Synthetic Creutz-Hubbard model: interacting topological insulators with ultracold atoms

Matteo Rizzi

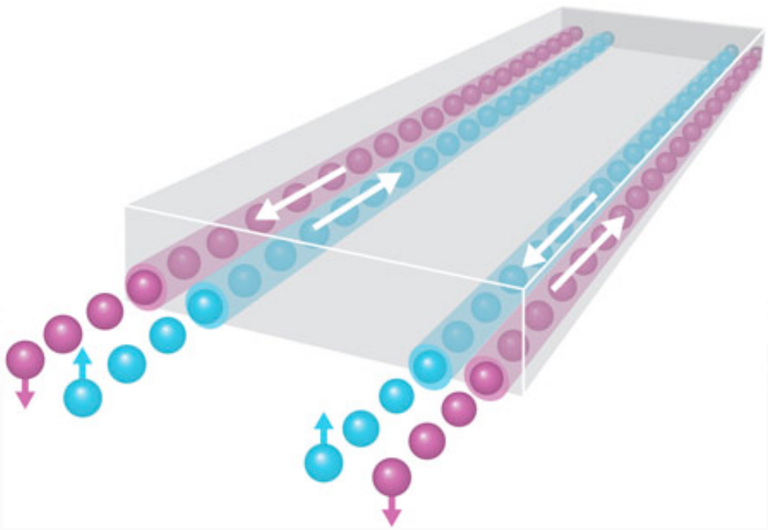
Johannes Gutenberg Universität Mainz

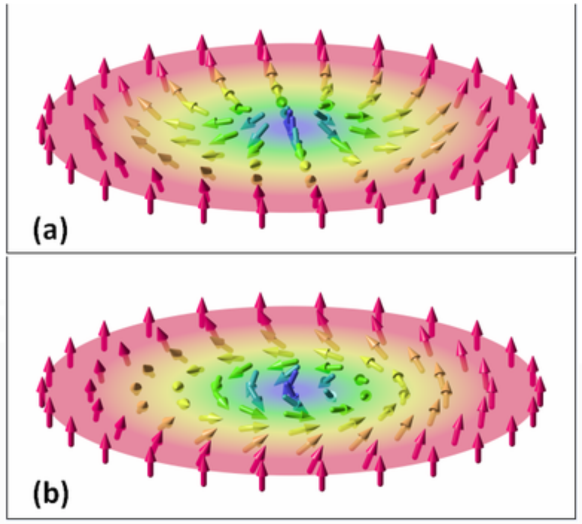
J. Jünemann, et al., arXiv:1612.02996 -- accepted on PRX

ICTP Trieste, 14 September 2017

Motivation

- Topological phases of matter: academic & practical interests !





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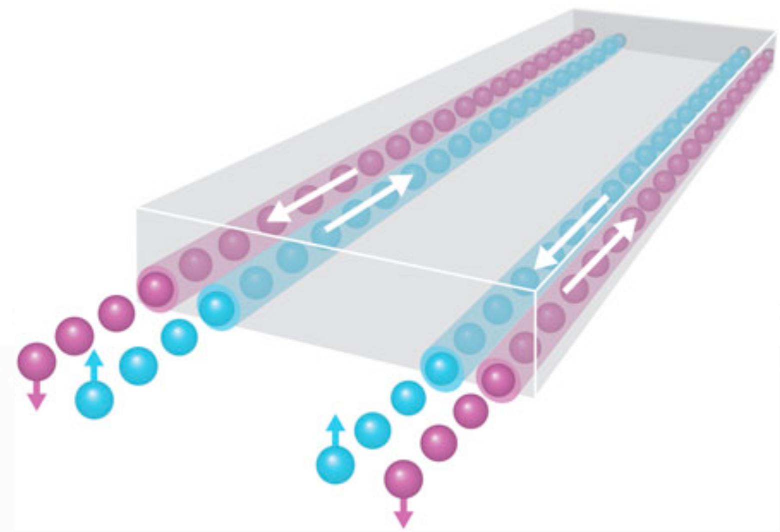

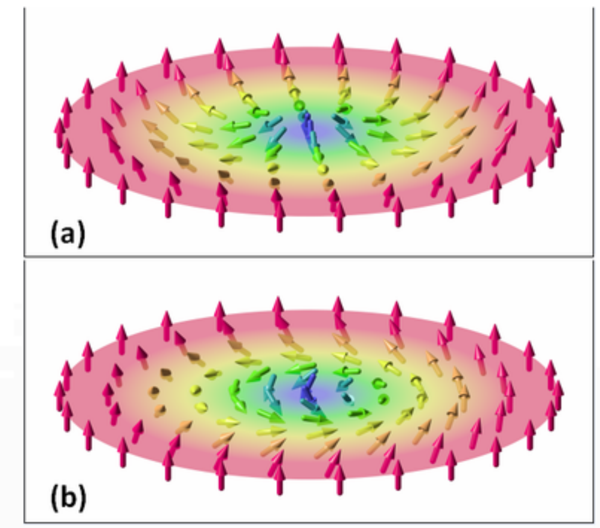

- Two open issues:
  i. paradigm models <==> real materials ?
  (e.g., Kitaev, Haldane, Kane-Mele, Harper-Hofstadter models, ...)

  ii. role of interactions ==> correlation effects ?
  (a.k.a., can one get generalizations of fractional quantum Hall effect?)

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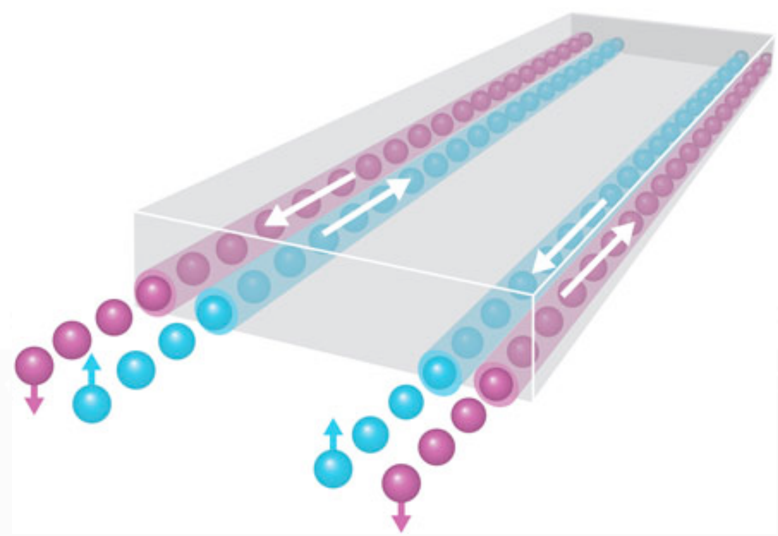

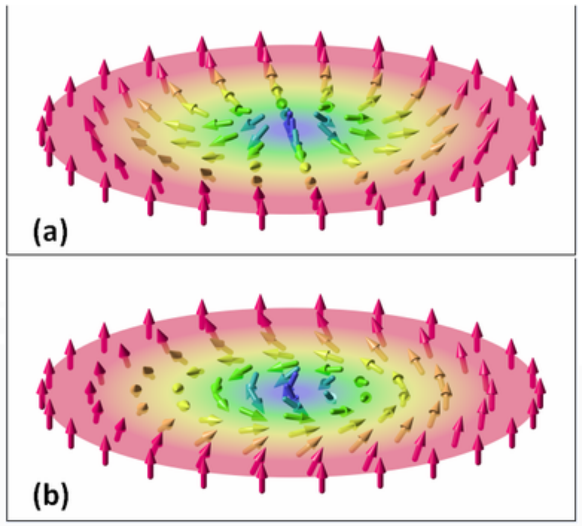

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experiments: Bloch, Esslinger, Fallani, Spielman, & many more !

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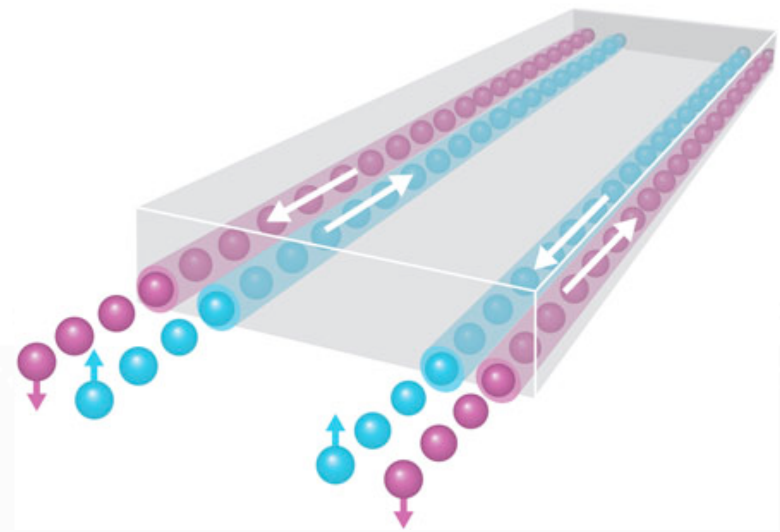

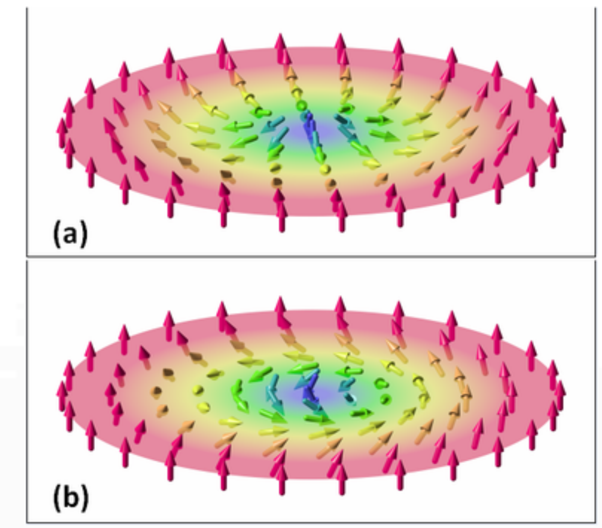

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- *Synthetic quantum matter* (e.g., via cold atoms) could help!
- *Quantum info. driven numerics (i.e., Tensor Networks), too!*

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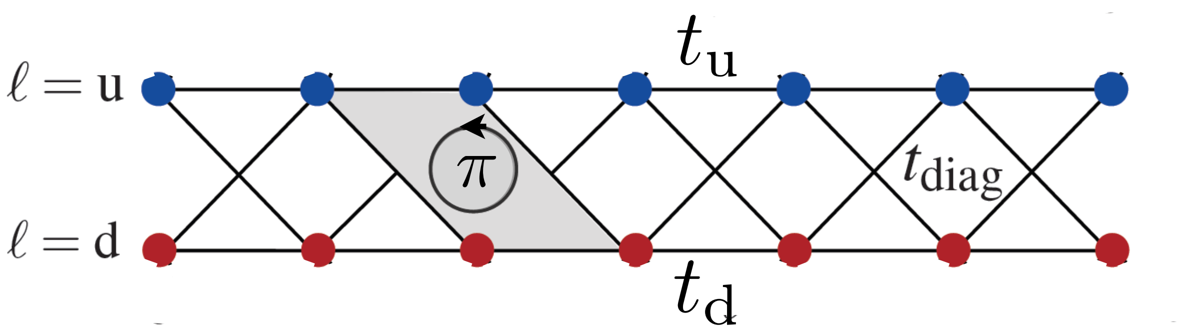
Imbalanced Creutz Ladder



Spin-dep., complex, tunnelling $\quad t_\ell = \mathrm{i}\, s_\ell\, \tilde{t}$, $\quad s_{\mathrm{u/d}} = \pm 1$

+

Spin-flipping, real, tunnelling $\quad t_\mathrm{diag}$

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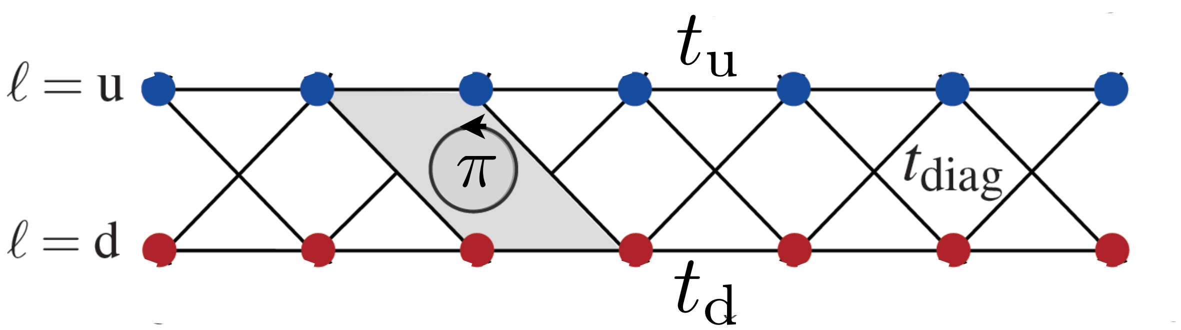

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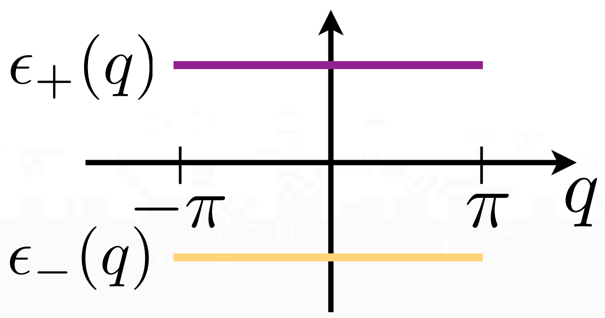

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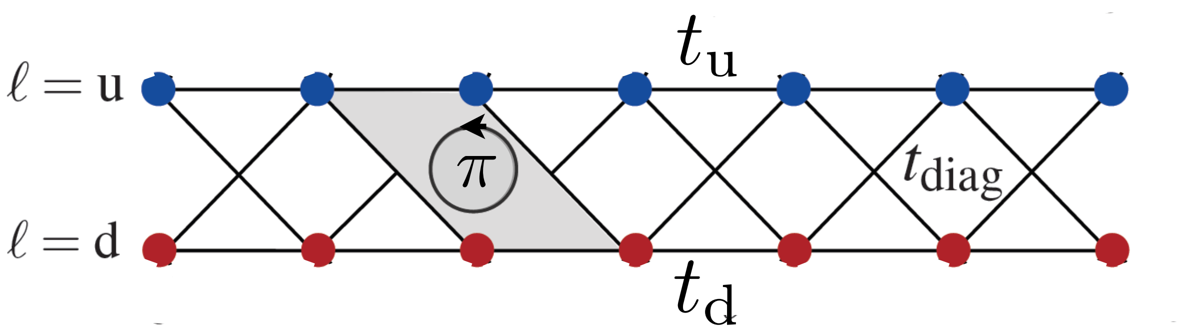

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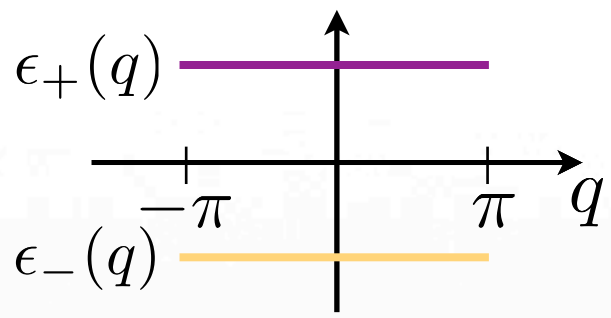


$$|t_\mathrm{diag}| = |t_\ell| = \tilde{t}$$

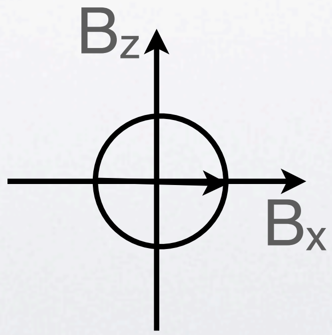
Topological character

$$\varphi_{\mathrm{Zak},\pm} = \pi$$

$$\mathcal{W} = \mathrm{sgn}(\tilde{t}) \neq 0$$

$$H_\mathrm{FB} = \sum_{q \in \mathrm{BZ}} \Psi^\dagger(q)[\boldsymbol{B}(q) \cdot \boldsymbol{\sigma}]\Psi(q)$$

$$\boldsymbol{B}(q) = 2\tilde{t}\,[-\cos(qa),\ 0,\ \ +\sin(qa)]$$



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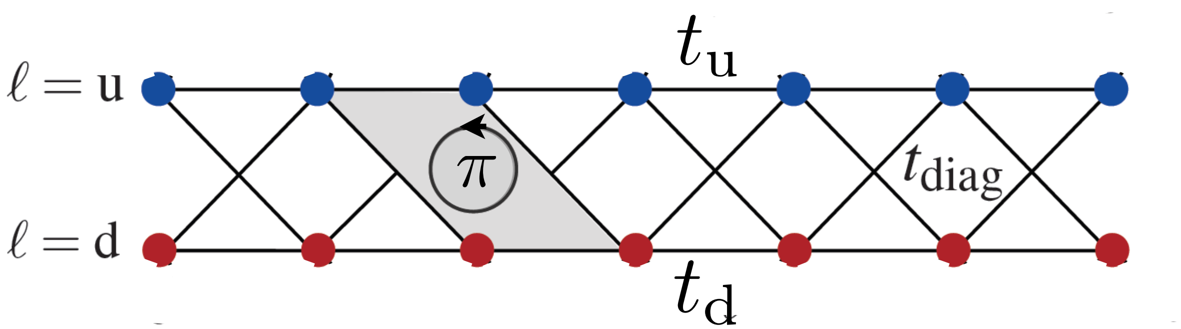

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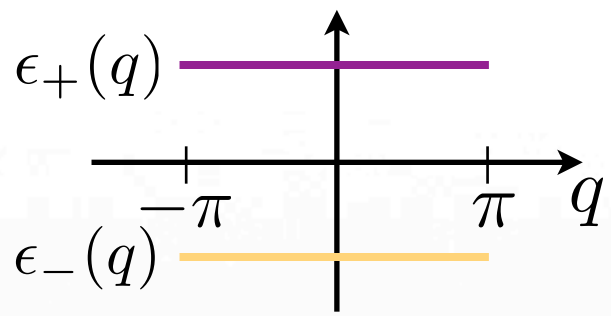

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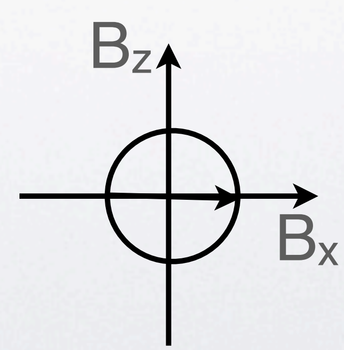

Measurement of Zak phase via Ramsey interferometry in ultracold gases (SSH model)

M. Atala, et al., Nat. Phys. 9, 795 (2013).

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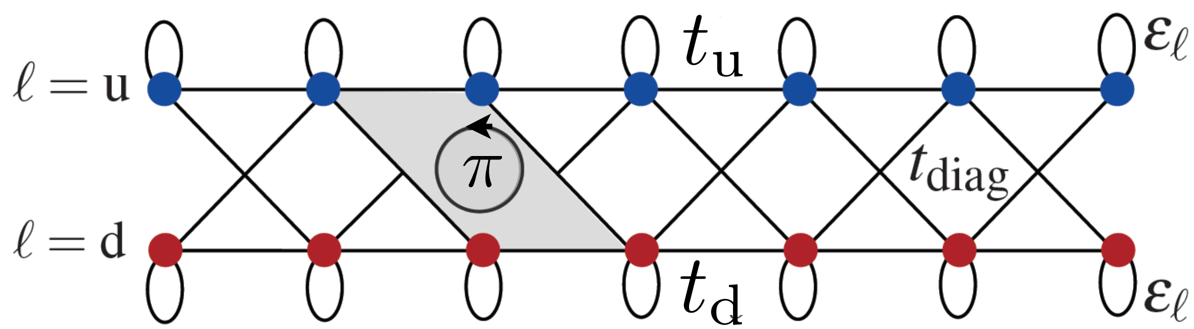

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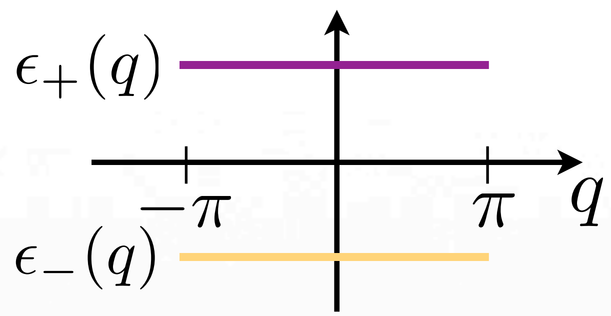
Zeeman splitting $\quad V_{\mathrm{imb}} = \sum_{j,\ell} \frac{\Delta\epsilon}{2} s_\ell n_{j,\ell}$



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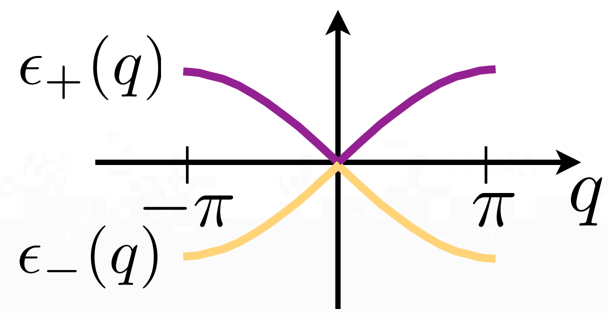

$|\Delta_\epsilon|/4\tilde{t} = 1$

(*undoubled*) Dirac point

M. Creutz, PRL **83** 2636 (1999)

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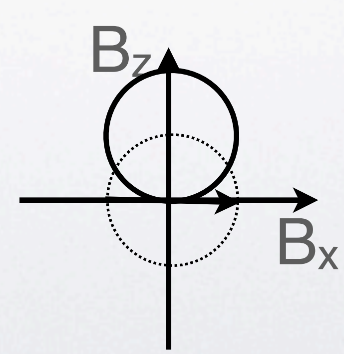

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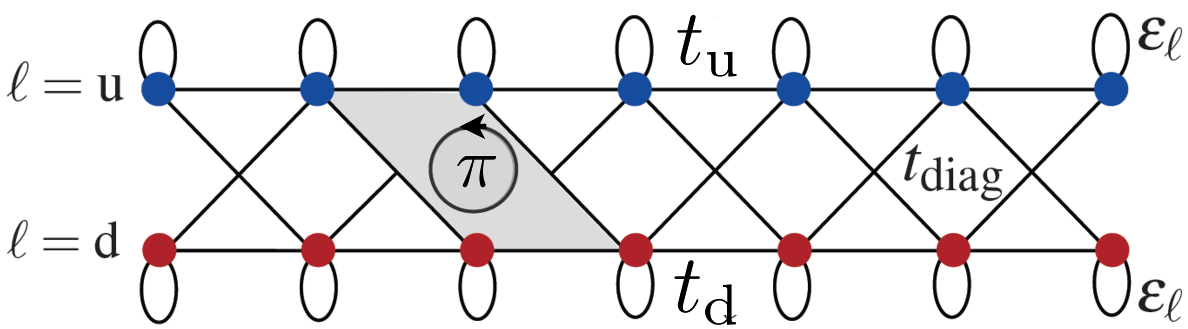

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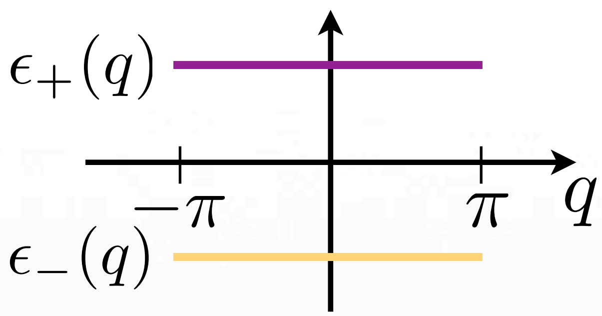

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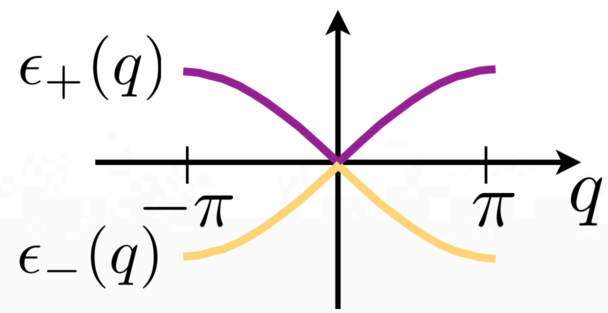

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Discrete chiral symmetry only! Class **AIII**

$$\sigma_y H(q) \sigma_y = -H(q)$$

$$\nexists U_{T/C} \text{ s.t. } \quad U_\alpha H(-q)^* U_C^\alpha = \pm H(q)$$

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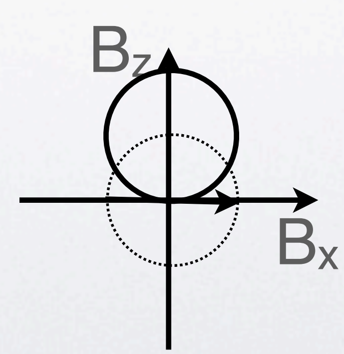

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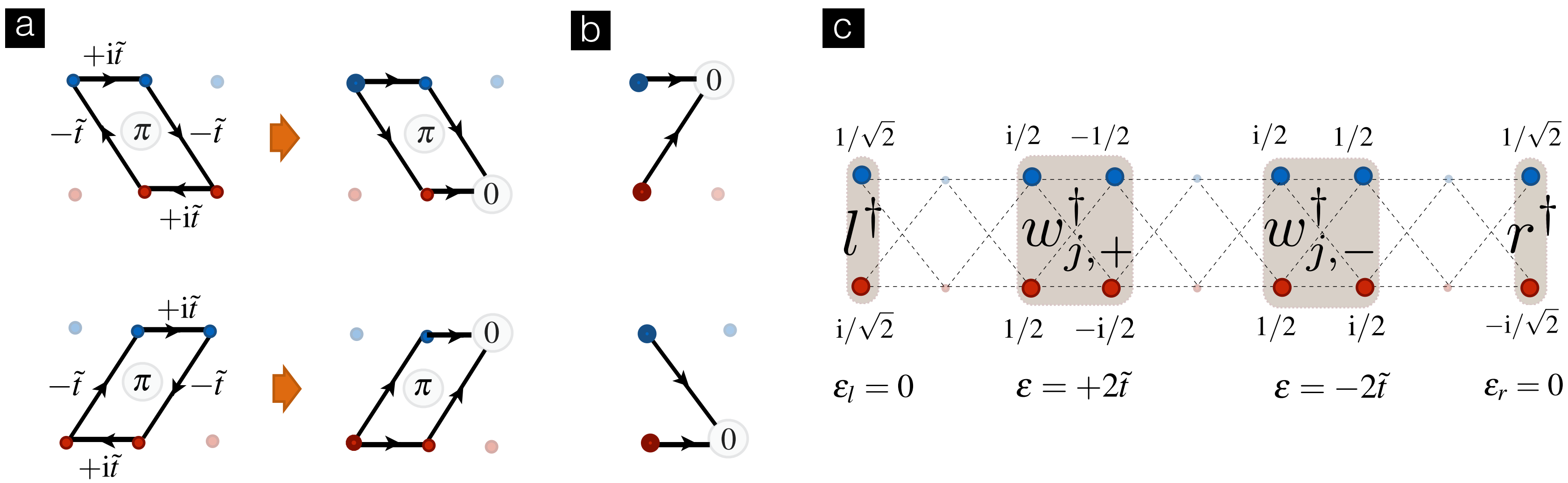
Flat bands, AB cages & Edge States



Flat bands <==> basis of localized states (Aharanov-Bohm cages) $w_{j,\alpha}$

Vidal, Mosseri, & Doucot, PRL 81, 5888 (1998)

Topological <==> zero-energy (mid-gap) edge states $\epsilon_\eta = 0 \qquad \eta = l, r$

$$H_{\rm FB} = \sum_{i=1}^{N-1} \sum_{\alpha=\pm} \epsilon_\alpha w^\dagger_{i,\alpha} w_{i,\alpha} + \sum_{\eta=l,r} \epsilon_\eta \eta^\dagger \eta$$

Doubly degenerate ground at half filling (N_p = N)

Tovmasyan, van Nieuwenburg, & Huber, PRB 88, 220510(R) (2013)
Takayoshi, Katsura, Watanabe, & Aoki, PRA 88, 063613 (2013)
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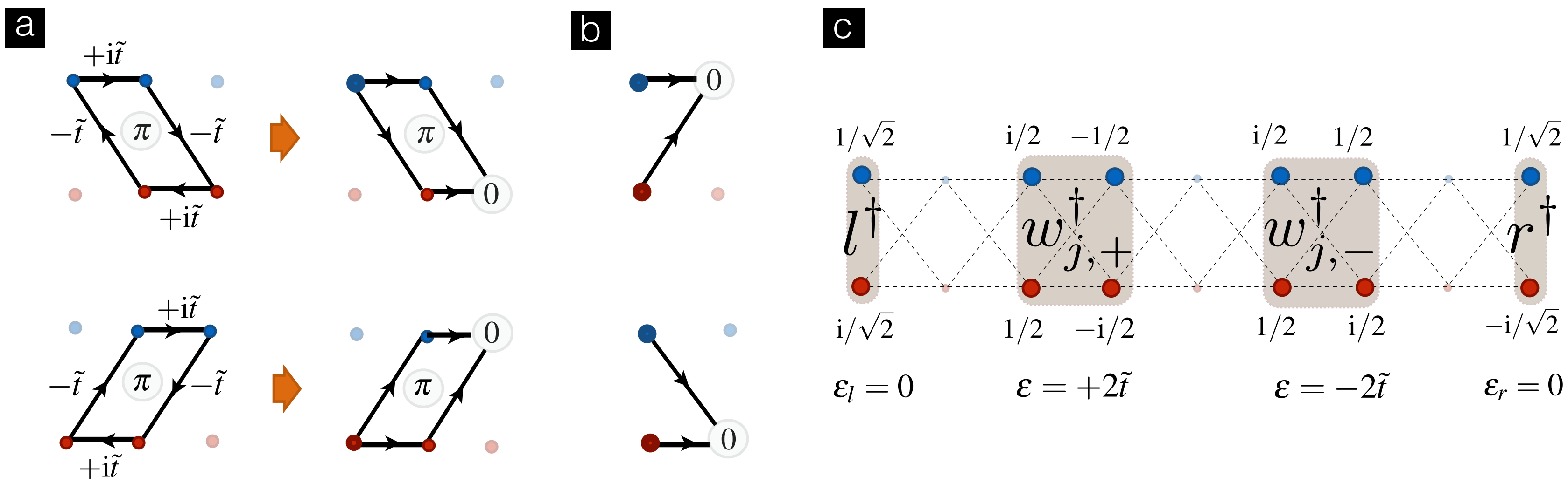

Bragg techniques
to measure edge states
in ultracold cold atoms
(with steep enough potential)

Goldman,Beugnon, Gerbier,
PRL 108, 255303 (2012).

