Radiation and radiation reaction in classical electrodynamics

75-80 minute

+ slides

In 1 hour and 15-20 minutes at the blackboard I did up to pg 18-19 without the derivation on pgs 7-9 and on pg. 12-16

Units and basic notation

Units with

$\hbar = c = 4\pi\epsilon_0 = 1$

↳ vacuum permittivity

are used unless explicitly indicated.

Thus, if

$e =$ electron charge $= -4.8 \times 10^{-10}$ statcoulomb

the fine-structure constant

$\alpha = e^2 \simeq \frac{1}{137}$

The electron mass is

$m = 9.1 \times 10^{-28}$ g

and the metric $g^{\mu\nu} = \mathrm{diag}(+1, -1, -1, -1)$ will be employed

I will often talk about a "charge" meaning an electron.
It is a well known fact that accelerated charges emit electromagnetic radiation. By emitting radiation, they loose energy at a rate (Larmor formula)

$$\frac{d\mathcal{E}}{dt} = \frac{2}{3} e^2 \dot{\vec{v}}^2$$ (in the non relativistic case)

$$\frac{d\mathcal{E}}{dt} = -\frac{2}{3}\frac{e^2}{m^2}\frac{dp^\mu}{ds}\frac{dp_\mu}{ds}$$ (relativistic case)

with

$$p^\mu = m\dot{x}^\mu = m(\gamma, \gamma\vec{v}) \qquad \gamma = \frac{1}{\sqrt{1-v^2}}$$

being the four-momentum of the charge and s being the proper time

$$\frac{dt}{ds} = \gamma \geq 1$$

Now, one usually describes the dynamics of an electron in a given electromagnetic field $F^{\mu\nu}(x)$ by means of the Lorentz equation

$$m \frac{du^\mu}{ds} = e F^{\mu\nu} u_\nu$$

The Lorentz equation, however, does not take into account the mentioned fact that the electron, while being accelerated, looses energy (and momentum) which changes its trajectory. The question then is: how can we modify the equation of motion of the electron in order to take into account this energy momentum loss? The only way to change the electron trajectory is via an electromagnetic field and, apart from the external electromagnetic field, the only other field we have is the field radiated by the electron. Thus, we have to account for the reaction of the field radiated by the electron on the dynamics of the electron itself (radiation reaction).

This means that we have to start from the general and complete set of equations describing an electron, its electromagnetic field and the background field generated by charges and currents outside the interaction region (the atoms producing the laser light via stimulated emission, for example)

These equations are

$\partial^\mu F_T^{\nu\rho} + \partial^\nu F_T^{\rho\mu} + \partial^\rho F_T^{\mu\nu} = 0$ (homogeneous Maxwell's equations)

$\partial_\mu F_T^{\mu\nu} = 4\pi J_e^\nu$ (In - " ")

$m_0 \frac{du^\mu}{ds} = e F_T^{\mu\nu} u_\nu$ (Lorentz equations)

Observations

1) $F_T^{\mu\nu}(x) = \underbrace{F^{\mu\nu}(x)}_{\text{external field}} + \overbrace{F_e^{\mu\nu}(x)}^{\text{field produced by the electron}}$

2) ~~the reason to~~ J_e^ν is the electron four-current (to be specified later)

3) the reason why we use "m_0" will be clear later

~~From the~~ The homogeneous Maxwell's equations are automatically satisfied by introducing the four-vector potential $A_T^\mu(x)$ such that

$F_T^{\mu\nu}(x) = \partial^\mu A_T^\nu(x) - \partial^\nu A_T^\mu(x)$

and we decide to work in the Lorenz gauge

$\partial_\mu A_T^\mu = 0$

such that the equations that we have to solve are

$$\begin{cases} \partial_\mu \partial^\mu A_T^\nu = 4\pi j_e^\nu \\ m_0 \dfrac{du^\mu}{ds} = e\left(\partial^\mu A_T^\nu - \partial^\nu A_T^\mu\right) u_\nu \end{cases}$$

We first decide to solve the Maxwell's equation by means of the Green's function method. The general solution of an inhomogeneous equation is given by the general solution of the homogeneous equation (which will describe the external field in our case) plus a special solution of the inhomogeneous equation (which will describe the field produced by the electron):

$$A_T^\mu(x) = A^\mu(x) + A_e^\mu(x)$$

and we know that we can write

$$A_e^\mu(x) = \int d^4y \, G(x, y)\, 4\pi j^\mu(y)$$

where

~~The reason is that, the energy radiated per unit time through a surface Σ is given by~~

$$\cancel{\frac{d\mathcal{E}_\Sigma}{dt} = \frac{1}{4\pi}\int_\Sigma \underbrace{d\vec{S}}_{R^2} \cdot \big(\underbrace{\vec{E}}_{\frac{1}{R}} \times \underbrace{\vec{B}}_{\frac{1}{R}}\big)}$$

→ 4bis

~~In general, the field produced by a given four-current density $j^\mu(x)$ can be determined by first evaluating the Green's function of the wave operator~~

$$\partial_\mu \partial^\mu G(x,y) = \delta^{(4)}(x-y)$$

It is clear that $G(x,y) = G(x-y)$ and we can seek a solution of the form

$$G(x-y) = \int \frac{d^4k}{(2\pi)^4}\, \bar{G}(k)\, e^{-ik(x-y)}$$

then

$$\int \frac{d^4k}{(2\pi)^4} \left(-k^2\right) \bar{G}(k)\, e^{-ik(x-y)} = \int \frac{d^4k}{(2\pi)^4}\, e^{-ik(x-y)}$$

which implies

$$\tilde{G}(k) = -\frac{1}{k^2} \quad \text{and} \quad G(x-y) = -\int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2} e^{-ik(x-y)}$$

It is clear that the integral in k^0 has two poles at $k^0 = \pm\sqrt{|\vec{k}|^2}$ (recall that $k^2 = k^{0\,2} - \vec{k}^2$) and, in order to perform the integral, we have to provide prescription to avoid the poles. We are interested here in the so-called "retarded" Green's function $G_R(x-y)$. Let us motivate this choice physically. Once we determine the Green's function, the solution of $A^\mu_{e,K}(x)$ of the wave equation will be

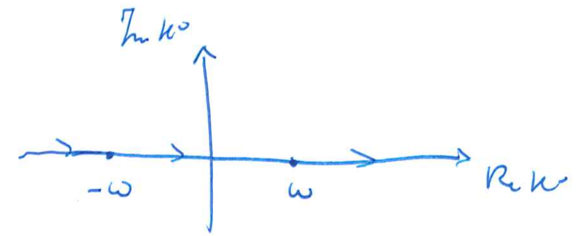
$$A^\mu_{e,K}(x) = 4\pi \int d^4y \, G_R(x-y) J^\mu_e(y)$$

As we will see, the retarded Green's function will ensure that the field at all time x^0 will be produced by the value of the current at "earlier" times y^0 as we would expect from causality considerations.

