# Fourier integral operators on Lie groupoids

Jean-Marie Lescure

In this talk, I will present the content of two papers (L-Manchon-Vassout: Journal of NCG, and L-Vassout: arXiv:1601.00932) and also results of a work under progress with Stéphane Vassout.

After reviewing the definition and basic examples of Lie groupoids, I will explain the main properties of the convolution product of distributions on Lie groupoids. This includes a discussion on sufficient transversality conditions and a formula describing the wave front set of the product. This will be the opportunity to explain the basic role played by the *cotangent symplectic groupoid* $T^*G$ associated with any Lie groupoid $G$ (Coste-Dazord-Weinstein).

Secondly, I will describe a suitable subclass of Lagrangian distributions on $G$, called $G$-FIOs. Then I will explain how to develop a calculus for $G$-FIOs, showing that this essentially boils down to a calculus on the Lagrangian submanifolds in the cotangent groupoid. In the same spirit, $C^*$-continuity results will be stated for suitable $G$-FIOs.

This calculus recovers the Hörmander's calculus when $G$ is the pair groupoid of a $C^\infty$-manifold as well as the Melrose's calculus when $G$ is the $b$-groupoid associated with a manifold with boundary, and produces a framework for Fourier integral operators for any singular manifold having a suitable groupoid (like, for instance, stratified pseudomanifolds). It also complete the calculus on Lie groups initiated by Nielsen-Stetkaer.

In the remaining time, I will show that the one parameter group $e^{itP}$, $t \in \mathbb{R}$, where $P$ is an order 1 positive elliptic $G$-pseudodifferential operator, consists of $G$-FIOs, and finally explain how this result could be used in future works to study generalizations of spectral asymptotics.