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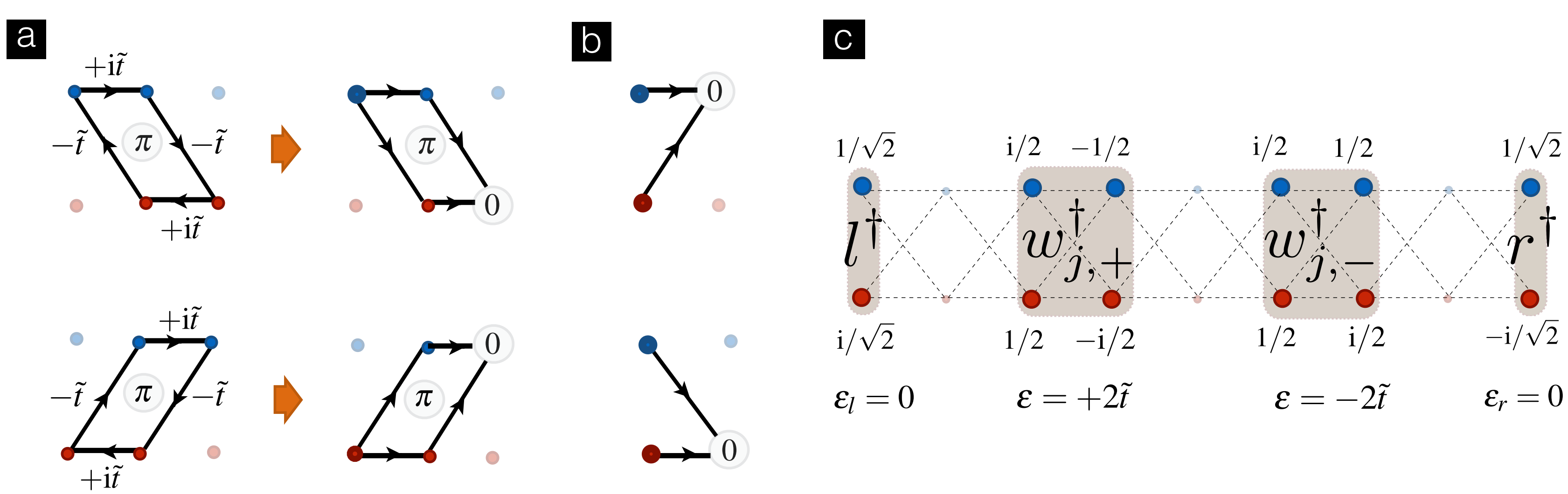

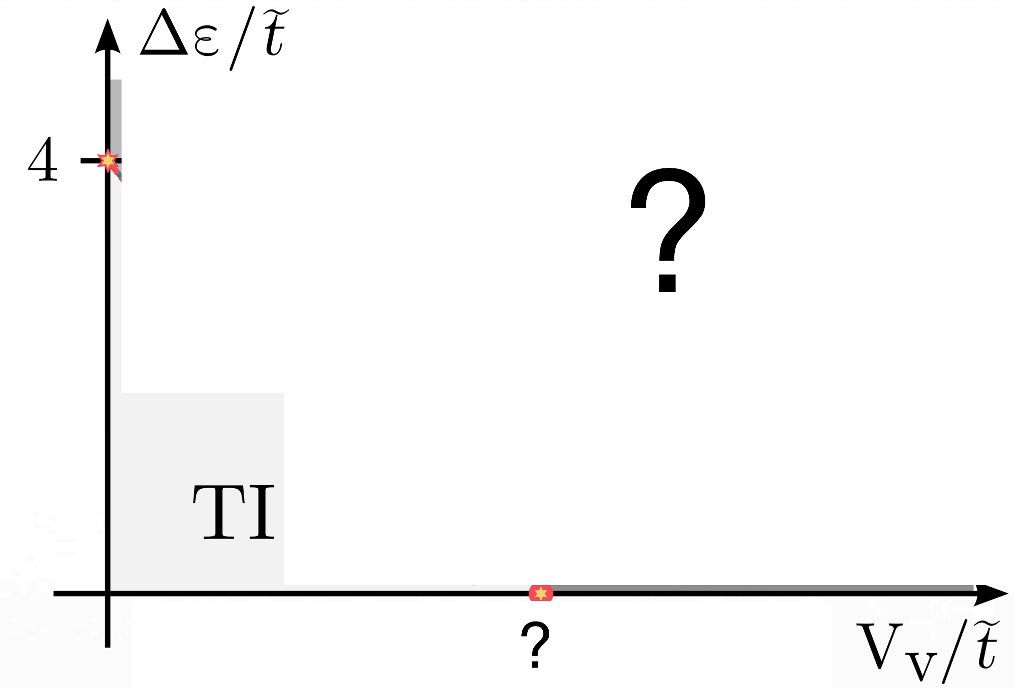

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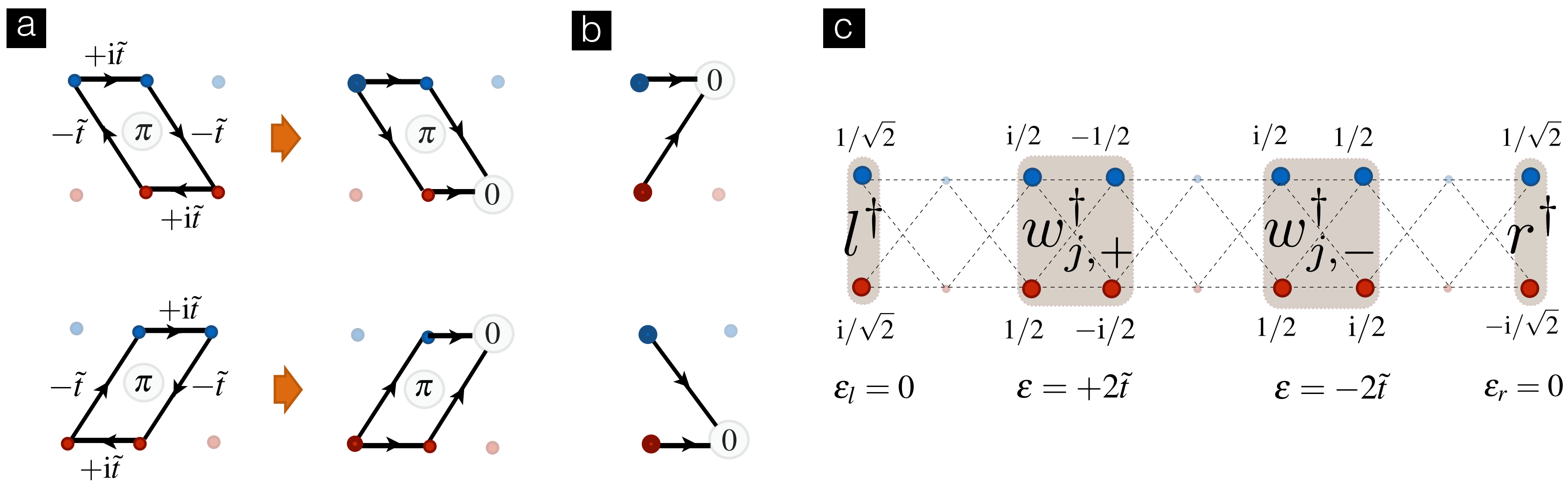
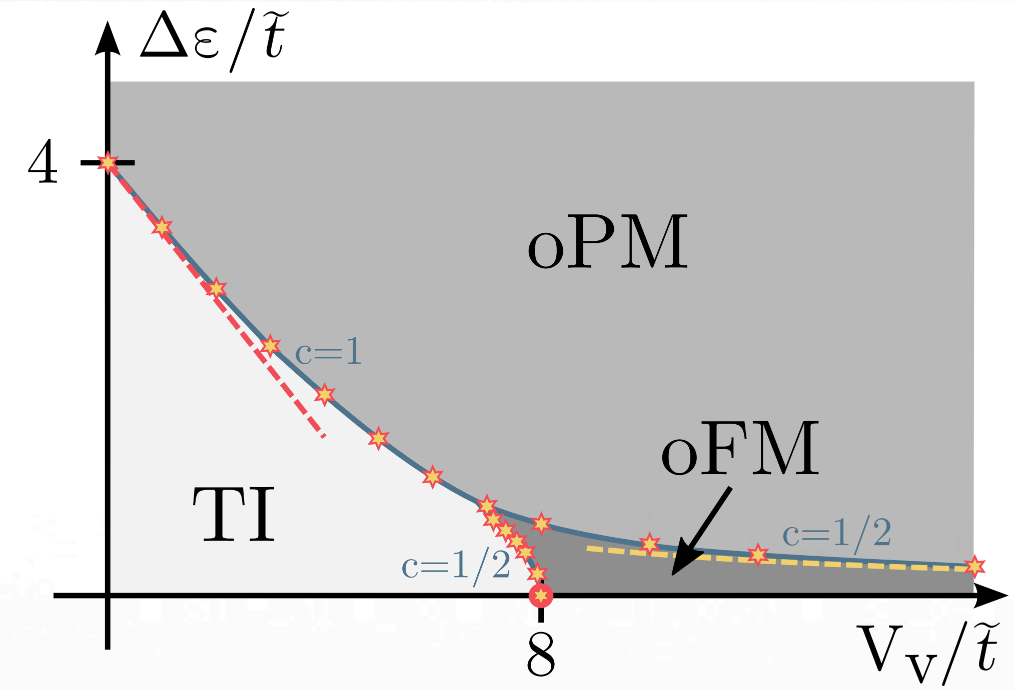
$$V_{\rm imb} = \sum_{j,\ell} \frac{\Delta\epsilon}{2} s_\ell n_{j,\ell} \qquad V_{\rm Hubb} = \sum_{j,\ell} \frac{V_{\rm v}}{2} n_{j,\ell} n_{j,\bar{\ell}}$$

==> bend bands / close gap & cancel edge modes ...

Flat bands, AB cages & Edge States





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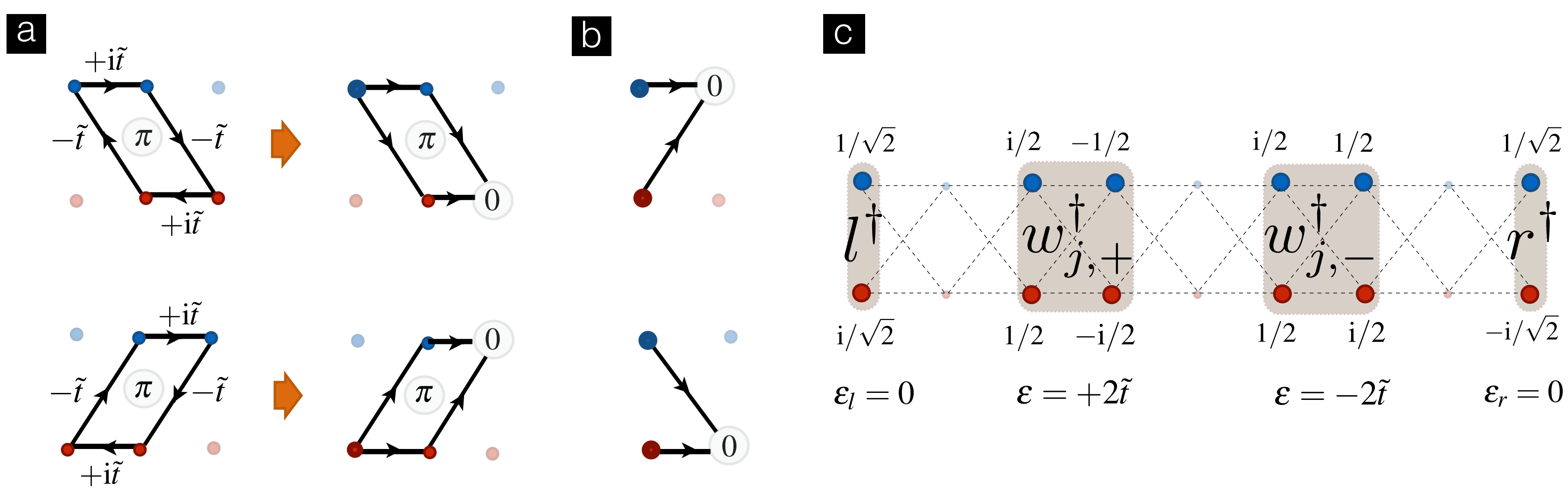

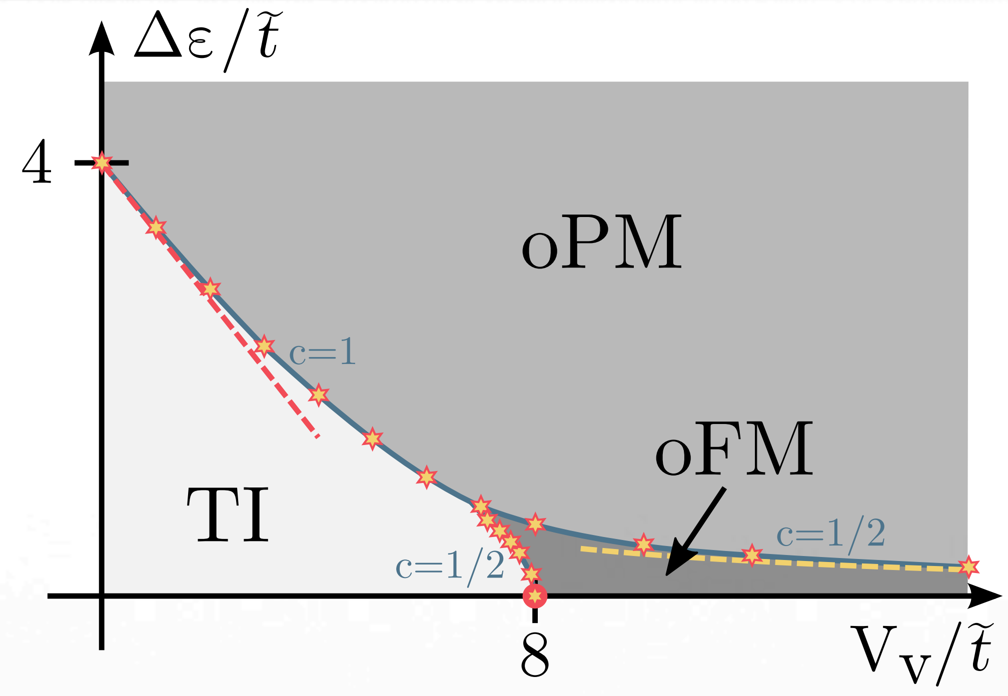

Outline of the attack plan:

1. Analytics: mappings onto effective Ising models for each transition
2. Numerics: matrix product states (MPS) & entanglement analysis
3. Experiments: sketch of proposal with assisted tunnelling in optical lattices

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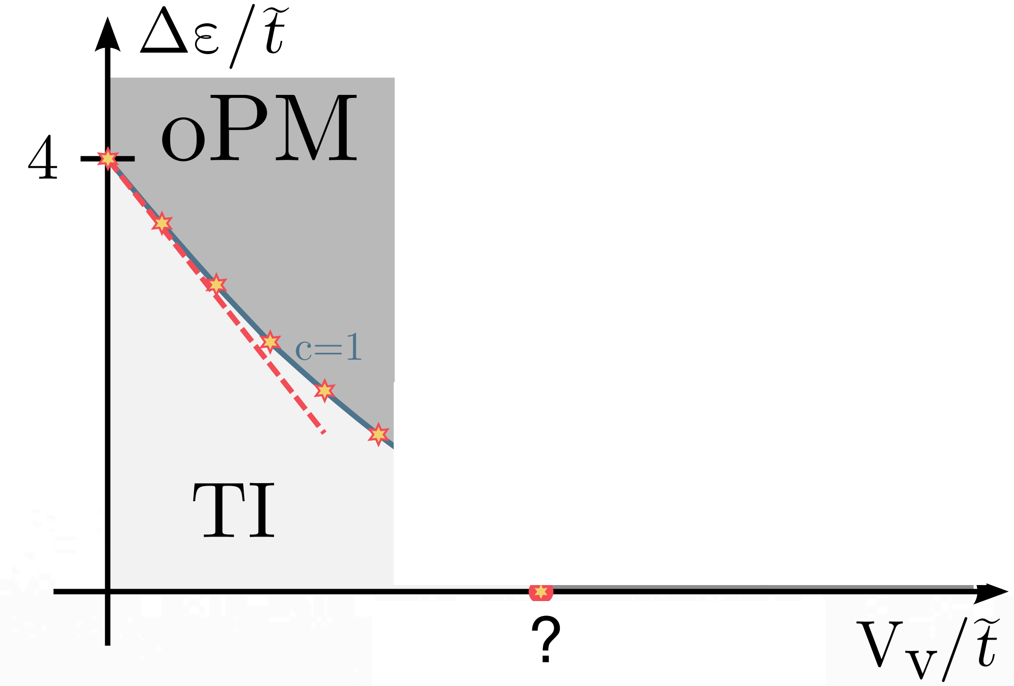
Weak interactions: gap behaviour

Indicator 1: compressibility gap vs. degeneracy split

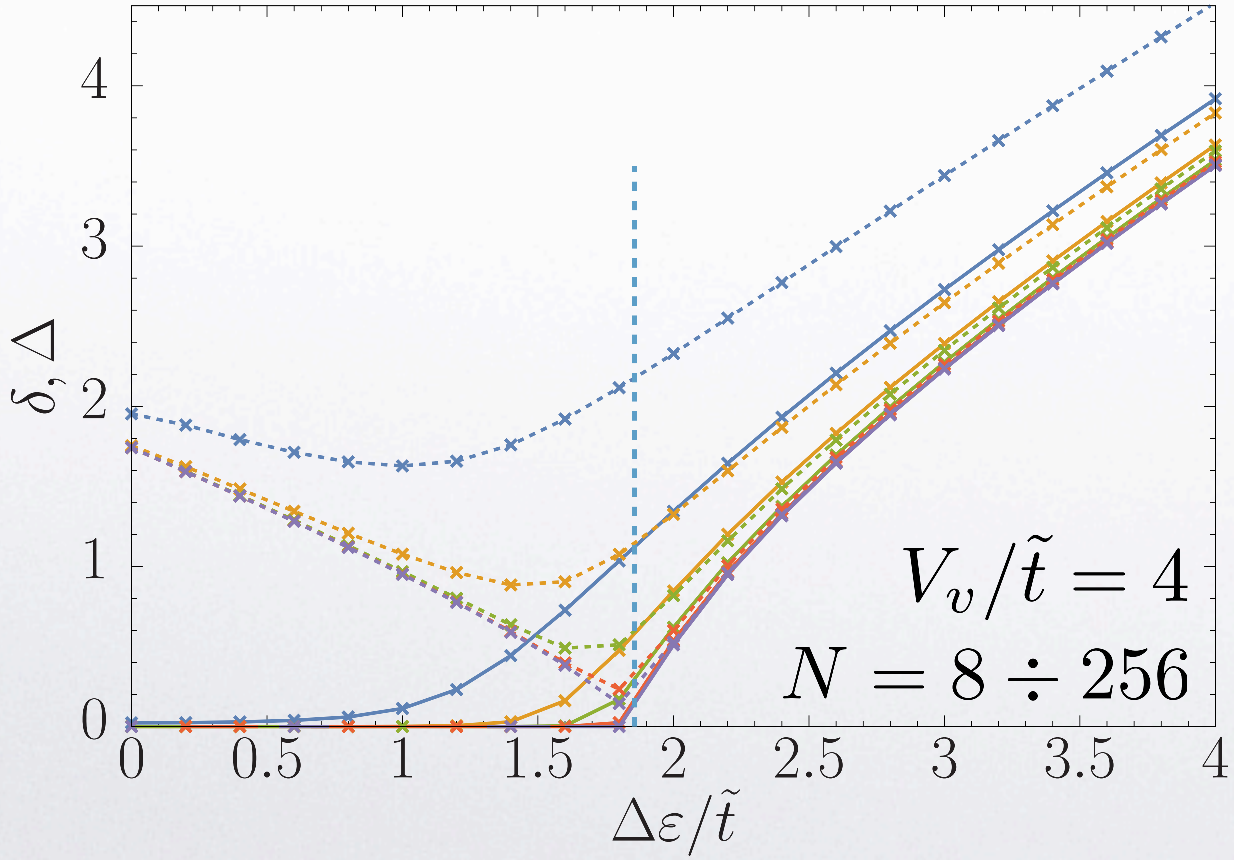
$$\delta = \lim_{N\to\infty} [E(N+1) + E(N-1) - 2E(N)]$$

$$\Delta = \lim_{N\to\infty} \tfrac{1}{2} [E(N+2) + E(N-2) - 2E(N)]$$



TI: $\delta = 0$, $\Delta \neq 0$

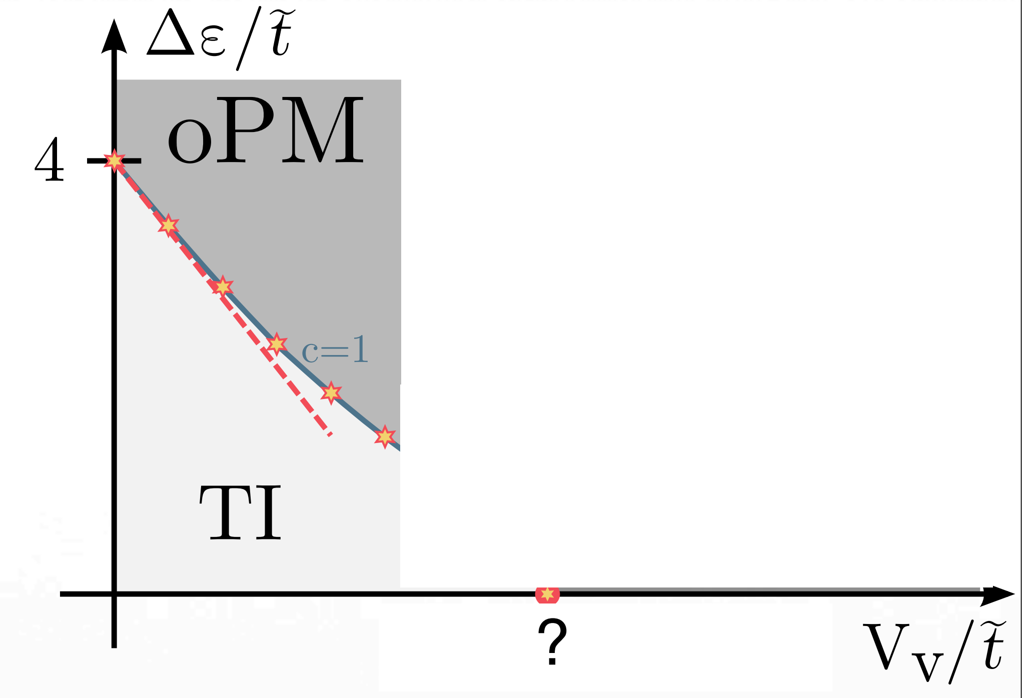
oPM: $\delta = \Delta \neq 0$

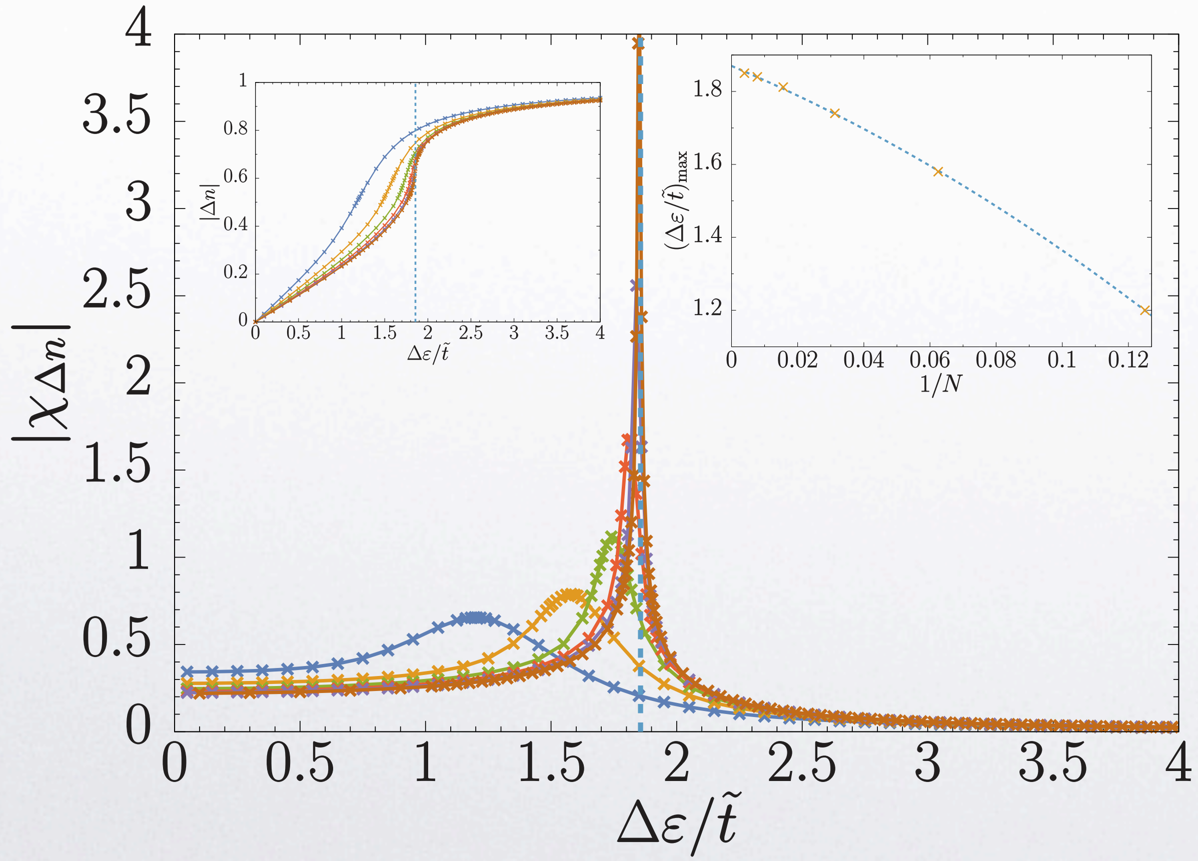


Weak interactions: coupled Ising chains

Indicator 2: density imbalance between ladder legs

$$\Delta n = \frac{1}{N}\sum_i \left(c^\dagger_{i,\mathrm{u}}c_{i,\mathrm{u}} - c^\dagger_{i,\mathrm{d}}c_{i,\mathrm{d}}\right)$$





