



# School on Synchrotron and Free-Electron-Laser Methods for Multidisciplinary Applications

Fu

## Radiation from Storage Rings and Free Electron Lasers

Giorgio Margaritondo

Ecole Polytechnique Fédérale de Lausanne (EPFL)

## Outline:

- Building an excellent x-ray source: a paradox?
- Essential properties of synchrotron radiation
- Free Electron Lasers (FELs): the basic “Italian salami” mechanism
- X-ray FELs: subtle points

# Origin of this presentation:

Must synchrotron radiation be so formal and complicated?

$$\vec{r}(t) = \left( \rho \sin \frac{\beta c}{\rho} t, \rho \left( 1 - \cos \frac{\beta c}{\rho} t \right), 0 \right).$$

In the limit of small angles we compute

$$\hat{n} \times (\hat{n} \times \vec{\beta}) = \beta \left[ -\vec{\epsilon}_{\parallel} \sin \left( \frac{\beta c t}{\rho} \right) + \vec{\epsilon}_{\perp} \cos \left( \frac{\beta c t}{\rho} \right) \sin \theta \right]$$

$$\omega \left( t - \frac{\hat{n} \cdot \vec{r}(t)}{c} \right) = \omega \left[ t - \frac{\rho}{c} \sin \left( \frac{\beta c t}{\rho} \right) \cos \theta \right]$$

Substituting into the radiation integral and introducing

$$\xi = \frac{\rho \omega}{3c\gamma^3} (1 + \gamma^2 \theta^2)^{3/2}$$

$$\frac{d^3 W}{d\Omega d\omega} = \frac{e^2}{16\pi^3 \epsilon_0 c} \left( \frac{2\omega\rho}{3c\gamma^2} \right)^2 (1 + \gamma^2 \theta^2)^2 \left[ K_{2/3}^2(\xi) + \frac{\gamma^2 \theta^2}{1 + \gamma^2 \theta^2} K_{1/3}^2(\xi) \right]$$

*J. Synchrotron Rad.* (1995). 2, 148–154

**A Primer in Synchrotron Radiation: Everything You Wanted to Know about SEX (Synchrotron Emission of X-rays) but Were Afraid to Ask**

G. Margaritondo

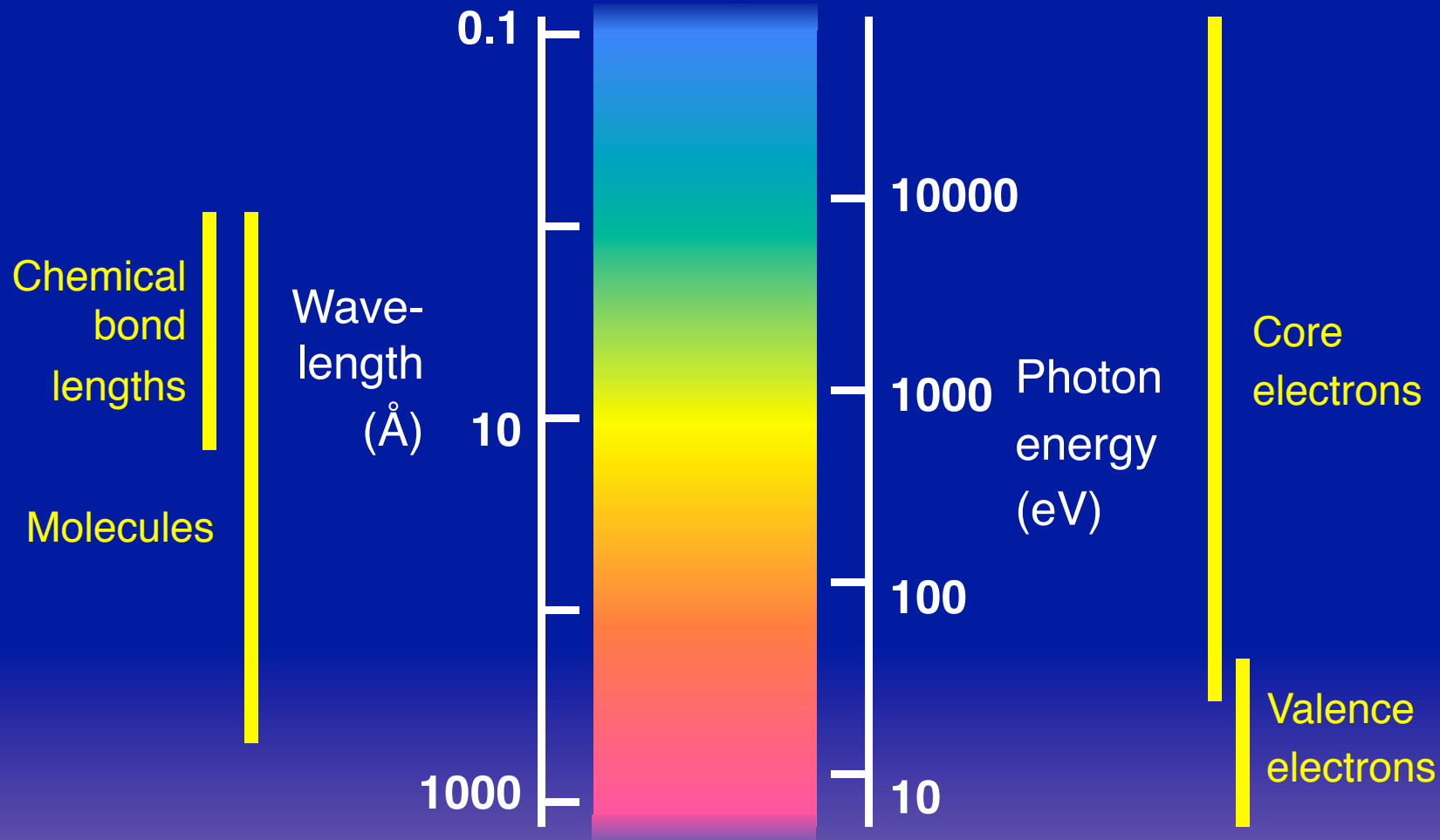
**A simplified description of X-ray free-electron lasers**

