ICTP Winter College on Extreme Non-linear Optics
Attosecond Science and High-field Physics
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Temporal Characterization of Ultrafast Laser Pulses

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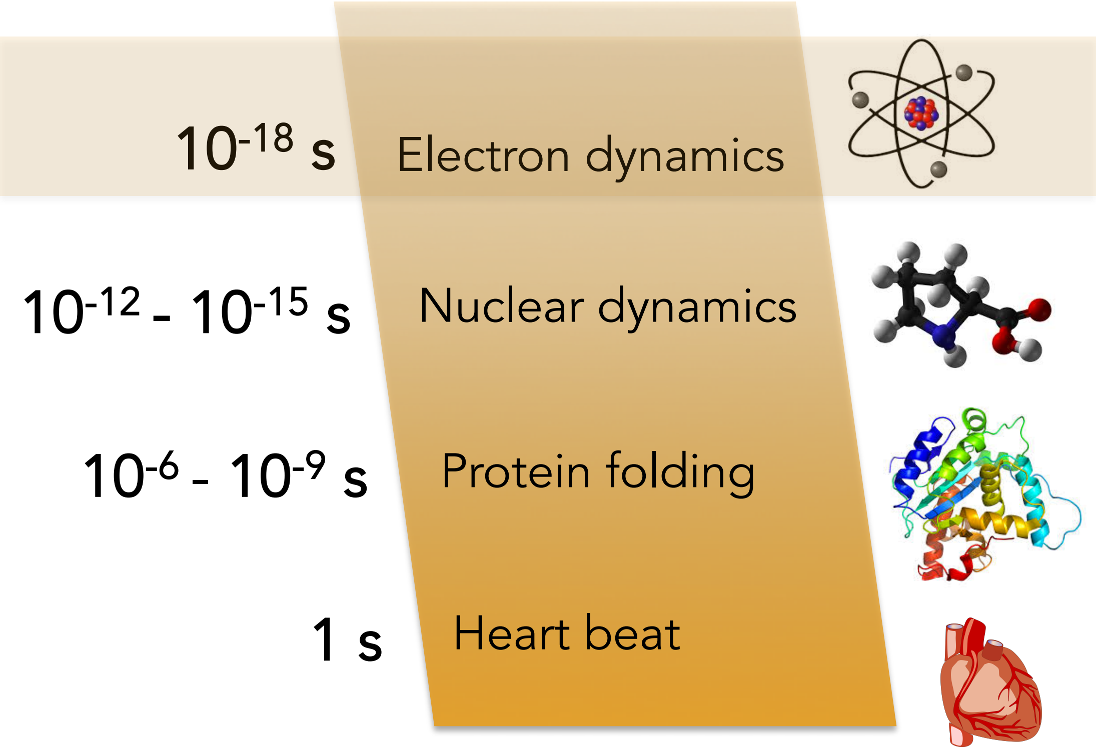
Time scale in matter

A journey in time…



10^{-18} s Electron dynamics

10^{-12} - 10^{-15} s Nuclear dynamics

10^{-6} - 10^{-9} s Protein folding

1 s Heart beat

Atomic unit of time:
24 attoseconds

Electron orbit time around the nucleus:
150 attoseconds

Attosecond Science for following and controlling electron dynamics in matter!

Time resolved measurement

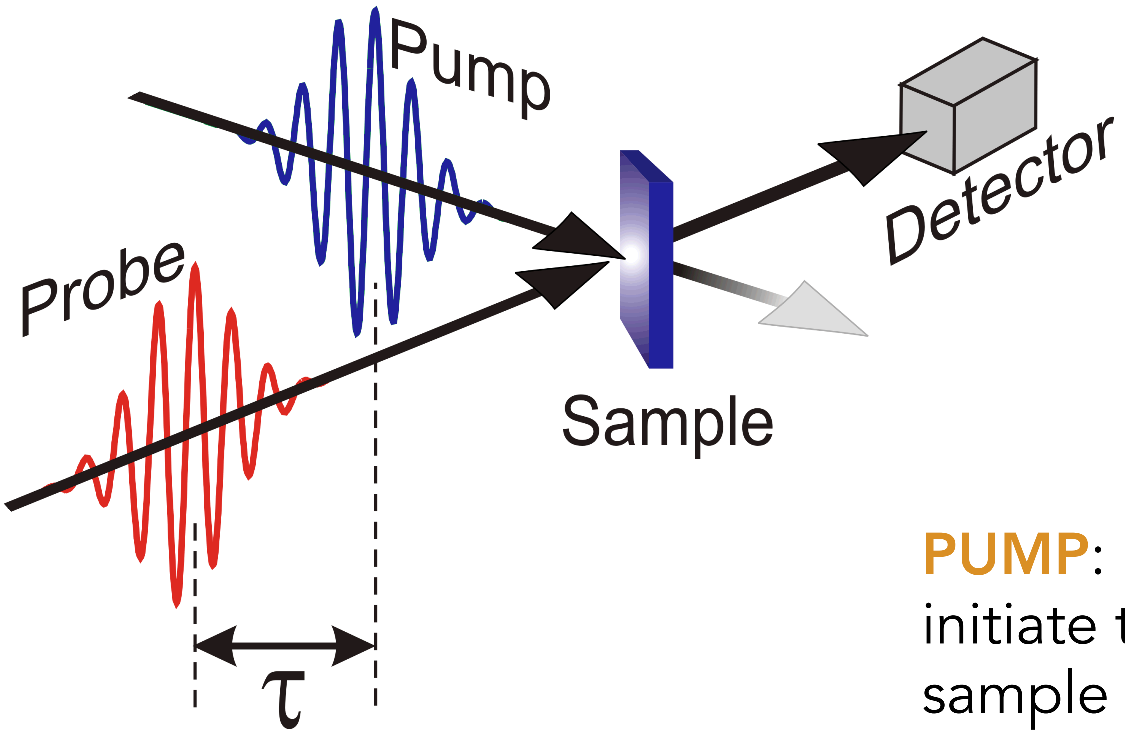
In order to measure an event in time, you need a shorter one.

We need a strobe light pulse short enough!

To measure the strobe light pulse,
you need a detector whose response time is even shorter.

How can we measure the shortest events?

Time resolved measurement

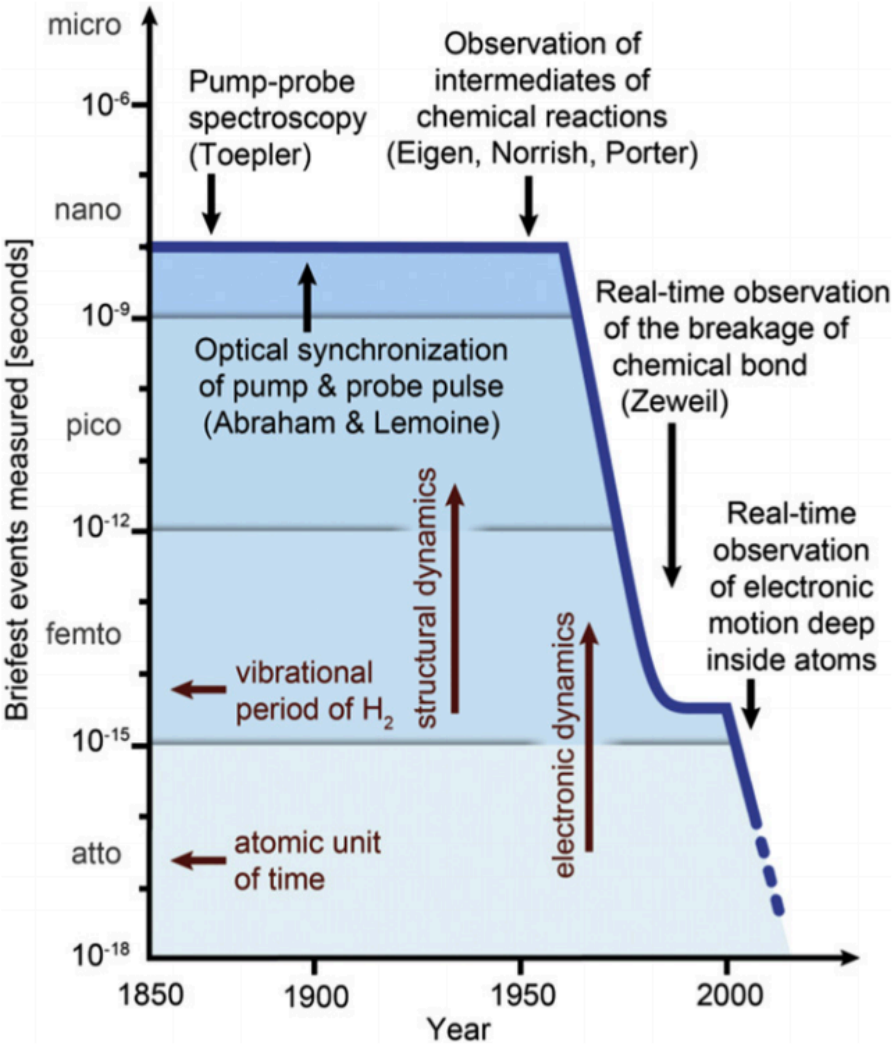


G. Cerullo et al., *Photochem. Photobiol. Sci.* 6, 135 (2007)

PUMP: a first laser pulse initiate the dynamics in the sample

PROBE: a second delayed laser pulse probe the dynamics

How fast can we measure?



F. Krausz Phys. Scr. 91, 063011 (2016)

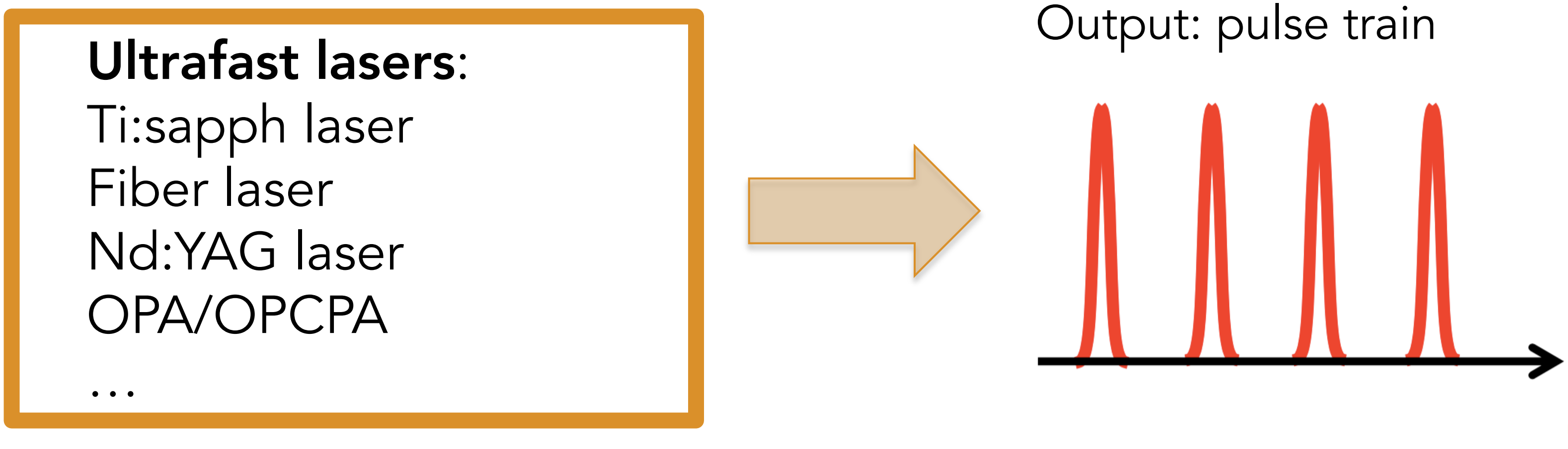
With **pico/femto**second laser pulses: real-time observation of nuclear dynamics & breakage of a chemical bond

With **atto**second laser pulses: real-time observation of electron dynamics

Summary of the lecture

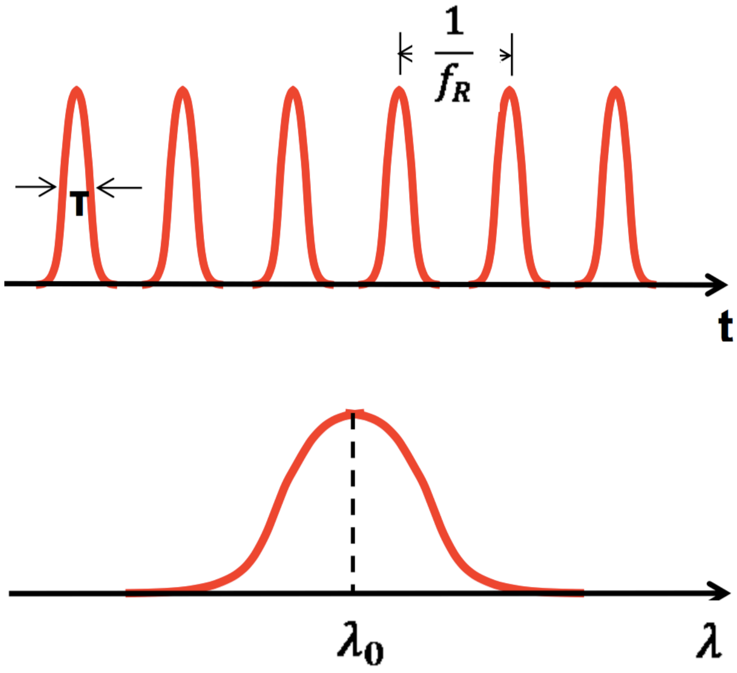
- Pulse characterization
- Intensity autocorrelation
- Interferometric Autocorrelation (IAC)
- Frequency Resolved Optical Gating (FROG)
- Spectral Phase Interferometry for Direct Electric-field Reconstruction (SPIDER)
- Attosecond pulse characterization

Ultrafast lasers



Ultrafast lasers:
Ti:sapph laser
Fiber laser
Nd:YAG laser
OPA/OPCPA
…

Output: pulse train

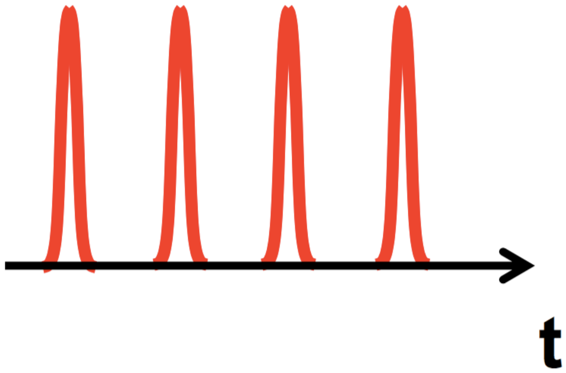


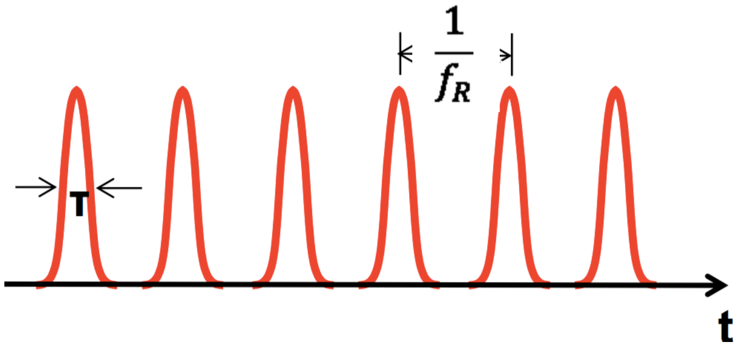
- Pulse duration T (fs-ns)
- Pulse energy E (pJ-mJ)
- Peak power $P_p \approx E/T$ (kW-PW)
- Repetition rate f_R (Hz-MHz)
- Average power $P=E*f_R$ (mW-W)
- Center wavelength λ_0 *(infrared-UV)*

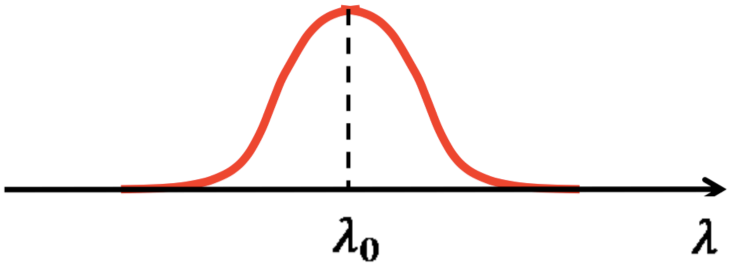
Measurement of pulse “physical quantities”

Ultrafast lasers:
Ti:sapph laser
Fiber laser
Nd:YAG laser
OPA/OPCPA
…

Output: pulse train







Physical quantity	Measuring device
Average power	Power meter
Repetition rate	RF spectrum analyzer
Spectrum	Spectrometer
Temporal duration	**Device???**

Full characterization of an optical pulse

Electric field of a laser pulse in time domain:

$$E(t) \sim \mathrm{Re}\,\{I(t)^{1/2} \exp\,[j\omega_0 t - j\phi(t)\,]\,\}$$

Intensity **Temporal phase**

…& in frequency domain:

$$\tilde{E}(\omega) \sim I(\omega - \omega_0)^{1/2} \exp\,[-j\varphi(\omega - \omega_0)]$$

Intensity **Spectral phase**

↓

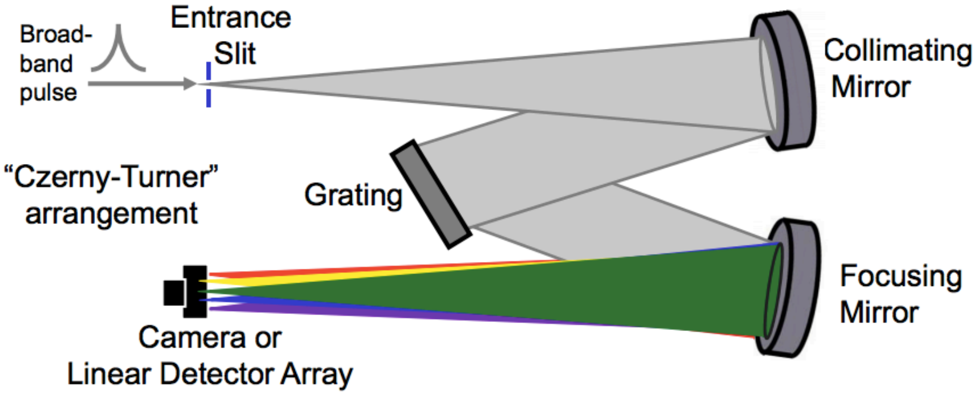
Can be measured
with a spectrometer

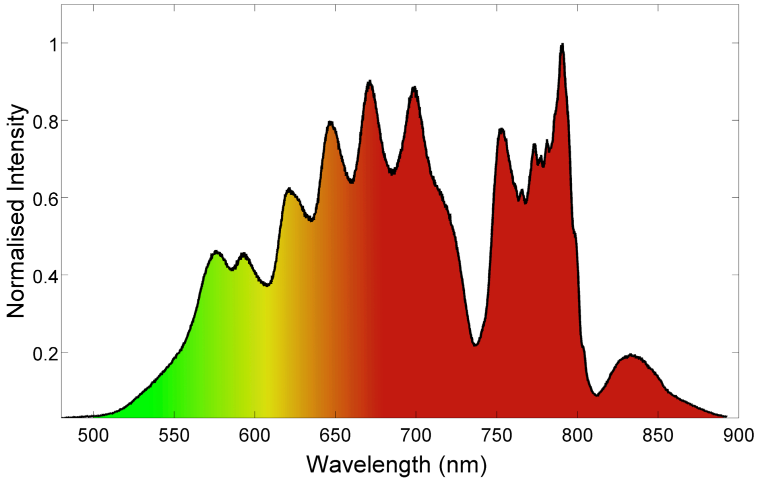
Measurement of the spectrum

$$\tilde{E}(\omega) \sim I(\omega-\omega_0)^{1/2} \; \exp\,[-j\varphi(\omega-\omega_0)]$$

Intensity

Spectral phase





- **Transform-limited pulse** can be obtained from the measured spectrum
- Spectral phase is missing!

