Energy as a detector of nonlocality of many-body spin systems

Jordi Tura

Max Planck Institute of Quantum Optics, Germany

Workshop on Quantum Science and Quantum Technologies | (smr 3183)
12th – September - 2017

MAX PLANCK INSTITUTE
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ICFO

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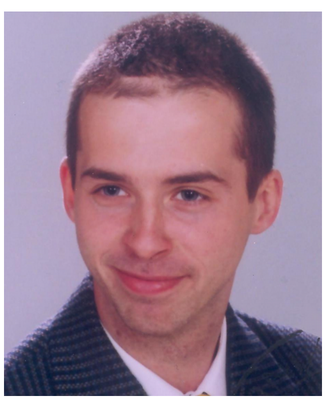
joint work with

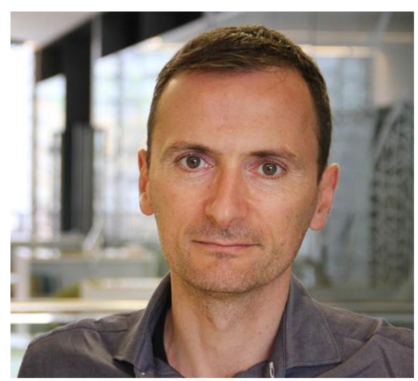
Gemma de las Cuevas

Maciej Lewenstein

J. Ignacio Cirac



Remigiusz Augusiak



Antonio Acín

The paper is available on
Phys. Rev. X **7**, 021005 (2017)

Outline

Outline

- Motivation

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- The idea, the setting

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- Conclusions and outlook

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The Device-Independent Approach

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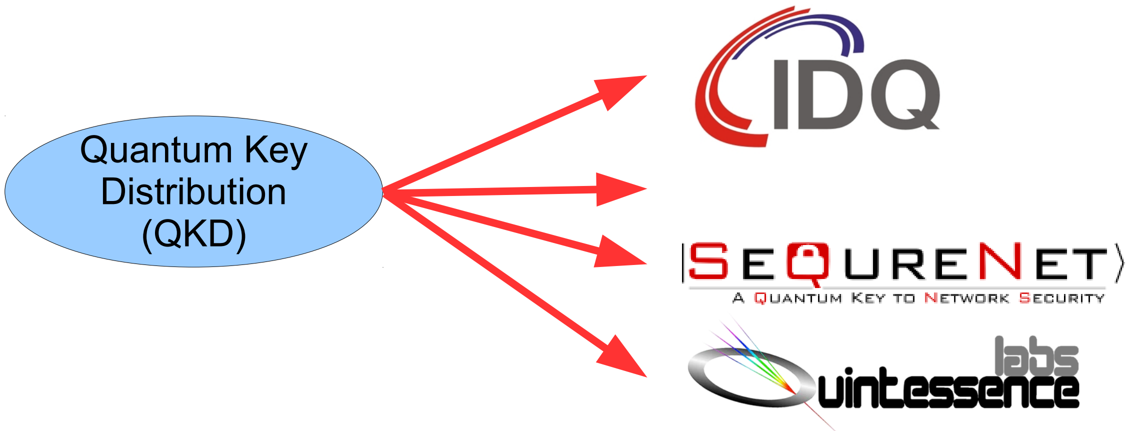
Quantum Key Distribution (QKD)

The Device-Independent Approach



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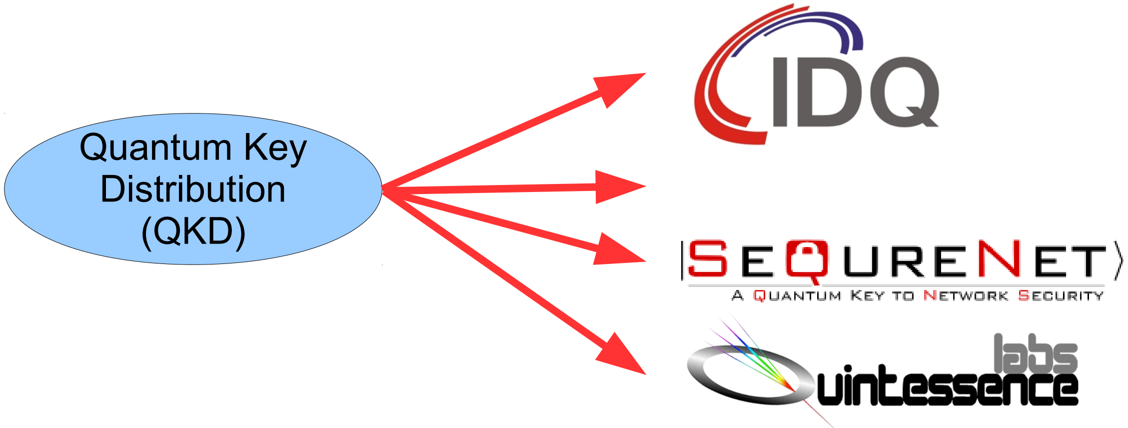

Current record: 1200 km! [J.Yin *et al. Science* **356** 1140 (2017)]

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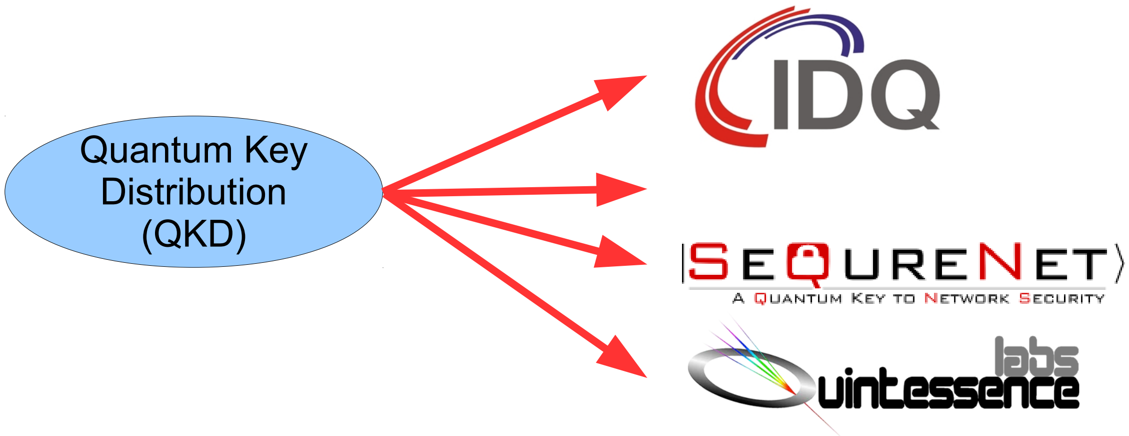

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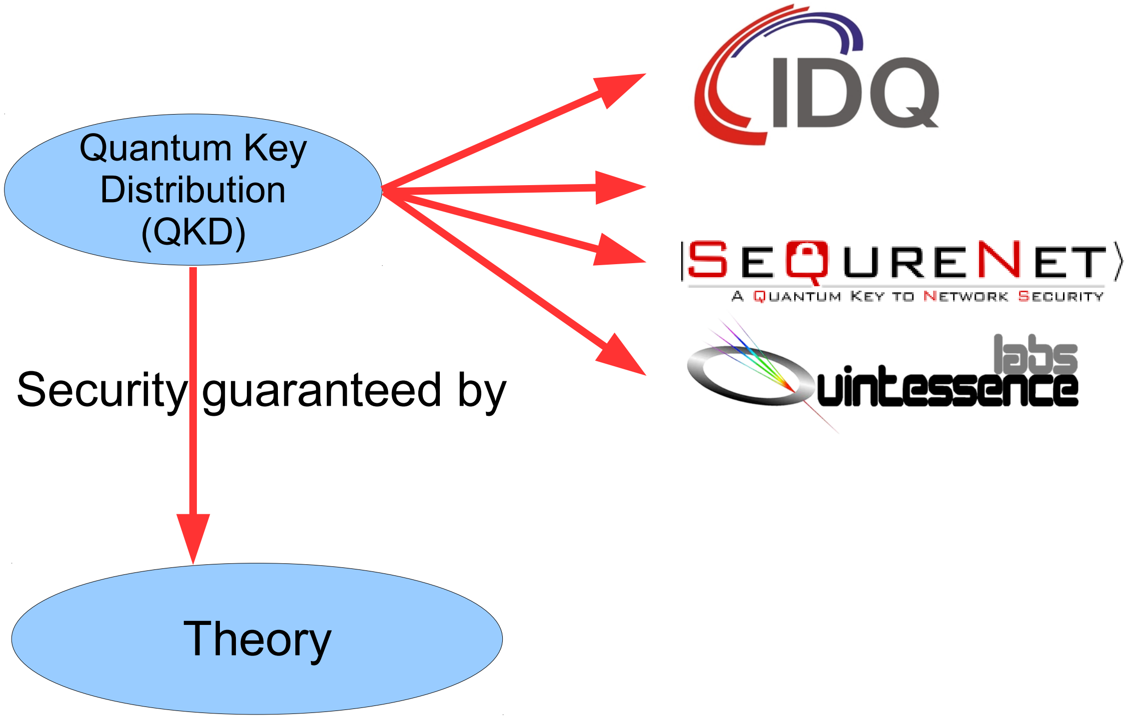

$$\mathcal{H} = \mathbb{C}^2$$

The Device-Independent Approach

Quantum Key Distribution (QKD)

IDQ

SeQureNet — A Quantum Key to Network Security

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Theory

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Implementation

The Device-Independent Approach

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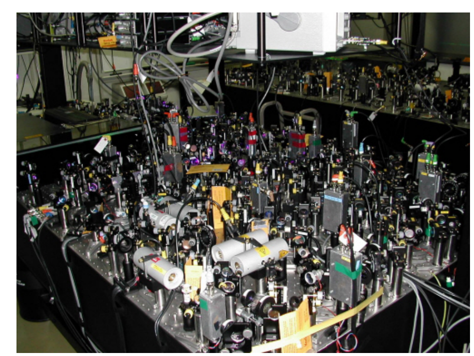

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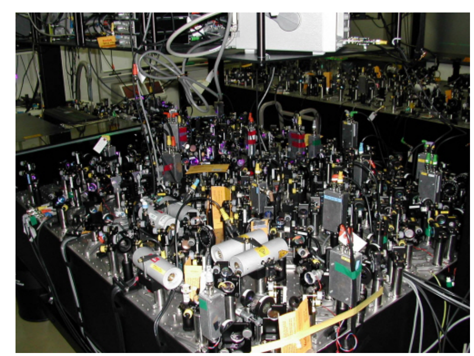

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[V. Makarov, *Quantum Hacker*]

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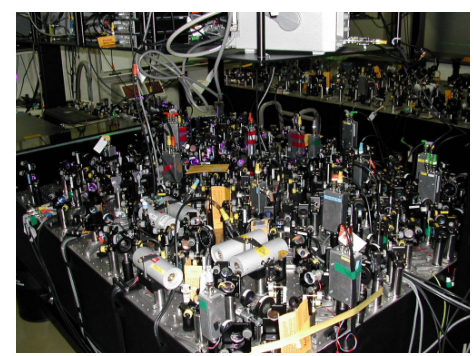

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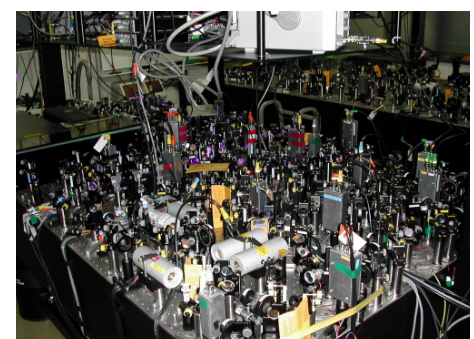

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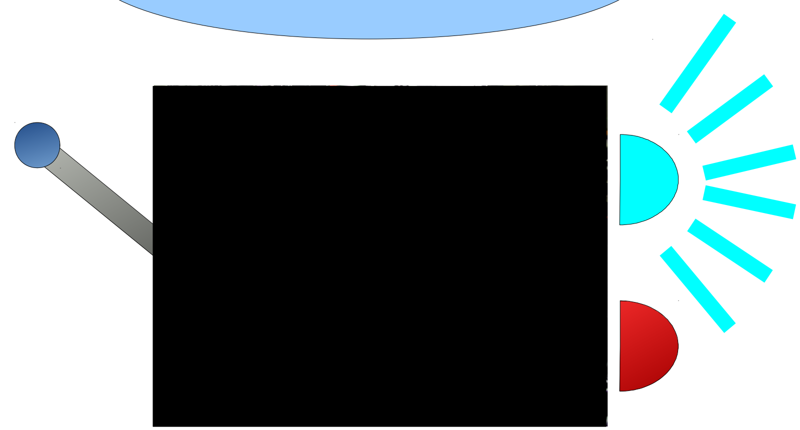

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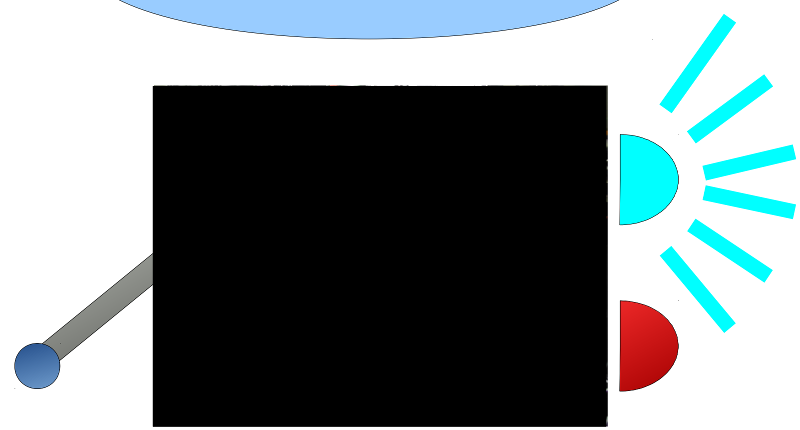

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#inputs, #outputs
Are the observed statistics *nonlocal*?

Why Bell correlations?

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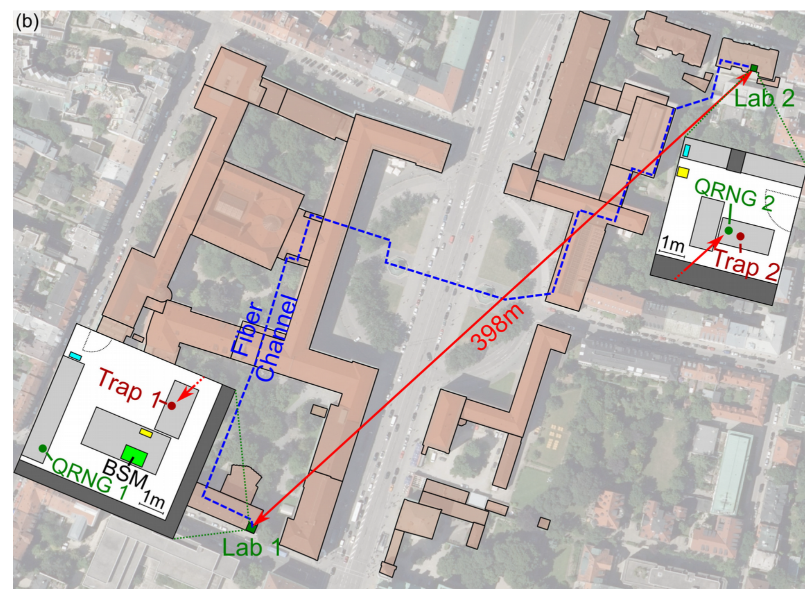

- Also at MPQ

[W. Rosenfeld, D.Burchardt, R. Garthoff, K. Redeker, N.Ortegel, M. Rau, H. Weinfurter **arXiv:1611.04604** [quant-ph] (2016)]

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- Do Bell correlations appear naturally in low-energy states of physical systems?

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[Tura et al, Science **344** 1256 (2014), Tura et al, Ann. Phys. **362**, 370-423 (2015)]
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- This talk: spin systems in one spatial dimension

A crash course on nonlocality

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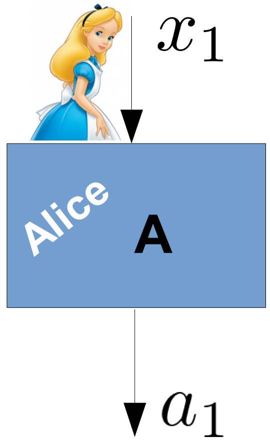

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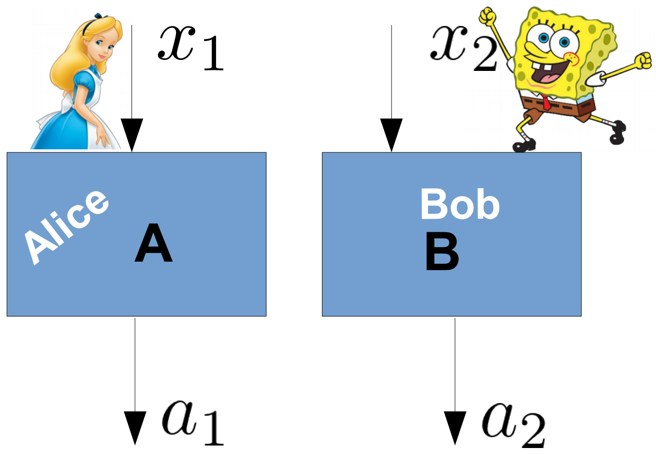

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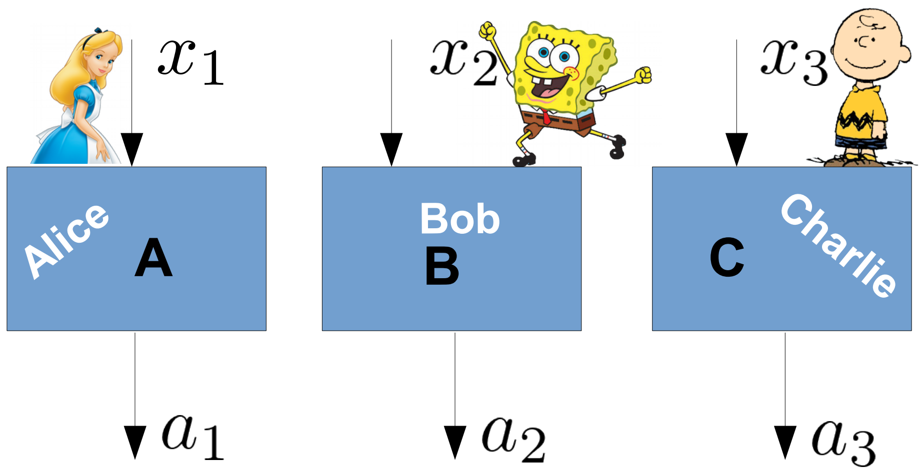

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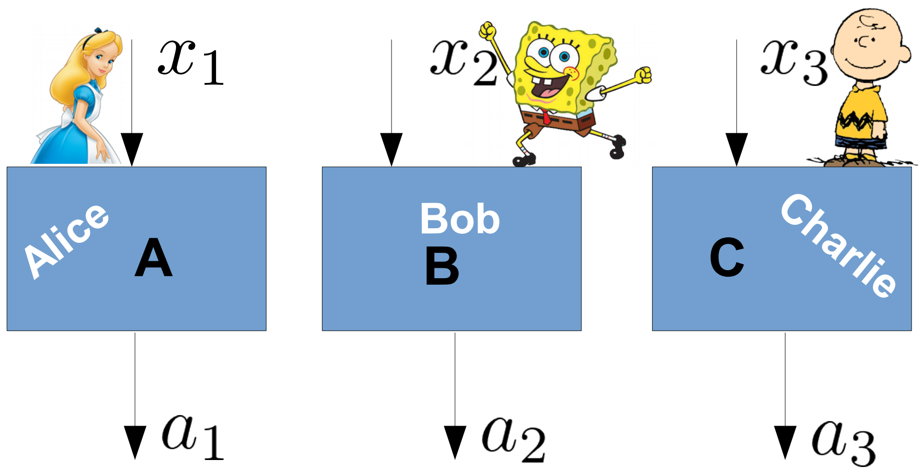

$$P(a_1 \dots a_n | x_1 \dots x_n)$$

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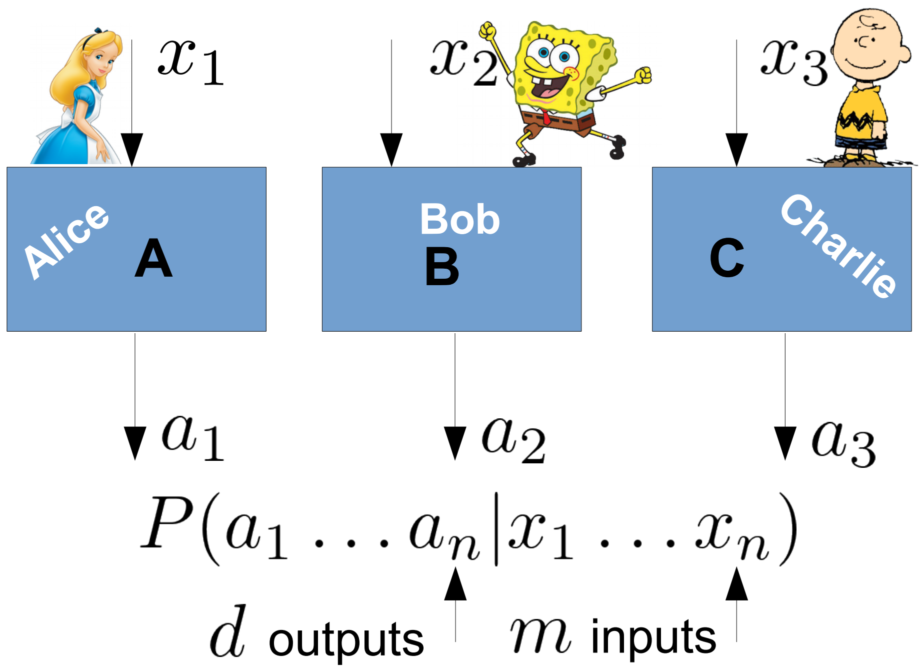

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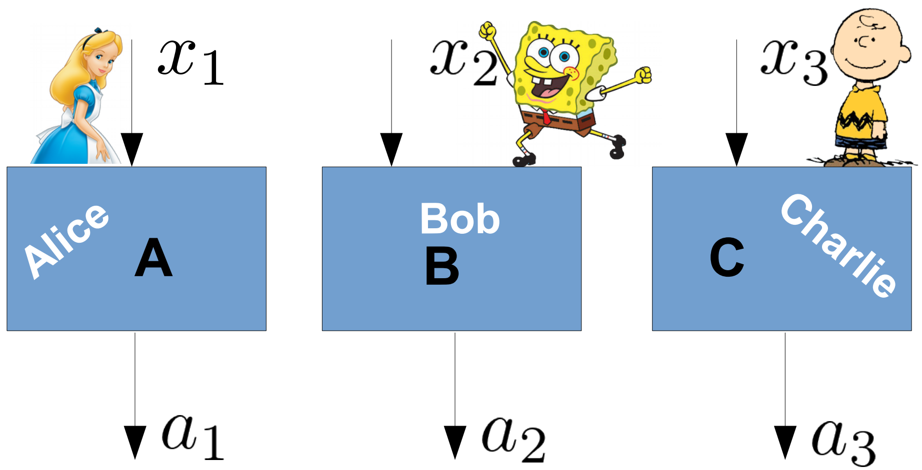

$$P(a_1 \ldots a_n | x_1 \ldots x_n)$$

d outputs $\quad$ m inputs

$$\vec{v} = \{P(\vec{a}|\vec{x}) \quad \forall \vec{a}, \vec{x}\}$$

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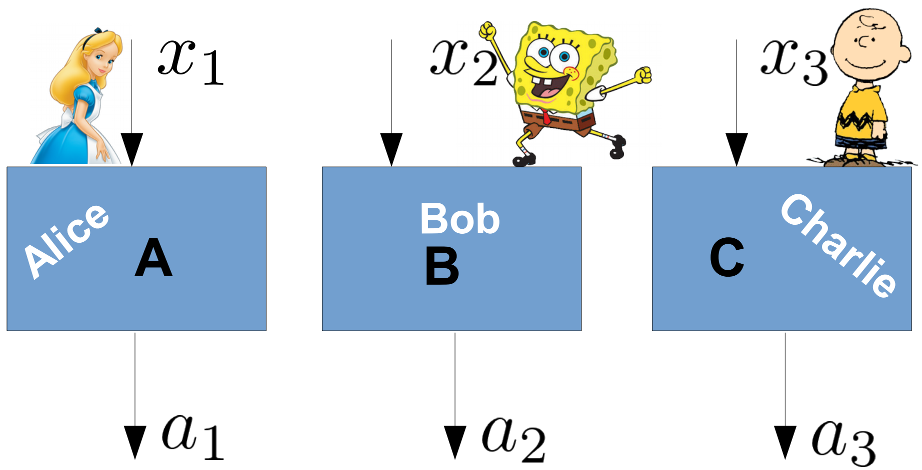

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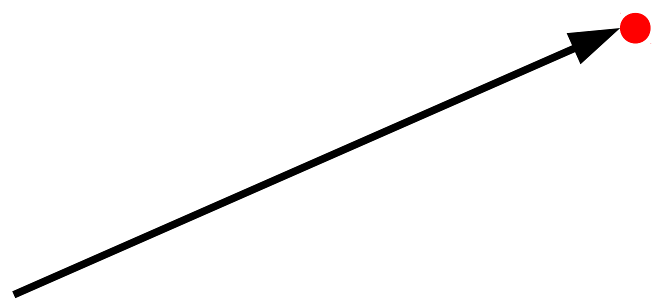

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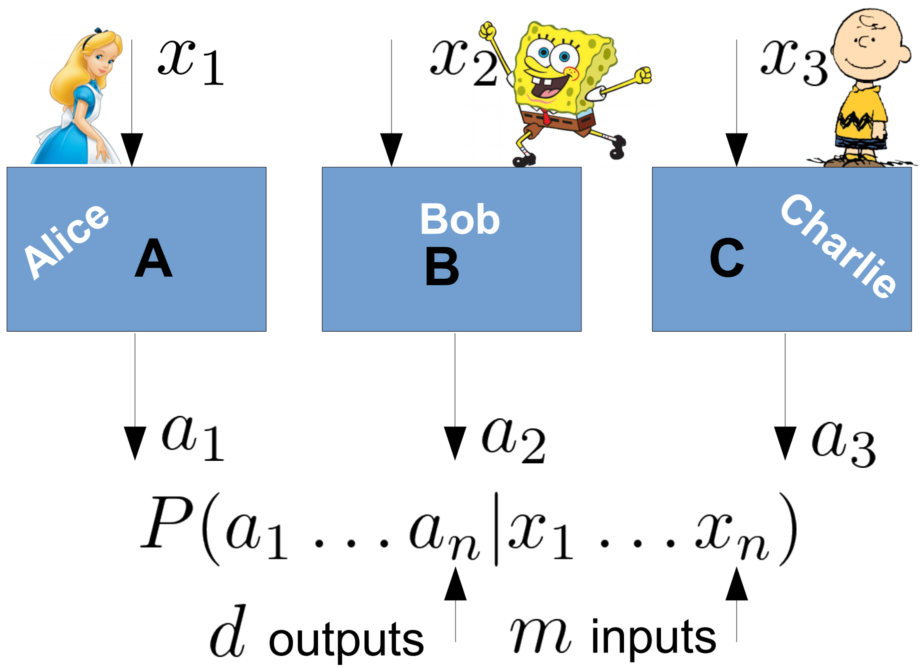

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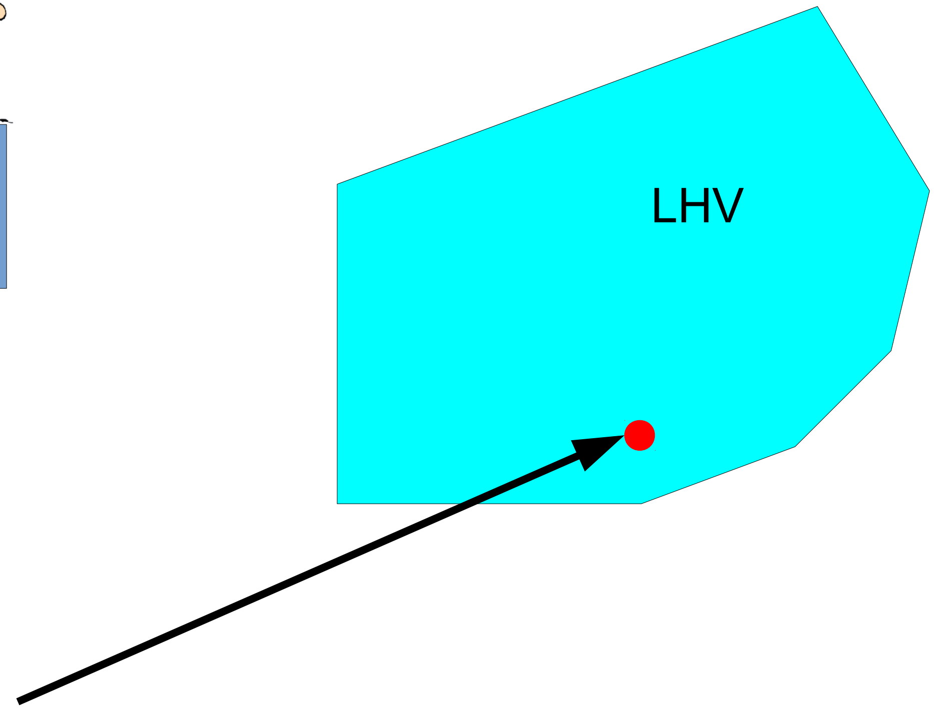
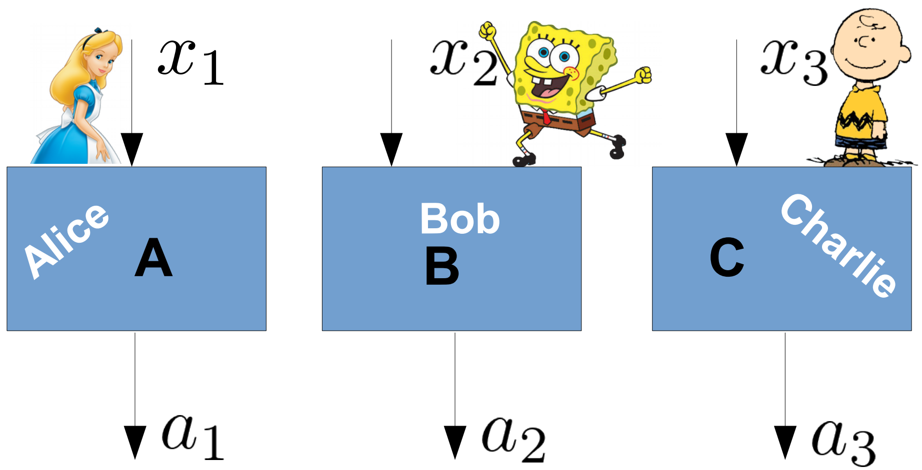


Local Polytope

$$\mathbb{P}_L$$

A crash course on nonlocality



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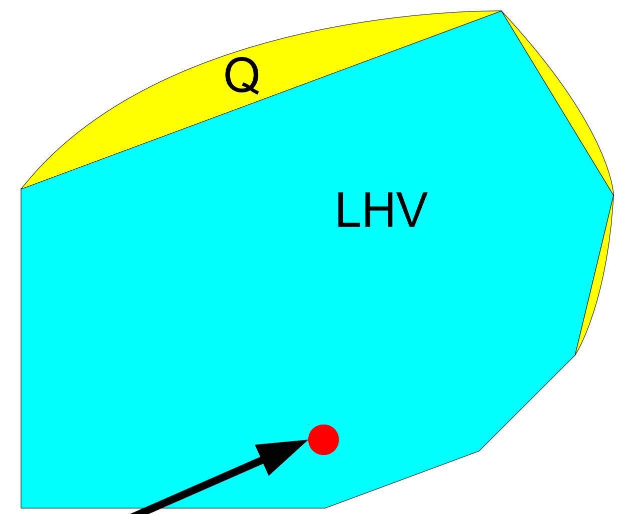
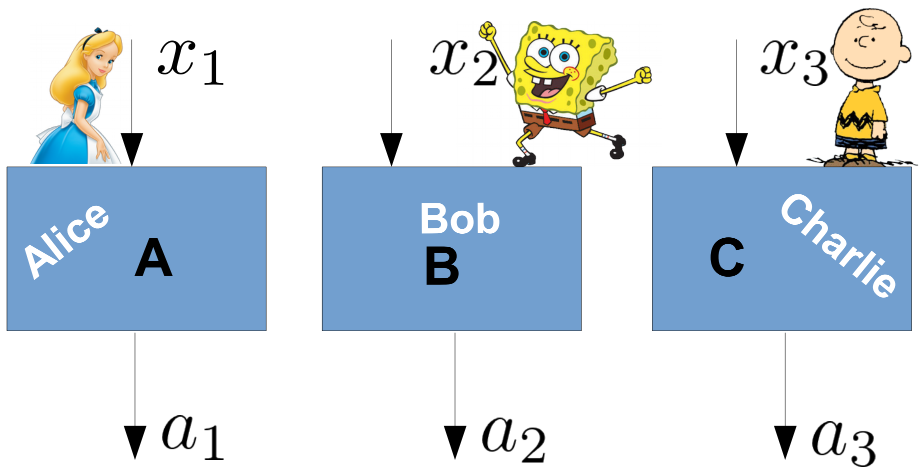


Local Polytope $\subset$ Quantum Set

$\mathbb{P}_L$ $\qquad$ $\mathcal{Q}$

A crash course on nonlocality



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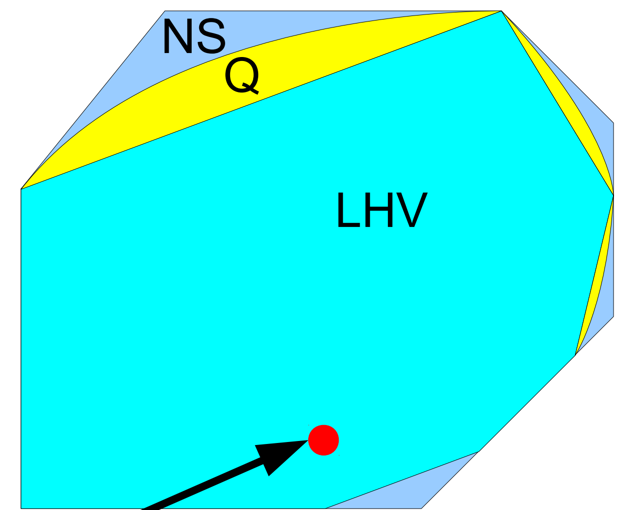

Local Polytope $\subset$ Quantum Set $\subset$ NS Polytope

$\mathbb{P}_L$ $\mathcal{Q}$ $\mathbb{P}_{NS}$

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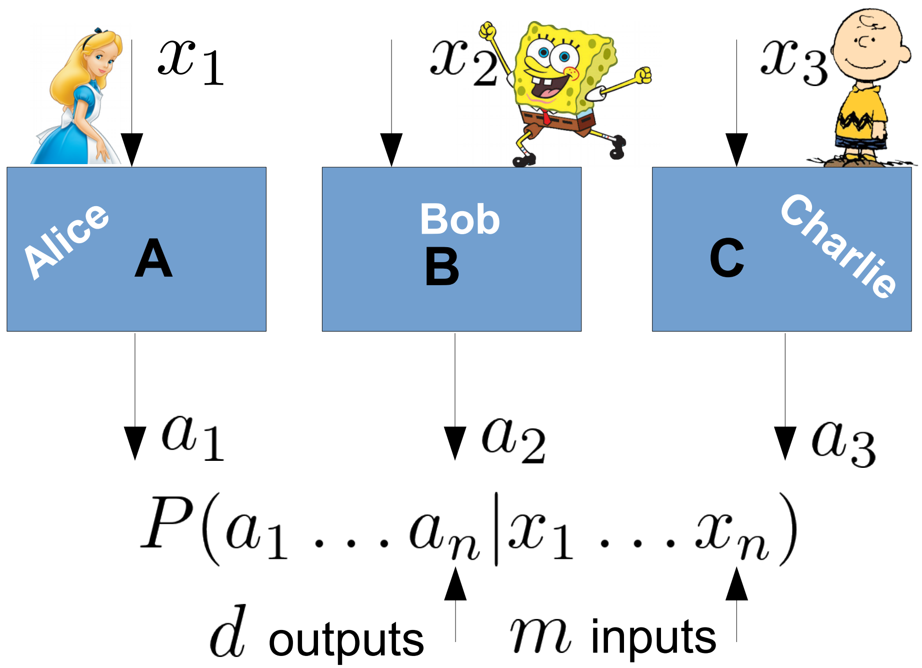

