Obstructions for Twist Star Products

Thomas Weber

In this talk we will discuss obstructions for star products that can be induced by Drinfel'd twists on Poisson manifolds. Examples include the symplectic two sphere and the higher genus Riemann surfaces. One aim of Deformation Quantization is to replace the commutative pointwise product on the Poisson algebra of smooth functions by a star product, i.e. a noncommutative product on its formal power series. This can be interpreted as a quantization of the classical observables of a physical system. Thus we require the noncommutative product to coincide with the original product in the classical limit. Furthermore, the correspondence principle ensures that the first order of the commutator with respect to the new product coincides with the Poisson bracket of the system. This is motivated by the canonical commutation relations. In the 1980's Drinfel'd introduced a special kind of Deformation Quantization of a physical system via its symmetries: a Drinfel'd twist on a Hopf algebra does not only provide a noncocommutative comultiplication but also a Deformation Quantization of any module algebra of the Hopf algebra. Moreover, the star product can explicitly be written in terms of the twist. As a consequence, the Poisson bivector is the image of a solution of the classical Yang-Baxter equation under a Lie algebra action. However, it is the goal of the talk to show that this desirable situation can not be obtained in general. In fact, it is quite rare. We state conditions on Poisson manifolds to inhabit twist star products and give concrete obstructions, e.g. for the symplectic two sphere and the higher genus Riemann surfaces. These results were obtained in joint work with P. Bieliavsky, C. Esposito and S. Waldmann.