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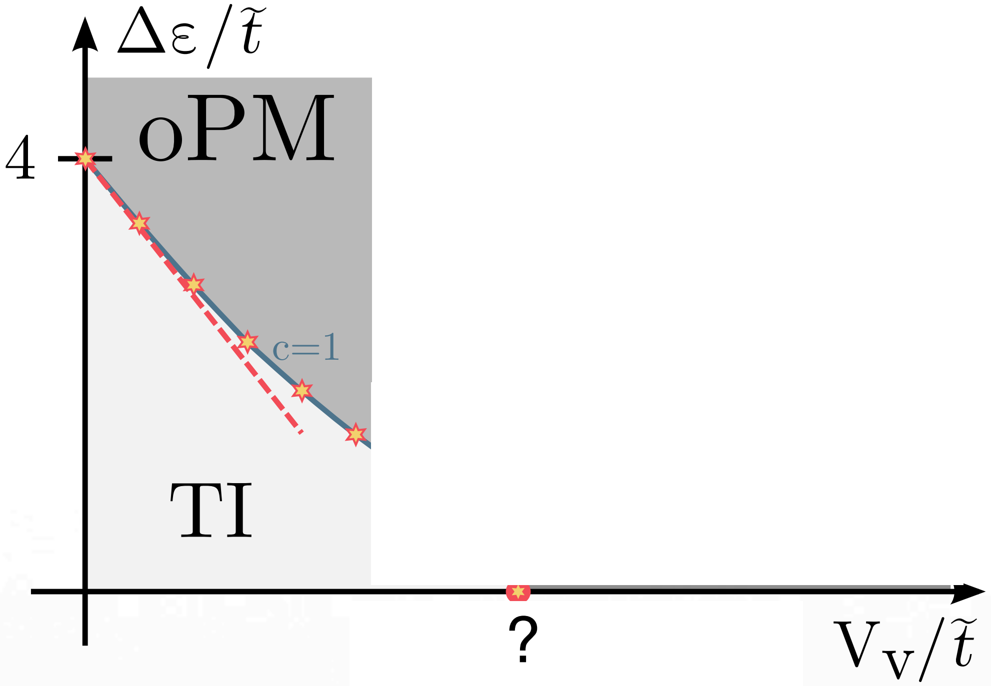
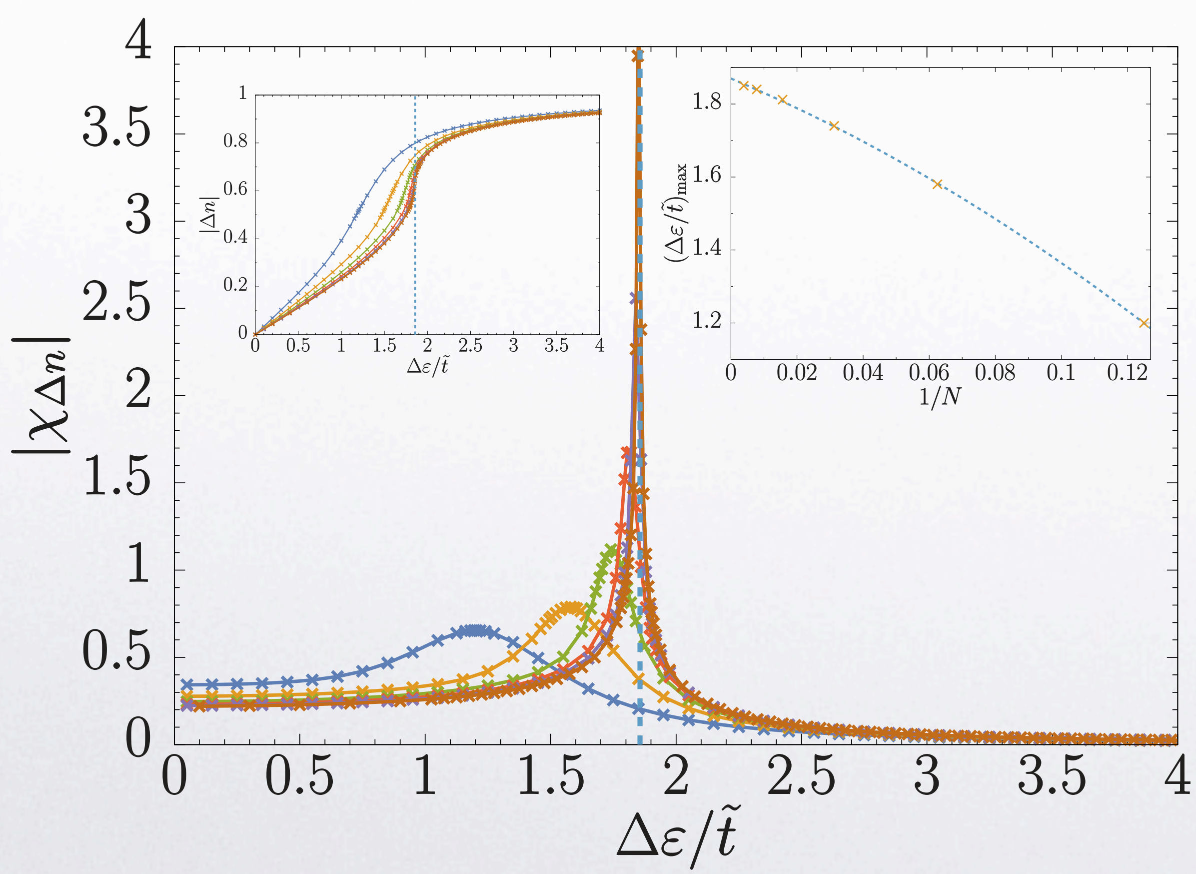
$$\Delta n = \frac{1}{N}\sum_i \left(c^\dagger_{i,\mathrm{u}}c_{i,\mathrm{u}} - c^\dagger_{i,\mathrm{d}}c_{i,\mathrm{d}}\right) = \frac{1}{2N}\sum_j\sum_n \sigma^z_{j,n}$$





Bogolubov + Jordan Wigner trafos

$$r_{j,1} = \frac{\mathrm{i}^j}{\sqrt{2}}\left(\mathrm{i}c_{j,\mathrm{u}} + (-1)^j c^\dagger_{j,\mathrm{d}}\right) \qquad r_{j,2} = \frac{\mathrm{i}^j}{\sqrt{2}}\left(c_{j,\mathrm{u}} + \mathrm{i}(-1)^j c^\dagger_{j,\mathrm{d}}\right)$$

$$r^\dagger_{j,n} = \prod_{i<j}(-\sigma^z_{i,n})\sigma^+_{j,n} = (r_{j,n})^\dagger, \quad r^\dagger_{j,n}r_{j,n} = \tfrac{1}{2}\sigma^z_{j,n} + \tfrac{1}{2}$$

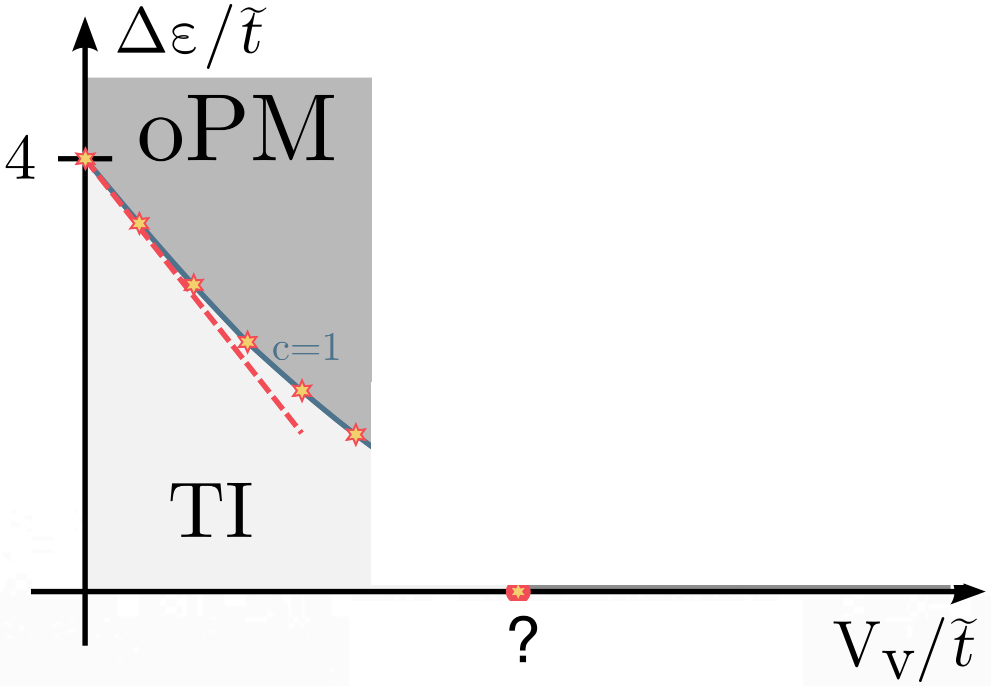
$$H_{\pi\mathrm{C}} = \sum_j \sum_{n=1,2}\left(-\tilde{t}\sigma^x_{j,n}\sigma^x_{j+1,n} + \frac{\Delta\varepsilon}{4}\sigma^z_{j,n}\right)$$

$$V_{\mathrm{Hubb}} = -\frac{V_\mathrm{v}}{4}\sum_j \sigma^z_{j,1}\sigma^z_{j,2} + \mathrm{const.}$$

Weak interactions: coupled Ising chains

Indicator 2: density imbalance between ladder legs

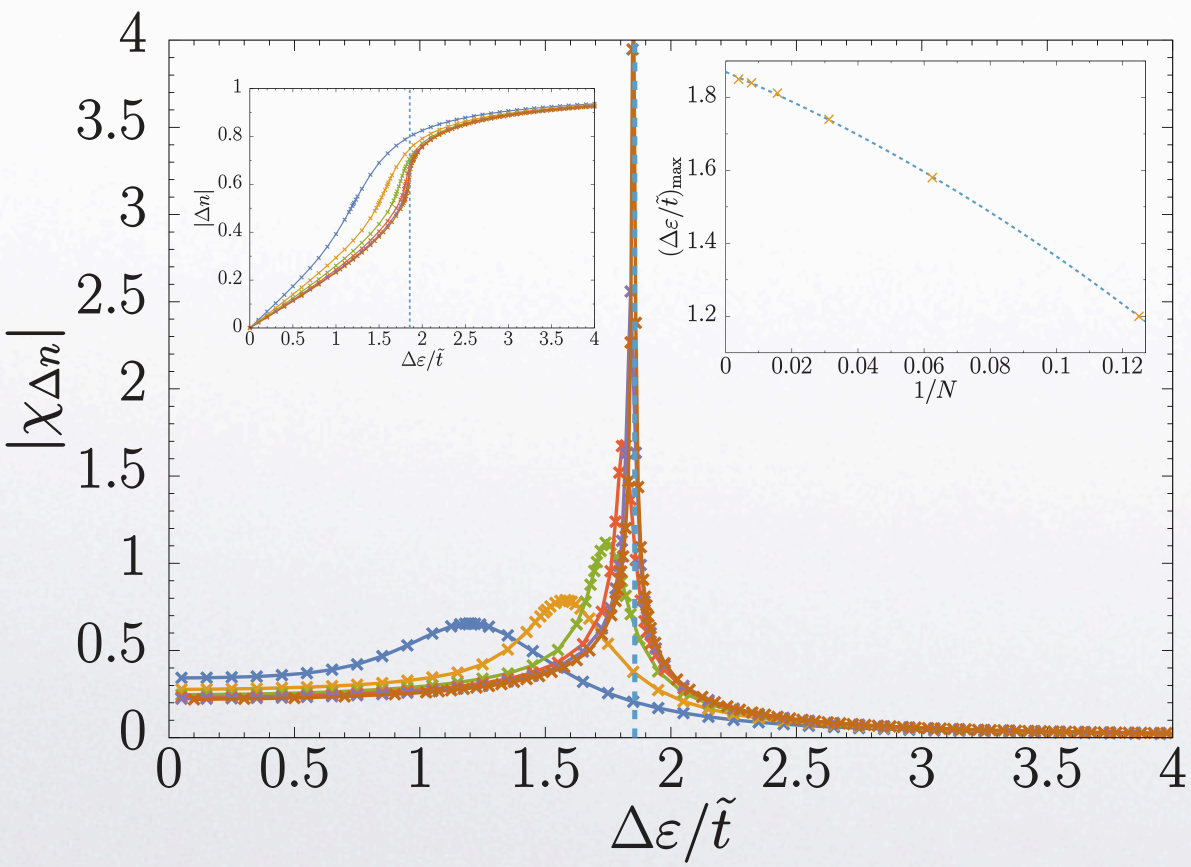
$$\Delta n = \frac{1}{N}\sum_i \left(c^\dagger_{i,\mathrm{u}}c_{i,\mathrm{u}} - c^\dagger_{i,\mathrm{d}}c_{i,\mathrm{d}}\right) = \frac{1}{2N}\sum_j\sum_n \sigma^z_{j,n}$$



self-consistent mean-field

$$\frac{\Delta\epsilon}{\tilde{t}} = 4 - \frac{2}{\pi}\frac{V_v}{\tilde{t}} + \mathcal{O}\left(\frac{V_v^2}{\tilde{t}^2}\right)$$

two simultaneous Ising transitions (**c=1**)



$$H_{\pi\mathrm{C}} = \sum_j \sum_{n=1,2}\left(-\tilde{t}\sigma^x_{j,n}\sigma^x_{j+1,n} + \frac{\Delta\varepsilon}{4}\sigma^z_{j,n}\right)$$

$$V_{\mathrm{Hubb}} = -\frac{V_{\mathrm{v}}}{4}\sum_j \sigma^z_{j,1}\sigma^z_{j,2} + \mathrm{const.}$$

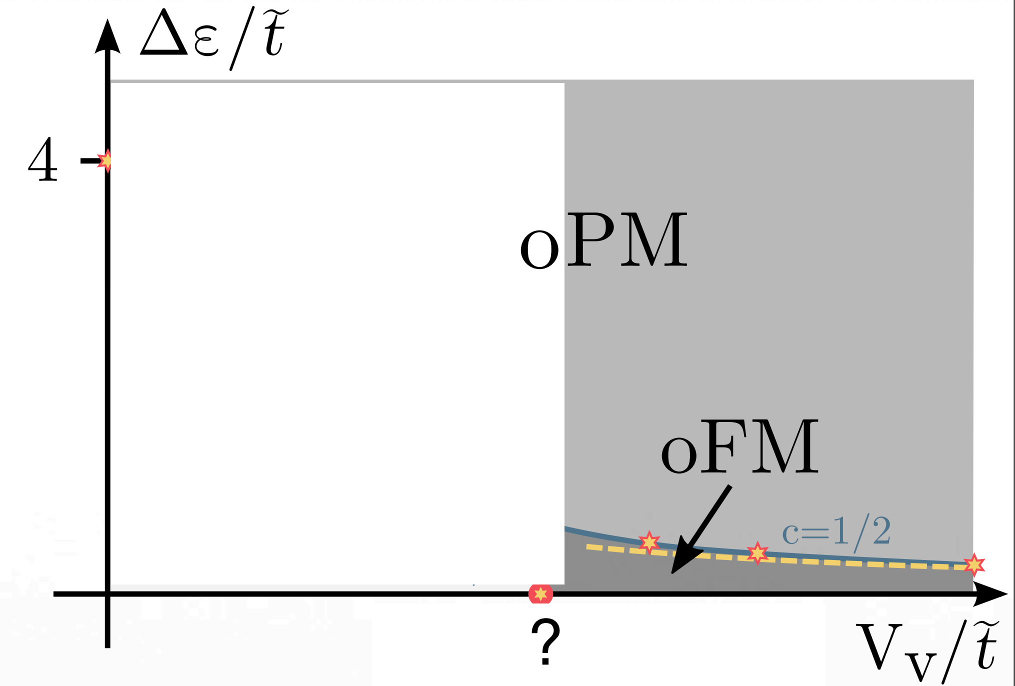
Strong interactions: orbital Ising model

Gutzwiller projector

$$\mathcal{P}_\mathrm{r} = \Pi_i (n_{i,\mathrm{u}} - n_{i,\mathrm{d}})^2$$

Super-exchange coupling

$$J = -8\tilde{t}^2/V_\mathrm{v}$$

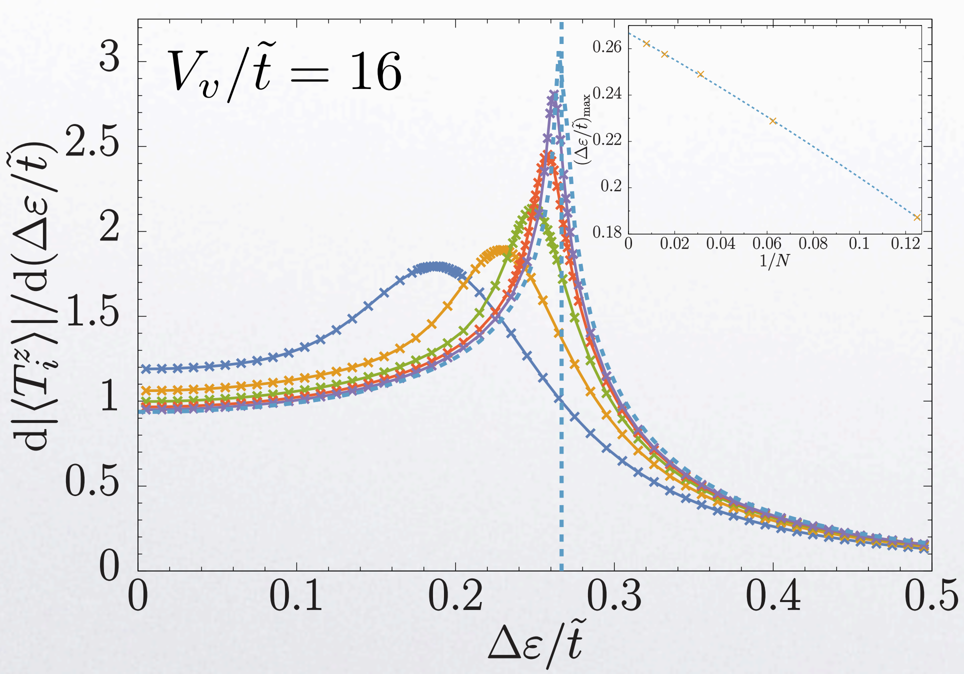


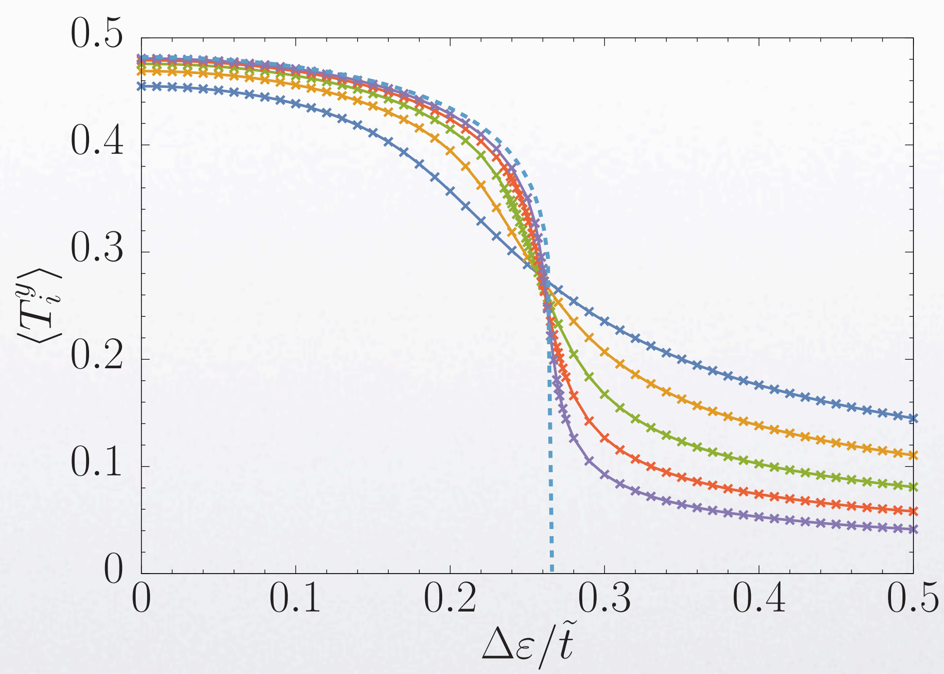
$$\mathcal{P}_\mathrm{r} H_{\pi\mathrm{CH}} \mathcal{P}_\mathrm{r} = \frac{1}{4} J N + J \sum_i T_i^y T_{i+1}^y + \Delta\epsilon \sum_i T_i^z$$

$$T_i^z = \frac{1}{2}\left(c_{i,\mathrm{u}}^\dagger c_{i,\mathrm{u}} - c_{i,\mathrm{d}}^\dagger c_{i,\mathrm{d}}\right) \qquad T_i^y = \frac{1}{2}\left(-\mathrm{i} c_{i,\mathrm{u}}^\dagger c_{i,\mathrm{d}} + \mathrm{i} c_{i,\mathrm{d}}^\dagger c_{i,\mathrm{u}}\right)$$

single Ising critical line (**c=1/2**)

$$\frac{2\Delta\epsilon}{|J|} = 1 \iff \frac{\Delta\epsilon}{\tilde{t}} = \frac{4\tilde{t}}{V_\mathrm{v}}$$





ICTP Workshop 2017
Quantum Technologies

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arXiv:1612.02996

Synthetic Creutz-Hubbard model: interacting topol. insul. with ultracold atoms

Strong interactions: orbital Ising model

Gutzwiller projector

$$\mathcal{P}_{\rm r} = \Pi_i (n_{i,\rm u} - n_{i,\rm d})^2$$

Super-exchange coupling

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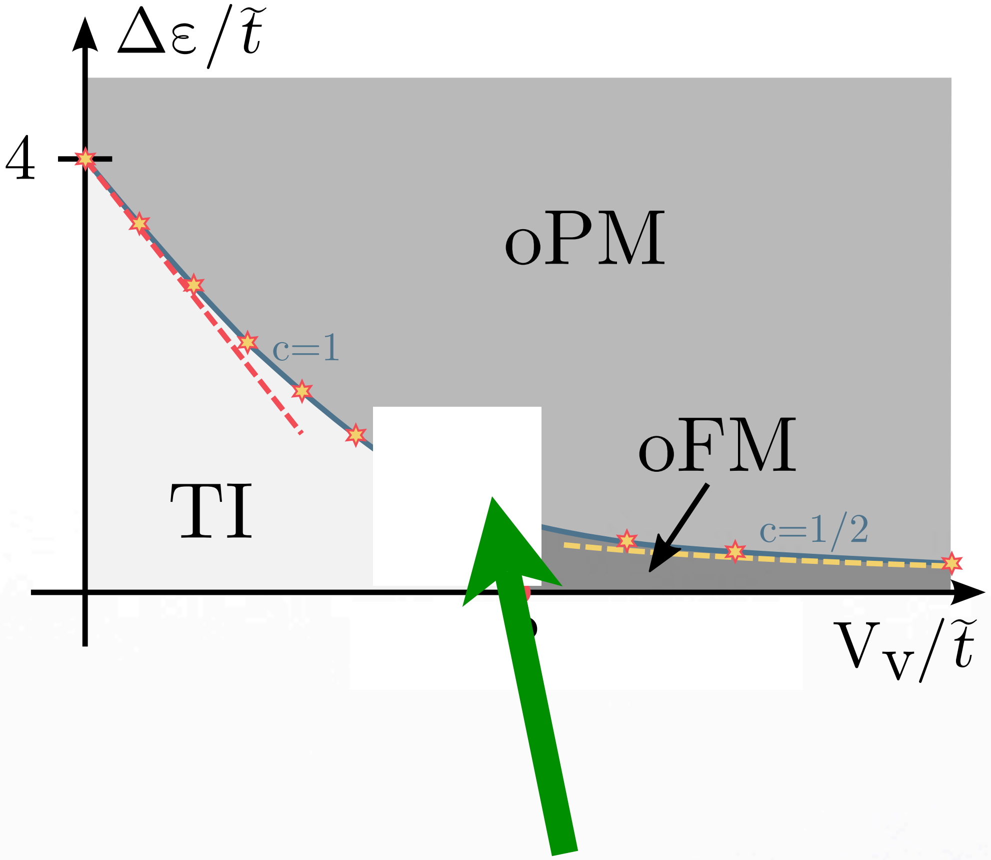

$$\mathcal{P}_{\rm r} H_{\pi\rm CH} \mathcal{P}_{\rm r} = \frac{1}{4} J N + J \sum_i T_i^y T_{i+1}^y + \Delta\epsilon \sum_i T_i^z$$

$$T_i^z = \frac{1}{2}\left(c_{i,\rm u}^\dagger c_{i,\rm u} - c_{i,\rm d}^\dagger c_{i,\rm d}\right) \qquad T_i^y = \frac{1}{2}\left(-{\rm i} c_{i,\rm u}^\dagger c_{i,\rm d} + {\rm i} c_{i,\rm d}^\dagger c_{i,\rm u}\right)$$