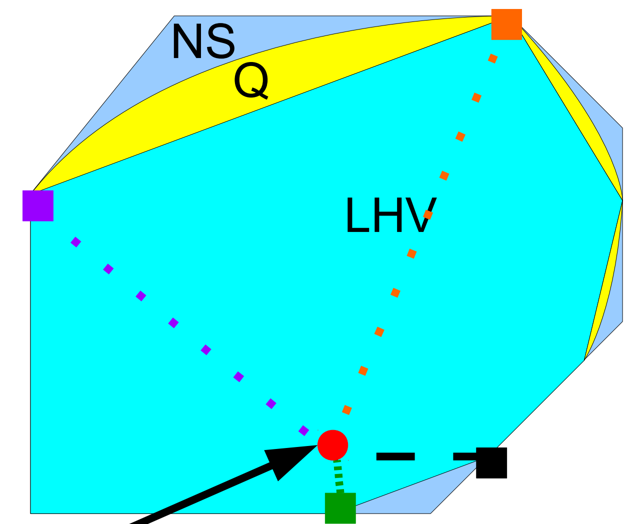

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Local Polytope $\subset$ Quantum Set $\subset$ NS Polytope

$\mathbb{P}_L$ $\quad \mathcal{Q}$ $\quad \mathbb{P}_{NS}$

Example:

Charlie's Instructions

$\lambda = \{1, 3, 1, 2, 4, 3, 1, 1 \ldots\}$

Output

$0, x_3, 0, 1, \overline{x_3}, x_3, 0, 0, \ldots$

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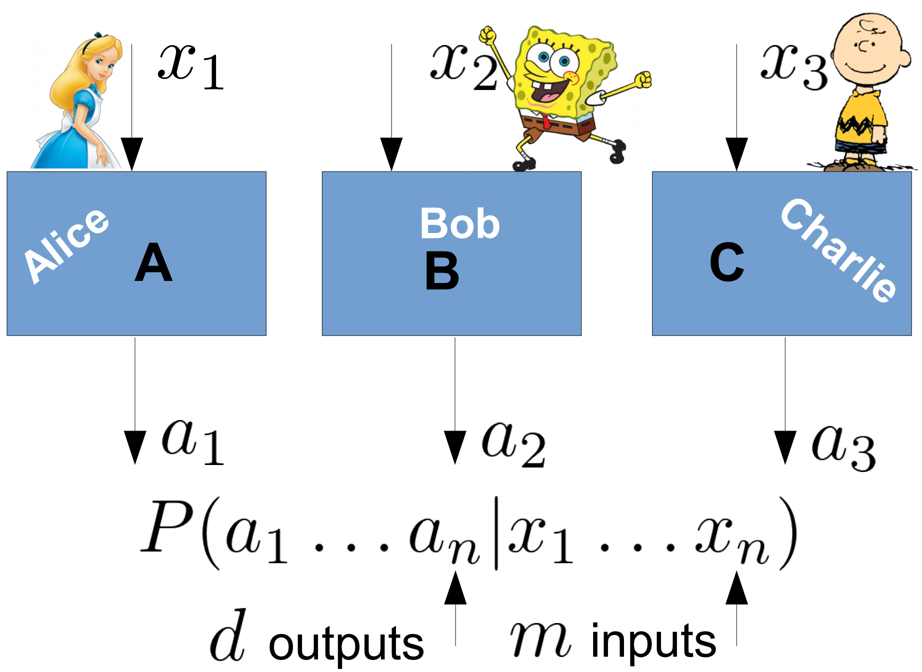

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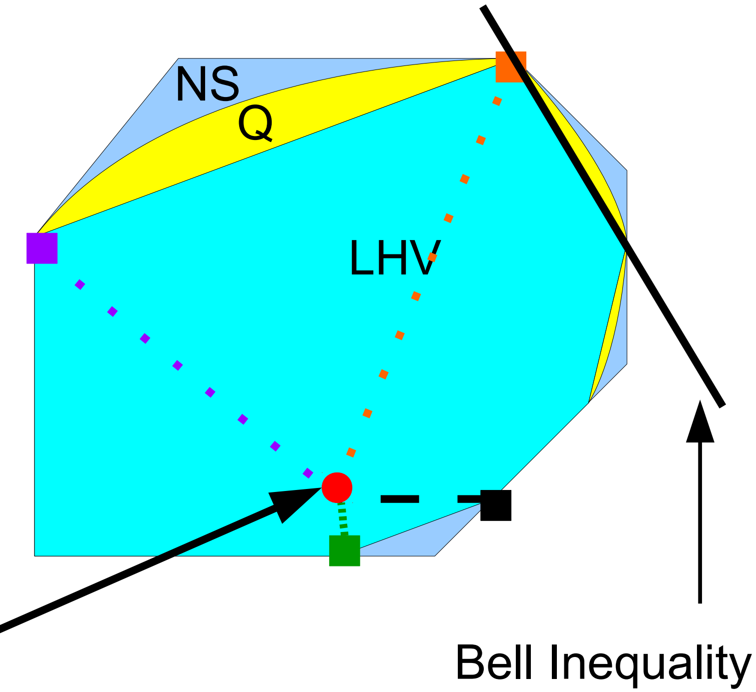

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- Quantum optimization
- Assigning a Bell inequality to a Hamiltonian
- Classical optimization
- Translational invariance
- Examples
- Conclusions and outlook

The idea

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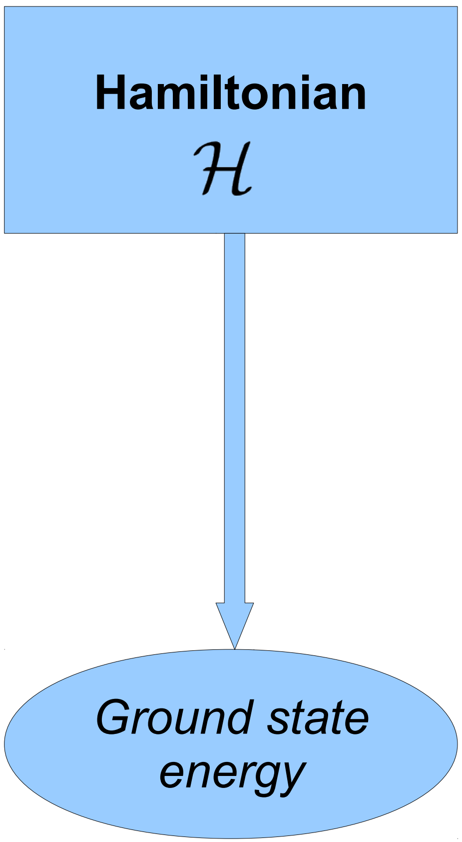
Hamiltonian

$\mathcal{H}$

The idea



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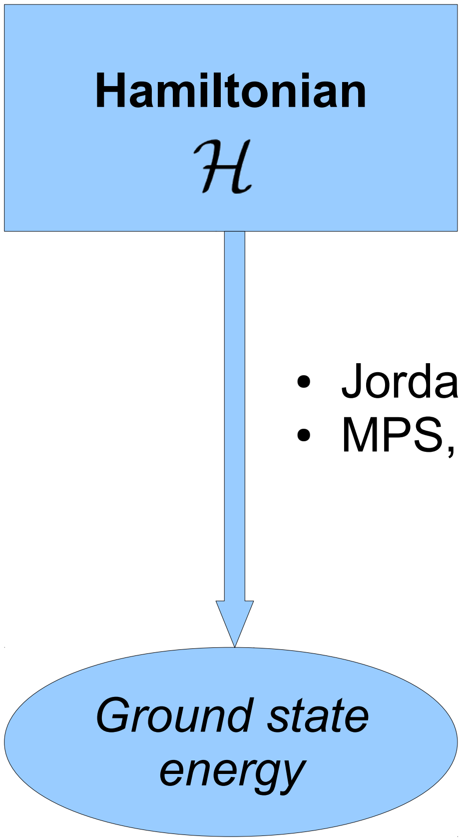

- Jordan-Wigner
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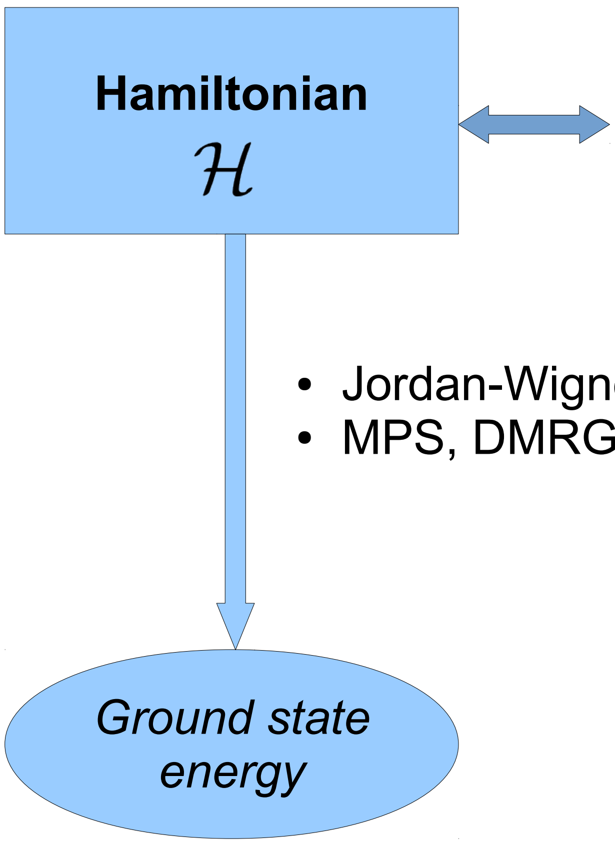

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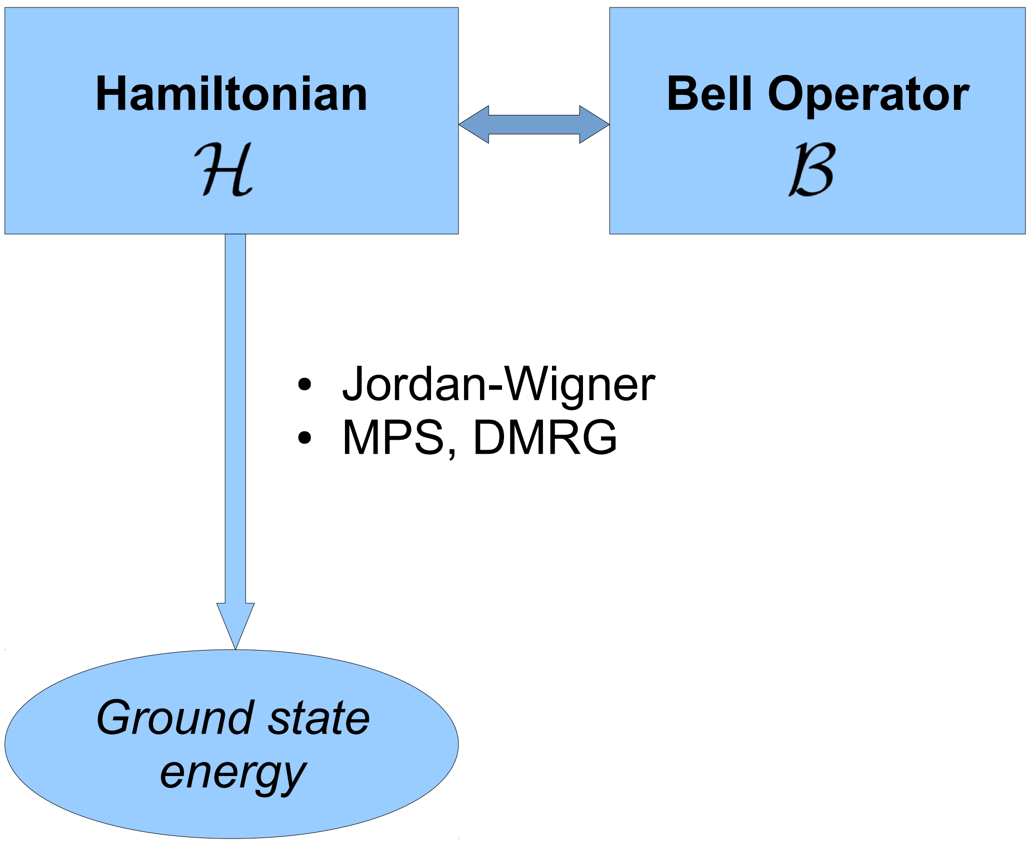

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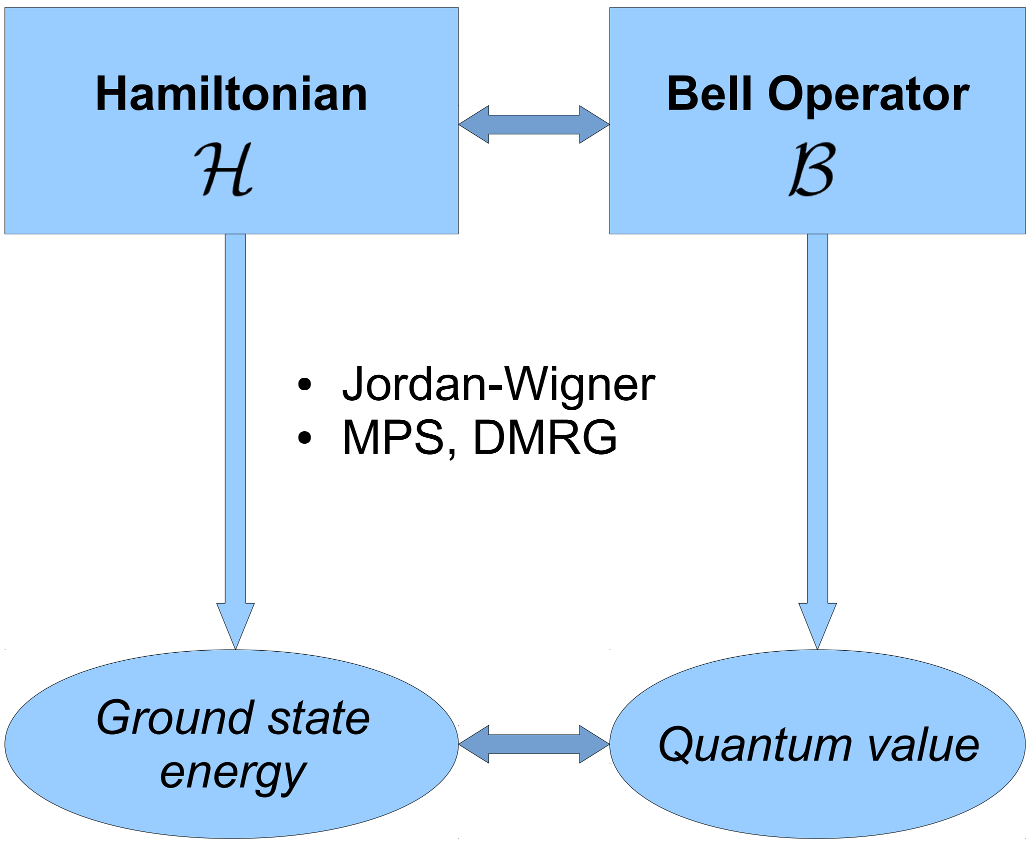
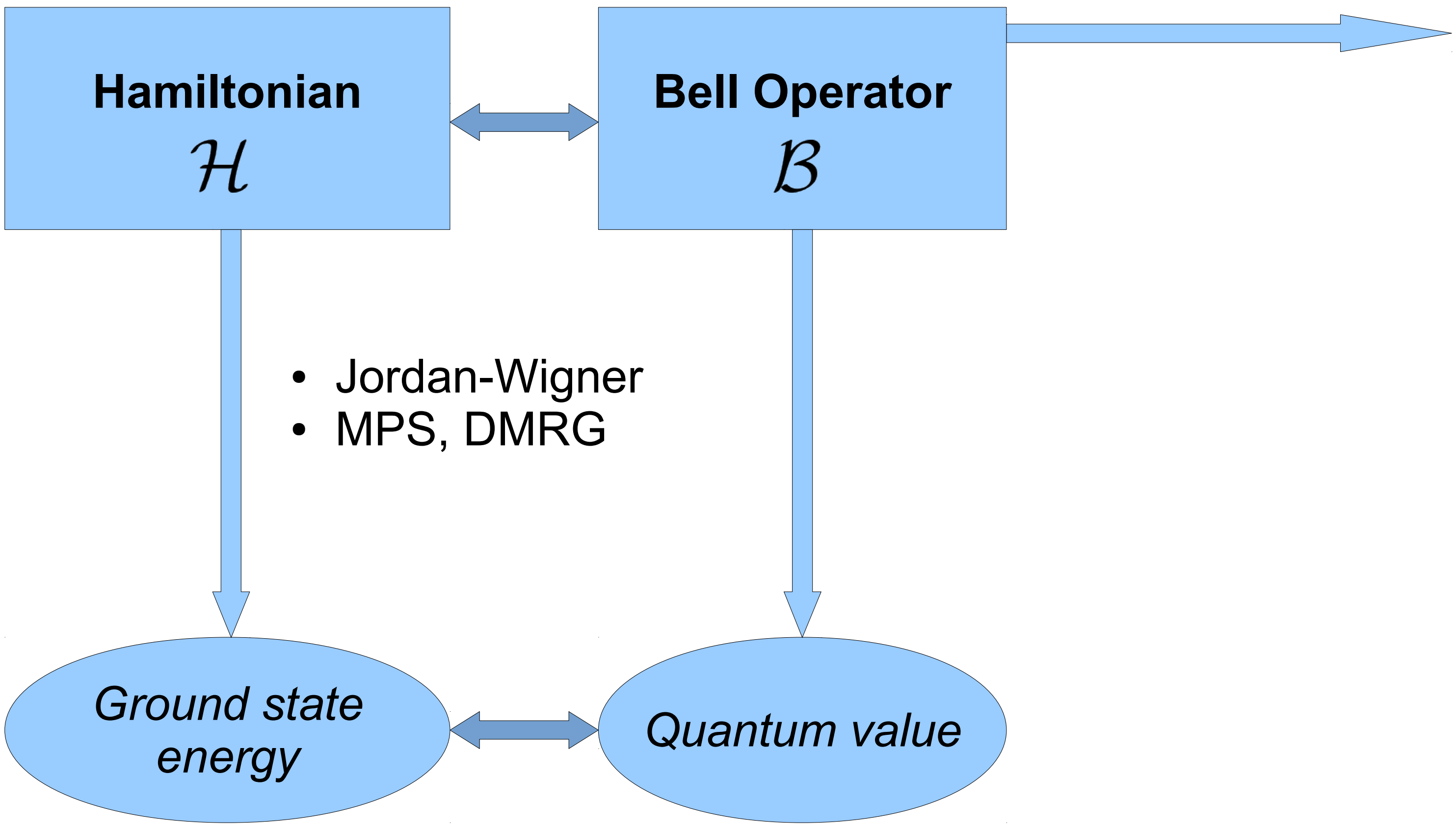
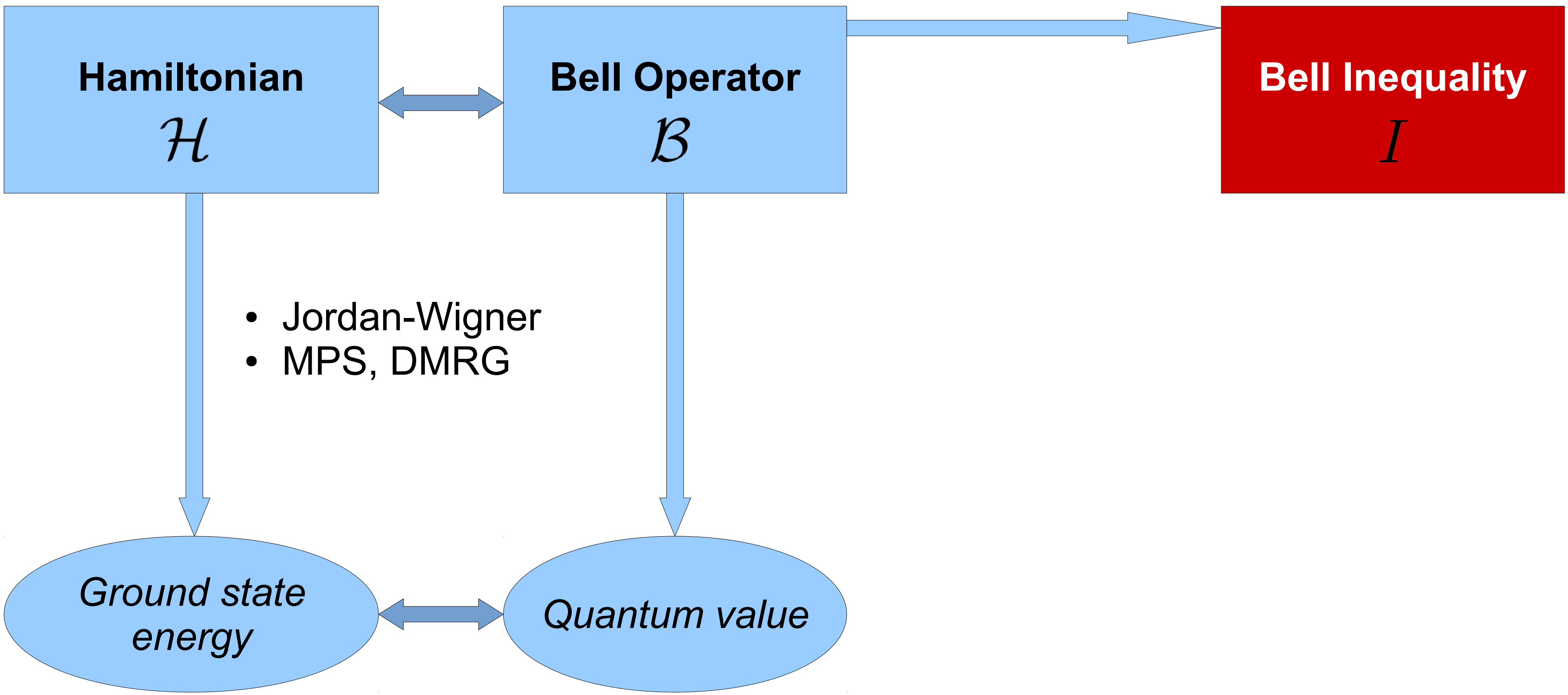

Hamiltonian
$\mathcal{H}$
Bell Operator
$\mathcal{B}$
• Jordan-Wigner
• MPS, DMRG
Ground state energy
Quantum value

The idea


Hamiltonian
$\mathcal{H}$
Bell Operator
$\mathcal{B}$
• Jordan-Wigner
• MPS, DMRG
Ground state energy
Quantum value

The idea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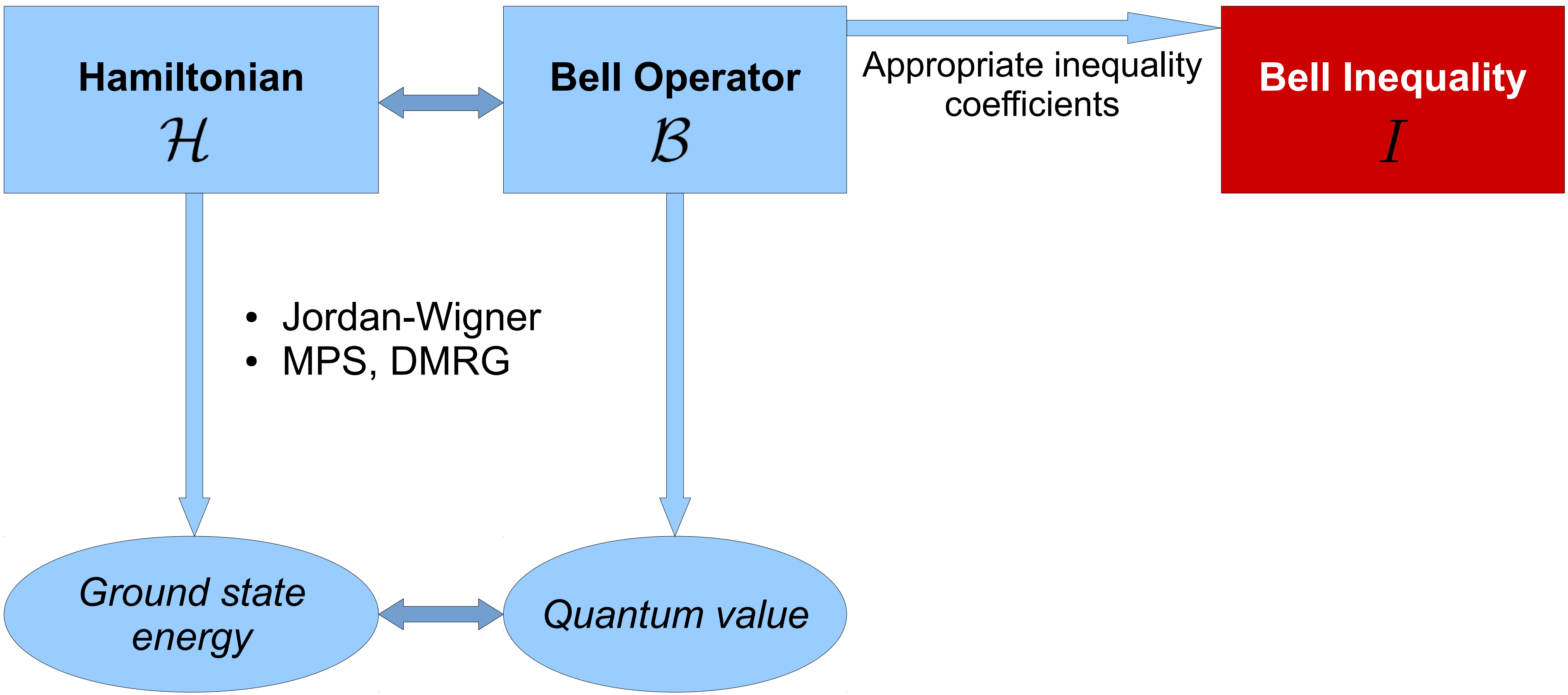
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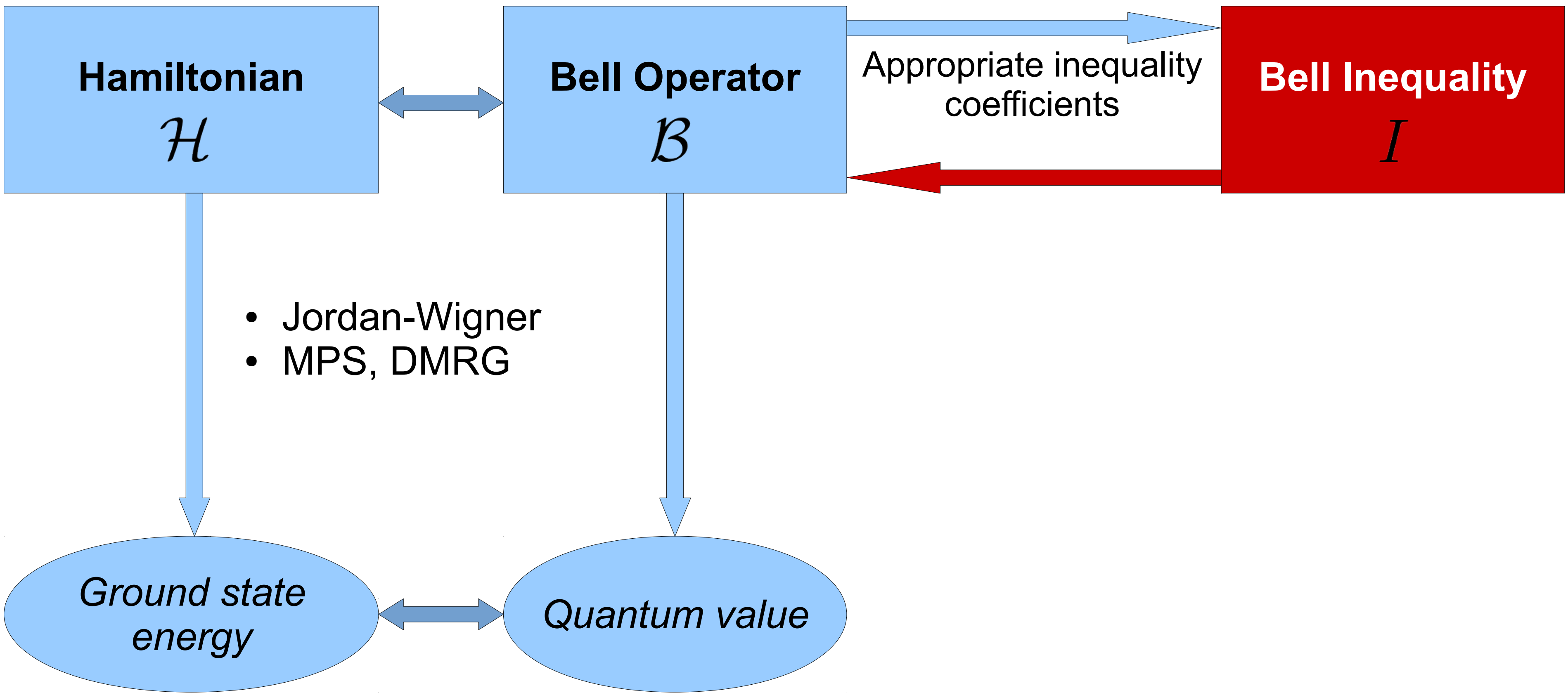
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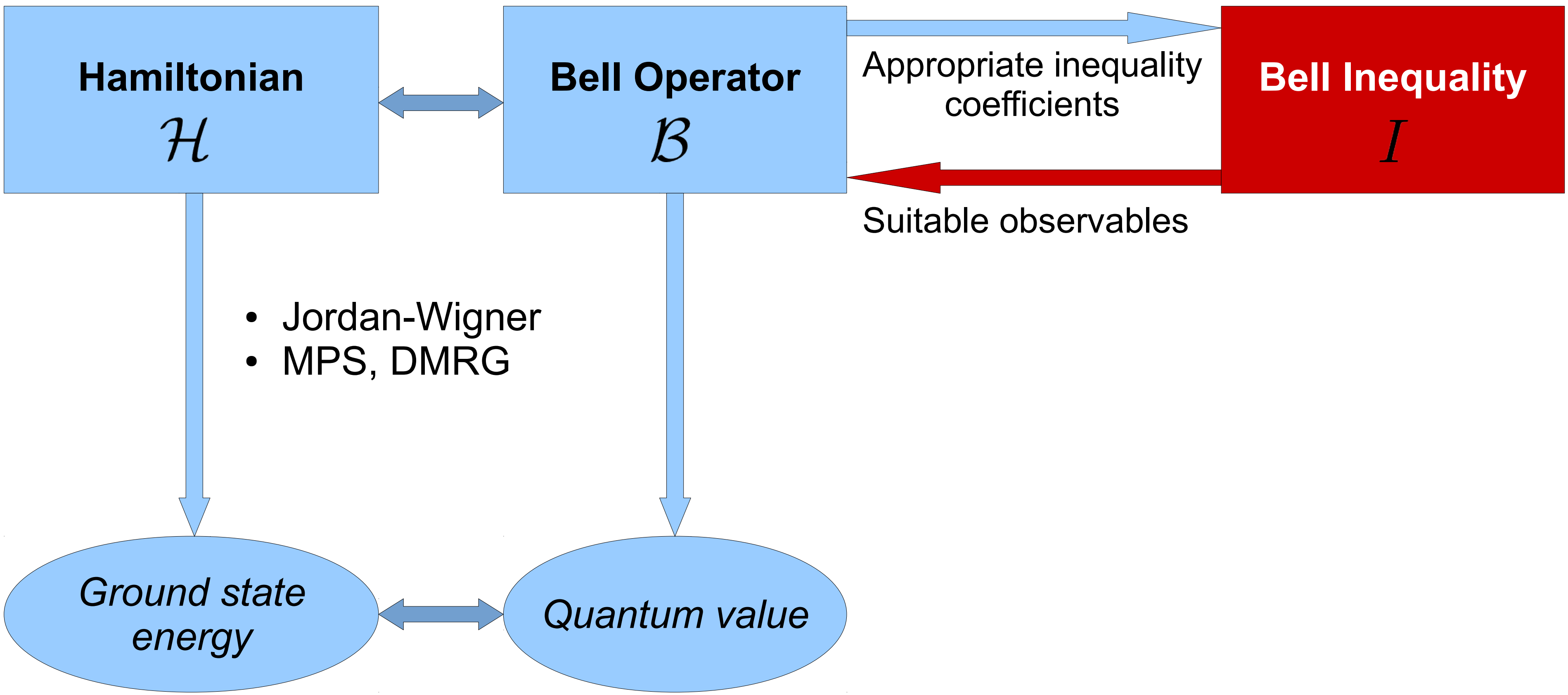
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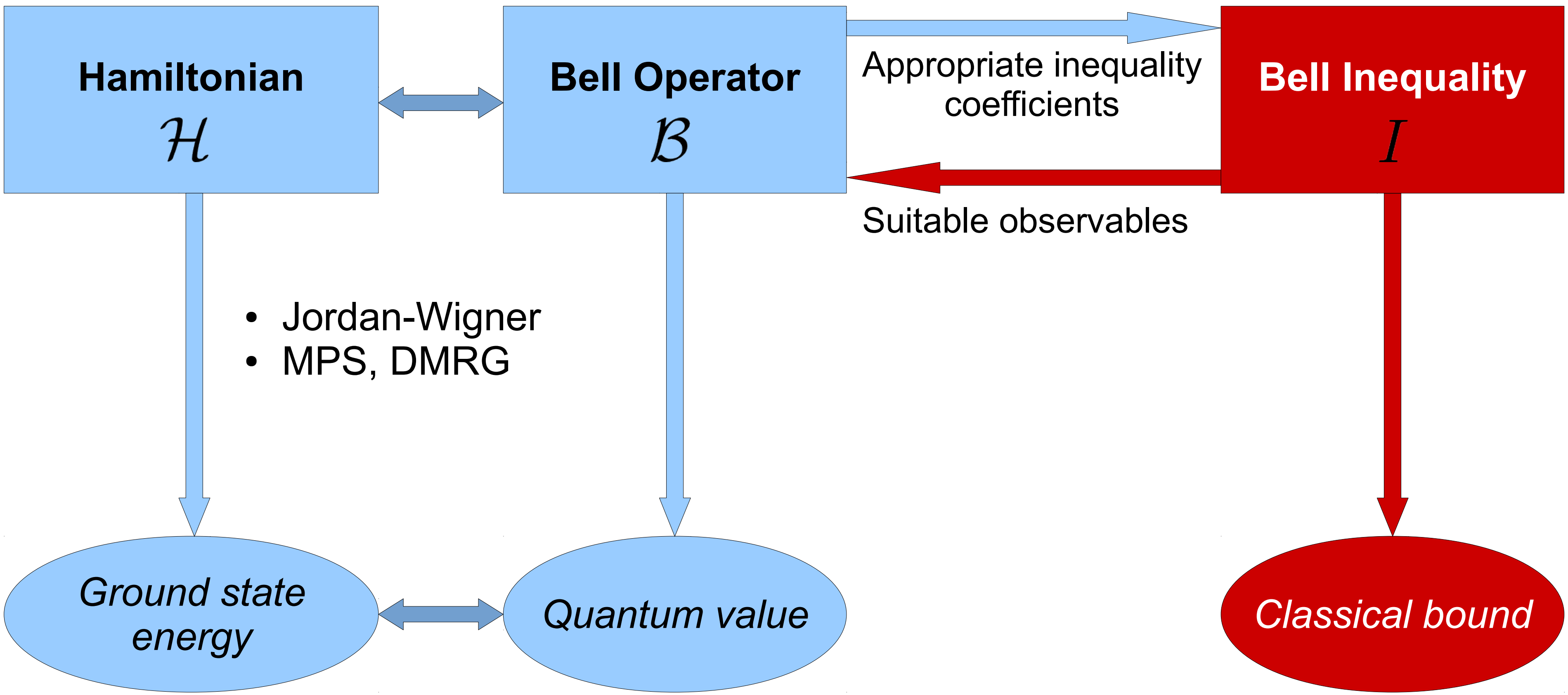
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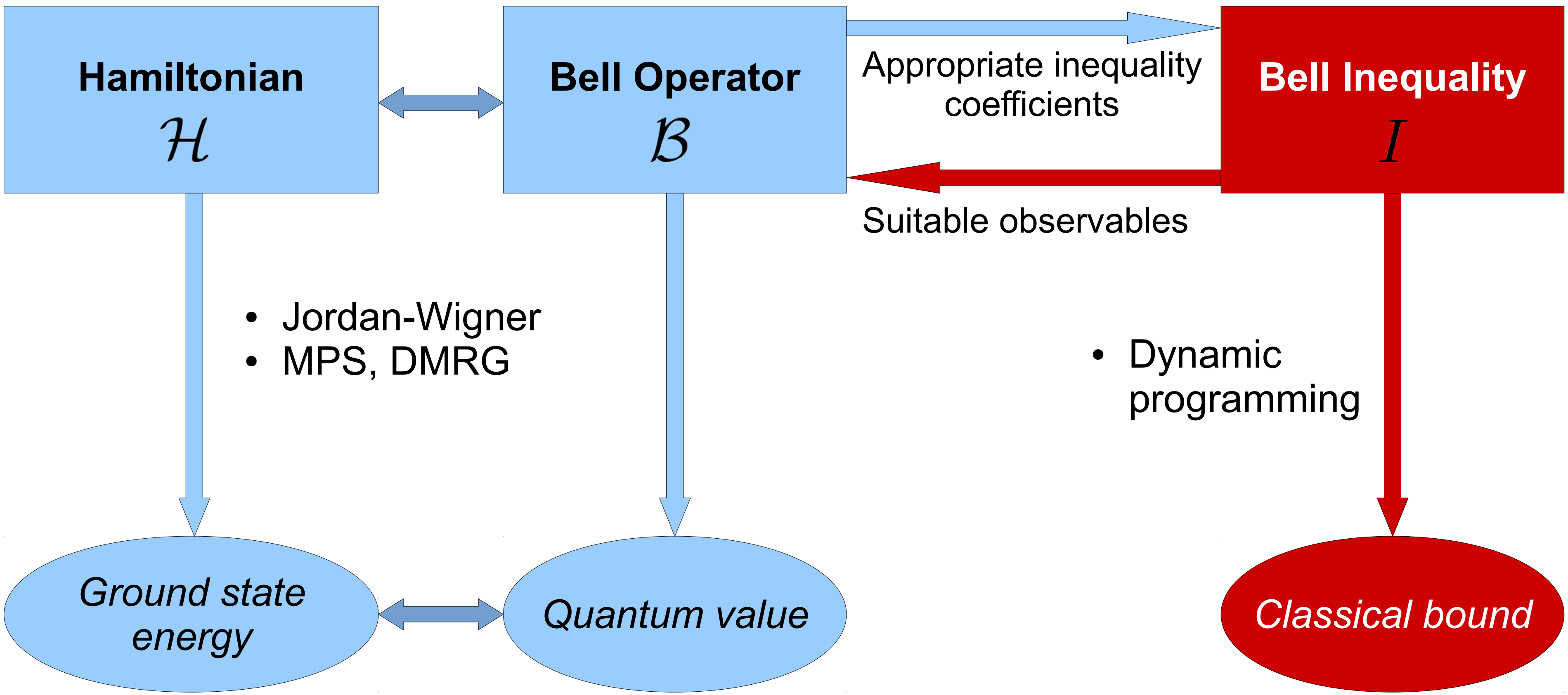