We have to evaluate the integral

$$I = \int \frac{dk^0}{2\pi} \frac{1}{k^{0\,2} - \vec{k}^2} e^{-ik^0(x^0-y^0)} = \text{by fixing } \omega = |\vec{k}| = \int \frac{dk^0}{(2\pi)} \frac{1}{k^{0\,2} - \omega^2} e^{-ik^0(x^0-y^0)}$$

Let's ~~consid~~ imagine to work in the complex k^0 plane



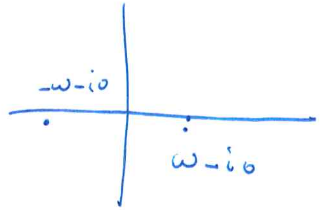
From what we said, we want the Green's function to vanish if $x^0-y^0<0$. Now, in this case

$$-i k^0(x^0-y^0) = +i\,(\mathrm{Re}\,k^0 + i\,\mathrm{Im}\,k^0)\,|x^0-y^0| = i(\mathrm{Re}\,k^0)\,|x^0-y^0| - (\mathrm{Im}\,k^0)\,|x^0-y^0|$$

Thus we can add to the integral along the real axis, an integral along a infinite semicircle with $\mathrm{Im}\,k^0>0$ without changing the value of the integral because the value of the integral along the semicircle is exponentially suppressed

$$\oint dk^0 \frac{e^{-i k^0(x^0-y^0)}}{k^{0\,2}-\omega^2} = \int_{(\mathbb{R})} dk^0 \frac{e^{-i k^0(x^0-y^0)}}{k^{0\,2}-\omega^2} + \int_{(\frown)} dk^0 \, \cancel{\frac{e^{i(\mathrm{Re}\,k^0)|x^0-y^0|}}{k^{0\,2}-\omega^2}}\, e^{-(\mathrm{Im}\,k^0)|x^0-y^0|}$$

Since the integral along the closed path can be evaluated by means of the Cauchy theorem and since we want it to be zero, we have to shift the poles ~~towards the negative~~ "below"



and $I \to \frac{1}{\pi}\int \frac{dk^0}{2\pi} \frac{1}{(k^0+i0)^2-\omega^2} e^{-ik^0(x^0-y^0)}$

then

$k^0 = \pm\omega - i0 \Rightarrow$ $k^{0\,2} = \omega^2 \mp (\mathrm{Sgn}\,\omega)\, i0$ $(k^0+i0)^2 = \omega^2 =$ $\cancel{k^{0\,2} + i k^0 0 = \omega^2 \Rightarrow k^{0\,2} - \omega^2 - i k^0 0}$

If now, $x^0-y^0>0$, we can evaluate the integral by closing the contours on the half plane

$\mathrm{Im}\, k^0 < 0$. In this case we have two poles and then

$$I_R = \vartheta(x^0-y^0)\,(-2\pi i)\,\frac{1}{2\pi}\left[\frac{1}{k^0+\omega}\,e^{-ik^0(x^0-y^0)}\Big|_{k^0=\omega} + \frac{1}{k^0-\omega}\,e^{-ik^0(x^0-y^0)}\Big|_{k^0=-\omega}\right] = \quad (x)$$

$$= \vartheta(x^0-y^0)\,\frac{1}{2i\omega}\left[e^{-i\omega(x^0-y^0)} - e^{i\omega(x^0-y^0)}\right] =$$

$$= \vartheta(x^0-y^0)\left(-\frac{1}{\omega}\right)\sin\omega(x^0-y^0)$$

and

$$G_R(x-y) = \int\frac{d^3k}{(2\pi)^3}\,\vartheta(x^0-y^0)\,\frac{1}{\omega}\,e^{i\vec{k}\cdot(\vec{x}-\vec{y})}\sin\omega(x^0-y^0) =$$

$$\cancel{-\int_0^\infty\frac{\omega^2 d\omega\, d\Omega}{(2\pi)^3}\,\vartheta(x^0-y^0)\sin\omega t}$$

$$\cancel{= \int\frac{d^3k}{(2\pi)^3}\,\frac{\vartheta(x^0-y^0)}{\omega}\,\frac{e^{i\omega(x^0-y^0)} - e^{-i\omega(x^0-y^0)}}{\omega}\,e^{i\vec{k}\cdot(\vec{x}-\vec{y})}} =$$

= by setting the z-axis in $\vec{k}$ along $\vec{x}-\vec{y}$ (the integral is a scalar so we can rotate the axes)

$=$ by changing $\vec{k} \to -\vec{k}$ in the first integral

$$= \vartheta(x^0-y^0)\int_0^\infty \frac{\omega^2 d\omega}{(2\pi)^3}\,\frac{1}{\omega}\,\sin\omega(x^0-y^0)\underbrace{\int d\Omega}_{=\int \sin\vartheta\, d\vartheta\, d\varphi}\, e^{i\omega|\vec{x}-\vec{y}|\cos\vartheta} =$$

$$= \vartheta(x^0-y^0)\int_0^\infty \frac{\omega\, d\omega}{(2\pi)^3}\,\sin\omega(x^0-y^0)\int_{-1}^{1} dq\; 2\pi\; e^{i\omega|\vec{x}-\vec{y}|q} =$$

$$= \vartheta(x^0-y^0)\int_0^\infty \cancel{\omega}\frac{d\omega}{4\pi^2}\,\sin\omega(x^0-y^0)\,\frac{1}{i\cancel{\omega}|\vec{x}-\vec{y}|}\left(e^{i\omega|\vec{x}-\vec{y}|}-e^{-i\omega|\vec{x}-\vec{y}|}\right) =$$

$$= \vartheta(x^0-y^0)\,\frac{1}{4\pi^2 i|\vec{x}-\vec{y}|}\left[\int_0^\infty d\omega\,\sin\omega(x^0-y^0)\,e^{i\omega|\vec{x}-\vec{y}|} + \int_0^{-\infty} -d\omega\,(-1)\sin\omega(x^0-y^0)\,(-1)\,e^{i\omega|\vec{x}-\vec{y}|}\right] =$$

$$= \vartheta(x^0-y^0)\,\frac{1}{4\pi^2 i|\vec{x}-\vec{y}|}\int_{-\infty}^{\infty} d\omega\,\sin\omega(x^0-y^0)\,e^{i\omega|\vec{x}-\vec{y}|} =$$

$$\vartheta(x^0-y^0)\frac{1}{4\pi^2 i|\vec{x}-\vec{y}|}\left[\frac{1}{2i}\int_{-\infty}^{\infty}d\omega\, e^{i\omega(x^0-y^0)}e^{i\omega|\vec{x}-\vec{y}|}-\frac{1}{2i}\int_{-\infty}^{\infty}d\omega\, e^{-i\omega(x^0-y^0)}e^{i\omega|\vec{x}-\vec{y}|}\right]=$$

$$=\vartheta(x^0-y^0)\frac{1}{4\pi^2|\vec{x}-\vec{y}|}\cdot\frac{1}{2}2\pi\,\delta(x^0-y^0-|\vec{x}-\vec{y}|)=$$

$$=\frac{1}{4\pi}\vartheta(x^0-y^0)\frac{\delta(x^0-y^0-|\vec{x}-\vec{y}|)}{|\vec{x}-\vec{y}|}=\frac{1}{2\pi}\vartheta(x^0-y^0)\,\delta((x-y)^2)$$

Analogously, one can calculate the "advanced" Green's function

$$G_A(x-y)=\frac{1}{2\pi}\vartheta(y^0-x^0)\,\delta((x-y)^2)=\frac{1}{4\pi}\vartheta(y^0-x^0)\frac{\delta(x^0-y^0+|\vec{x}-\vec{y}|)}{|\vec{x}-\vec{y}|}$$

At this point, in order to calculate the field produced by a charge ~~we need~~

$$A^\mu_{e,R}(x)=4\pi\int d^4y\, G_R(x-y)\,J^\mu_e(y)$$

We need the current density associated to a pointlike particle moving along a given trajectory $\vec{r} = \vec{r}(t)$:

$$j^\mu_e(x) = (\rho_e, \rho_e \vec{v}) = e\, \delta^{(3)}(\vec{r} - \vec{r}(t))\, [1, \vec{v}]$$