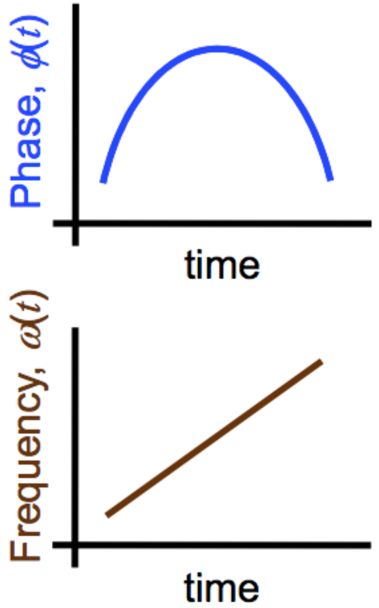
The spectral phase

$$\tilde{E}(\omega) \sim I(\omega-\omega_0)^{1/2} \exp[-j\varphi(\omega-\omega_0)]$$

Intensity

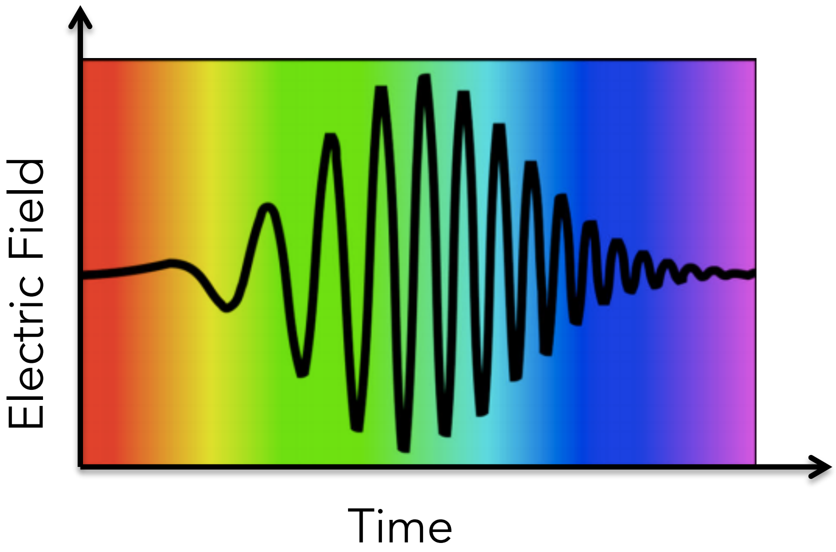
Spectral phase

The **instantaneous frequency** (frequency vs time) can be retrieved from the spectral phase



Example: parabolic phase, linear chirp

$$\omega(t) = \omega_0 - d\phi / dt$$

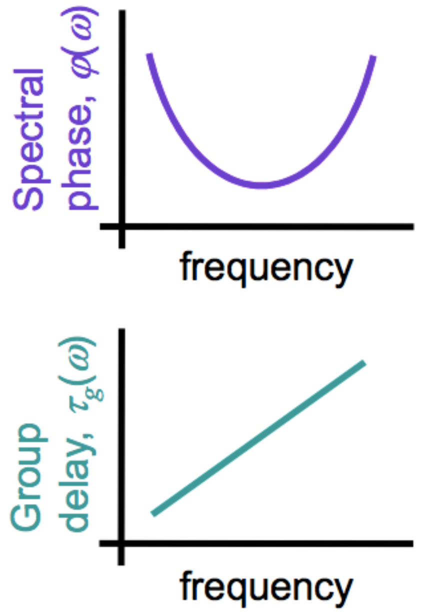


The spectral phase

$$\tilde{E}(\omega) \sim I(\omega-\omega_0)^{1/2} \quad \exp\left[-j\varphi(\omega-\omega_0)\right]$$

Intensity **Spectral phase**

The **group delay** can be retrieved from the spectral phase



Example: parabolic phase, linear chirp

$$\tau_g(\omega) = d\varphi / d\omega$$

The group delay vs. frequency is approximately the inverse of the instantaneous frequency vs. time

We should be able to measure, pulses with arbitrarily complex phases and frequencies vs. time!

Measurement in the time domain

Is there a device to measure the duration of the pulse?

Photo-detectors: photodiodes & photomultipliers

- Photo-detectors are devices that emit electrons in response to photons
- The detector output voltage is proportional to the pulse energy

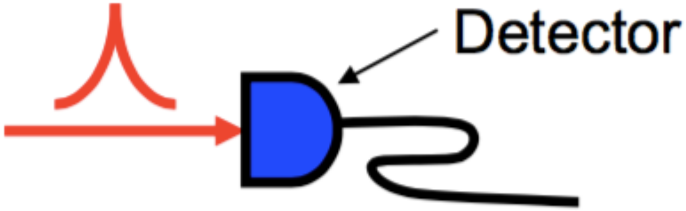
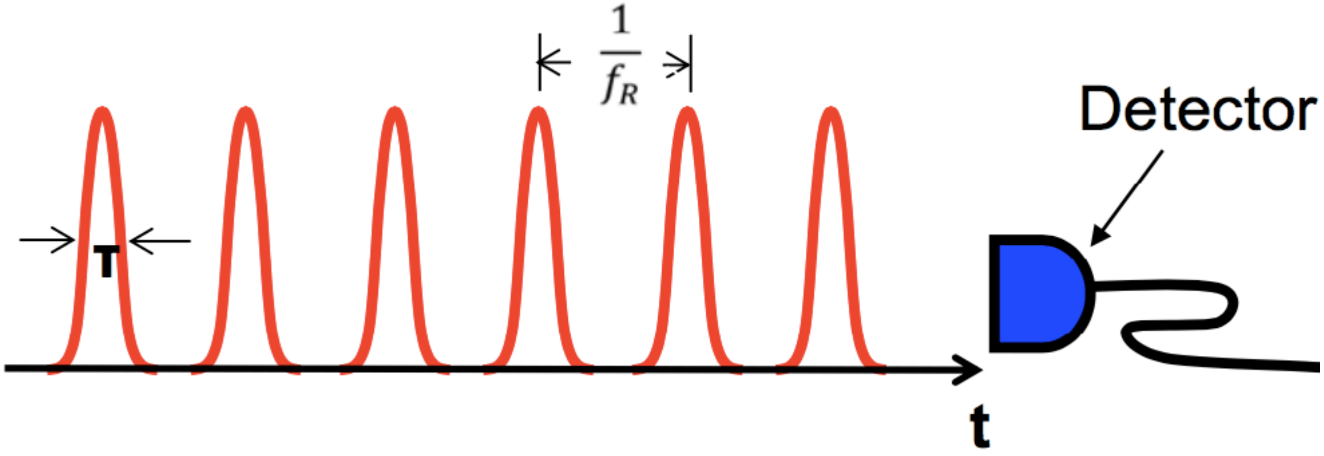


Photo-detectors measure the time integral of the pulse intensity:

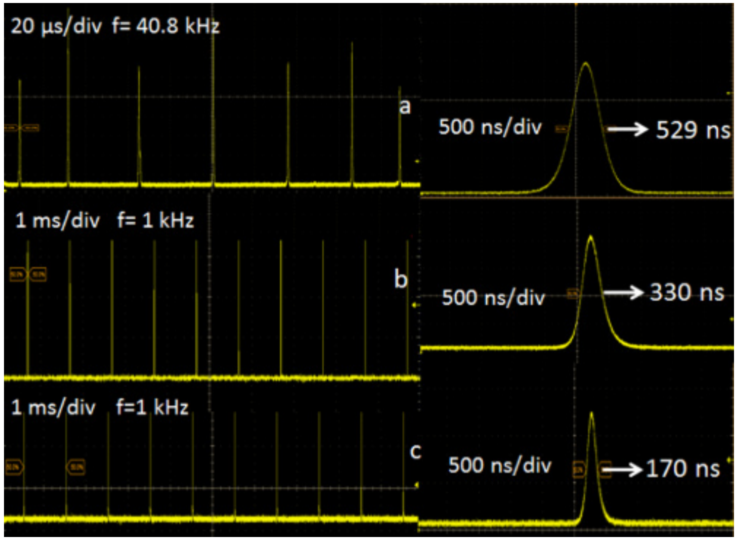
$$V_{detector} \propto \int_{-\infty}^{\infty} |E(t)|^2 \, dt$$

The detector response is too slow for ultrafast pulses (typically nanoseconds)!

Measurement in the time domain

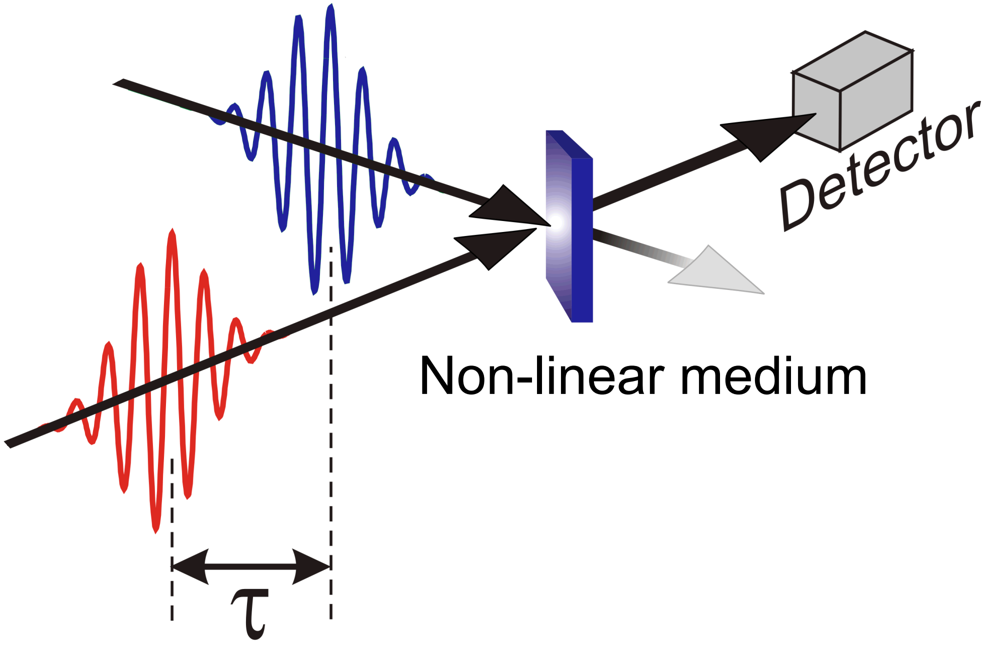


Fast photo-detectors allow the laser pulse train to be observed on the oscilloscope:



Measurement in the time domain

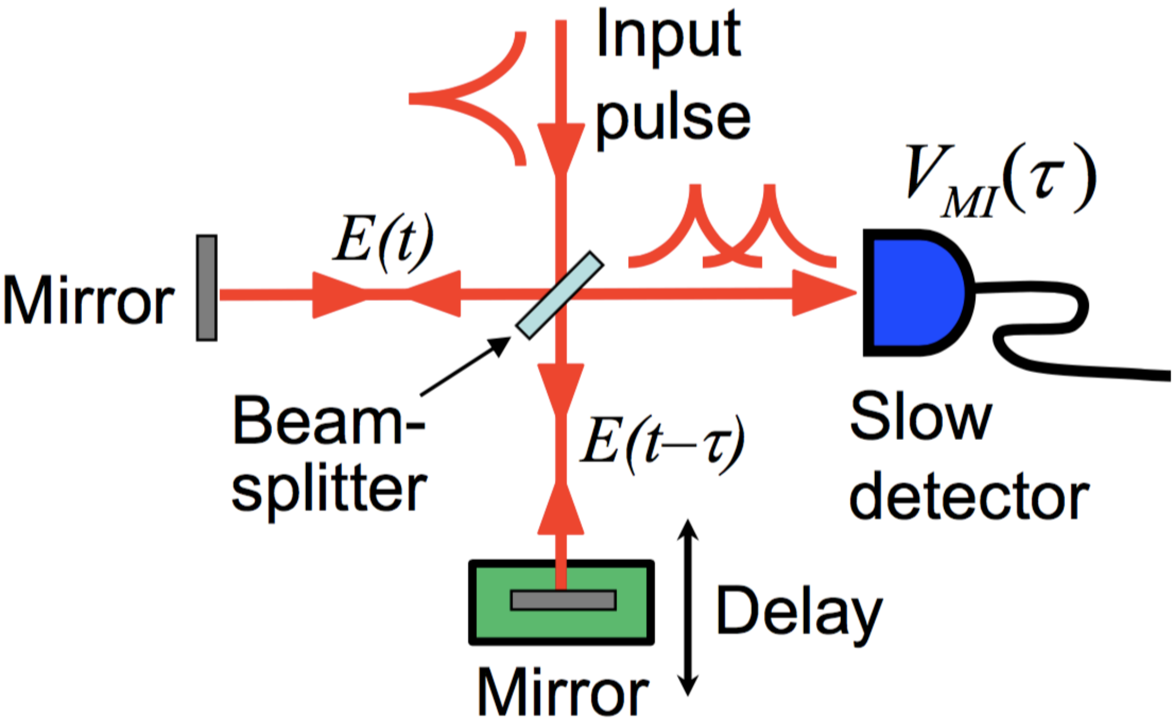
Photo-detectors tell us only a very little about the pulse



The best way to temporally characterize a laser pulse is to use the pulse itself (or a reference pulse)

All-optical methods!

Field autocorrelation



$$V_{MI}(\tau) \propto \int_{-\infty}^{\infty} \left|E(t) - E(t-\tau)\right|^2 \, dt$$

$$= \int_{-\infty}^{\infty} \left|E(t)\right|^2 + \left|E(t-\tau)\right|^2 - 2\,\mathrm{Re}[E(t)E^*(t-\tau)] \; dt$$

Field autocorrelation

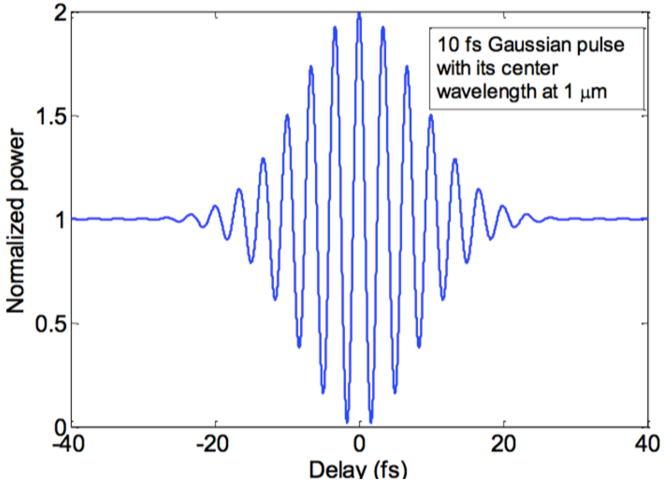
$$V_{MI}(\tau) \propto \int_{-\infty}^{\infty} \left|E(t)-E(t-\tau)\right|^2 dt$$

$$= \int_{-\infty}^{\infty} \left|E(t)\right|^2 + \left|E(t-\tau)\right|^2 - 2\,\mathrm{Re}[E(t)E^*(t-\tau)]\; dt$$

$$\Rightarrow \quad V_{MI}(\tau) \propto 2\int_{-\infty}^{\infty} \left|E(t)\right|^2 dt \; - \; 2\,\mathrm{Re}\int_{-\infty}^{\infty} E(t)E^*(t-\tau)\; dt$$

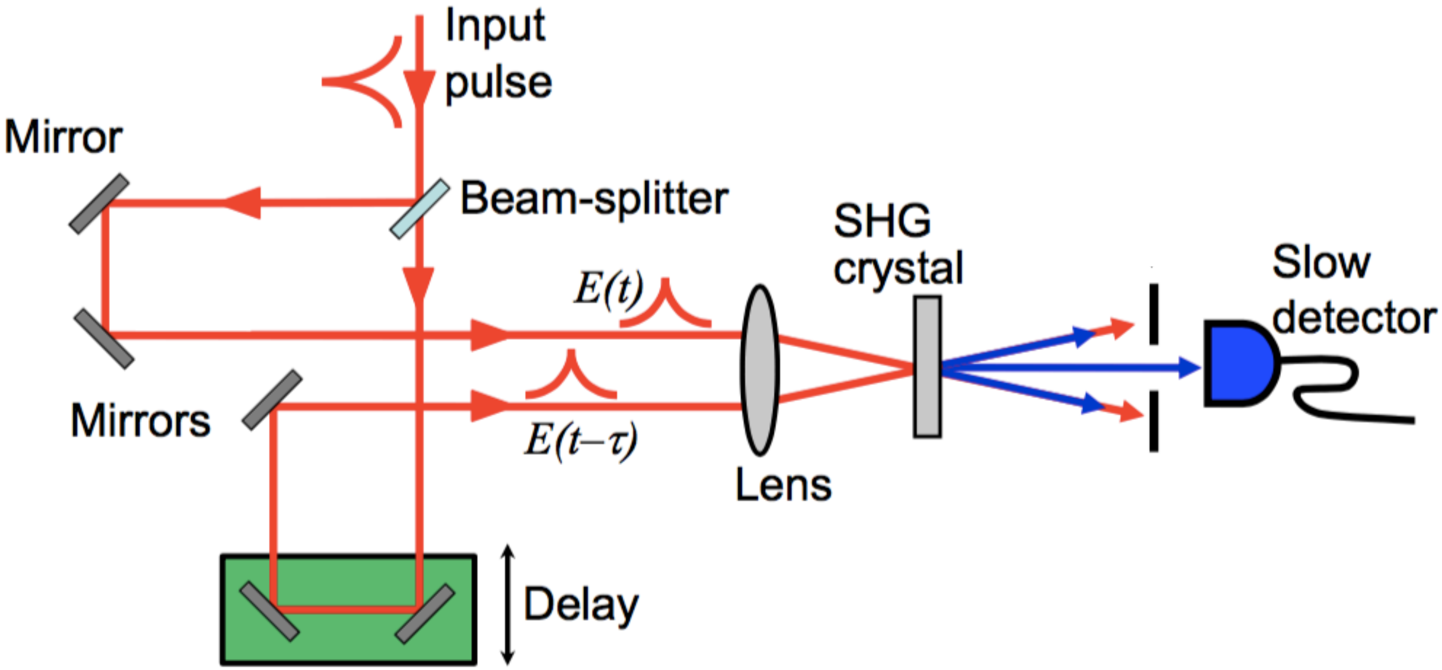
$\propto$ **Pulse energy**

Field autocorrelation (interferogram)



- Measuring the interferogram is equivalent to measuring the spectrum
- Field autocorrelation measurement gives no information about the spectral phase
- Field autocorrelation measurement cannot distinguish a transform-limited pulse from a longer chirped pulse with the same bandwidth

