Quantum simulation and spectroscopy of entanglement Hamiltonians

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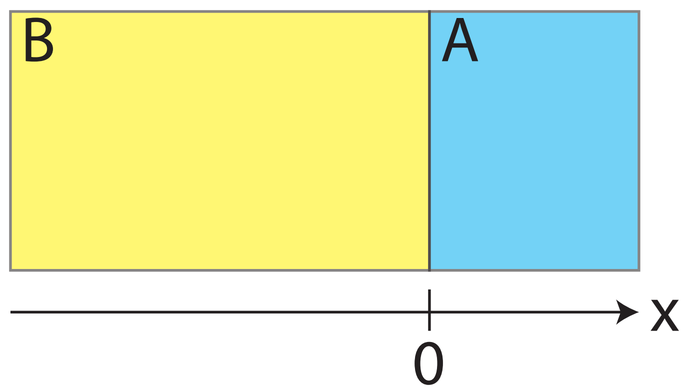
arxiv.1707.04455

Main question

Challenge: develop a protocol to measure ***entanglement spectra*** in atomic physics experiments

$$H = H_A + H_B + H_{AB}$$

$\{\lambda_\alpha\}$



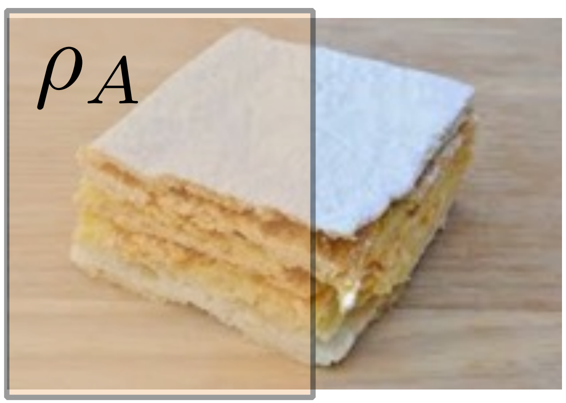
$$\rho_A = e^{-\tilde{H}} = \sum_\alpha \lambda_\alpha |\varphi_\alpha\rangle\langle\varphi_\alpha|$$

modular (or entanglement) Hamiltonian

Main result

Shift the paradigm: **not probing the density matrix** but directly the **modular (entanglement) Hamiltonian**

Instead of building a cake (ρ_A) and try to extract ingredients (λ_α), just look at the shopping bag ($\tilde{H}$)



realize a cake and then look inside

λ_α

$\tilde{H}$

realize the shopbag - much easier to inspect

Main result

Shift the paradigm: **not probing the density matrix** but directly the **modular (entanglement) Hamiltonian**

Applicable to

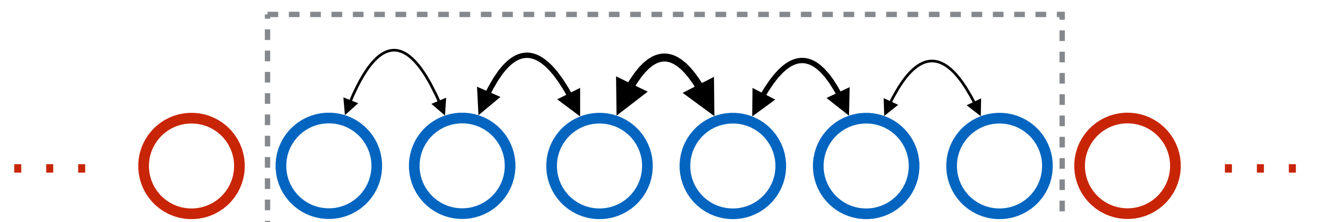
- most field theories, including *topological phases, CFTs, symmetry-broken, gauge theories*, …
- 1D, 2D, 3D equally difficult
- lattice and continuum
- no copies, no *in situ* needed
- all universal information

Concrete implementation schemes only require **light-induced interactions**:

- Rydberg-dressing
- light-assisted tunneling
- ….

$$J \propto \Omega_i \Omega_j / \omega$$

$$\Omega = \frac{d^2 - R^2}{R}$$



Outline

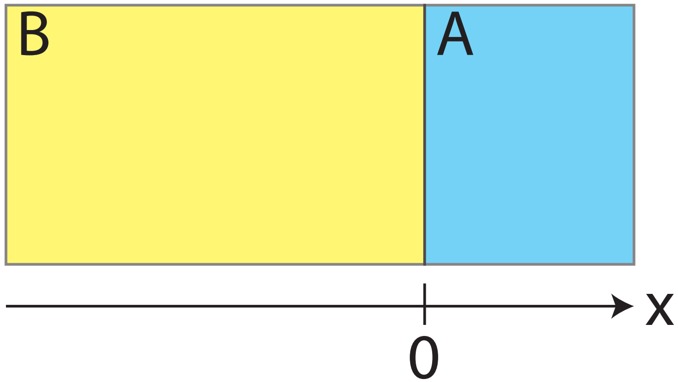
Entanglement spectrum:
- what it is, why it is interesting

Entanglement Hamiltonian:
- naïve reasoning
- exploiting *axiomatic field theory* / **Bisognano-Wichmann theorem**(s)

Quantum engineering of Entanglement Hamiltonians
- quantum field theory and lattice systems- some examples: Haldane chain, CFTs, free theories, 2D Topological insulators
- implementations

Another view at the entanglement spectrum

$$H = H_A + H_B + H_{AB}$$



$$\rho_A = e^{-\tilde{H}} = \sum_\alpha \lambda_\alpha |\varphi_\alpha\rangle\langle\varphi_\alpha|$$

What is this useful for?