Now, we know that relativistically, if s is the proper time of the particle then

$$\frac{dt}{ds} = \gamma \geq 1$$

which means that we can parametrize the curve $\vec{r} = \vec{r}(t)$ as $\begin{matrix} t = t(s) \\ \vec{r} = \vec{r}(s) \end{matrix}$ because to each s corresponds one and only one t (depending on the initial conditions, which are not important here). Then,

$$j^\mu_e(x) = \int dt\, \delta(t - t(s))\, e\, \delta^{(3)}(\vec{r} - \vec{r}(s))\, (1, \vec{v}(s)) =$$

$$= e \int ds\, \gamma\, \delta(t - t(s))\, \delta^{(3)}(\vec{r} - \vec{r}(s))\, (1, \vec{v}(s)) =$$

$$= e \int ds\, \delta^{(4)}(x - x(s))\, u^\mu(s) \qquad u^\mu = (\gamma, \gamma\vec{v})$$

10bis

The procedure to obtain the current at a given space-time point is ~~[illegible]~~

1) We fix t_0 and $\vec{x}_0$

2) We scan ~~with s~~ all possible values of s

3) Since

$$\frac{dt}{ds} = \gamma \Rightarrow ds = \frac{dt}{\gamma}$$

if we fix the initial conditions such that $s_i = s_0(t_i)$ then there is only one s_0 corresponding to t_0, which is

$$s_0 = s_i + \int_{t_i}^{t_0} \frac{dt'}{\gamma(t')} = s_i + \int_{t_i}^{t_0} dt' \sqrt{1 - \vec{v}^2(t')}$$

4) Then

$$j_e^\mu(t_0, \vec{x}_0) = e\, \delta^{(3)}(\vec{x}_0 - \vec{x}(s_0))\, u^\mu(s_0) = e\, \delta^{(3)}(\vec{x}_0 - \vec{x}(s_0(t_0)))\, u^\mu(s_0(t_0))$$

The field produced by the charge is the ~~[illegible]~~

$$A^{\mu}_{e,R}(x) = 4\pi \int d^4y \; G_R(x-y)\, e \int ds\, \delta^{(4)}(y - X(s))\, u^{\mu}(s)$$

$$F^{\mu\nu}_{e,R}(x) = \partial^{\mu} A^{\nu}_{e,R}(x) - \partial^{\nu} A^{\mu}_{e,R}(x)$$

Now, in order to determine exactly the dynamics of the charge, we have to include the reaction of the field radiated by the charge on the charge itself, so the charge is driven not only by the external field $F^{\mu\nu}(x)$ but also by the field produced by the charge itself

$$m_0 \frac{du^{\mu}}{ds} = e F^{\mu\nu} u_{\nu} + e F^{\mu\nu}_{e,R} u_{\nu}$$

~~(the value for m_0 will be clear later)~~

Here, however, we have a problem because we have to calculate the field produced by the charge at the position of the charge such that we will encounter a "Coulomb" like divergence.
We proceed in the following way. We write

$$A^{\mu}_{e,R}(x) = \frac{1}{2}\left[A^{\mu}_{e,R}(x) + A^{\mu}_{e,A}(x)\right] + \frac{1}{2}\left[A^{\mu}_{e,R}(x) - A^{\mu}_{e,A}(x)\right]$$

where $A^{\mu}_{A}(x)$ is the advanced field

$$A^{\mu}_{e,A}(x) = 4\pi \int d^4y \; G_A(x-y)\, e \int ds\, \delta^{(4)}(y - X(s))\, u^{\mu}$$

and notice that the field $A^\mu_{i,R}(x) - A^\mu_{i,A}(x)$ is a solution of the equation $\partial_\mu \partial^\mu [A^\mu_{i,R} - A^\mu_{i,A}] = 0$, i.e., it does not involve the electron current explicitly and it will be regular at the charge position. The divergence is only in $[A^\mu_{i,R}(x) + A^\mu_{i,A}(x)]/2$.

We first deal with the regular term and we set $G(x-y) = G_R(x-y) - G_A(x-y)$, then

$$A^\mu_{i,r}(x) = \frac{1}{2}[A^\mu_{i,R}(x) - A^\mu_{i,A}(x)] = \frac{1}{2}\cdot 4\pi \int d^4y\, G(x-y)\, e\int ds\, \delta^{(4)}(y - x(s))\, u^\mu =$$

$$= 2\pi e \int ds\, G(x - x(s))\, u^\mu$$

$$F^{\mu\nu}_{i,r}(x) = \partial^\mu A^\nu_{i,r} - \partial^\nu A^\mu_{i,r} = 2\pi e \int ds \left[\partial^\mu G(x-x(s))\, u^\nu - \partial^\nu G(x-x(s))\, u^\mu\right] =$$

$$= 2\pi e \int ds \left\{ \partial^\mu \left[\frac{1}{2\pi}\left(\vartheta(x^0 - x^0(s)) - \vartheta(x^0(s) - x^0)\right) \delta((x-x(s))^2)\right] u^\nu - \mu \leftrightarrow \nu \right\} =$$

$$= e \int ds \left\{ \delta^\mu_0\, 2\delta(x^0 - x^0(s))\, \delta((x-x(s))^2)\, u^\nu + \left(\vartheta(x^0 - x^0(s)) - \vartheta(x^0(s) - x^0)\right) \delta'((x-x(s))^2)\, 2(x-x(s))^\mu u^\nu - \mu\leftrightarrow\nu \right]$$

$$\delta(-|\vec{x} - \vec{x}(s)|^2)$$

$$= e \int ds \left[\left(\vartheta(x^0 - x^0(s)) - \vartheta(x^0(s) - x^0)\right) \frac{d}{d(x-x(s))^2} \delta((x-x(s))^2)\, 2(x-x(s))^\mu u^\nu - \mu\leftrightarrow\nu\right]$$

$$= e\int ds\left[\left[\vartheta(x^0 - x^0(s)) - \vartheta(x^0(s) - x^0)\right]\frac{d}{ds}\,\delta\big((x-x(s))^2\big)\,\frac{1}{\frac{d}{ds}(x-x(s))^2}\;2(x-x(s))^\mu u^\nu - \mu\leftrightarrow\nu\right] =$$

$$= e\int ds\left[\left[\vartheta(x^0 - x^0(s)) - \vartheta(x^0(s) - x^0)\right]\frac{d}{ds}\,\delta\big((x-x(s))^2\big)\,\frac{1}{(x-x(s))^\sigma(-u_\sigma)}\;(x-x(s))^\mu u^\nu - \mu\leftrightarrow\nu\right] =$$

= by ~~neglecting~~ ignoring the finite term which vanish ~~and~~ the terms proportional to $\frac{d}{ds}\left[\vartheta(x^0 - x^0(s)) - \vartheta(x^0(s)-x^0)\right] = -2\delta(x^0 - x^0(s))\,\gamma =$

$$= 2e\int ds\; G(x-x(s))\left[\frac{d}{ds}\,\frac{(x-x(s))^\mu u^\nu}{u_\sigma(x-x(s))^\sigma} - \mu\leftrightarrow\nu\right]$$

