TOPOLOGICAL DYNAMICS OF PIECEWISE λ-AFFINE MAPS OF THE INTERVAL

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Abstract. Let $0 < a < 1$, $0 \leq c < 1$ and $I = [0, 1)$. We call *contracted rotation* the interval map $\phi_{a,c} : x \in I \mapsto ax + c \mod 1$. Once a is fixed, we are interested in the dynamics of the one-parameter family $\phi_{a,c}$, where c runs on the interval interval $[0, 1)$. Any contracted rotation has a rotation number $\rho_{a,c}$ which describes the asymptotic behavior of $\phi_{a,c}$. In the first part of the talk, we analyze the numerical relation between the parameters a, c and $\rho_{a,c}$ and discuss some applications of this map. Next, we introduce a generalization of contracted rotations. Let $-1 < \lambda < 1$ and $f : [0, 1) \to \mathbb{R}$ be a piecewise λ-affine contraction, that is, there exist points $0 = c_0 < c_1 < \cdots < c_{n-1} < c_n = 1$ and real numbers $b_1, \ldots, b_n$ such that $f(x) = \lambda x + b_i$ for every $x \in [c_{i-1}, c_i)$. We prove that, for Lebesgue almost every $\delta \in \mathbb{R}$, the map $f_\delta = f + \delta \pmod 1$ is asymptotically periodic. More precisely, f_δ has at most $n + 1$ periodic orbits and the ω-limit set of every $x \in [0, 1)$ is a periodic orbit.