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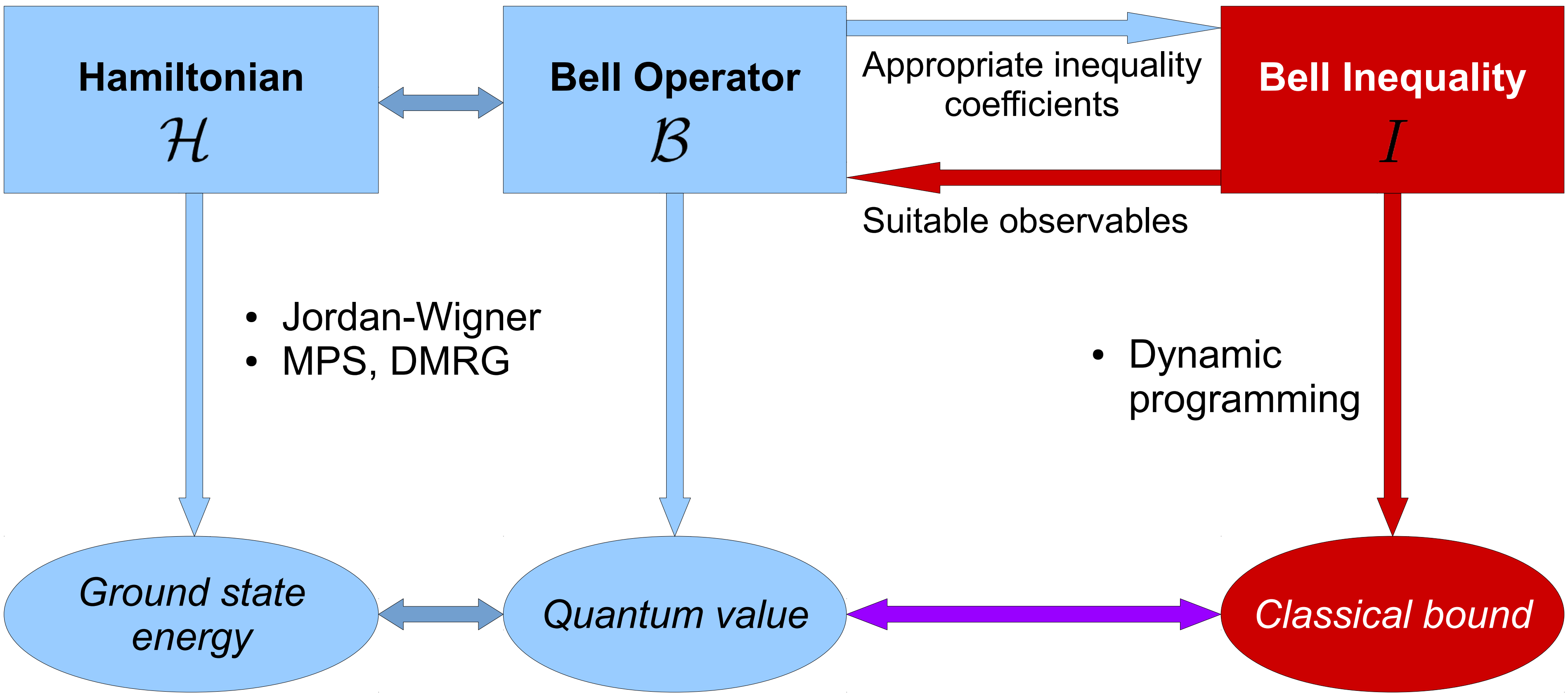

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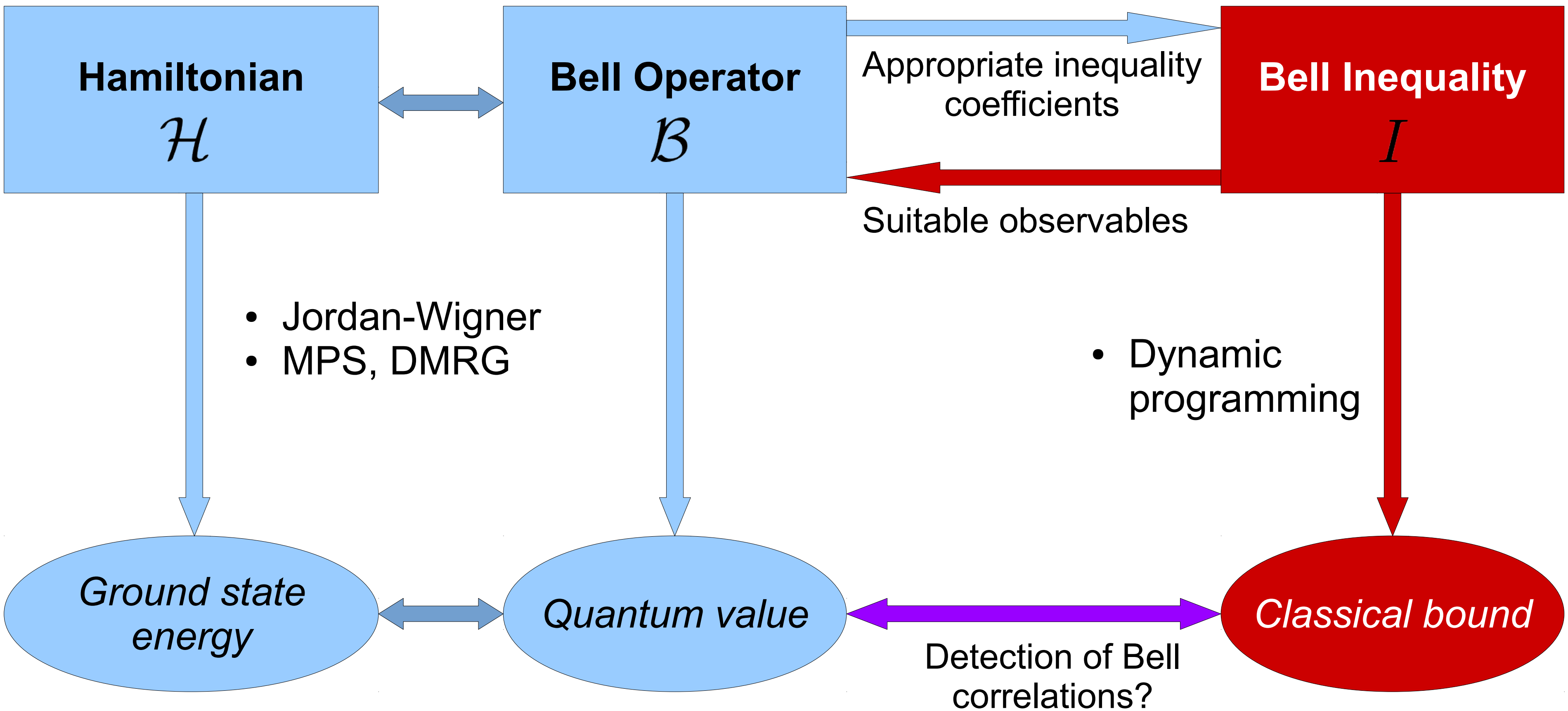

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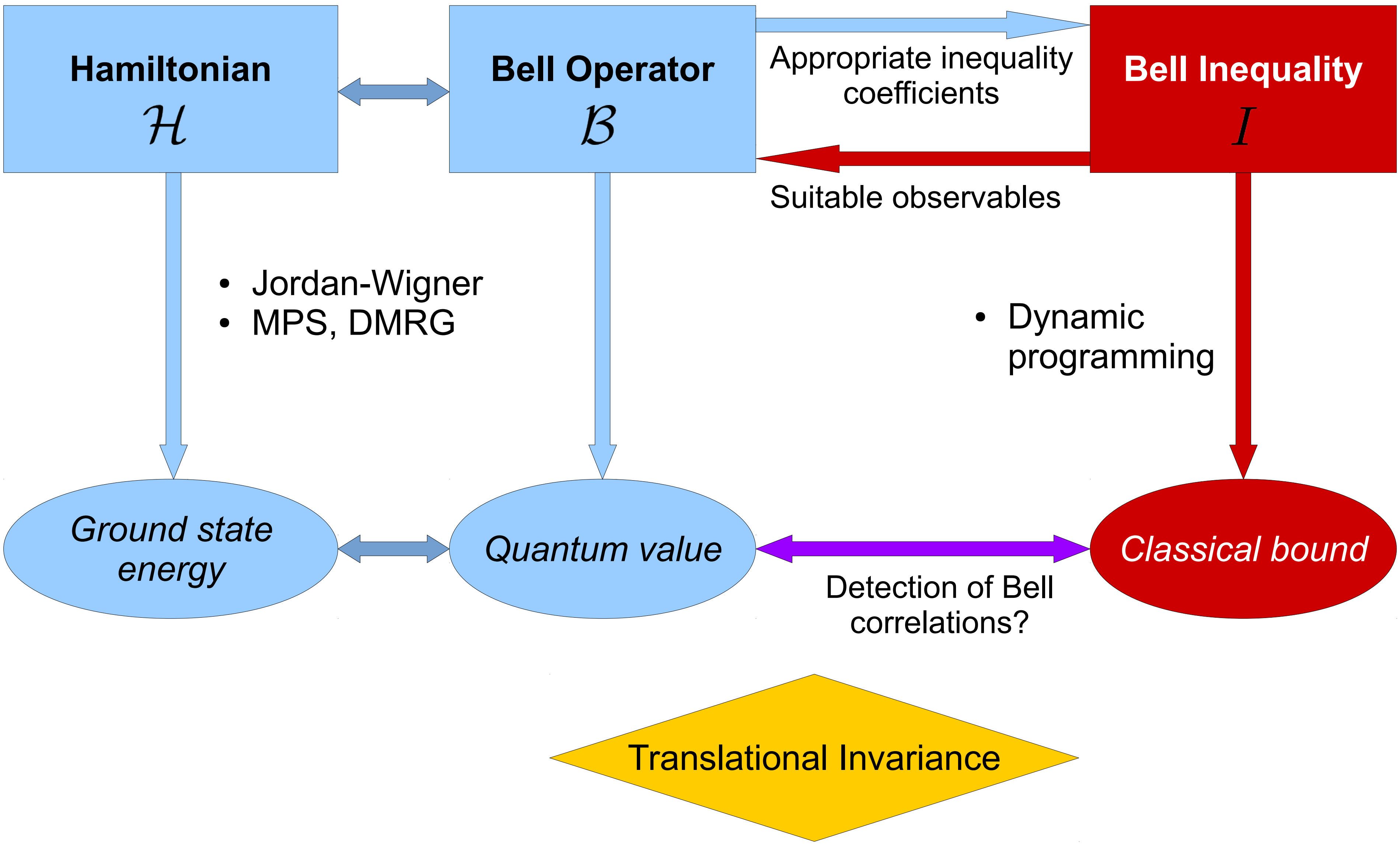
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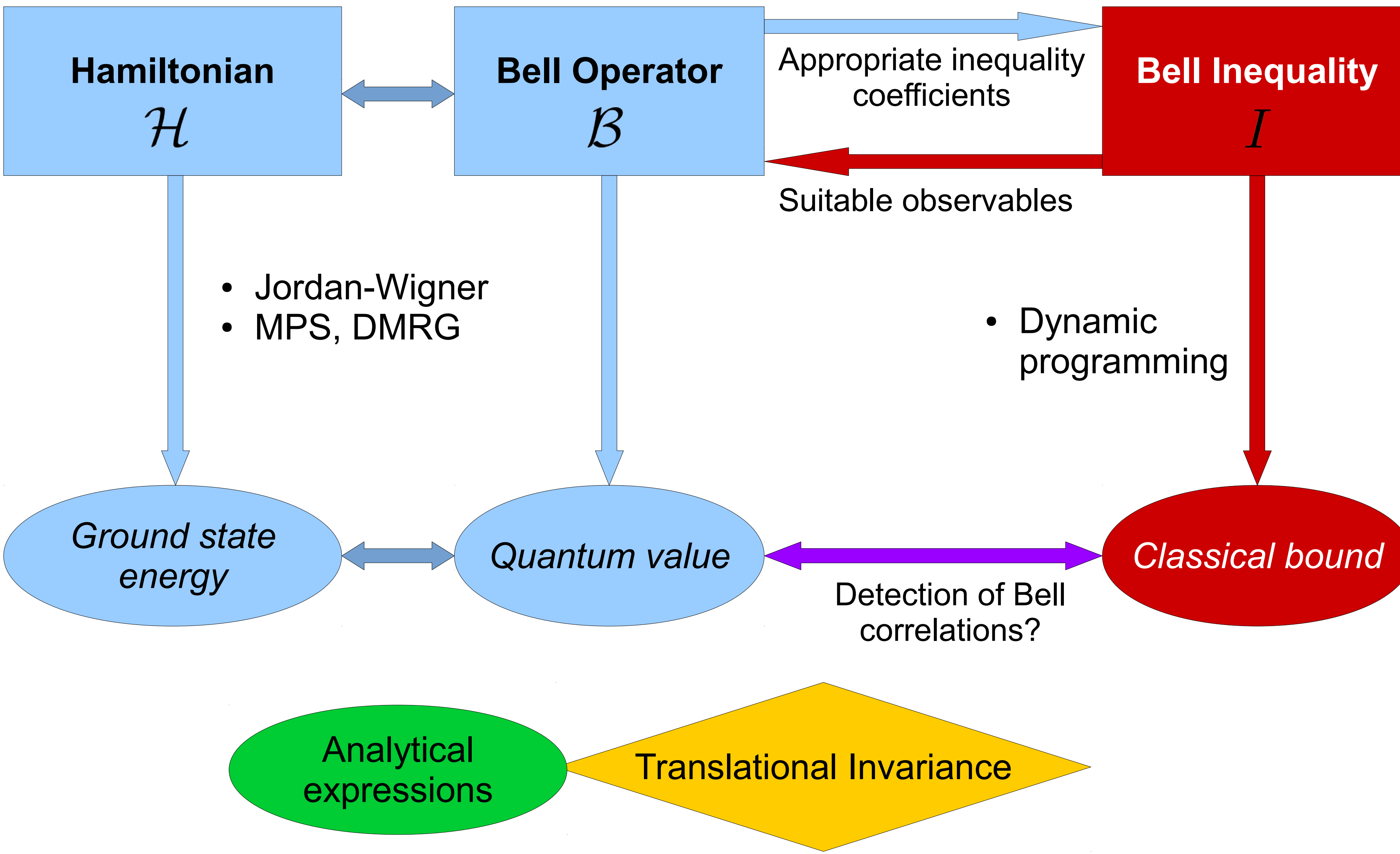
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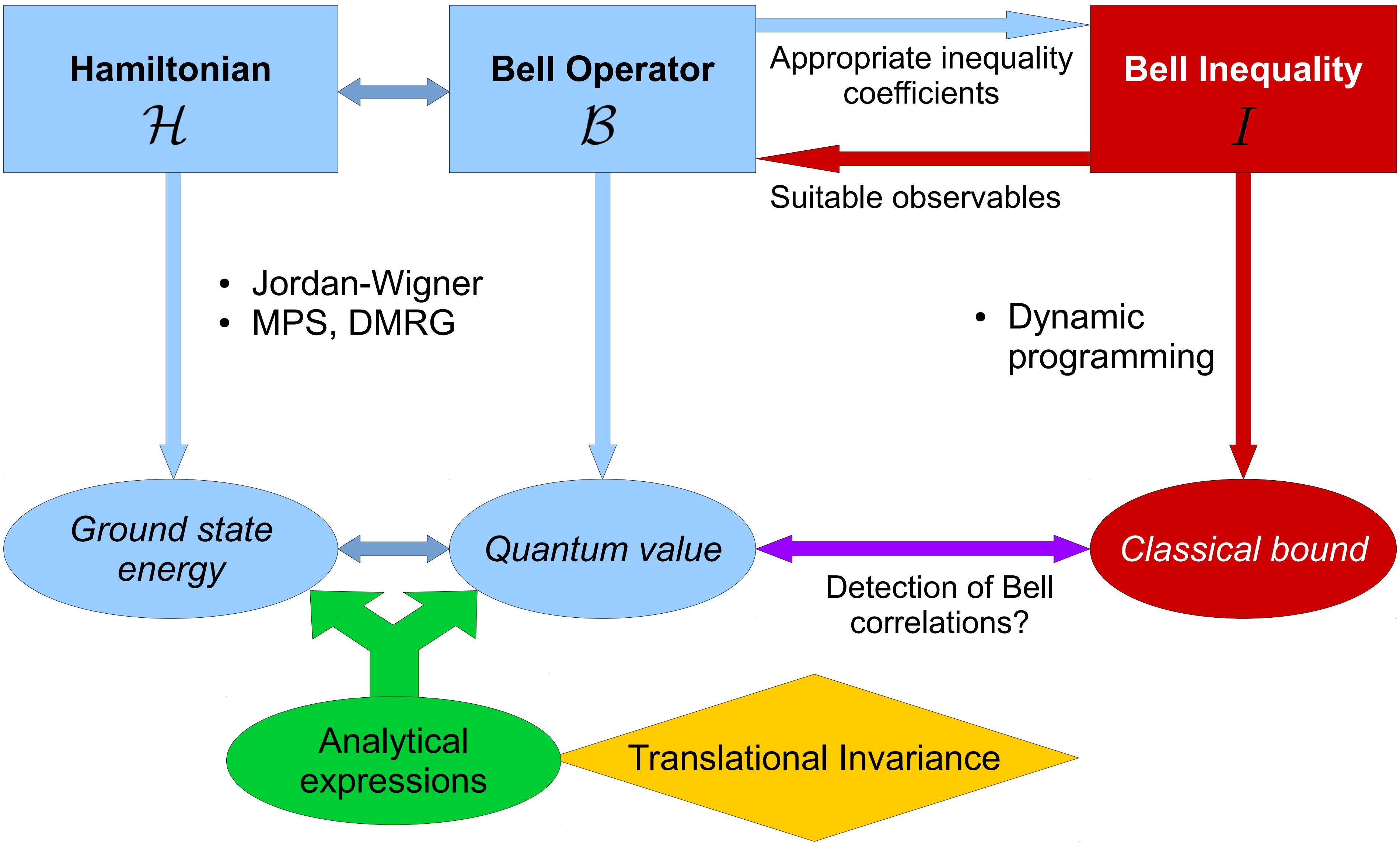
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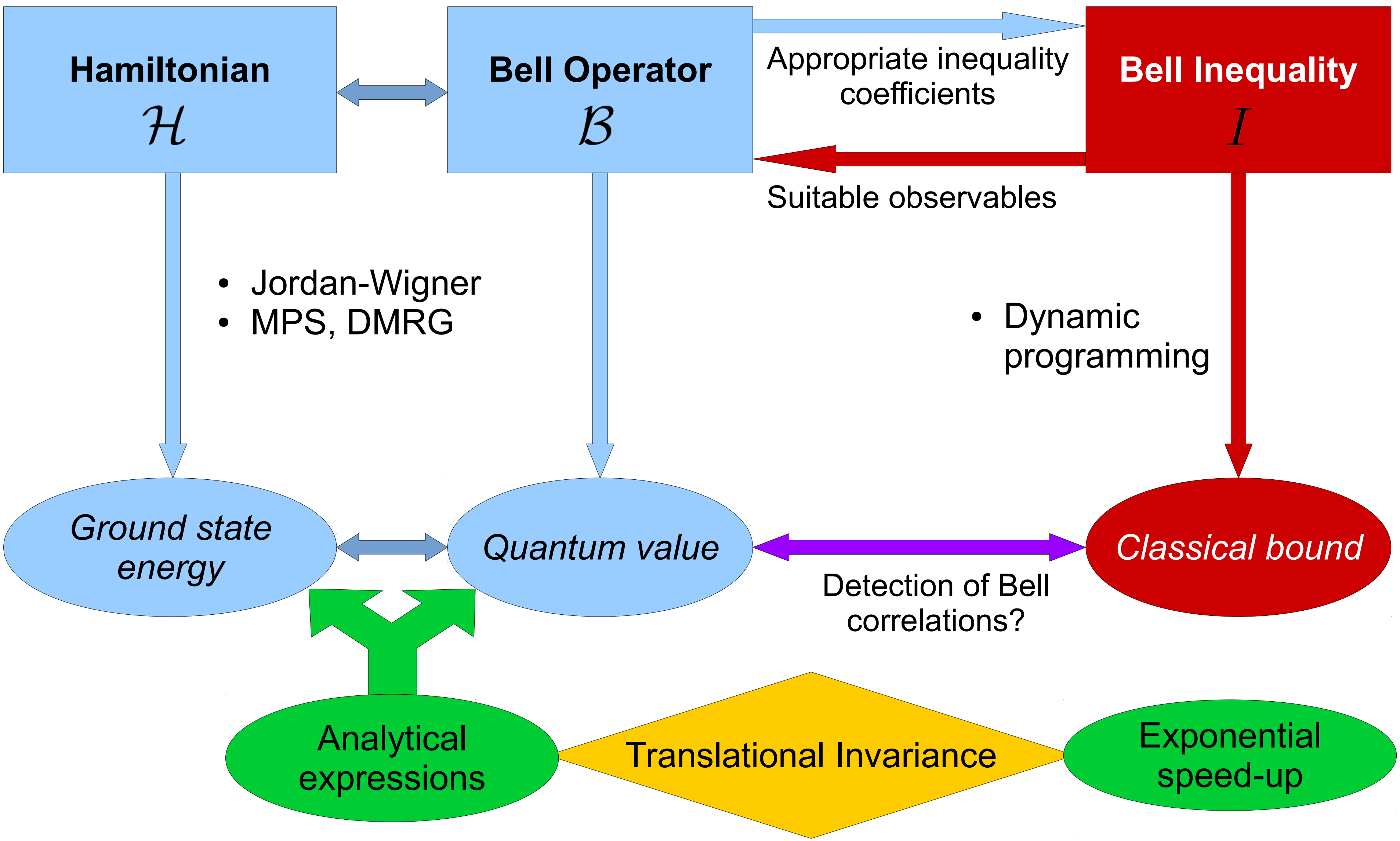
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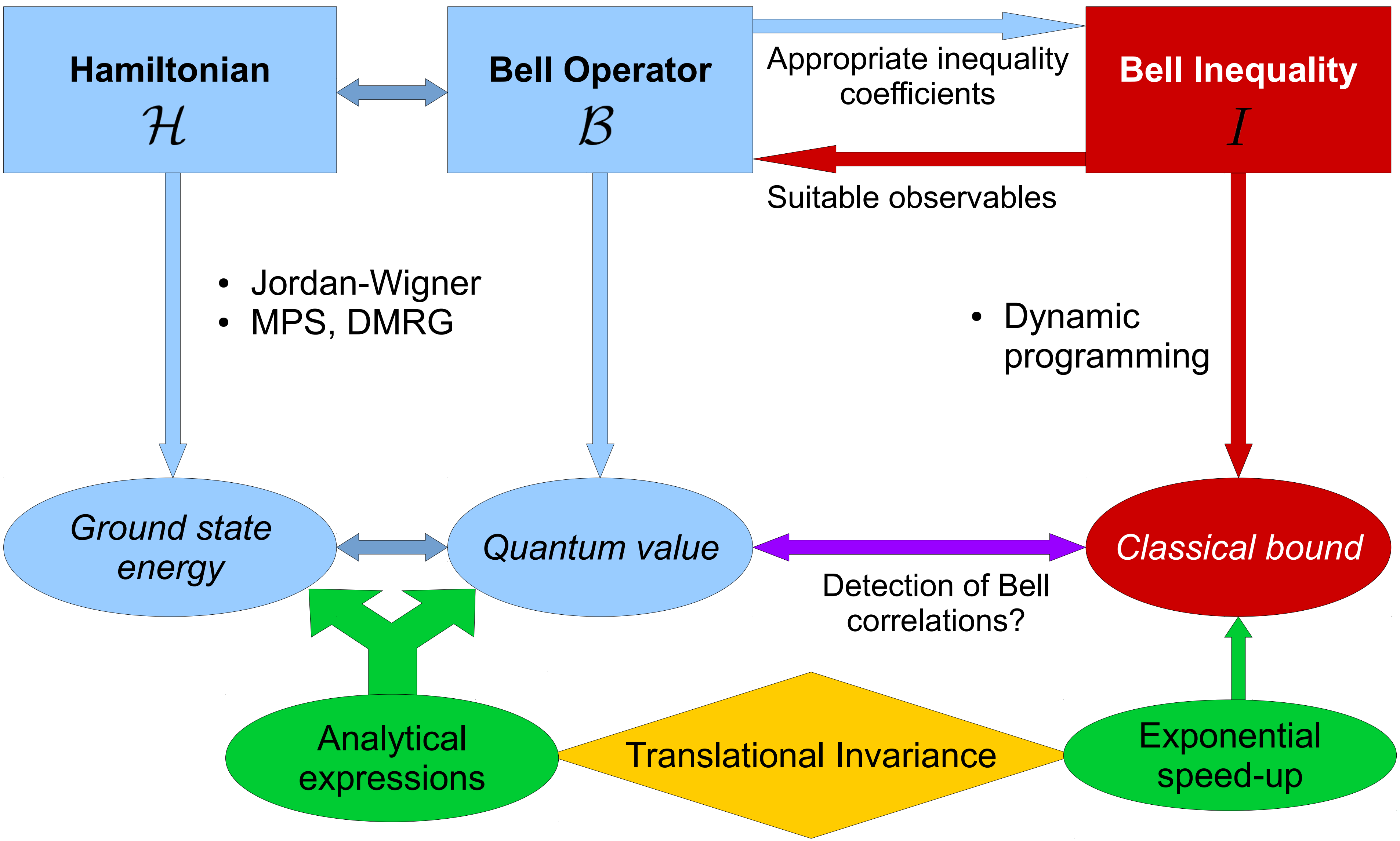
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The setting

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- Spin – 1/2 Hamiltonians

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- n particles

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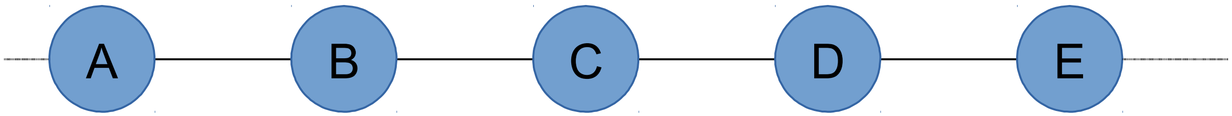

- One spatial dimension

The setting

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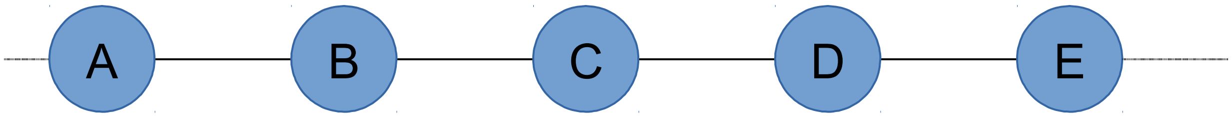

- One spatial dimension
- Open/Periodic boundary conditions

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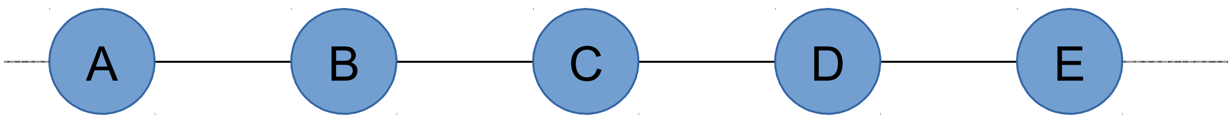

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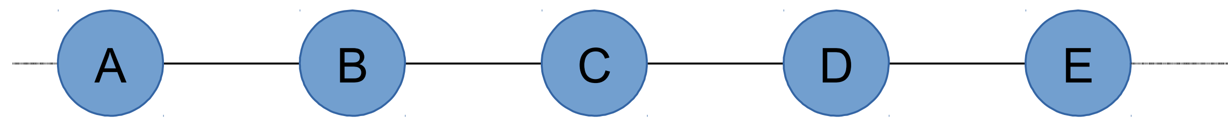

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$$\mathcal{H} = \sum_{i=0}^{n-1} \left(t^{(i)} \sigma_z^{(i)} + \sum_{r=1}^{R} \sum_{\alpha,\beta \in \{x,y\}} t_{\alpha,\beta}^{(i,r)} \mathrm{Str}_{\alpha,\beta}^{(i,r)} \right)$$

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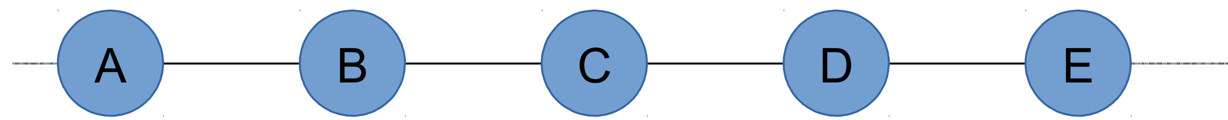

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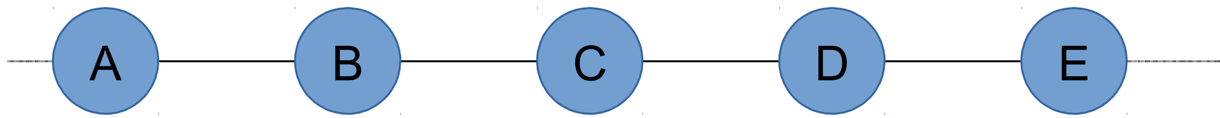

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e.g. XY-model in a transverse magnetic field

Outline

- Motivation
- The idea, the setting
- **Quantum optimization**
- Assigning a Bell inequality to a Hamiltonian
- Classical optimization
- Translational invariance
- Examples
- Conclusions and outlook

Finding the ground state energy (I)

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- Exact diagonalization

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  - Jordan – Wigner transformation: Spins to fermions

$$\hat{c}_{i,0} \leftrightarrow \prod_{j=0}^{i-1} \sigma_z^{(j)} \sigma_x^{(i)}, \qquad \hat{c}_{i,1} \leftrightarrow -\prod_{j=0}^{i-1} \sigma_z^{(j)} \sigma_y^{(i)}$$

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Real, antisymmetric

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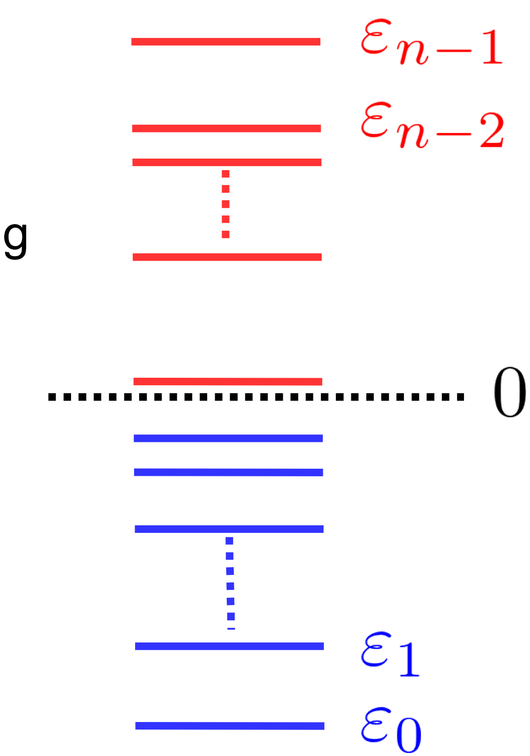

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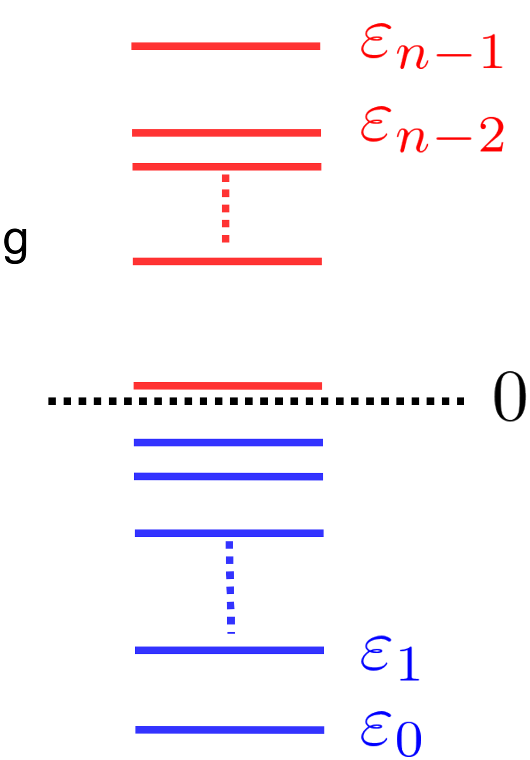

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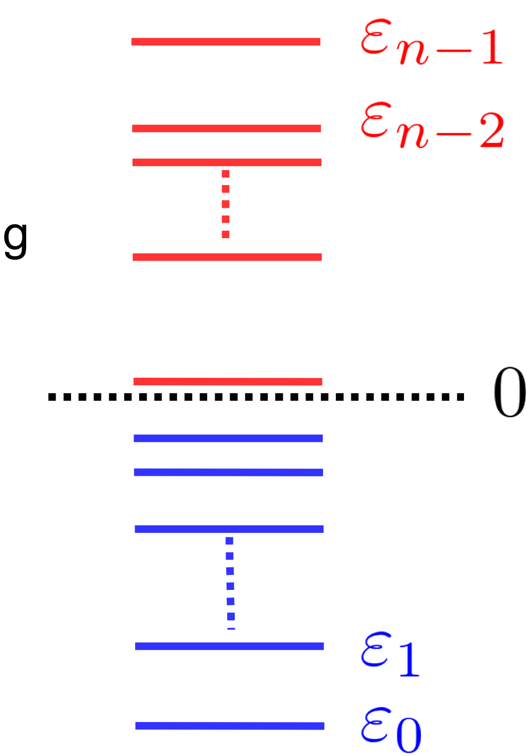
$$\beta_Q = \sum_{k=0}^{n-1} s_k \varepsilon_k$$