G. Margaritondo\* and Primoz Rebernik Ribic

*J. Synchrotron Radiation* 18, 101 (2011)

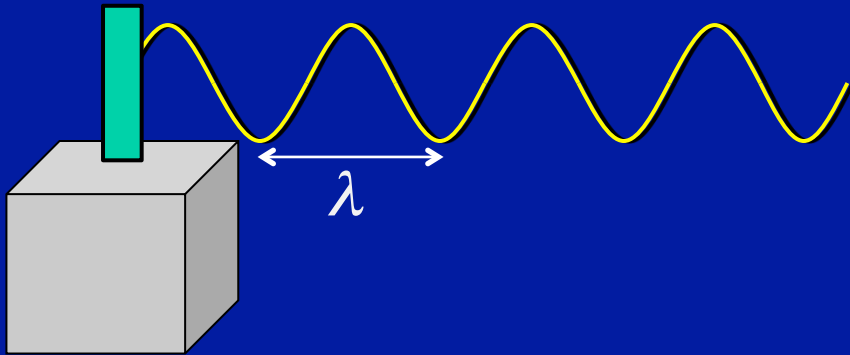
**NO!!!**  
What matters is the underlying physics!

# Starting point: why do we need x-ray sources?

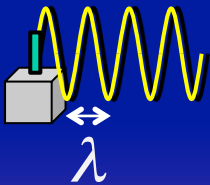


...x-rays are ideal probes of chemical bonds, where most of science is rooted

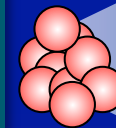
# Building good x-ray sources is a paradox and a huge problem: why?



for VHF radio waves, the source size is similar to the wavelength  $\lambda \approx 1$  meter



for x-rays, the source should shrink to  $\approx 1$  angstrom: one atom – indeed, conventional x-ray sources are based on atoms; but are very bad