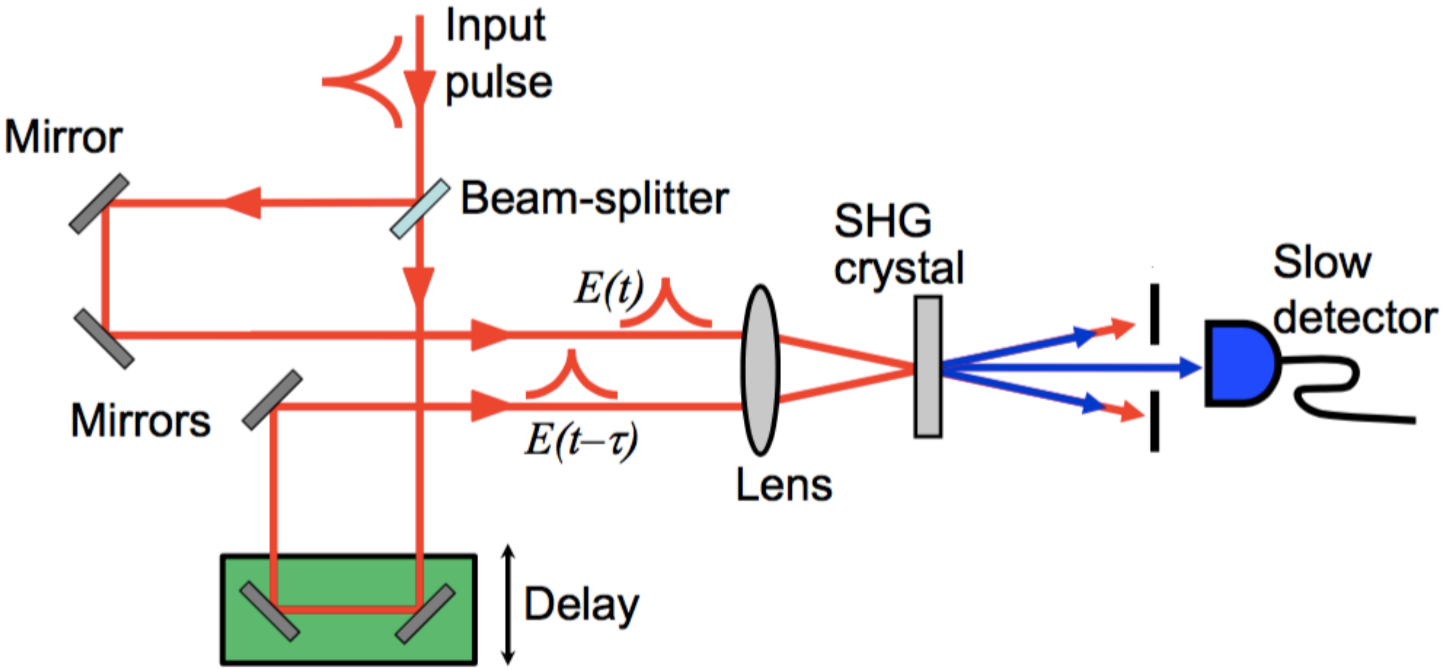
Intensity autocorrelation



Intensity Autocorrelation:

- create a delayed replica of the pulse
- cross beams in an second-harmonic generation (SHG) crystal
- vary the delay between the two pulses
- measure the second-harmonic (SH) pulse energy vs. delay

Intensity autocorrelation



$$E_{SH}(t,\tau) \propto E(t)E(t-\tau)$$

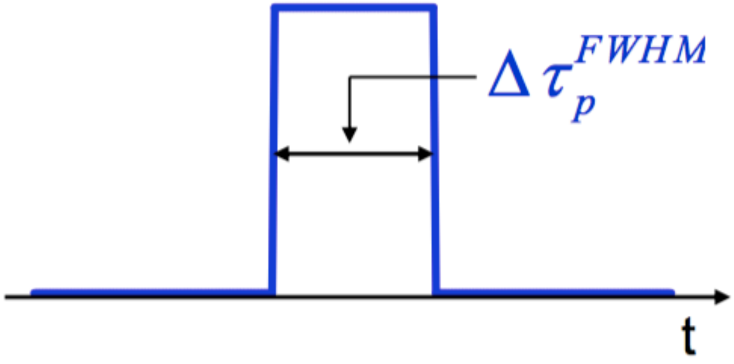
$$\Rightarrow I_{AC}(\tau) \propto \int_{-\infty}^{\infty} \left|E(t)E(t-\tau)\right|^2 dt$$

$$I_{AC}(\tau) \propto \int_{-\infty}^{\infty} I(t)I(t-\tau)dt$$

Intensity autocorrelation: squared pulse

Pulse

$$I(t) = \begin{cases} 1; \ |t| \leq \Delta\tau_p^{FWHM}/2 \\ 0; \ |\mathrm{t}| > \Delta\tau_p^{FWHM}/2 \end{cases}$$



Autocorrelation

$$A^{(2)}(\tau) = \begin{cases} 1 - \left|\dfrac{\tau}{\Delta\tau_A^{FWHM}}\right|; \ |\tau| \leq \Delta\tau_A^{FWHM} \\ 0; \ |\tau| > \Delta\tau_A^{FWHM} \end{cases}$$

$$\Delta\tau_A^{FWHM} = \Delta\tau_p^{FWHM}$$

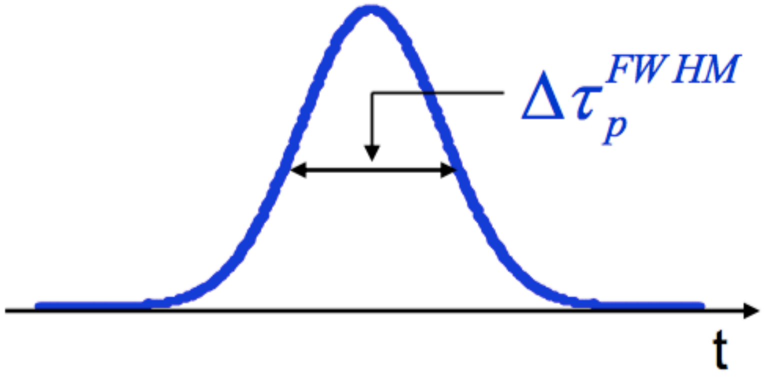
Intensity autocorrelation: gaussian pulse

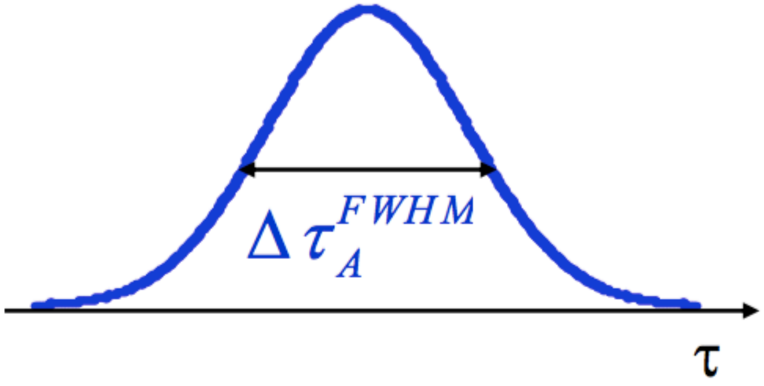
Pulse

Autocorrelation

$$I(t) = \exp\left[-\left(\frac{2\sqrt{\ln 2}\,t}{\Delta\tau_p^{FWHM}}\right)^2\right]$$

$$A^{(2)}(\tau) = \exp\left[-\left(\frac{2\sqrt{\ln 2}\,\tau}{\Delta\tau_A^{FWHM}}\right)^2\right]$$





$$\Delta\tau_A^{FWHM} = 1.41\,\Delta\tau_p^{FWHM}$$

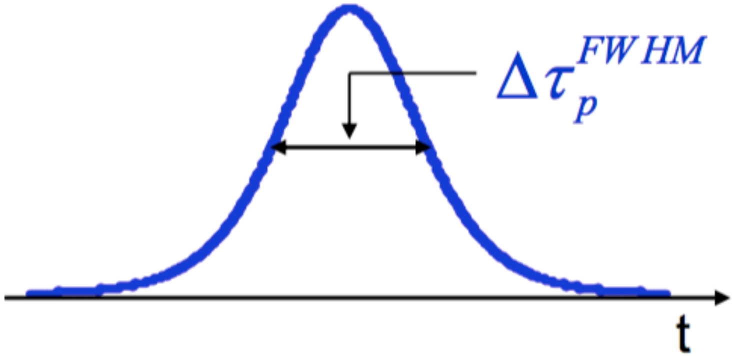
Intensity autocorrelation: sech^2 pulse

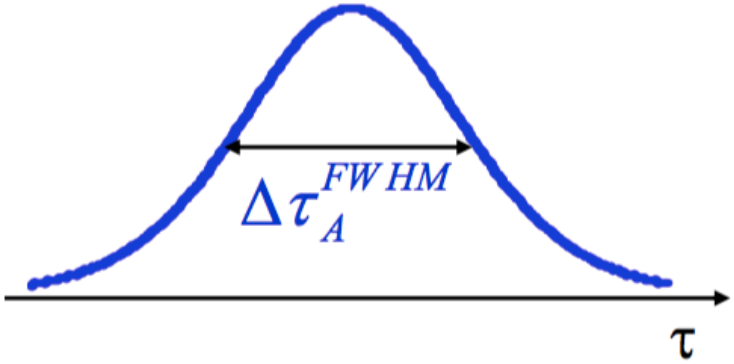
Pulse

$$I(t) = \operatorname{sech}^2\left[\frac{1.7627t}{\Delta t_p^{FWHM}}\right]$$

Autocorrelation

$$A^{(2)}(\tau) = \frac{3}{\sinh^2\left(\frac{2.7196\tau}{\Delta\tau_A^{FWHM}}\right)}\left[\frac{2.7196\tau}{\Delta\tau_A^{FWHM}}\coth\left(\frac{2.7196\tau}{\Delta\tau_A^{FWHM}}\right)-1\right]$$





$$\Delta\tau_A^{FWHM} = 1.54\,\Delta\tau_p^{FWHM}$$

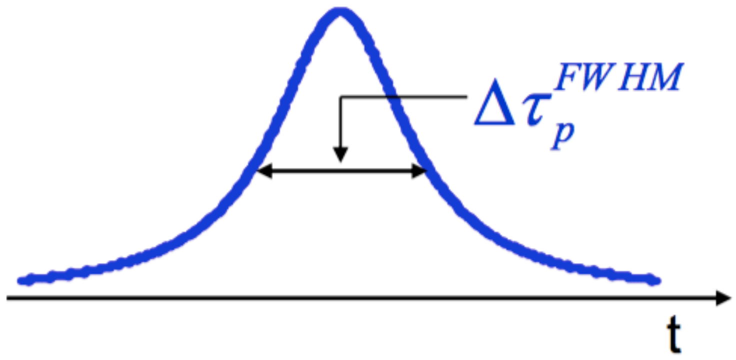
Intensity autocorrelation: Lorentzian pulse

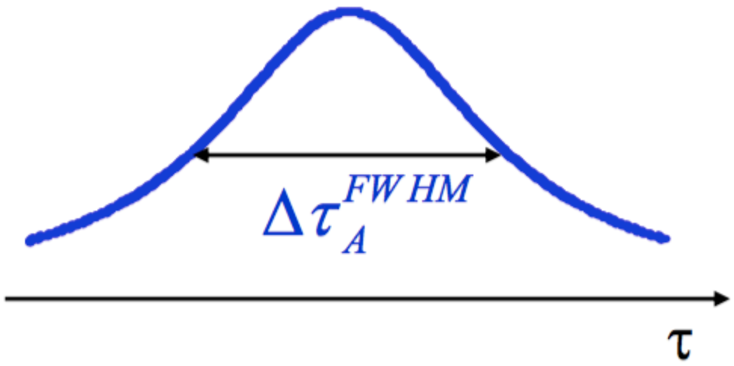
Pulse

Autocorrelation

$$I(t) = \frac{1}{1+\left(2t/\Delta\tau_p^{FWHM}\right)^2}$$

$$A^{(2)}(\tau) = \frac{1}{1+\left(2\tau/\Delta\tau_A^{FWHM}\right)^2}$$





$$\Delta\tau_A^{FWHM} = 2.0\,\Delta\tau_p^{FWHM}$$

Intensity autocorrelation:

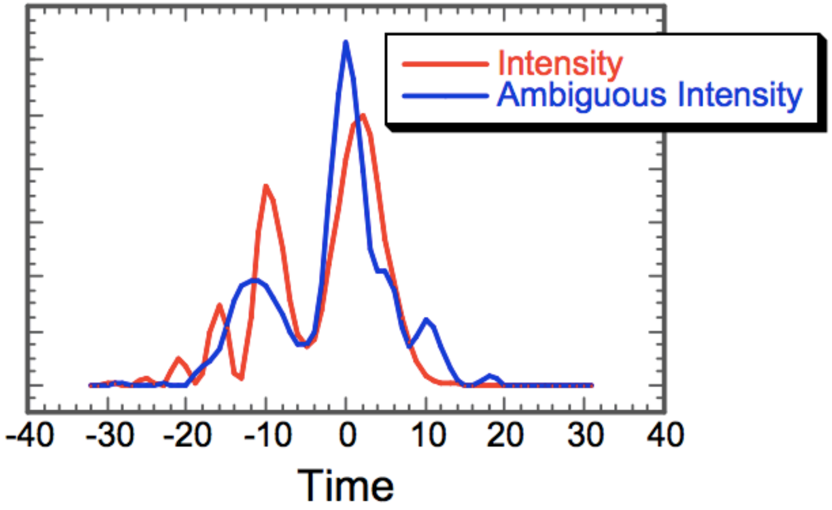
- It is always symmetric, and assumes its maximum value at τ = 0.

$$I_{AC}(\tau) \propto \int_{-\infty}^{\infty} I(t) I(t-\tau) dt \qquad I_{AC}(\tau) = I_{AC}(-\tau)$$

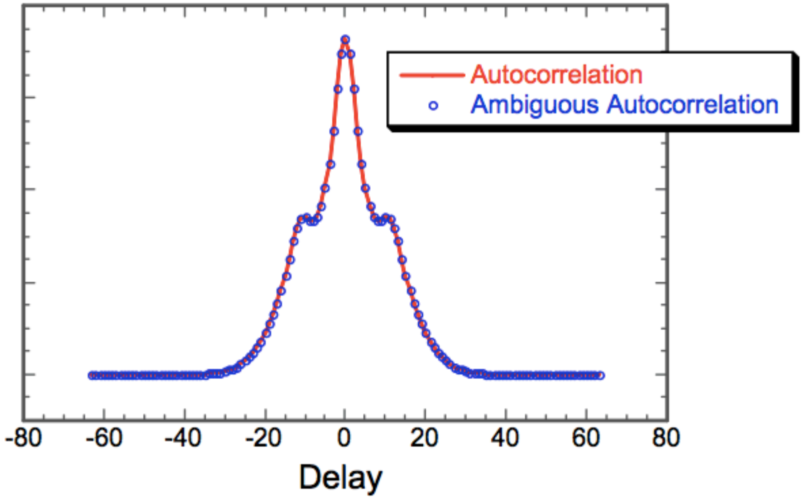
- Width of the correlation peak gives information about the pulse width
- Pulse phase information is missing
- To get the pulse duration, it is necessary to assume a pulse shape, and to use the corresponding deconvolution factor
- For short pulses, very thin crystals must be used to guarantee enough phase- matching bandwidth
- **The intensity autocorrelation is not sufficient to determine the pulse intensity profile**

Autocorrelations of more complex intensities

Autocorrelations nearly always have considerably less structure than the corresponding intensity



Intensity

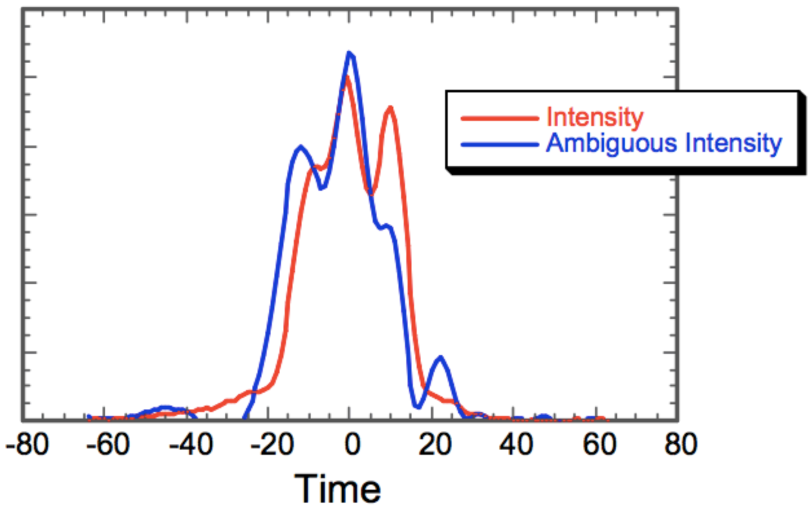


Autocorrelation

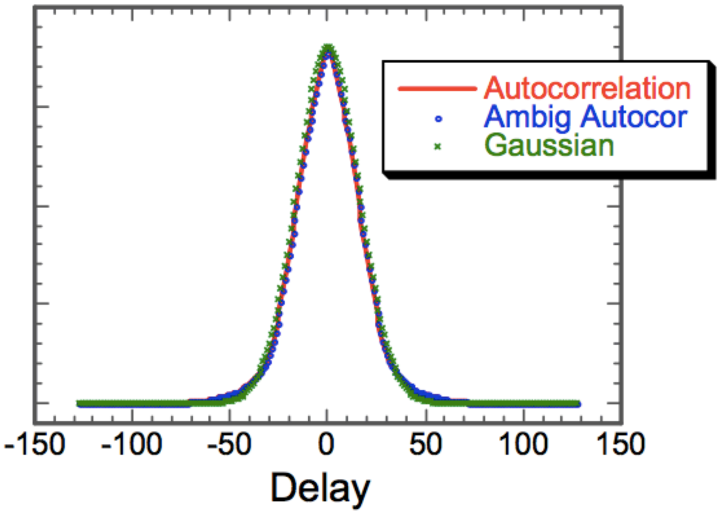
An autocorrelation typically corresponds to many different intensities ⟶ the autocorrelation does not uniquely determine the intensity

Autocorrelations of more complex intensities

These complex intensities have nearly Gaussian autocorrelations



Intensity

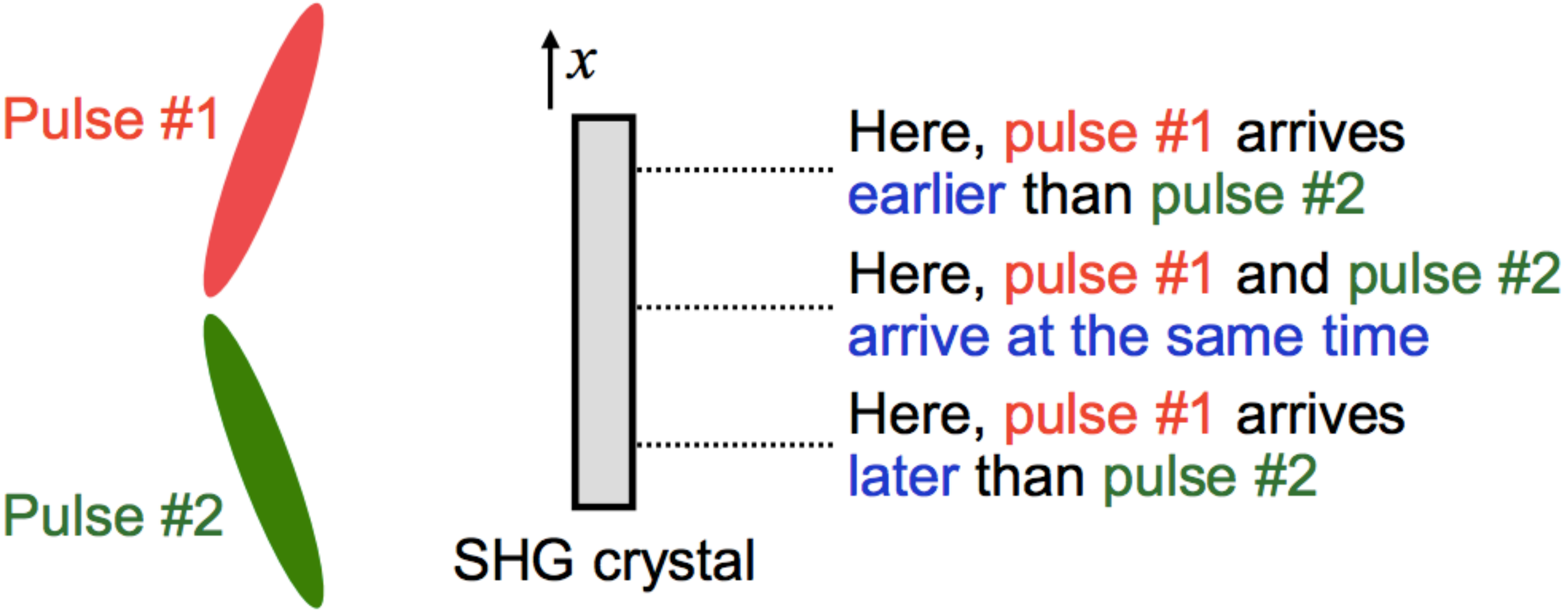


Autocorrelation

Autocorrelation has many nontrivial ambiguities!