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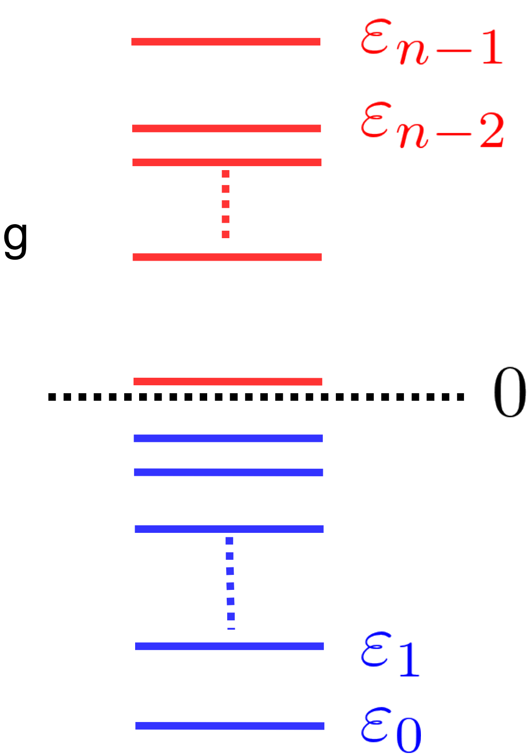

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The parity imposes a superselection rule

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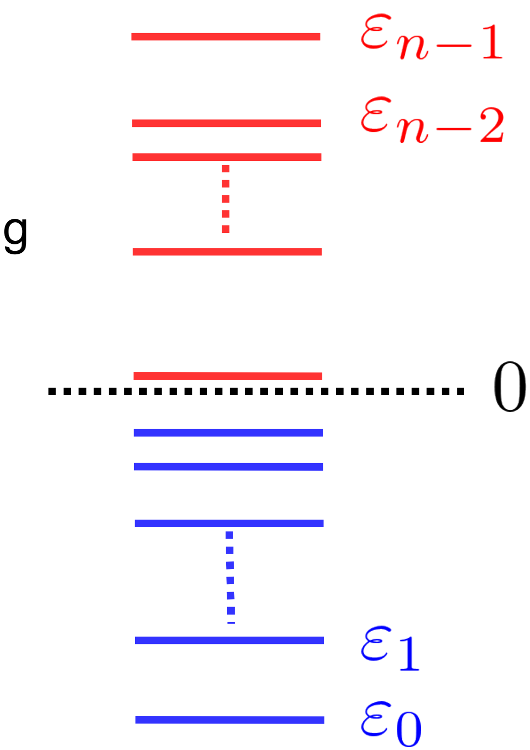

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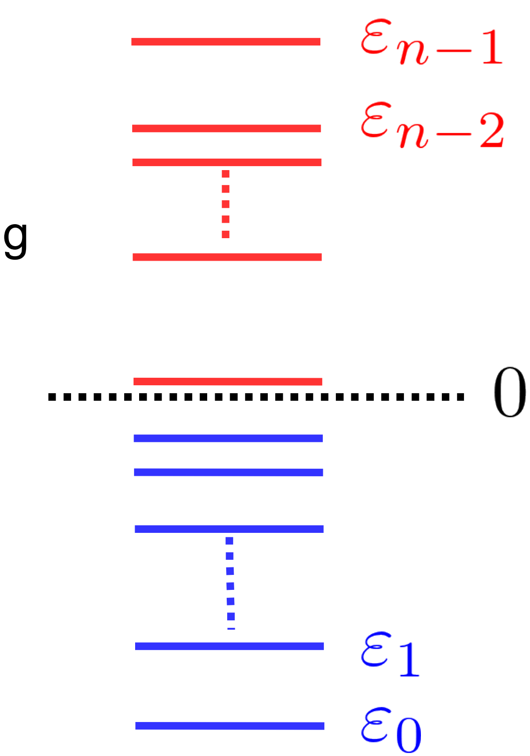

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The parity imposes a superselection rule

$$p = (\det O) \prod_{k=0}^{n-1} s_k$$

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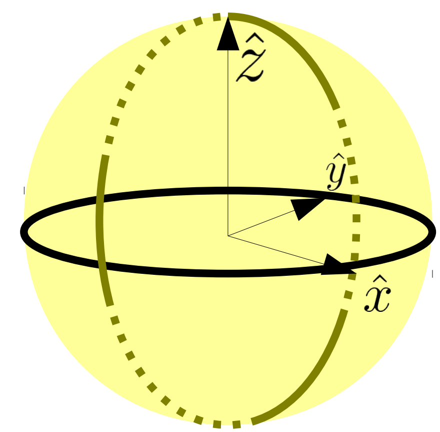
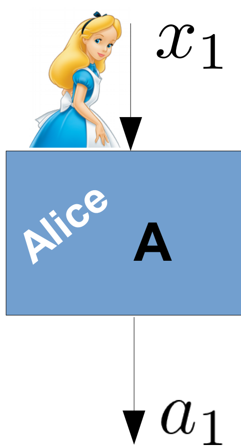
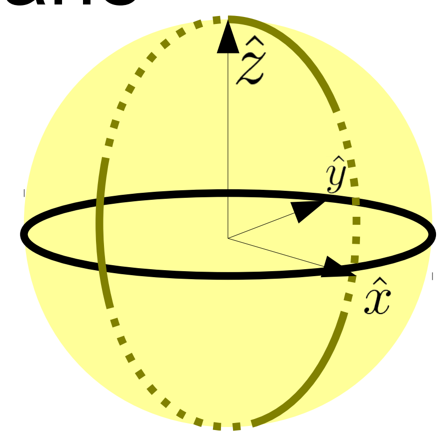
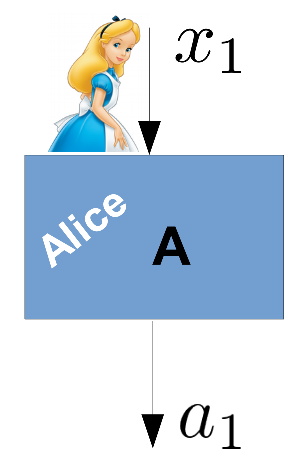




Assigning a Bell inequality

- We want a Bell operator of the form $\mathcal{B} = \beta_C \mathbb{1} + \mathcal{H}$
- Taking m measurements in the X-Y plane
  - Extra measurement in the Z direction





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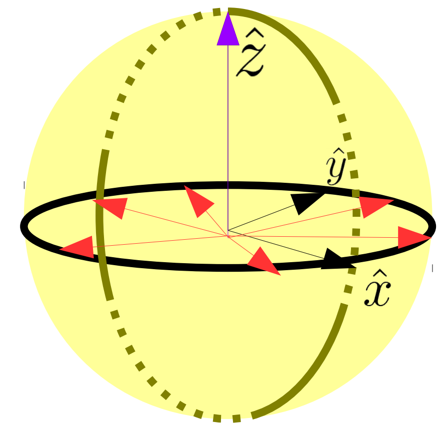

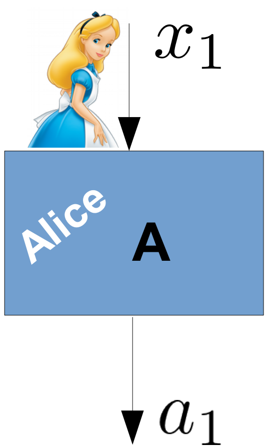

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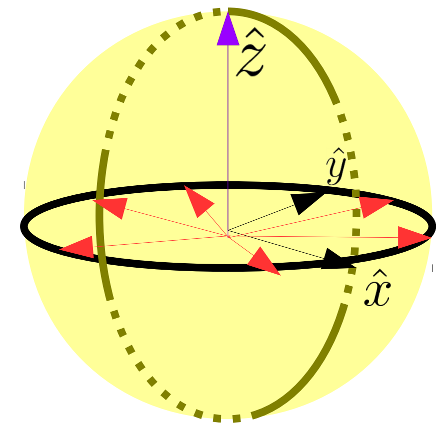
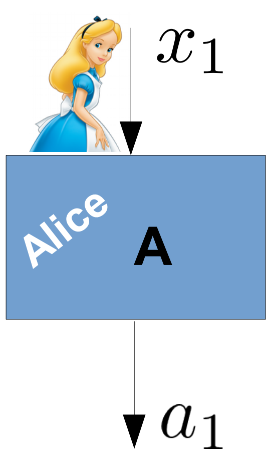


$$\mathcal{H} = \sum_{i=0}^{n-1} \left(t^{(i)} \sigma_z^{(i)} + \sum_{r=1}^{R} \sum_{\alpha,\beta \in \{x,y\}} t_{\alpha,\beta}^{(i,r)} \mathrm{Str}_{\alpha,\beta}^{(i,r)} \right)$$



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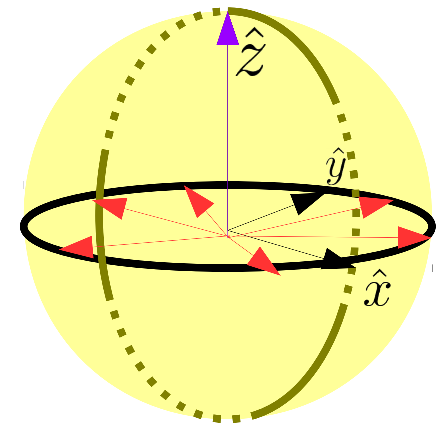

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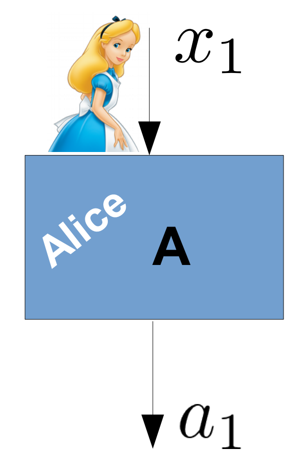

$$I = \sum_{i=0}^{n-1} \left(\gamma^{(i)} M_m^{(i,0)} + \sum_{r=1}^{R} \sum_{k,l=0}^{m-1} M_{(k,m,\ldots,m,l)}^{(i,r)} \right)$$

Outline

- Motivation
- The idea, the setting
- Quantum optimization
- Assigning a Bell inequality to a Hamiltonian
- **Classical optimization**
- Translational invariance
- Examples
- Conclusions and outlook

Finding the classical bound

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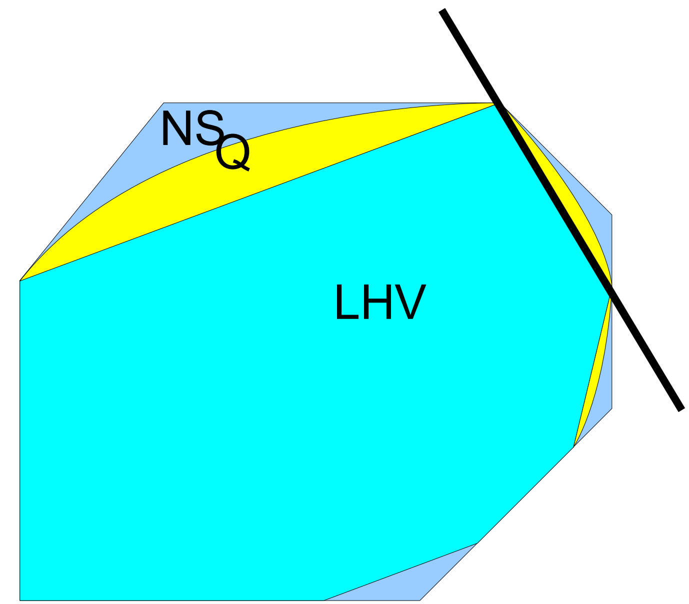

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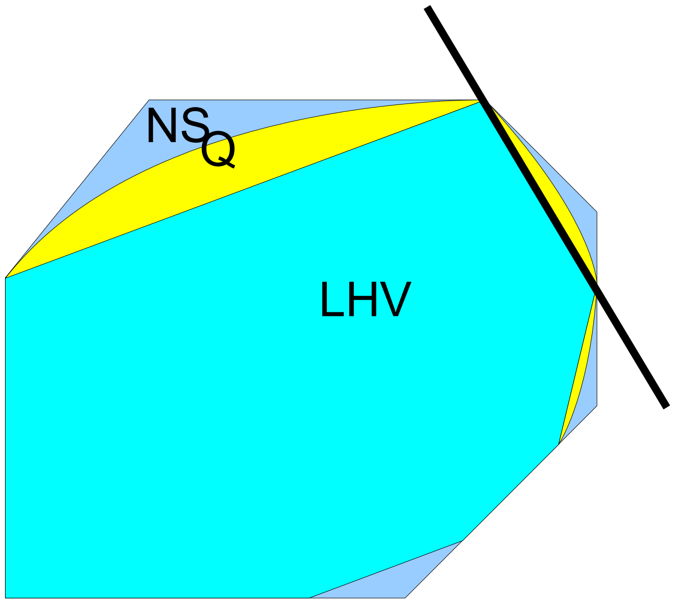

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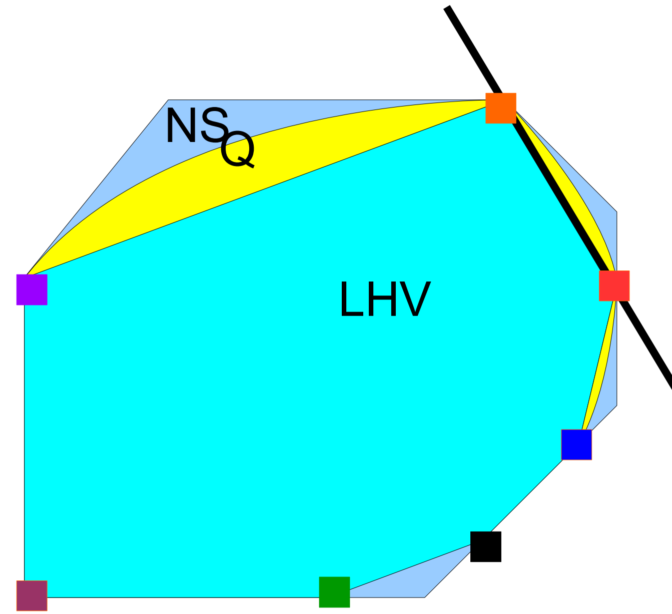

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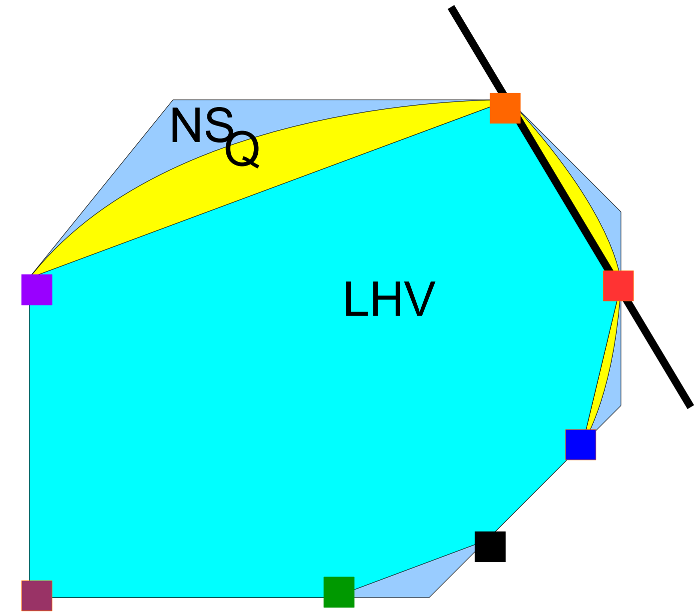

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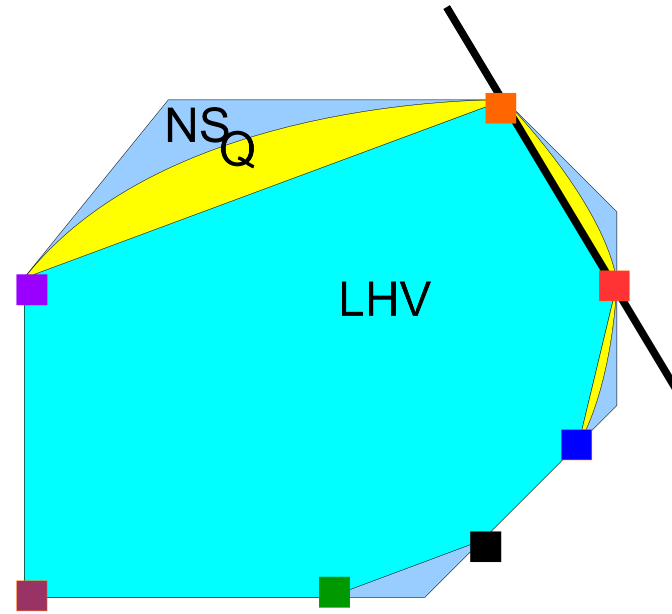

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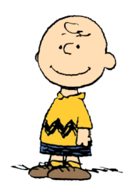
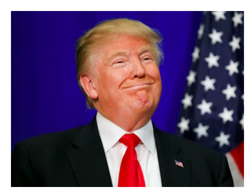

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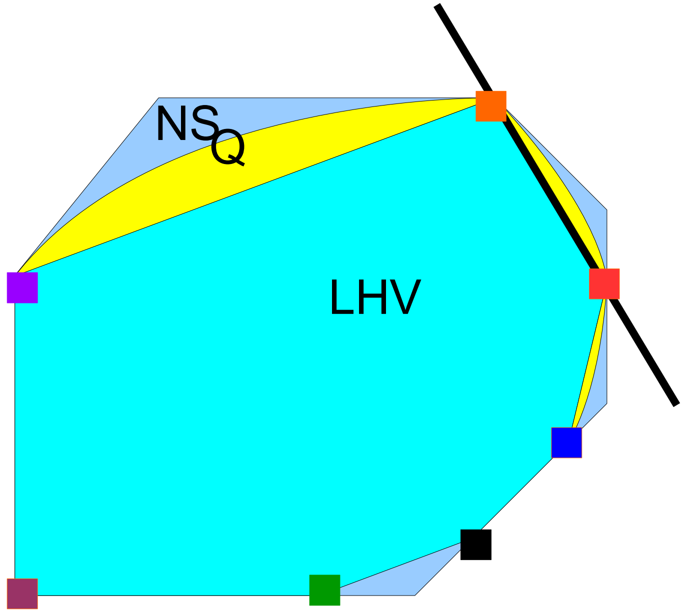

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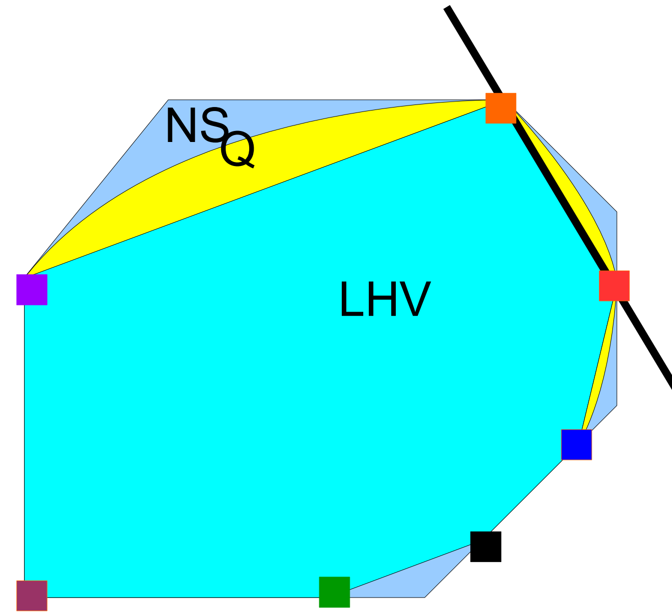

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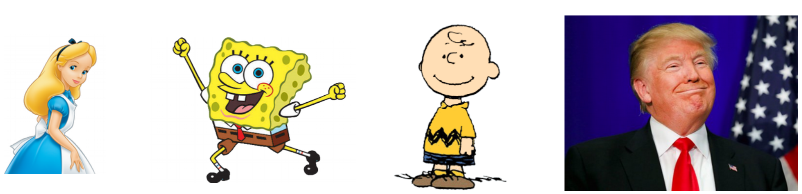

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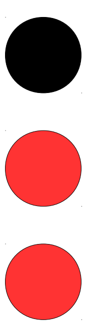

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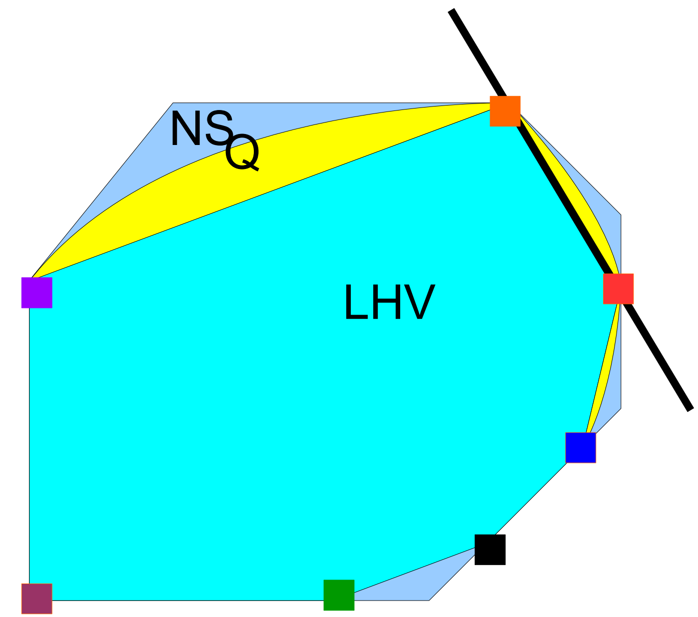

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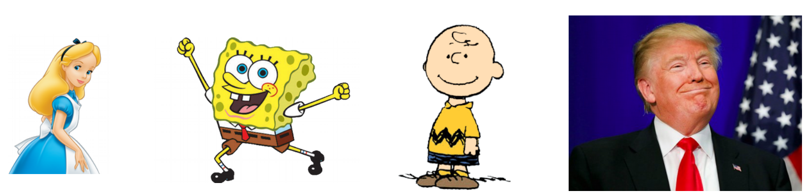

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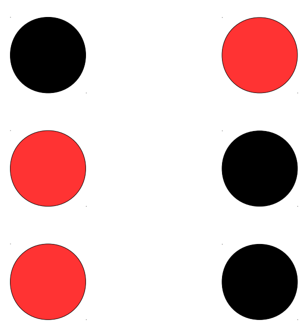

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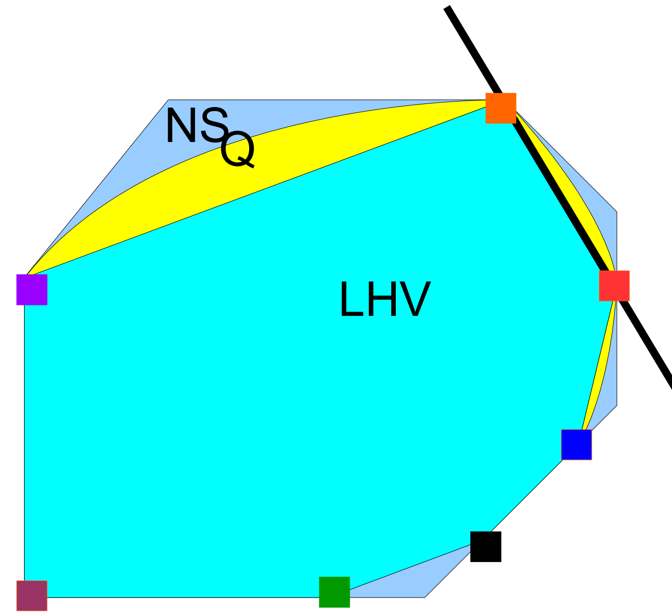

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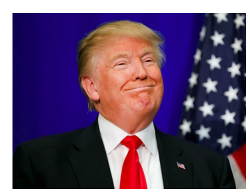

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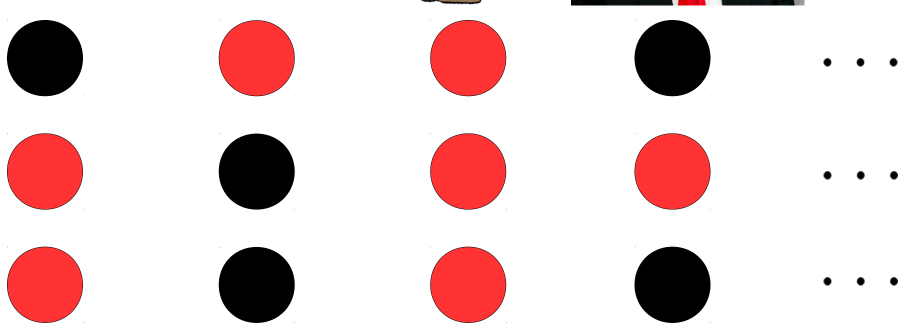

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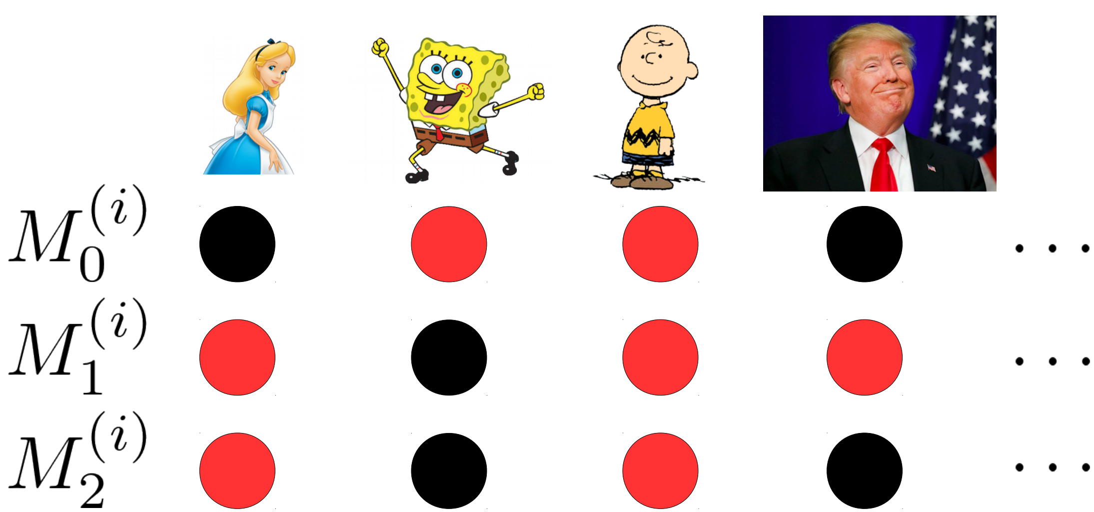

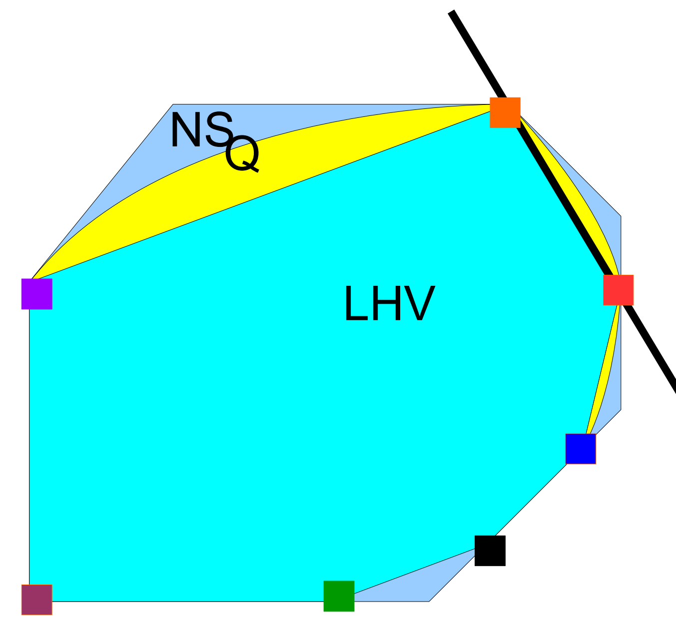

Problem

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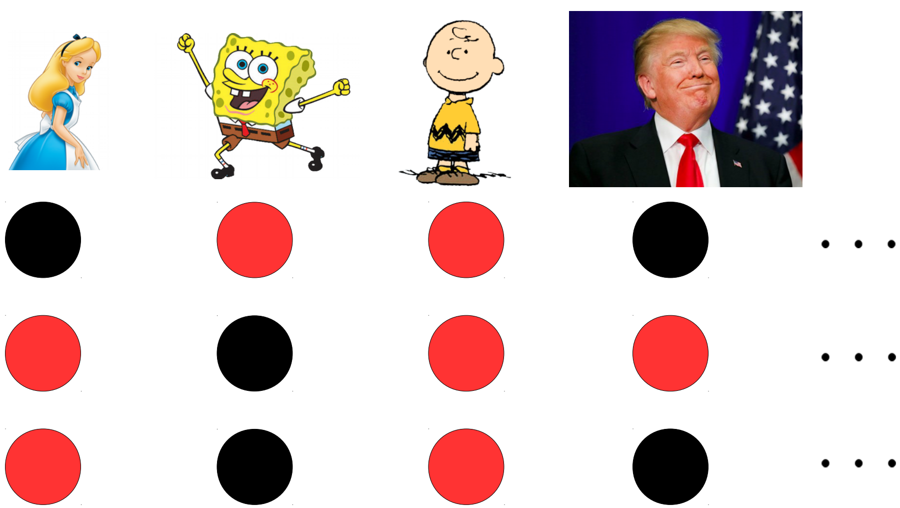

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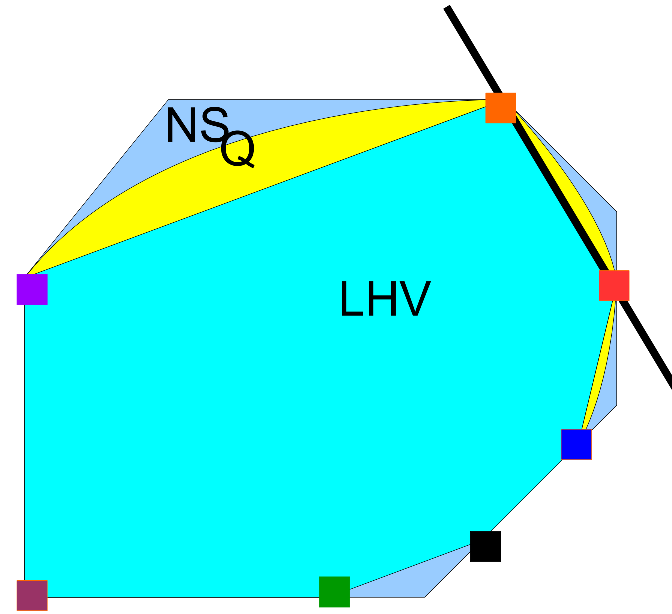

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2^{mn} vertices

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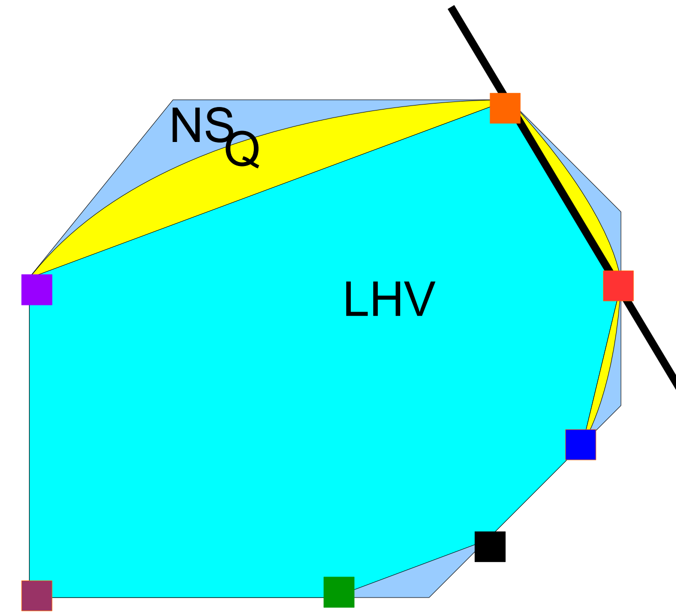

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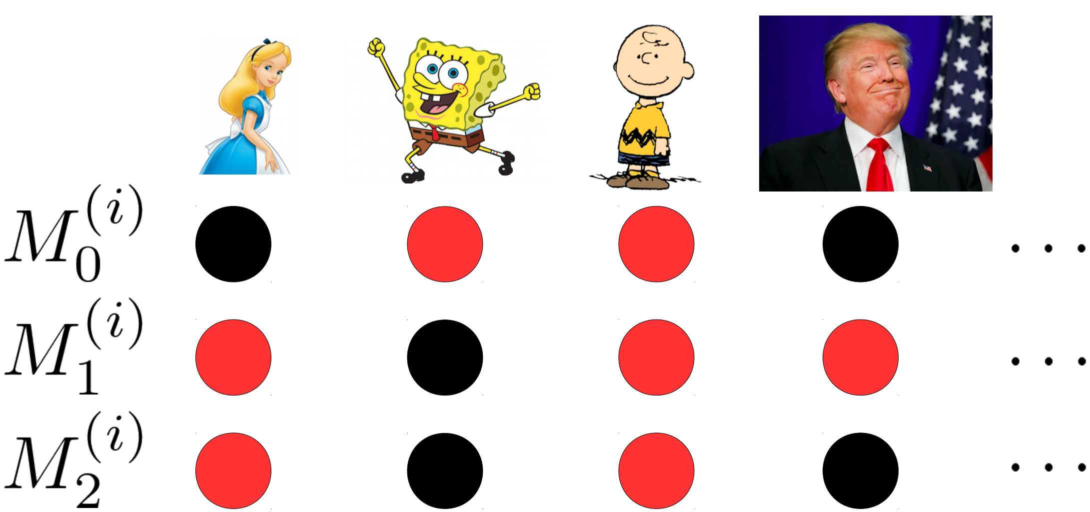

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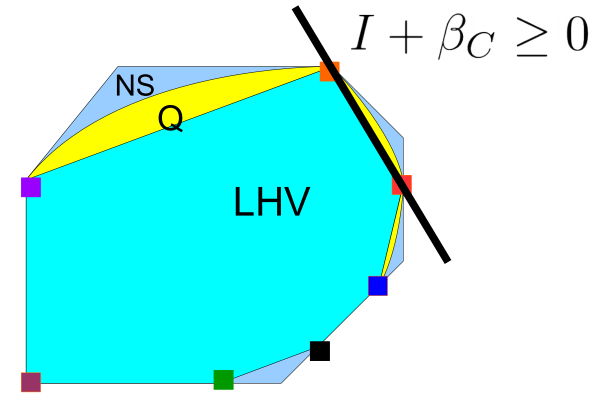
For our Bell inequalities

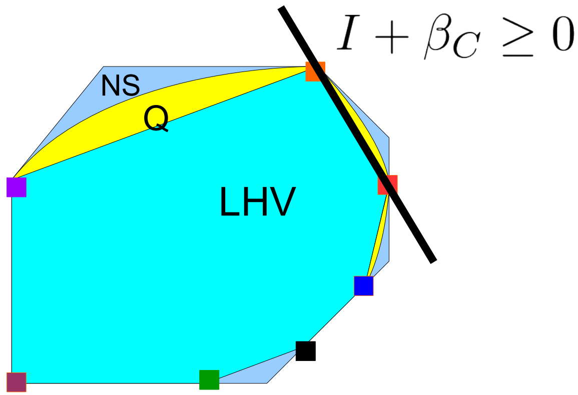
Dynamic programming

Finding the classical bound



Finding the classical bound

- Optimization over all LHV models



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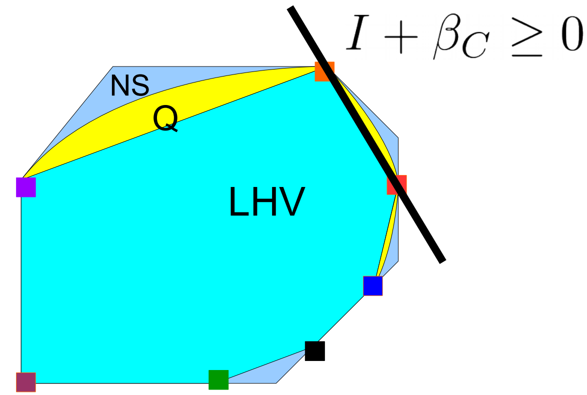

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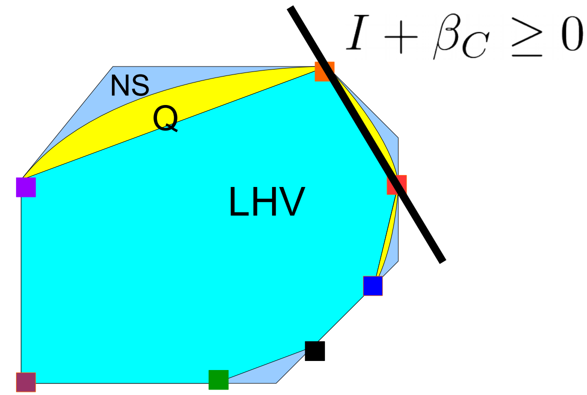

