Entanglement and coherence in quantum state merging

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Outline

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- A quantum state is called *incoherent* if it is diagonal in some preferred basis:

$$\sigma = \sum_i p_i \, |i\rangle \langle i|, \tag{1}$$

and any other state is called *coherent*. The set of all incoherent states will be called $\mathcal{I}$.

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- A quantum operation is called incoherent if it can be written as

$$\Lambda[\rho] = \sum_i K_i \rho K_i^\dagger \tag{2}$$

with incoherent Kraus operators K_i such that

$$K_i \mathcal{I} K_i^\dagger \subseteq \mathcal{I}. \tag{3}$$

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Alternative frameworks of coherence

- **Maximally incoherent operations (MIO)**[1]: most general set, contains all operations which cannot create coherence: $\Lambda[\rho_i] \in \mathcal{I}$, where $\mathcal{I}$ is the set of all incoherent states.

[1] J. Åberg, arXiv (2006)

[2] A. Winter and D. Yang, PRL (2016); B. Yadin, J. Ma, D. Girolami, M. Gu, V. Vedral, PRX (2016)

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- Translationally invariant operations (TIO)[3]: Quantum operations which commute with time translations, i.e., $e^{-iHt}\Lambda[\rho]e^{iHt} = \Lambda[e^{-iHt}\rho e^{iHt}]$ for a given Hamiltonian H.

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- Dephasing-covariant incoherent operations (DIO)[4]: Quantum operations which commute with dephasing, i.e., $\Delta[\Lambda(\rho)] = \Lambda[\Delta(\rho)]$ with the dephasing operation Δ.

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Quantifying coherence

Coherence monotones have the following properties[1]:

1. $C(\rho) \geq 0$, and equality holds if and only if ρ is incoherent,

2. $C(\rho)$ is nonincreasing under incoherent operations:

$$C(\rho) \geq C(\Lambda[\rho]). \tag{4}$$

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Many coherence monotones are additionally nonincreasing on average under selective incoherent operations:

$$C(\rho) \geq \sum_i q_i C(\sigma_i) \tag{5}$$

with $q_i = \mathrm{Tr}[K_i \rho K_i^\dagger]$ and $\sigma_i = K_i \rho K_i^\dagger / q_i$.

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Quantifying coherence

Two important coherence monotones are[1]:

- Coherence cost: quantifies the rate of maximally coherent states $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ required to create a state ρ via incoherent operations in the asymptotic limit.

$$C_c(\rho) = C_f(\rho) = \min \sum_i p_i C_r(|\psi_i\rangle). \qquad (6)$$

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- Distillable coherence: quantifies the maximal rate for extracting maximally coherent states $|+\rangle$ via incoherent operations in the asymptotic limit.

$$C_d(\rho) = S(\Delta[\rho]) - S(\rho) \qquad (7)$$

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- The quantities differ for different frameworks of coherence.

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Quantifying coherence

Other important coherence monotones:

- Relative entropy of coherence[1]

$$C_r(\rho) = \min_{\sigma \in \mathcal{I}} S(\rho \| \sigma). \qquad (8)$$

Note that $C_r(\rho) = C_d(\rho) = S(\Delta[\rho]) - S(\rho)$.

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- Robustness of coherence[2]

$$R_c(\rho) = \min_\tau \left\{ s \geq 0 \left| \frac{\rho + s\tau}{1 + s} \in \mathcal{I} \right. \right\}.$$ (9)

Operational interpretation via interferometric visibility[3].

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Quantum coherence in distributed scenarios[123]

Similar to LOCC operations in entanglement theory, it is possible to define <u>local quantum-incoherent operations and classical communication (LQICC)</u>.

