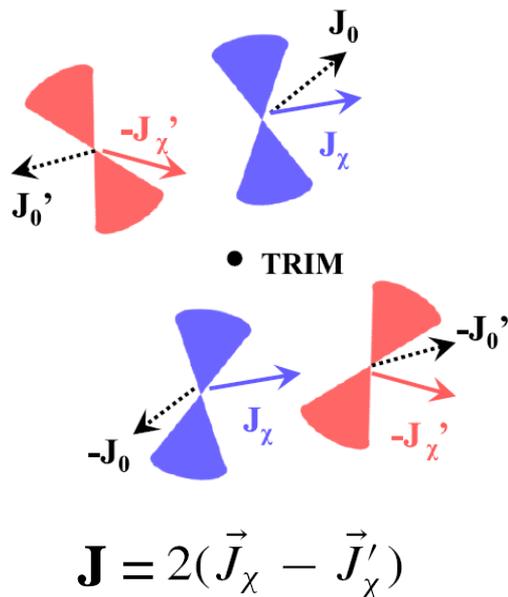
In general,

$$\vec{J}\left(\frac{v_t}{|v_t|}, \chi\right) = \frac{v_t}{|v_t|} \times \vec{J}_0 + \chi \times \vec{J}_\chi$$



# Minimal model of 4 Weyl nodes with TR symmetry

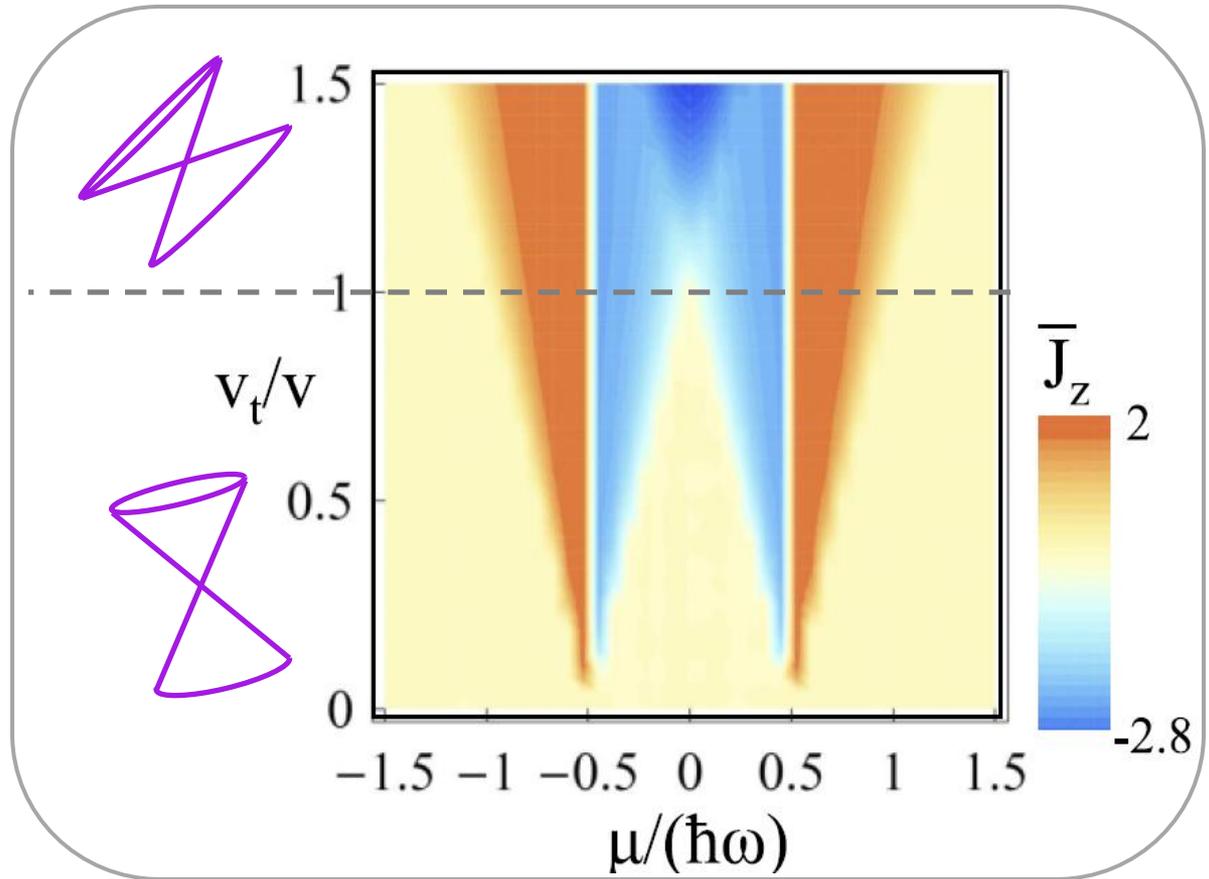
Example plot of photocurrent generated by 4 Weyl nodes driven along some direction