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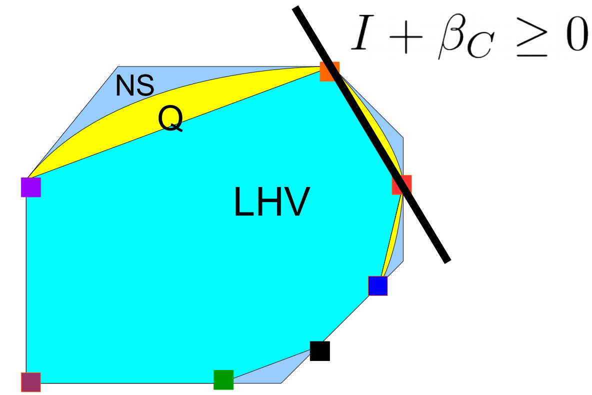
[N. Schuch, J. I. Cirac, Phys. Rev. A. **82**, 012314 (2010)]



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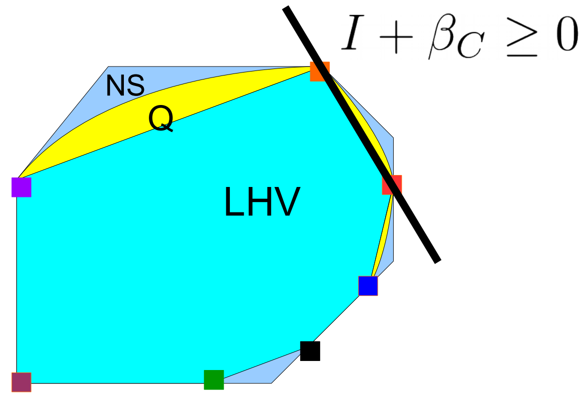

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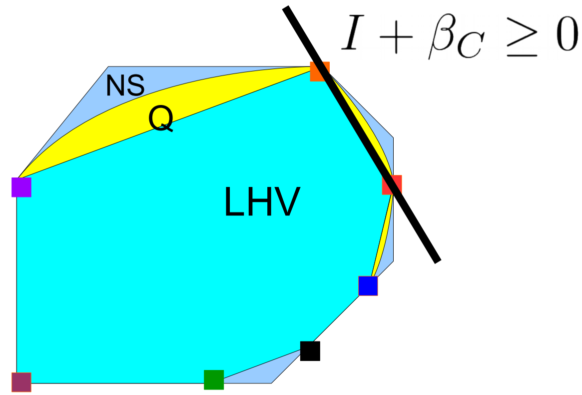

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•Recurrence relation

Finding the classical bound

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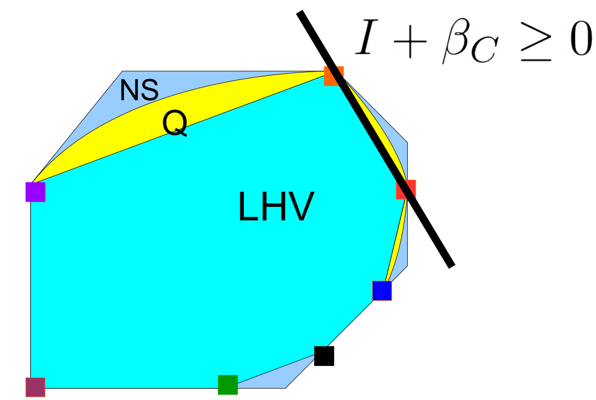

Ingredients
•Recurrence relation
•Compute & store intermediate sub-solutions

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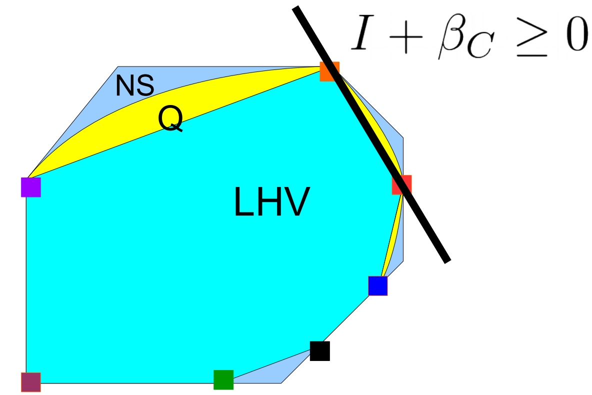

Ingredients

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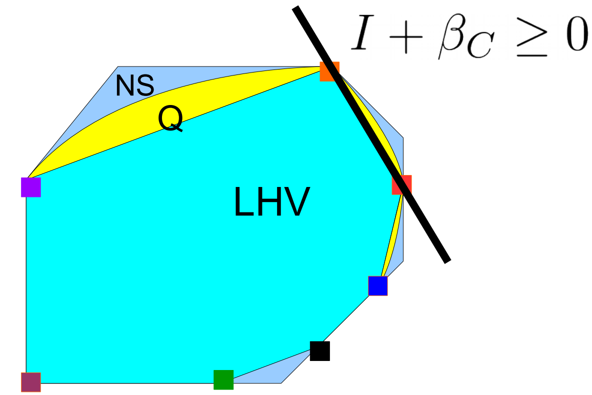

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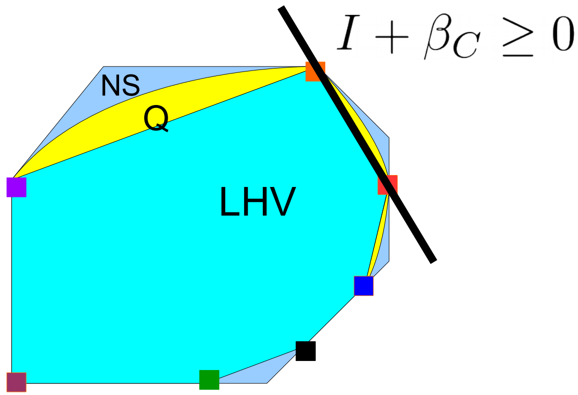

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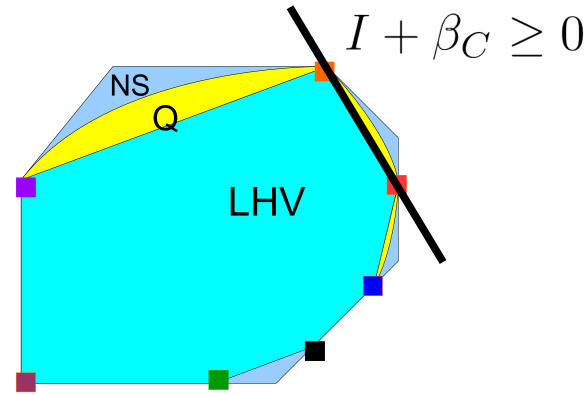

Ingredients
•Recurrence relation
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Result
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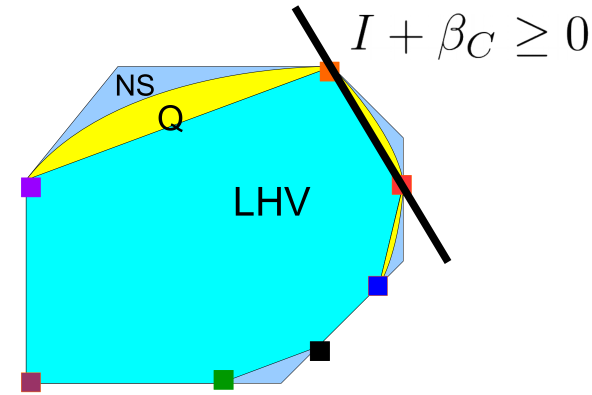

Ingredients
•Recurrence relation
•Compute & store intermediate sub-solutions
•Ordering of sub-solutions

Result
•Polynomial scaling
•Constructive method of 1 optimal solution

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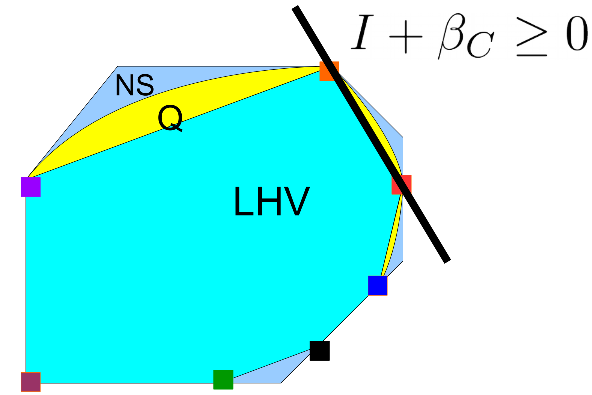

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Ingredients
•Recurrence relation
•Compute & store intermediate sub-solutions
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Result
•Polynomial scaling
•Constructive method of 1 optimal solution
•Much better than backtracking/brute force

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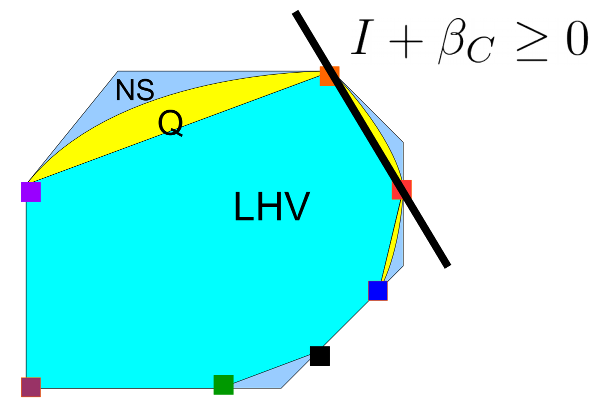

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# of problems by category at [TopCoder]®

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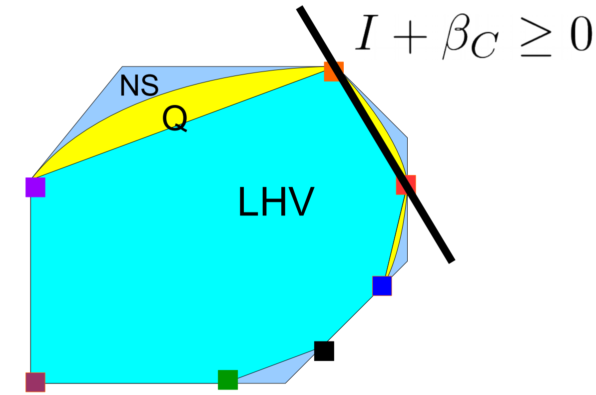

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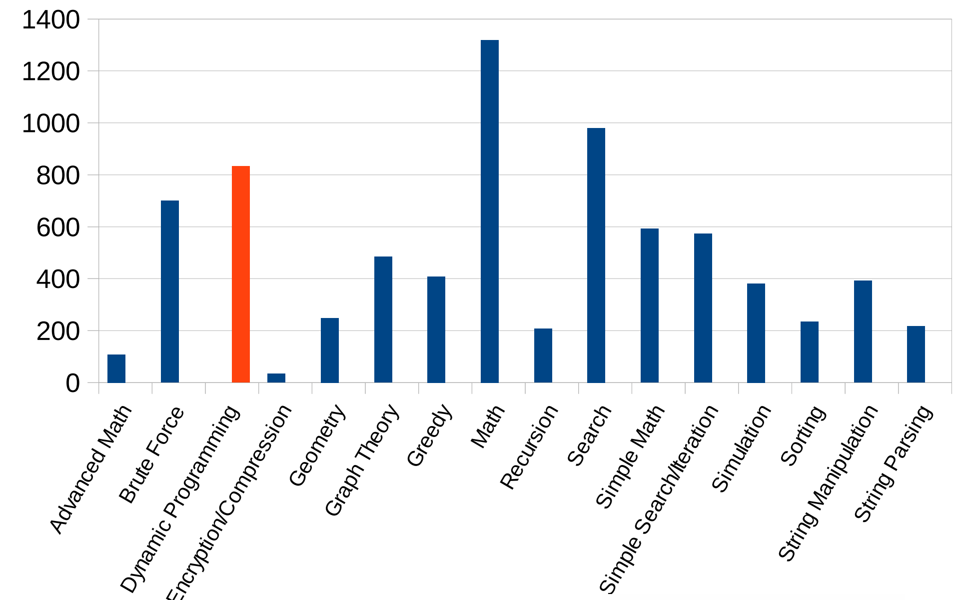
•Compute & store intermediate sub-solutions
•Ordering of sub-solutions

Result
•Polynomial scaling
•Constructive method of 1 optimal solution
•Much better than backtracking/brute force

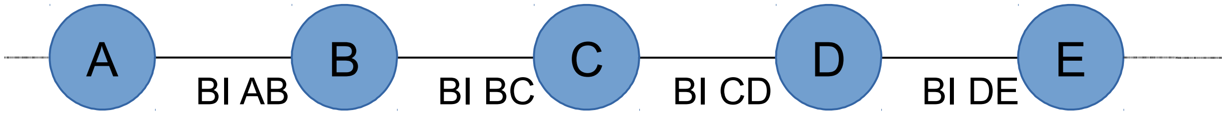


# of problems by category at [TopCoder]®

Dynamic programming

Dynamic programming

- The Bell Inequality as a sum of smaller BI



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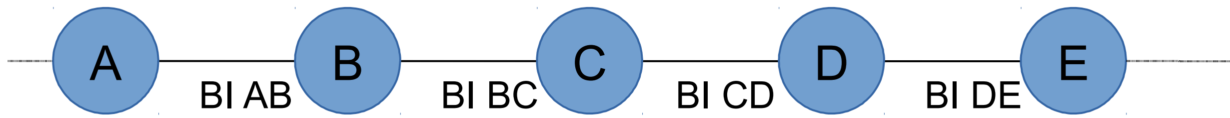

- The optimization

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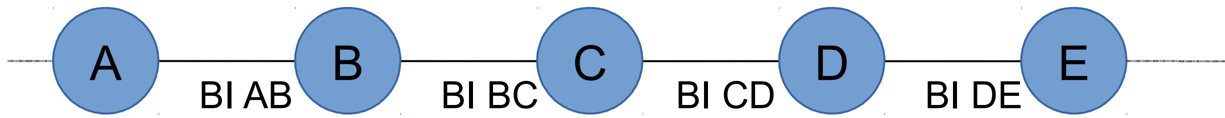

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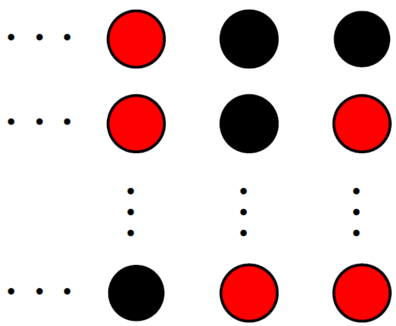

$$0$$
$$1$$
$$m-1$$

Dynamic programming

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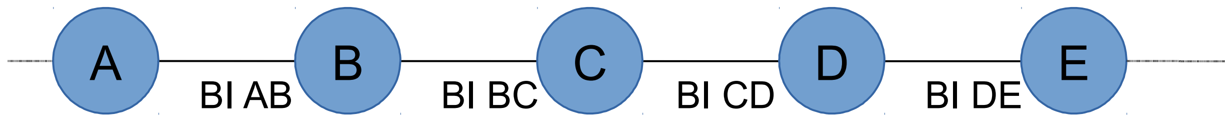

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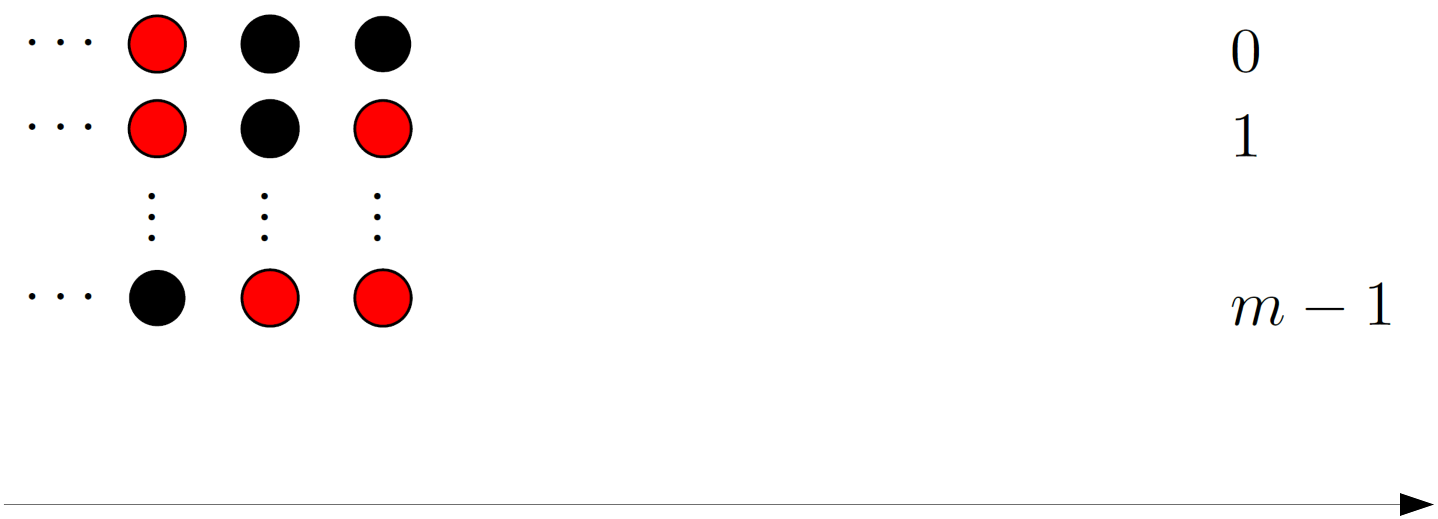

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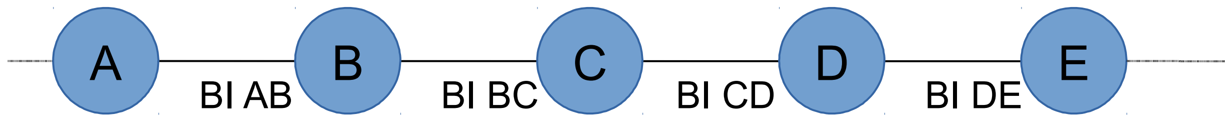

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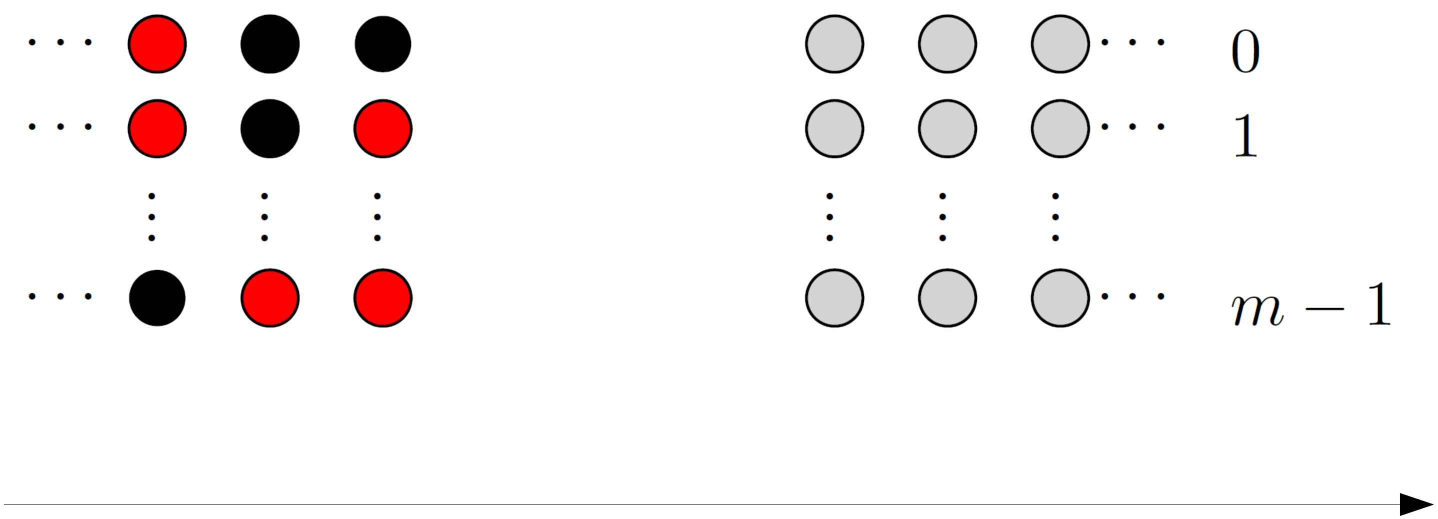

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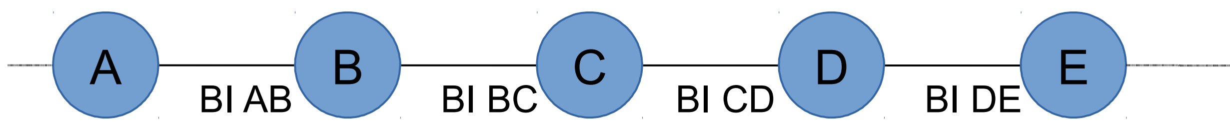

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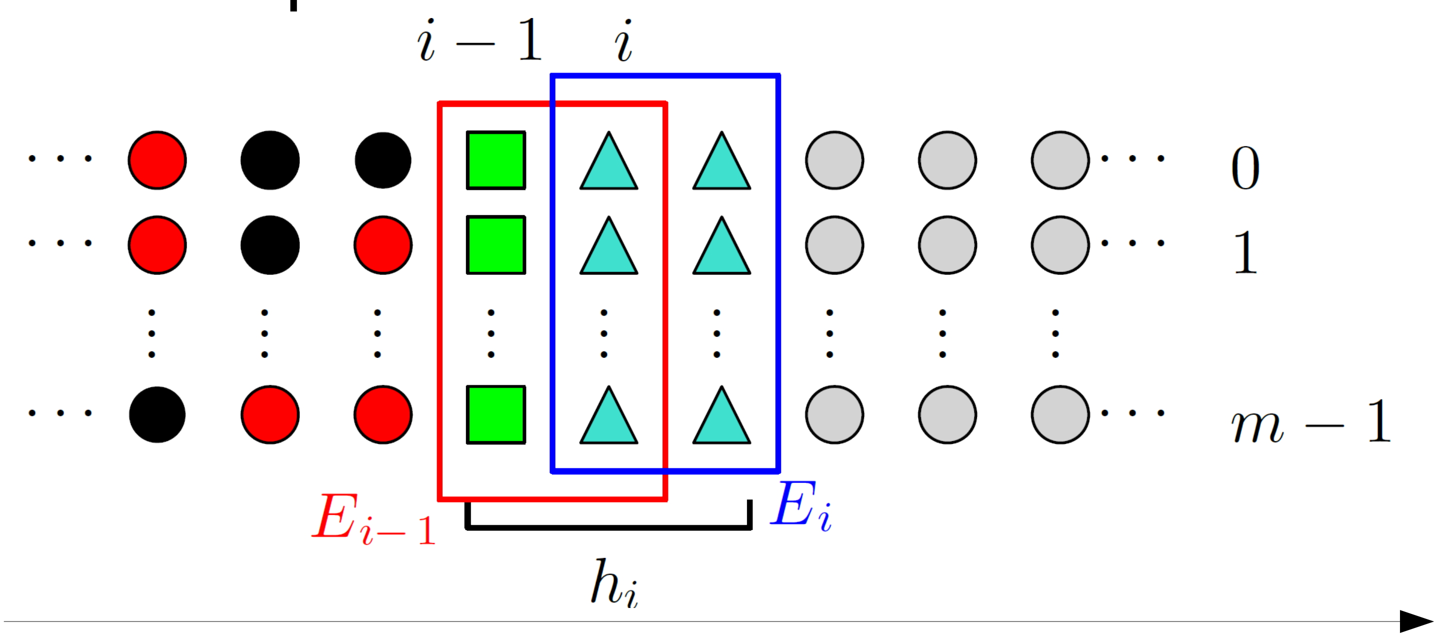

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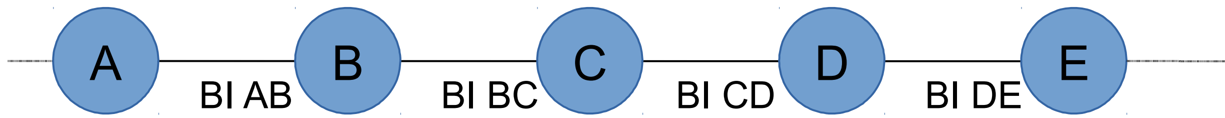

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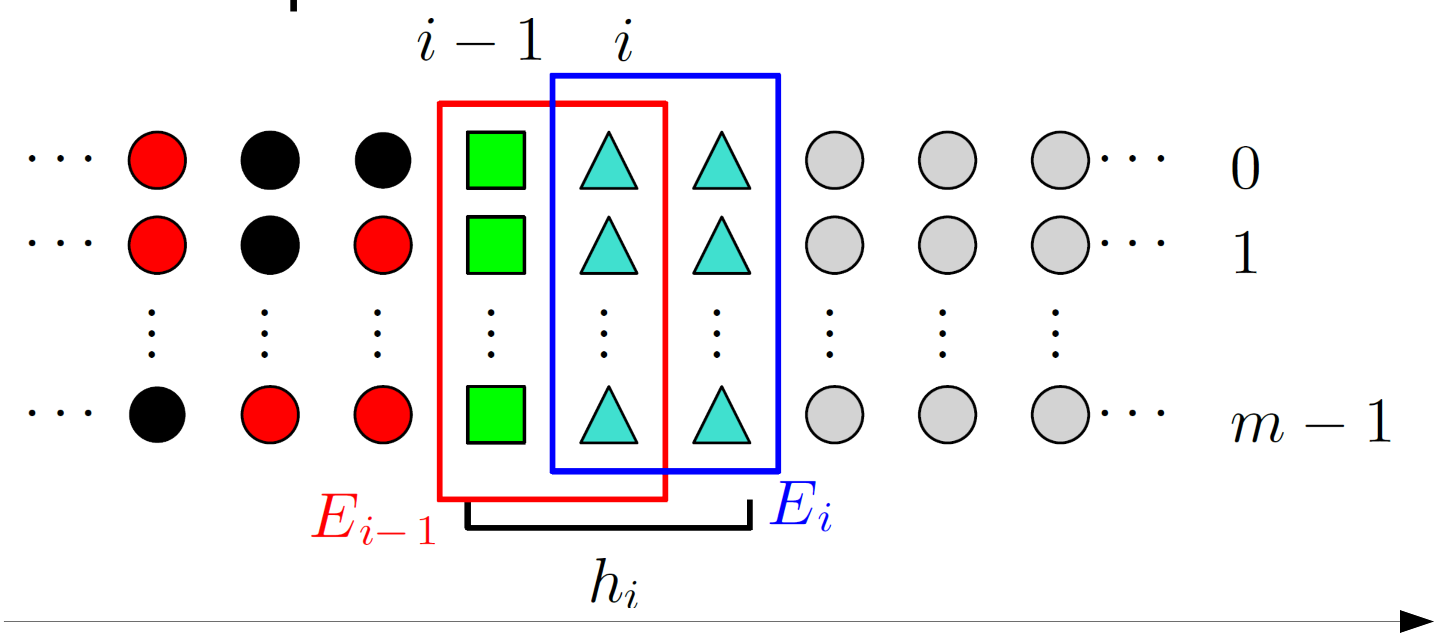

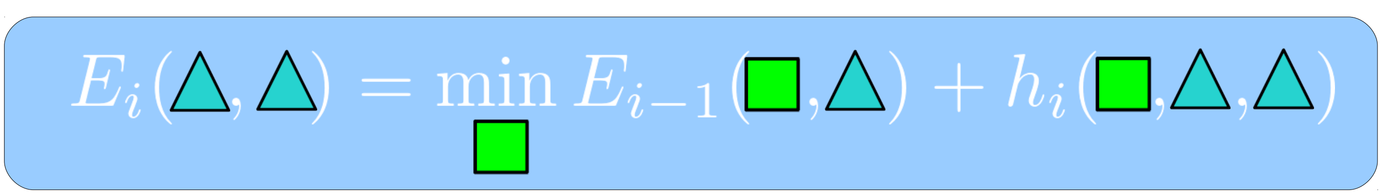

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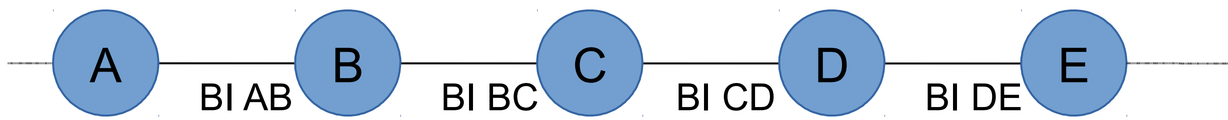

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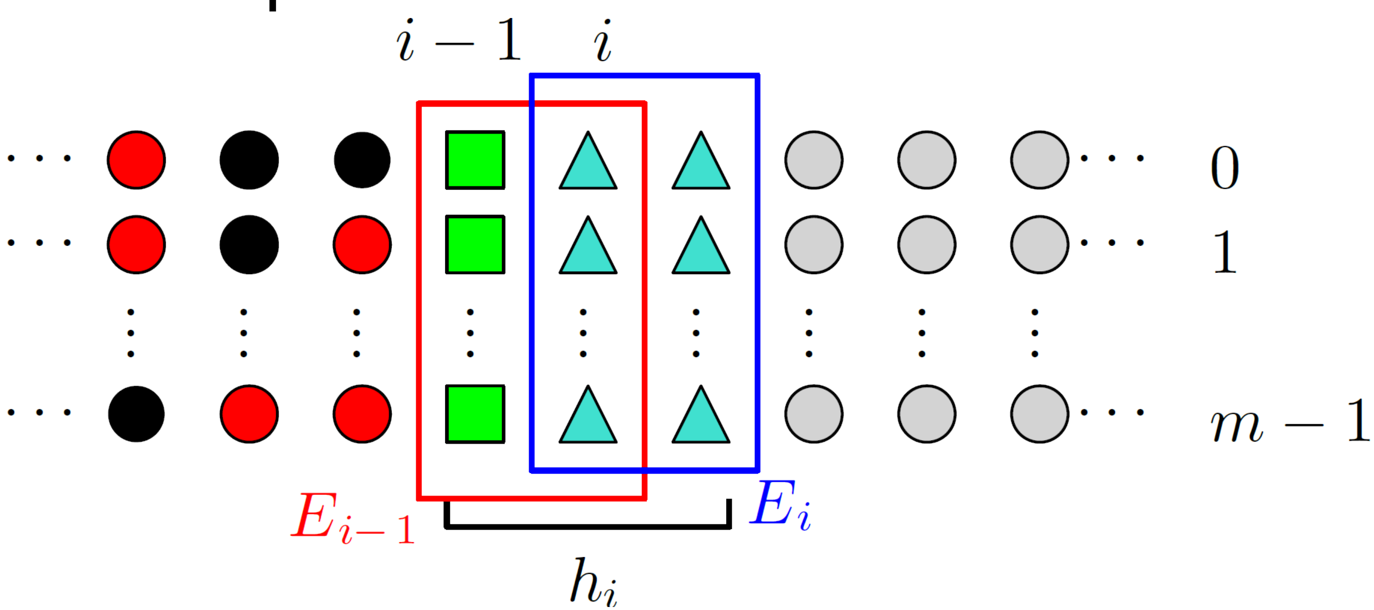

$$E_i(\triangle,\triangle) = \min_{\square} E_{i-1}(\square,\triangle) + h_i(\square,\triangle,\triangle)$$

Classical bound at

$$\beta_C := E_n$$

Dynamic programming

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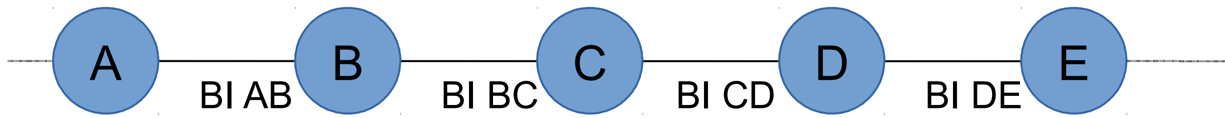

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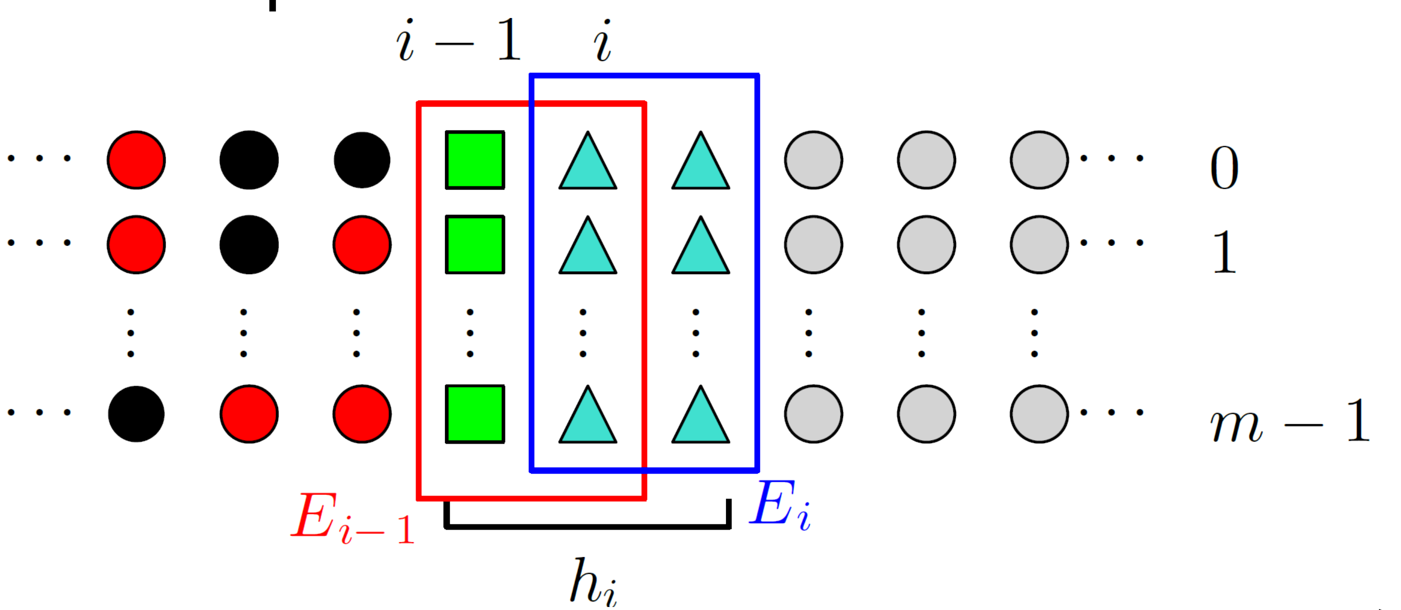

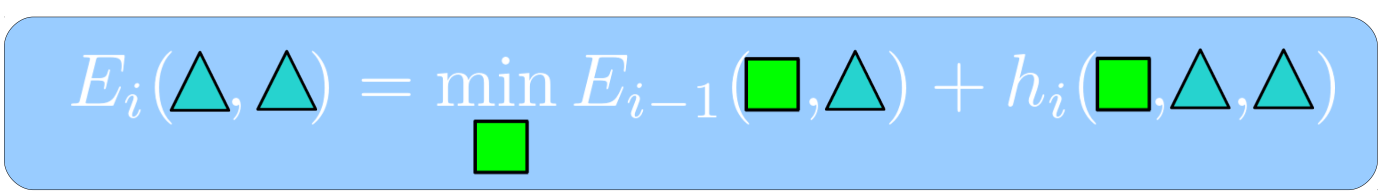

Classical bound at

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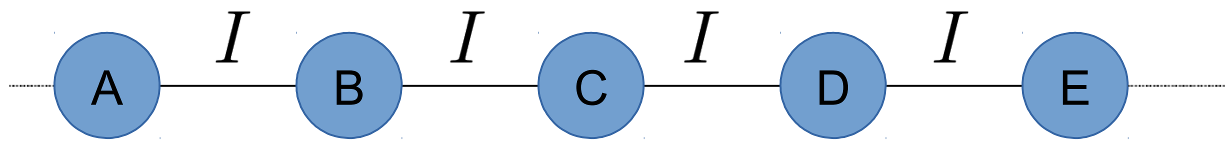
Overall complexity $O(n)$

Outline

- Motivation
- The idea, the setting
- Quantum optimization
- Assigning a Bell inequality to a Hamiltonian
- Classical optimization
- **Translational invariance**
- Examples
- Conclusions and outlook