Geometrical distortions in autocorrelation

When crossing beams at an angle, the delay varies across the beam



Pulse #1

Pulse #2

x

SHG crystal

Here, pulse #1 arrives earlier than pulse #2

Here, pulse #1 and pulse #2 arrive at the same time

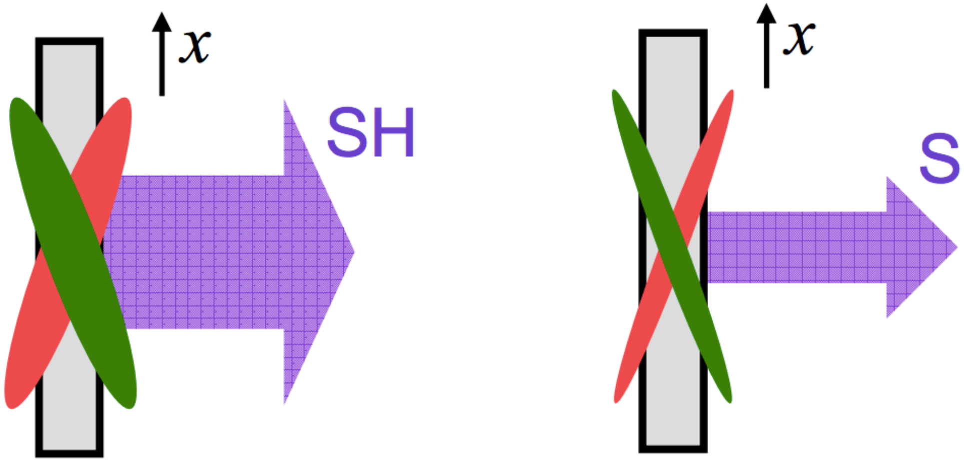
Here, pulse #1 arrives later than pulse #2

This effect causes a range of delays to occur at a given time and could cause geometrical smearing with a broadening of the autocorrelation width

Single-shot autocorrelation

Crossing beams at an angle also maps delay onto transverse position

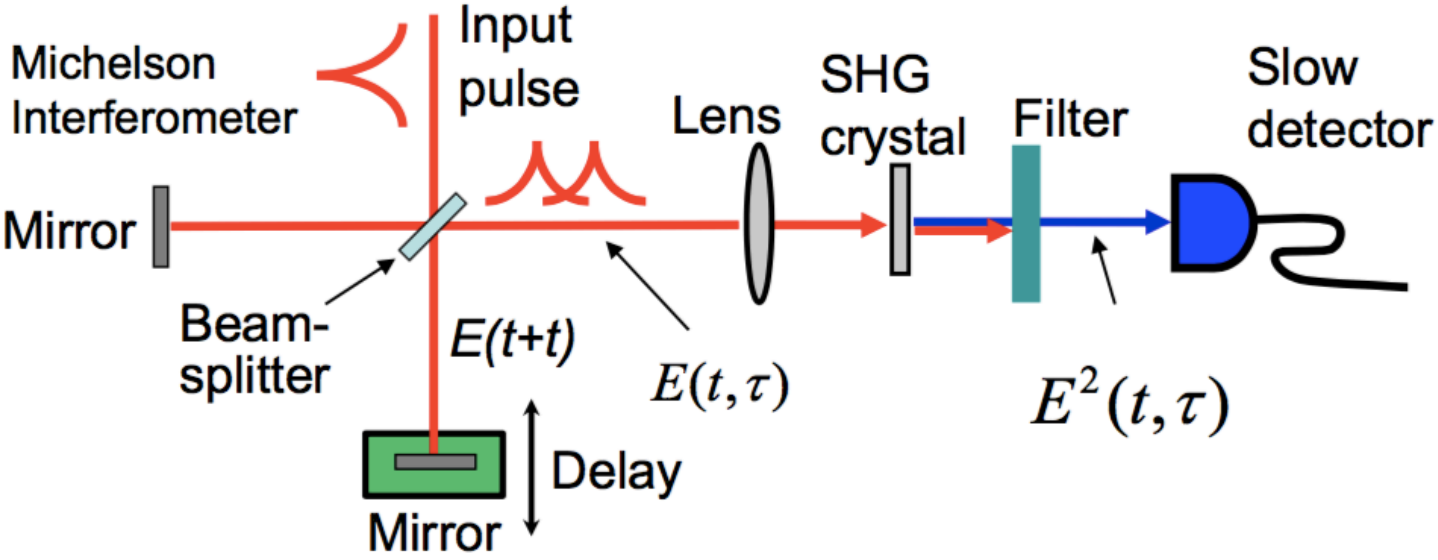
$$\tau(x) = 2(x/c)\sin(\theta/2) \approx x\theta/c$$



Large beams and a large angle allows to achieve the desired range of delays in a single-shot. No-need for delay scan!

Single-shot SHG AC has no geometrical smearing

Interferometric autocorrelation



An alternative approach is to use a collinear beam geometry, and allow the autocorrelator signal light to interfere with the SHG from each individual beam

$$IA^{(2)}(\tau) \equiv \int_{-\infty}^{\infty} \left| [E(t)-E(t-\tau)]^2 \right|^2 dt$$

$$IA^{(2)}(\tau) \equiv \int_{-\infty}^{\infty} \left| \underbrace{E^2(t)+E^2(t-\tau)}_{\text{New terms}} - 2\underbrace{E(t)E(t-\tau)}_{\text{Autocorrelation term}} \right|^2 dt$$

Interferometric autocorrelation

$$IA^{(2)}(\tau) \equiv \int_{-\infty}^{\infty} \left[E^2(t)+E^2(t-\tau)-2E(t)E(t-\tau)\right]\left[E^{*2}(t)+E^{*2}(t-\tau)-2E^*(t)E^*(t-\tau)\right] dt$$

$$\begin{aligned}IA^{(2)}(\tau) = \int_{-\infty}^{\infty} \Big\{ & \left|E^2(t)\right|^2 + E^2(t)E^{*2}(t-\tau) - 2E^2(t)E^*(t)E^*(t-\tau) + \\ & E^2(t-\tau)E^{*2}(t) + \left|E^2(t-\tau)\right|^2 - 2E^2(t-\tau)E^*(t)E^*(t-\tau) + \\ & -2E(t)E(t-\tau)E^{*2}(t) - 2E(t)E(t-\tau)E^{*2}(t-\tau) + 4\left|E(t)\right|^2\left|E(t-\tau)\right|^2 \Big\}\, dt\end{aligned}$$

$$\begin{aligned}= \int_{-\infty}^{\infty} \Big\{ & I^2(t) + E^2(t)E^{*2}(t-\tau) - 2I(t)E(t)E^*(t-\tau) + \\ & E^2(t-\tau)E^{*2}(t) + I^2(t-\tau) - 2I(t-\tau)E^*(t)E(t-\tau) + \\ & -2I(t)E(t-\tau)E^*(t) - 2I(t-\tau)E(t)E^*(t-\tau) + 4I(t)I(t-\tau) \Big\}\, dt\end{aligned}$$

Where: $I(t) \equiv \left|E(t)\right|^2$

Interferometric autocorrelation

From the math we can extract 4 terms:

$$= \int_{-\infty}^{\infty} I^2(t) + I^2(t-\tau)\, dt \quad = I_{back}$$

Background

$$+ \ 4\int_{-\infty}^{\infty} I(t)I(t-\tau)\, dt \quad = I_{int}$$

Intensity autocorrelation

$$- \ 2\int_{-\infty}^{\infty} \left[I(t) + I(t-\tau)\right] E(t)E^*(t-\tau)\, dt \ + \ c.c = I_{\omega}$$

Interferogram of E(t), oscillating at ω

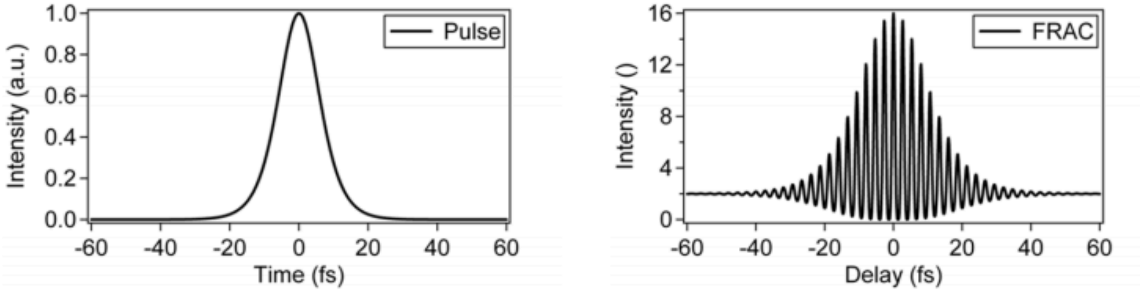
$$+ \int_{-\infty}^{\infty} E^2(t)E^{2*}(t-\tau)\, dt \ + \ c.c. \ = I_{2\omega}$$

Interferogram of the SH oscillating at 2ω

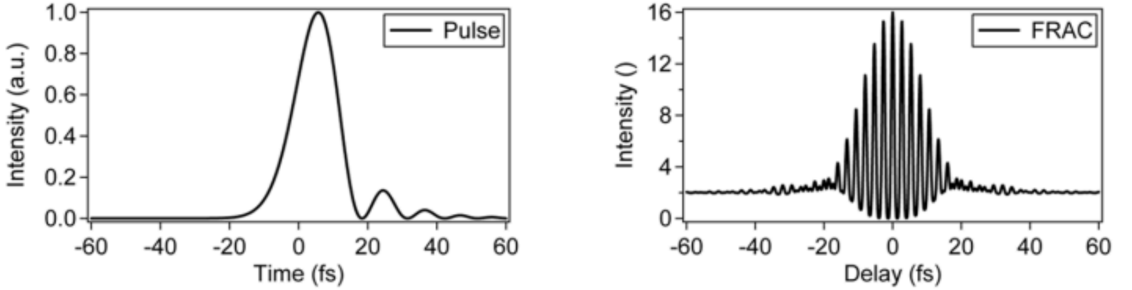
$IA^{(2)}(\tau = 0) = 8$ $\qquad$ $IA^{(2)}(\tau \rightarrow \infty) = 1$

Interferometric autocorrelation

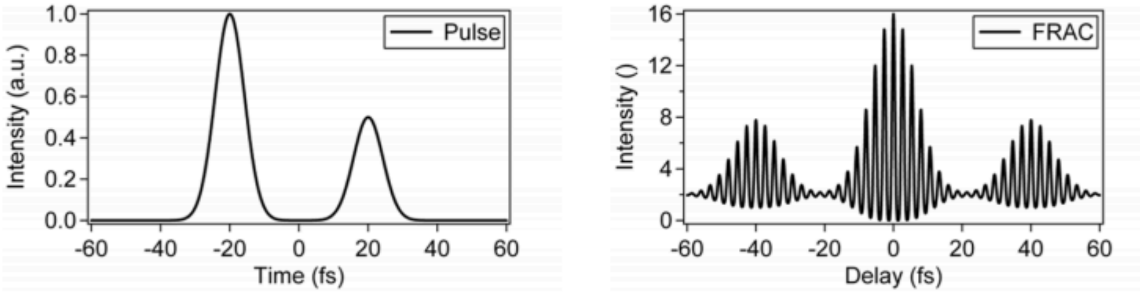
7-fs sech pulse



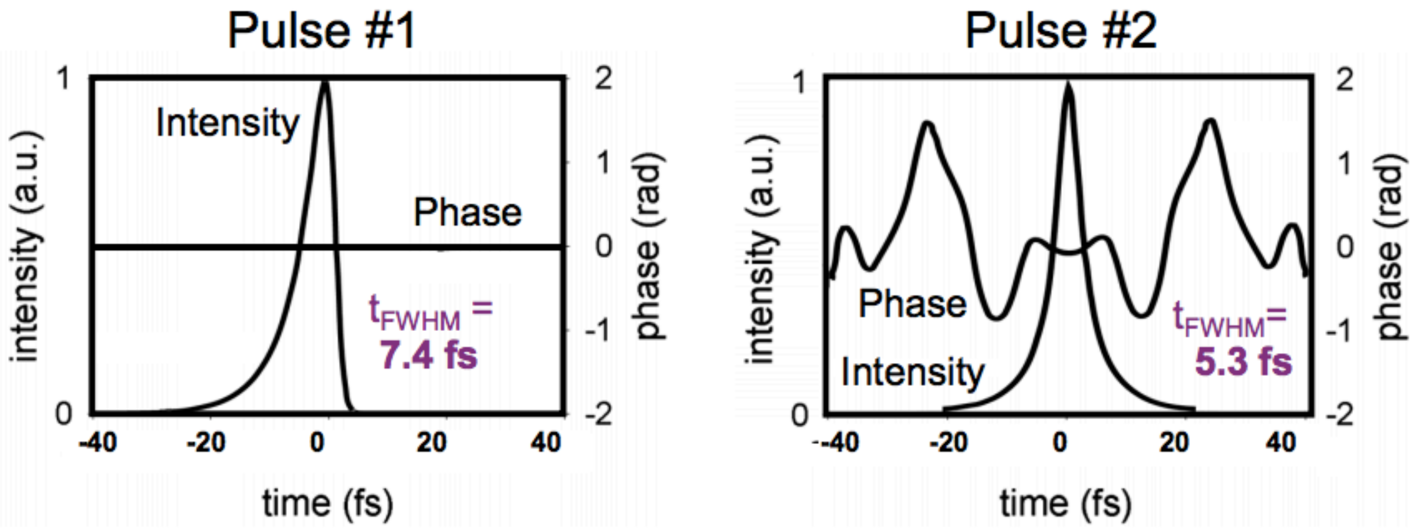
Pulse with cubic spectral phase



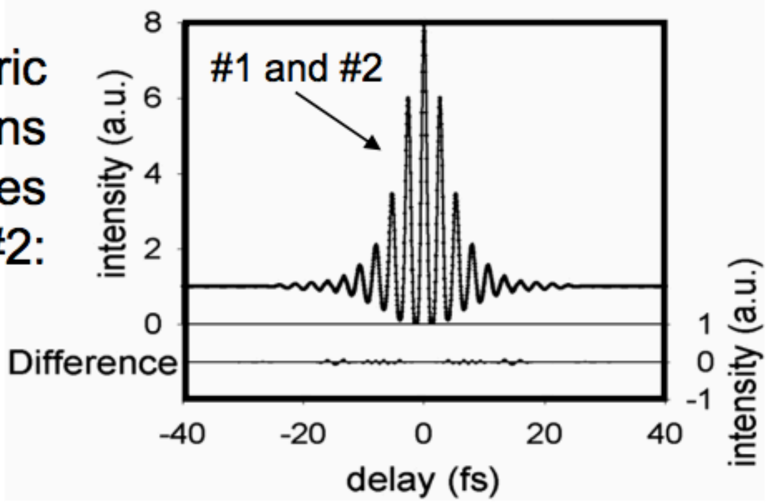
Double pulse



Interferometric autocorrelation



Interferometric Autocorrelations for Pulses #1 and #2:



Chung and Weiner, IEEE JSTQE, 2001.

Interferometric autocorrelation also have ambiguities

Interferometric autocorrelation

- It is always symmetric and the peak-to-background ratio should be 8.
- This device is difficult to align; there are five very sensitive degrees of freedom in aligning two collinear pulses.
- Dispersion in each arm must be the same, so it is necessary to insert a compensator plate in one arm.
- Using optical spectrum and background-free intensity autocorrelator can determine the presence or absence of strong chirp. The interferometric autocorrelation serves as a clear visual indication of moderate to large chirp.
- It is difficult to distinguish between different pulse shapes and, especially, different phases from interferometric autocorrelations.
- Like the intensity autocorrelation, it must be curve-fit to an assumed pulse shape and so should only be used for rough estimates.

How to measure both pulse intensity profile and phase?

- A pulse can be represented by two arrays of data with length N, one for the amplitude/intensity and the other for the phase. Totally we have 2N degrees of freedom (corresponding to the real and imaginary parts for the electric field)

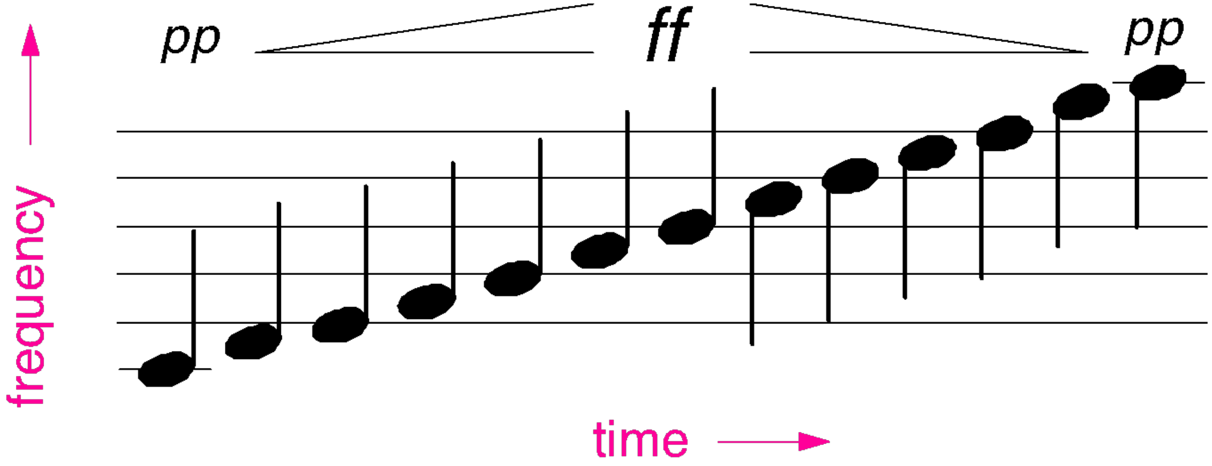
- Intensity autocorrelator provides only one array of data with length N. Optical spectrum measurement can provide another array of data with length N. However some information, especially about phase, is missing from both measurements

- Need to have more data, providing enough redundancy to recover the both the amplitude and phase

How about measuring the spectrum of the autocorrelation pulse at each delay? NxN data points

How to measure both pulse intensity profile and phase?