# Type I vs type II Weyl cone

-Larger tilt increases the “active” region ( $\mu/\hbar\omega$ )

-Magnitude of photocurrent is insensitive to tilt



## Some notable features

- Photocurrent magnitude is **independent of frequency**  $\vec{J} = \left( \frac{e^3 \tau I}{16\pi^2 \hbar^2 \epsilon_0 c} \right) \vec{J}$
- Photocurrent magnitude is **independent of Fermi velocity** ( $v_F$ )
- Photocurrent direction is determined by lattice crystal symmetry

$$J_\alpha(\omega = 0) = \eta_{\alpha\beta\gamma}(\omega, -\omega) E_\beta(\omega) E_\gamma^*(\omega)$$

# Room temperature IR photodetector

Weyl semimetal candidate: TaAs  
Long relaxation time  $\sim 45\text{ps}$   
Tilt  $\sim 60\%$   
 $\mu \sim 20\text{meV}$

+

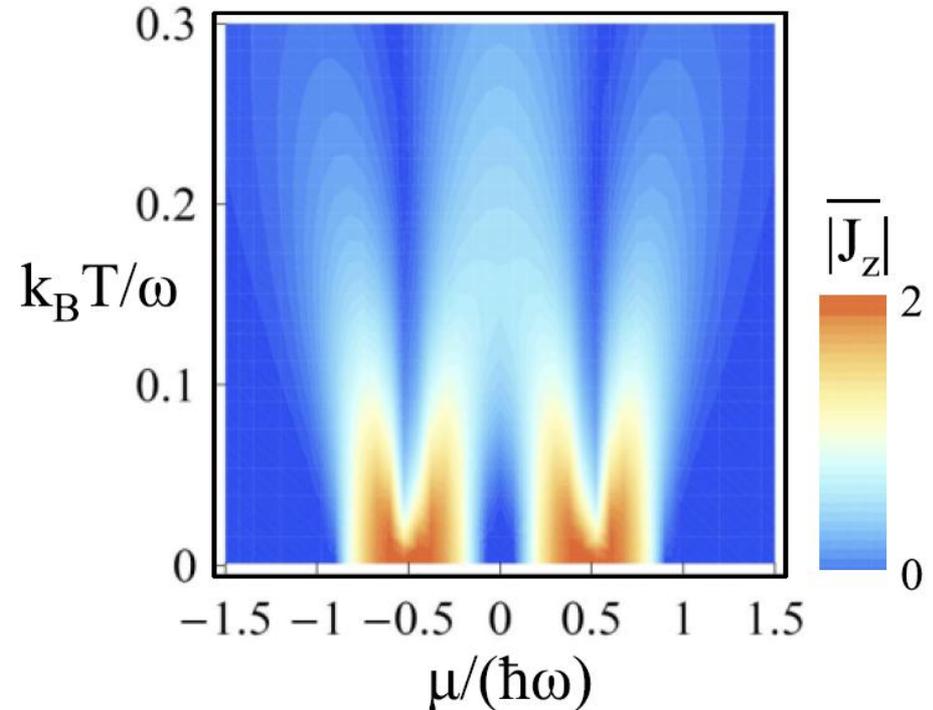
CO<sub>2</sub> laser:  $\hbar\omega = 120\text{meV}$   
Intensity:  $I \sim 10^6 \text{ Wm}^{-2}$

➤ Current density  $\sim 4 \times 10^7 \text{ Am}^{-2}$   
at low temperature

- **Gigantic photocurrent density**  
can be generally induced in Weyl  
semimetals

- **Several orders of efficiency  
improvement** when compared to  
graphene on substrates

(CKC, Lindner, Refael and Lee, PRB, 2017)