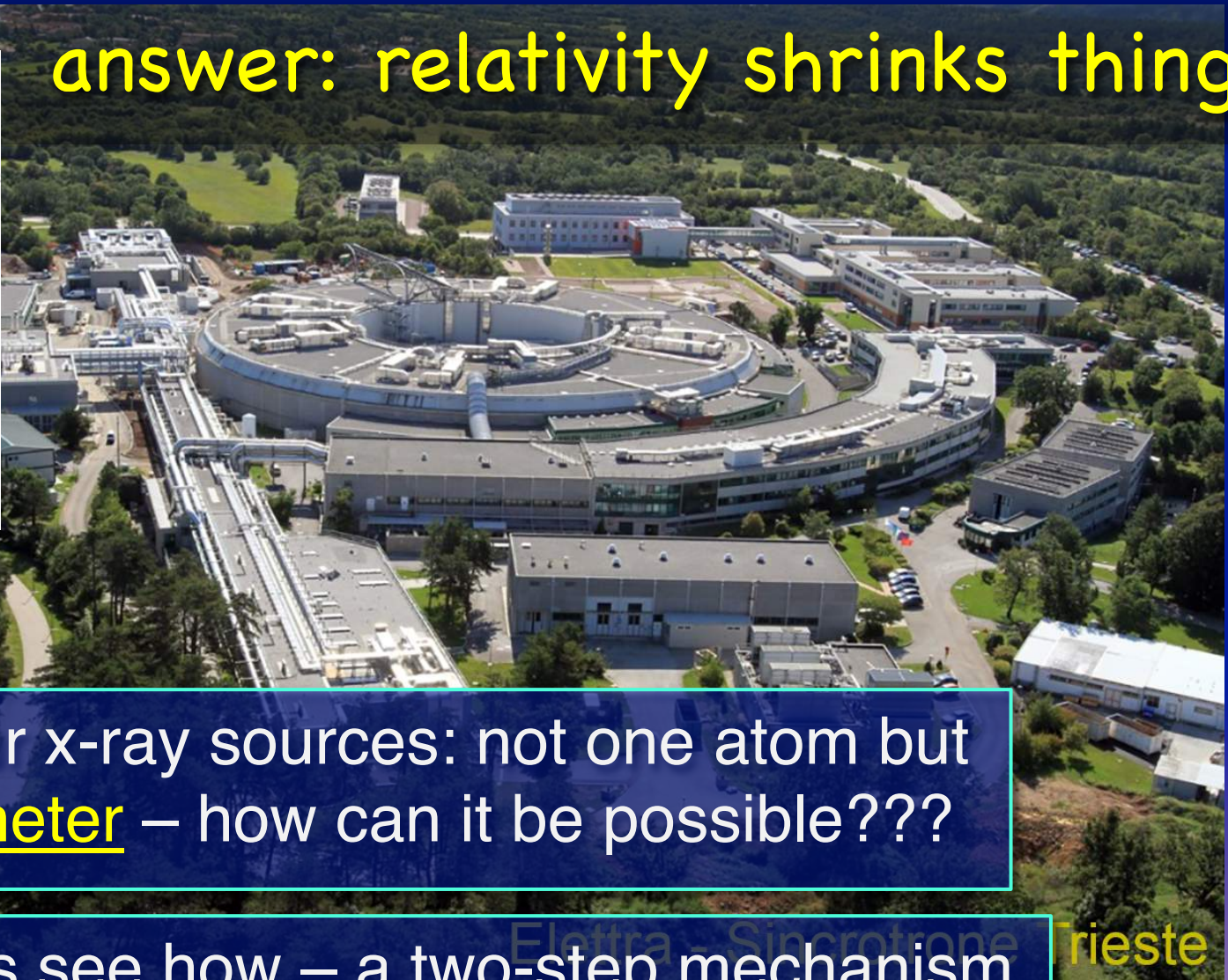


weak flux,  
large spread

to improve them, we would need artificial devices with size  $\approx 1$  angstrom: no way!

...instead, this is what we use:

answer: relativity shrinks things!



one of our x-ray sources: not one atom but one kilometer – how can it be possible???

let's see how – a two-step mechanism

An electron with an accelerated transverse motion emits electromagnetic waves

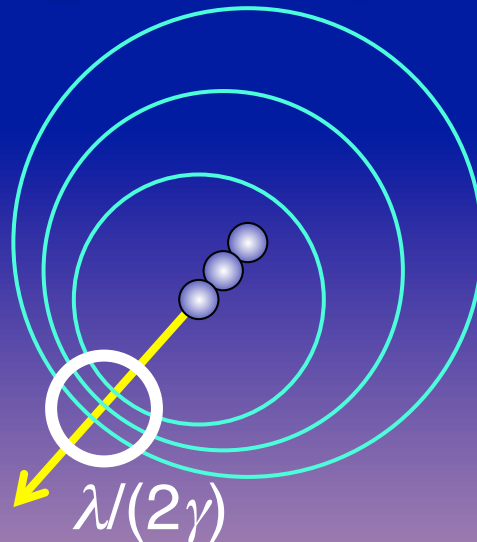
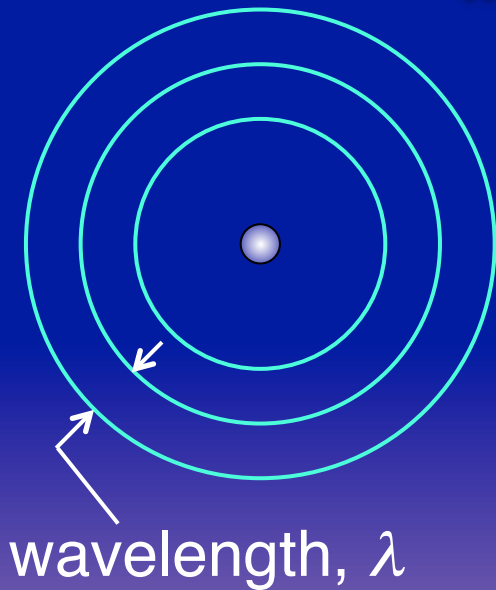