1) *you get most of entanglement measures*
2) *paramount importance for topological phases*
3) *contains much more information than entropies*
4) *it is crazy hard to get via numerical experiments*

Why Entanglement spectra?

Obvious reason: you get a lot of entanglement measures:

Example: entanglement entropies

$$\{\lambda_\alpha\} \qquad\qquad S = -\sum_\alpha \lambda_\alpha \log[\lambda_\alpha]$$

that are good for:

Diagnosing topological order

Classify quantum field theories

measure entanglement

Why Entanglement spectra?

topological phases: the entanglement spectrum reveals edge and excitations properties just from the wave-functions!

Li and Haldane, PRL 2008.

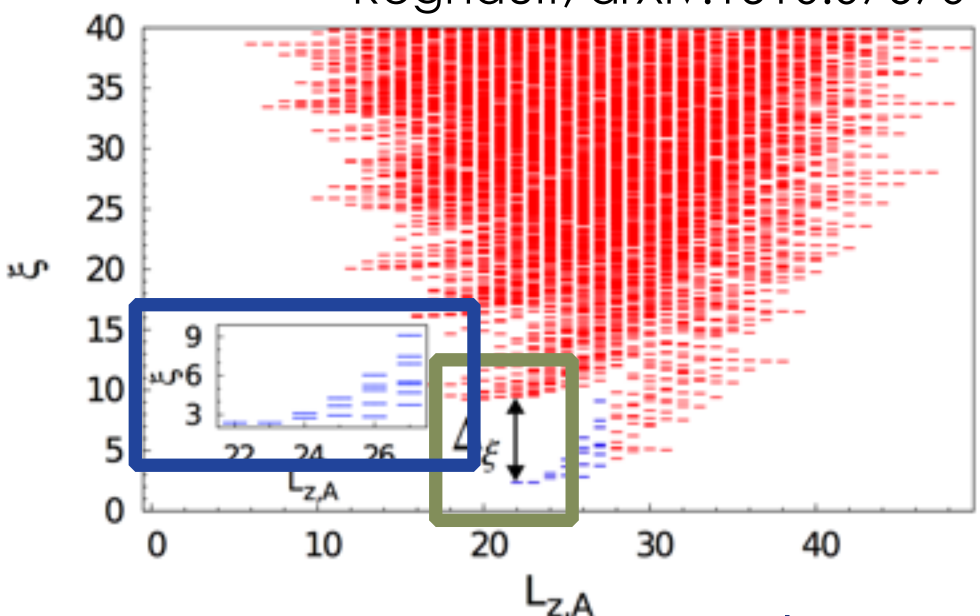
Example: Coulomb gas, sphere

$$n_f = 12, N_\Phi = 33$$

1)**Finite entanglement gap**

2)**edge state counting**

Regnault, arXiv.1510.07670



angular momentum

Why Entanglement spectra?

very hard to get via numerics / much, much harder than entropies

No universal method to calculate it.

H. A. Carteret, PRL **94**, 040502 (2005), H. Song, et al. PRB **85**, 035409 (2012),
C.-M. Chung, et al. PRB **89**, 195147 (2014) - illustrates challenges with MC methods

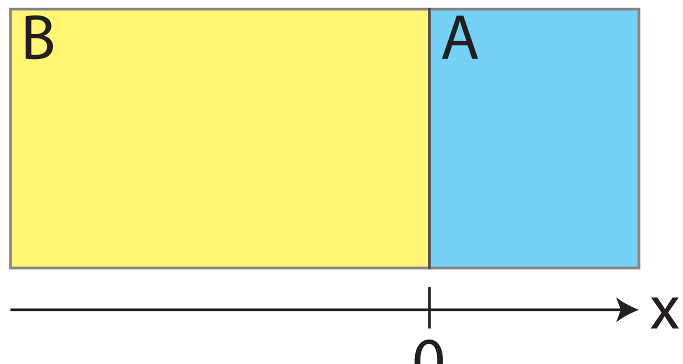
Instead, entropies can be calculated (conventional replica trick, nowadays routinely implemented)

Melko, Roscilde, Isakov, ….

Accessible only with full knowledge of the wave function - via ED, DMRG, ..

Entanglement spectrum

$$H = H_A + H_B + H_{AB}$$



$$\rho_A = e^{-\tilde{H}} = \sum_\alpha \lambda_\alpha |\varphi_\alpha\rangle\langle\varphi_\alpha|$$

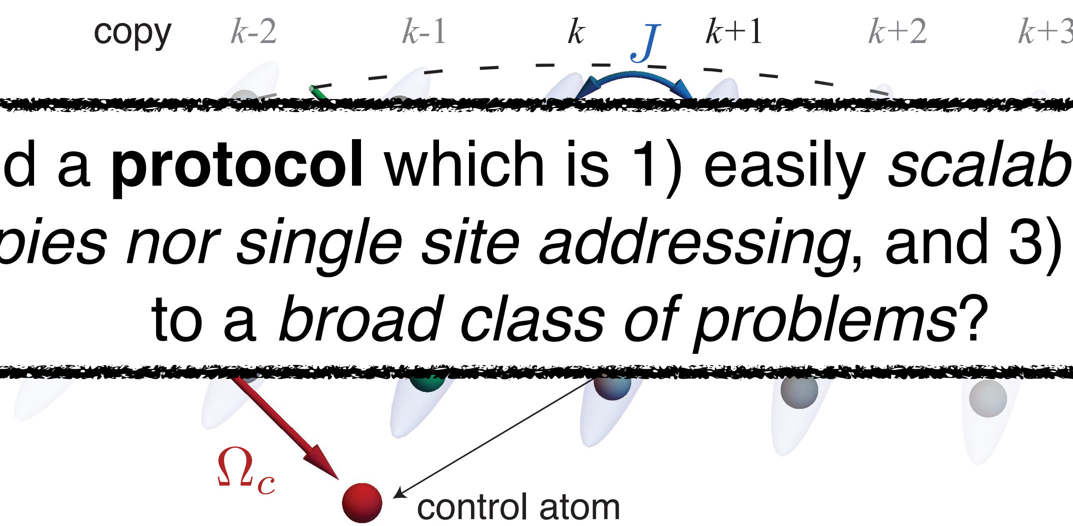
What is this useful for?

