The discovery of high-temperature superconductivity in the cuprates has stimulated intense study of the Hubbard and $t$-$J$ models on a square lattice. However, the accurate simulation of these models is one of the major challenges in computational physics. In this talk I report on recent progress in simulating the Hubbard model at a particularly challenging point in the phase diagram, $U/t = 8$, and doping $\delta = 1/8$, at which an extremely close competition between a uniform d-wave superconducting state and different types of stripe states is found [1]. Here I mostly focus on results obtained with infinite projected-entangled pair states (iPEPS) - a variational tensor network approach where the accuracy can be systematically controlled by the so-called bond-dimension $D$. Systematic extrapolations to the exact, infinite $D$ limit show that the fully-filled stripe ordered state is the lowest energy state. Consistent results are obtained with density matrix embedding theory, the density matrix renormalization group, and constrained-path auxiliary field quantum Monte Carlo [1], demonstrating the power of current state-of-the-art numerical methods to solve challenging open problems.