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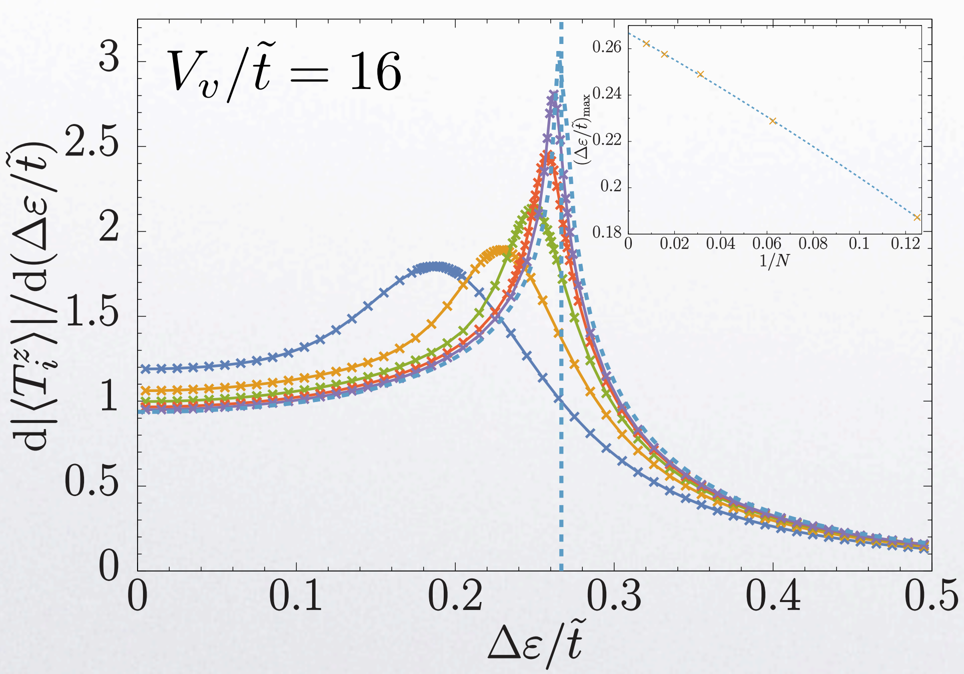
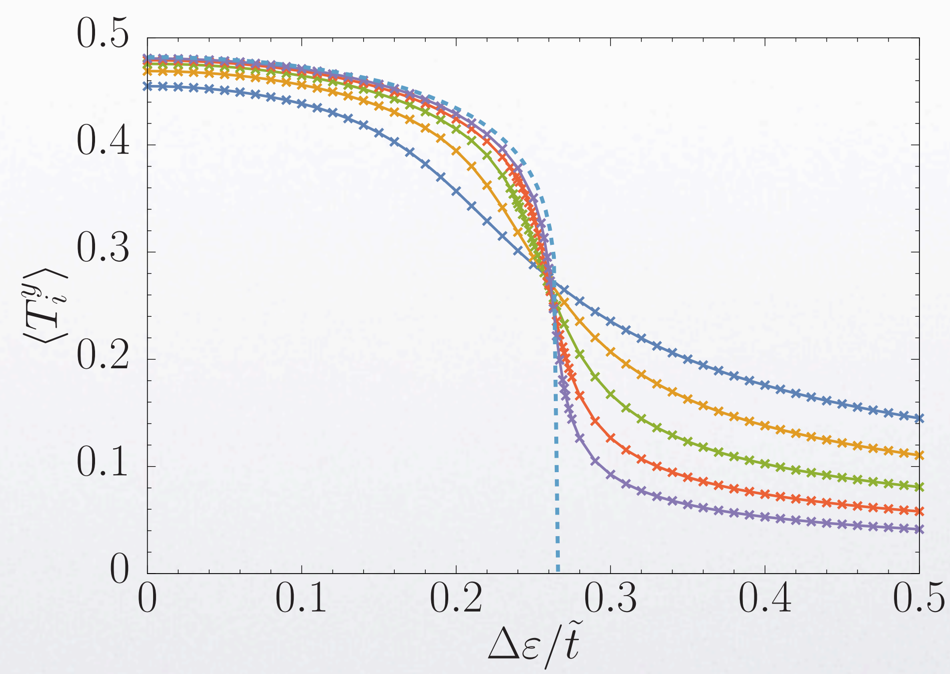




$$\langle T_i^y T_{i+r}^y \rangle_{\rm TI} = 0$$

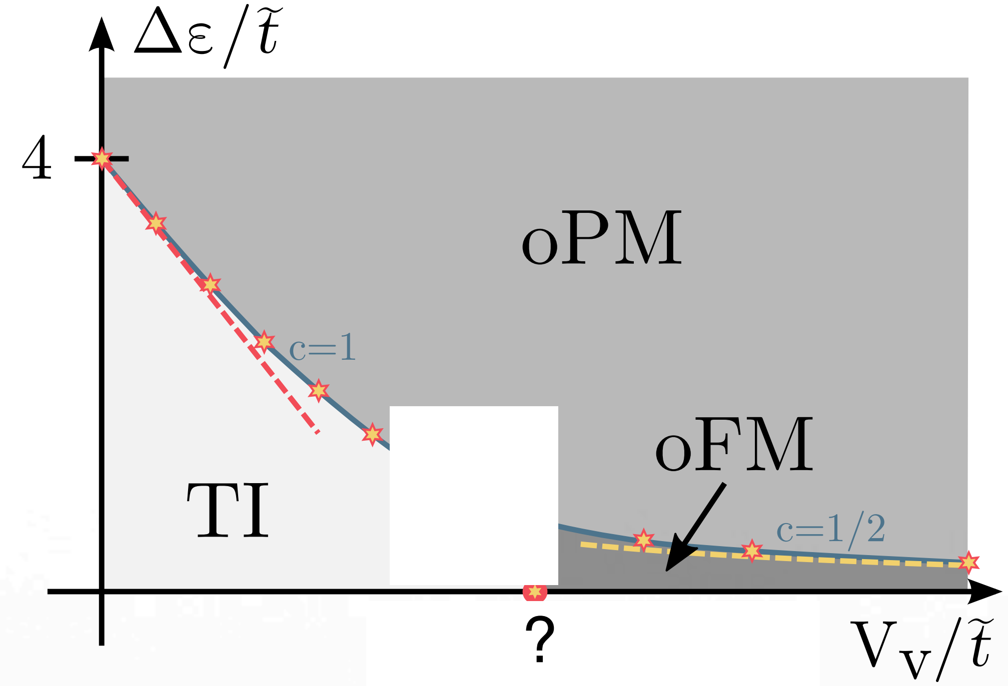
oFM ≠ TI

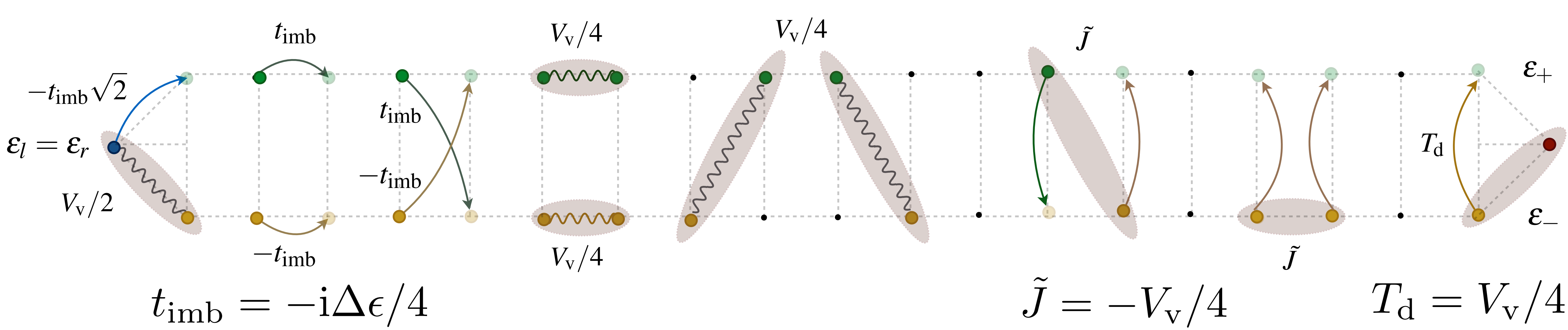
another transition
at low imbalance !

AB-cages: *exotic* Hubbard

NNN interactions + dens. dep. tunnelling
without dipolar atoms or strange schemes !!

$$V_{\mathrm{Hubb}} = V_{\mathrm{nn}} + V_{\mathrm{pt}} + V_{\mathrm{dt}}$$





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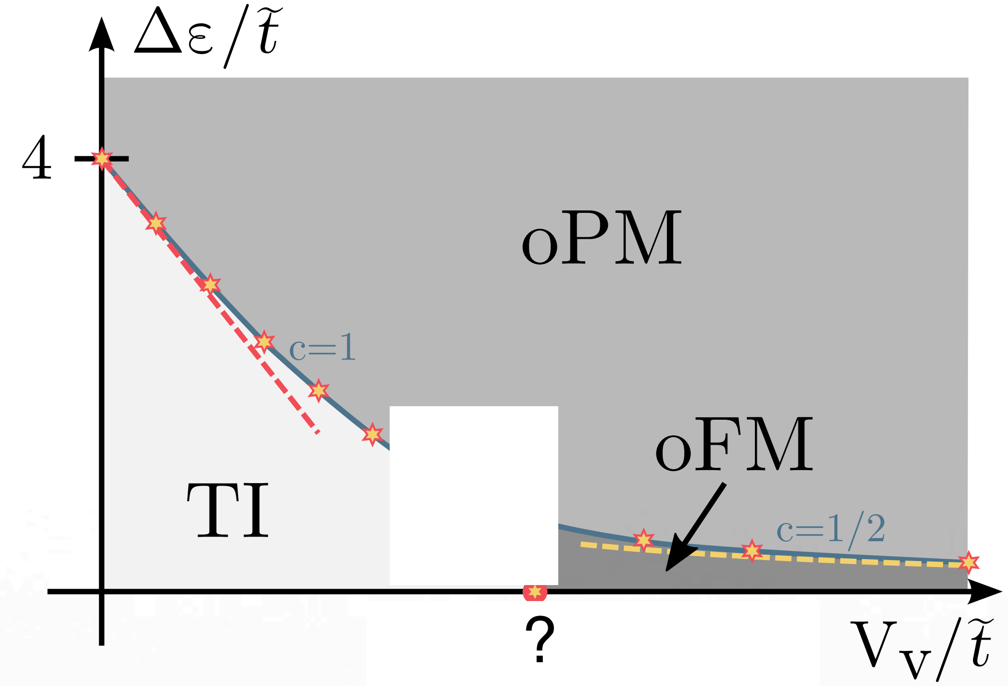
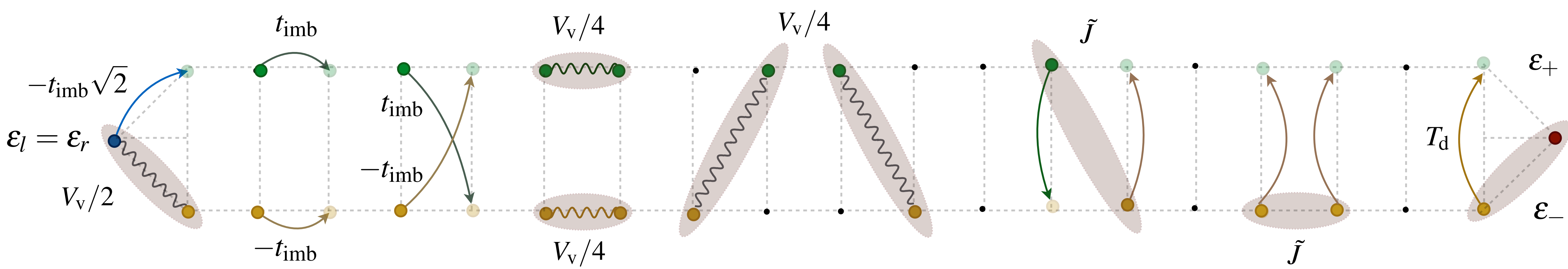
CRUCIAL: we keep both bands! no projection!





$t_{\mathrm{imb}} = -\mathrm{i}\Delta\epsilon/4$ $\tilde{J} = -V_v/4$ $T_d = V_v/4$

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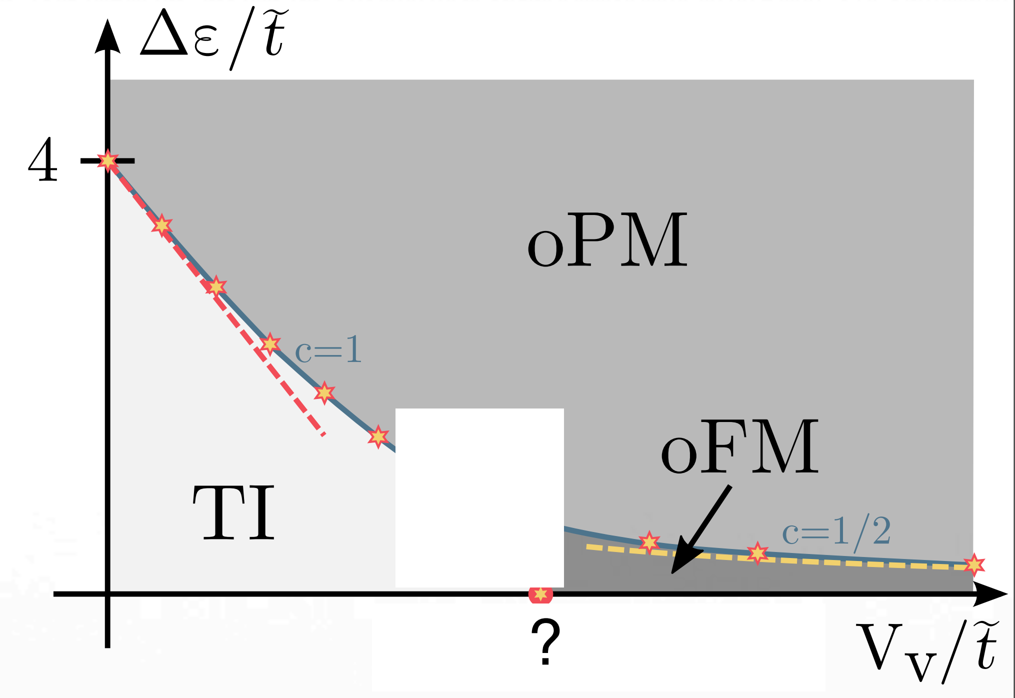

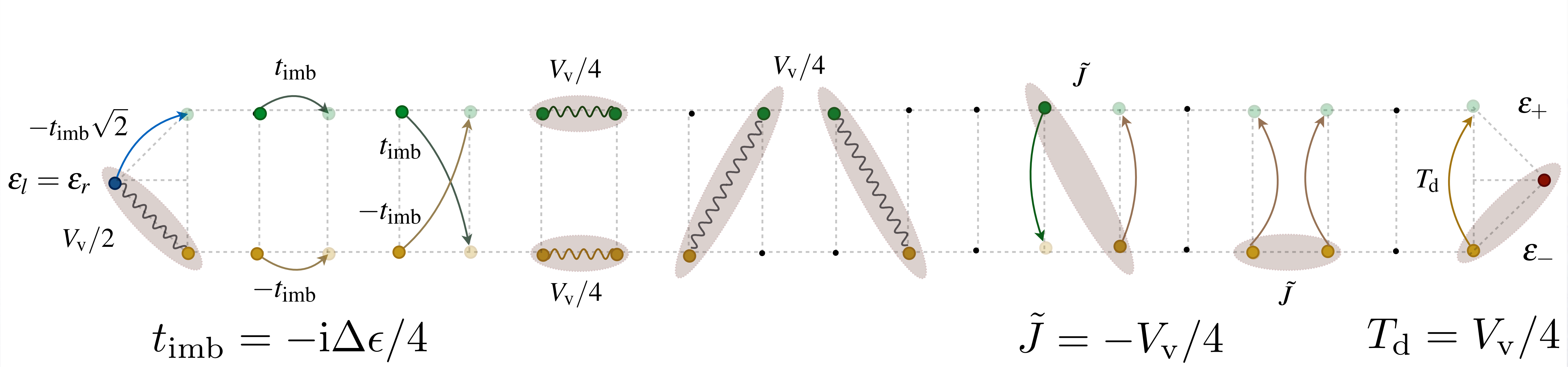

leads to bulk mediated edge-edge interaction à la Fano-Anderson ...

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Weak imbalance: yet another Ising

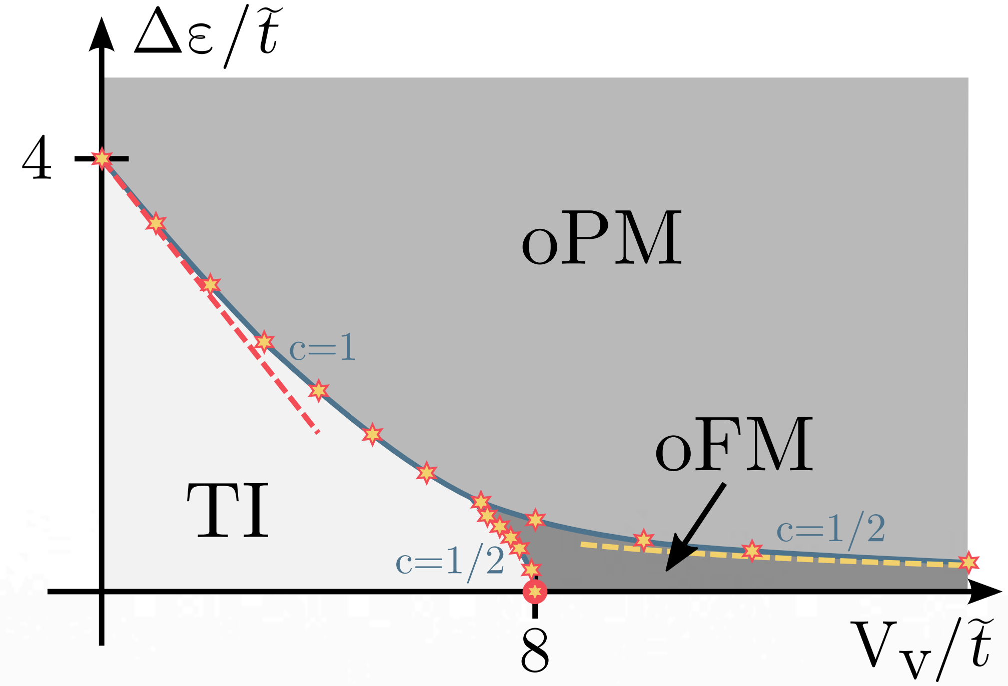
… restrict to singly occupied AB-c manifold ...

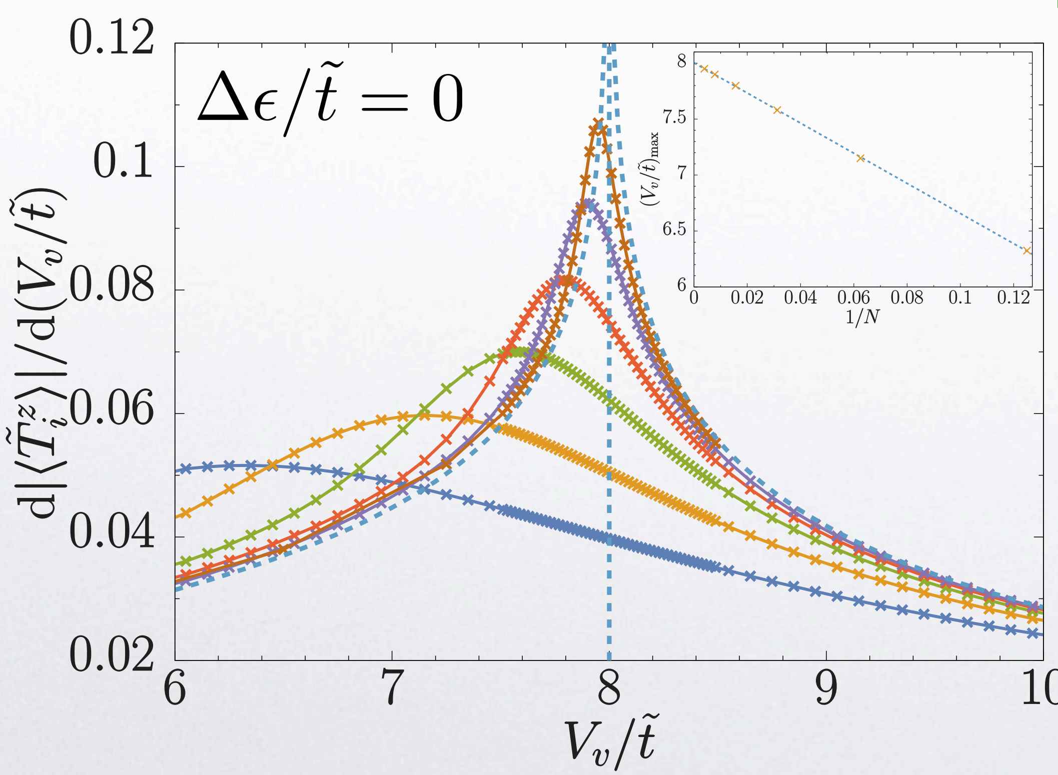
$$H_{\text{bulk}} = \frac{V_{\text{v}}}{4} N + \sum_{i=1}^{N-1} 4\tilde{t}\tilde{T}_i^z + \sum_{i=2}^{N-1} 4\tilde{J}\tilde{T}_{i-1}^x \tilde{T}_i^x$$

$$\tilde{T}_i^x = \frac{1}{2}\left(w_{i,+}^\dagger w_{i,-} + w_{i,-}^\dagger w_{i,+}\right) \qquad \tilde{T}_i^z = \frac{1}{2}\left(w_{i,+}^\dagger w_{i,+} - w_{i,-}^\dagger w_{i,-}\right)$$

single Ising critical line (**c=1/2**)

$$\frac{2\tilde{t}}{|\tilde{J}|} = 1 \iff \frac{V_{\text{v}}}{\tilde{t}} = 8$$





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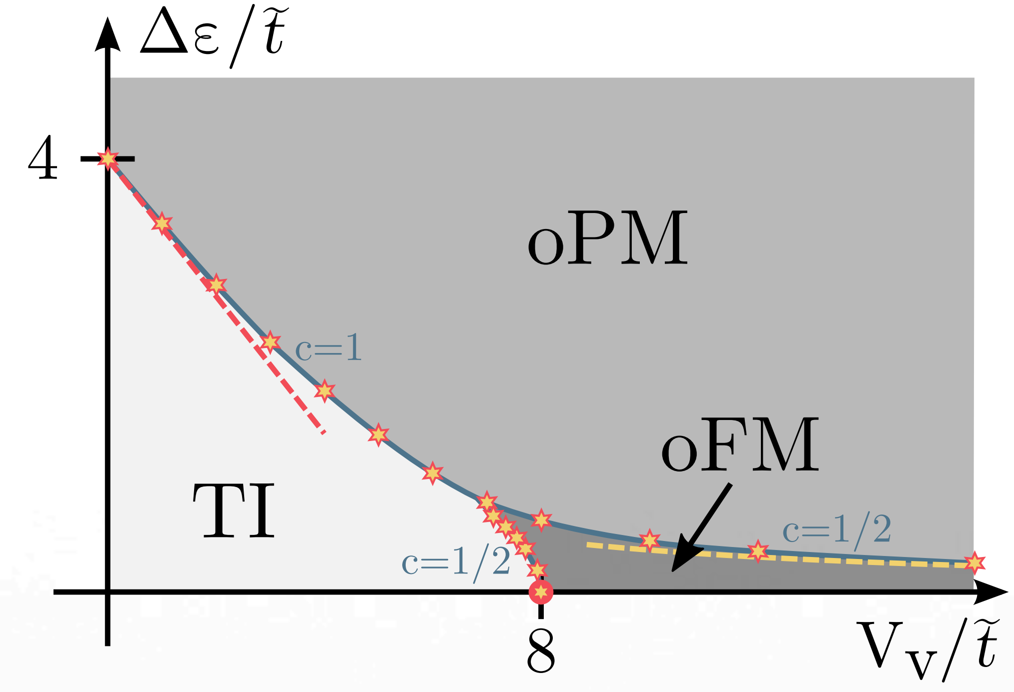

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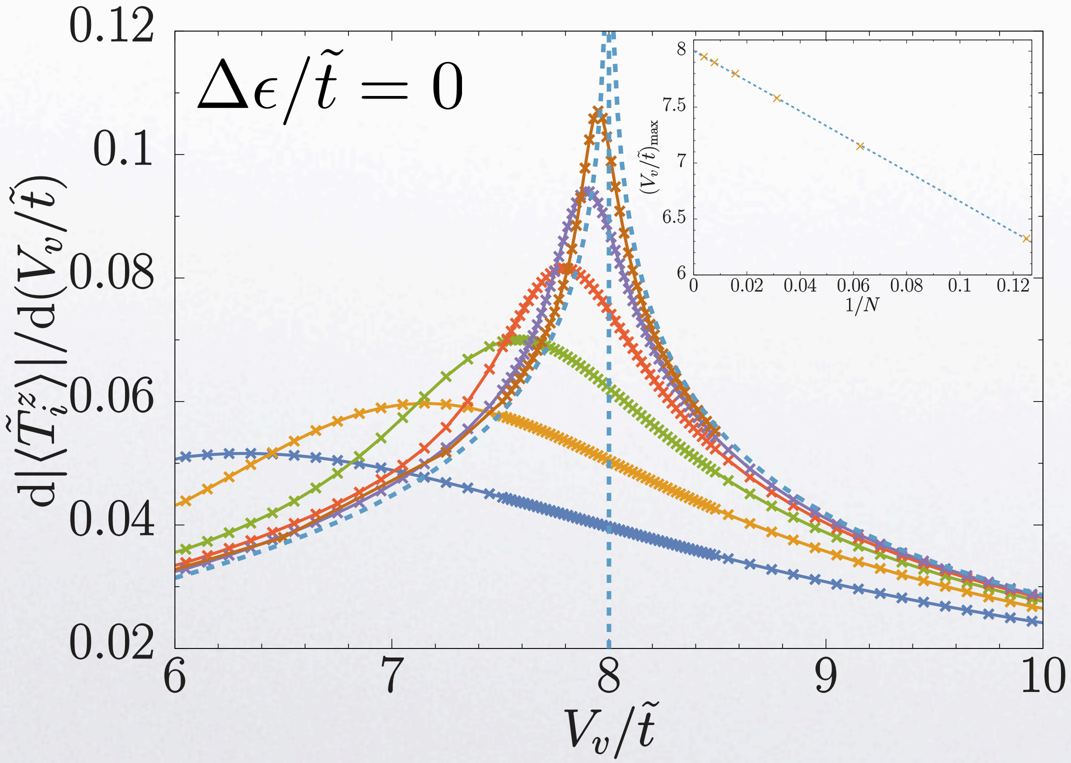

Jordan-Wigner gives "effective" bands $\tilde{\mathfrak{f}} = V_{\text{v}}/8\tilde{t}$

$$\tilde{\epsilon}_\pm(q) = \pm\tilde{\epsilon}(q) = \pm 2|\tilde{J}|\sqrt{1 + \tilde{\mathfrak{f}}^{-2} - 2\tilde{\mathfrak{f}}^{-1}\cos q}$$

dual to the original ones … $\mathfrak{f} = 4\tilde{t}/\Delta\epsilon$

$$\epsilon_\pm(q) = \pm\epsilon(q) = \pm 2\tilde{t}\sqrt{1 + \mathfrak{f}^{-2} + 2\mathfrak{f}^{-1}\sin q}$$

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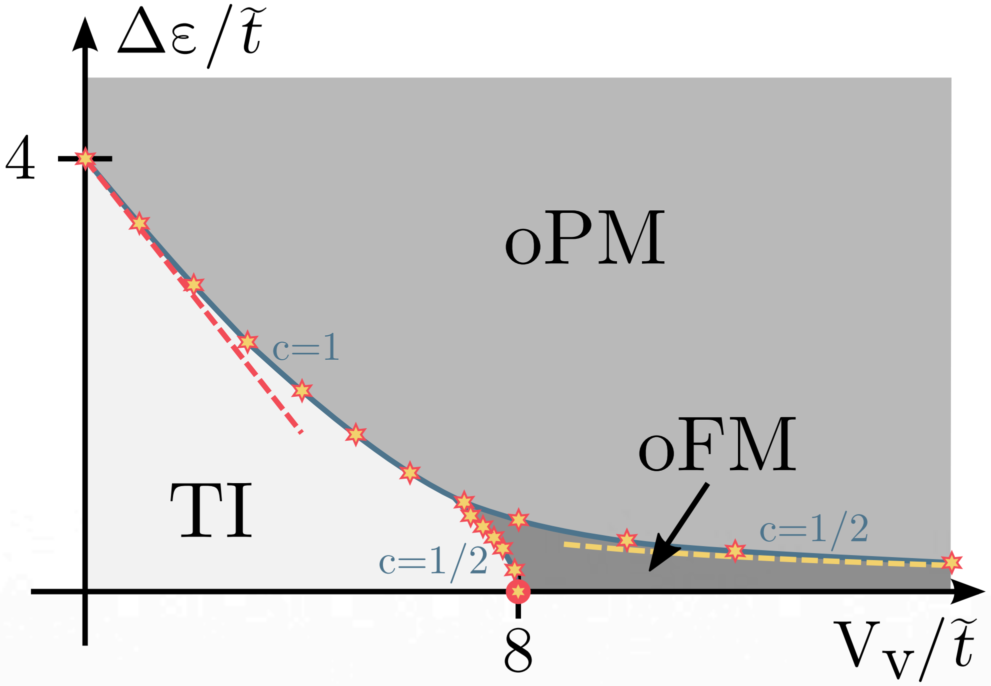

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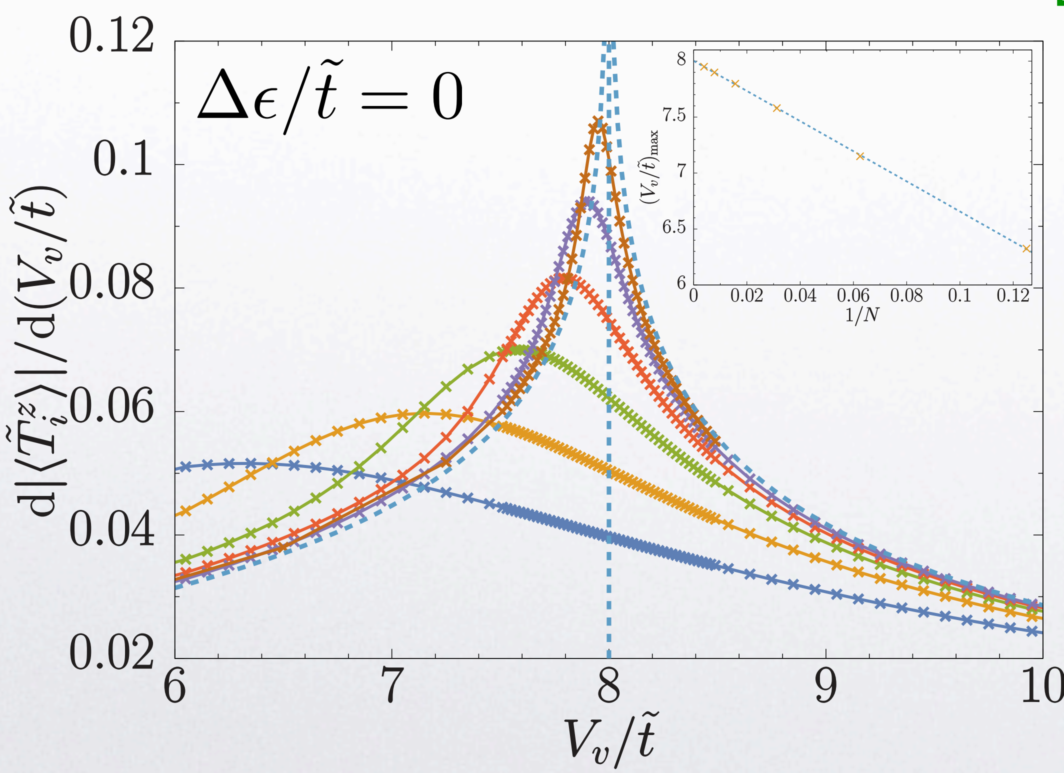

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… what happens then to the second Ising?