Paradigmatic quantity in many-body theory

Is this measurable at all?

How to measure it? Real experiments

General protocols exist - see Pichler et al., PRX 2016



Can we find a **protocol** which is 1) easily *scalable,* 2)*does no require copies nor single site addressing*, and 3) is applicable to a *broad class of problems*?

However, due to generality, very resource expensive - many-copies needed, Rydberg gates, accurate spectroscopy, hard to scale up, only on lattice (?).

e.g., to resolve the ES degeneracy of the Haldane chain, some 150 copies are required.

Shifting the paradigm: from density matrices to modular Hamiltonian

Our strategy here: focus directly on **entanglement Hamiltonians**!

1) immediate **experimental protocols** to measure entanglement spectra

2) novel theoretical route which might be more amenable to numerics, and also useful for analytics / **entanglement field theories**

Key element from axiomatic field theory

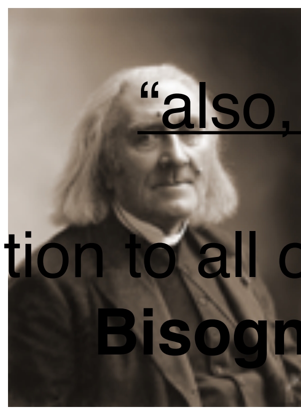
<u>**Problem**</u>: entanglement Hamiltonians? $\rho = e^{-\tilde{H}}$

<u>“they might be **highly non-local**”</u>

<u>“in principle, **many-body interactions**”</u>

<u>“also, how can you get them?”</u>

H



$\tilde{H}$

A solution to all of these problems is provided by the **Bisognano-Wichmann theorem**

<u>The funkiest Hamiltonian - this is scary</u>

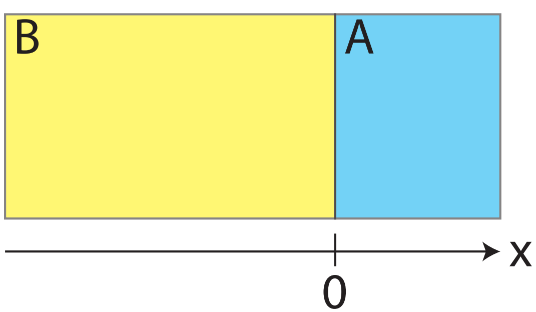
The Bisognano-Wichmann theorem

Well-established result in axiomatic field theory - series of papers in 1975/76.

For our purposes:

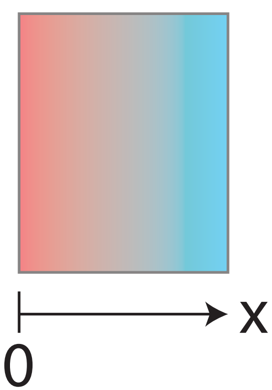
$$H(x)$$

Hamiltonian density, <u>must</u> be Lorentz invariant



Given a bipartition A, the entanglement (modular) Hamiltonians is:

$$\tilde{H}_A = 2\pi \int_{x\in A} dx(xH(x)) + c'$$



Local, few-body Hamiltonian with spatially dependent couplings

$$\{\lambda_\alpha\} = \{\exp[-\epsilon_\alpha]\}$$

Bisognano and Wichmann, J. Math. Phys. 17, 303 (1976); review: Guido, Cont. Math 534, 97 (2011)

Experimental strategy

1) find the entanglement Hamiltonian

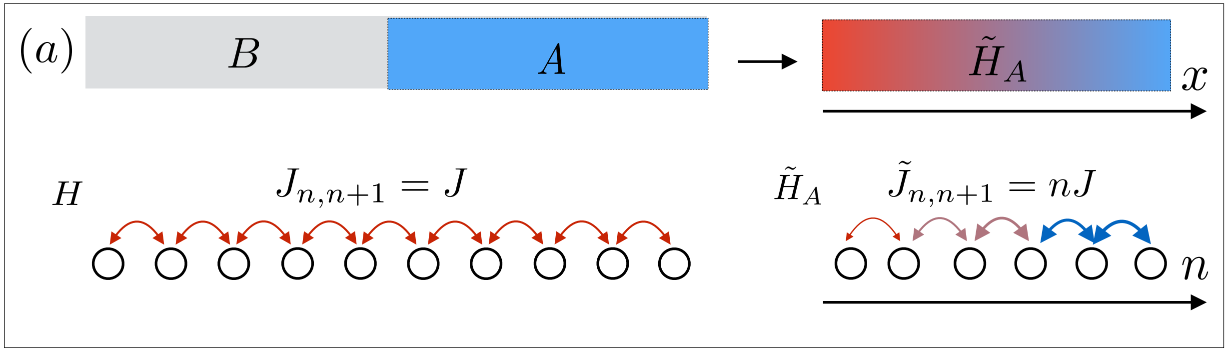
2) devise a protocol to realize it

3) use spectroscopy, and get the entanglement spectrum

$$\{\lambda_\alpha\} = \{\exp[-\epsilon_\alpha]\}$$

<u>Real issue</u> - does BW theorem really hold for lattice model, finite size, etc…?