Frequency vs Time → SPECTROGRAM
A spectrogram can be seen as a musical score!



How about measuring the spectrum of the autocorrelation pulse at each delay? NxN data points

The spectrogram

If $E(t)$ is the waveform of interest, its spectrogram is:

$$\Sigma_E(\omega,\tau) \equiv \left| \int_{-\infty}^{\infty} E(t)\, g(t-\tau) \exp(-i\omega t)\, dt \right|^2$$

where $g(t-t)$ is a variable-delay gate function and t is the delay

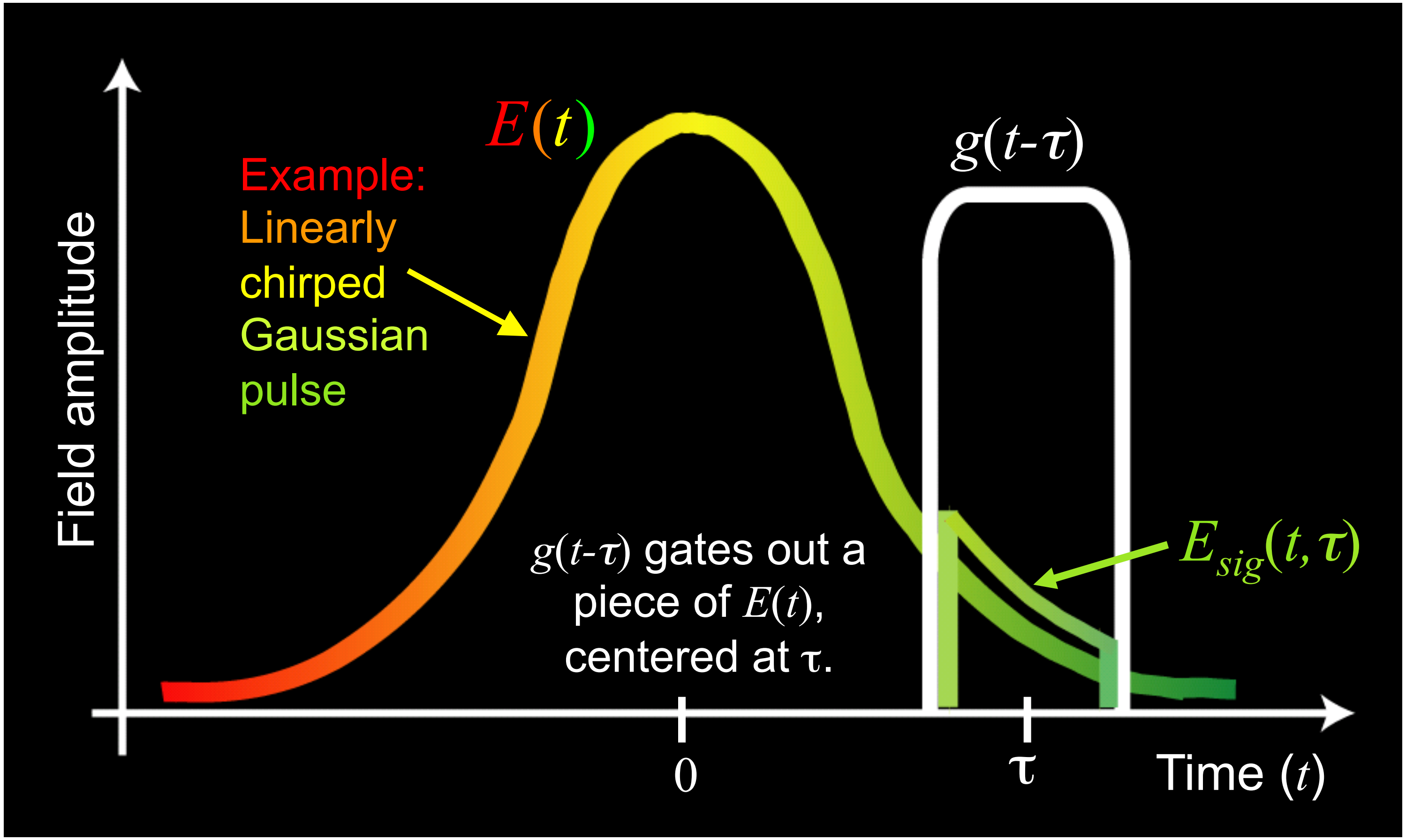
Without $g(t-t)$, $\Sigma_E(\omega,\tau)$ would simply be the spectrum

The spectrogram is a function of ω and t

It is the set of spectra of all temporal slices of $E(t)$

The spectrogram

We must compute the spectrum of the product: $E_{sig}(t,\tau) = E(t)\, g(t-\tau)$

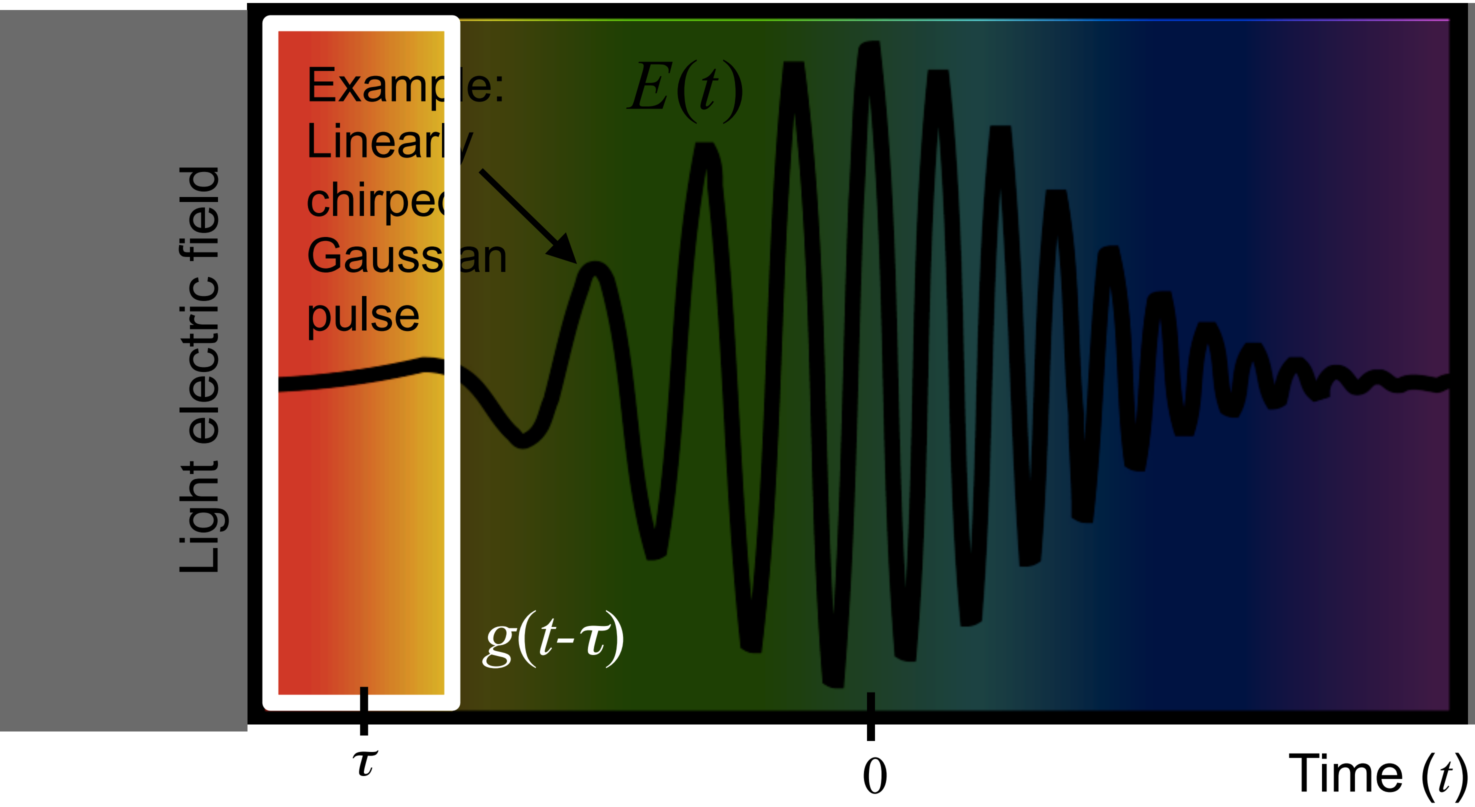


The spectrogram contains the color and intensity of $E(t)$ at each time t

The spectrogram

It's the spectrum of the product: $E(t)\ g(t\text{-}\tau)$:

$$\Sigma_E(\omega,\tau) \equiv \left| \int_{-\infty}^{\infty} E(t)\, g(t-\tau) \exp(-i\omega t)\, dt \right|^2$$

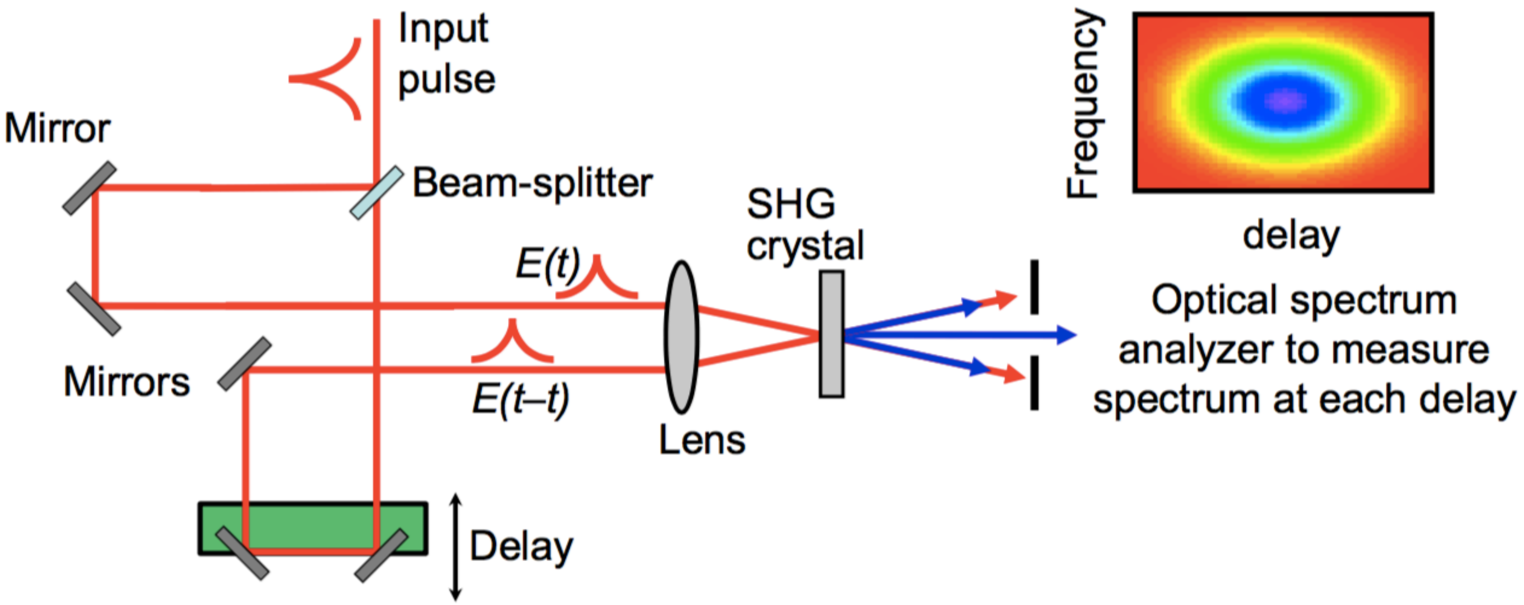


$g(t\text{-}t)$ gates out a portion of $E(t)$, centered at t.

The spectrogram yields the color and intensity of $E(t)$ at the time, t.

Frequency-Resolved Optical Gating (FROG): SHG-FROG

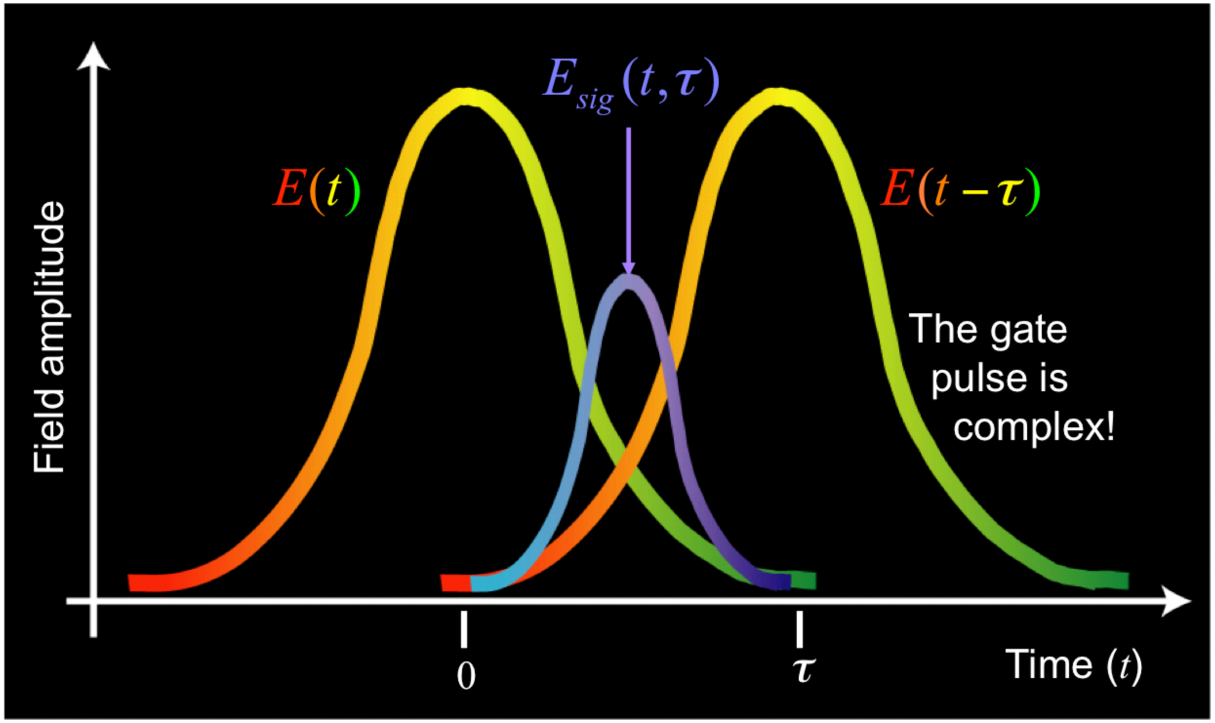
Background-free intensity autocorrelator + optical spectrum analyzer



FROG provides N X N data points. **With an iterative algorithm it is possible to retrieve both the amplitude and phase of the measured optical pulse.**

Frequency-Resolved Optical Gating (FROG): SHG-FROG

$$I_{FROG}(\omega,\tau) = \left| \int_{-\infty}^{\infty} E_{sig}(t,\tau)\exp(-i\omega t)\,dt \right|^2$$