Now, we should evaluate the field at $x^\mu = x^\mu(s)$. In order to treat carefully the divergences we first calculate it at $x^\mu = x^\mu(s')$ with $s' = s - \eta$ and $\eta \to 0$ (in this way the procedure will also be manifestly covariant). We ~~have~~ expand now everything around $x^\mu(s')$

$$x^\mu(s') - x^\mu(s) \simeq -\dot{x}^\mu(s')\eta - \frac{1}{2}\ddot{x}^\mu(s')\eta^2 - \frac{1}{6}\dddot{x}^\mu(s')\eta^3 = -u^\mu(s')\eta - \dot{u}^\mu(s')\frac{\eta^2}{2} - \ddot{u}^\mu(s')\frac{\eta^3}{6}$$

$$u^\mu(s) \simeq u^\mu(s') + \dot u^\mu(s')\eta + \ddot u^\mu(s')\frac{\eta^2}{2}$$

$$(x(s')-x(s))^2 \simeq u^2(s')\eta^2 = \eta^2 \qquad \text{because } u^2 = 1 \text{ and } (u\dot u) = 0.$$

$$u^{(s)}_\sigma(x(s')-x(s))^\sigma \simeq \left[u_\sigma(s') + \dot u_\sigma(s')u + \ddot u_\sigma(s')\frac{u^2}{2}\right]\left[-u^\sigma(s')\eta - \dot u^\sigma(s')\frac{\eta^2}{2} - \ddot u^\sigma(s')\frac{\eta^3}{2}\right] \simeq -\eta$$

Also, we have that

$$G(x(s')-x(s)) = \frac{1}{2\pi}\left[\vartheta(x^0(s')-x^0(s)) - \vartheta(x^0(s)-x^0(s'))\right]\delta((x(s')-x(s))^2) \simeq$$

$$\simeq \frac{1}{2\pi}\left[\vartheta(-u^0\eta) - \vartheta(u^0\eta)\right]\delta(\eta^2) =$$

$$= \frac{1}{2\pi}\left[\vartheta(-\eta) - \vartheta(\eta)\right]\delta(\eta^2) = \frac{1}{2\pi}\left[\vartheta(-\eta) - \vartheta(\eta)\right]\delta(\eta^2) = -\frac{1}{4\pi}\delta'(\eta)$$

see pg 14 bis

In this way

$$F^{\mu\nu}_{e,-}(x(s')) \simeq +2\pi e\int d\eta\,\frac{1}{2\pi}\,\delta'(\eta)\frac{d}{d\eta}\left\{\frac{1}{\eta}\left[u^\mu(s')\eta + \dot u^\mu(s')\frac{\eta^2}{2} + \ddot u^\mu(s')\frac{\eta^3}{6}\right]\left[u^\nu(s') + \dot u^\nu(s')\eta + \ddot u^\nu(s')\frac{\eta^2}{2}\right] - \mu\leftrightarrow\nu\right\}$$

Now,

$$\frac{1}{2\pi}\left[\vartheta(-\eta)-\vartheta(\eta)\right]\delta(\eta^2) = \lim_{a\to 0^+}\frac{1}{2\pi}\left[\vartheta(-\eta)-\vartheta(\eta)\right]\delta(\eta^2-a^2) =$$

$$= \lim_{a\to 0}\frac{\left[\vartheta(-\eta)-\vartheta(\eta)\right]}{\pi}\frac{1}{2a}\left[\delta(\eta+a)+\delta(\eta-a)\right] =$$

$$= \frac{1}{2\pi}\lim_{a\to 0}\frac{1}{2a}\left[\delta(\eta+a)-\delta(\eta-a)\right] = \frac{1}{2\pi}\delta'(\eta)$$

$$= -\frac{e}{\ }\int d\xi\, \frac{\delta(\xi)}{\ }\frac{\partial^2}{\partial\xi^2}\Big\{ \cancel{u^\mu u^\nu} + \cancel{u^\mu \dot{u}^\nu \xi} + u^\mu \ddot{u}^\nu \frac{\xi^2}{2} + \cancel{\dot{u}^\mu \frac{\xi}{2} u^\nu} + \cancel{\dot{u}^\mu \frac{\xi^2}{2}\dot{u}^\nu} + \dot{u}^\mu \frac{\xi}{2}\ddot{u}^\nu \frac{\xi^2}{2} + \ddot{u}^\mu \frac{\xi^2}{6} u^\nu + \ddot{u}^\mu \frac{\xi^3}{6}\dot{u}^\nu +$$

$$+ \cancel{\ddot{u}^\mu \ddot{u}^\nu \frac{\xi^4}{12}} - \mu \leftrightarrow \nu \Big\} =$$

$$= -\frac{e}{\ }\left[\frac{u^\mu \ddot{u}^\nu}{\ } + \cancel{\dot{u}^\mu \dot{u}^\nu \frac{3}{2}} + + \frac{\ddot{u}^\mu u^\nu}{3} - \frac{u^\nu \ddot{u}^\mu}{\ } - \frac{\ddot{u}^\nu u^\mu}{3} \right] =$$

$$= -\frac{e}{\ }\frac{2}{3}\left[u^\mu \ddot{u}^\nu - u^\nu \ddot{u}^\mu \right]$$

Therefore, the equation of motion becomes

$$m_0 \frac{du^\mu}{ds} = e F^{\mu\nu} u_\nu + e F^{\mu\nu}_{R,+} u_\nu + e F^{\mu\nu}_{e-} u_\nu =$$

$$= e F^{\mu\nu} u_\nu + e F^{\mu\nu}_{e+} u_\nu + \frac{2}{3} e^2 \left[\ddot{u}^\mu - u^\mu \ddot{u}^\nu u_\nu \right] =$$

$$= e F^{\mu\nu} u_\nu + e F^{\mu\nu}_{e+} u_\nu + \frac{2}{3} e^2 \left[\ddot{u}^\mu + \dot{u}^2 u^\mu \right]$$

If we carry out the same calculation but with $\frac{A_{ret}^{\mu}(x) + A_{adv}^{\mu}(x)}{2}$ at a certain point we have that instead of

$$G(x(s') - x(s)) \approx \frac{1}{4\pi} [\vartheta(-\eta) - \vartheta(\eta)]\,\delta(\eta^2) = \frac{1}{2\pi}\,\delta'(\eta)$$

We have

$$\tilde{G}(x(s') - x(s)) = \frac{1}{2\pi}[\vartheta(-\eta) + \vartheta(\eta)]\,\delta(\eta^2) = \frac{1}{2\pi} \lim_{a \to 0} \frac{1}{a}\,\delta(\eta)$$

and

$$F_{et}^{\mu\nu}(x(s')) = + \lim_{a\to 0}\frac{1}{a}\, e \int d\eta\, \delta(\eta) \frac{d}{d\eta}\left[u^{\mu}\dot{u}^{\nu}\eta + \dot{u}^{\mu}u^{\nu}\frac{\eta}{2} - u^{\nu}\dot{u}^{\mu}\eta - \dot{u}^{\nu}u^{\mu}\frac{\eta}{2}\right] =$$

$$= + \frac{e}{2} \lim_{a\to 0} \frac{1}{a} \left(u^{\mu}\dot{u}^{\nu} - u^{\nu}\dot{u}^{\mu}\right)$$

such that

$$m_0 \frac{du^{\mu}}{ds} = e F^{\mu\nu}u_{\nu} + \frac{e^2}{2}\lim_{a\to 0}\frac{1}{a}(-1)\,\dot{u}^{\mu} + \frac{2}{3}e^2\left[\ddot{u}^{\mu} + \dot{u}^2 u^{\mu}\right]$$

It is important that the divergent part is proportional to $\dot{u}^\mu = \frac{du^\mu}{ds}$ such that we can absorb the divergent term as

$$\underbrace{\left(m_0 + \frac{e^2}{2a}\right)}_{\substack{\text{mass renormalization} \\ m}} \frac{du^\mu}{ds} = eF^{\mu\nu}u_\nu + \frac{2}{3}e^2\left(\ddot{u}^\mu + \dot{u}^2 u^\mu\right)$$

The near field contributes to the inertia of the particle.