BW: Does it work?



Numerical results

Ising Hamiltonians (including ‘long-ranged’)

Haldane chain

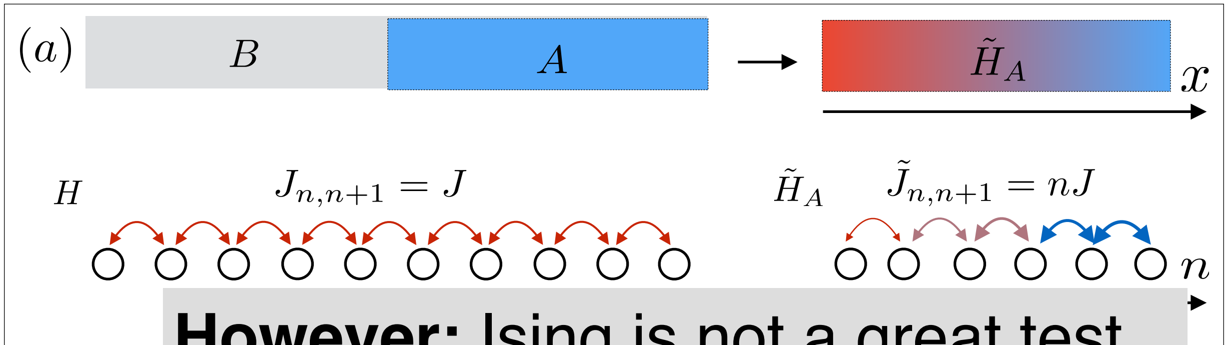
Conformal field theories on lattices (free fermions, XXZ chain)

Two-dimensions: free theories, topological insulators

Analytical intuition

Fractional Quantum Hall and Chern-Simons theories

Ising check



<u>However:</u> Ising is not a great test, even mean field works!

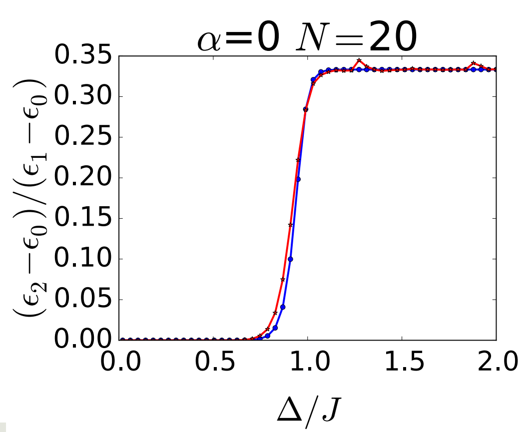
?

Entanglement spectrum of the GS of

$$H = \sum_n [\sigma_n^z + \lambda \sigma_n^x \sigma_{n+1}^x]$$

Physical spectrum of

$$\mathcal{H}_\alpha = -C \left[\sum_{n\geq1} (2n-1)\sigma_n^z + \lambda \sum_{n\geq1} 2n \sigma_n^x \sigma_{n+1}^x \right]$$



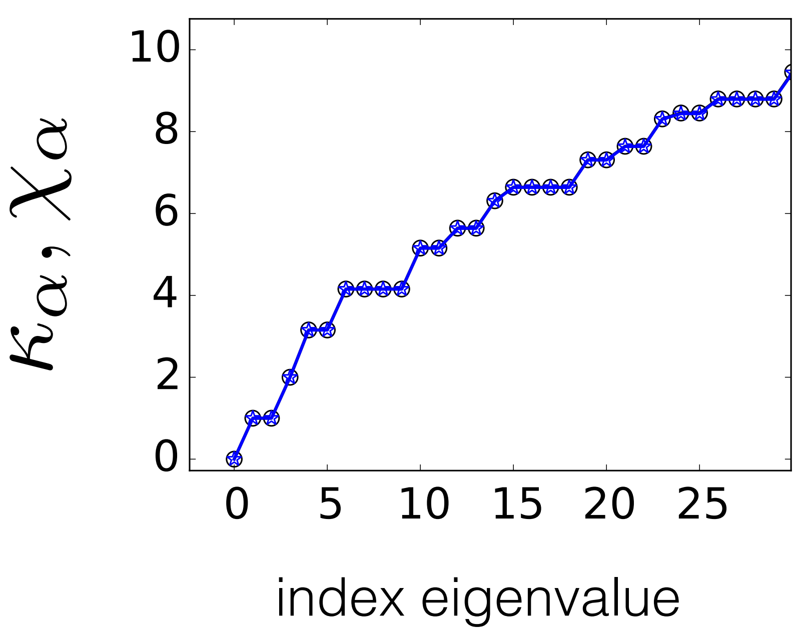
Exact match even at very small sizes!