to profit from relativity, we add a longitudinal motion, with speed  $v \approx c$

this is the relativistic “Doppler effect”, shrinking the emitted wavelengths by a factor  $2\gamma$ , where

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

here is the effect of the longitudinal speed:

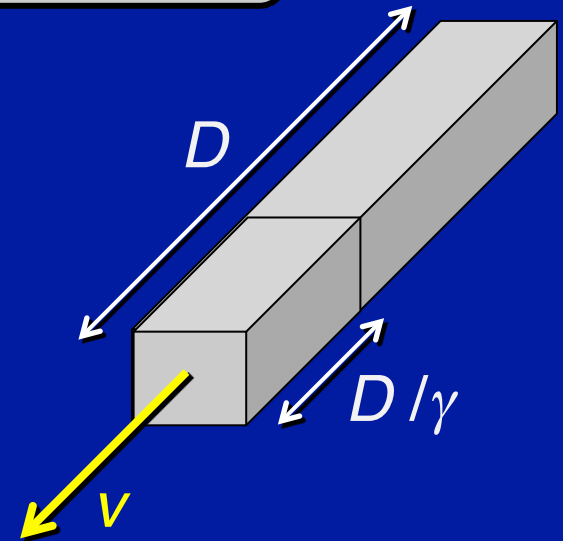


large  $v$ ,  
large  $\gamma$ ,  
small  $\lambda$ 's!

...and this is not all!!!



relativistic “Lorentz contraction”: an object of length  $D$  moving with speed  $v$  shrinks to  $D/\gamma$



this contraction affects the wavelength – overall, the combination of the two relativistic effects shrinks the wavelength from the non-relativistic value  $\lambda$  to:

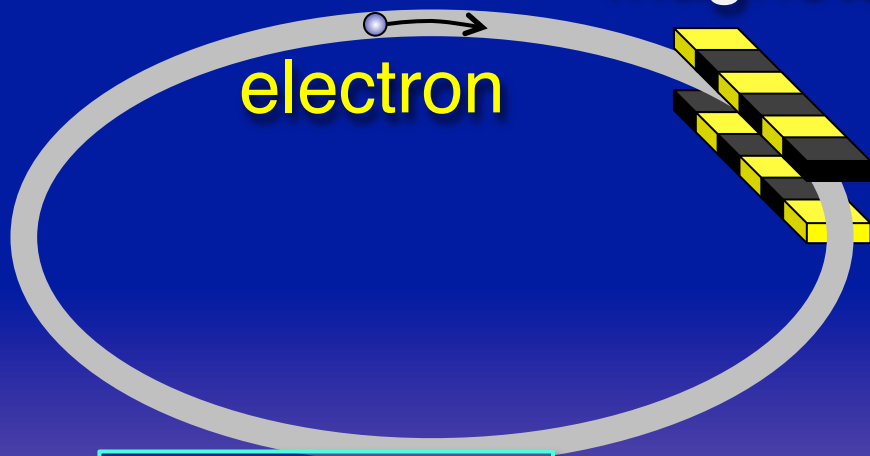
$$\frac{\lambda/\gamma}{2\gamma} = \frac{\lambda}{2\gamma^2}$$



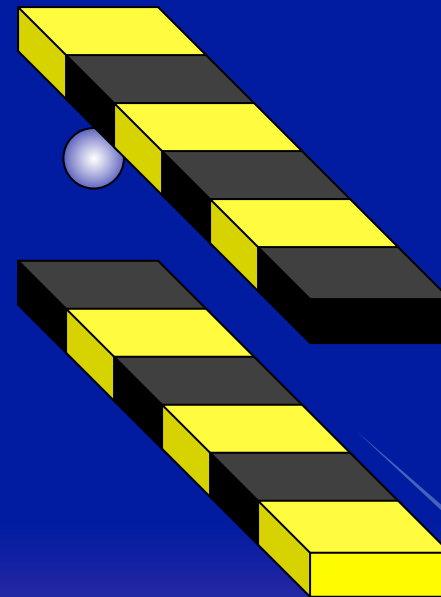
# Realizing this relativistic strategy:

“undulator” (a periodic series of magnets)

electron



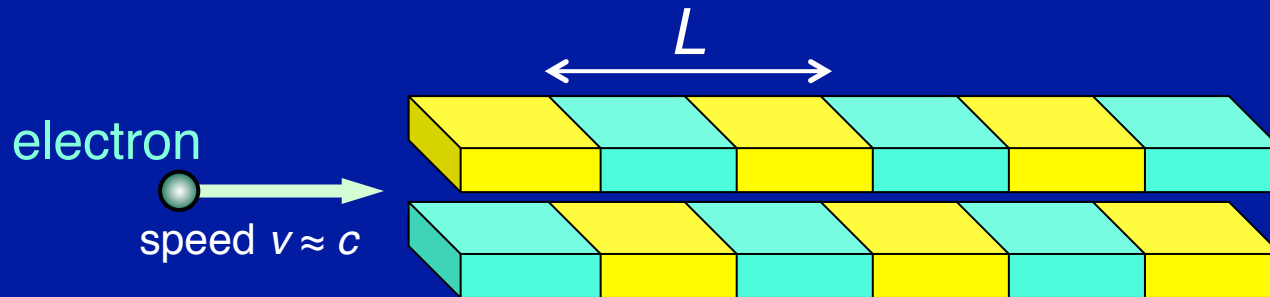
electron  
accelerator



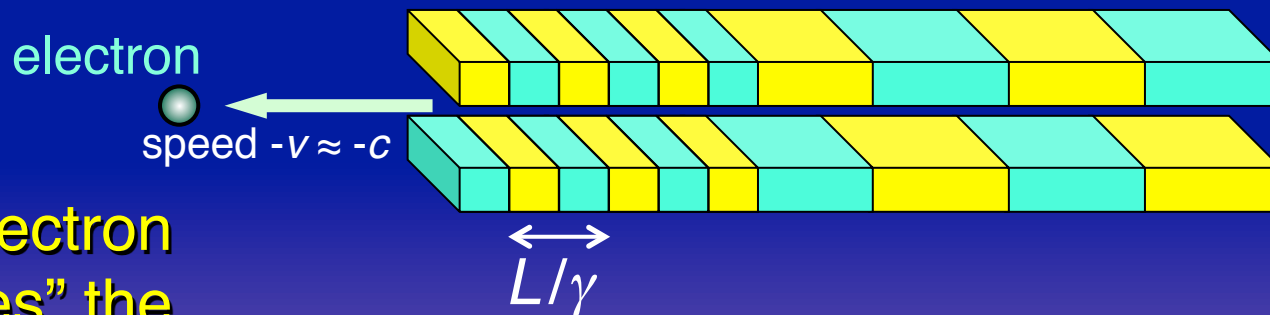
synchrotron  
radiation



# Let us see how an undulator works:



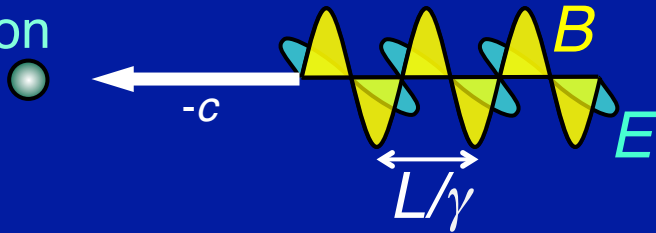
to find out how the electron “sees” the undulator, we apply the relativistic Lorentz transformations from the laboratory reference frame to the electron reference frame:



First, the electron “sees” the undulator moving at speed  $-v \approx -c$

Second, the electron “sees” the undulator period shrinking to  $\approx L/\gamma$  due to the relativistic “Lorentz contraction”

electron

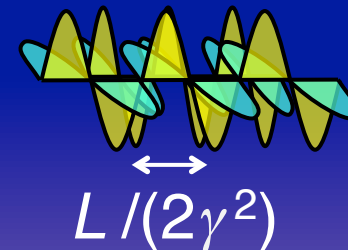


Third, the Lorentz transformation adds to the periodic, transverse magnetic field of the undulator a periodic, transverse electric field