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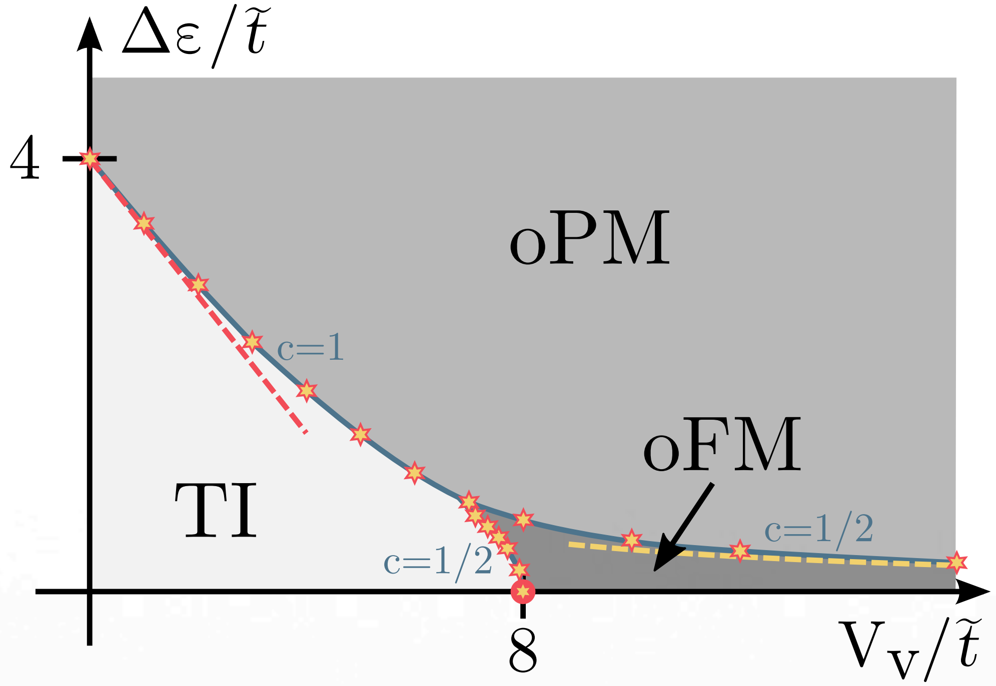
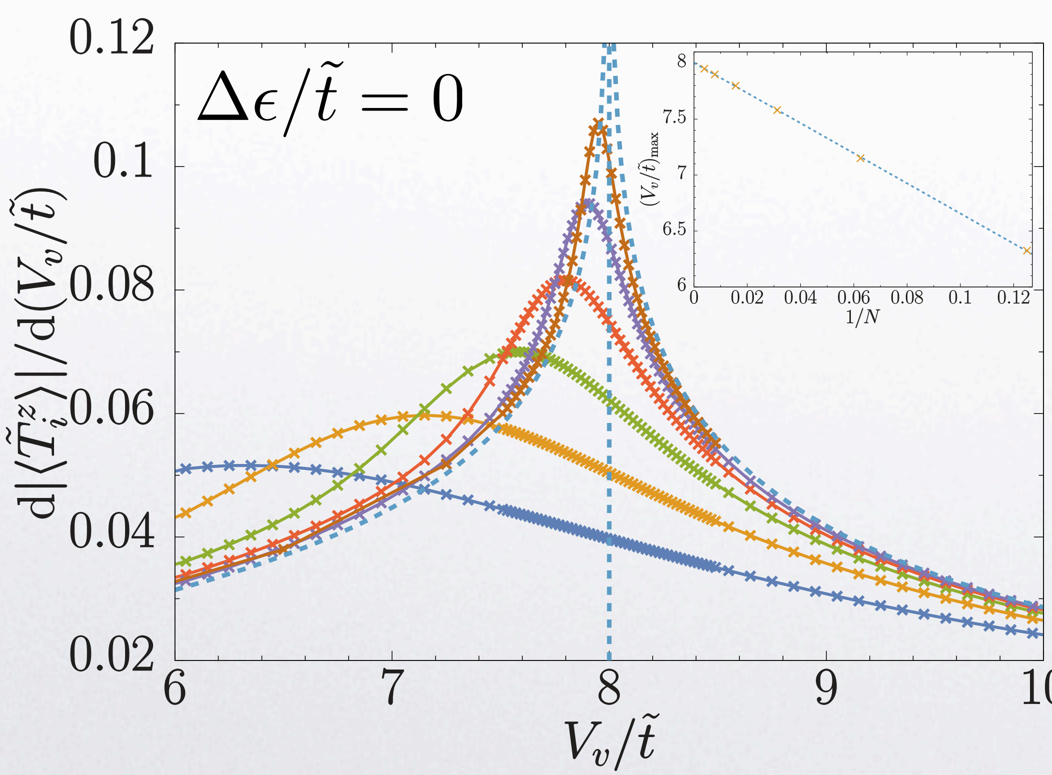




impurity model <==> topological character

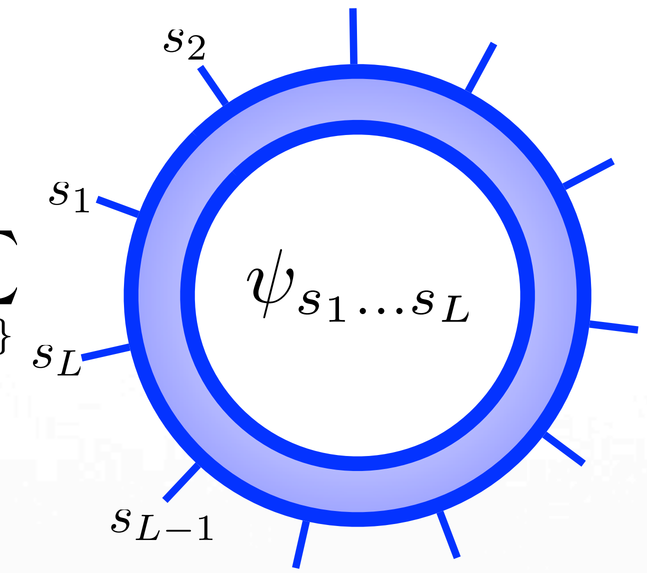
i) bulk mediated edge-edge interaction
shift energies but does *not* lift degeneracy

ii) no dephasing if Bogoliubov modes gapped
vs. dephasing if gapless ==> $|\tilde{J}| = 2\tilde{t}$
(i.e., not well defined edge modes!)

Matrix Product States

Generic description of a many-body Hilbert space is exponentially expensive

$H_j = \{|1\rangle \dots |d\rangle\}$ $H = \bigotimes_{j=1}^{L} H_j$ $O(d^L)$ numbers

$|\psi\rangle = \sum_{\{s\}}$  $|s_1 s_2 \dots s_L\rangle$

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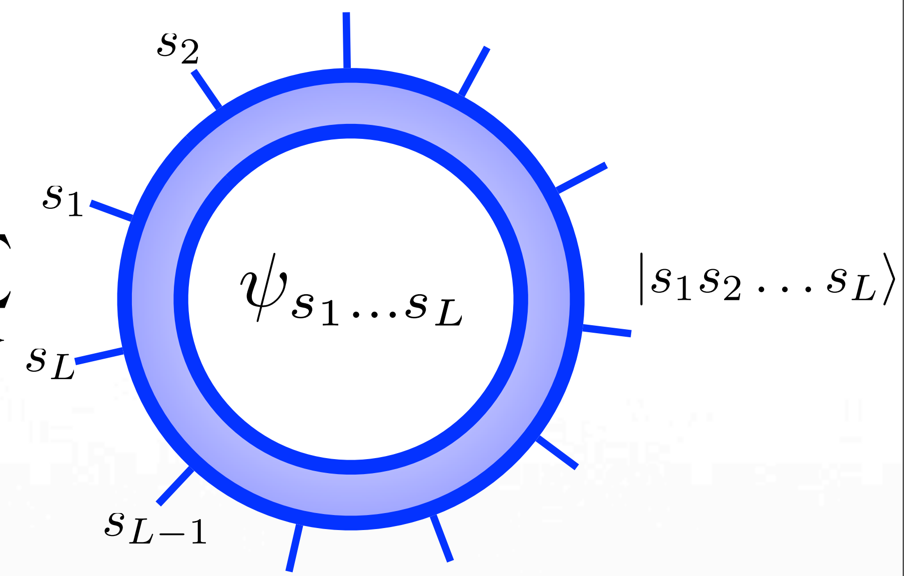
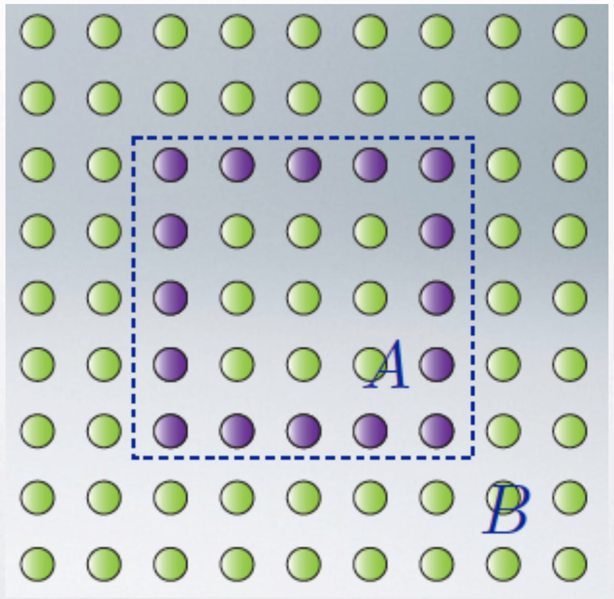


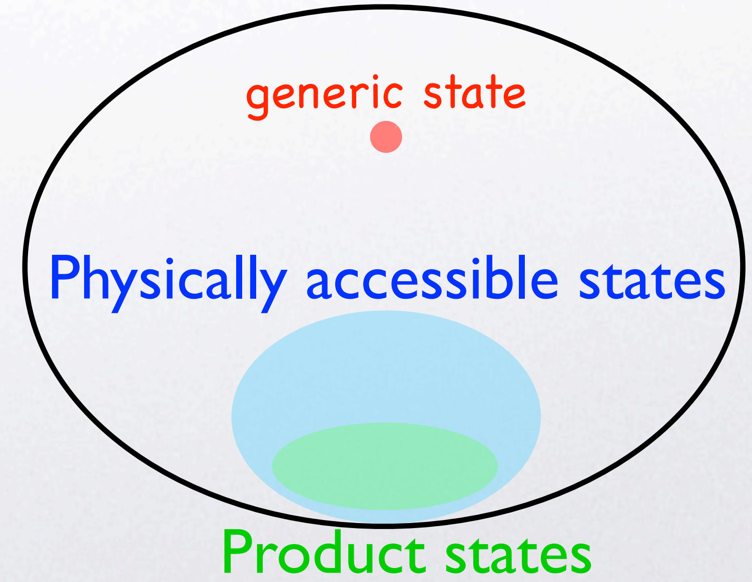
Area-law for entanglement entropy

$$\rho_A = \mathrm{Tr}_B \, |\psi\rangle\langle\psi|$$

$$S(\rho_A) \propto |\delta_A|$$

Eisert, Cramer, Plenio RMP 82, 277 ('10)





Matrix Product States

generic state

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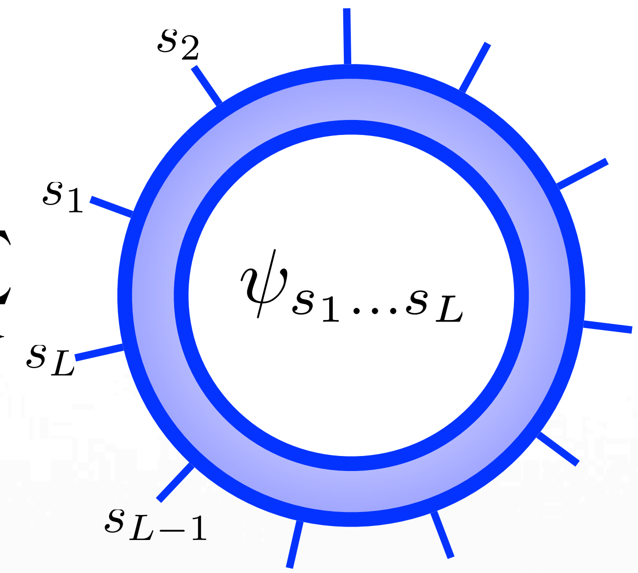

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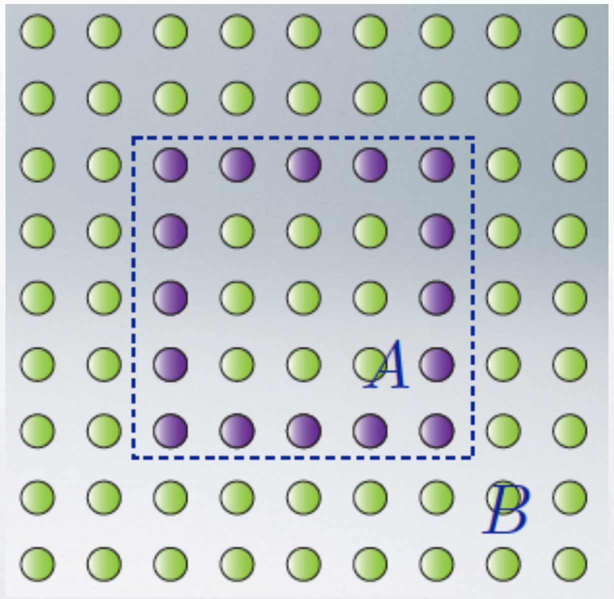

Physically accessible states

Product states

generic state

Matrix Product States

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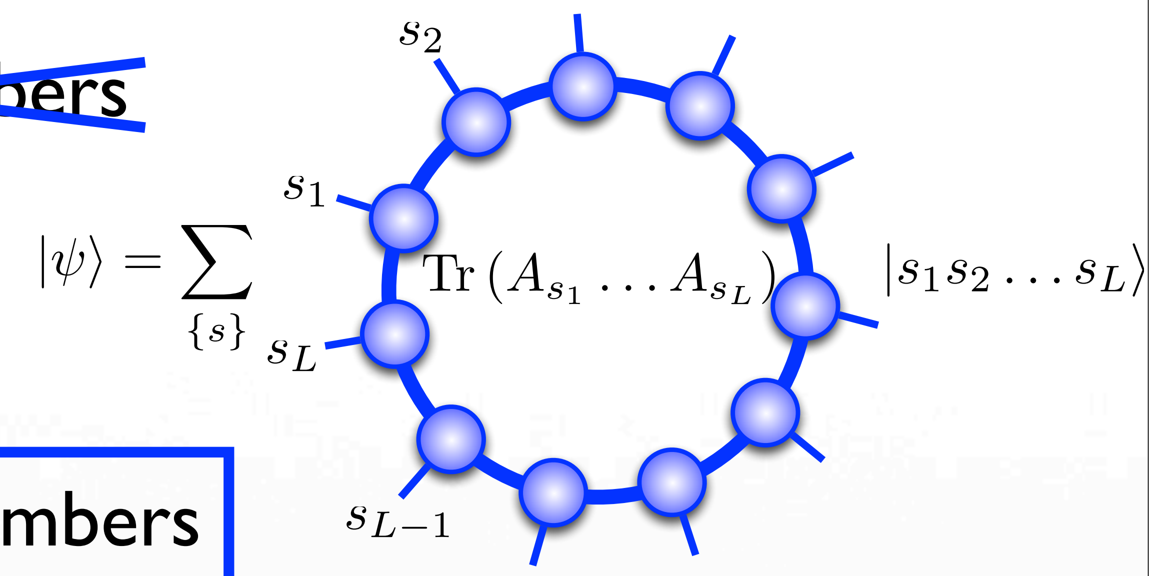
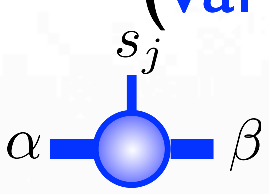


Economic description by "Tensor Networks":
(variational RG schemes, DMRG)



$A^{[j]}_{s_j \alpha\beta}$ $\quad s_j \in \{0, \ldots, n_j^{\max}\}$ $\quad \alpha, \beta = 1 \ldots m$

$O(L\, d\, m^2)$ numbers

Schollwock, Ann. Phys. 326, 96 (2011)

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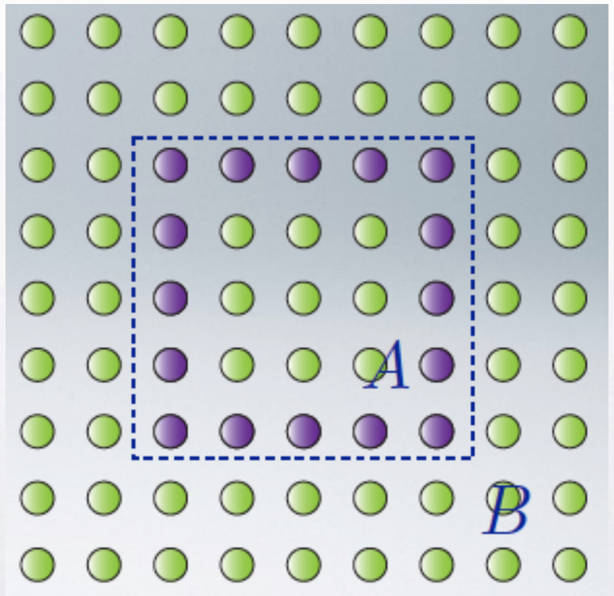

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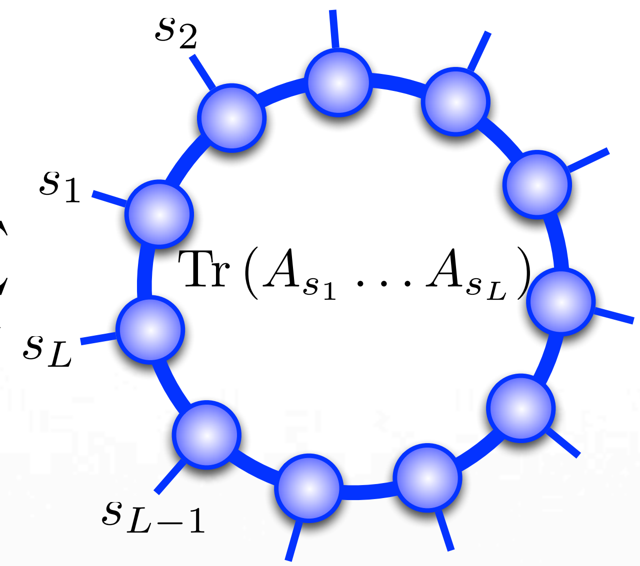

$$|\psi\rangle = \sum_{\{s\}} \mathrm{Tr}\left(A_{s_1} \dots A_{s_L}\right) |s_1 s_2 \dots s_L\rangle$$

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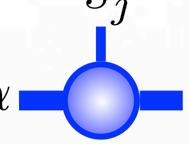

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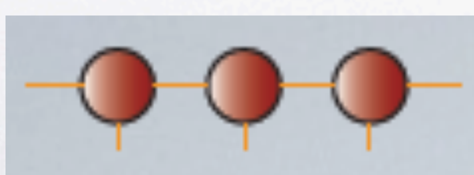
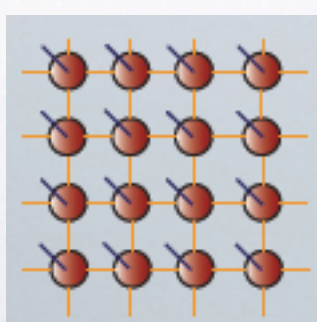
Schollwock, Ann. Phys. 326, 96 (2011)

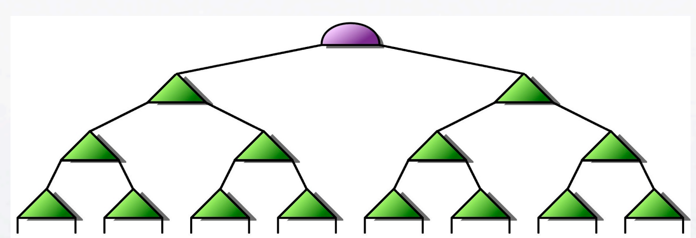
… plenty of different decompositions in tensor products:



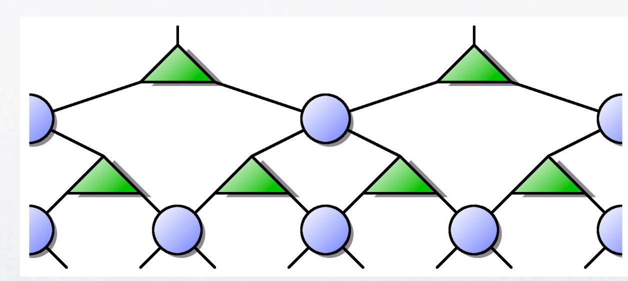
MPS

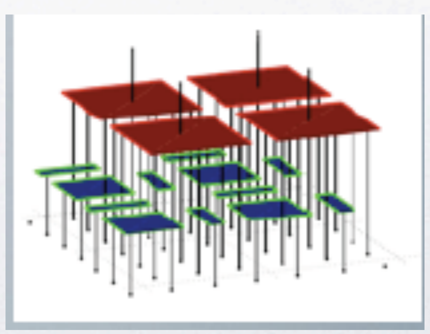


PEPS



TTN





MERA

Entanglement signatures (MPS)

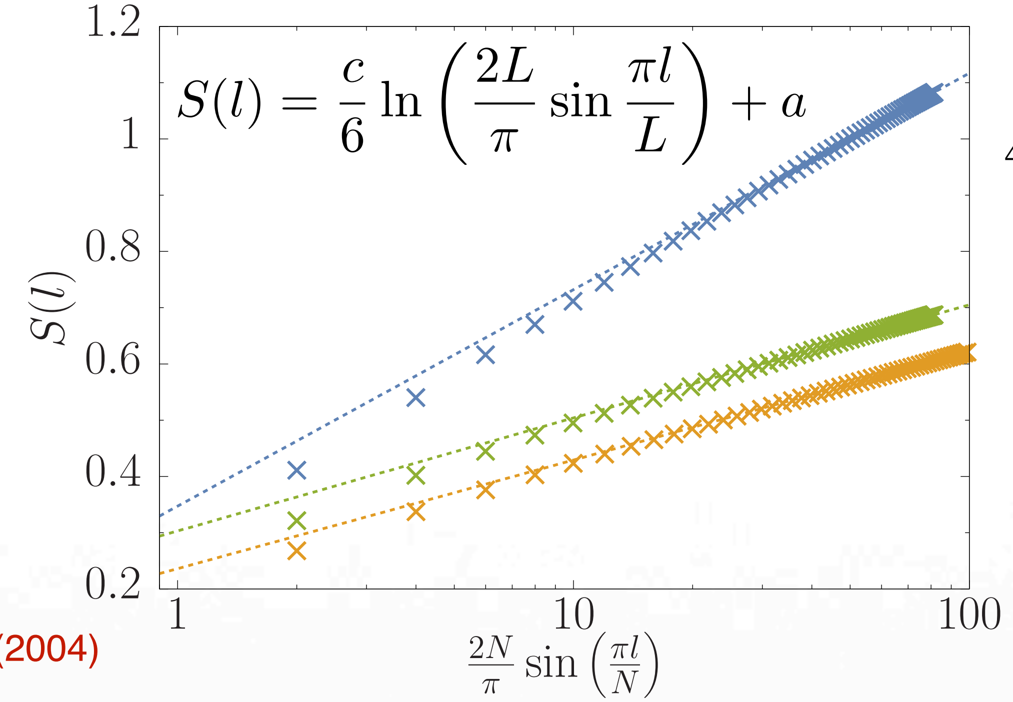
entanglement entropy

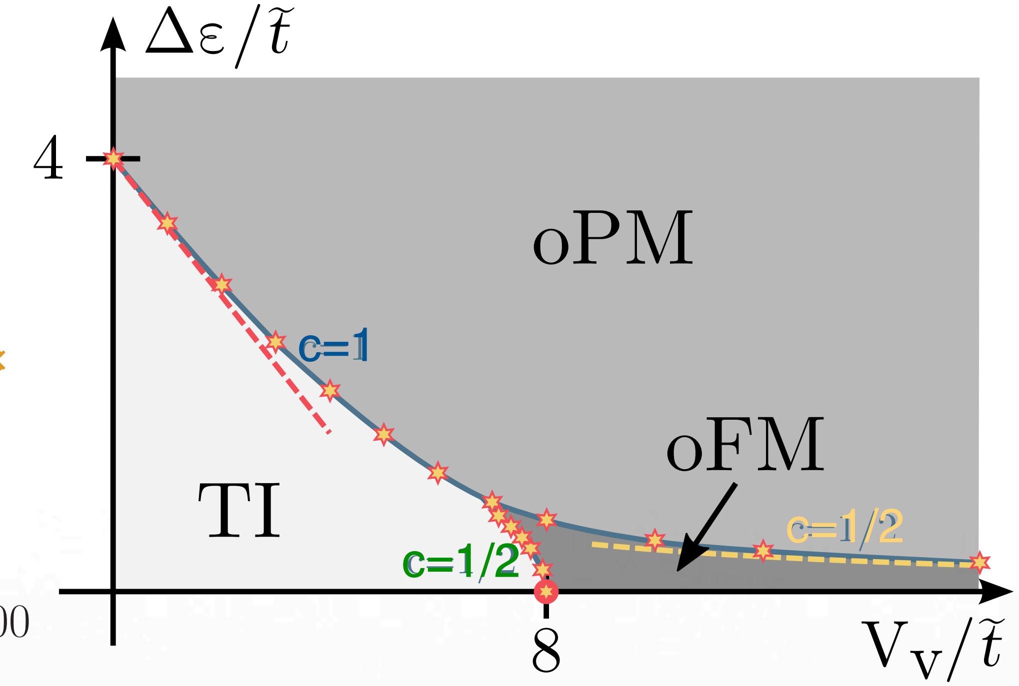
$S(l) = -\mathrm{Tr}\{\rho_l \log \rho_l\}$

⇓

CFT central charge

Vidal, Latorre, Rico & Kitaev, PRL 90, 227902 (2003);
Calabrese & Cardy, J. Stat. Mech. P06002 (2004)