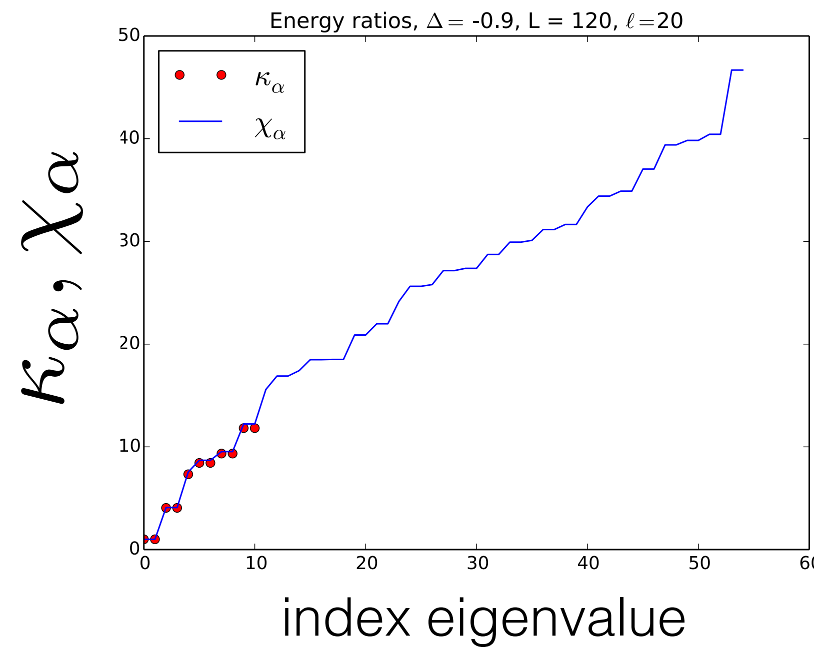
can be proved analytically: Peschel and Eisler, arxiv. 0905.1663 [sublime review]

Luttinger liquids

$$H = J \sum_i [(S_i^x S_{i+1}^x + S_i^y S_{i+1}^y) + \Delta S_i^z S_{i+1}^z]$$

Free fermions, L= 32





<u>However:</u> maybe CFTs are a bit too simple…

$$\frac{\log[\lambda_\alpha/\lambda_0]}{\log[\lambda_1/\lambda_0]} = \kappa_{\alpha 1} \qquad \chi_{\alpha 1} = \frac{\epsilon_\alpha - \epsilon_0}{\epsilon_1 - \epsilon_0}$$

Haldane chain (Delta = 0.6)

Question: can we resolve **topological degeneracies?**

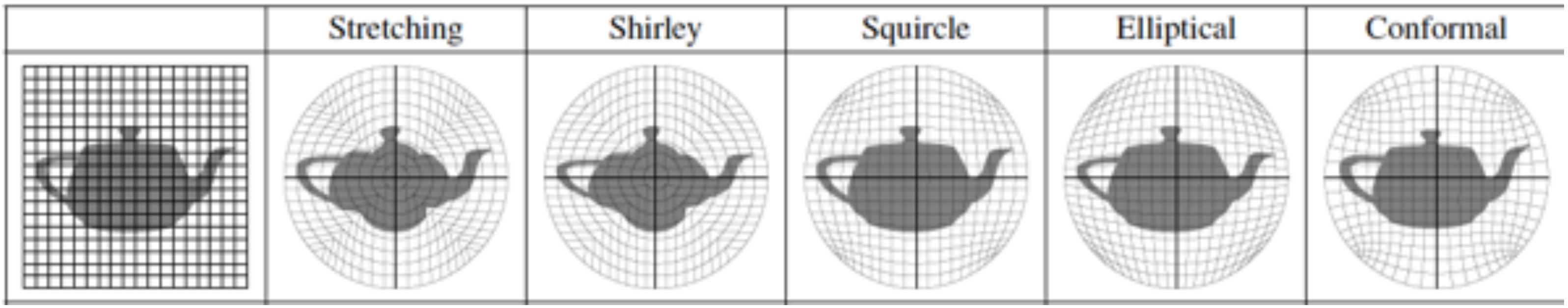
α	OBC $\chi_{\alpha 1} = \frac{\epsilon_\alpha - \epsilon_0}{\epsilon_1 - \epsilon_0}$	$\frac{\log[\lambda_\alpha/\lambda_0]}{\log[\lambda_1/\lambda_0]} = \kappa_{\alpha 1}$	PBC $\chi_{\alpha 1} = \frac{\epsilon_\alpha - \epsilon_0}{\epsilon_1 - \epsilon_0}$
0	0.00759986651826	0.000591291392465	0.00320061840297
1	0.0077067246213	0.000591291392465	0.00320061840297
2	0.00781075678774	0.000591393359787	0.00320113816346
3	1.0	1.0	1.0
4	1.00005039179	1.00000139174	1.0
5	1.00010232951	1.00000192193	1.00000111145
6	1.0001556862	1.00000271721	1.00000111145

All degeneracies are resolved with 10^-3 accuracy.

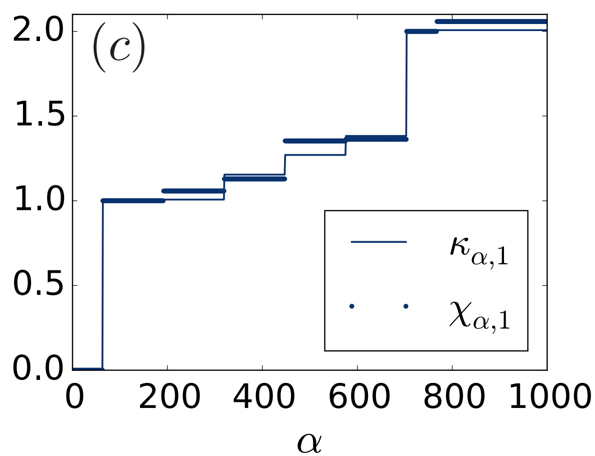
DMRG up to L=108 sites (PBC); multitargeting up to 170 excited states (10 per sector). Accuracy around 10^-6

2D: Free fermions

In 2D, we use the conformal mapping to get the distance function - it preserves angles