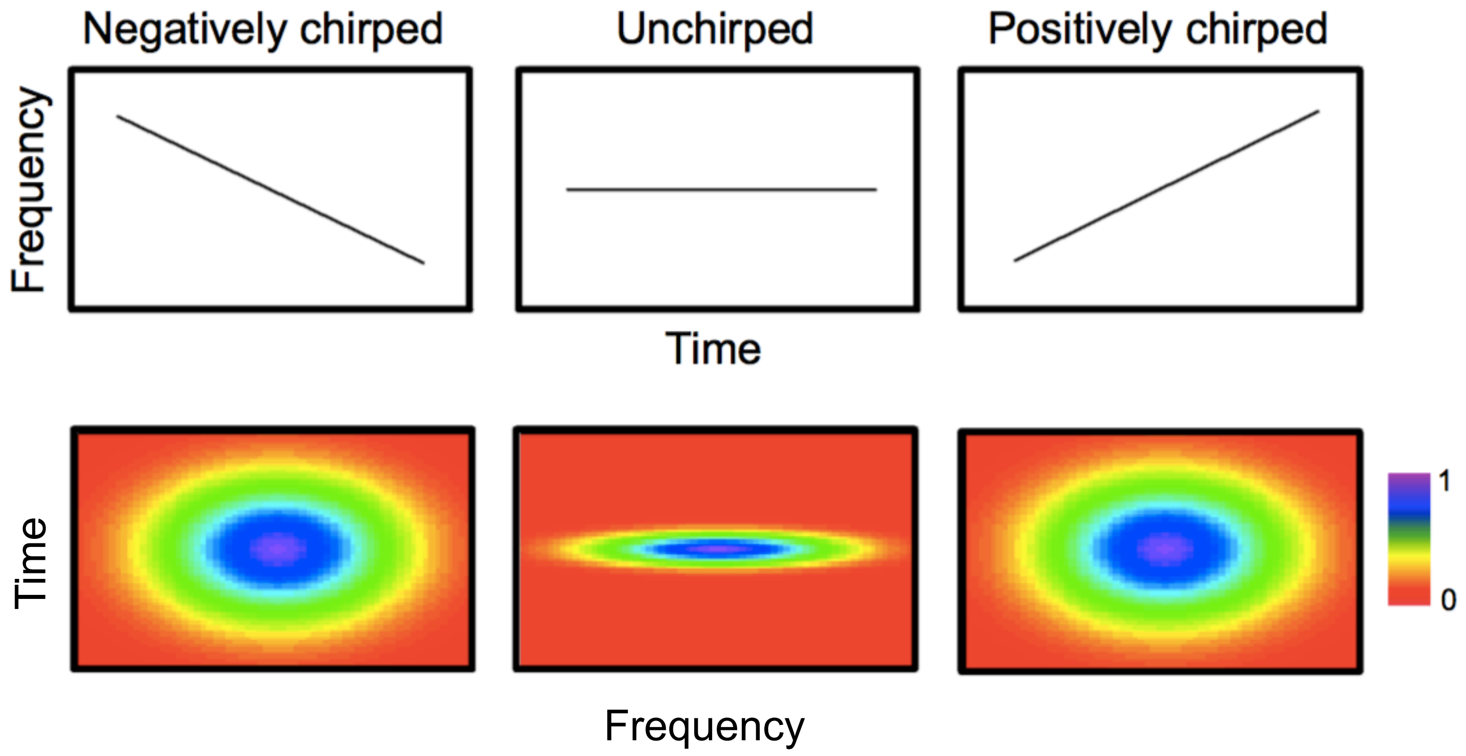


$E_{sig}(t,\tau) = E(t)\, g(t-\tau)$

$g(t-\tau) = E(t-\tau)$

Frequency-Resolved Optical Gating (FROG): SHG-FROG

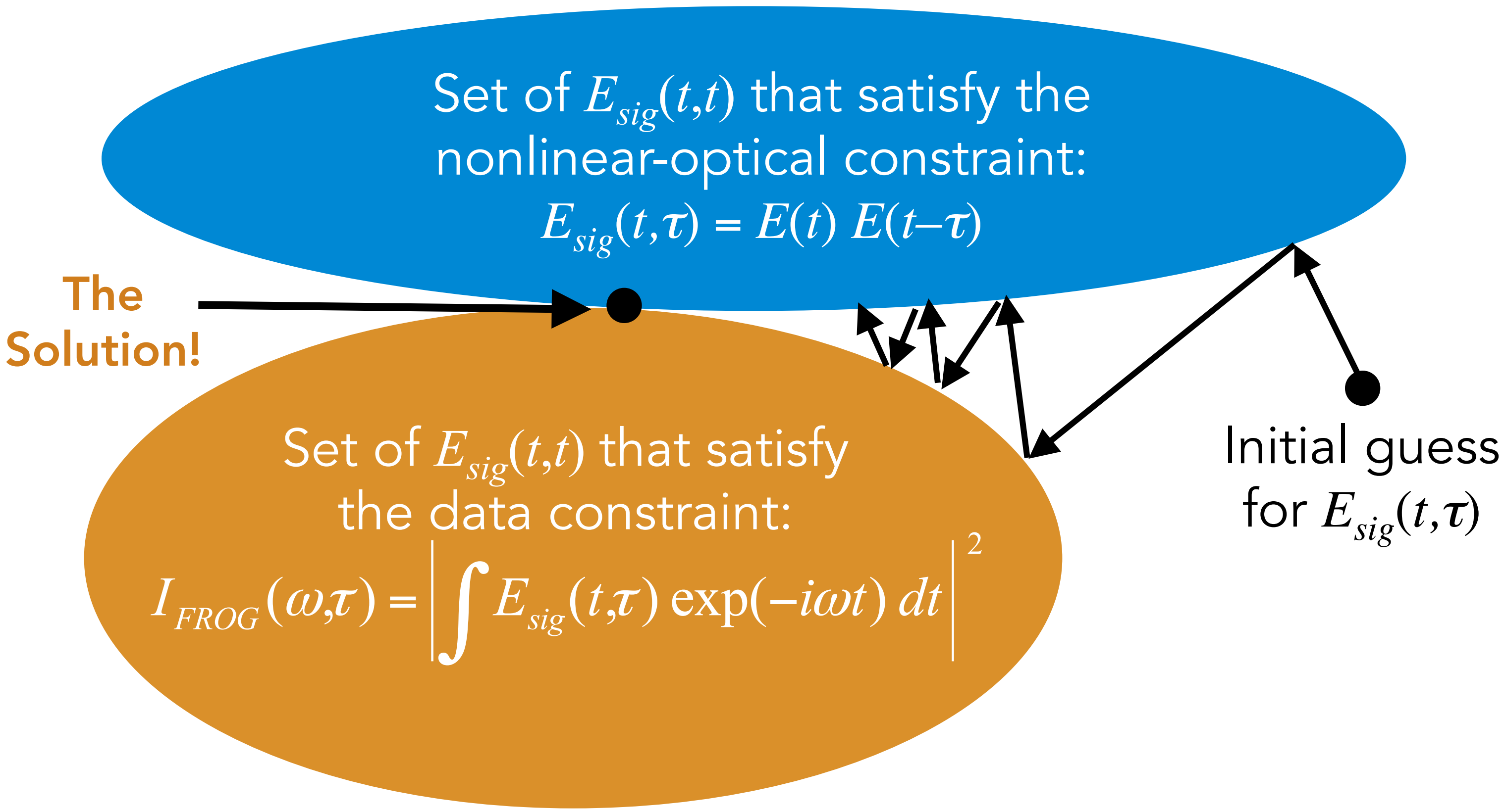
SHG FROG traces are symmetrical with respect to delay



SHG FROG has an ambiguity in the direction of time, but it can be removed

Generalized projections algorithm

$E(t)$ can be fully retrieved from the measured spectrogram by applying iterative reconstruction algorithms



Set of $E_{sig}(t,t)$ that satisfy the nonlinear-optical constraint:

$$E_{sig}(t,\tau) = E(t)\,E(t-\tau)$$

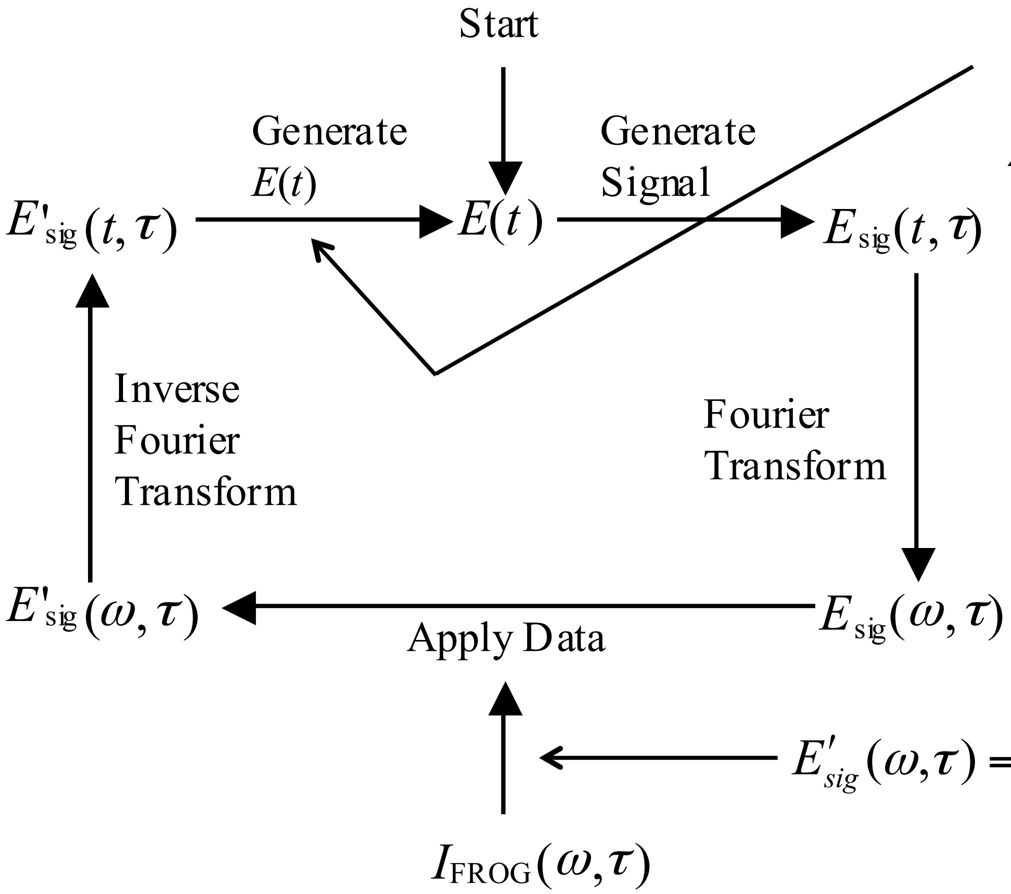
The Solution!

Set of $E_{sig}(t,t)$ that satisfy the data constraint:

$$I_{FROG}(\omega,\tau) = \left|\int E_{sig}(t,\tau)\exp(-i\omega t)\,dt\right|^2$$

Initial guess for $E_{sig}(t,\tau)$

FROG algorithm



Minimize Z w.r.t. $E^{(k+1)}(t_i)$:

$$Z = \sum_{i,j=1}^{N} \left| E_{sig}^{(k)}(t_i,\tau_j) - E_{sig}^{(k+1)}(t_i,\tau_j) \right|^2$$

$$= \sum_{i,j=1}^{N} \left| E_{sig}^{(k)}(t_i,\tau_j) - E^{(k+1)}(t_i) \cdot \left| E^{(k+1)}(t_i-\tau_j) \right|^2 \right|^2$$

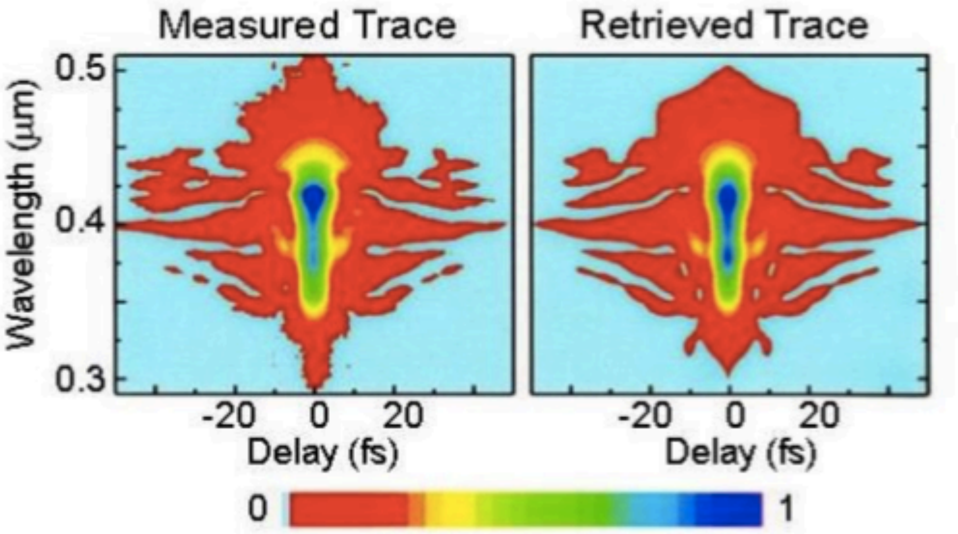
$$E'_{sig}(\omega,\tau) = \frac{E_{sig}(\omega,\tau)}{\left|E_{sig}(\omega,\tau)\right|}\sqrt{I_{FROG}(\omega,\tau)}$$

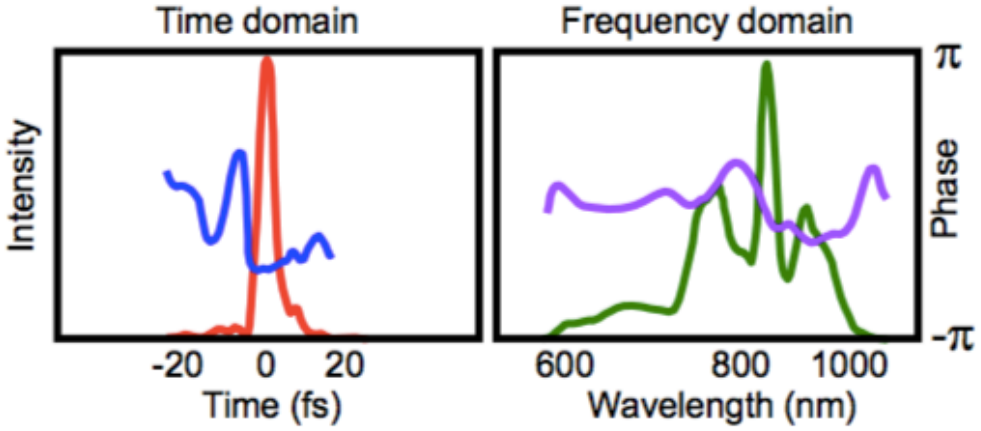
Find the value of μ that minimizes G.

Measure of fit quality, the "FROG Error":

$$G^{(k)} = \sqrt{\frac{1}{N^2}\sum_{i,j=1}^{N} \left| I_{\mathrm{FROG}}(\omega_i,\tau_j) - \mu\, I_{\mathrm{FROG}}^{(k)}(\omega_i,\tau_j) \right|^2}$$

SHG FROG measurement of a 4.5-fs pulse

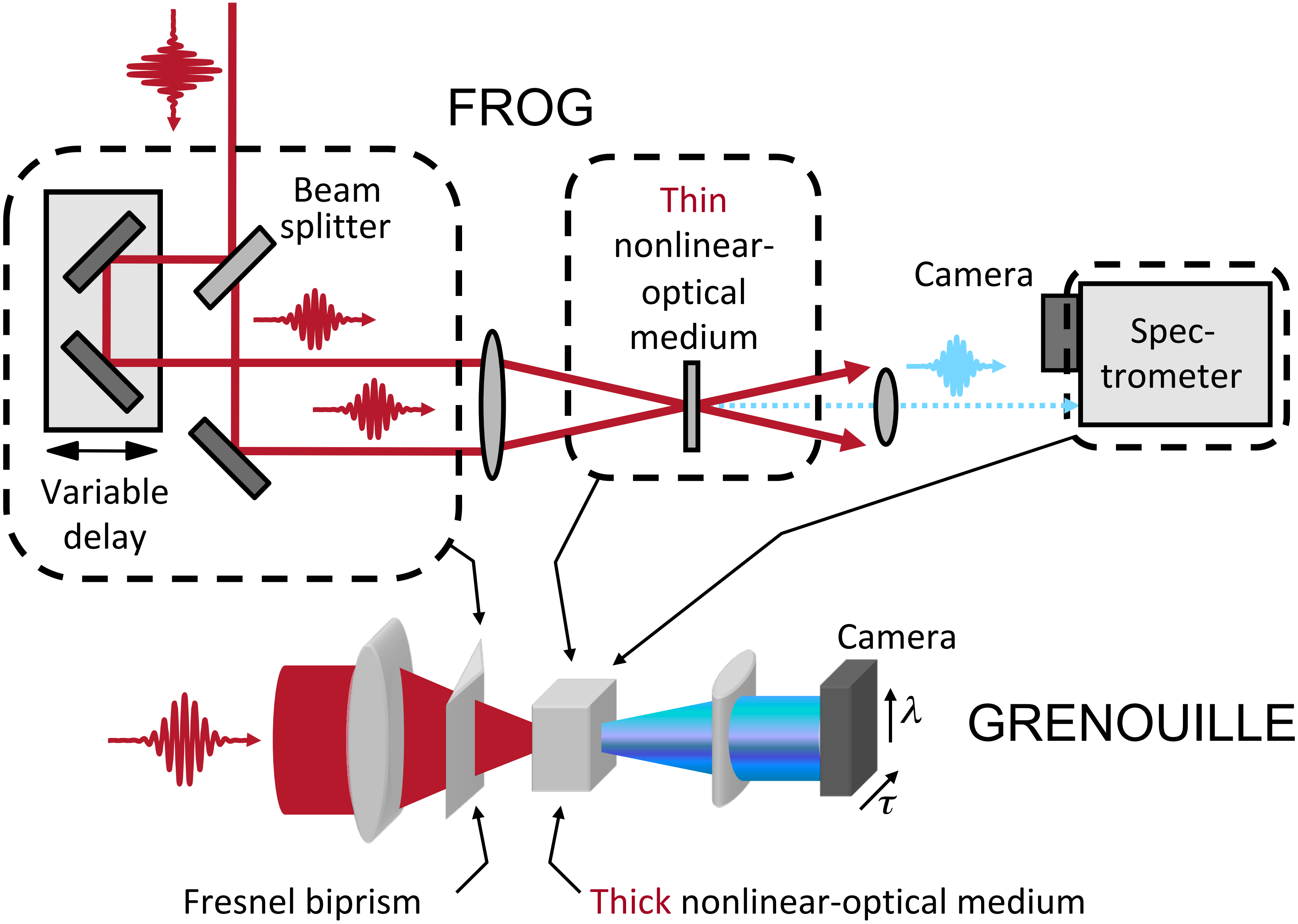




Agreement between the experimental and reconstructed FROG traces provides a nice check on the measurement.

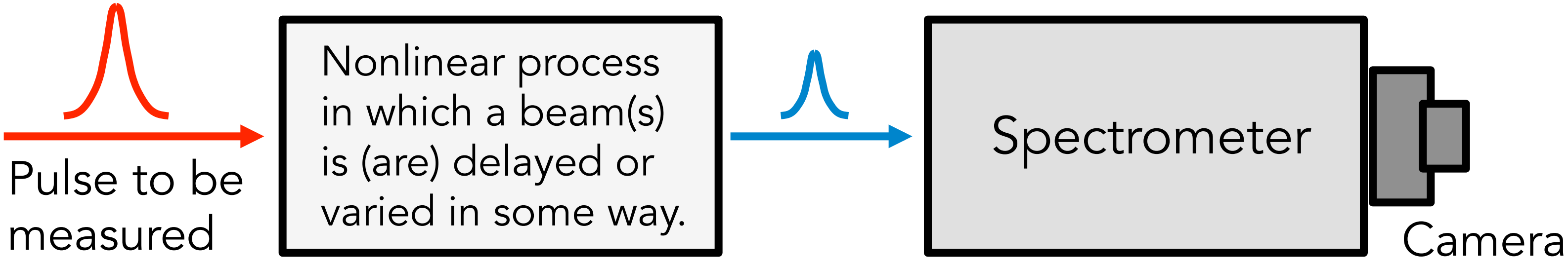
Baltuska, Pshenichnikov, and Weirsma, J. Quant. Electron., 35, 459 (1999).

GRating-Eliminated No-nonsense Observation of Ultrafast Incident Laser Light E-fields (GRENOUILLE)



FROG using arbitrary nonlinear-optical interactions

FROG is simply a frequency-resolved nonlinear-optical signal that's a function of time and delay (or another variable).



$$E_{sig}(t,\tau) = \begin{cases} E(t)E(t-\tau) & \text{SHG} \\ E(t)\left|E(t-\tau)\right|^2 & \text{PG} \\ E(t)^2 E^*(t-\tau) & \text{SD} \\ E(t)^2 E(t-\tau) & \text{THG} \end{cases}$$

Use any nonlinear-optical process that is fast enough.