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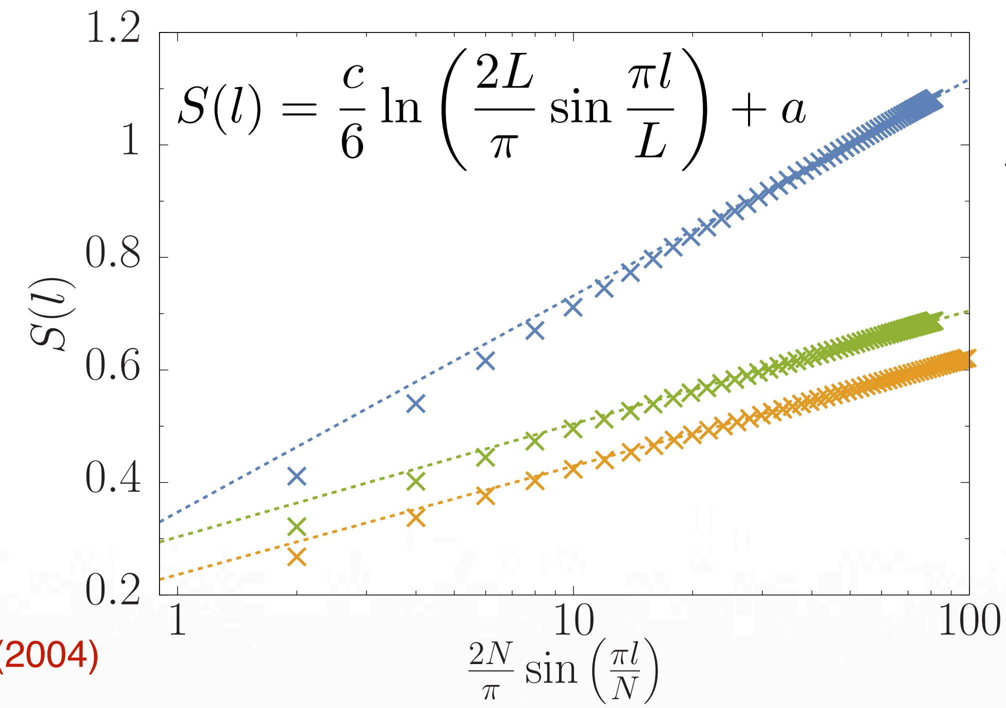
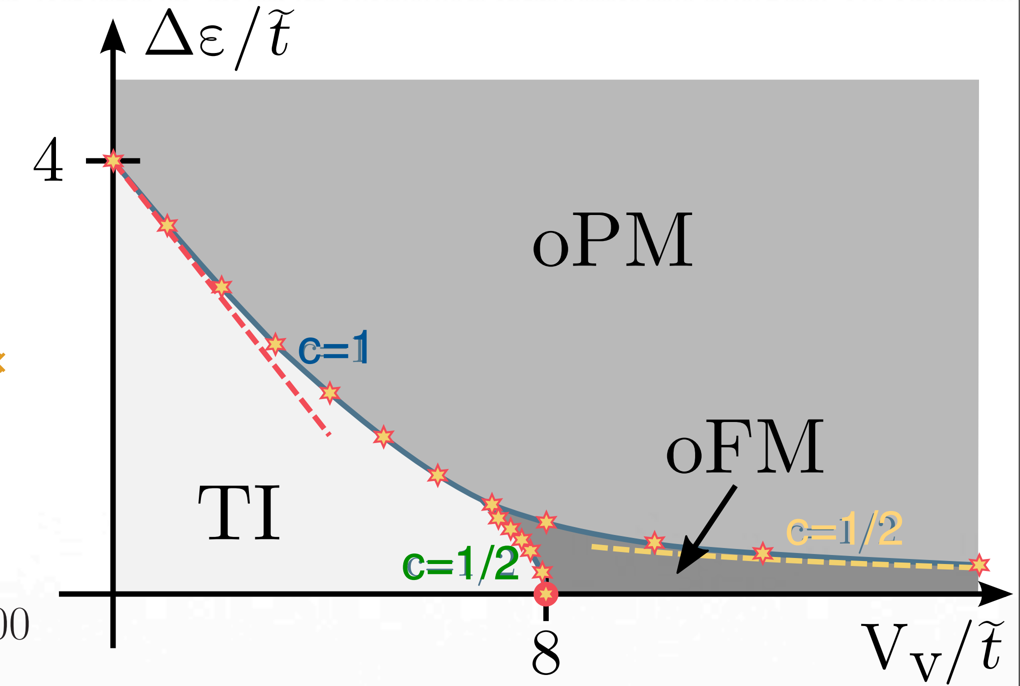
Vidal, Latorre, Rico & Kitaev, PRL 90, 227902 (2003);
Calabrese & Cardy, J. Stat. Mech. P06002 (2004)



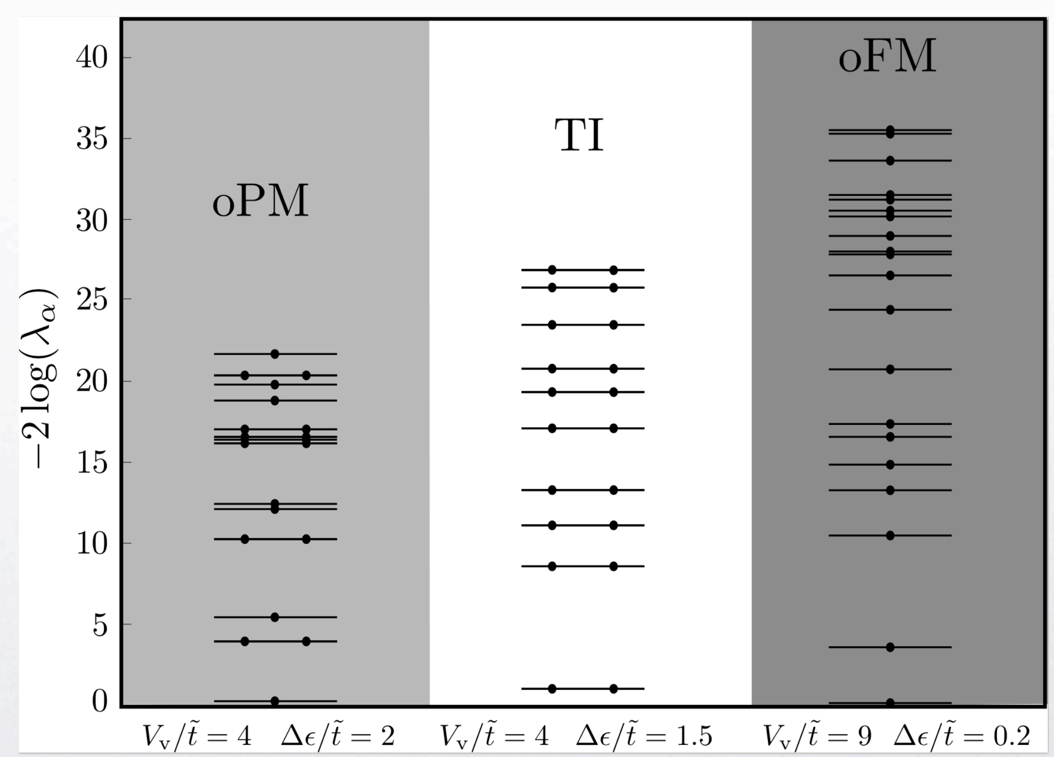


MPS gives out more:

$$|\psi\rangle = \sum_i \lambda_i \, |\psi_l^i\rangle \otimes |\psi_{L-l}^i\rangle$$

degeneracy pattern in entanglement spectrum !

H. Li and F. D. M. Haldane, PRL **101** 010504 (2015)
F. Pollmann, A. M. Turner, E. Berg, and M. Oshikawa PRB **81** 064439 (2010)



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arXiv:1612.02996

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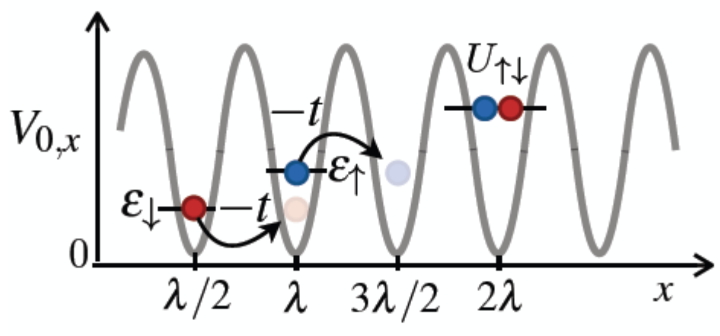

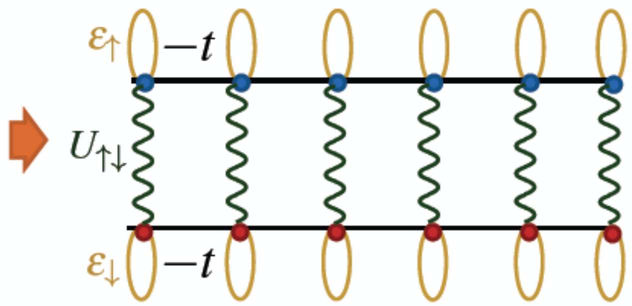

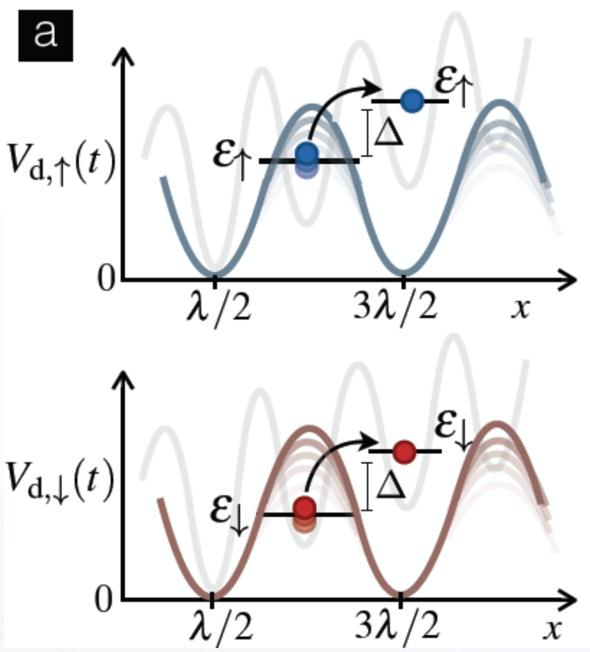

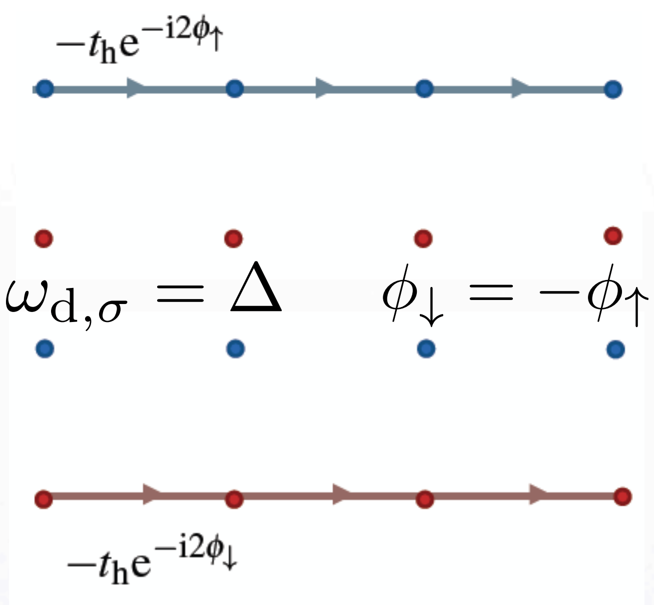

$$\omega_{\mathrm{d},\sigma} = \Delta \qquad \phi_\downarrow = -\phi_\uparrow = -\frac{\pi}{4}$$

state indep. gradient

+

intensity modulated OL

Holthaus, PRL **69**, 351 (1992)
Jaksch & Zoller, NJP **5**, 56 (2003)
Gerbier & Dalibard, NJP **12**, 033007 (2010)
Eckardt, RMP **89**, 011004 (2017)

$$V_{\mathrm{d},\sigma}(t) = V_{\mathrm{d},0}\sin(\omega_{\mathrm{d},\sigma} t - \phi_\sigma)$$

$$t_\mathrm{h} = t\,\mathfrak{J}_2\left(\frac{V_{\mathrm{d},0}}{\Delta}\right)$$

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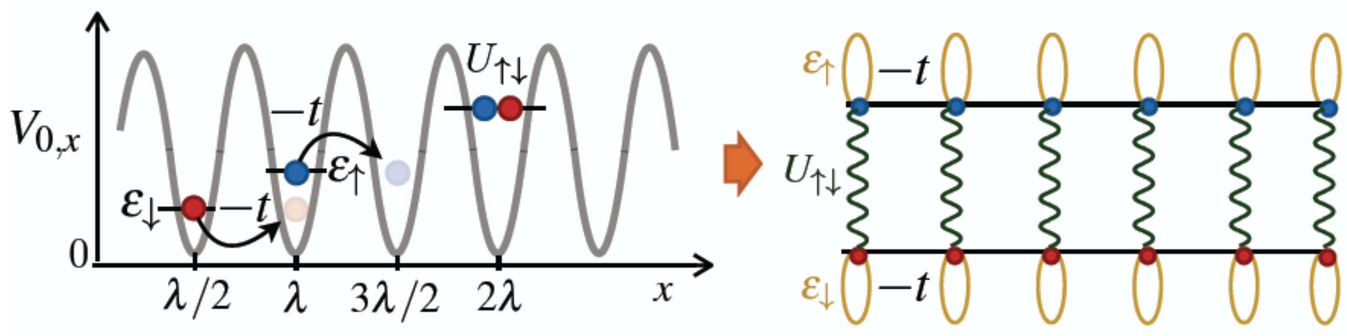

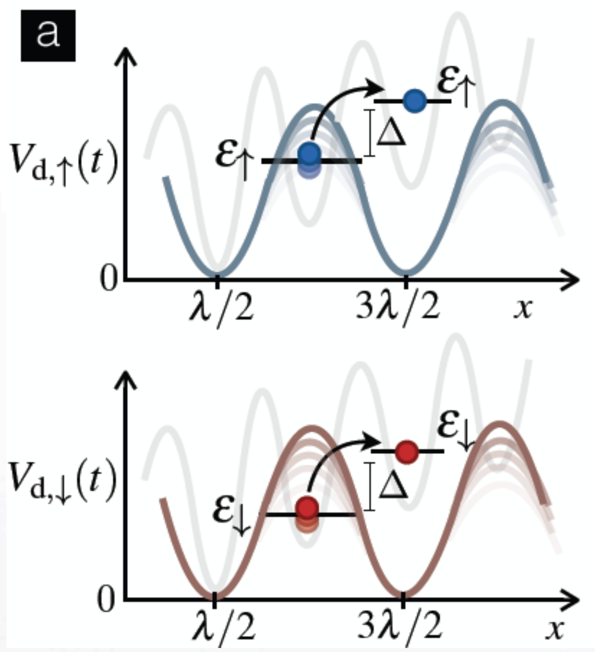

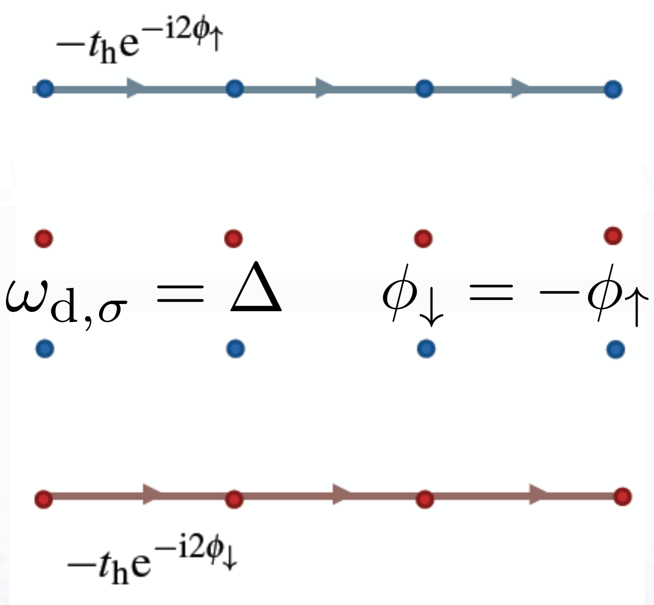

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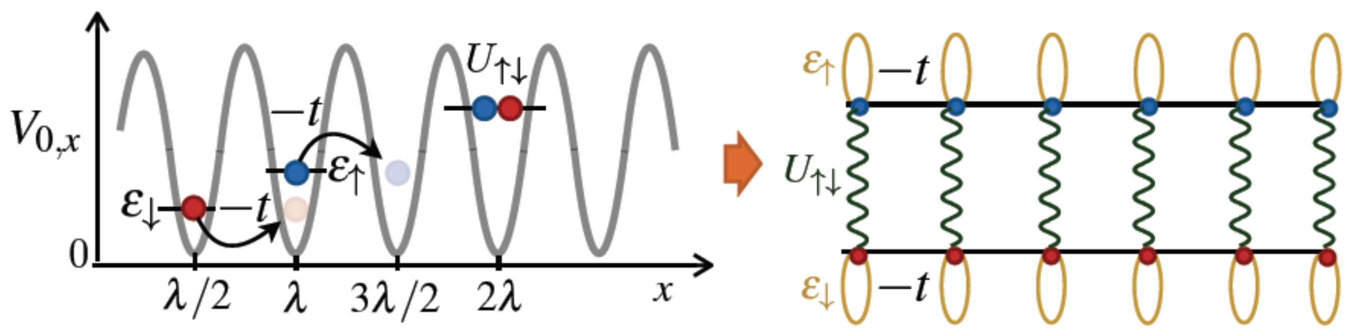
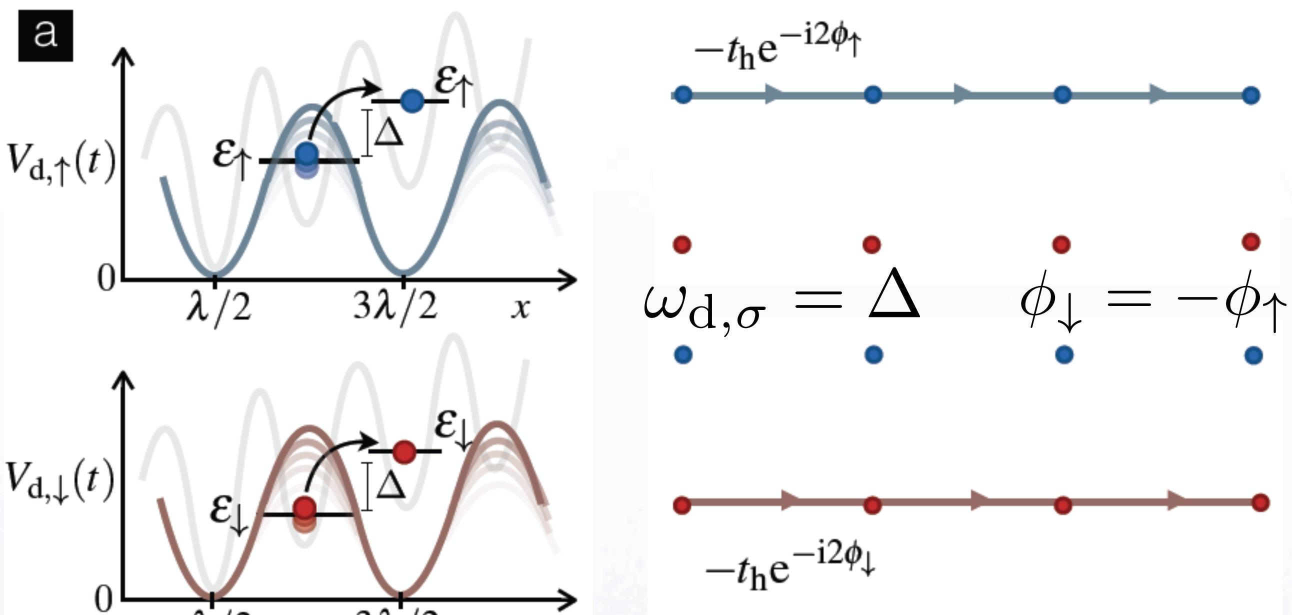
Interactions away from driving-induced resonances

Ma, et al. (Greiner), PRL 107, 095301 (2011)
Chen, et al. (Bloch), PRL 107, 210405 (2011)

Experimental ingredients





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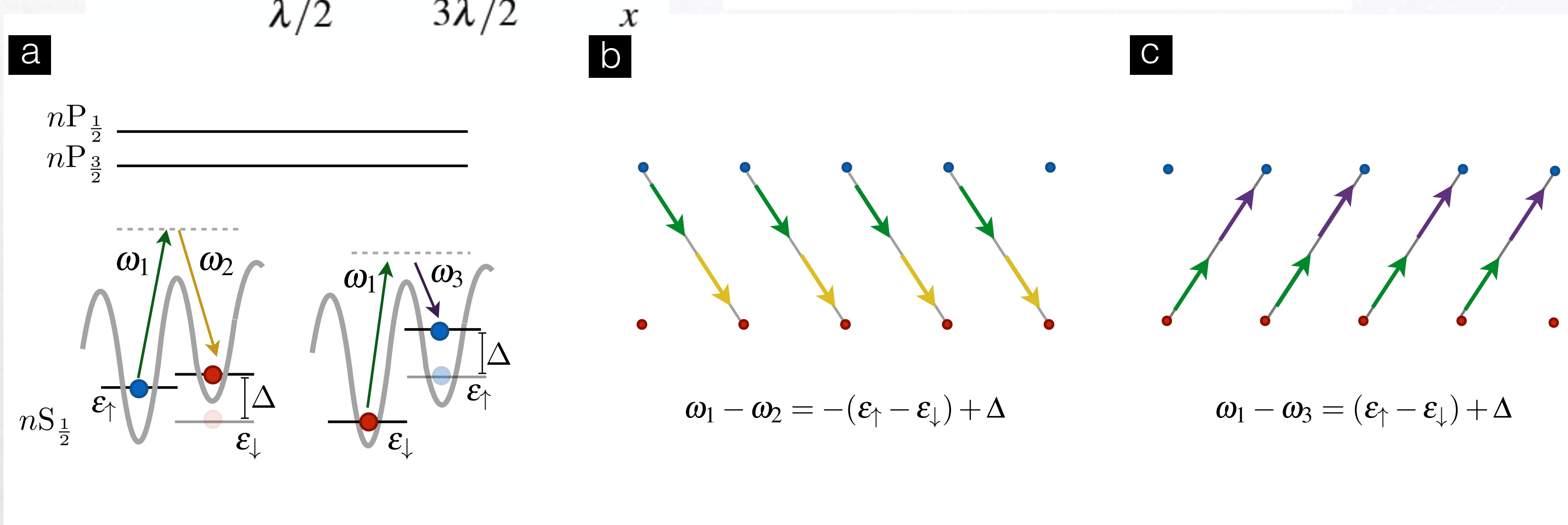

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Chen, et al. (Bloch), PRL 107, 210405 (2011)

\+

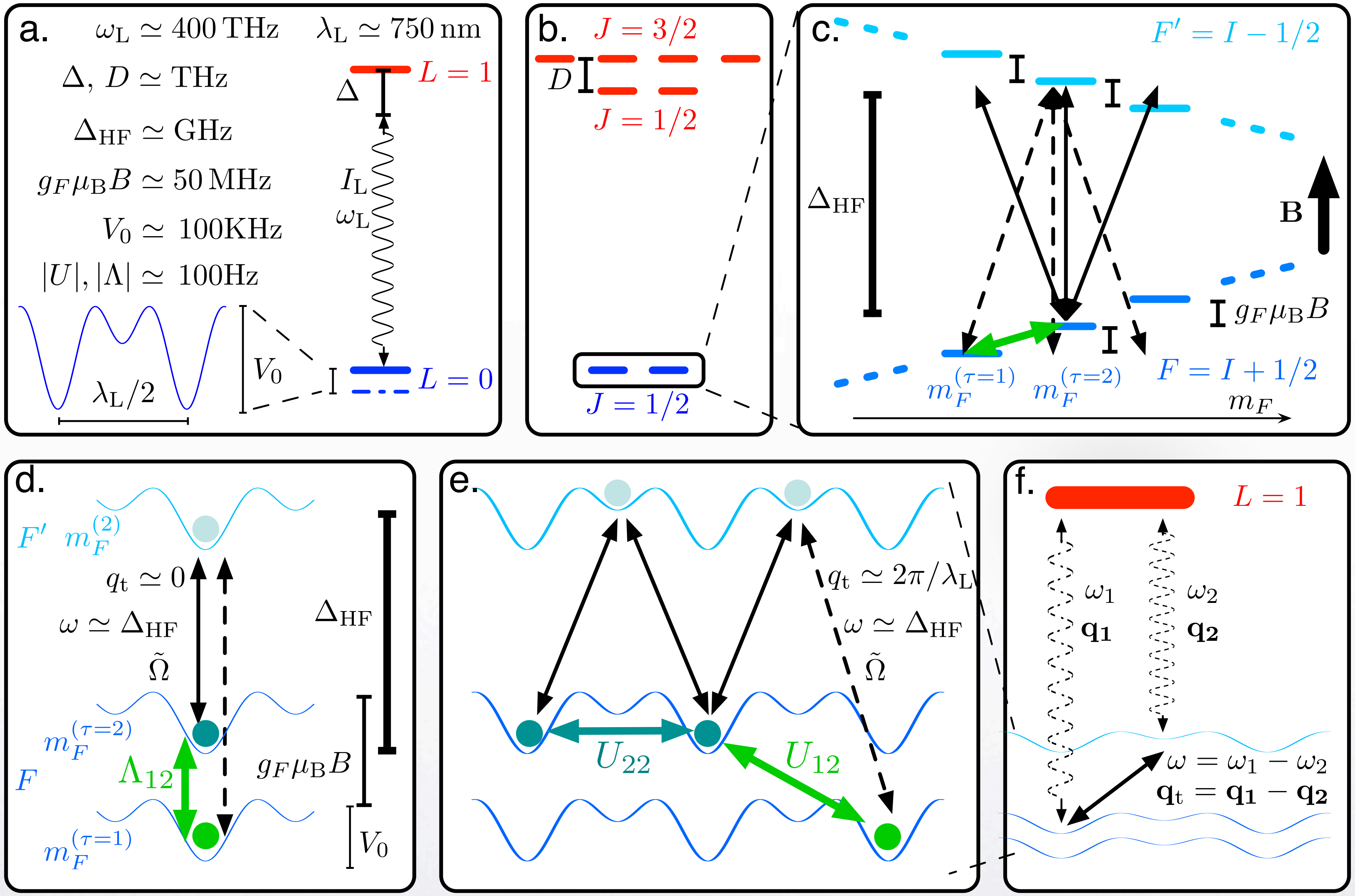
Raman assisted tunnelling

$$t_d = \frac{\Omega}{2}\mathfrak{J}_0\left(\frac{V_{d,0}}{\Delta}\right)$$

Aidelsburger, et al. (Bloch), PRL 111, 185301 (2013)
Miyake, et al. (Ketterle), PRL 111, 185302 (2013)

Experimental ingredients

Alternative: Superlattice “toolbox” for topological insulators !



L. Mazza, A. Bermudez, MR, et al., PRL 105, 190404 (‘10), NJP 14 015007 (‘12)
MR, PoS - SISSA 193, 036 (2014)

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Overall picture & perspectives

- Interaction driven split of a Dirac line (c=1) into two Majorana ones (c=1/2)
  ==> consequences for topological excitations!? nature of the tricritical point?
- Mappings onto impurity problems & broadening of edge modes (not shown)
  ==> new elements for understanding bulk-edge correspondence!
- **High feasibility & detectability in current experimental setups :-)**
- How stable is the picture for imperfect flux?
  (*intuition*: similar to imbalance)
- Away from half-filling: resonances? fractional effects? dualities?
  (among others: emergent local $\mathbb{Z}_2$ gauge symmetry, etc.)