Translationally invariant BI

Translationally invariant BI



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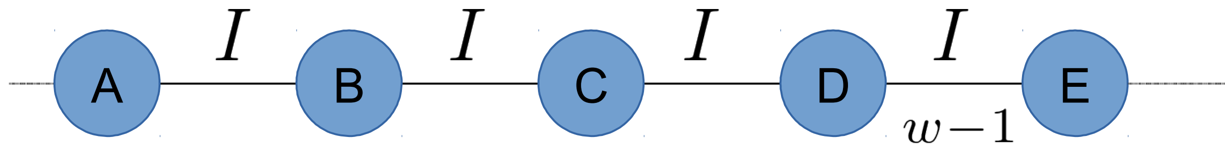

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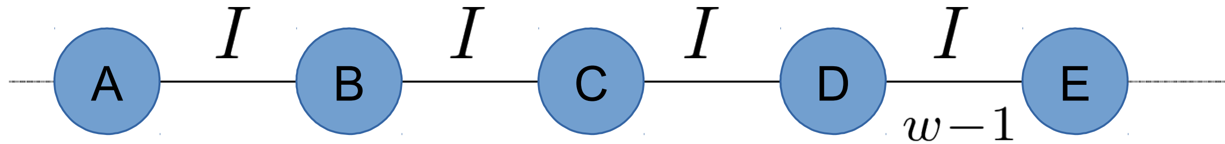

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  by eliminating half of the variables at each step

$$f^{(t+1)}(x,y) = \min_z (f^{(t)}(x,z) + f^{(t)}(z,y))$$

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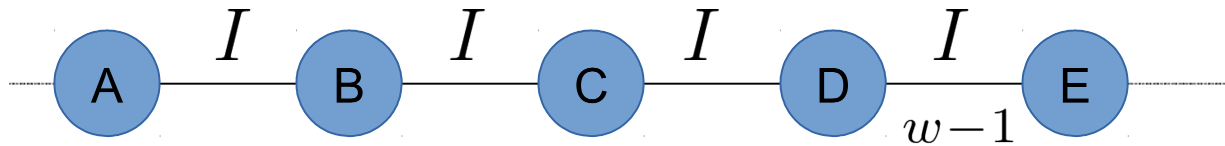

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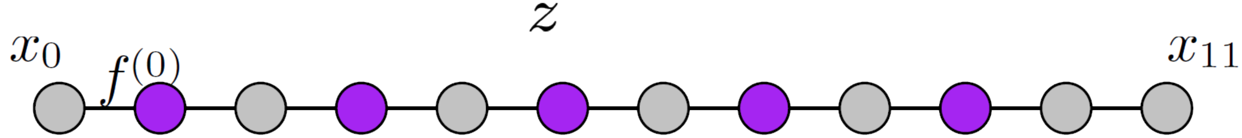

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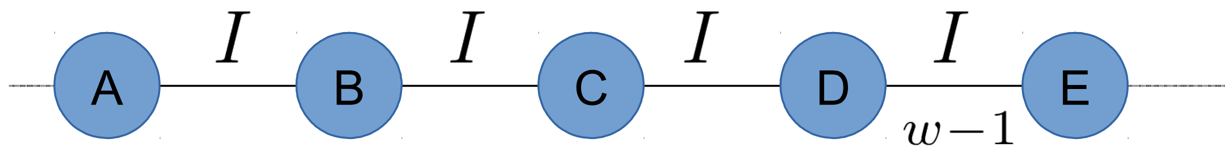

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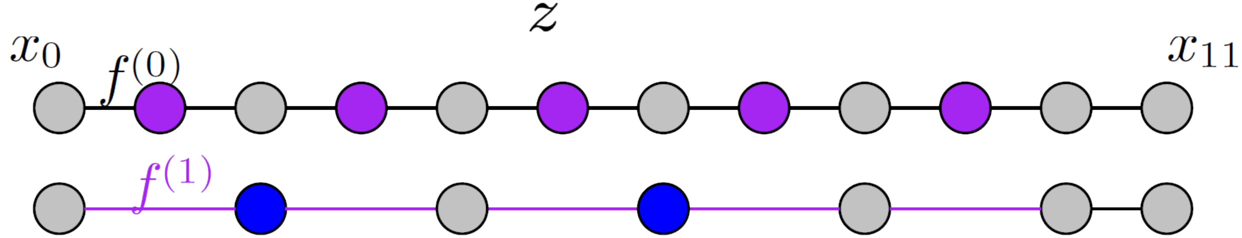

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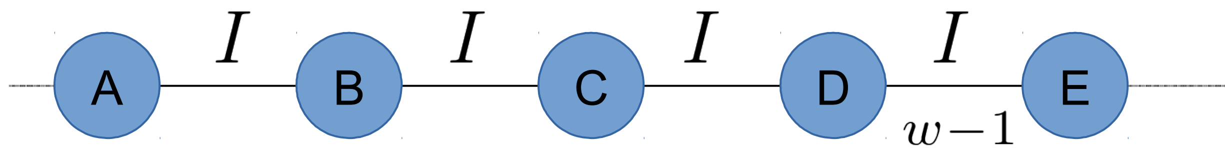

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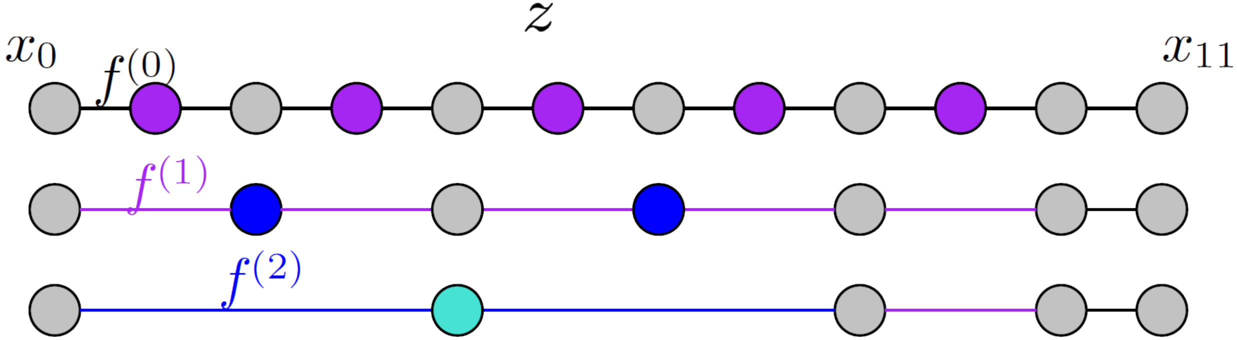

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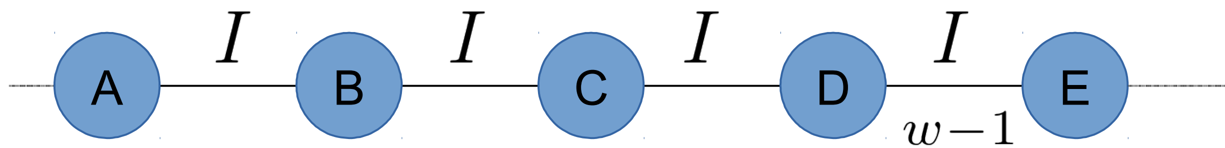

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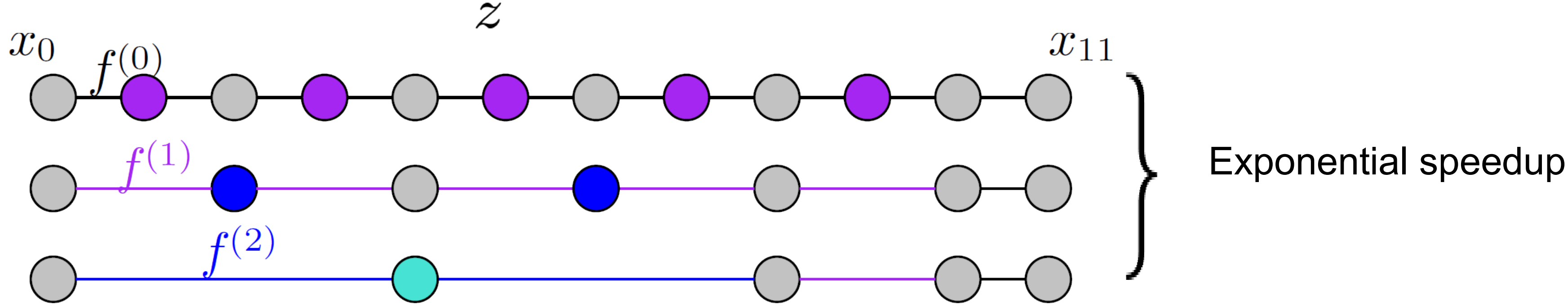

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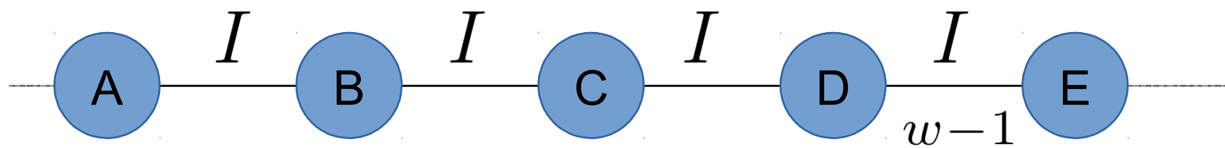

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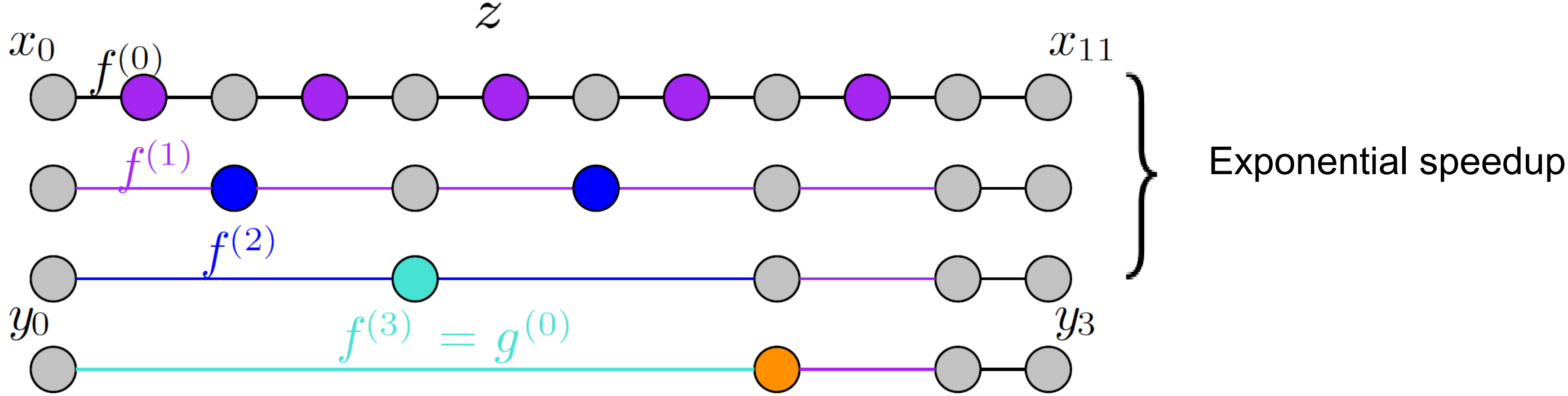

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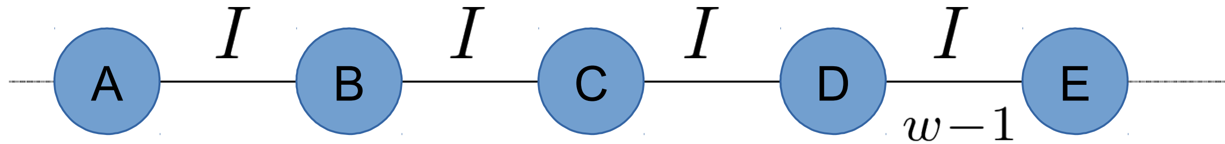

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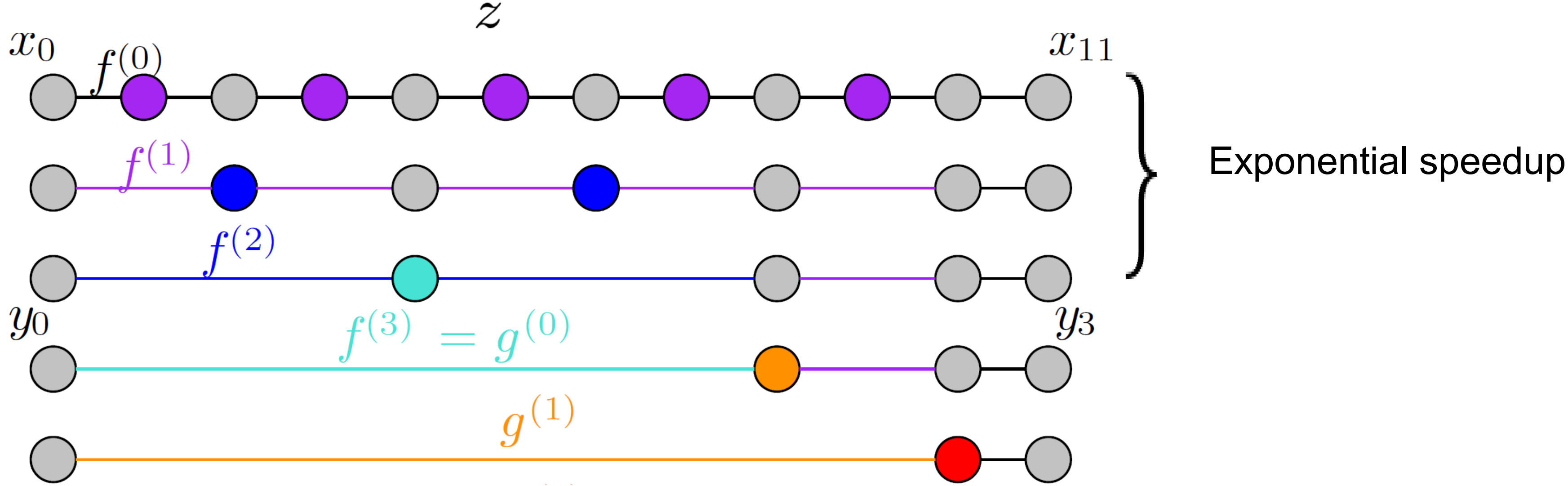

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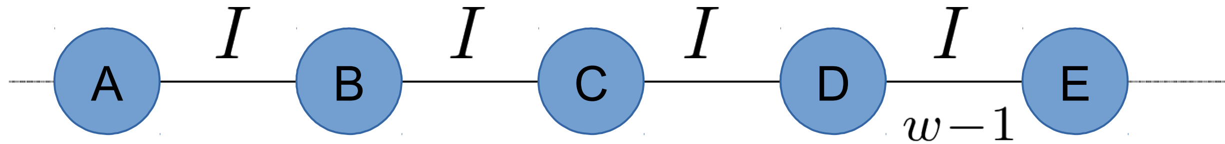

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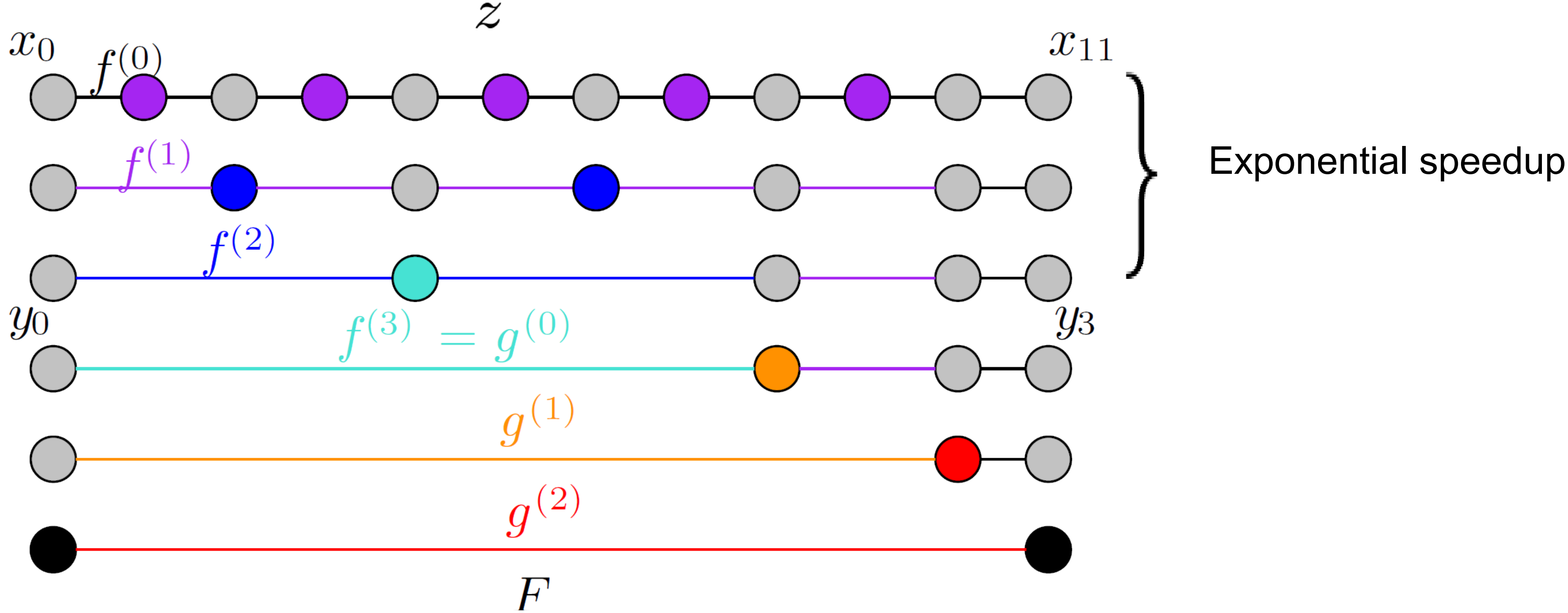

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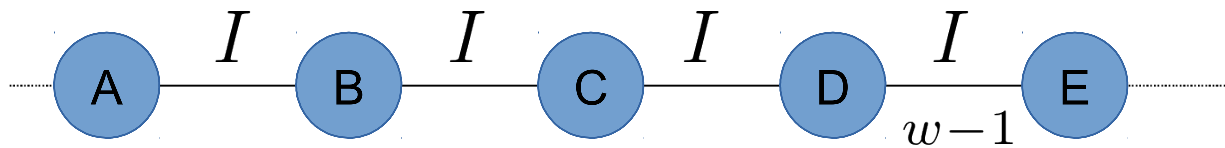

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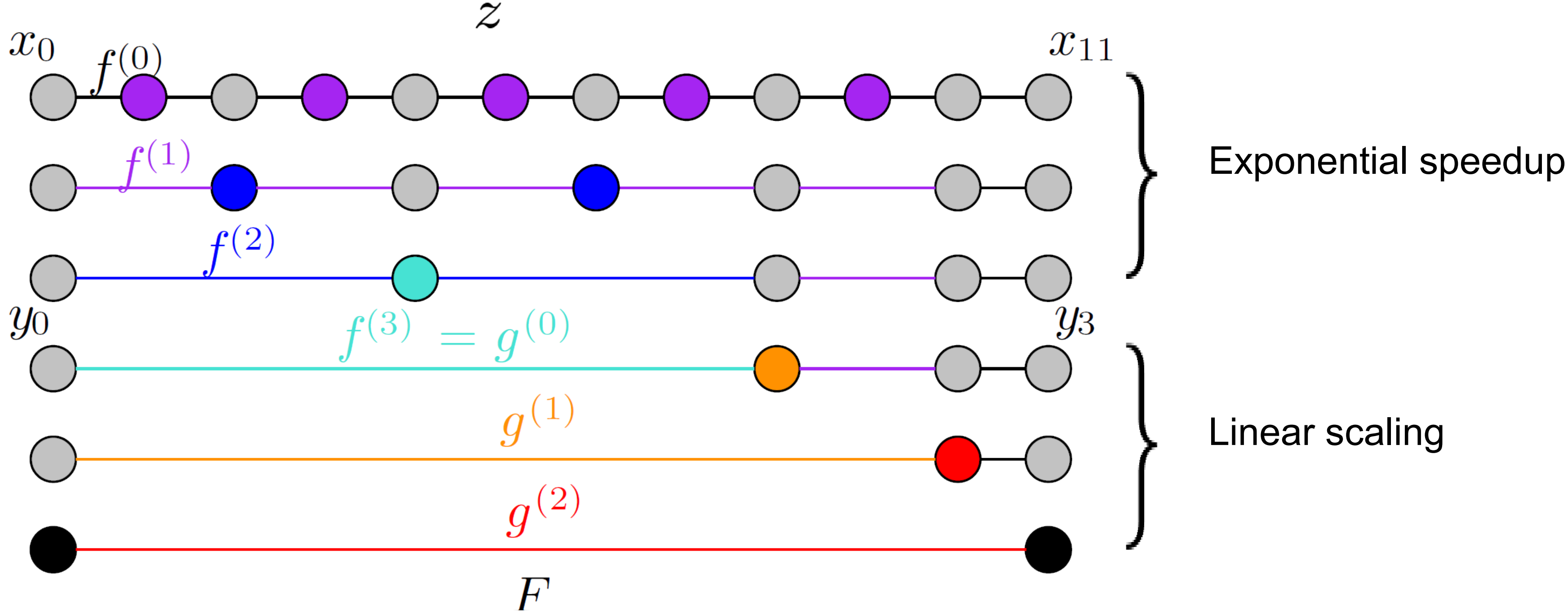

Application to an inequality with $R > 1$

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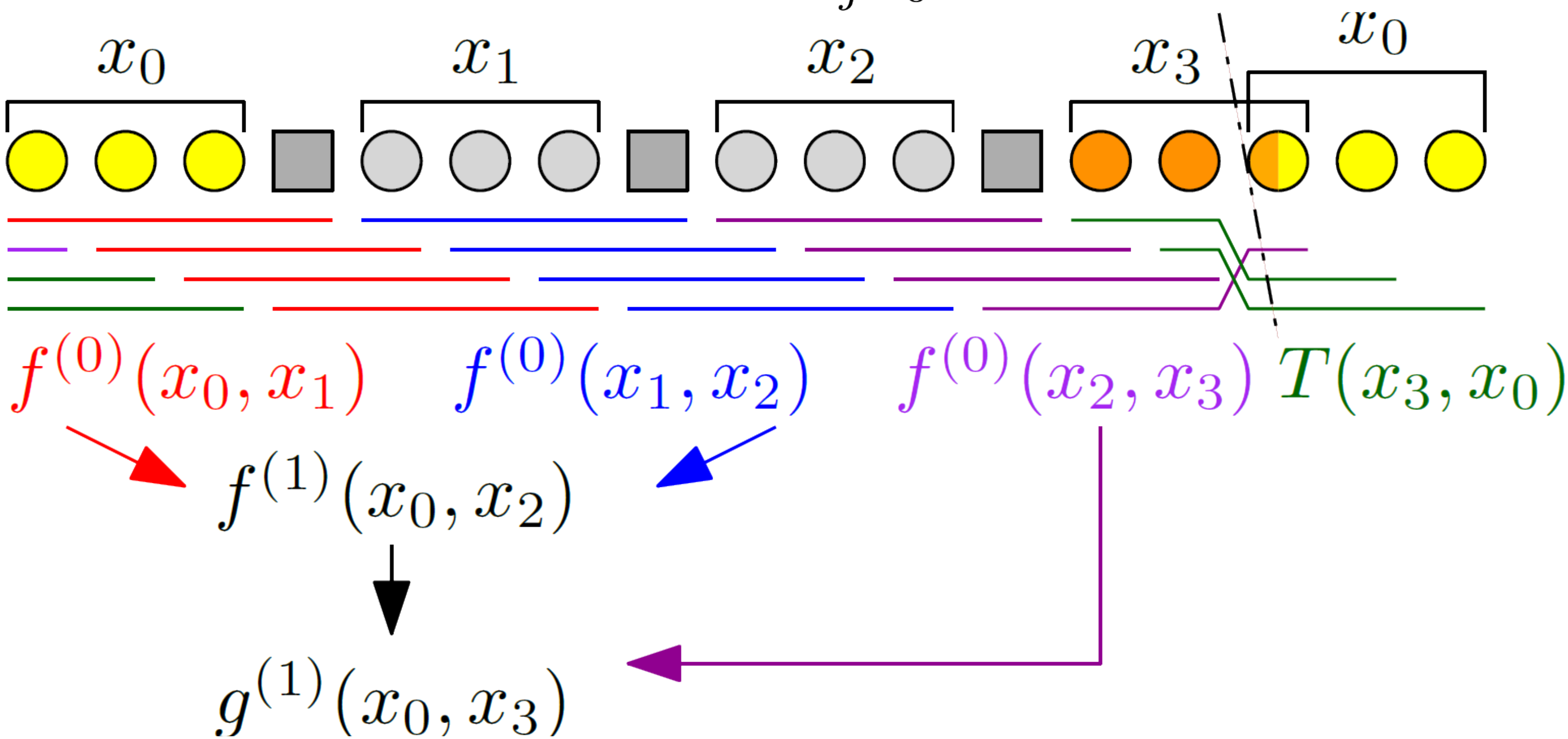

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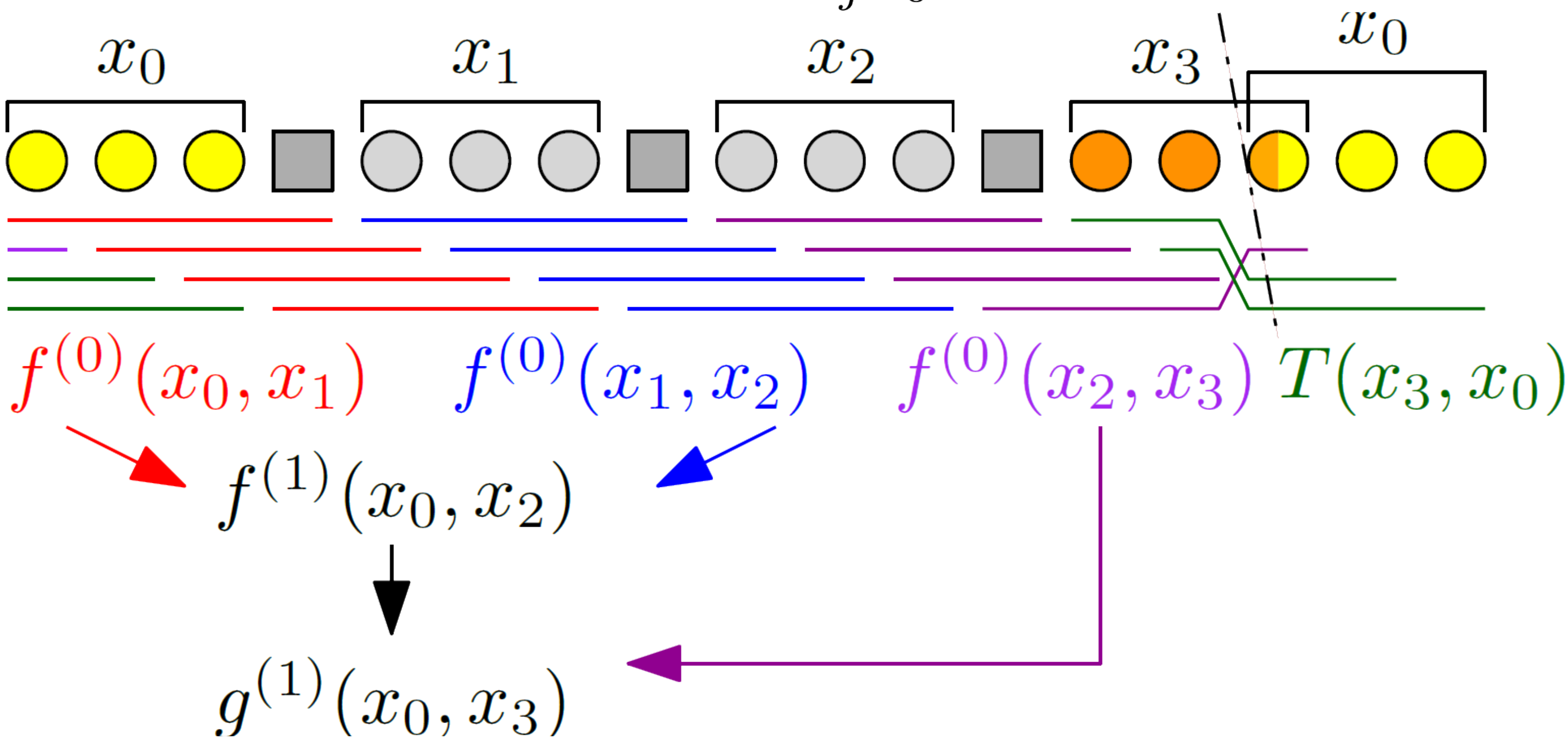

$O(\log n)$ overall complexity

Translationally invariant Hamiltonian (I)

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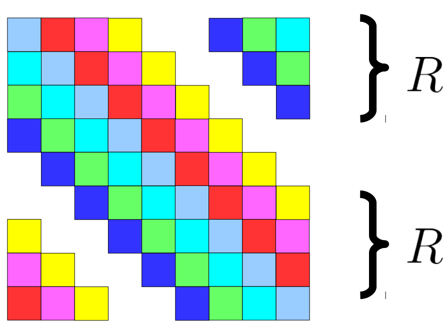

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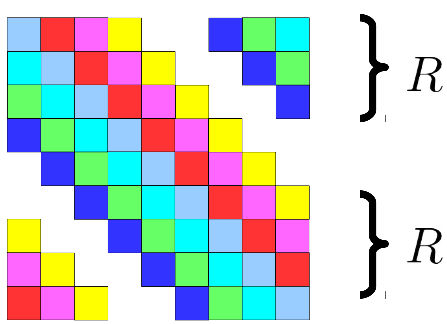

If the fermion system has parity -1

Translationally invariant Hamiltonian (I)

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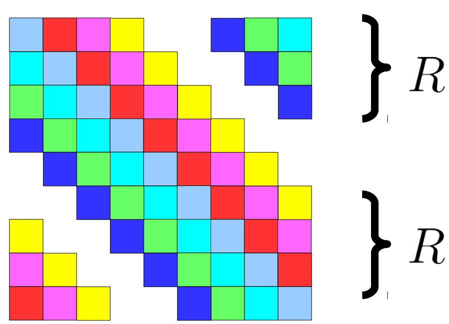

If the fermion system has parity -1
Discrete Fourier Transform will diagonalize it

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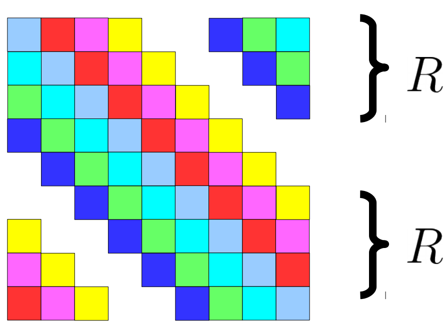

If the fermion system has parity -1
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$$(\mathcal{F}_n)_{kl} := \frac{1}{\sqrt{n}} \omega^{k \cdot l}, \qquad \omega^n = 1$$

Translationally invariant Hamiltonian (II)

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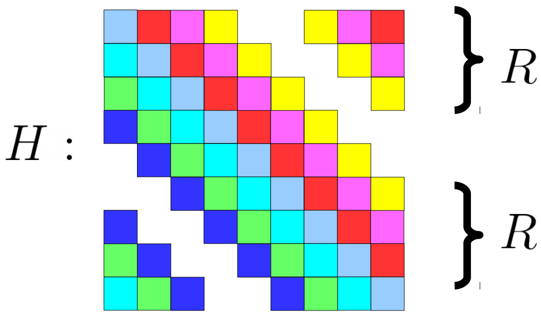
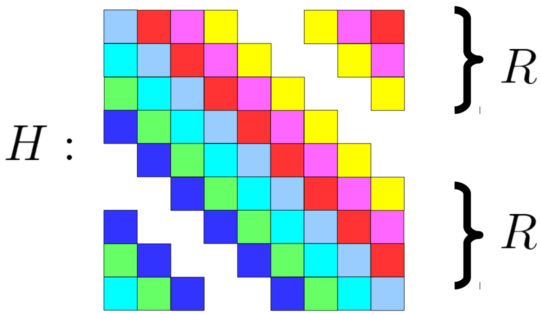
If the fermion system has parity 1

Translationally invariant Hamiltonian (II)



If the fermion system has parity 1
it is no longer circulant, but

Translationally invariant Hamiltonian (II)



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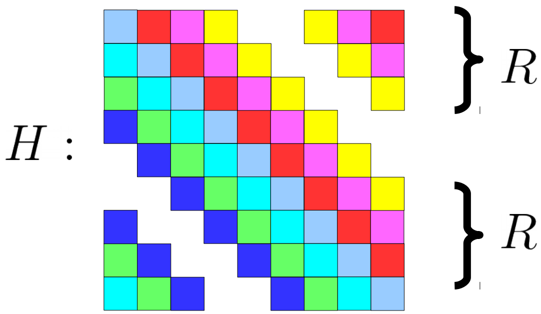

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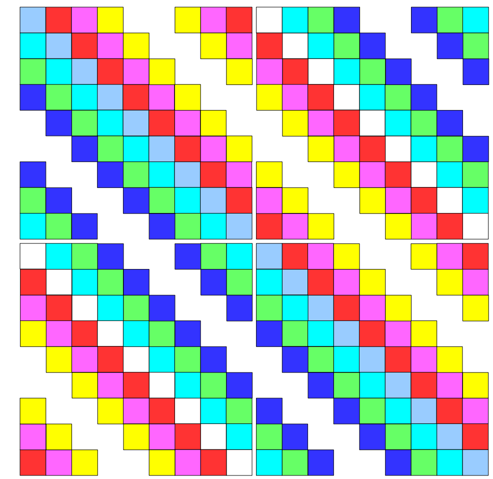

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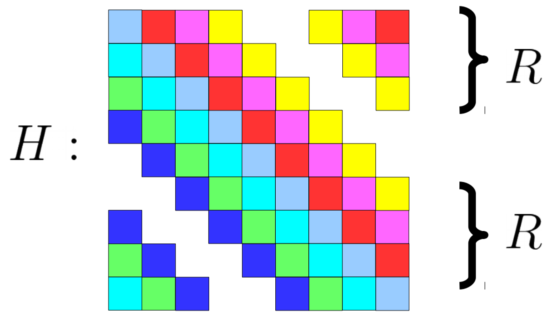

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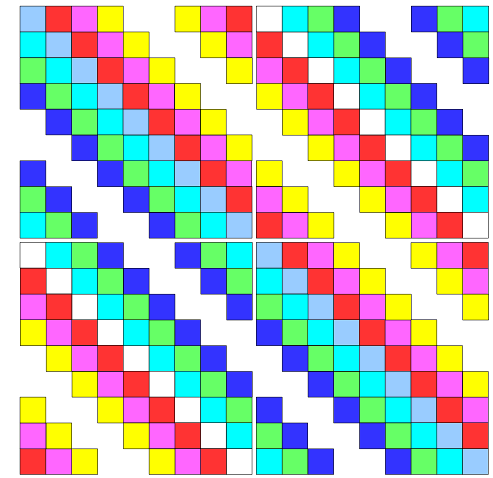

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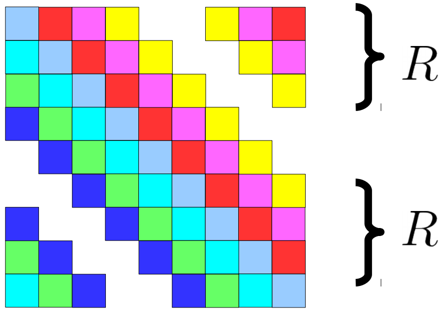

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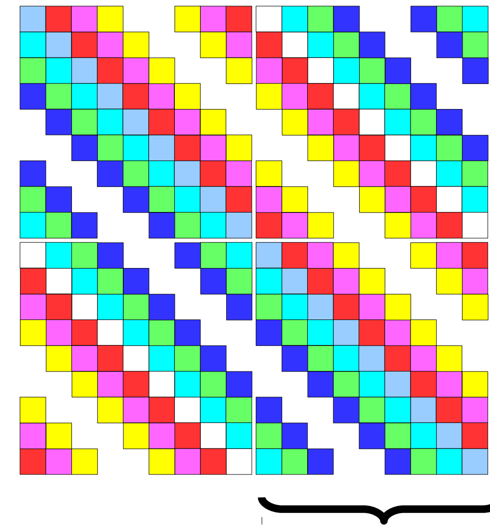