Room temperature  
reduction:  $\sim 30$

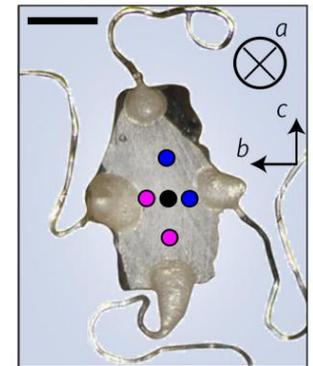
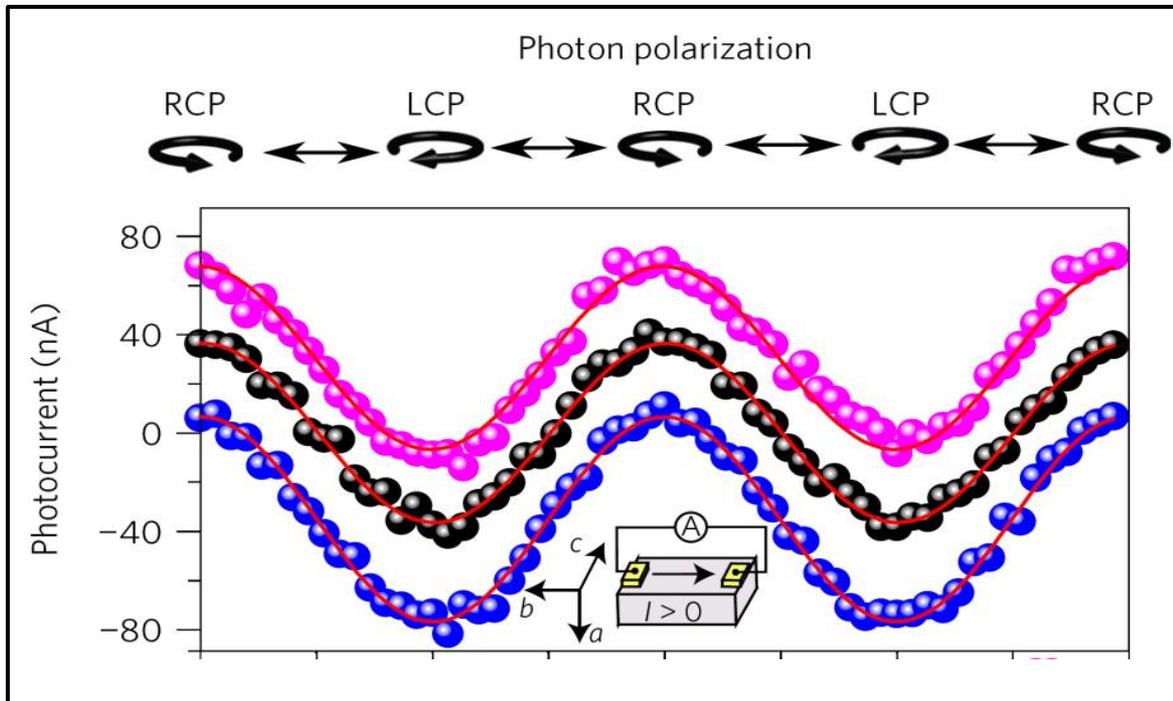
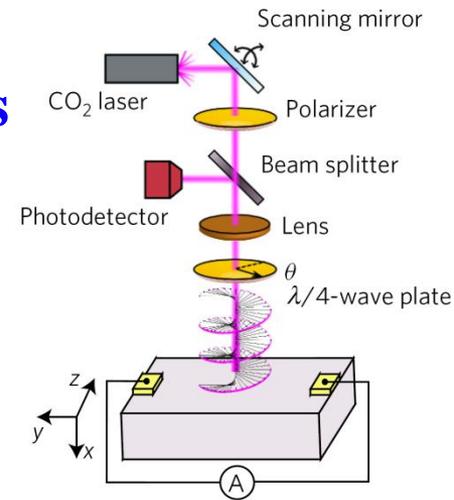
# Observation of circular photogalvanic effect in TaAs

## Setup:

-CO<sub>2</sub> laser  $\hbar\omega = 120\text{meV}$

-TaAs Weyl dispersion tilting  $\sim 72\%$  and  $\mu \sim 18\text{meV}$

Observation of sizeable photocurrent amplitude  $\sim 40\text{ nA}$ .