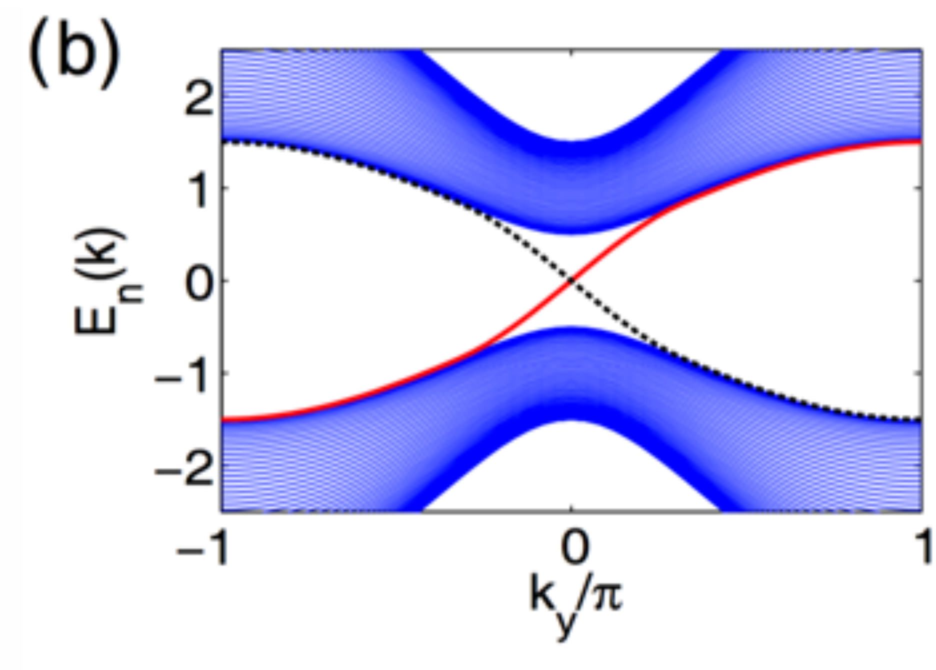


Good agreement up to ~1000 eigenvalues

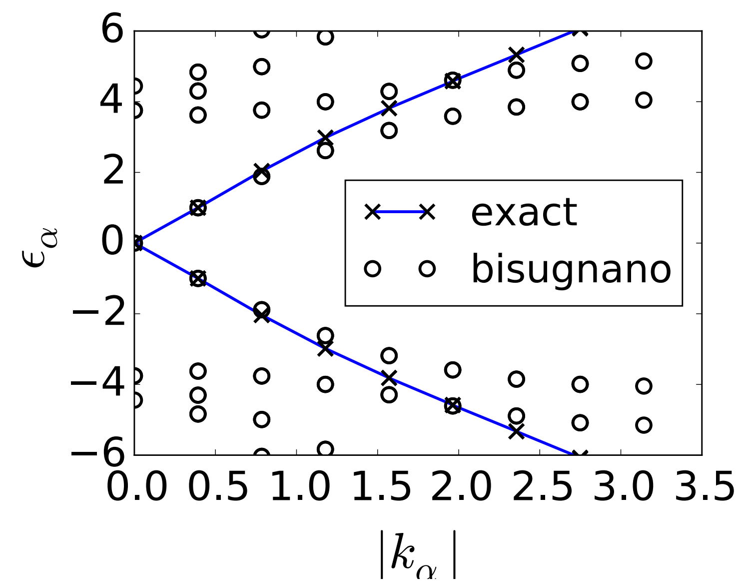


2D Dirac model

$$H_A = \sum_{\mathbf{i}} \mathbf{c}_{\mathbf{i}}^\dagger \frac{\sigma_z - i\sigma_x}{2} \mathbf{c}_{\mathbf{i}+\mathbf{x}} + \mathbf{c}_{\mathbf{i}}^\dagger \frac{\sigma_z - i\sigma_y}{2} \mathbf{c}_{\mathbf{i}+\mathbf{y}} + \text{h.c.} + m \sum_{\mathbf{i}} \mathbf{c}_{\mathbf{i}}^\dagger \sigma_z \mathbf{c}_{\mathbf{i}},$$