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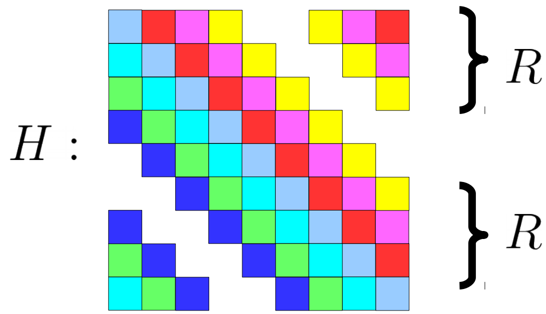

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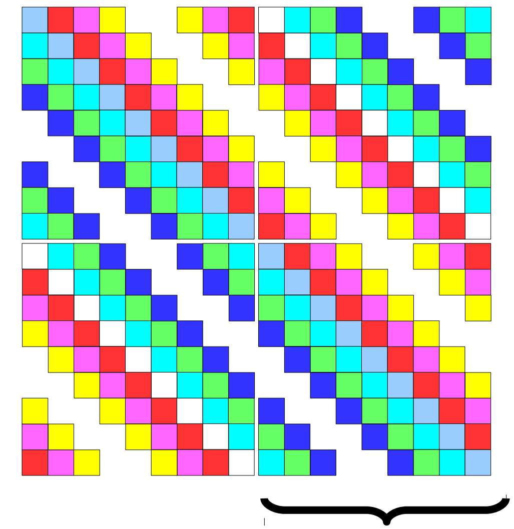

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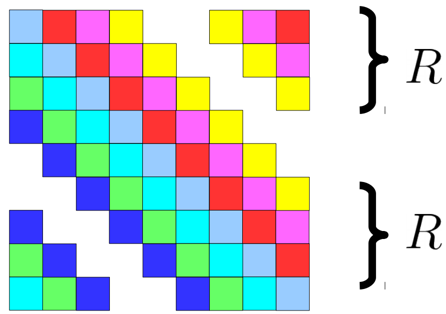
$H :$ 

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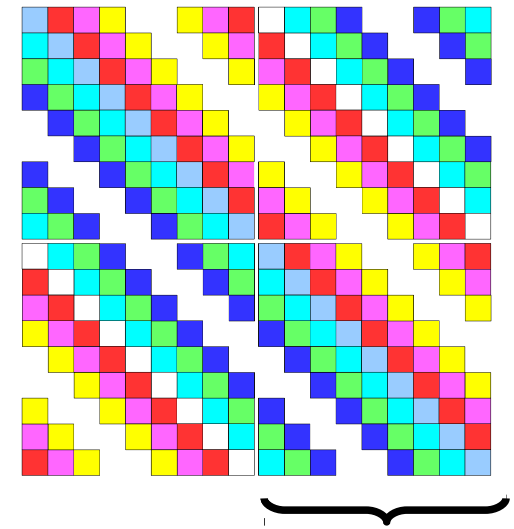

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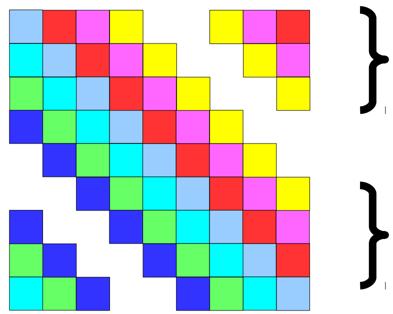

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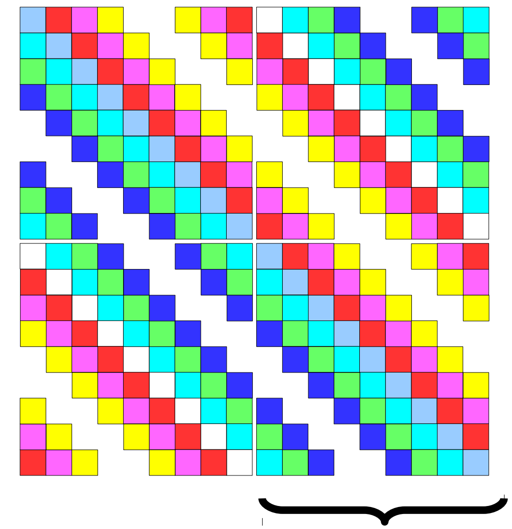

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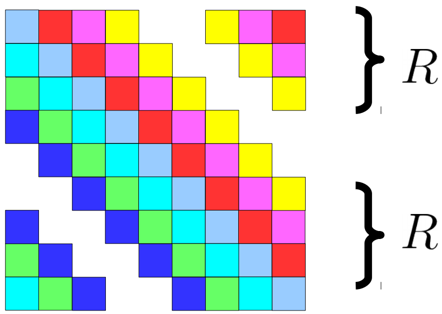
$H:$ 

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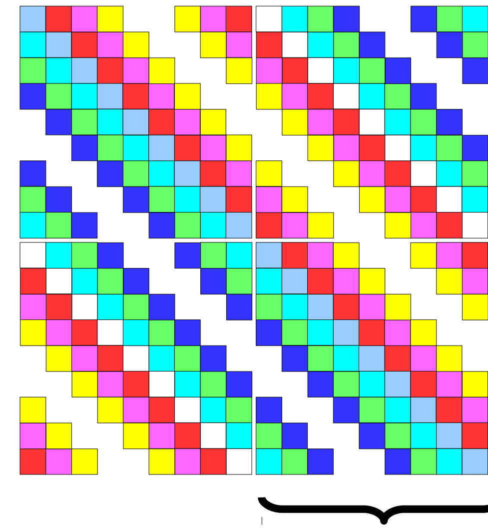

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$$\left\{ \begin{array}{lcl} x_k & = & H_{00;01} + \sum_{r=1}^{R} \cos(\nu_{k,r})(H_{00;r1} - H_{01;r0}) \\ a_k & = & -2\sum_{r=1}^{R} \sin(\nu_{k,r}) H_{00;r0} \\ b_k & = & -\sum_{r=1}^{R} \sin(\nu_{k,r})(H_{00;r1} + H_{01;r0}) \\ c_k & = & -2\sum_{r=1}^{R} \sin(\nu_{k,r}) H_{11;r0} \end{array} \right.$$

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Outline

- Motivation
- The idea, the setting
- Quantum optimization
- Assigning a Bell inequality to a Hamiltonian
- Classical optimization
- Translational invariance
- **Examples**
- Conclusions and outlook

Examples (Ia)

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- The projected polytope approach

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Finding all Bell inequalities ⟷ Convex Hull problem

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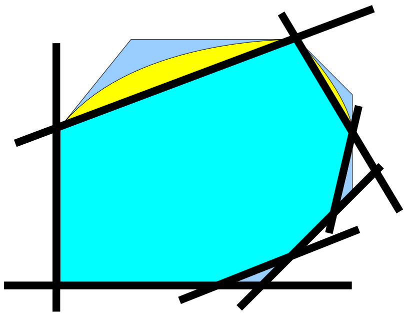
Finding all Bell inequalities ◄——► Convex Hull problem



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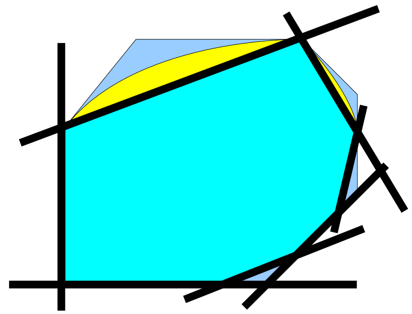
Finding all Bell inequalities ⟷ Convex Hull problem

(n, m, d) scenario



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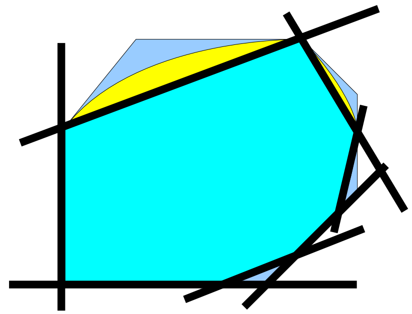

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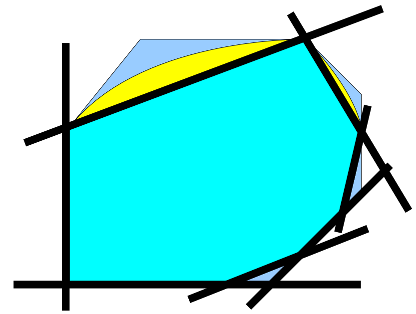

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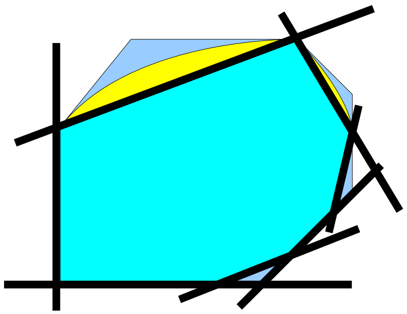

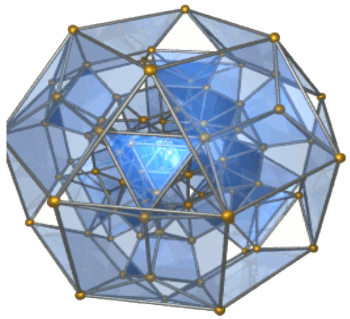

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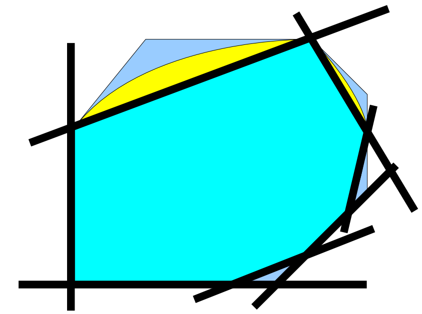

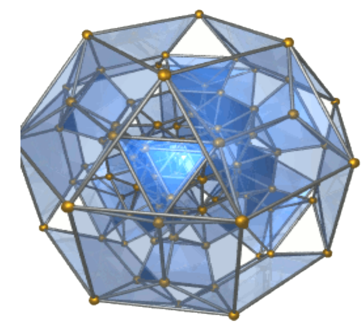

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[B. Chazelle, *An optimal convex hull algorithm in any fixed dimension, Discrete Comput. Geom.* **10** 377409 (1993)]

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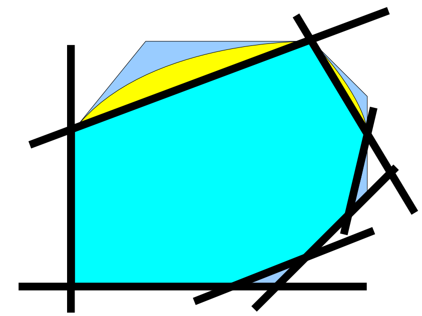

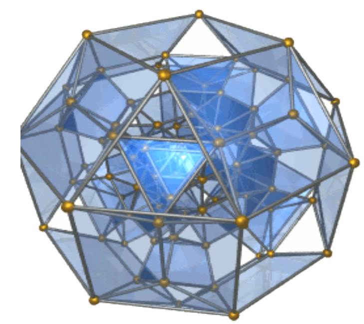

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[redacted]

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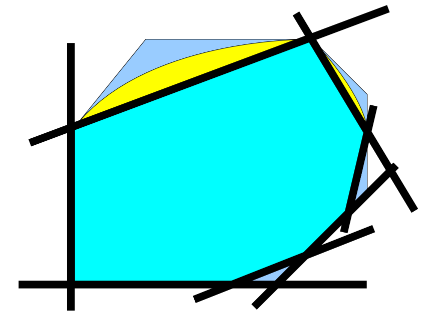

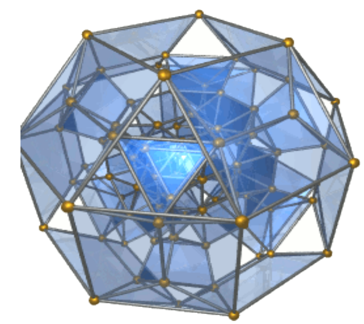

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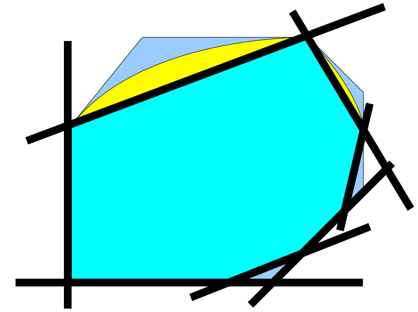

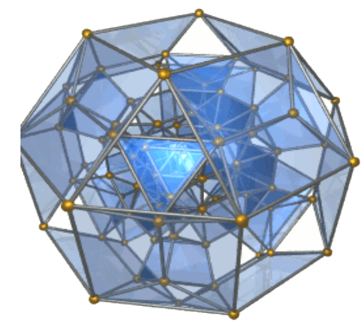

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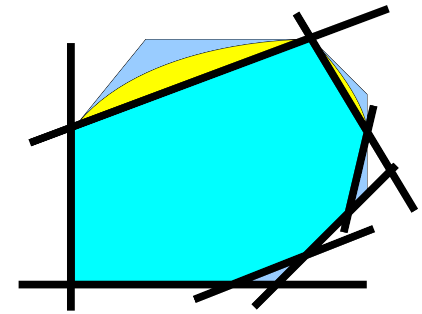

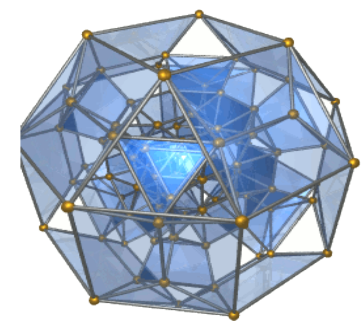

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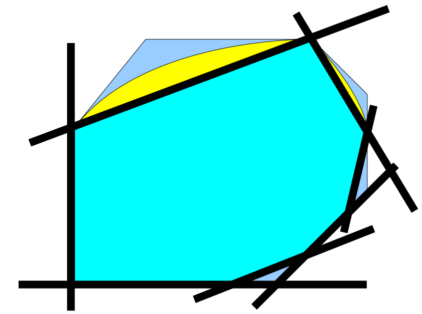

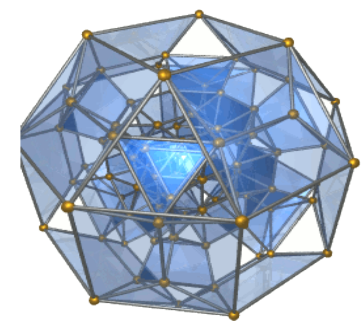

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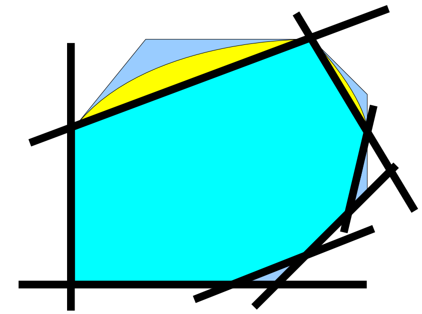

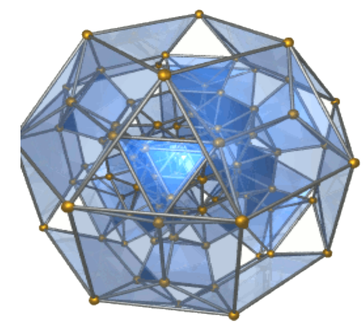

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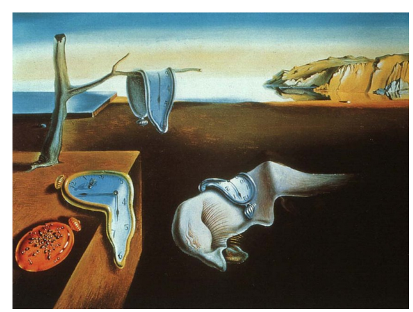
$(10^4,2,2) \longrightarrow 10^{10^{10^{4.67867\ldots}}}$ basically any timescale you want



[S. Dalí *The persistence of memory (1931)*]

Examples (Ib)

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- Projecting $\mathbb{P}_L$ to the space of few-body, TI BI

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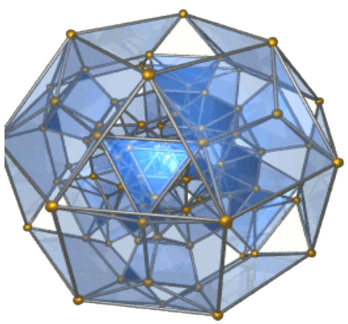

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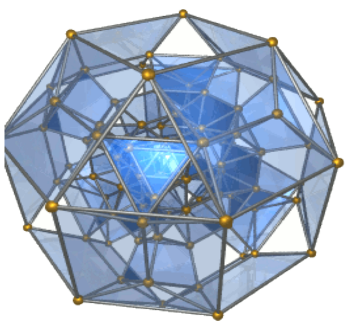

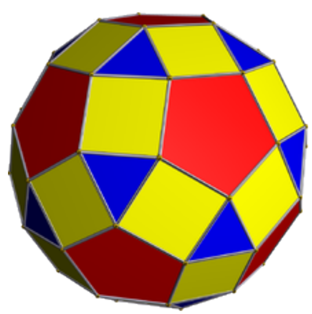

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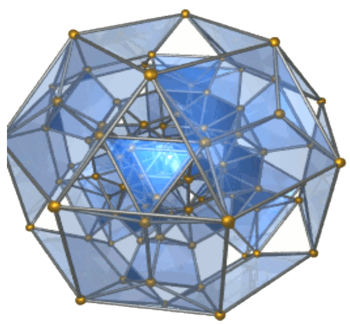
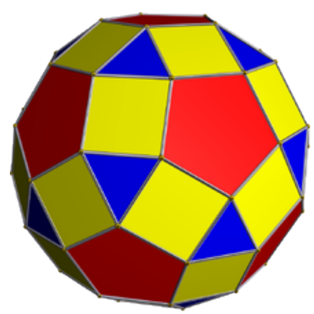
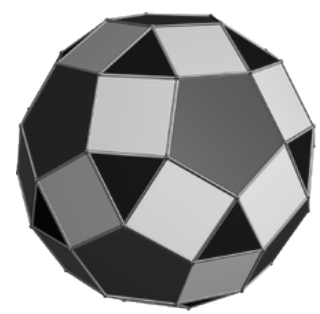

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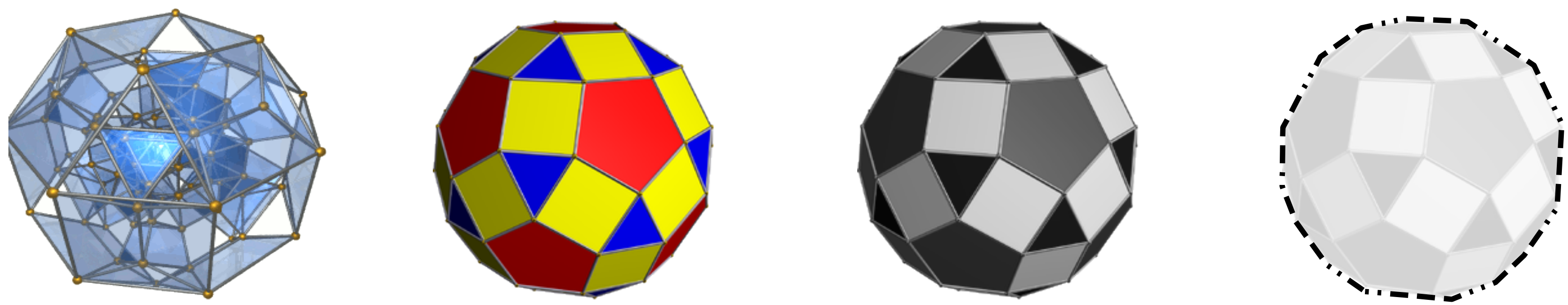

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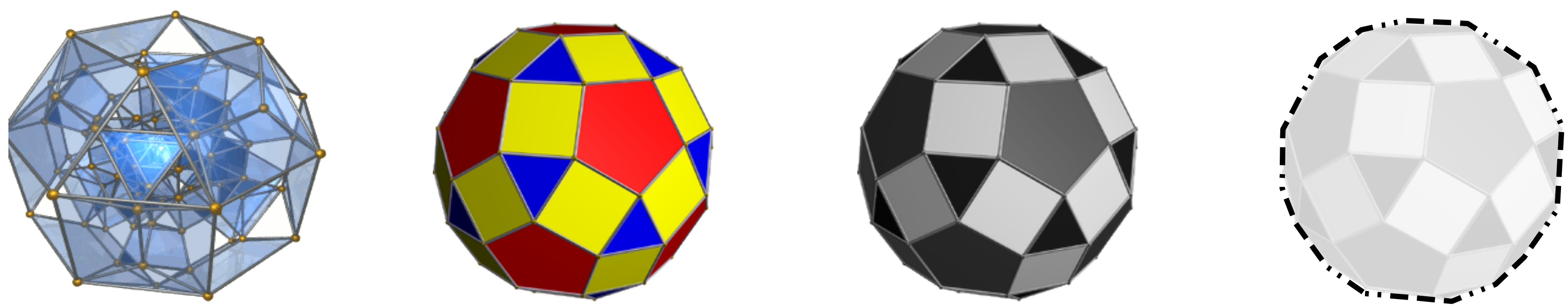

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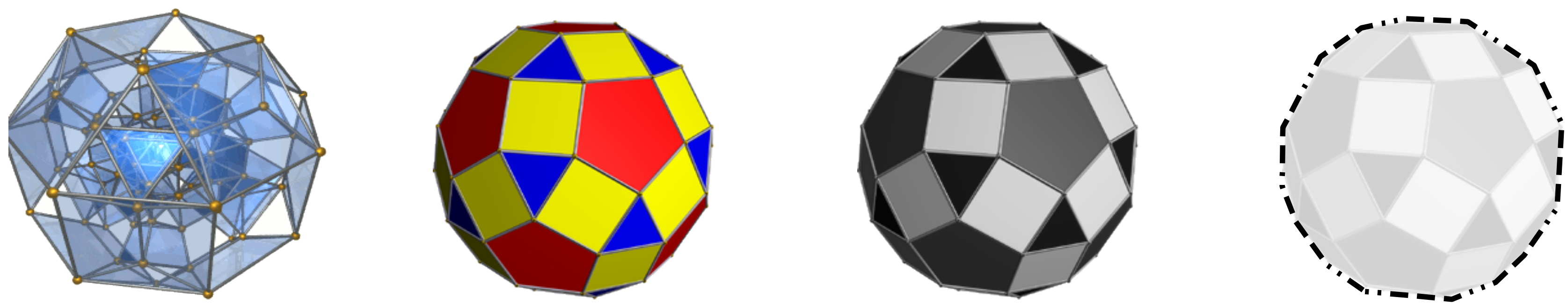

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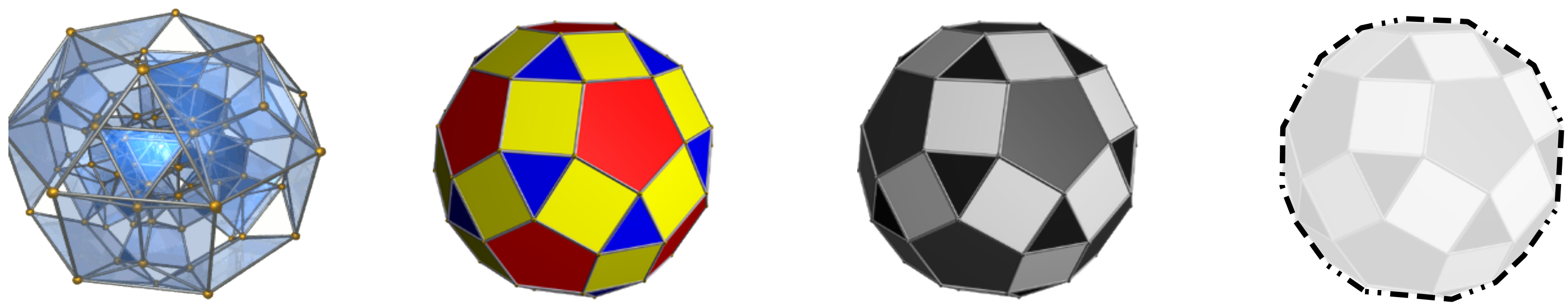

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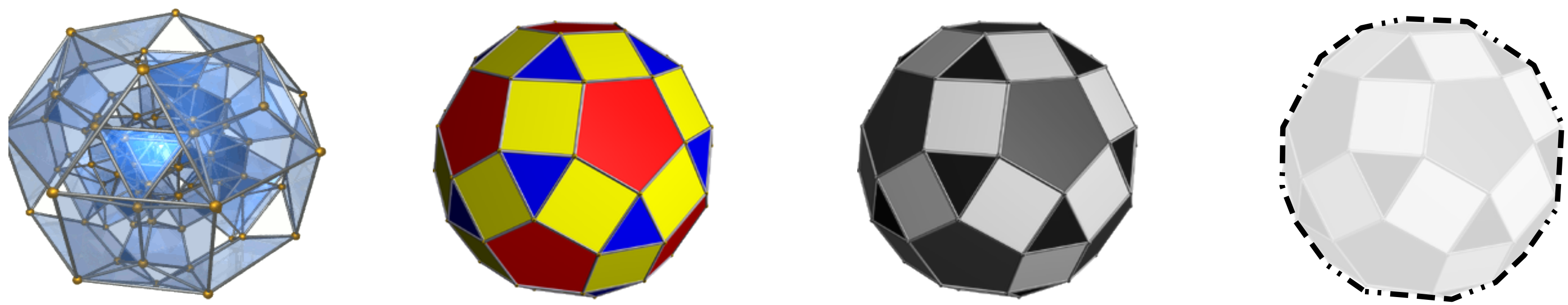

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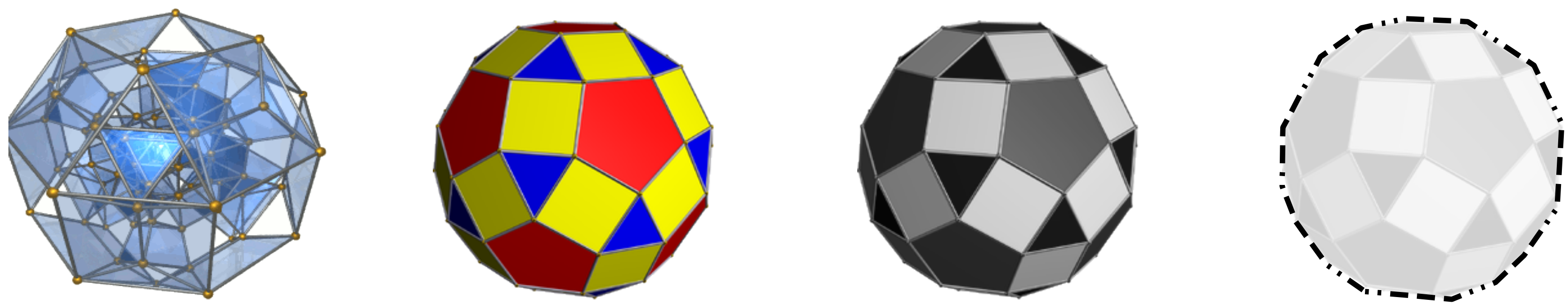

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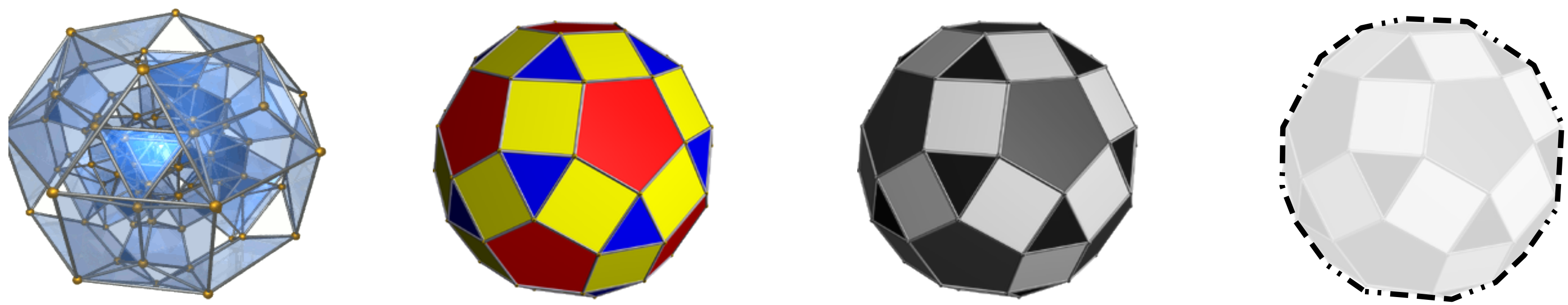

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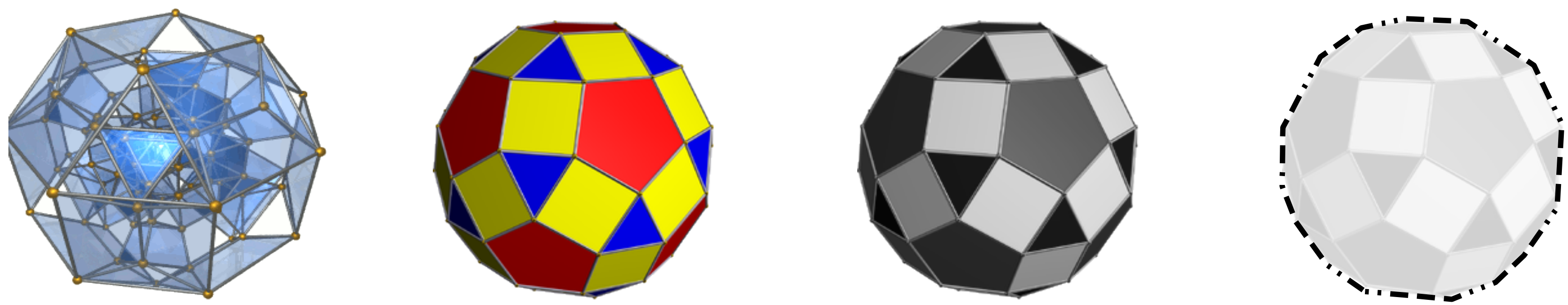

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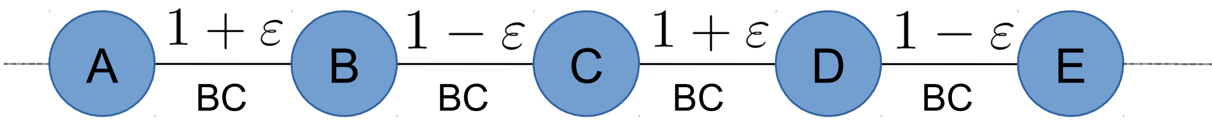

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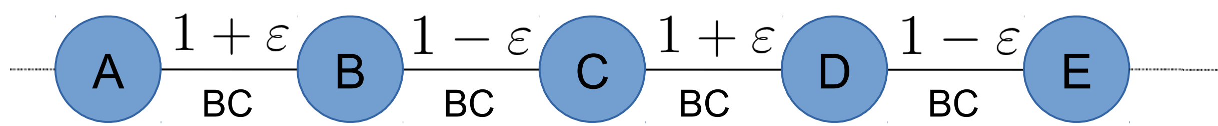

Take the Braunstein-Caves (BC) chained inequality for n measurement setting

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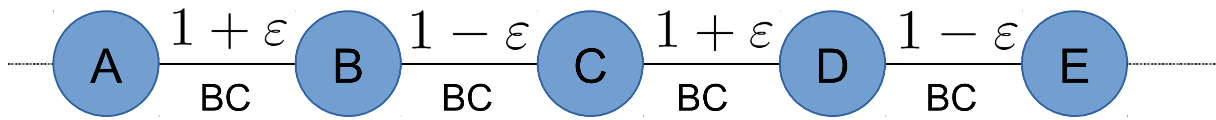

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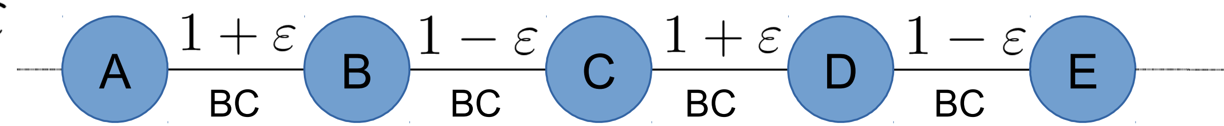

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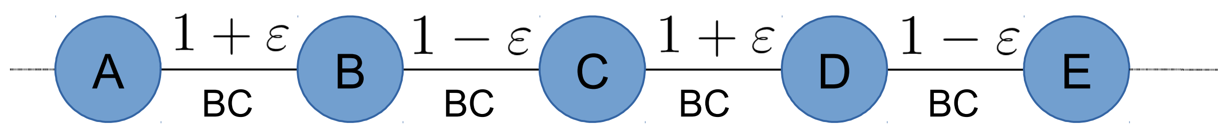

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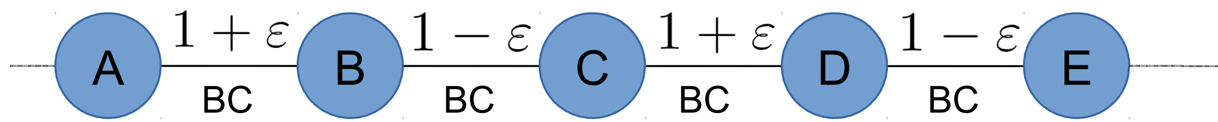

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[Wang et al., arXiv:1608.03485v3 (2016)]

Examples (IIb)

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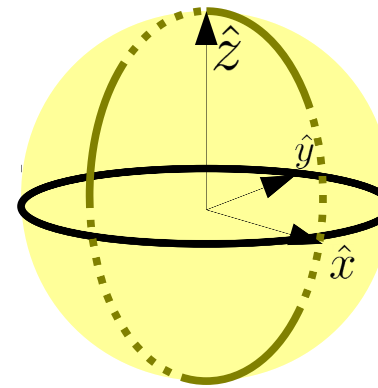

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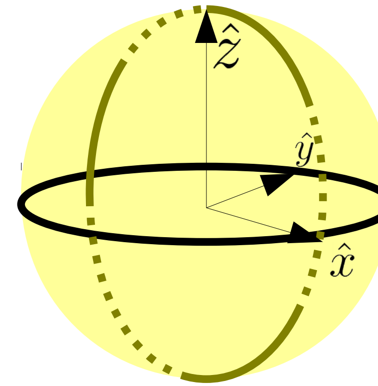

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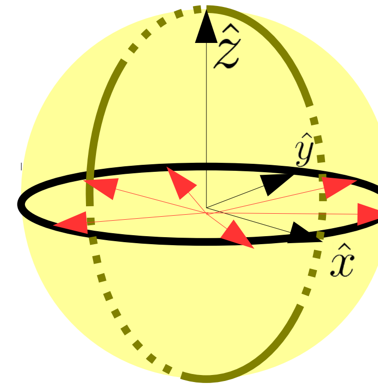

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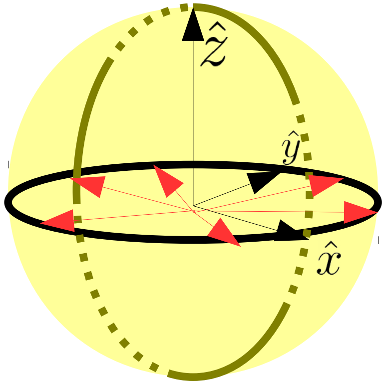

Asymptotic contributions per particle to quantum value and classical bound

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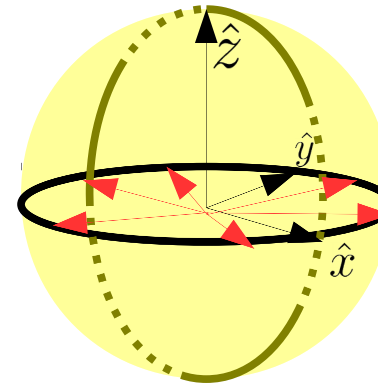