J. Jünemann


A. Piga


S.J. Ran


M. Lewenstein


A. Bermudez

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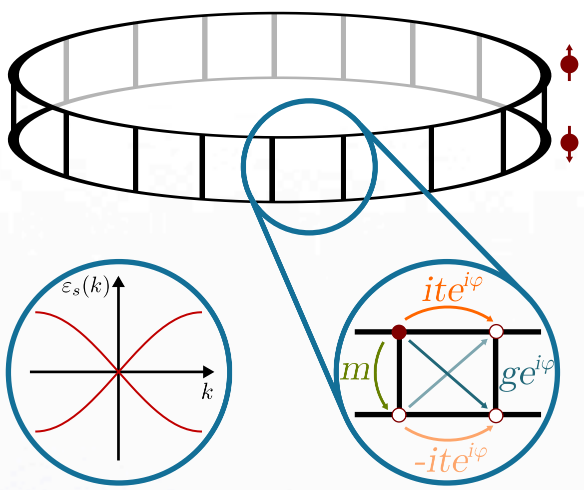

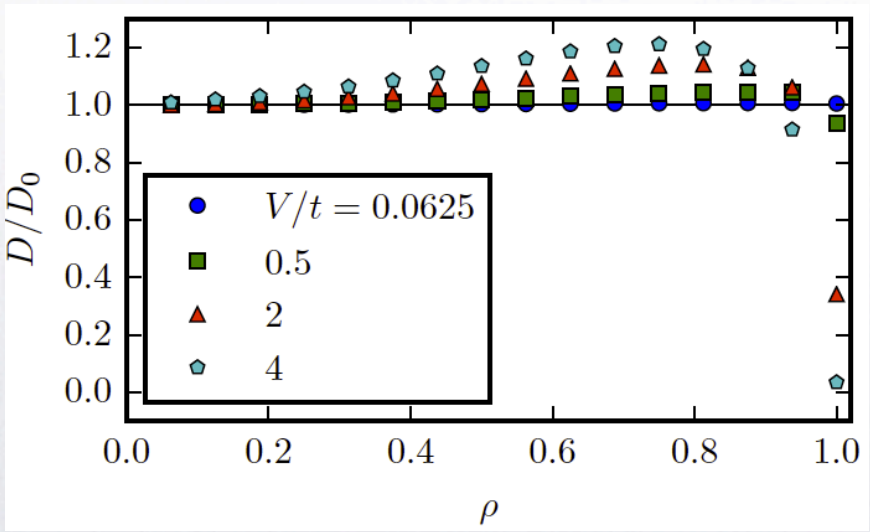

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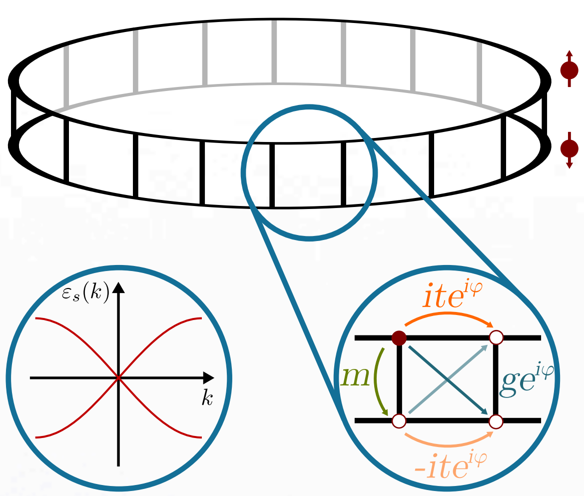

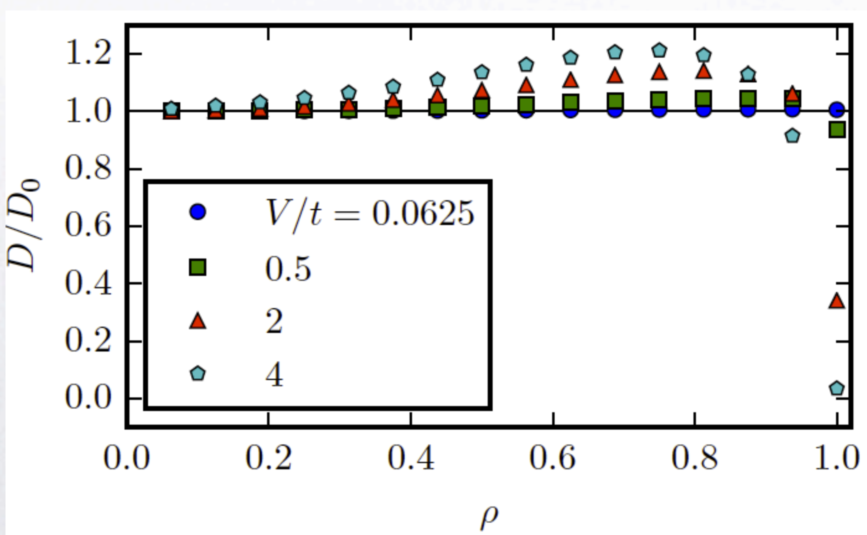

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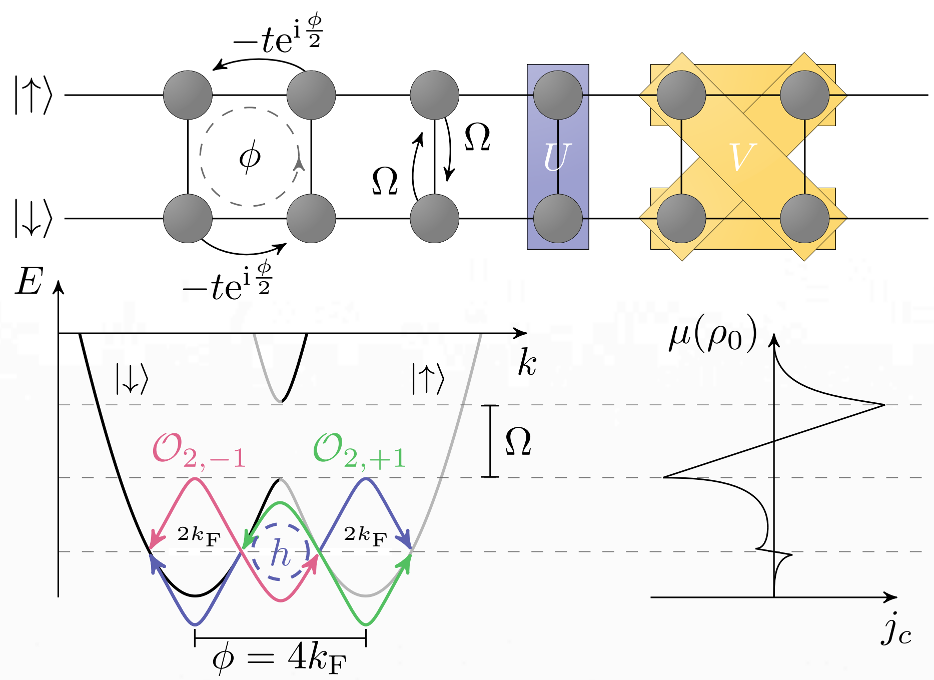
A. Haller, MR, M. Burrello, arXiv:1707.05715



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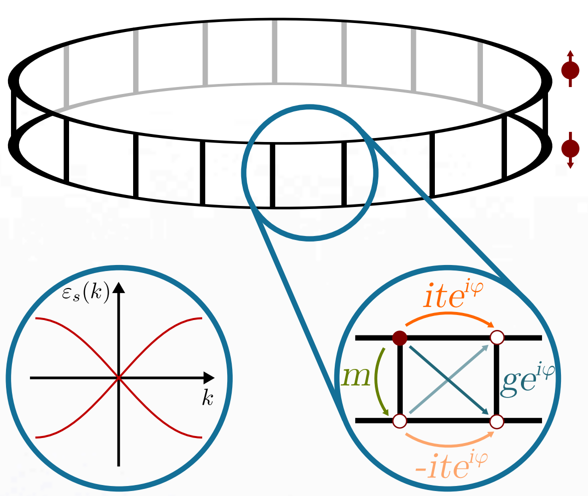

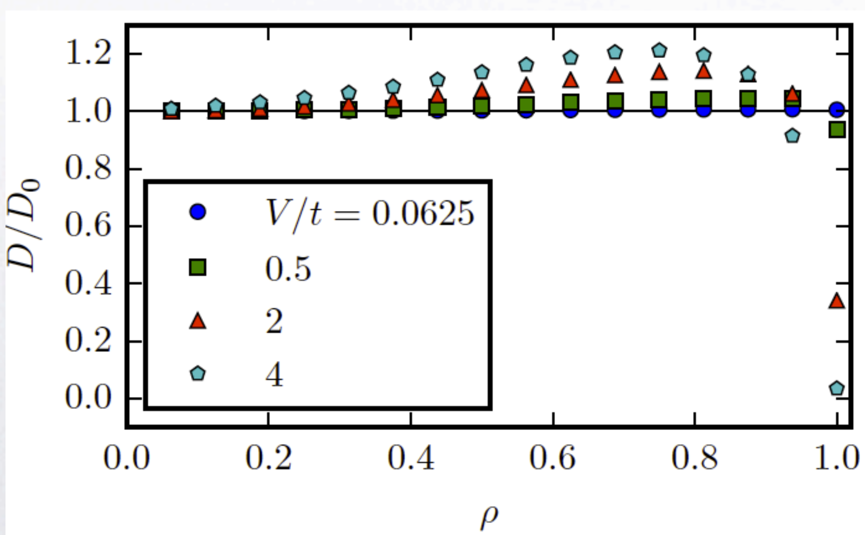

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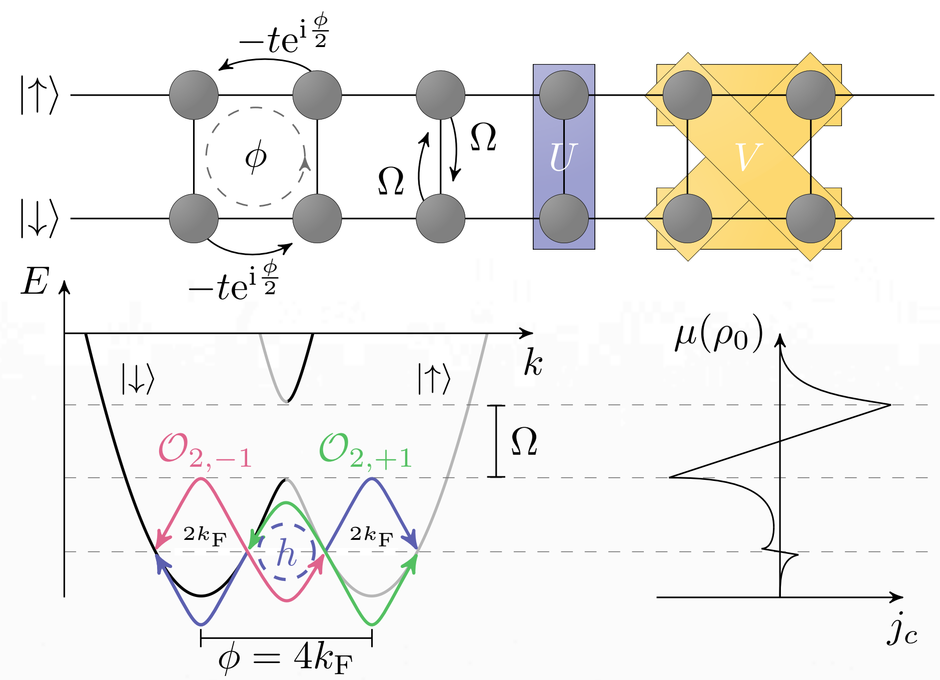

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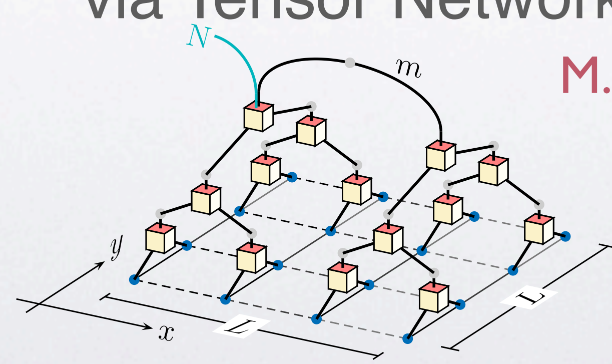

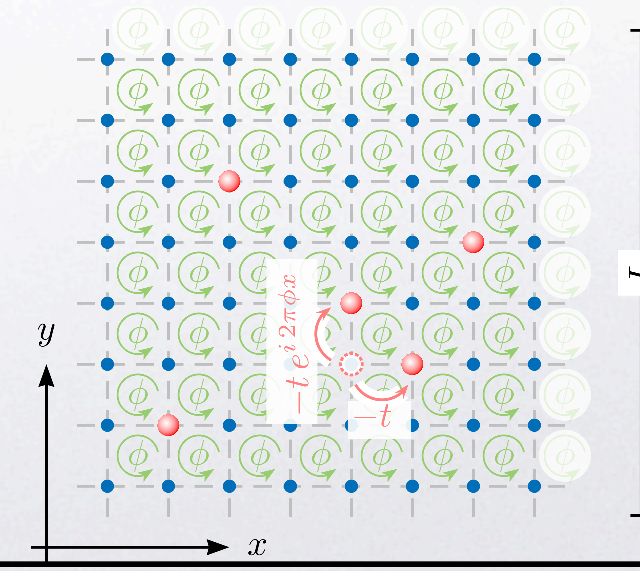

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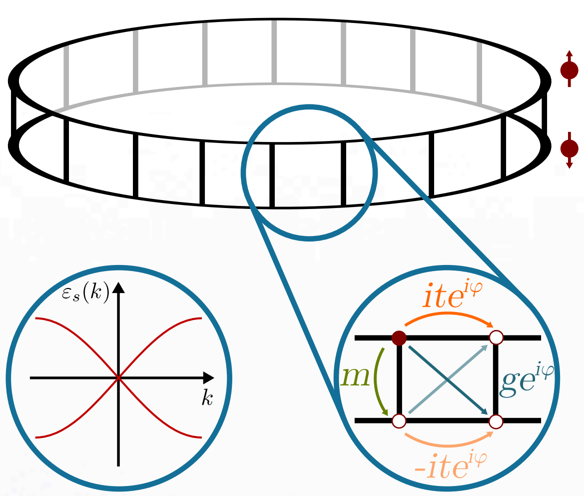

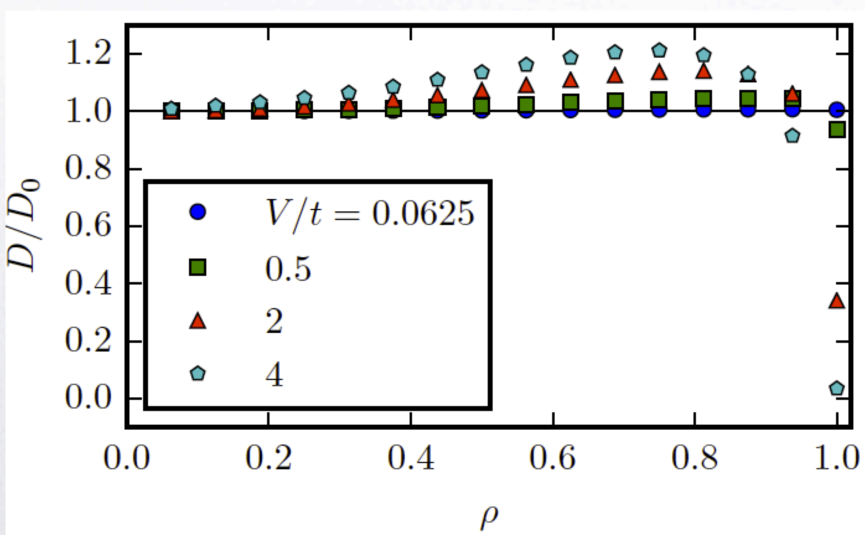

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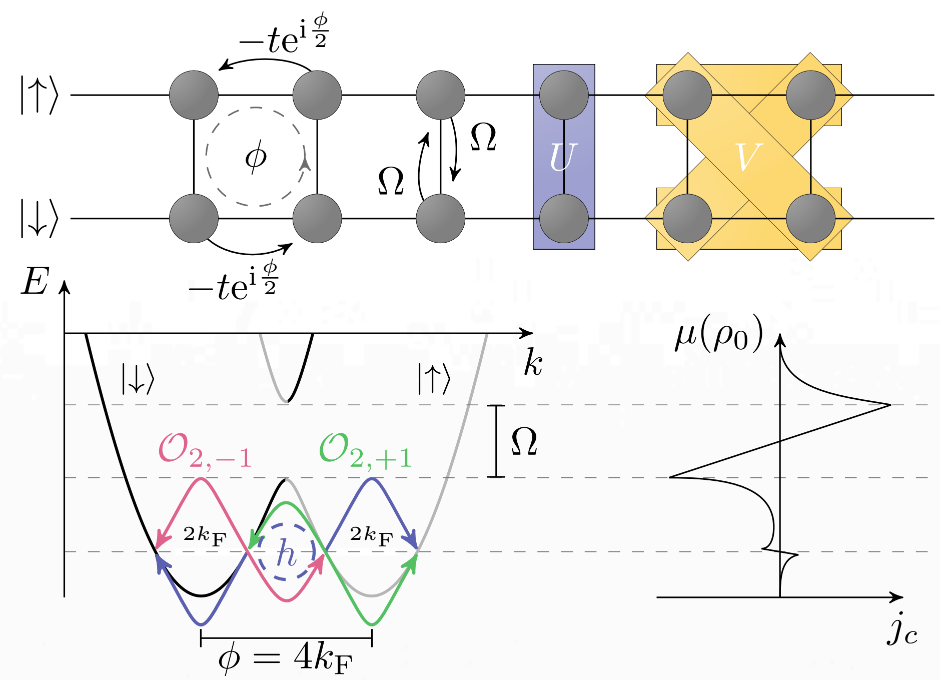

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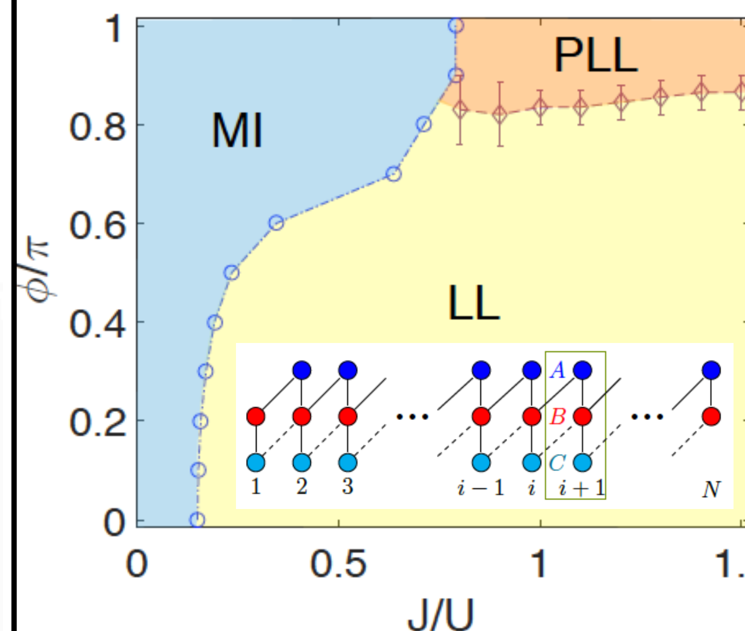

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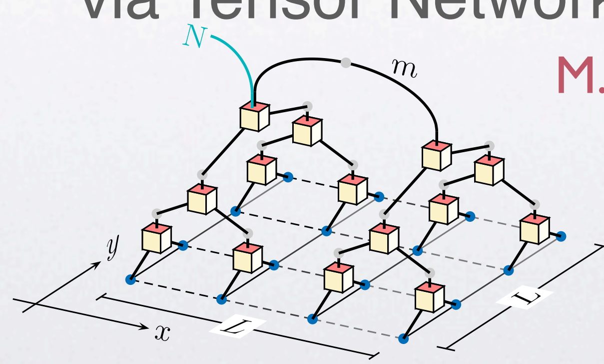

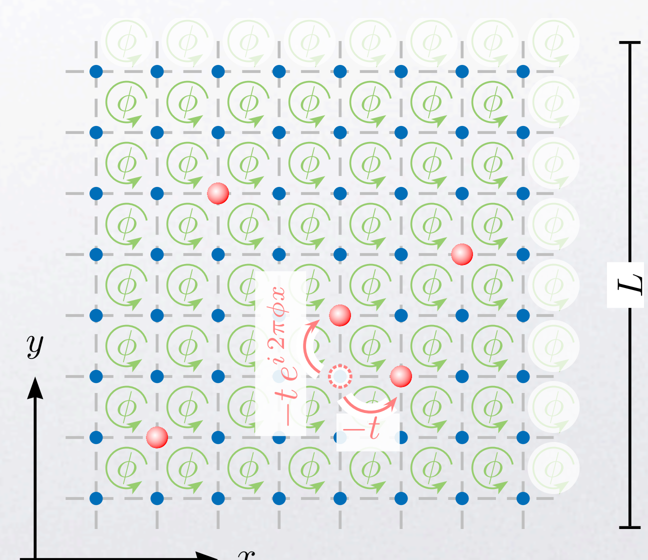

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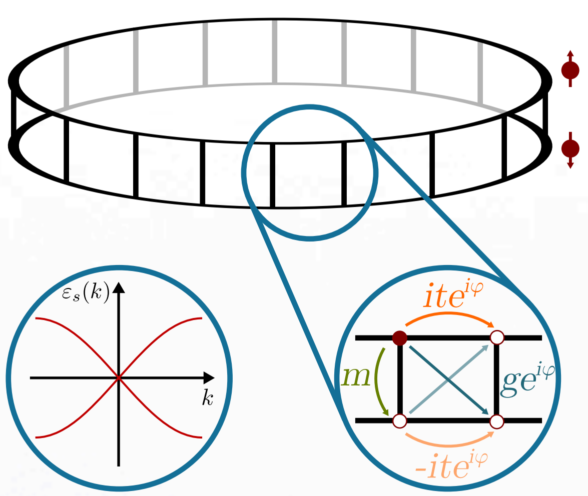

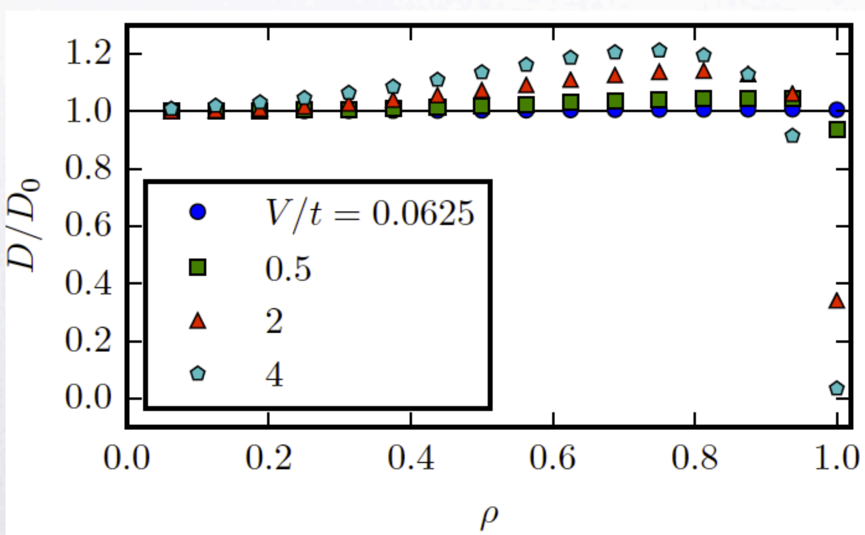

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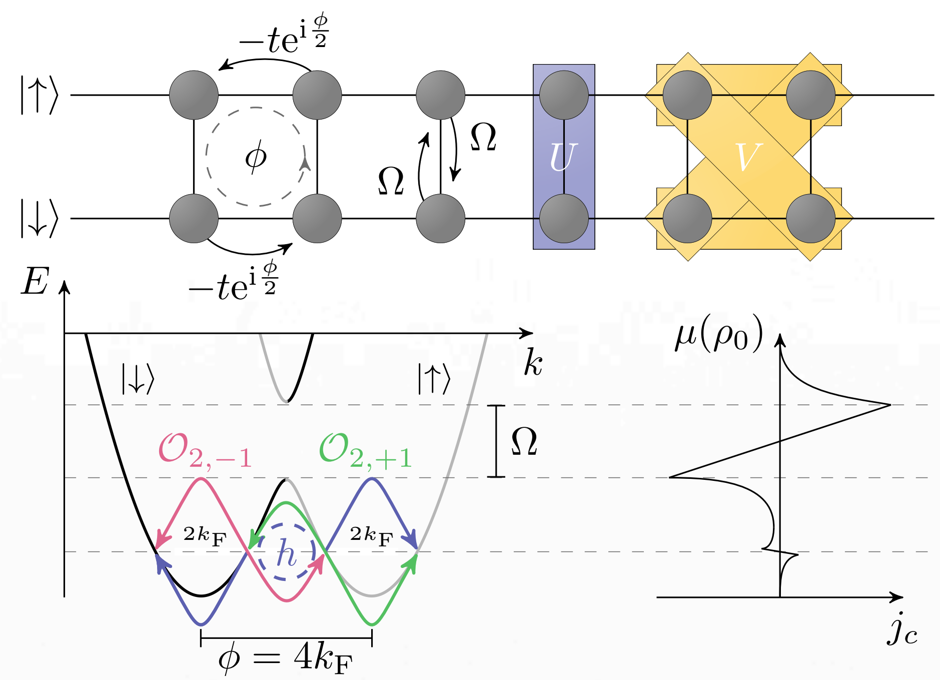

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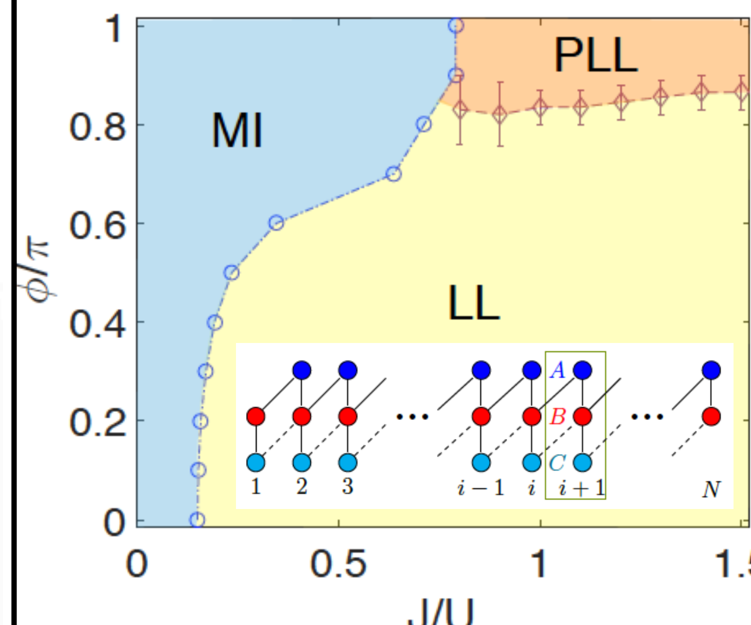

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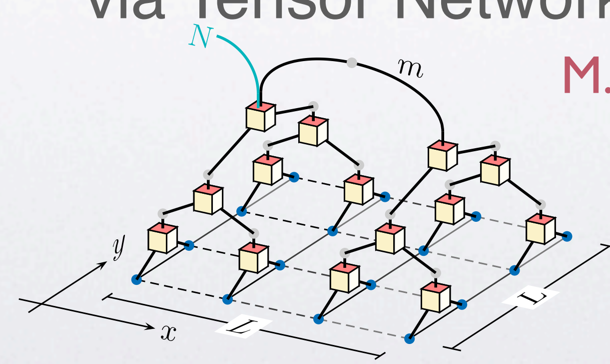

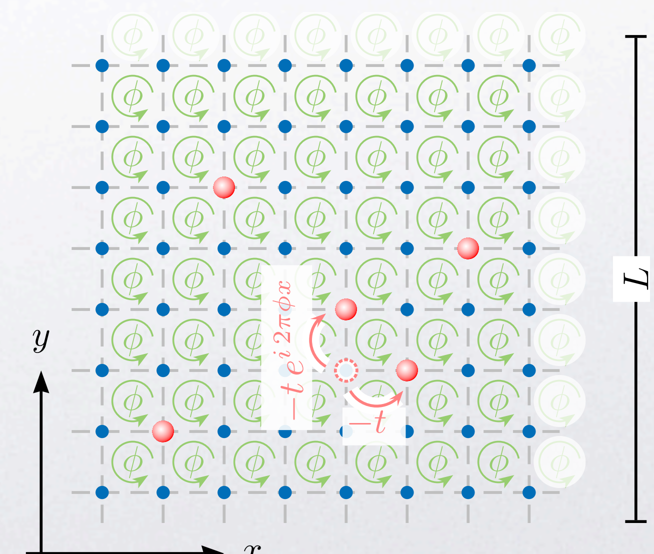

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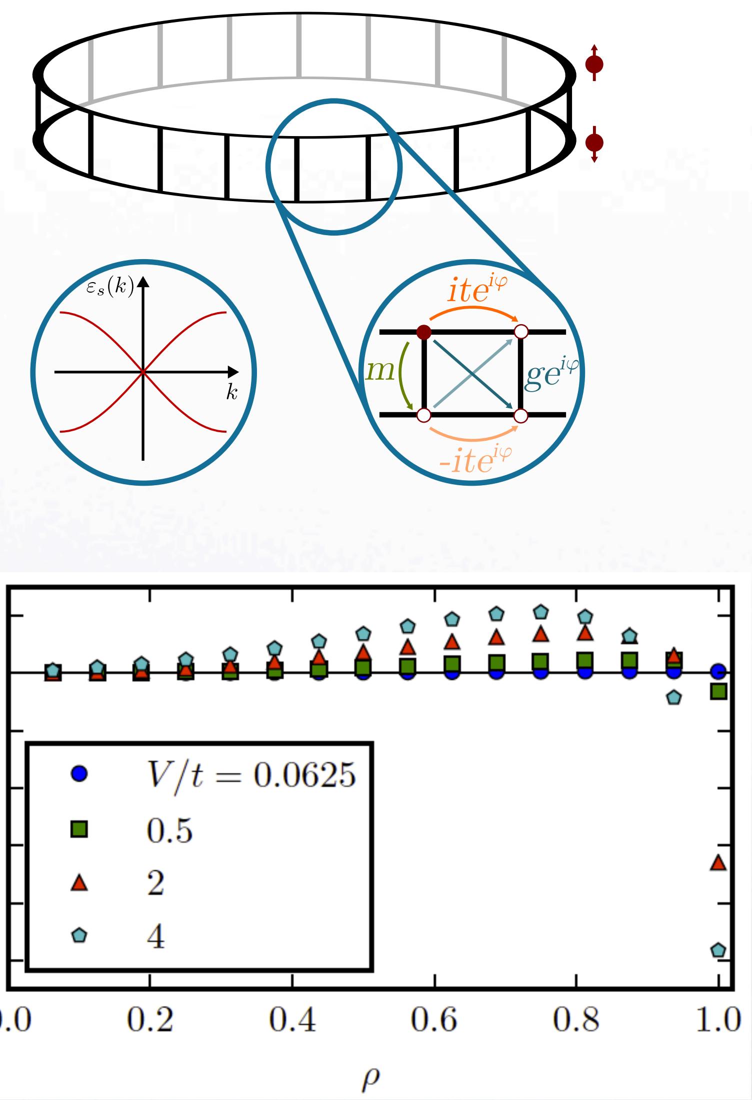