[1] E. Chitambar, A. S., S. Rana, M. N. Bera, G. Adesso, and M. Lewenstein, PRL 2016

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Similar to LOCC operations in entanglement theory, it is possible to define <u>local quantum-incoherent operations and classical communication (LQICC)</u>.
Properties of LQICC operations:

- LQICC operations preserve the set of <u>quantum-incoherent states (QI)</u>:

$$\rho_{\mathrm{qi}}^{AB} = \sum_i p_i \sigma_i^A \otimes |i\rangle \langle i|^B. \tag{10}$$

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- LQICC operations cannot increase the QI relative entropy

$$C_r^{A|B}(\rho^{AB}) = \min_{\sigma^{AB} \in \mathcal{QI}} S(\rho^{AB}\|\sigma^{AB}) = S(\Delta^B[\rho^{AB}]) - S(\rho^{AB}). \tag{11}$$

QI relative entropy is additive in the input state.

[1] E. Chitambar, A. S., S. Rana, M. N. Bera, G. Adesso, and M. Lewenstein, PRL 2016

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Alice and Bob share many copies a bipartite state ρ^{AB} and can perform bipartite LQICC operations.

<u>Aim of the task</u>: asymptotic distillation of maximally coherent states $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ on Bob's side.

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- The figure of merit is the distillable coherence of collaboration which is bounded as follows:

$$C_d^{A|B}(\rho^{AB}) \leq C_r^{A|B}(\rho^{AB}) = S(\Delta^B[\rho^{AB}]) - S(\rho^{AB}). \qquad (12)$$

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$$C_d^{A|B}(|\psi\rangle^{AB}) = C_r^{A|B}(|\psi\rangle^{AB}) = S(\Delta[\rho^B]). \qquad (13)$$

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- Without assistance Bob can distill coherence at rate

$$C_d(\rho^B) = S(\Delta[\rho^B]) - S(\rho^B). \tag{14}$$

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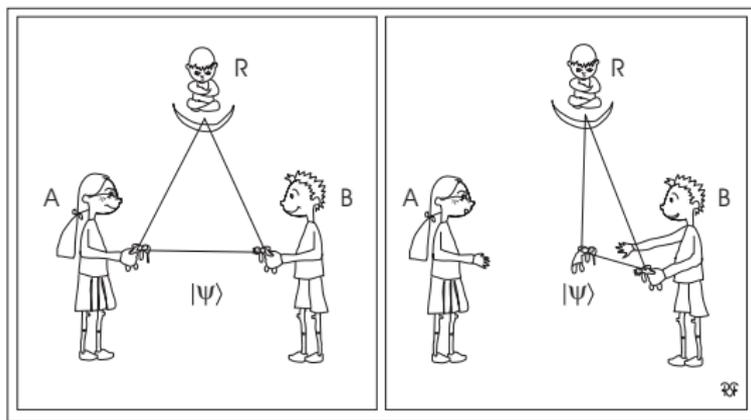[3] K.-D. Wu, Z. Hou, H.-S. Zhong, Y. Yuan, G.-Y. Xiang, C.-F. Li, G.-C. Guo, Nonlinear Optics 2017

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- Alice, Bob, and a referee share many copies of a pure state $|\psi\rangle^{RAB}$.
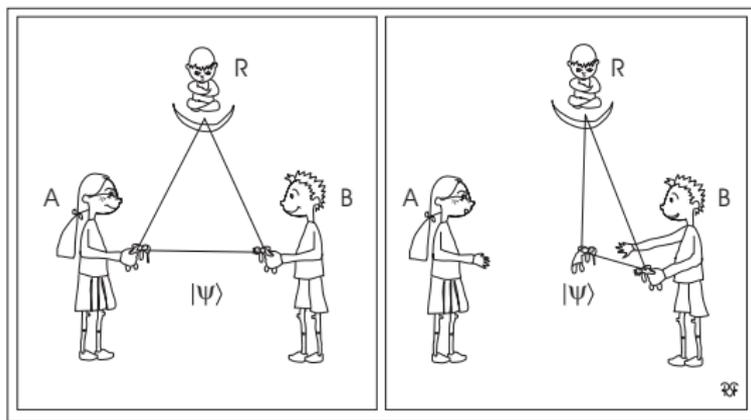
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- Alice, Bob, and a referee share many copies of a pure state $|\psi\rangle^{RAB}$.