This is the Lorentz-Abraham-Dirac (LAD) equation

$$m\frac{du^\mu}{ds} = eF^{\mu\nu}u_\nu + \frac{2}{3}e^2\left(\ddot{u}^\mu + \dot{u}^2u^\mu\right)$$

Two additional terms:
1) $\frac{2}{3}e^2\ddot{u}^\mu$ (Schott term related to the near field) This is important because it guarantees that $u^\mu \dot{u}_\mu = 0$
2) $\frac{2}{3}\frac{e^2}{c^2}\left(\frac{du}{ds}\right)^2 u^\mu$ damping term (Larmor with opposite sign)

This equation is very controversial already because it is of the third order in time (term in $\ddot{u}^\mu$), so we need to give also the initial acceleration of the electron in order to solve it. The worst inconsistency of this equation, though, is the admittance of so-called runaway solutions, where the acceleration exponentially increases with time even if there is no external field. We can see this by looking at the non-relativistic limit where $\ddot{u}^\mu + \dot{u}^2u^\mu = \ddot{u}^\mu - (u_\nu \ddot{u}^\nu)u^\mu \approx$ since $|u^\mu| \ll 1$ $\approx \ddot{u}^\mu \Rightarrow \dot{\vec{a}}$

$\hookrightarrow v^2/c^2$

(18)

then

$$m\frac{d\vec{v}}{dt} = e\,(\vec{E} + \underbrace{\vec{v}\times\vec{B}}_{\text{small only } \frac{\vec{v}}{c} \ (\text{if } \vec{E}\sim\vec{B})}) + \frac{2}{3}e^2\,\ddot{\vec{v}}$$

Even if the external field vanishes identically, we see that this equation admits not only the physical solution $\vec{v} = \vec{v}_0$, but also the solution

$$\vec{v}(t) = \vec{v}_0\, e^{\frac{3}{2}\frac{m}{e^2}t}$$

It is worth noticing ~~that the typical~~ two things

1) The runaway solution shows a non-perturbative dependence on the electron charge

2) The typical growth time is

$$\tau = \frac{2}{3}\frac{e^2}{m} = \frac{2}{3}\underbrace{\frac{e^2}{\hbar c}}_{\text{fine-structure constant}}\cdot\underbrace{\frac{\hbar}{mc^2}}_{\text{Compton wavelength } \frac{\hbar}{mc} = 3.9\times10^{-11}\text{ cm divided by } c} = \frac{2}{3}\cdot\frac{1}{137}\,\frac{3.9\times10^{-11}}{3\cdot10^{+10}}\,s \simeq \frac{2}{3}\cdot 9.5\times10^{-24}\,s$$

The important observation is that this time τ is much smaller than the typical QED time $\frac{\lambda_c}{c}$. So it is like the unphysical runaway behaviour occurs at scales where already quantum effects should play a role. We will see that this is the very observation that will let us understand the origin of the inconsistencies of the LAD equation. (19)

Let us go back to the nonrelativistic equation

$$m\frac{d\vec{v}}{dt} = e\left(\vec{E}+\vec{v}\times\vec{B}\right)+\frac{2}{3}e^2\ddot{\vec{v}}$$

We want to show that the RR term is always much smaller than the Lorentz term. By assuming that this is the case, we estimate

see pg. 19 bis

$$\frac{2}{3}e^2\ddot{\vec{v}} = \frac{2}{3}e^2\frac{d}{dt}\left(\frac{d\vec{v}}{dt}\right) \approx \frac{2}{3}\frac{e^2}{m}\left[e\dot{\vec{E}}+e\dot{\vec{v}}\times\vec{B}+e\vec{v}\times\dot{\vec{B}}\right] \approx$$

small because $|\vec{v}|$

$$\approx \frac{2}{3}\frac{\alpha}{m}\left[e\dot{\vec{E}}+e\,\frac{e\vec{E}\times\vec{B}}{m}\right] = \frac{2}{3}\alpha\lambda_c\frac{d}{dt}(e\vec{E}) \pm \frac{2}{3}\alpha\frac{e\vec{E}\times e\vec{B}}{B_c}$$

where B_c

$$E_c = \frac{m^2c^3}{\hbar|e|} = 1.3\times10^{16}\ \mathrm{V/cm} \qquad B_c = \frac{m^2c^3}{\hbar|e|} = 4.4\times10^{13}\ \mathrm{G}$$

known as critical fields of QED

Let's go to the instantaneous rest-frame of the electron to investigate this problem of the quantum effects more in detail (the conditions, we will obtain, will be invariant). The LAD equation becomes

$$m\frac{d\gamma\vec{v}}{dt} = m\frac{d\gamma}{dt}\vec{v} + m\gamma\frac{d\vec{v}}{dt} \longrightarrow m\frac{d\vec{v}}{dt}$$

$e(\vec{E} + \vec{v}\times\vec{B}) \longrightarrow$ we first keep the term in $\vec{v}\times\vec{B}$ for a reason that will be clear

$$\cancel{\frac{2}{3}e^2\left[\ddot{\vec{u}}^{\mu} + (\dot{u})^2\vec{u}^{\mu}\right] \longrightarrow \frac{2}{3}e^2}$$

$$u^{\mu} = (\gamma, \gamma\vec{v}) \longrightarrow (1, 0)$$

$$\dot{u}^{\mu} = \frac{du^{\mu}}{ds} = \frac{du^{\mu}}{dt}\frac{dt}{ds} = \gamma\left(\frac{d\gamma}{dt}, \frac{d\gamma}{dt}\vec{v} + \gamma\frac{d\vec{v}}{dt}\right) \Rightarrow \quad \frac{d\gamma}{dt} = \frac{d}{dt}\frac{1}{\sqrt{1-v^2}} = \left(-\frac{1}{2}\right)\frac{1}{(1-v^2)^{3/2}}(-2)\vec{v}\cdot\frac{d\vec{v}}{dt}$$

$$= \gamma\left(\gamma^3\vec{v}\cdot\frac{d\vec{v}}{dt}, \gamma^3\left(\vec{v}\cdot\frac{d\vec{v}}{dt}\right)\vec{v} + \gamma\frac{d\vec{v}}{dt}\right) \longrightarrow \left(0, \frac{d\vec{v}}{dt}\right) \qquad = \gamma^3\vec{v}\cdot\frac{d\vec{v}}{dt}$$

$$\ddot{u}^\mu = \frac{d}{ds}\dot{u}^\mu = \gamma \frac{d}{dt}\dot{u}^\mu = \gamma \frac{d}{dt}\left(\gamma^4 \vec{v}\cdot\frac{d\vec{v}}{dt},\ \gamma^4\left(\vec{v}\cdot\frac{d\vec{v}}{dt}\right)\vec{v} + \gamma^2\frac{d\vec{v}}{dt} \right) =$$

$$= \gamma\left(4\gamma^3 \frac{d\gamma}{dt}\,\vec{v}\cdot\frac{d\vec{v}}{dt} + \gamma^4 \frac{d\vec{v}}{dt}\cdot\frac{d\vec{v}}{dt} + \gamma^4 \vec{v}\cdot\frac{d^2\vec{v}}{dt^2}, \right.$$

$$4\gamma^3\frac{d\gamma}{dt}\left(\vec{v}\cdot\frac{d\vec{v}}{dt}\right)\vec{v} + \gamma^4\left(\frac{d\vec{v}}{dt}\cdot\frac{d\vec{v}}{dt}\right)\vec{v} + \gamma^4\left(\vec{v}\cdot\frac{d^2\vec{v}}{dt^2}\right)\vec{v} + \gamma^4\left(\vec{v}\cdot\frac{d\vec{v}}{dt}\right)\frac{d\vec{v}}{dt} +$$

$$\left. + 2\gamma\frac{d\gamma}{dt}\frac{d\vec{v}}{dt} + \gamma^2\frac{d^2\vec{v}}{dt^2} \right) \longrightarrow \left(\left(\frac{d\vec{v}}{dt}\right)^2,\ \frac{d^2\vec{v}}{dt^2} \right)$$

$$\frac{2}{3}e^2\left(\ddot{\vec{u}} + \dot{u}^2\vec{u}\right) \longrightarrow \frac{2}{3}e^2\frac{d^2\vec{v}}{dt^2}$$