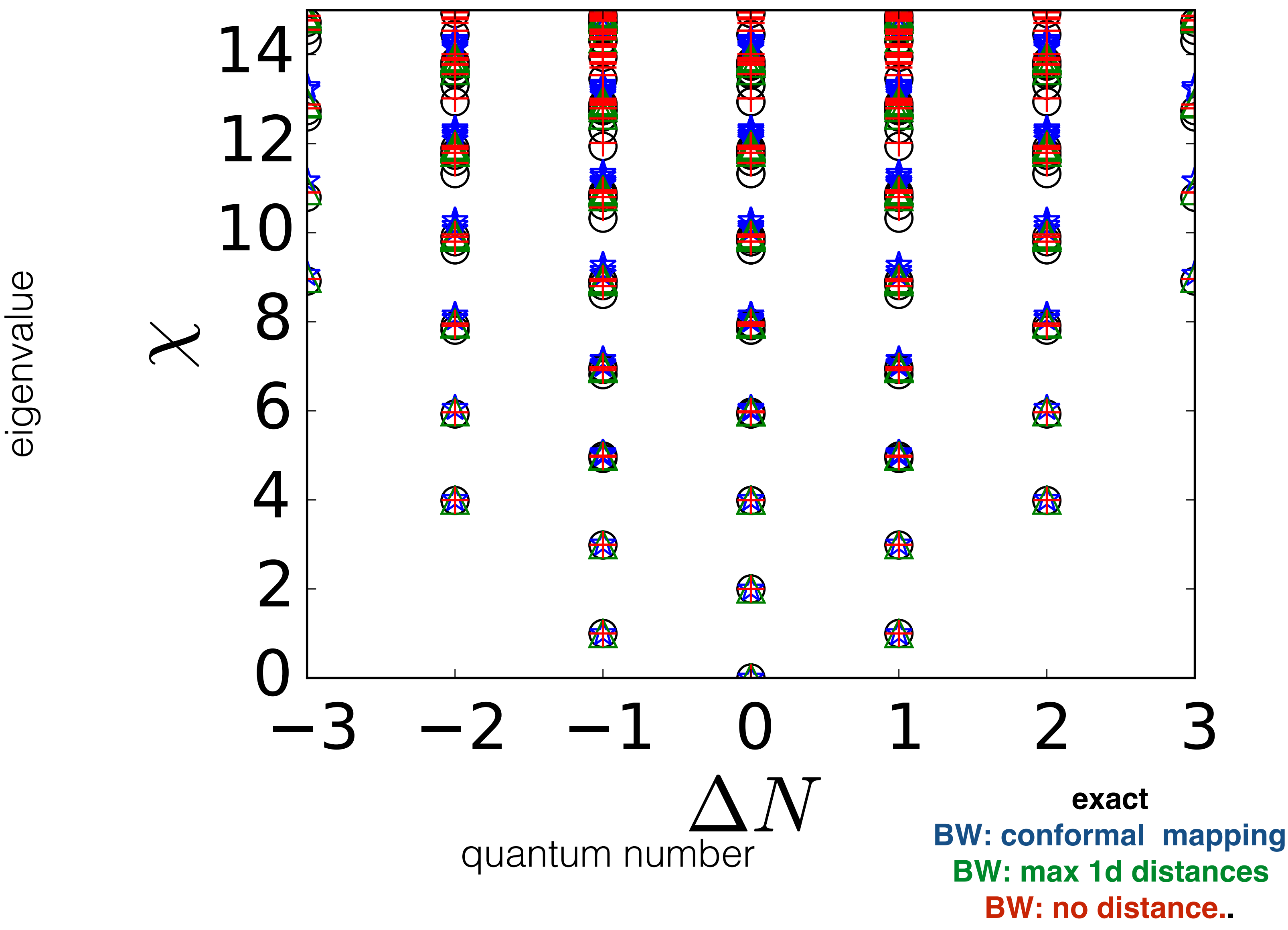


Qi et al., PRB 2008



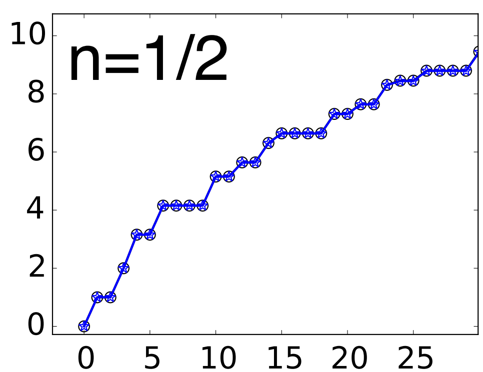
‘Single particle’ entanglement spectrum

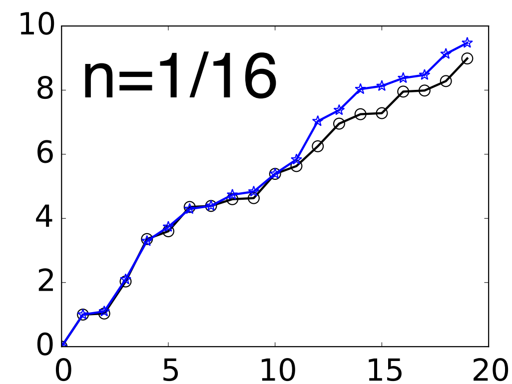
Massive dirac model (m=-1), subsystem 10x10



Beware of limitations

NB: we know that BW **<u>will fail</u>** for certain models, e.g., ferromagnets, and free fermions at very low filling:





NB: finite size effects are not easily predictable, but in all the cases of interest, they seem well under control. **Scaling entanglement theory** will soon be needed

Experimental strategy

1) find the entanglement Hamiltonian

<u>Bottom line is</u>: using the BW theorem, it is possible to access the entanglement Hamiltonian of a very broad class of physical phenomena

2) devise a protocol to realize it

3) use spectroscopy, and get the entanglement spectrum

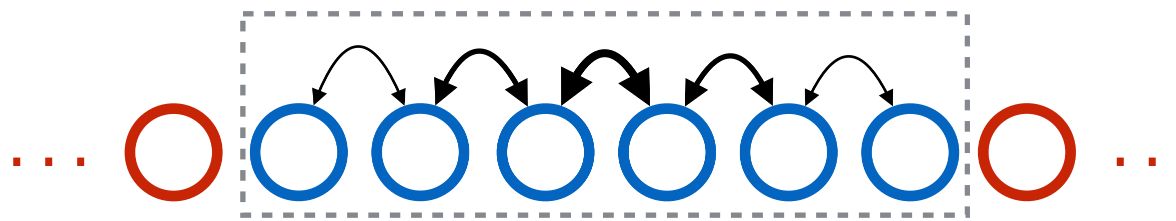
How to realize entanglement Hamiltonians?