- Aim of Alice and Bob: merge their systems on Bob's side while preserving the total state, i.e., the final state $|\psi\rangle^{RBB'}$ is the same as $|\psi\rangle^{RAB}$ up to relabeling A and B'.

Standard quantum state merging[1]

- For this purpose, Alice and Bob have access to <u>shared singlets and a classical channel</u>.

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- The <u>minimal number of singlets</u>, asymptotically needed per copy of the state $|\psi\rangle^{RAB}$, is given by the <u>conditional entropy</u>:

$$S(A|B) = S(\rho^{AB}) - S(\rho^B). \tag{15}$$

- Recall that in quantum theory the conditional entropy can be either positive or negative.

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- If $S(A|B)$ is positive, merging is possible with singlets at rate $S(A|B)$, and merging is not possible if less singlets are available.

- If $S(A|B)$ is negative, merging is possible without singlets, and Alice and Bob can additionally gain singlets at rate $-S(A|B)$.

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Incoherent quantum state merging[1]

- In standard quantum state merging, shared entanglement is considered as an expensive resource, while <u>local coherence</u> is available at no additional cost.

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- In particular, for a tripartite state ρ^{RAB}, we consider state merging via LQICC operations, where additional singlets and maximally coherent states on Bob's side are provided at rates E and C.

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- We consider <u>optimal entanglement-coherence pairs</u> (E, C): these are pairs of entanglement and coherence rate for which merging is possible, but neither E nor C can be reduced.

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- We consider <u>optimal entanglement-coherence pairs</u> (E, C): these are pairs of entanglement and coherence rate for which merging is possible, but neither E nor C can be reduced.

- The main problem is to determine all such optimal pairs (E, C) for a given state ρ^{RAB}.

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Incoherent quantum state merging[1]

Theorem

Given a tripartite quantum state ρ^{RAB}, any achievable pair (E, C) fulfills the following inequality:

$$E + C \geq S\left(\Delta^{AB}[\rho^{RAB}]\right) - S\left(\Delta^{B}[\rho^{RAB}]\right). \tag{16}$$

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Here, S is the von Neumann entropy and Δ^{X} denotes full decoherence of a (possibly multipartite) subsystem X:

$$\Delta^{X}[\rho] = \sum_{i} |i\rangle \langle i|^{X} \rho |i\rangle \langle i|^{X}. \tag{17}$$

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$$\Delta^X[\rho] = \sum_i |i\rangle\langle i|^X \rho |i\rangle\langle i|^X. \qquad (17)$$

Since the right-hand side of Eq. (16) is nonnegative, the sum $E + C$ is also nonnegative: **no merging procedure can gain coherence and entanglement at the same time**.

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For pure states $|\psi\rangle^{RAB}$ we have the following bounds:

$$E \geq E_{\min} = S(\rho^{AB}) - S(\rho^{B}), \qquad (18)$$

$$E + C \geq S(\Delta^{AB}[\rho^{AB}]) - S(\Delta^{B}[\rho^{B}]). \qquad (19)$$

Crucially, the bound in Eq. (19) is achievable for all pure states.

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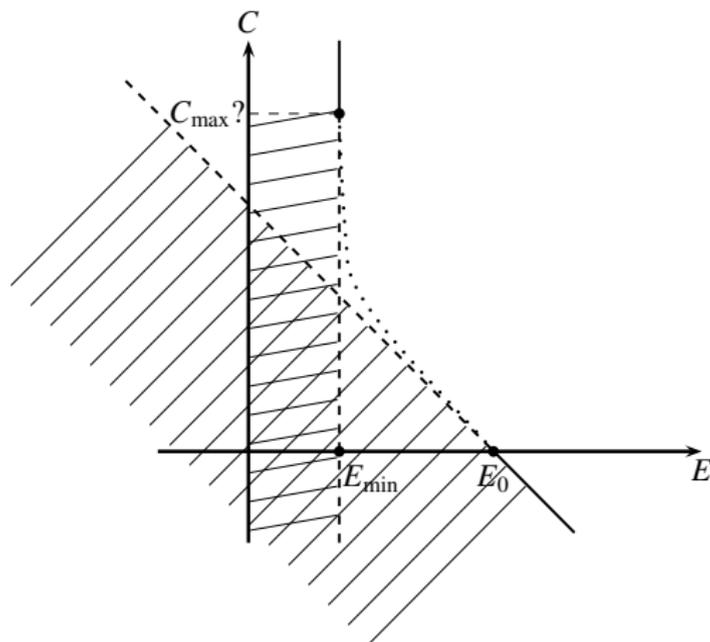Crucially, the bound in Eq. (19) is achievable for all pure states.