Thus, in the instantaneous rest frame of the electron the LAD equation becomes

$$m\frac{d\vec{v}}{dt} = e\left(\vec{E} + \vec{v}\times\vec{B}\right) + \frac{2}{3}e^2\frac{d^2\vec{v}}{dt^2}$$

Let us imagine that in this frame the RR force is much smaller than the Lorentz force. Then we can estimate the RR force as

$$\frac{2}{3}e^2\frac{d}{dt}\left(\frac{d\vec{V}}{dt}\right) \approx \frac{2}{3}e^2\frac{d}{dt}\left[\frac{e}{m}(\vec{E}+\vec{V}\times\vec{B})\right] = \frac{2}{3}\frac{e^3}{m}\left(\dot{\vec{E}}+\dot{\vec{V}}\times\vec{B}+\cancel{\vec{V}\times\dot{\vec{B}}}\right) \approx$$

$$\approx \frac{2}{3}\frac{e^3}{m}\dot{\vec{E}} + \frac{2}{3}\frac{e^3}{m}\frac{e}{m}\vec{E}\times\vec{B}$$

It is convenient to introduce the quantities

$$\frac{2}{3}e^2\frac{1}{m} = \frac{2}{3}\frac{e^2}{\hbar c}\frac{\hbar}{mc}\frac{1}{c} = \frac{2}{3}\alpha\frac{\lambda_c}{c}$$

$$\frac{2}{3}e^2\frac{|e|}{m^2} = \frac{2}{3}\frac{e^2}{\hbar c}\frac{|e|\hbar}{m^2c^3} = \frac{2}{3}\alpha\frac{1}{E_{cr}}$$

$$E_{cr} = \frac{m^2c^3}{\hbar|e|} = 1.3\times10^{16}\ \mathrm{V/cm}$$

$$B_{cr} = \frac{m^2c^3}{\hbar|e|} = 4.4\times10^{13}\ \mathrm{G}$$

23

By comparing this term with the main term of the Lorentz force we realize that it is much smaller than it in general if ~~the~~ fields (in cgs units)

$$\alpha \frac{\lambda_c}{c} \left| \frac{d\vec{E}}{dt} \right| \ll |\vec{E}| \quad , \quad \alpha \frac{\lambda_c}{c} \left| \frac{d\vec{B}}{dt} \right| \ll |\vec{B}|$$

$$\alpha \frac{|\vec{E}|}{E_{cr}} \ll 1 \quad , \quad \alpha \frac{|\vec{B}|}{B_{cr}} \ll 1$$

Now, we have that in order quantum effects to be negligible, it must be

$$\frac{|\vec{E}|}{E_{cr}} \ll 1 \quad \cancel{\text{and}}, \quad \frac{|\vec{B}|}{|B_{cr}|} \ll 1 \qquad \left\{ \begin{aligned} &\lambda_c \left| \frac{d\vec{E}}{dt} \right| \ll |\vec{E}| \\ &\lambda_c \left| \frac{d\vec{B}}{dt} \right| \ll |\vec{B}| \end{aligned} \right\} \text{ see my ~~3~~ bis}$$

Since $\alpha \simeq \frac{1}{137}$ these conditions are automatically fulfilled within ~~class~~ CED (it is clear that we are neglecting all quantum effects here which are ~~larger~~ $\frac{1}{\alpha} \simeq 137$ larger than those discussed). Another important observation is that to consider the non-relativistic limit

3b i)

The reason why quantum effects become important at fields (in the rest frame of the electron) of the order of E_{cr}/B_{cr} and at ~~frequencies of the~~ wavelengths of the order of $\lambda_c = \frac{\hbar}{mc}$ can be understood because

$|e| E_{cr} \frac{\hbar}{mc} = mc^2$ (pair creation)

$\underbrace{\frac{|e|\hbar}{mc}}_{\text{Bohr magneton}} B_{cr} = mc^2$ (spin interaction of the order of mc^2)

Instead the wavelength λ_c corresponds to photon energies in the field of the order of mc^2 and the recoil is essential.

~~of the LAD equation can be interpreted as performing a boost into the instantaneous rest frame of the electron (where $\vec{v}=0$ rigorously speaking).~~ Thus, our conclusion is that in the instantaneous rest frame of the electron, RR effects are ~~negligible~~ small (~~much smaller than quantum effects, which have been neglected~~) and we can approximate the LAD equation as

$$m\frac{d\vec{V}}{dt} = e(\vec{E} + \frac{2}{3}\frac{e^3}{m}\dot{\vec{E}} + \frac{2}{3}\frac{e^4}{m^2}\vec{E}\times\vec{B}$$

The relativistic generalization of this equation is obtained by starting from the LAD equation

$$m\frac{du^\mu}{ds} = eF^{\mu\nu}u_\nu + \frac{2}{3}e^2\left(\frac{d^2u^\mu}{ds^2} + \frac{du^\nu}{ds}\frac{du_\nu}{ds}u^\mu\right)$$

and by replacing

$$\frac{du^\mu}{ds} = \frac{eF^{\mu\nu}u_\nu}{m}$$

in the ~~rest~~ RR terms in such a way that

$$m\frac{du^\mu}{ds} = eF^{\mu\nu}u_\nu + \frac{2}{3}e^2\frac{d}{ds}\left(\frac{eF^{\mu\nu}u_\nu}{m}\right) + \frac{2}{3}e^2\frac{eF^{\nu\lambda}u_\lambda}{m}\frac{F_{\nu\rho}u^\rho}{m}u^\mu =$$

$$= eF^{\mu\nu}u_\nu + \frac{2}{3}e^2\,\frac{e}{m}(\partial_\lambda F^{\mu\nu})u^\lambda u_\nu + \frac{2}{3}\frac{e^3}{m}F^{\mu\nu}\frac{du_\nu}{ds} + \frac{2}{3}\frac{e^4}{m^2}F^{\nu\lambda}u_\lambda F_{\nu\rho}u^\rho u^\mu \simeq$$

$$\simeq eF^{\mu\nu}u_\nu + \frac{2}{3}\frac{e^3}{m}(\partial_\lambda F^{\mu\nu})u^\lambda u_\nu + \frac{2}{3}\frac{e^4}{m^2}F^{\mu\nu}F_{\nu\lambda}u^\lambda + \frac{2}{3}\frac{e^4}{m^2}F^{\nu\lambda}u_\lambda F_{\nu\rho}u^\rho u^\mu$$