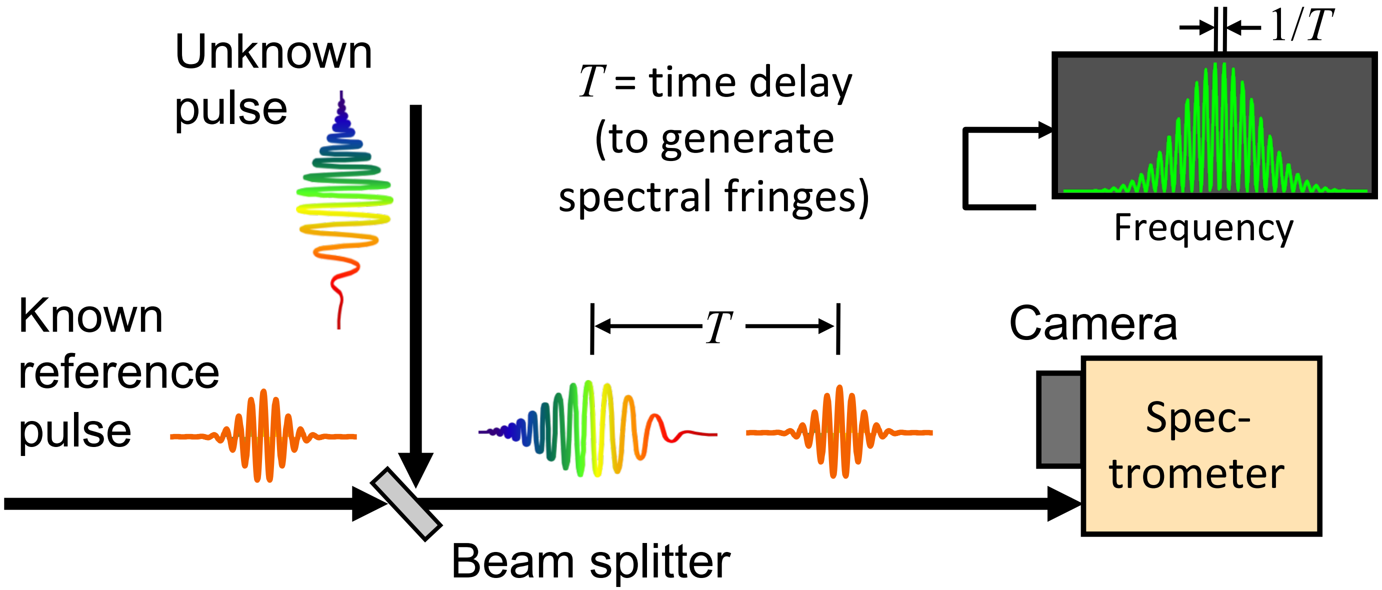
Pulse retrieval remains equivalent to the 2D phase-retrieval problem.

$$I_{FROG}(\omega,\tau) = \left| \int_{-\infty}^{\infty} E_{sig}(t,\tau) \exp(-i\omega t)\, dt \right|^2$$

Spectral interferometry

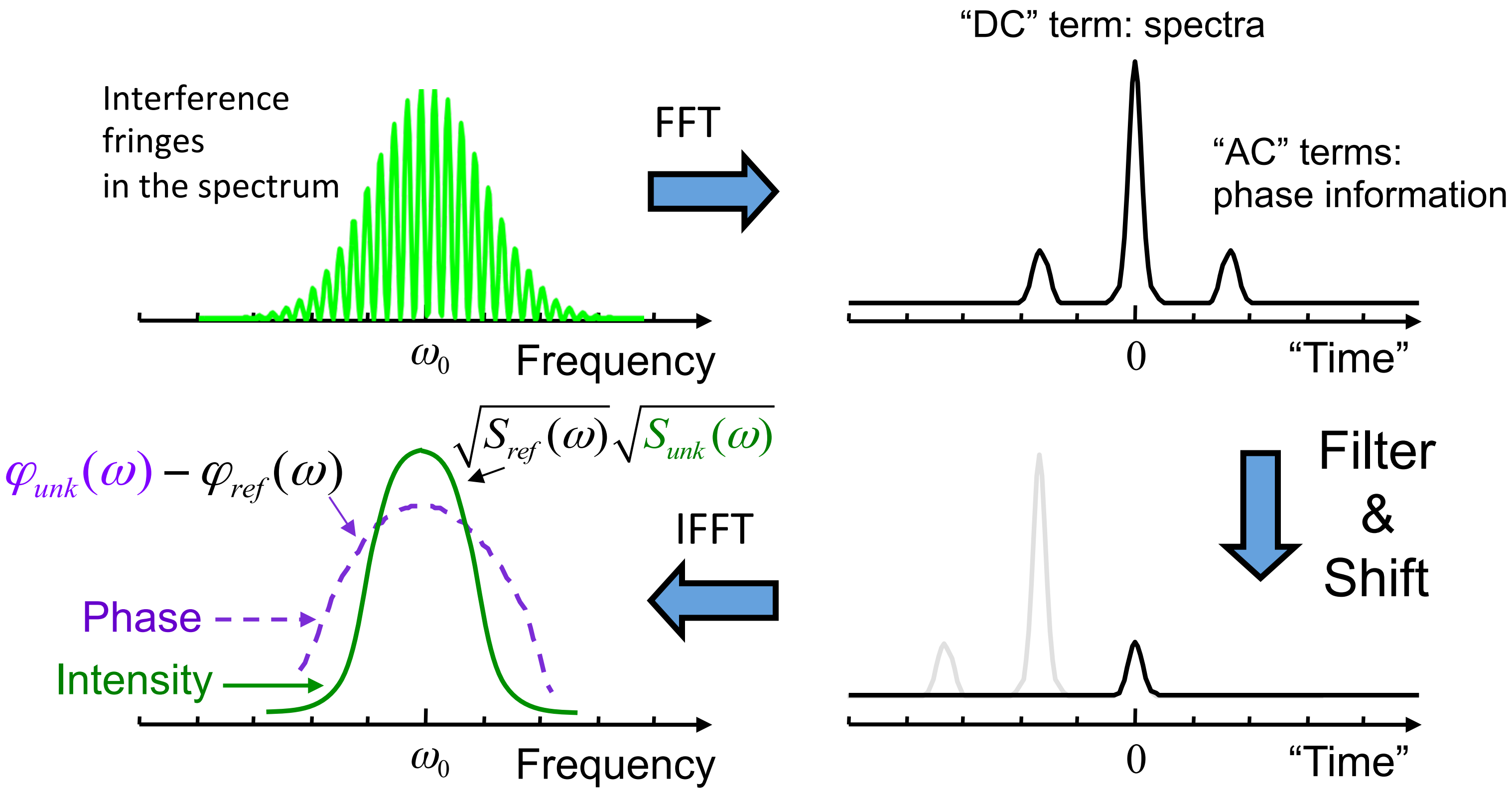
Measure the spectrum of the sum of a known and unknown pulse

Retrieve the unknown pulse from the spectral fringes



$$S_{SI}(\omega) = S_{ref}(\omega) + S_{unk}(\omega) + 2\sqrt{S_{ref}(\omega)}\sqrt{S_{unk}(\omega)}\cos[\varphi_{unk}(\omega) - \varphi_{ref}(\omega) + \omega T]$$

Spectral interferometry



This retrieval algorithm is quick, direct, and reliable

A reference pulse is usually not available!

Spectral Phase Interferometry for Direct Electric-Field Reconstruction (SPIDER)

If we perform spectral interferometry between a pulse and itself, the spectral phase cancels out. Perfect sinusoidal fringes always occur:

$$S_{SI}(\omega) = S_{unk}(\omega) + S_{unk}(\omega) + 2\sqrt{S_{unk}(\omega)}\sqrt{S_{unk}(\omega)}\cos[\cancel{\varphi_{unk}(\omega)} - \cancel{\varphi_{unk}(\omega)} + \omega T]$$

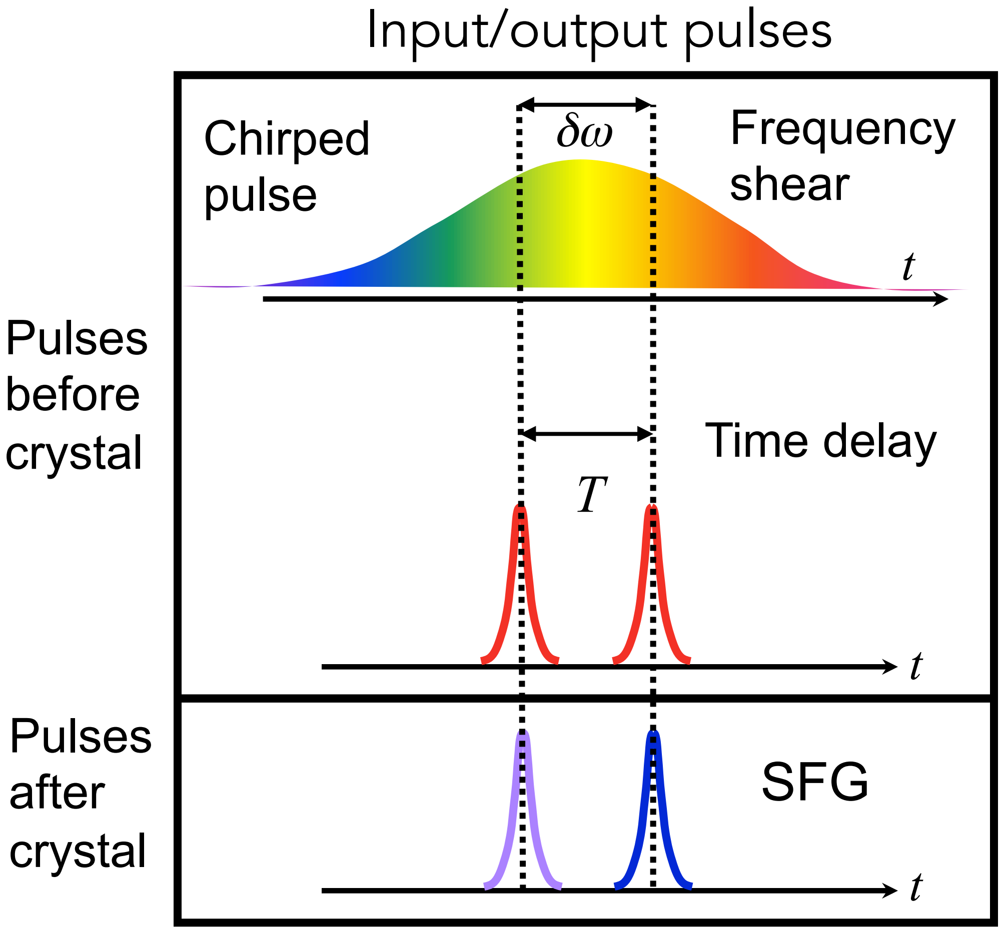
However if we frequency shift one pulse replica compared to the other:

$$S_{SI}(\omega) = S(\omega) + S(\omega+\delta\omega) + 2\sqrt{S(\omega)}\sqrt{S(\omega+\delta\omega)}\cos[\varphi(\omega+\delta\omega) - \varphi(\omega) + \omega T]$$

$$\phi_{SPIDER} = \varphi(\omega+\delta\omega) - \varphi(\omega) + \omega T = \overbrace{\delta\omega}^{\textbf{frequency shear}} \overbrace{\frac{d\varphi}{d\omega}}^{\textbf{group delay vs. }\omega} + \overbrace{\omega T}^{\textbf{Time delay}}$$

This measures the derivative of the spectral phase (the group delay)

Spectral Phase Interferometry for Direct Electric-Field Reconstruction (SPIDER)

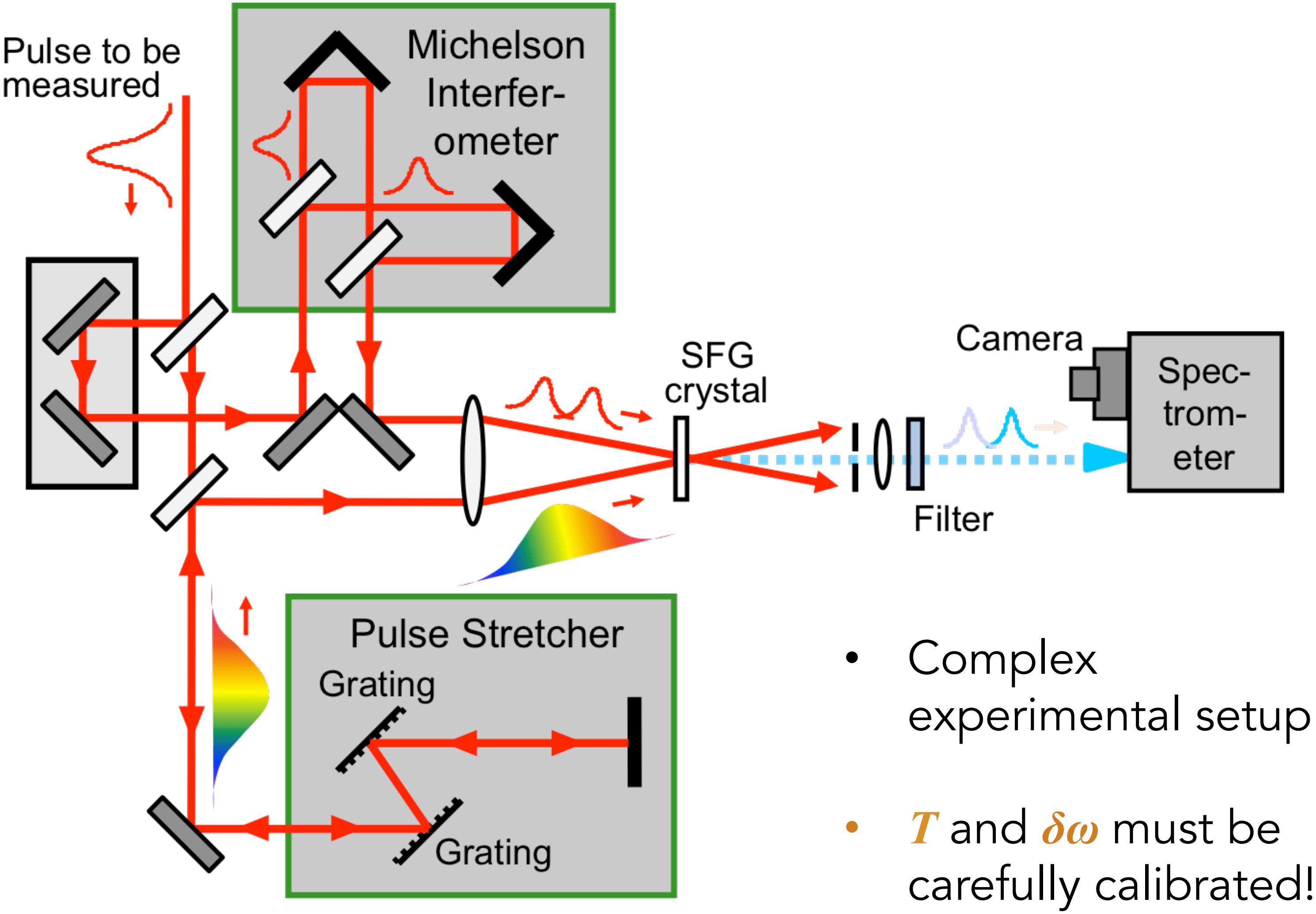


1) Make a very chirped pulse

2) Create two replicas of the pulse

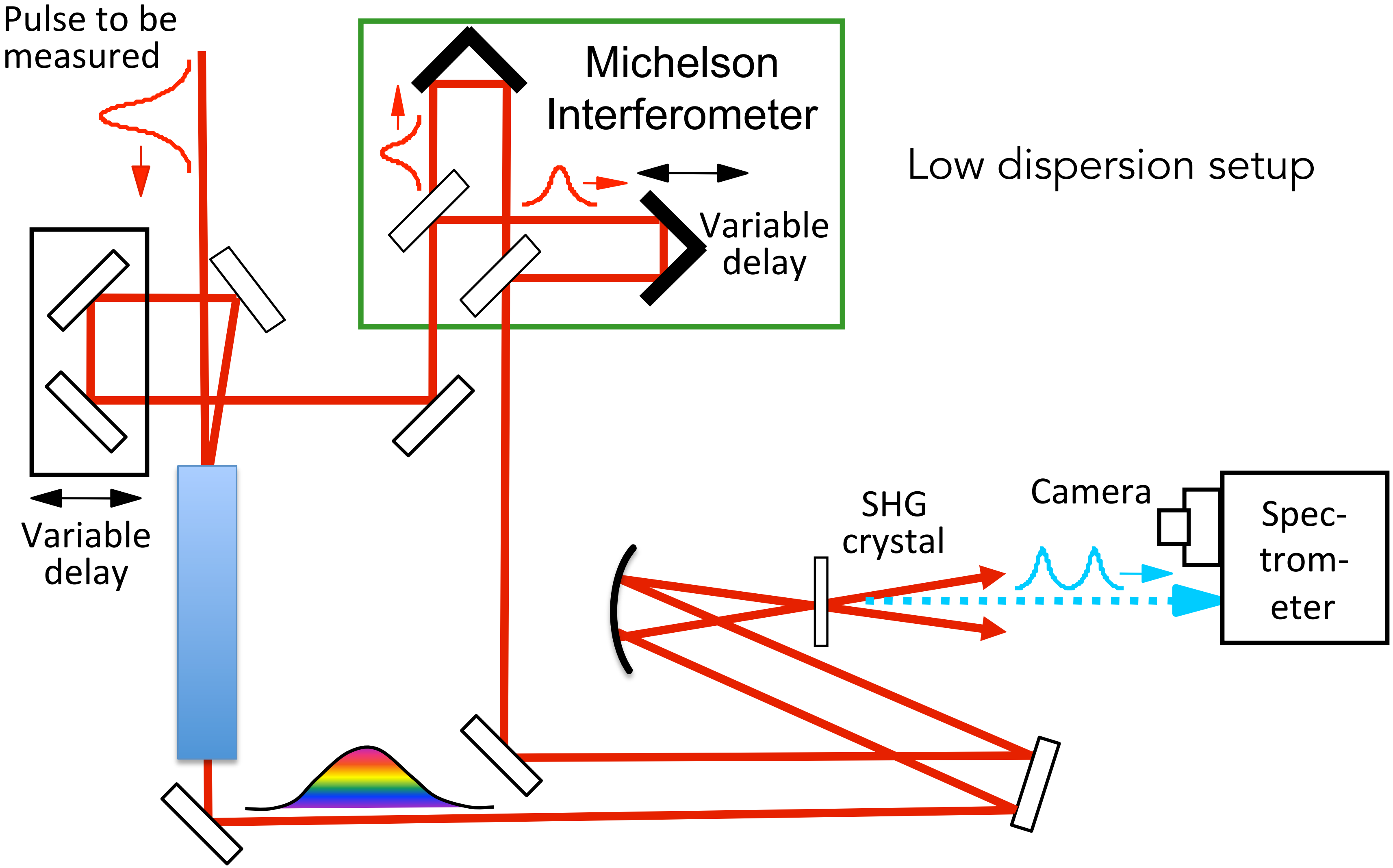
3) Frequency shift the 2 replicas by SFG with the broadband pulse and perform SI

Spectral Phase Interferometry for Direct Electric-Field Reconstruction (SPIDER)



- Complex experimental setup
- T and $\delta\omega$ must be carefully calibrated!