Theorem

Any pure state $|\psi\rangle^{RAB}$ can be merged without local coherence by using singlets at rate

$$E_0 = S(\Delta^{AB}[\rho^{AB}]) - S(\Delta^B[\rho^B]). \qquad (20)$$

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$$E_{\min} = S(\rho^{AB}) - S(\rho^B), \tag{21}$$

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For some mixed states ρ^{RAB} it is possible to find all optimal pairs. Example: states of the form

$$\rho^{RAB} = \sum_{i,j} p_{ij} |ij\rangle \langle ij|^R \otimes |\psi_{ij}\rangle \langle \psi_{ij}|^A \otimes |i\rangle \langle i|^B, \tag{23}$$

where $|\psi_{ij}\rangle$ are mutually orthogonal for different j. Note that this state can be merged without entanglement.

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state can be merged without entanglement.
All optimal pairs are given by

$$(E, C) = (aC_{\max}, [1 - a]C_{\max}) \qquad (24)$$

with $a \geq 0$ and $C_{\max} = \sum_{i,j} p_{ij} S(\Delta[\psi_{ij}])$.

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Incoherent quantum state merging[1]

There is evidence that a large amount of local coherence can be saved by using little extra entanglement, i.e., that for some states the pairs $(E, C \gg 0)$ and $(E' = E + \varepsilon, C' \ll C)$ are both optimal for small ε.

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Possible candidate for such states:

$$\rho = \frac{1}{d_B} \sum_{i=0}^{d_B-1} |i\rangle \langle i|^R \otimes |\phi_i\rangle \langle \phi_i|^A \otimes |\psi_i\rangle \langle \psi_i|^B, \tag{25}$$

where $|\psi_i\rangle$ are mutually orthogonal maximally coherent states of arbitrary dimension d_B, and $|\phi_i\rangle$ are single-qubit states.

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Incoherent quantum state merging[1]

There is evidence that a large amount of local coherence can be saved by using little extra entanglement, i.e., that for some states the pairs $(E, C \gg 0)$ and $(E' = E + \varepsilon, C' \ll C)$ are both optimal for small ε.

Possible candidate for such states:

$$\rho = \frac{1}{d_B} \sum_{i=0}^{d_B-1} |i\rangle\langle i|^R \otimes |\phi_i\rangle\langle\phi_i|^A \otimes |\psi_i\rangle\langle\psi_i|^B, \qquad (25)$$

where $|\psi_i\rangle$ are mutually orthogonal maximally coherent states of arbitrary dimension d_B, and $|\phi_i\rangle$ are single-qubit states.

- This state can be merged without entanglement if Bob performs a local von Neumann measurement in the basis $\{|\psi_i\rangle\}$, and conditionally prepares the states $|\phi_i\rangle$. However, this process requires a large amount of coherence.
- Alice and Bob can use one singlet for teleporting Alice's system to Bob, in which case no coherence is needed.

[1] A. S., E. Chitambar, S. Rana, M. N. Bera, A. Winter, and M. Lewenstein, PRL 2016.

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- This implies that <u>no merging procedure can gain entanglement and coherence at the same time.</u>

- The bound is tight for all pure states: any pure state can be merged without coherence by using singlets at rate $E_0 = S(\Delta^{AB}[\rho^{AB}]) - S(\Delta^{B}[\rho^{B}])$.

- Our results imply an <u>incoherent version of Schumacher compression</u>: $S(\Delta[\rho])$ is the optimal compression rate for a state ρ if the decompression has to be performed via incoherent operations only.