This equation is known as Landau-Lifshitz (LL) equation, it is Newtonian in form and it can be shown not to have the runaway solutions of the LAD equation but only the physical ones (Spohn, Europhys. Lett. 50, 287 (2000)). Just to have an idea, the three dimensional version of this equation reads

$$m\frac{d\vec{u}}{ds} = e(\vec{E}+\vec{v}\times\vec{B}) + \frac{2}{3}\frac{e^3}{m}\gamma\left[\left(\frac{\partial}{\partial t}+\vec{v}\cdot\vec{\nabla}\right)\vec{E} + \vec{v}\times\left(\frac{\partial}{\partial t}+\vec{v}\cdot\vec{\nabla}\right)\vec{B}\right] +$$

$$+\frac{2}{3}\frac{e^4}{m^2}\left[\vec{E}\times\vec{B} + \vec{B}\times(\vec{B}\times\vec{v}) + \vec{E}(\vec{v}\cdot\vec{E})\right] -$$

$$-\frac{2}{3}\frac{e^4}{m^2}\gamma^2\,\vec{v}\left[(\vec{E}+\vec{v}\times\vec{B})^2 - (\vec{E}\cdot\vec{v})^2\right]$$

Very recently the new terms of this equation have been tested experimentally by colliding a laser beam with an ultrarelativistic electron bunch (see slide)

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Radiation and radiation reaction in classical and quantum electrodynamics

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Background

- Special relativity and basics of electromagnetism
  - Lorentz and gauge invariance
  - Maxwell's equations
  - electromagnetic field generated by a moving charge
- Covariant formulation of classical electrodynamics
- Basic knowledge of quantum electrodynamics
  - Dirac equation and gamma matrices "technology"
  - Feynman diagrams

BASIC REFERENCES

Classical Part

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Quantum Part

1. V. B. Berestetskii et al., Quantum Electrodynamics (Elsevier, Oxford, 1982)
2. V. N. Baier et al., Electromagnetic Processes at High Energies in Oriented Crystals (World Scientific, Singapore, 1998)
3. E. S. Fradkin et al., Quantum Electrodynamics with Unstable Vacuum (Springer, Berlin, 1991)
4. V. I. Ritus, J. Sov. Laser Res. **6**, 497 (1985)
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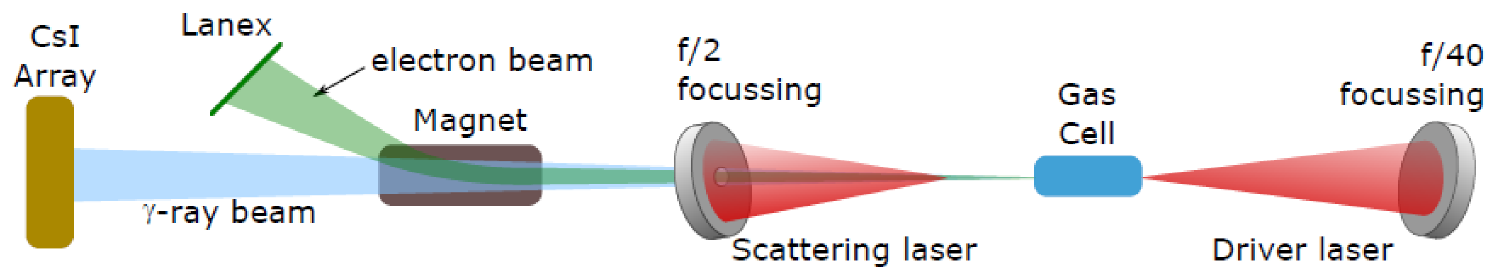
Outline (Part I)

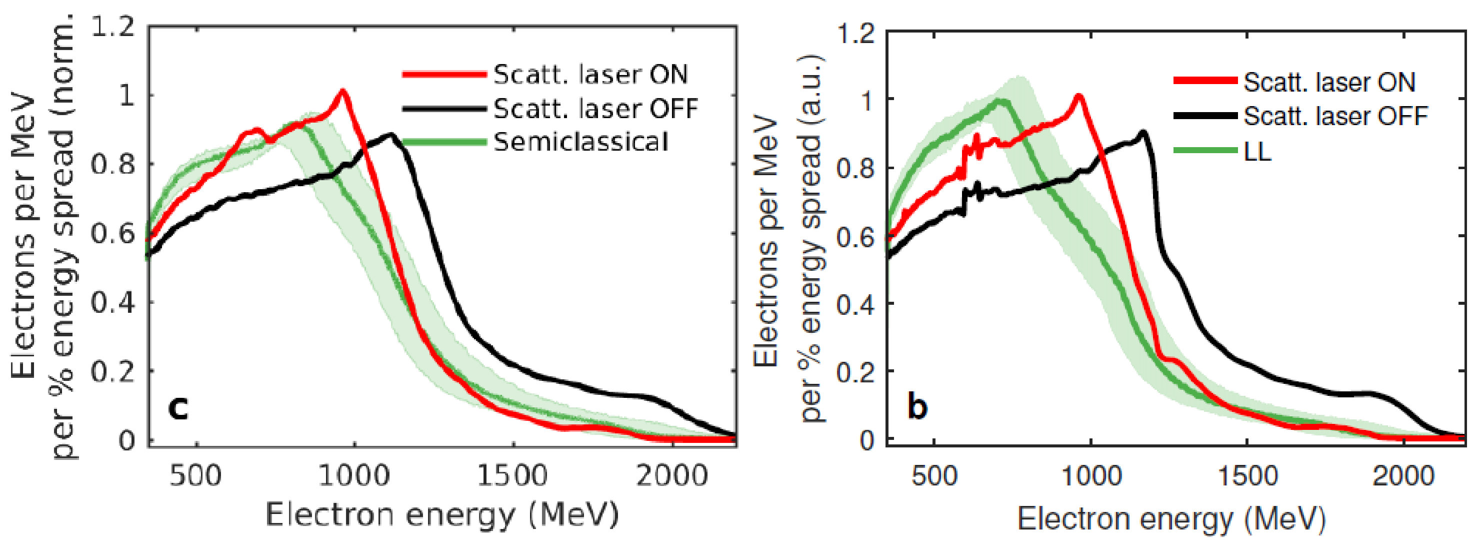
- Radiation by accelerated charges
- Necessity of introducing radiation-reaction terms in the Lorentz equation
- Thorough derivation of the Lorentz-Abraham-Dirac (LAD) equation
- Physical inconsistencies of the LAD equation
- The Landau-Lifshitz (LL) equation
- Recent experimental tests of the LL equation
- Conclusions

Experimental observation of radiation reaction

- Experiment carried out at Astra Gemini (UK)
- Electron energy: up to 2.0 GeV, Laser intensity: 2×10^{20} W/cm^2 (Poder, Tamburini et al. arXiv:1709.01861)



- Experimental results:



Outline (Part II)

- QED in the presence of a strong background electromagnetic field
- The Furry picture and the Volkov states
- Nonlinear single Compton scattering
- Radiation reaction in QED
- Conclusions