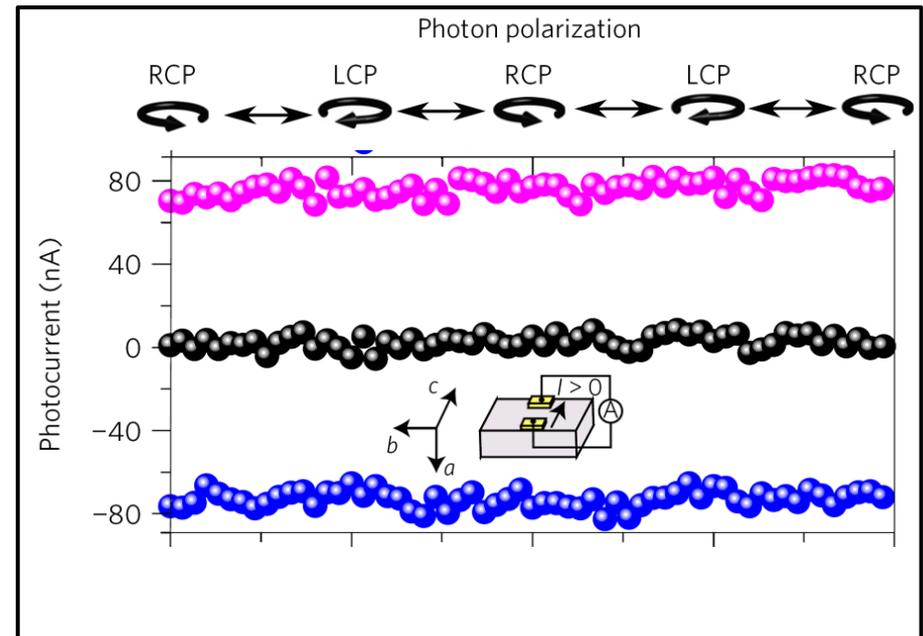
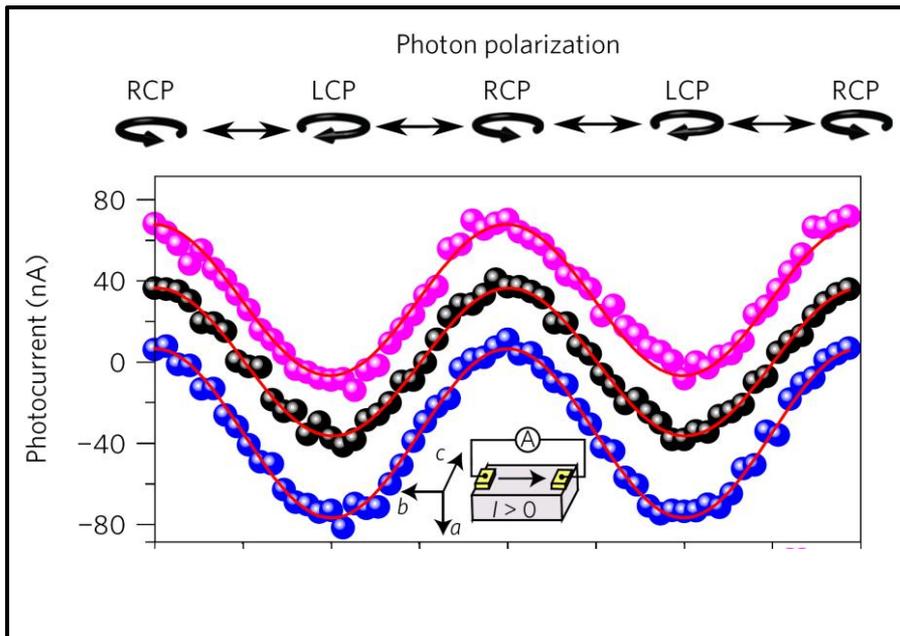
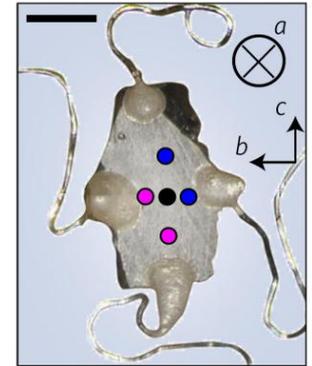