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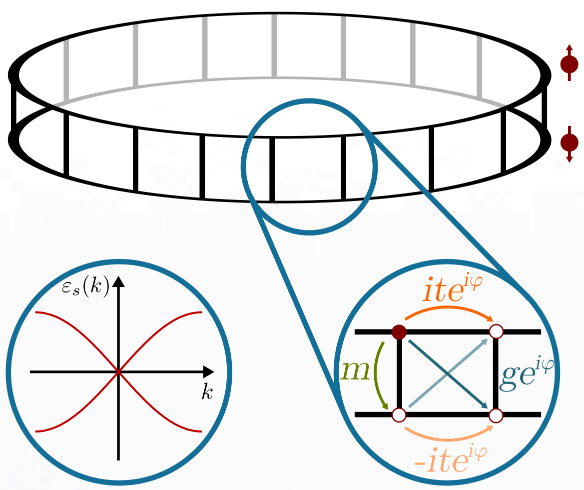

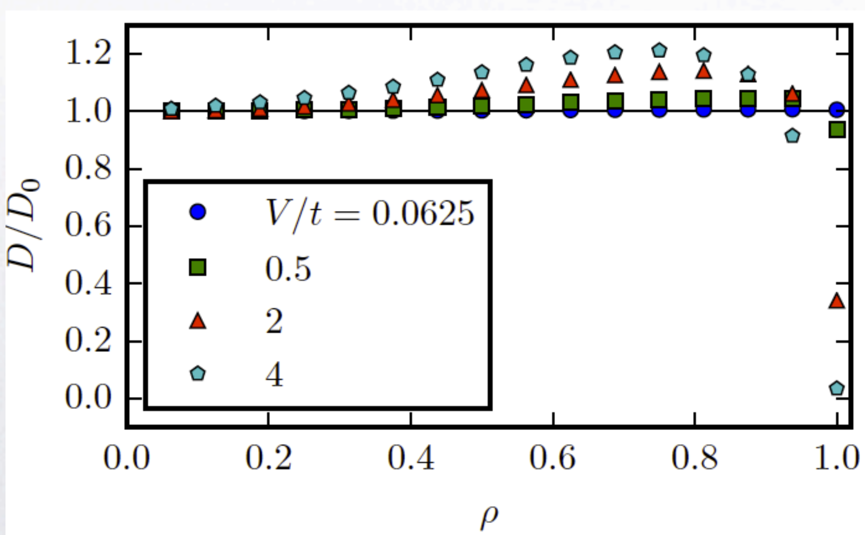

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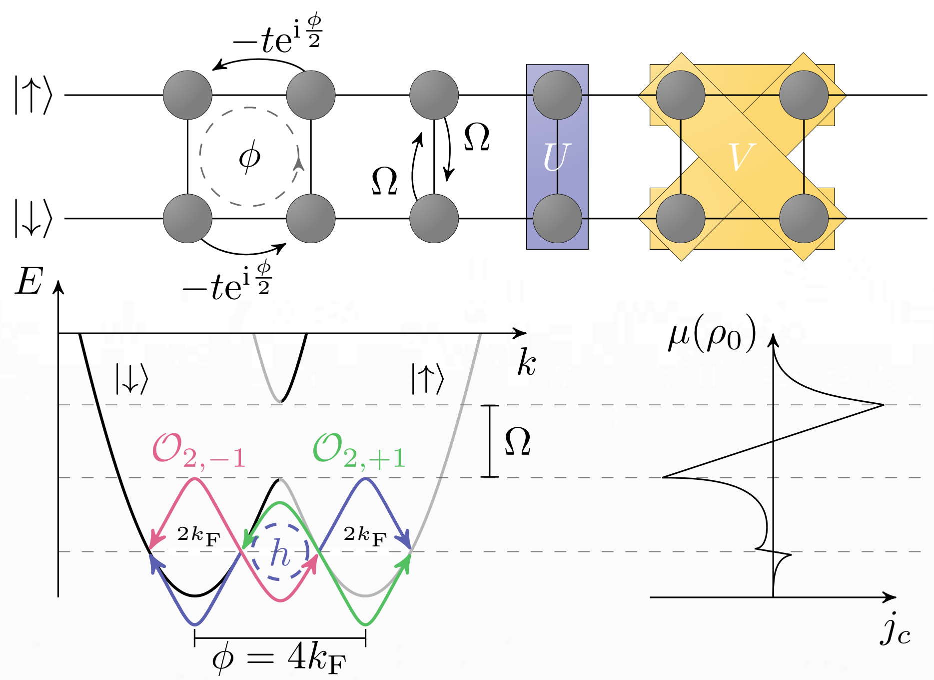

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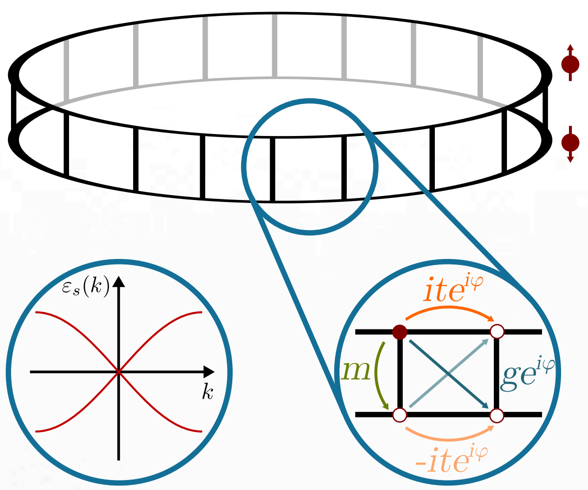

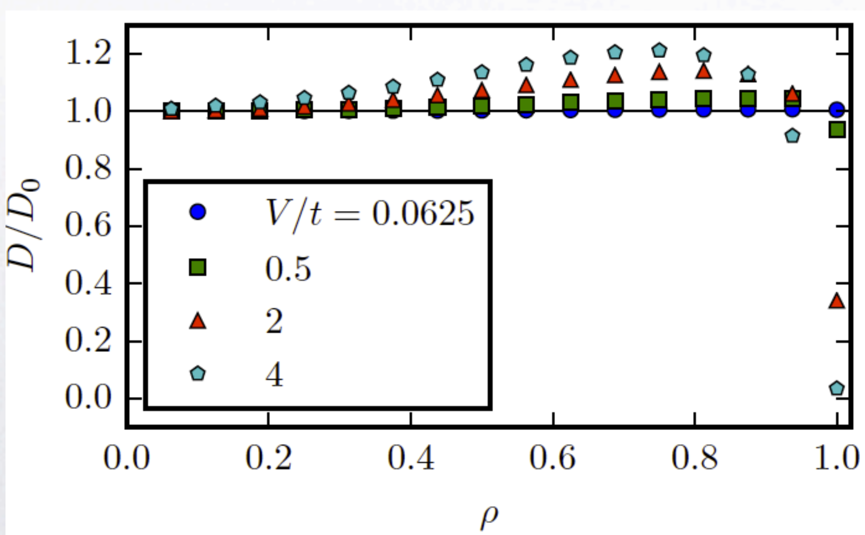

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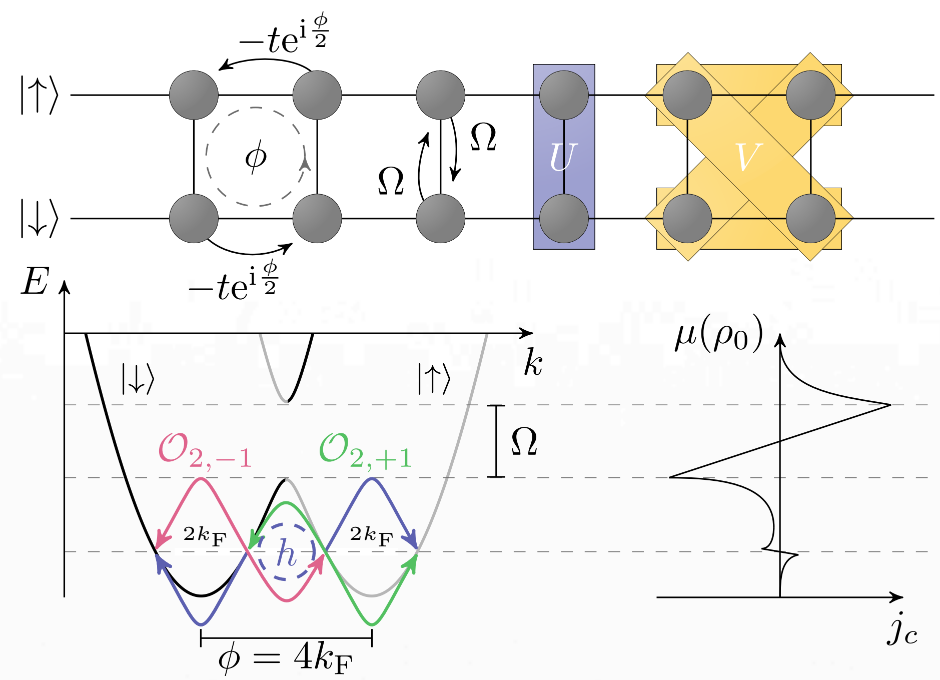

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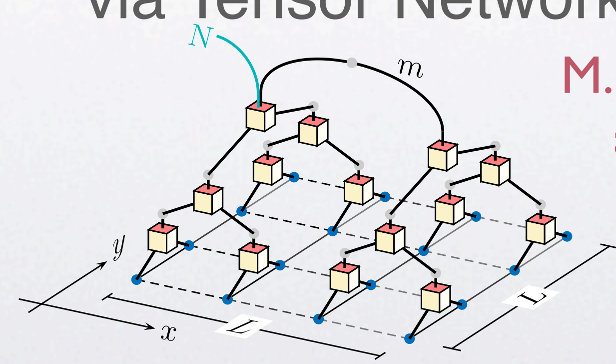

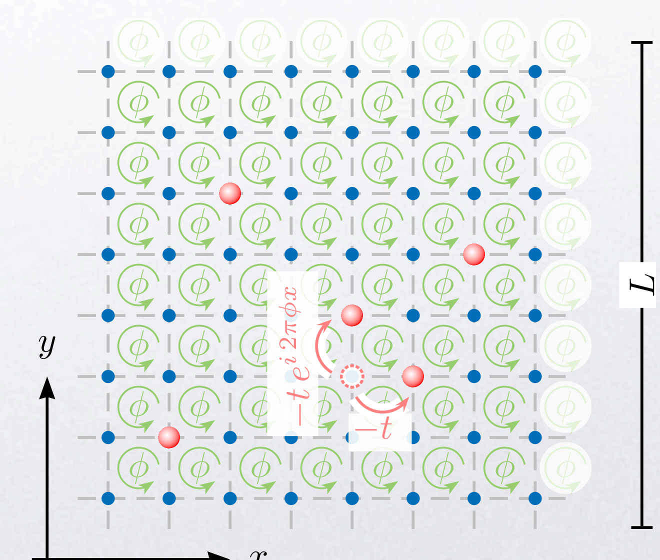

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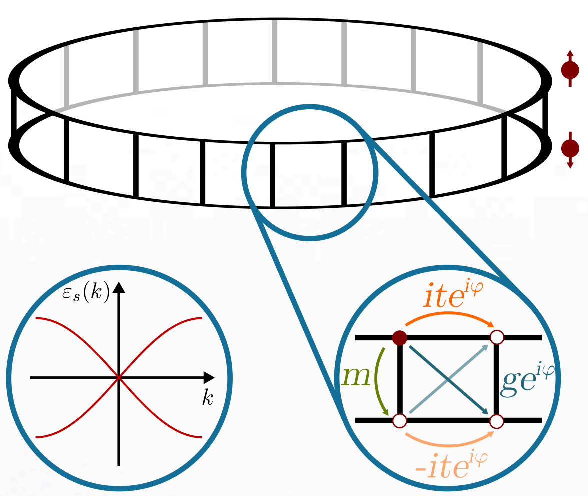

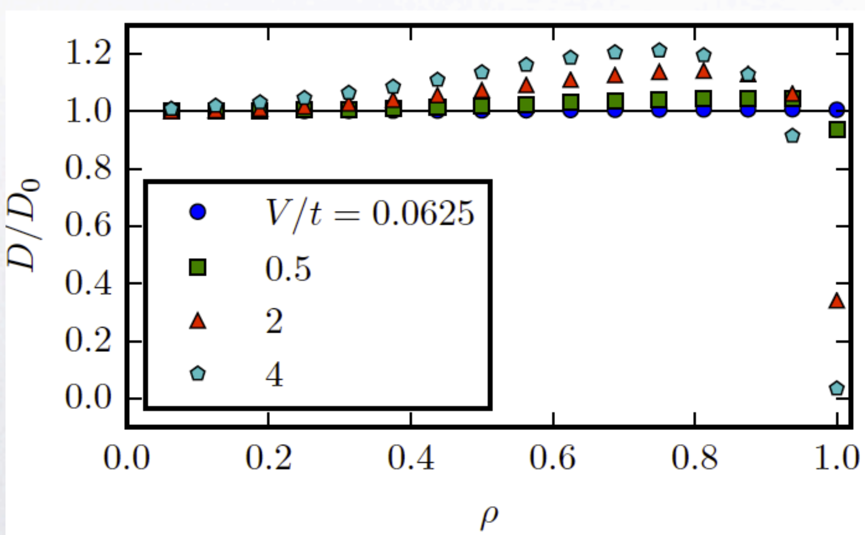

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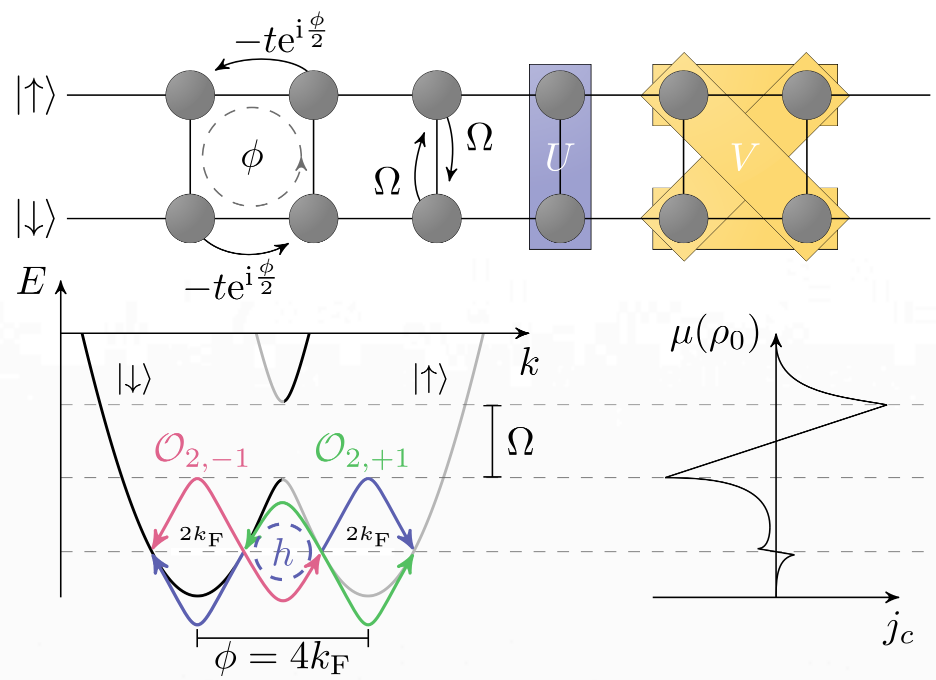

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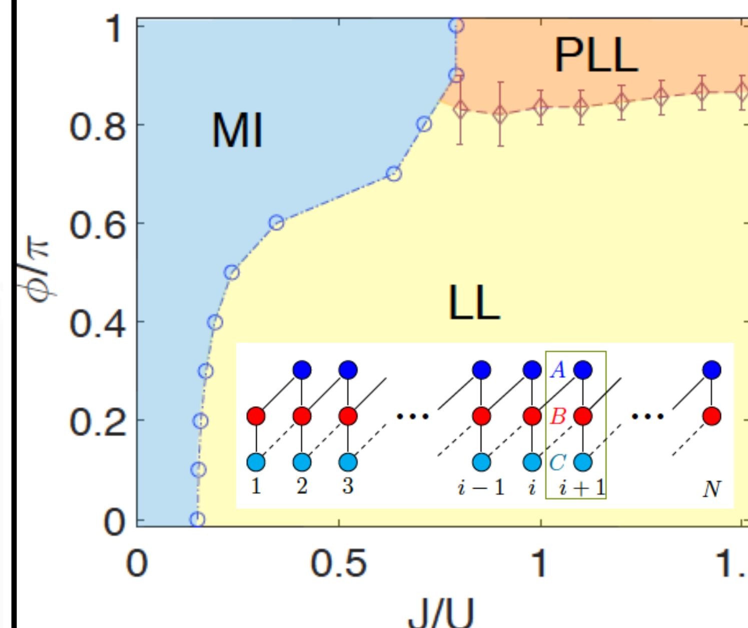

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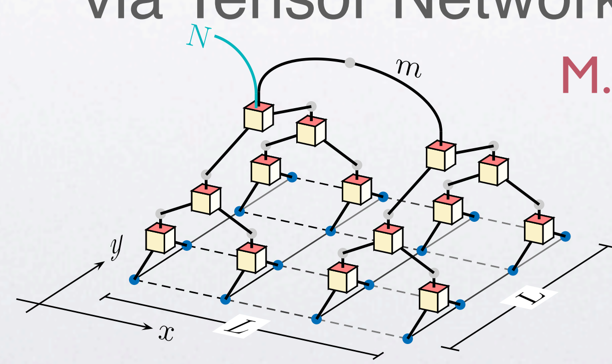

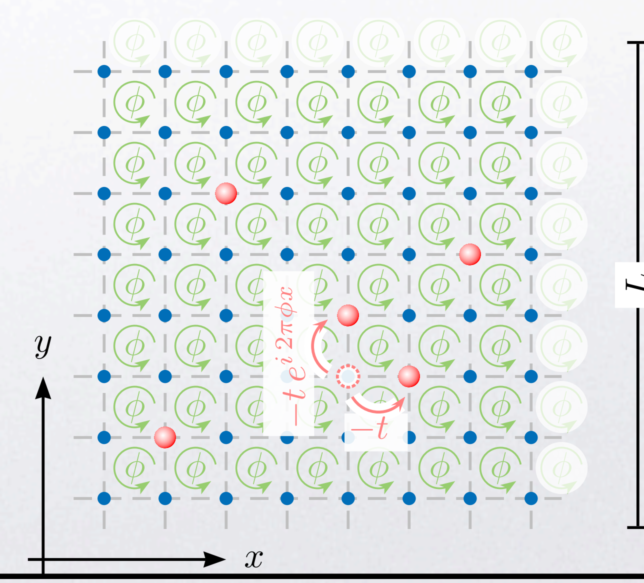

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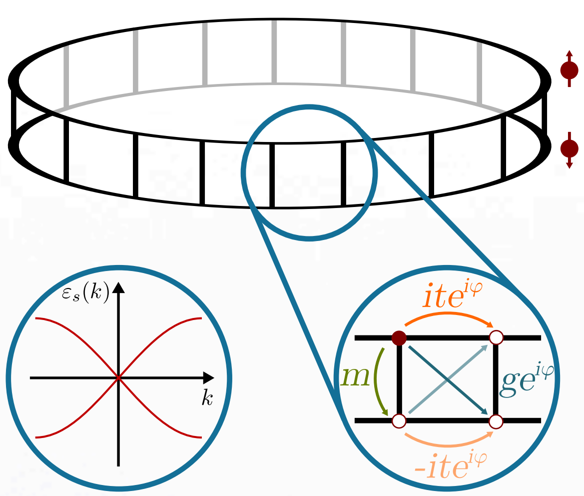
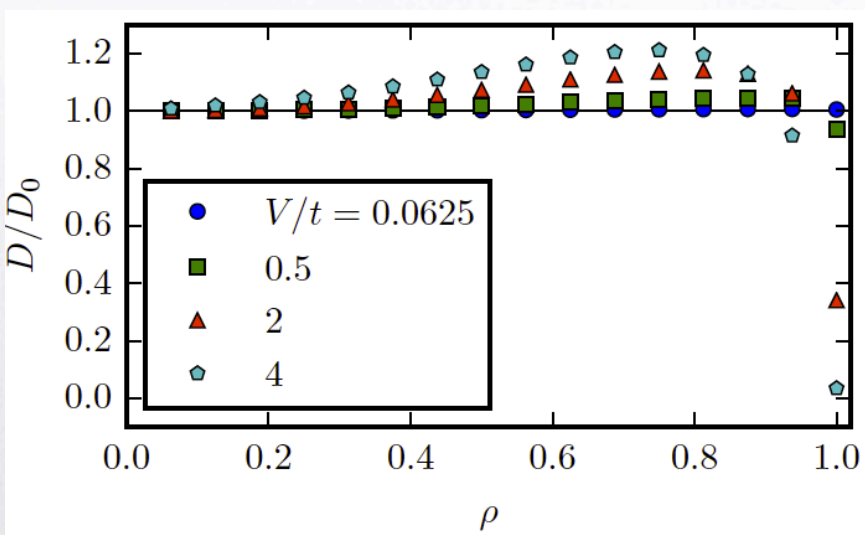
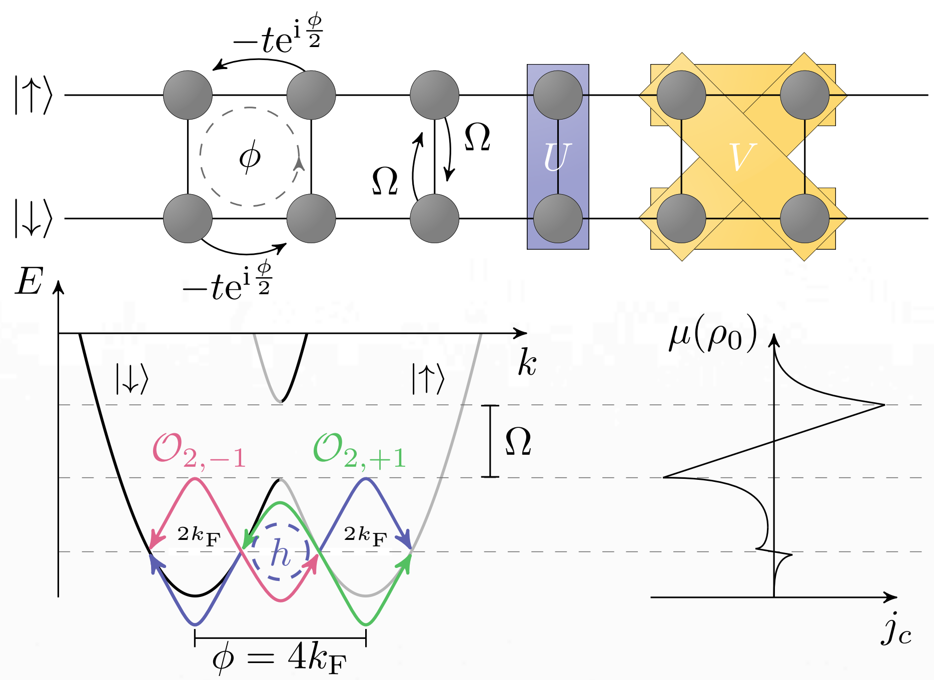




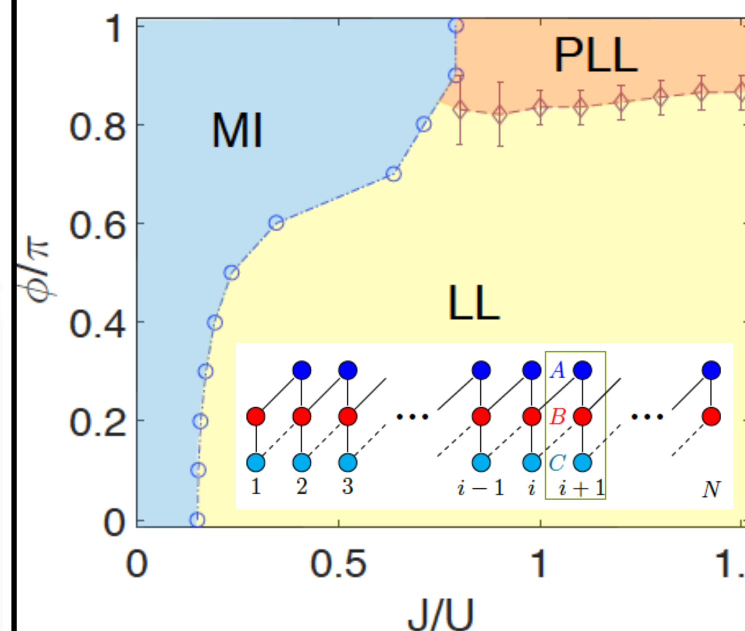
$\nu = 1/2$ resonance in chiral fermionic ladders

A. Haller, MR, M. Burrello, arXiv:1707.05715



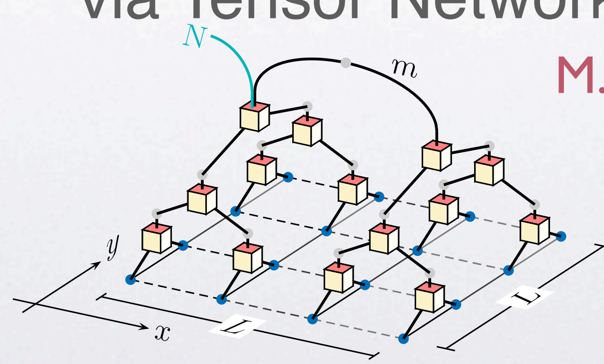
Pair Luttinger Liquid in Bosonic Flat Bands

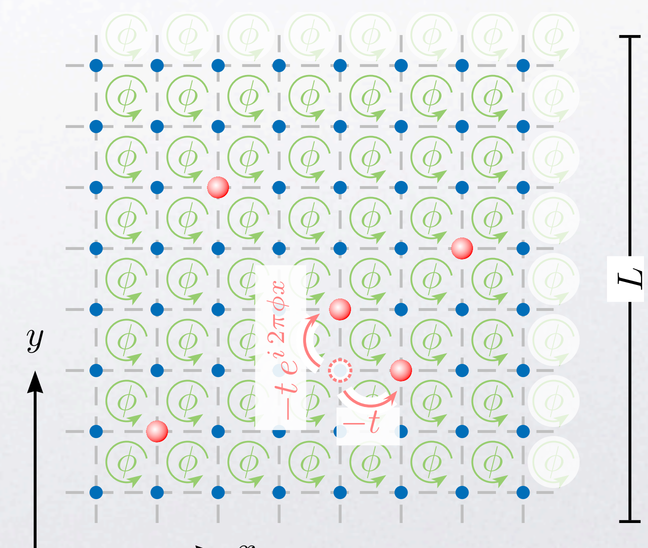
C. Cartwright, MR, G. deChiara,
soon on arXiv



FQHE in hard-core bosons: Harper-Hofstadter model via Tensor Networks

M. Gerster, et al. (MR)
arXiv:1705.06515





THANKS FOR YOUR ATTENTION