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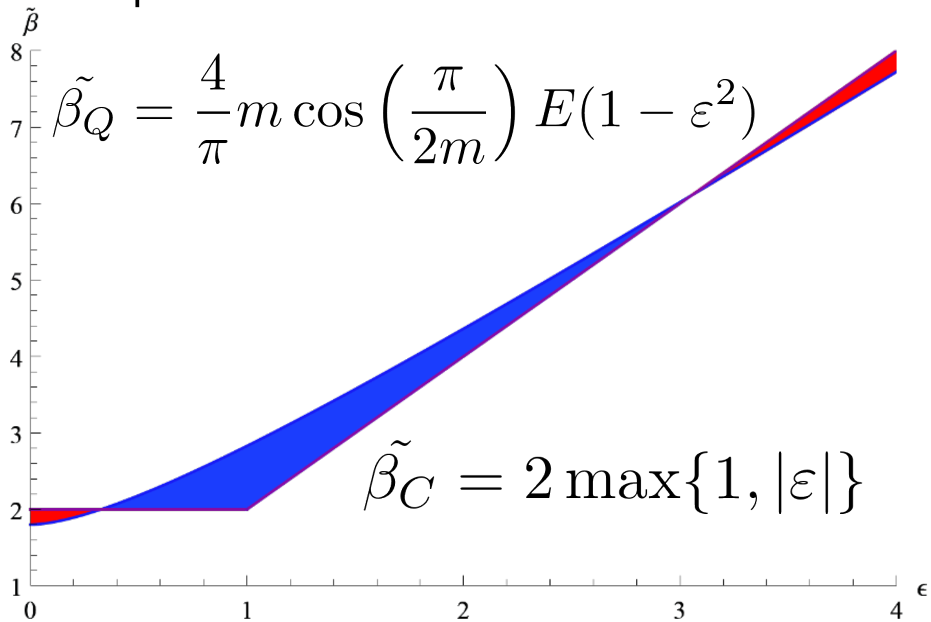

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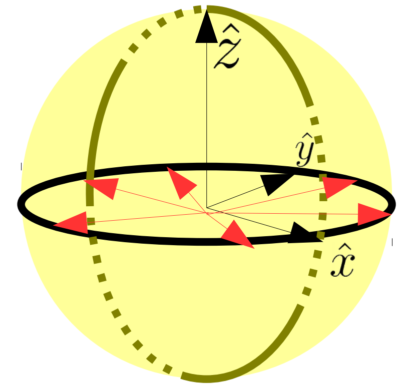

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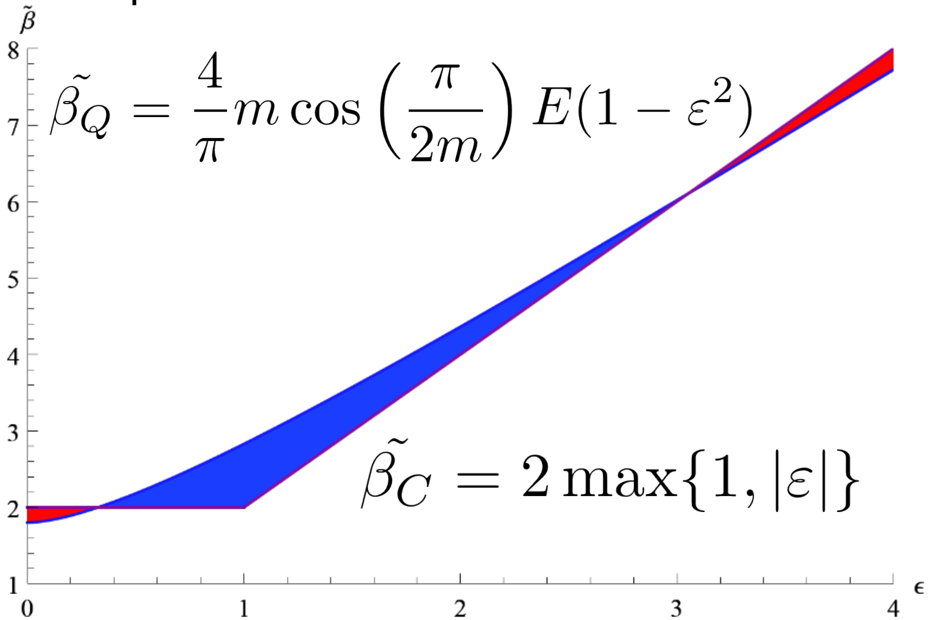

E(t) → Elliptic integral of
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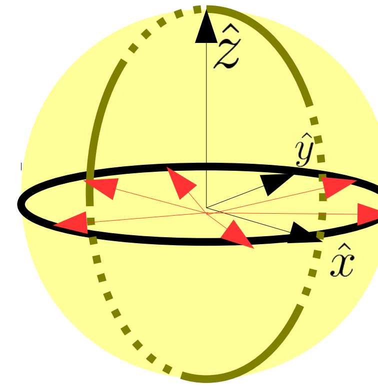

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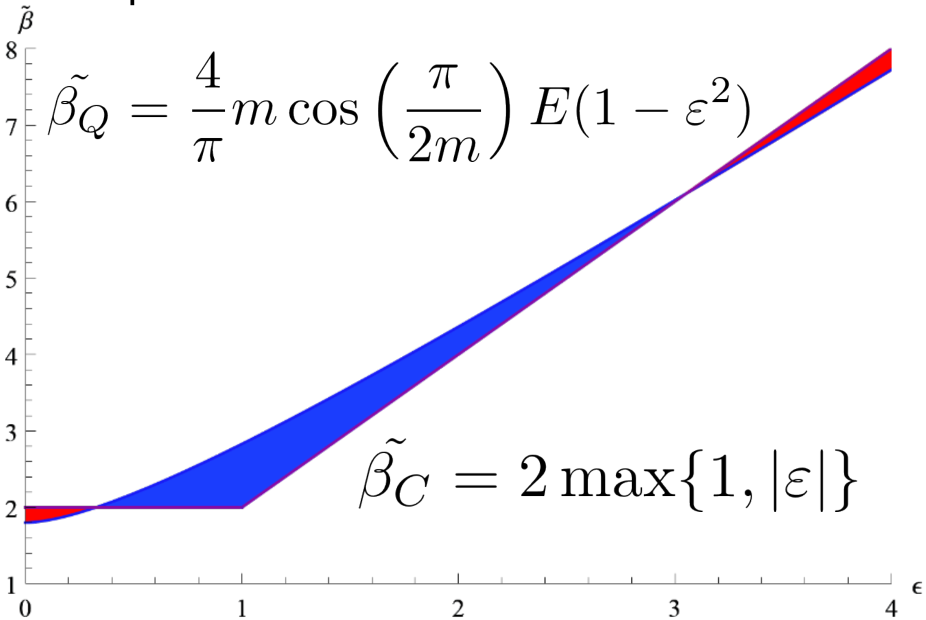
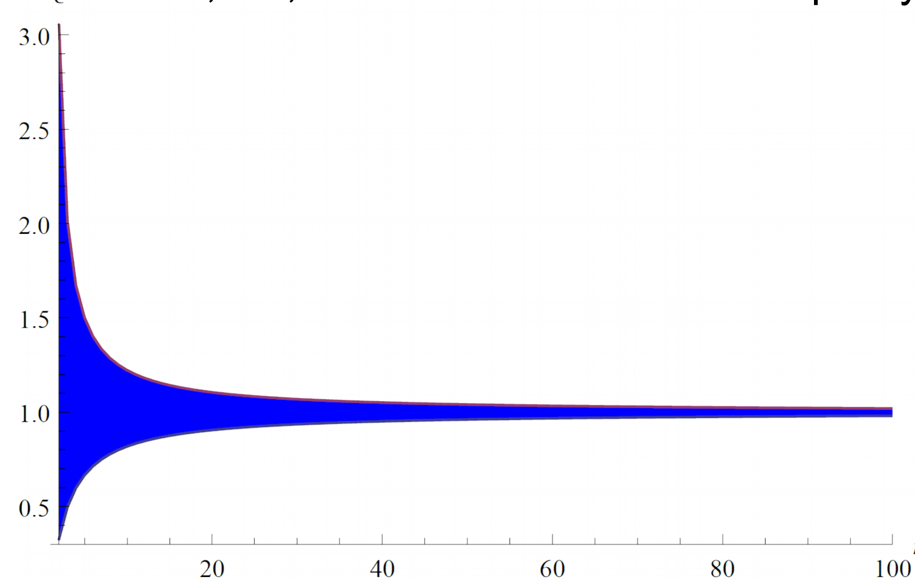


The optimal number of measurements is m=2, i.e., when BC is the CHSH inequality



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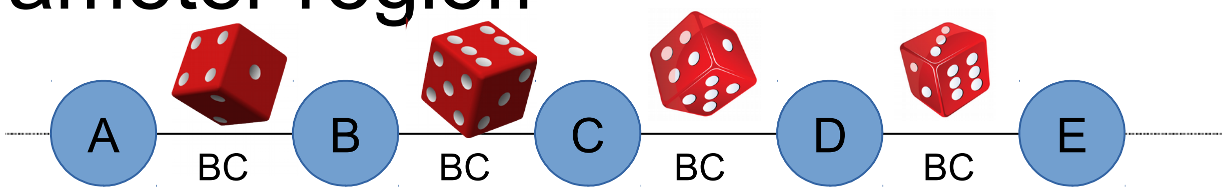

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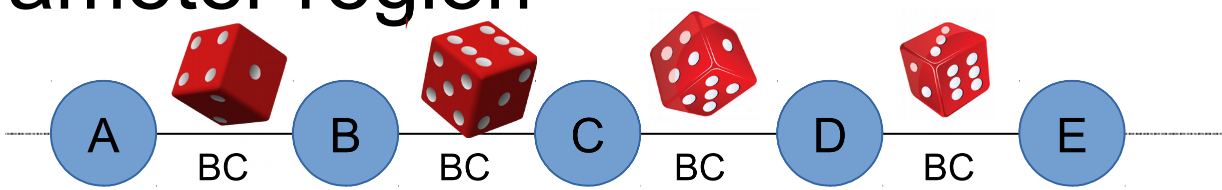

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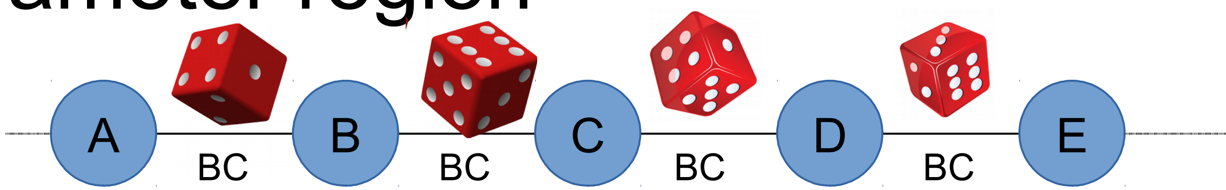

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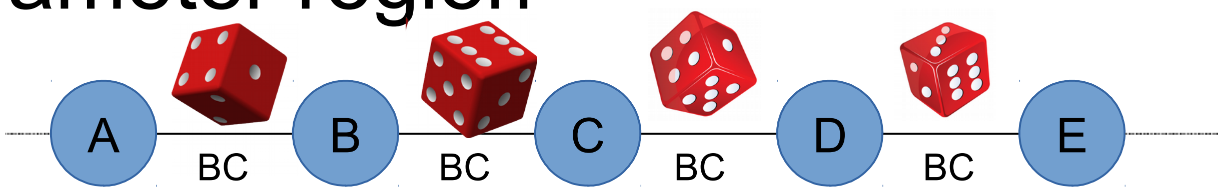

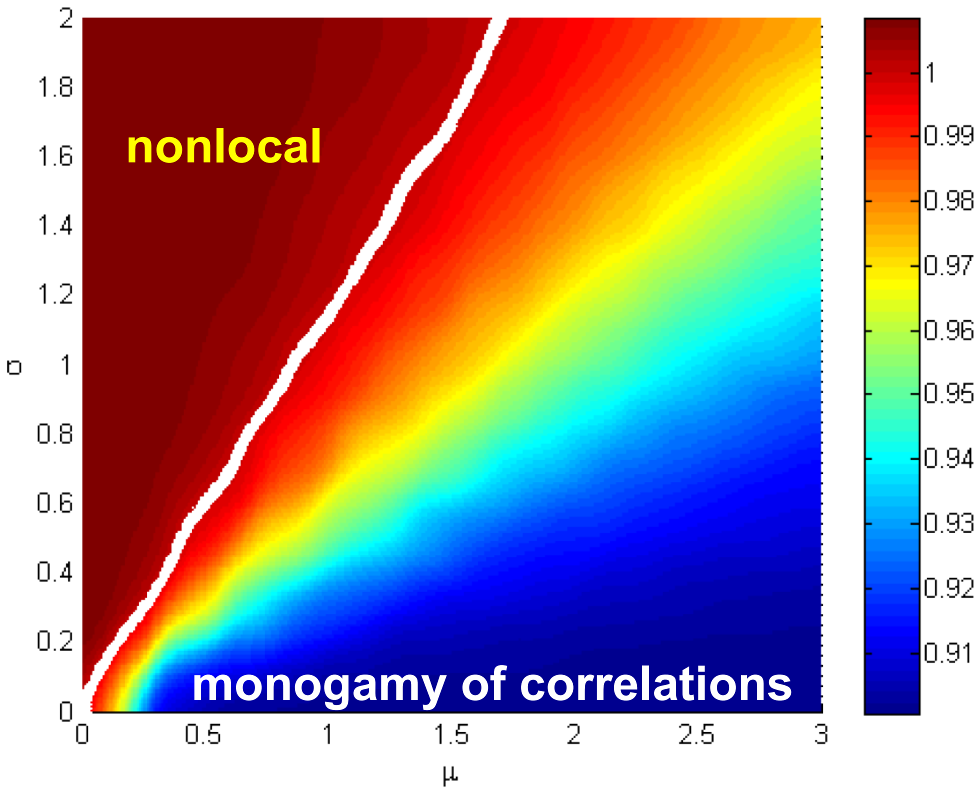

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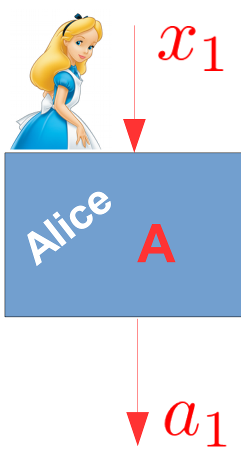

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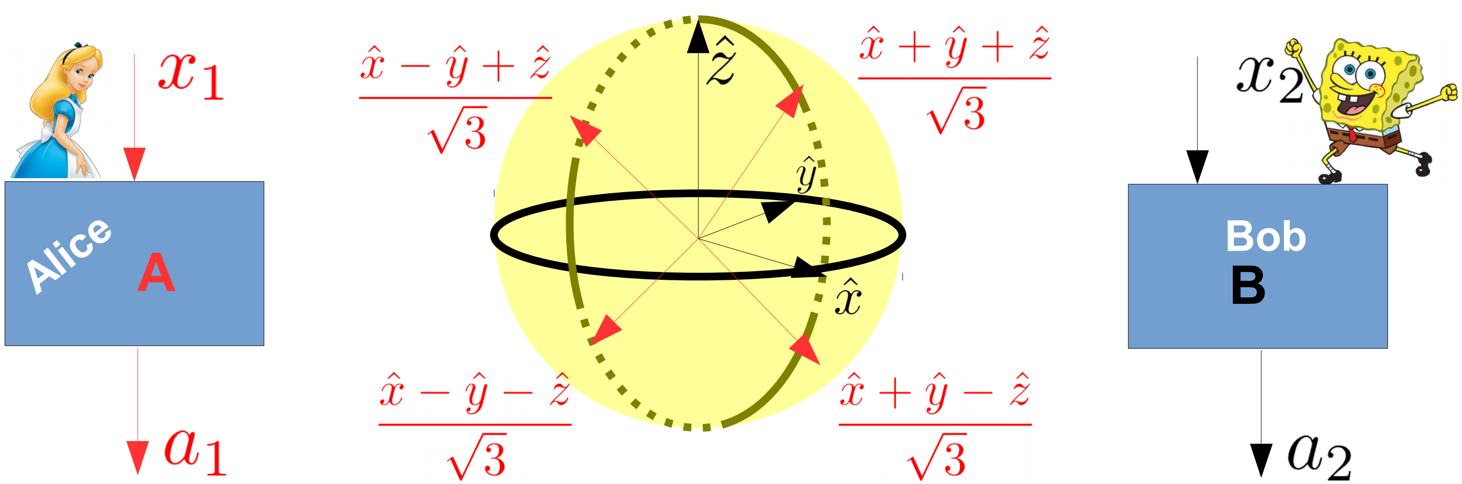

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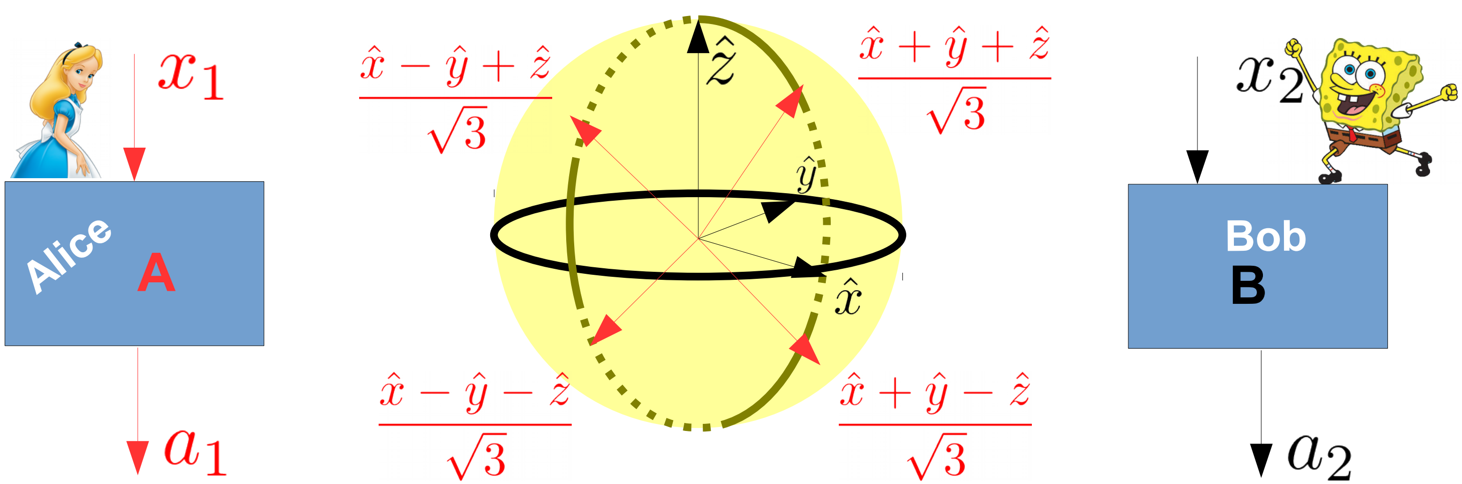

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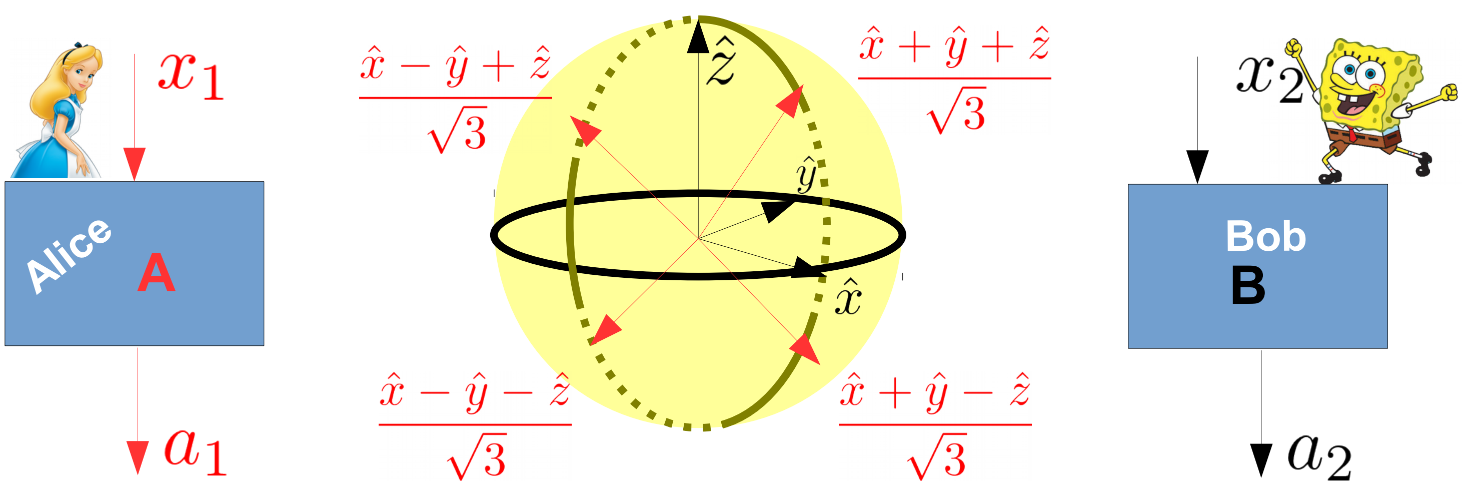

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Bell operator **is** permutationally invariant

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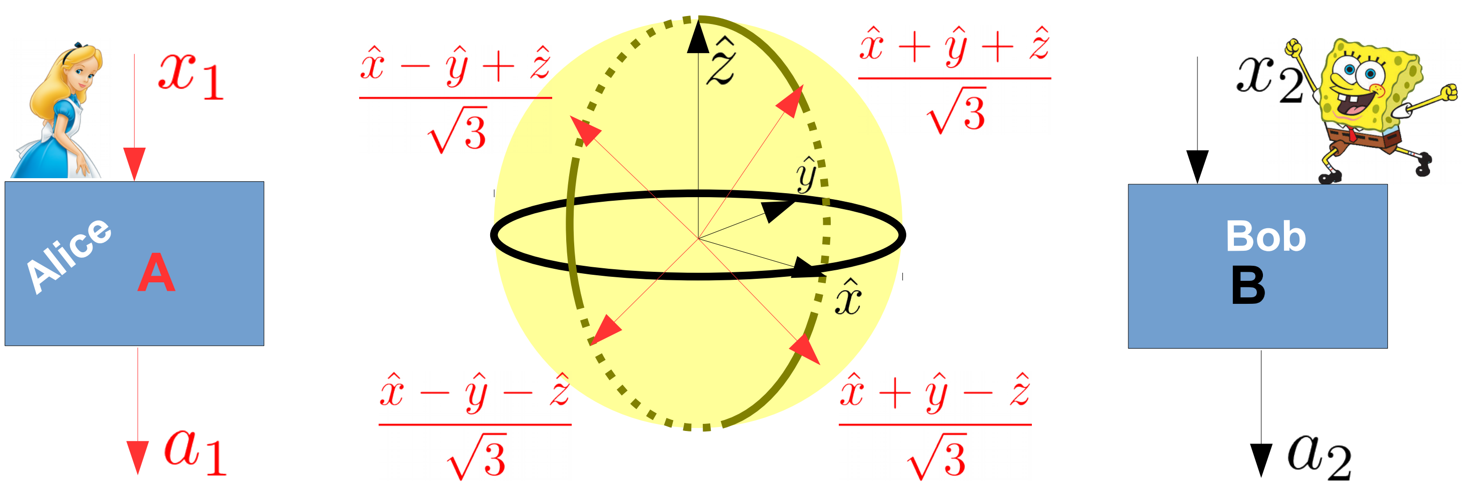
$$I = \begin{pmatrix} A_0 & A_1 & A_2 & A_3 \end{pmatrix} \begin{pmatrix} 1 & 1 & \Delta \\ 1 & -1 & -\Delta \\ -1 & 1 & -\Delta \\ -1 & -1 & \Delta \end{pmatrix} \begin{pmatrix} B_0 \\ B_1 \\ B_2 \end{pmatrix}$$

Not symmetric (even a different number of measurements)



$$\mathcal{B} = \sigma_x\sigma_x + \sigma_y\sigma_y + \Delta\sigma_z\sigma_z$$

Bell operator **is** permutationally invariant

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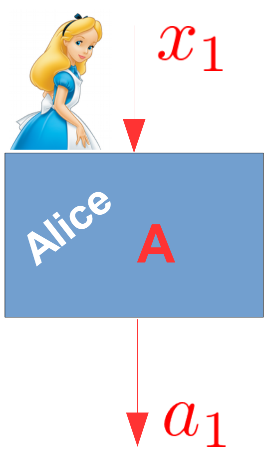

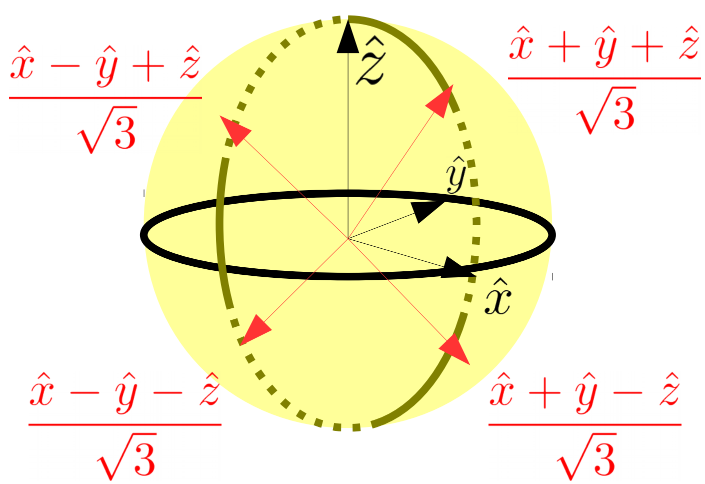

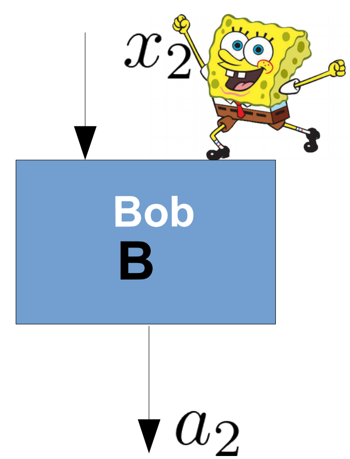

Correspondence can be non-obvious

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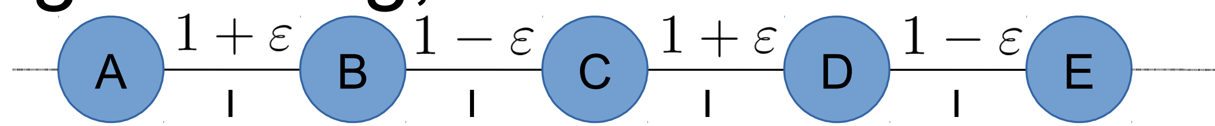

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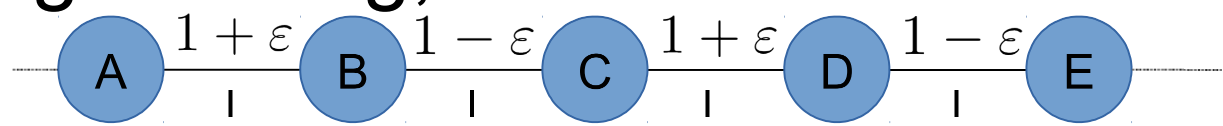

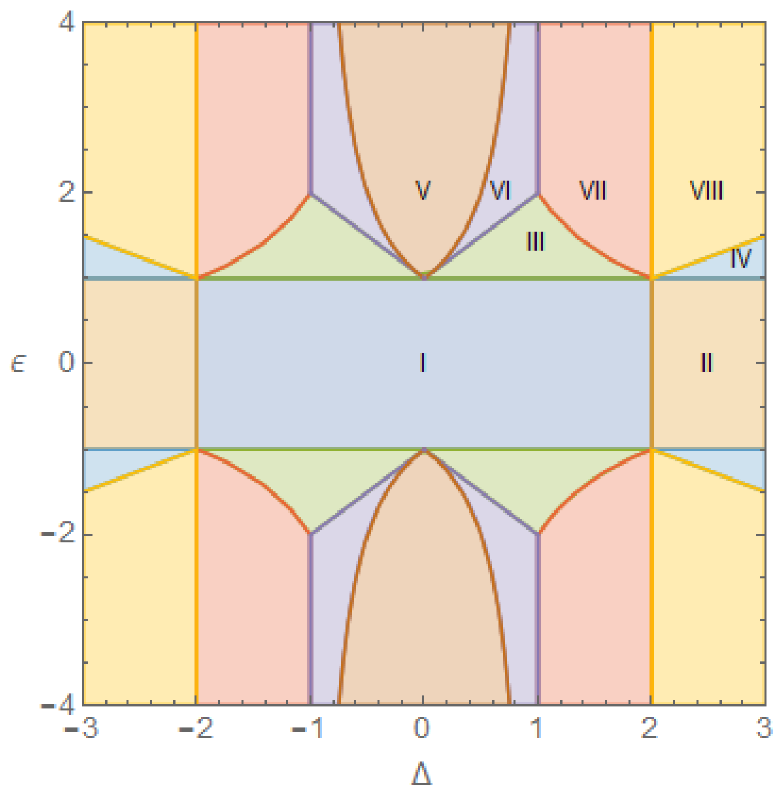

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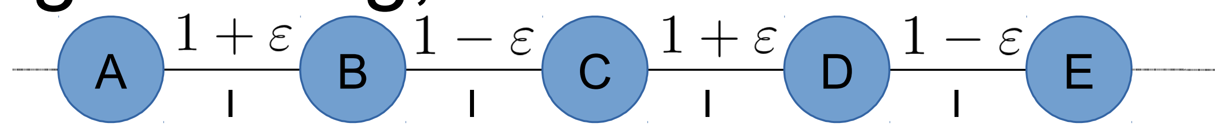

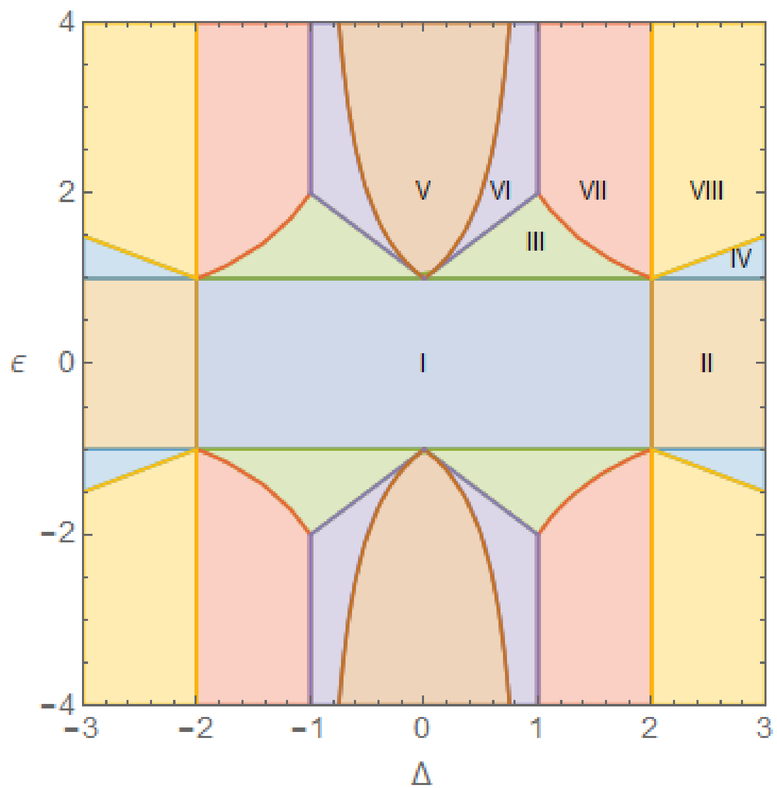

$$
\begin{aligned}
\beta_{C,\mathrm{I}} &= -n(4+2|\Delta|)\\
\beta_{C,\mathrm{II}} &= -4n|\Delta|\\
\beta_{C,\mathrm{III}} &= -8-4|\Delta|-(4n-8)|\epsilon|-(2n-4)|\Delta||\epsilon|\\
\beta_{C,\mathrm{IV}} &= -8|\Delta|-(4n-8)|\epsilon||\Delta|\\
\beta_{C,\mathrm{V}} &= -4n|\epsilon|-(2n-8)|\epsilon||\Delta|\\
\beta_{C,\mathrm{VI}} &= -4-(4n-4)|\epsilon|-(2n-4)|\epsilon||\Delta|\\
\beta_{C,\mathrm{VII}} &= -4|\Delta|-(4n-8)|\epsilon|-2n|\epsilon||\Delta|\\
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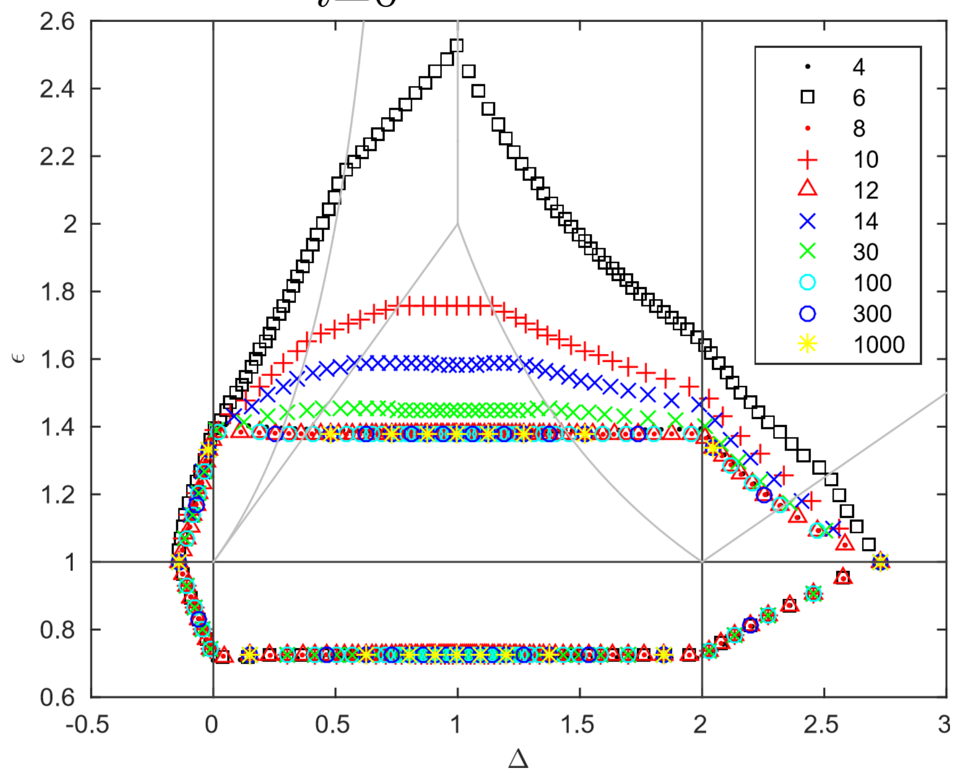

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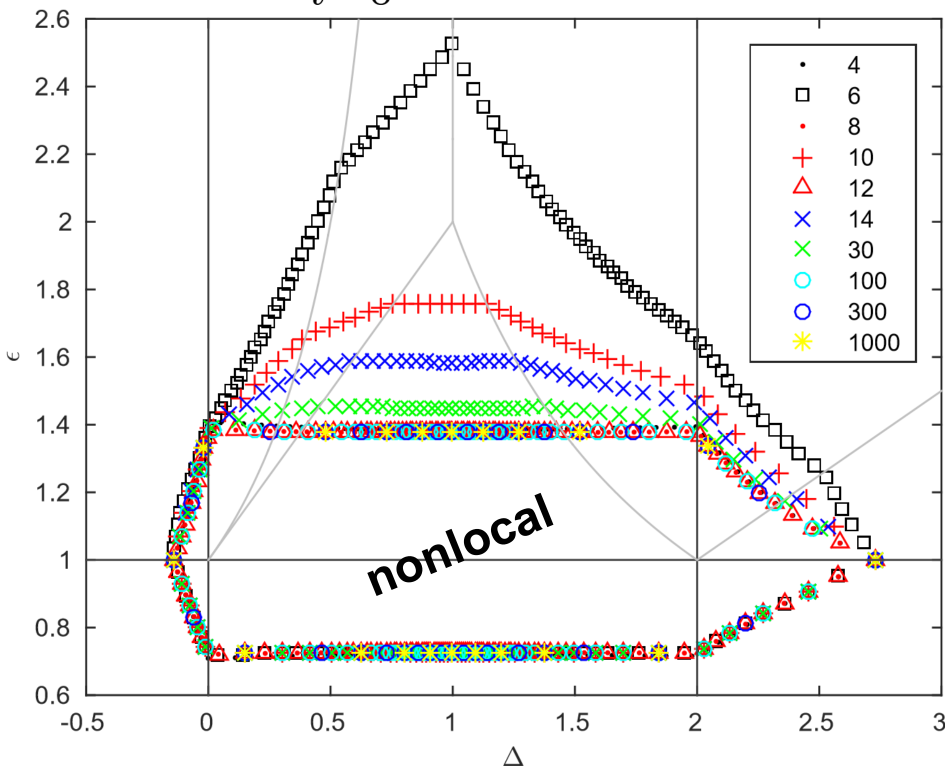

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Outline

- Motivation
- The idea, the setting
- Quantum optimization
- Assigning a Bell inequality to a Hamiltonian
- Classical optimization
- Translational invariance
- Examples
- **Conclusions and outlook**

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Thanks for your attention!

MAX-PLANCK-GESELLSCHAFT

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Thanks for your attention!