# Observation of circular photogalvanic effect in TaAs

Photocurrent response tensor respects crystal symmetry

$$J_{\alpha}(\omega = 0) = \eta_{\alpha\beta\gamma} E_{\beta}(\omega) E_{\gamma}^{*}(\omega)$$

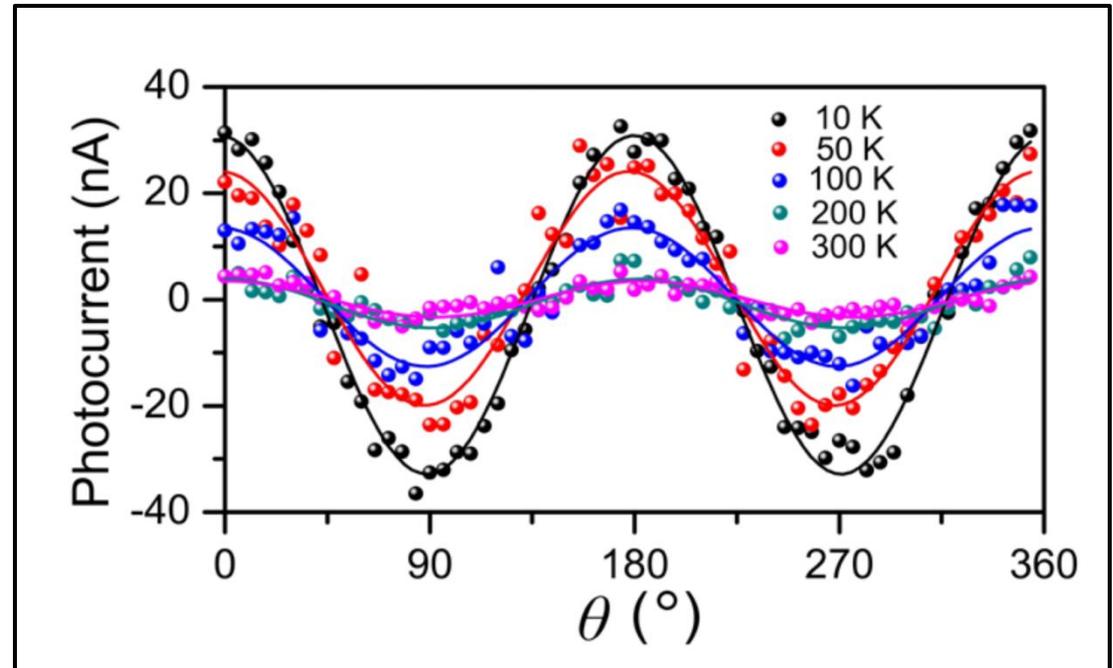
$$\eta_{aca} = \eta_{aac}^{*} = \eta_{bcb} = \eta_{bbc}^{*} \neq 0$$



# Observation of circular photogalvanic effect in TaAs

Photocurrent drops by increasing temperature due to

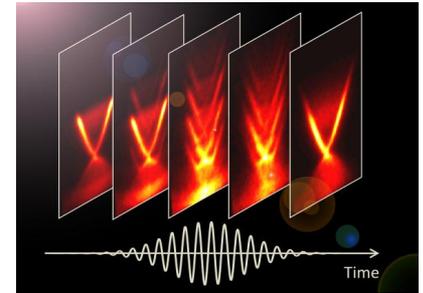
- reduced relaxation time
- population of excited states in Fermi-Dirac distribution



# Conclusion

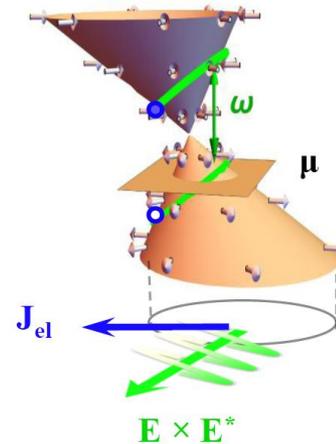
## - Tunable Floquet-Bloch bands

- ➔ Floquet band replica, open gaps by breaking TR, manipulate Weyl spectra



## - Circular photogalvanic effect

- ➔ Generic and large effect in noncentrosymmetric Weyl semimetals, promising candidate for room temperature infrared photodetector



## -Experimental relevance

- ➔ TRARPES, photoinduced AHE, Faraday effects, photovoltaic effects, etc..