Every system where interaction is light-induced is good (atoms, superconducting circuits, ions, …)

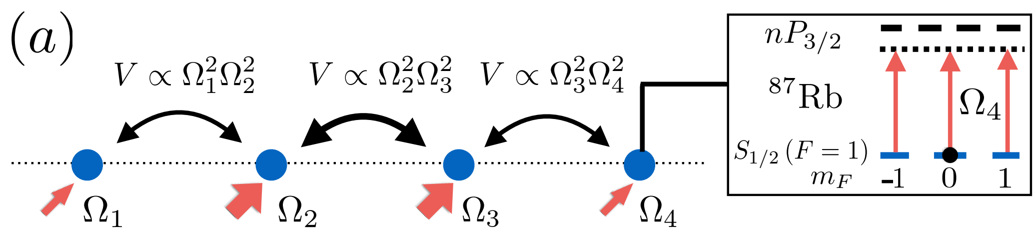
Example: **Rydberg-dressed atoms**

$$J \propto \Omega_i \Omega_j / \omega$$

$$\Omega = \frac{d^2 - R^2}{R}$$



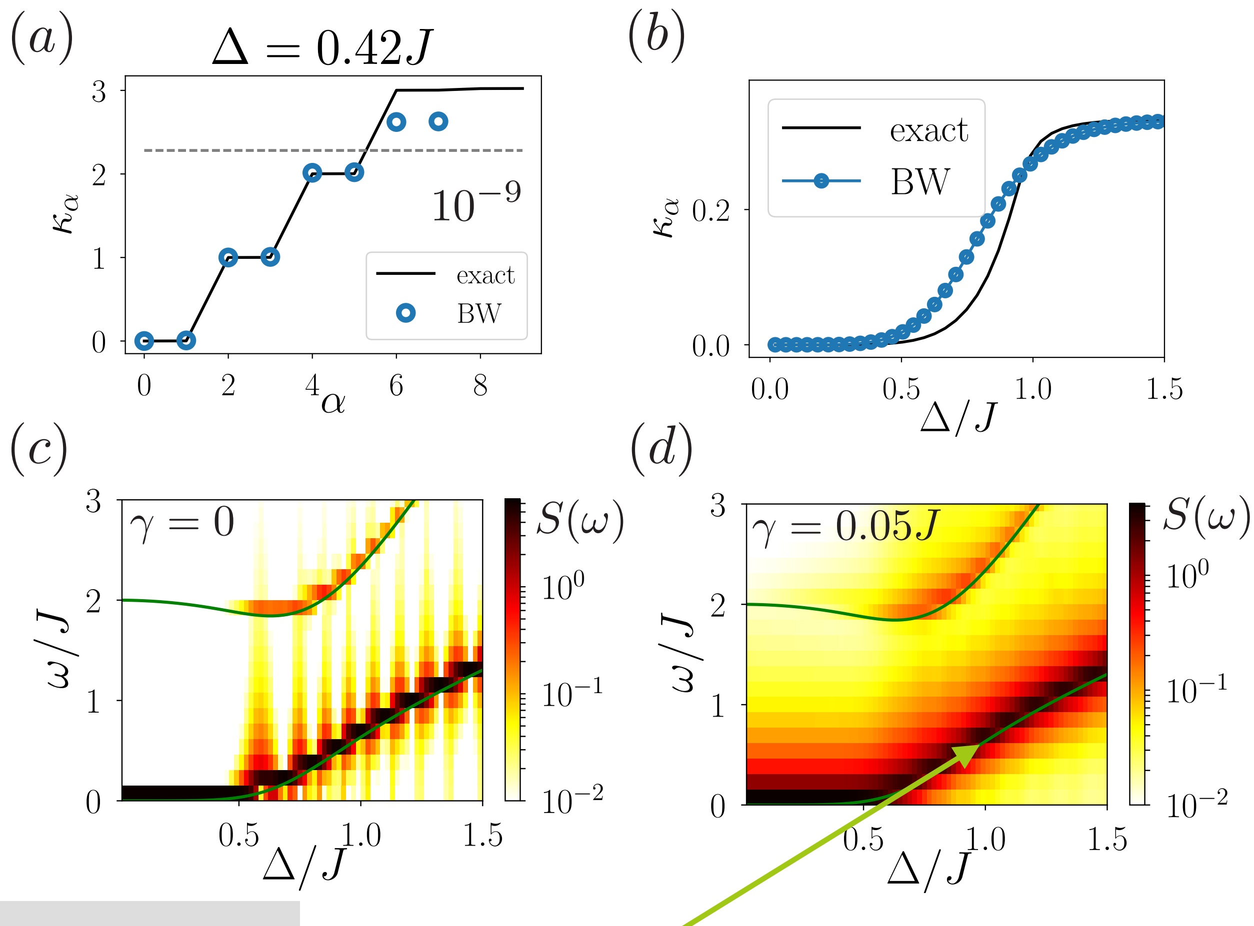
(a)



A. W. Glätzle et al., PRL 2015; Van Bijnen and Pohl, PRL 2015; Zeiher et al., NatPhys. 2016; Jau et al., NatPhys. 2016

How to extract the gaps? Spectroscopy

$L = 6$



Full spectroscopic simulations, including noise in state preparation and during measurement

Green line: exact result

Scheme **resilient to imperfections** (no surprise)

Conclusions

Entanglement Hamiltonians are local, few-body, and can be written in a closed form for a broad class of models

[see recent PEPS works by Schuch et al., PRL2013, PRB 2015] for an interesting relation between BW and Wegner gauge theory

Use **synthetic quantum systems** for the **direct realization of entanglement Hamiltonians**!

One just requires: locally tailored interactions + spectroscopy.

Very robust to imperfections, including finite-size, etc…

Adaptable to many platforms - Rydbergs, ions, more?

and outlook

Entanglement field theories

Useful also for diagnosing topological order in 1D (no true topology)? Quantum Frustration [Illuminati et al., PRL2012, PRL2013] and BW

Entanglement Hamiltonians for real time dynamics

2D interacting systems / connections to lattice gauge theories (see Schuch's talk)

Beyond bipartite entanglement?

Entanglement field theories offer a brand new look to understand (bipartite) entanglement in many-body systems using standard statistical mechanics tools

IQOQI / ITP Univ Innsbruck

Benoit

Peter

arxiv.1707.04455

Thank you