Certain weak Hopf C*-algebras associated to modular categories of type A

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Drinfeld-Jimbo quantum groups $U_q(\mathfrak{g})$ at certain roots of unity $q = e^{i\pi/\ell d}$ are non-semisimple algebras giving rise to semisimple rigid braided tensor categories $\mathcal{F}(\mathfrak{g},\ell)$ via the so called quotient construction $\mathcal{F}(\mathfrak{g},\ell) =$ Tilting/Negligible (Andersen, Gelfand-Kazhdan). $\mathcal{F}(\mathfrak{g},\ell)$ is known to have deep connections with WZW models of CFT described as an equivalence of two braided tensor categories (Kohno, Drinfeld, Kazhdan-Lusztig, Finkelberg). Furthermore $\mathcal{F}(\mathfrak{g},l)$ has the structure of a tensor C^*-category with unitary braiding (Kirillov, Wenzl, Xu). Wenzl's proof can be regarded as the construction of a certain functor $W : \mathcal{F}(\mathfrak{g},\ell) \to$ Hilbert spaces. The aim of my talk is to introduce a notion of weak tensor structure on a functor between tensor C^*-categories and illustrate the construction of such a structure on Wenzl's functor in the case $\mathfrak{g} = \mathfrak{sl}_N$. Since, by Tannaka-Krein duality, the datum of a weak tensor structure on a functor $\mathcal{C} \to$ Hilb of a tensor category admits an algebraic formulation, we correspondingly introduce a notion of weak Hopf algebra. The coalgebra structure of a weak Hopf algebra is not necessarily strictly coassociative, and this is a special subclass of the weak quasi-Hopf algebras of Drinfeld, Mack-Schomerus which is closed under Drinfeld twist by the analogue of a 2-cocycle. Our result can equivalently be stated as the construction of a f.d. ribbon weak Hopf C^*-algebra $\mathcal{G}$ and an equivalence of braided tensor C^*-categories $E : \mathcal{F}(\mathfrak{sl}_N,\ell) \to \text{Rep}(\mathcal{G})$ such that the composite of E with the forgetful functor $\text{Rep}(\mathcal{G}) \to$ Hilb is isomorphic to Wenzl's functor. As an algebra $\mathcal{G}$ is a semisimple quotient of $U_q(\mathfrak{sl}_N)$ by an ideal $\mathcal{I}$ which parallels the negligible morphisms at the categorical level. (Joint work with S. Ciamprone)