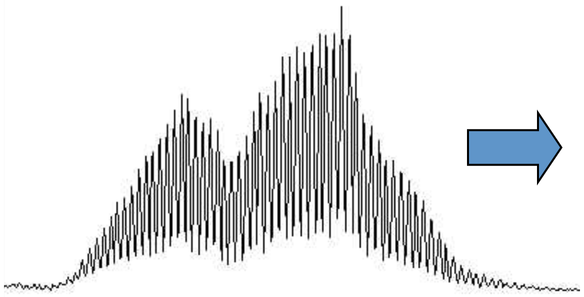
ZAP-SPIDER


Pulse to be measured
Michelson Interferometer
Variable delay
Low dispersion setup
Variable delay
SHG crystal
Camera
Spec-trom-eter

Spectral Phase Interferometry for Direct Electric-Field Reconstruction (SPIDER)

Extraction of the spectral phase

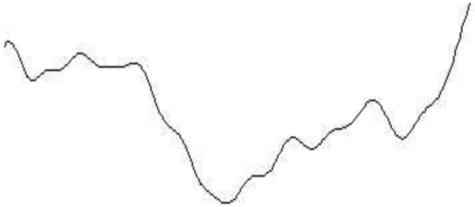
Measurement of the interferogram



Extraction of their spectral phase difference using spectral interferometry

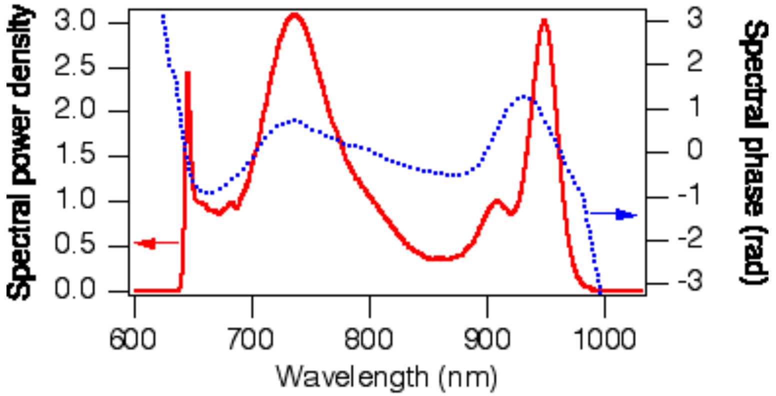
$\varphi(\omega+\delta\omega)-\varphi(\omega)$

Integration of the phase

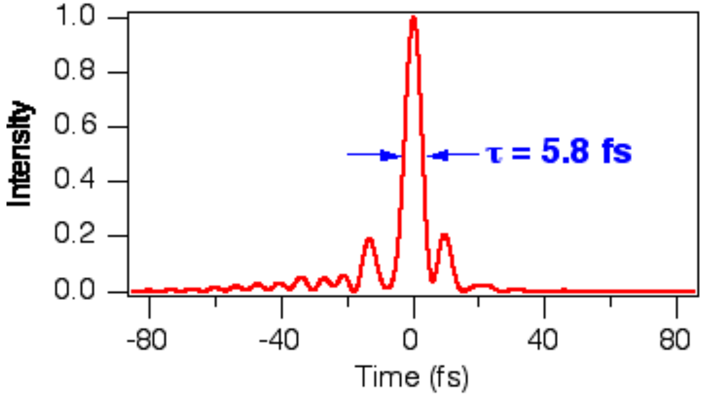


$\varphi(\omega)$

Frequency domain



Time domain

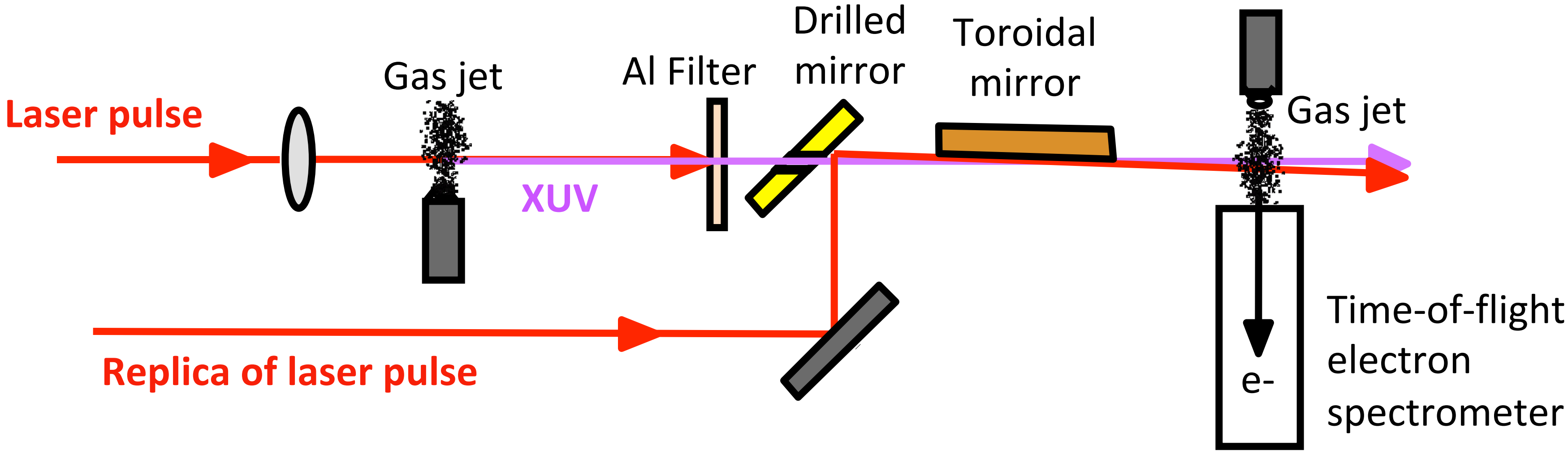


Many more methods exist…

- 2DSI: Two Dimensional Spectral Shearing Interferometer
- STRUT: Spectrally and Temporally Resolved Upconversion Technique
- TURTLE: Tomographic Ultrafast Retrieval of Transverse Light *E* fields Reconstruction
- TADPOLE: Temporal Analysis by Dispersing a Pair Of Light E-fields

FROG for Complete Reconstruction of Attosecond Bursts (FROG CRAB)

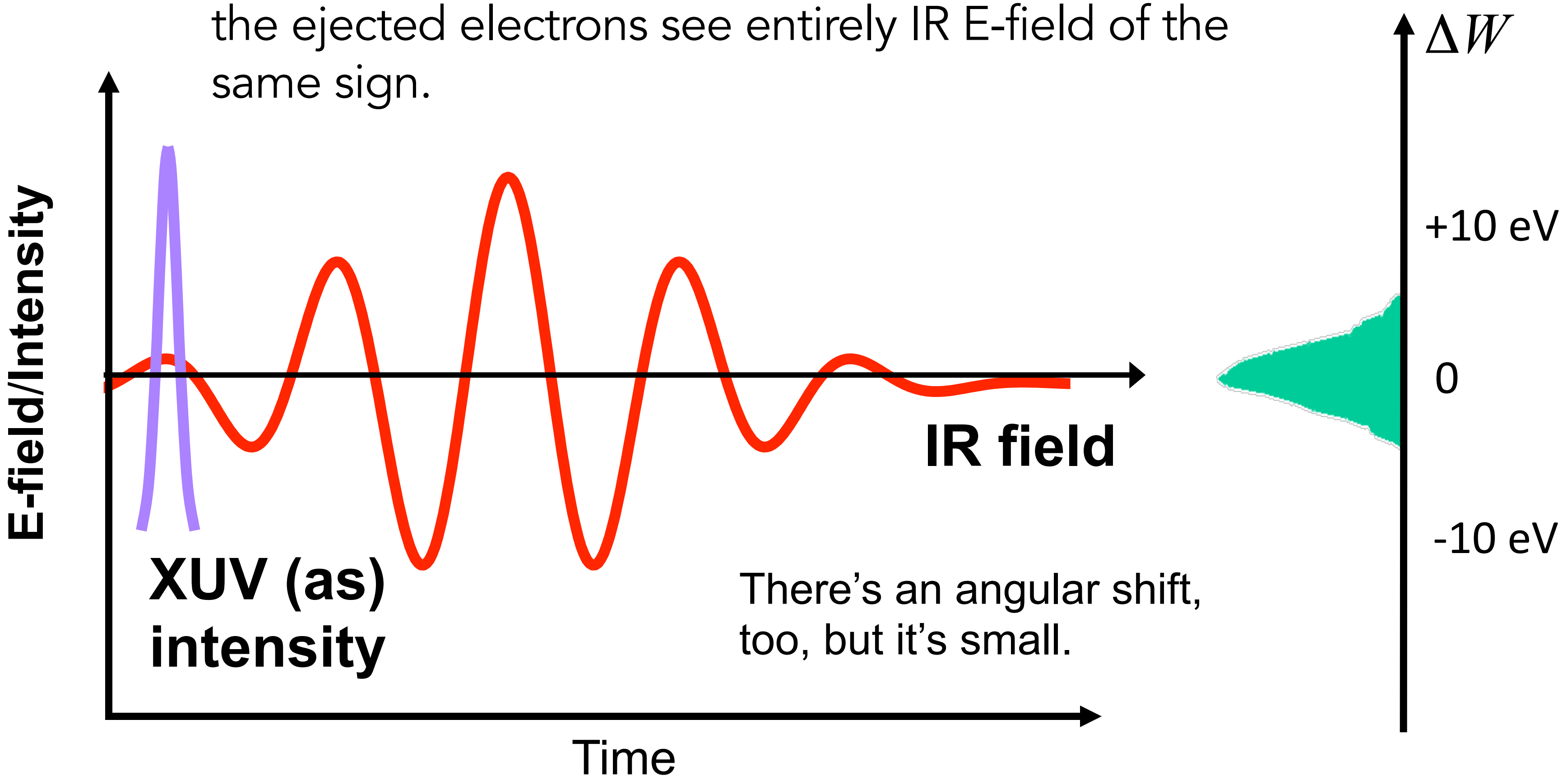
Use a second gas jet and photoionization to produce a cross-correlation with the input pulse



Energy-resolve the photoelectrons to generate a spectrally resolved cross-correlation. This generates a type of XFROG trace, which yields the intensity and phase of the attosecond pulse.

FROG for Complete Reconstruction of Attosecond Bursts (FROG CRAB)

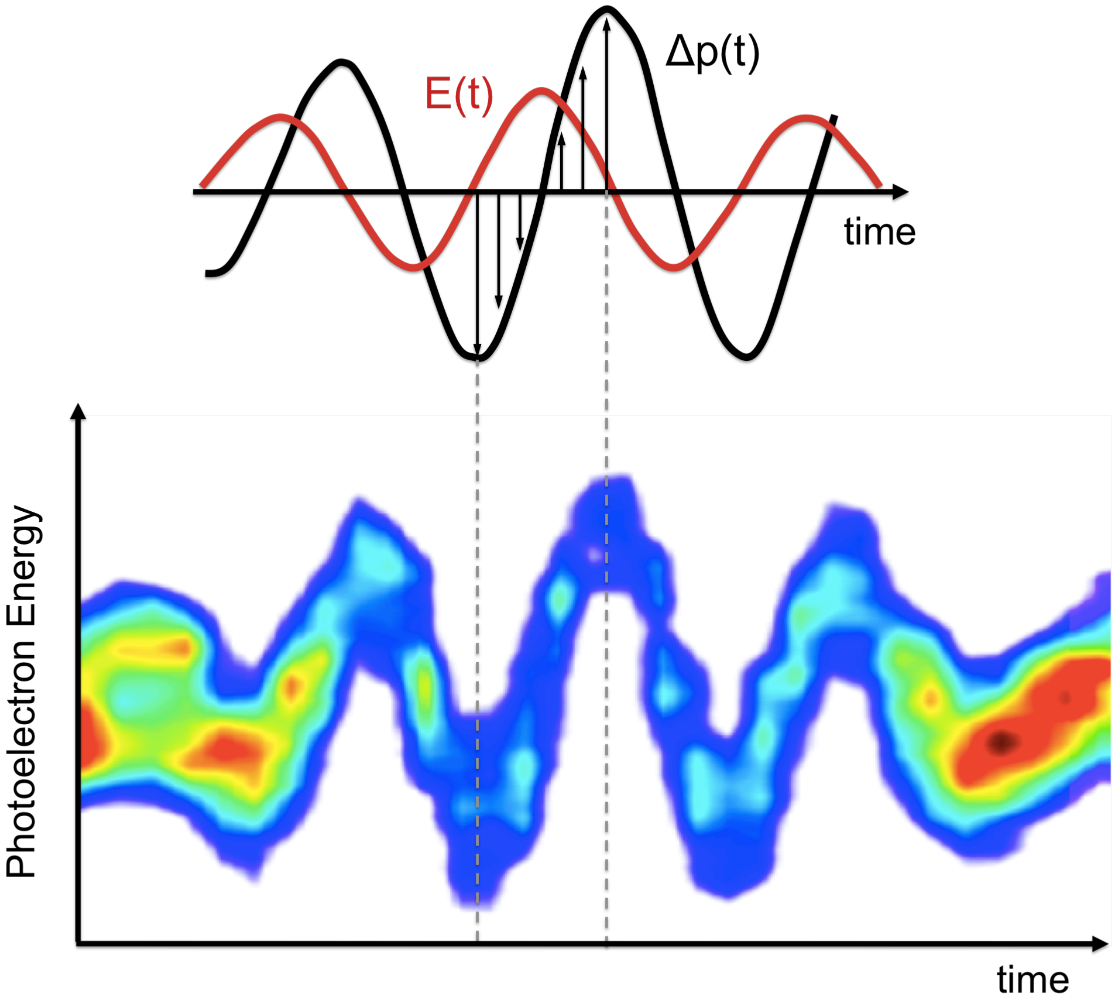
As the relative delay between the XUV pulse and the 800nm field varies, the **added energy** (ΔW) of the emitted **electron packet** will vary.



The added energy will be greatest or least when the ejected electrons see entirely IR E-field of the same sign.

There's an angular shift, too, but it's small.

FROG for Complete Reconstruction of Attosecond Bursts (FROG CRAB)



FROG for Complete Reconstruction of Attosecond Bursts (FROG CRAB)

$$S(\vec{\mathrm{v}},\tau) = \left| \int \exp(i\Phi(t))\vec{d}_{\vec{p}-\vec{A}(t)} \cdot \vec{E}_{XUV}(t-\tau)\exp(i(W+I_p)t)\,dt \right|^2$$

$$\Phi(t) = -\int_t^{+\infty} dt' \left[\vec{\mathrm{v}}\cdot\vec{\mathrm{A}}(t') + \vec{\mathrm{A}}^2(t')/2\right] =$$

$$= -\int_t^{+\infty} dt' U_p(t') + \frac{\sqrt{8WU_p}}{\omega}\cos\theta\cos\omega t - (U_p/2\omega)\sin 2\omega t$$

In FROG CRAB the gate function is a modulation of the phase of the electronic wave packet: phase gate!

$$\Sigma_E(\omega,\tau) \equiv \left| \int_{-\infty}^{\infty} E(t)\, g(t-\tau) \exp(-i\omega t)\, dt \right|^2$$

FROG for Complete Reconstruction of Attosecond Bursts (FROG CRAB)

$$S(\vec{\mathrm{v}},\tau) = \left| \int \exp(i\Phi(t))\vec{d}_{\vec{p}-\vec{A}(t)} \cdot \vec{E}_{XUV}(t-\tau)\exp(i(W+I_p)t)\,dt \right|^2$$

$$\Phi(t) = -\int_t^{+\infty} dt'\left[\vec{\mathrm{v}} \cdot \vec{\mathrm{A}}(t') + \vec{\mathrm{A}}^2(t')/2\right] =$$

$$= -\int_t^{+\infty} dt' U_p(t') + \frac{\sqrt{8WU_p}}{\omega}\cos\theta\cos\omega t - (U_p/2\omega)\sin 2\omega t$$

$S(\mathbf{v},\tau) = I_{\text{FROG-CRAB}}$ spectrogram

$\Phi(t)$ = phase of electron wavepacket modulated by the external IR field

$\mathbf{A}(t)$ = vector potentialof the IR pulse

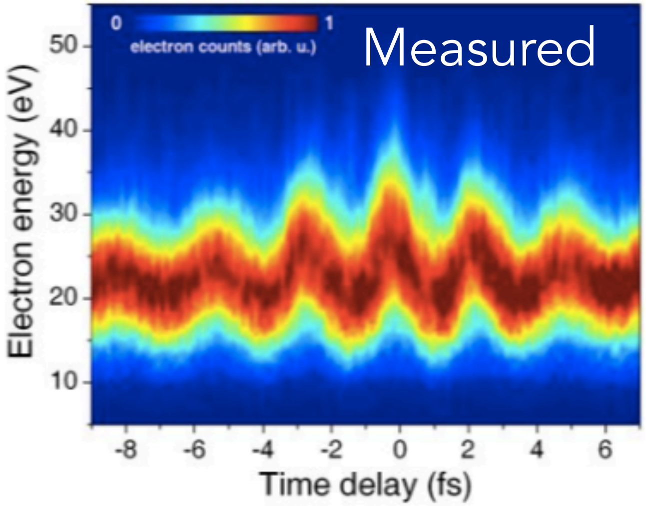
W = kinetic energy of the ejected electron

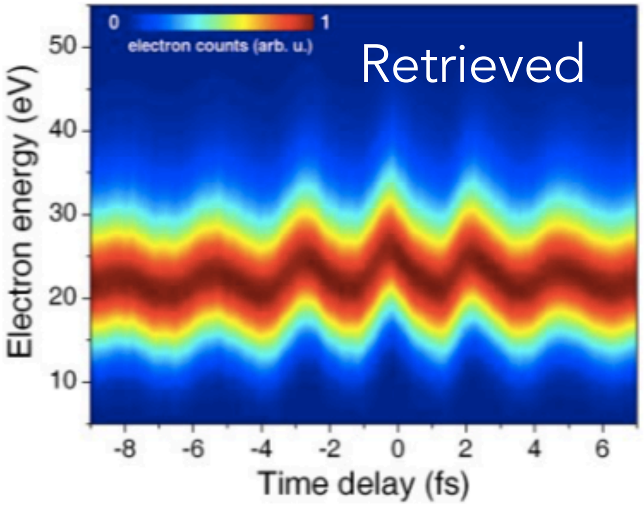
ω = frequency of the IR field

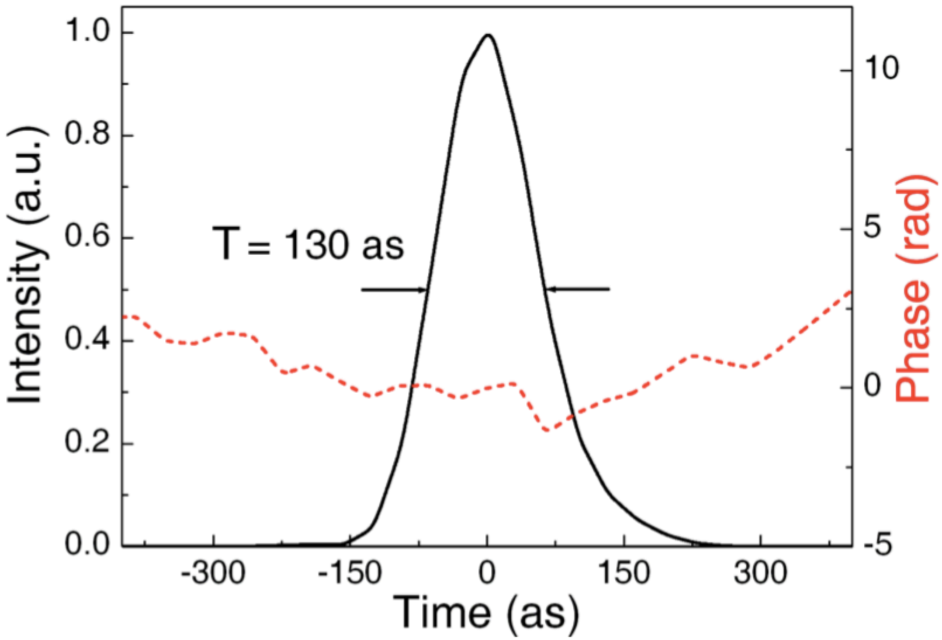
U_p = ponderomotive potential of the IR pulse

θ = angle between the electron velocity **v** and the vector potential **A**

FROG CRAB


Measured
0
1
electron counts (arb. u.)
Electron energy (eV)
50
40
30
20
10
-8
-6
-4
-2
0
2
4
6
Time delay (fs)


Retrieved
0
1
electron counts (arb. u.)
Electron energy (eV)
50
40
30
20
10
-8
-6
-4
-2
0
2
4
6
Time delay (fs)


T = 130 as
Intensity (a.u.)
1.0
0.8
0.6
0.4
0.2
0.0
Phase (rad)
10
5
0
-5
-300
-150
0
150
300
Time (as)

Bibliography

- C. Manzoni et al., Laser Photonics Rev. 9, No. 2, 129–171 (2015)
- R. Trebino et al., Rev. Sci. Instrum. 68 (9), 3277 (1997)
- C. Iaconis and I. A. Walmsley, Opt. Lett. 23(10), 792–794 (1998)
- Y. Mairesse and F. Quére, Phys. Rev. A 71, 011401(R) (2005)
- E. Goulielmakis, et al, Science 320, 1614 (2008)

Lecture slides from R. Trebino on the following website:
http://frog.gatech.edu/lectures.html