Optical laser technology

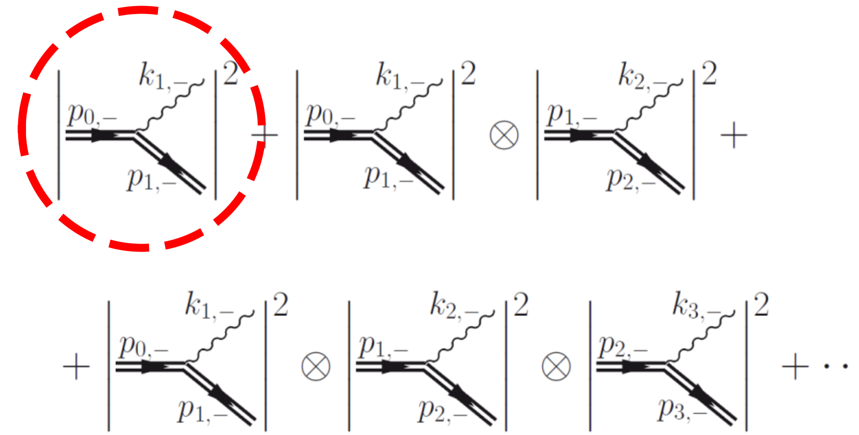
Optical laser technology ($\hbar\omega_0$=1 eV, λ_0=1 μm)	Energy (J)	Pulse duration (fs)	Spot radius (μm)	Intensity (W/cm^2)
State-of-art (Yanovsky et al., Opt. Express 2008)	10	30	1	2×10^{22}
Soon (APOLLON, ELI Beamlines, ELI Nuclear Physics etc...)	$10\div100$	$10\div100$	1	$10^{22}\div10^{23}$
Near future (ELI 4$^{\text{th}}$ pillar, XCELS)	10^4	10	1	$10^{25}\div10^{26}$

Electron accelerator technology

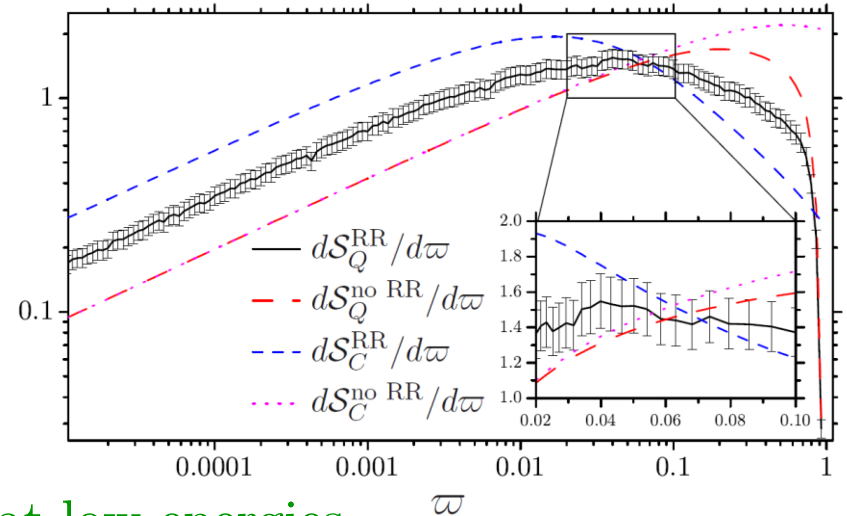
Electron accelerator technology	Energy (GeV)	Beam duration (fs)	Spot radius (μm)	Number of electrons
Conventional accelerators (PDG)	$10\div50$	$10^3\div10^4$	$10\div100$	$10^{10}\div10^{11}$
Laser-plasma accelerators (Leemans et al., Phys. Rev. Lett. 2014)	4.2	40	50	8×10^8

Present technology allows in principle the experimental investigation of strong-field QED

- We have calculated the average energy emitted per unit of electron energy (emission spectrum), by taking into account the emission of N>1 photons (quantum radiation reaction)

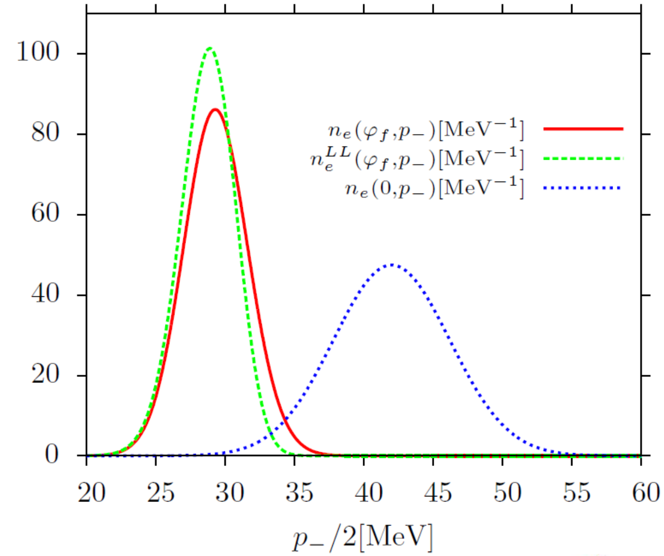


- Numerical parameters: electron energy 1 GeV, laser wavelength 0.8 μm, laser intensity 5×10^{22} W/cm^2 (ξ=150, χ=1.8), laser pulse duration 5 fs (the spectra converged after the inclusion of the emission of 13 photons)

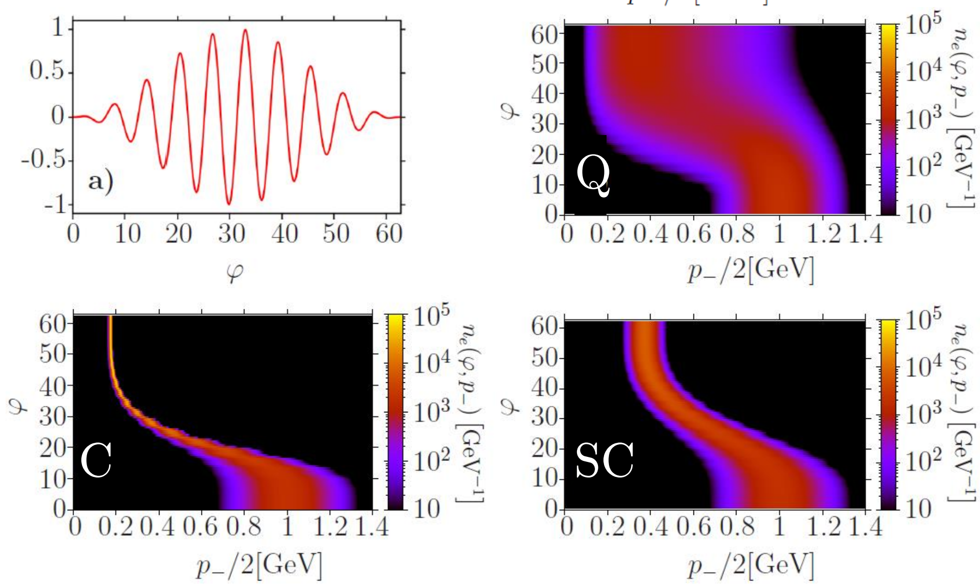


- Effects of radiation reaction:
  1. increase of the spectrum yield at low energies
  2. shift to lower energies of the maximum of the spectrum yield
  3. decrease of the spectrum yield at high energies
- Classical radiation reaction artificially amplifies all the above effects
- Classical spectra both without and with radiation reaction give unphysical results at high photon energies

- Numerical example: $\sin^2$-like optical (λ_0=0.8 μm) pulse with I_0=4.3×10^{20} W/cm^2 (ξ=10), and an electron bunch initially with ε^*=42 MeV (χ^*=5×10^{-3})
- QED effects are small and both classical and quantum equations predict a reduction of the energy width



- Numerical parameters as above except I_0=2.2×10^{22} W/cm^2 (ξ=68) and ε^*=1 GeV (χ^*=0.8)
- SC: classical formulas with quantum intensity of radiation



- Classical and quantum approaches give opposite results
- The semiclassical approach does not include stochasticity effects and cannot explain the broadening of the distribution function