The electron, therefore, “sees” the undulator as an electromagnetic “wave” with its transverse magnetic and electric fields and with wavelength  $\approx L/\gamma$

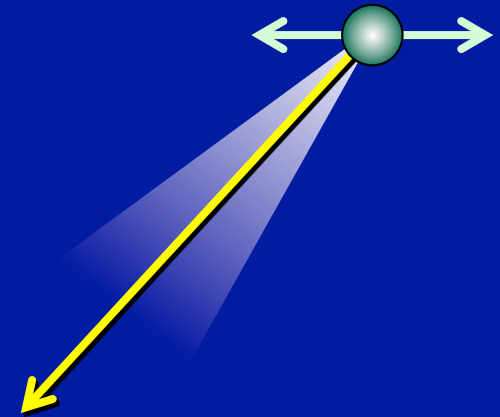
The electron backscatters this “wave”, producing “synchrotron radiation”, also of wavelength  $L/\gamma$

electron



In the laboratory, as we have seen, this wavelength is Doppler shifted to  $\approx (L/\gamma)/(2\gamma) = L/(2\gamma^2)$

...in general terms, the Lorentz transformation determines how the electron “sees” the (magnetic) device that causes the transverse acceleration and the emission, decreasing the emitted wavelengths in the electron reference frame by a factor  $\gamma$



...and the Doppler-shifted wavelengths detected in the laboratory by  $(\gamma)(2\gamma) = 2\gamma^2$

So, high-energy electron accelerators produce x-rays: but why is this interesting?  
Consider fireplaces and torchlights:



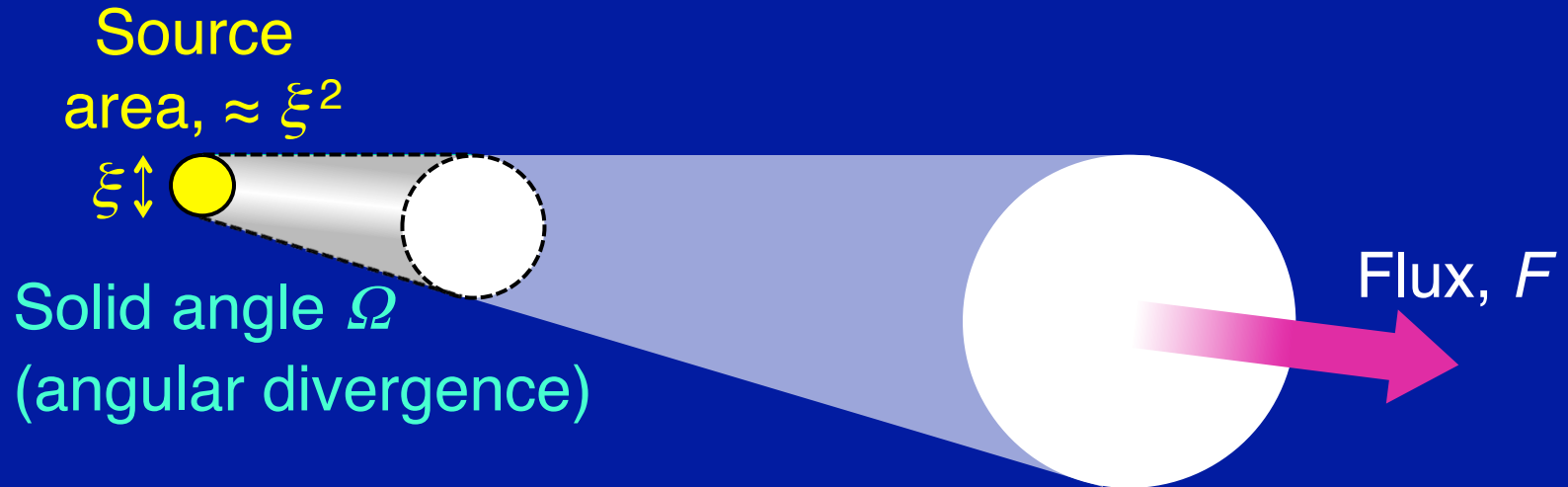
A fireplace is not very effective in "illuminating" a specific target: its emitted power is spread in all directions



A torchlight is much more effective: it is a small-size source with emission concentrated within a narrow angular spread

This can be quantitatively expressed using the "brightness" (or "brilliance")

The “brightness” (or “brilliance”) of  
a source of waves:

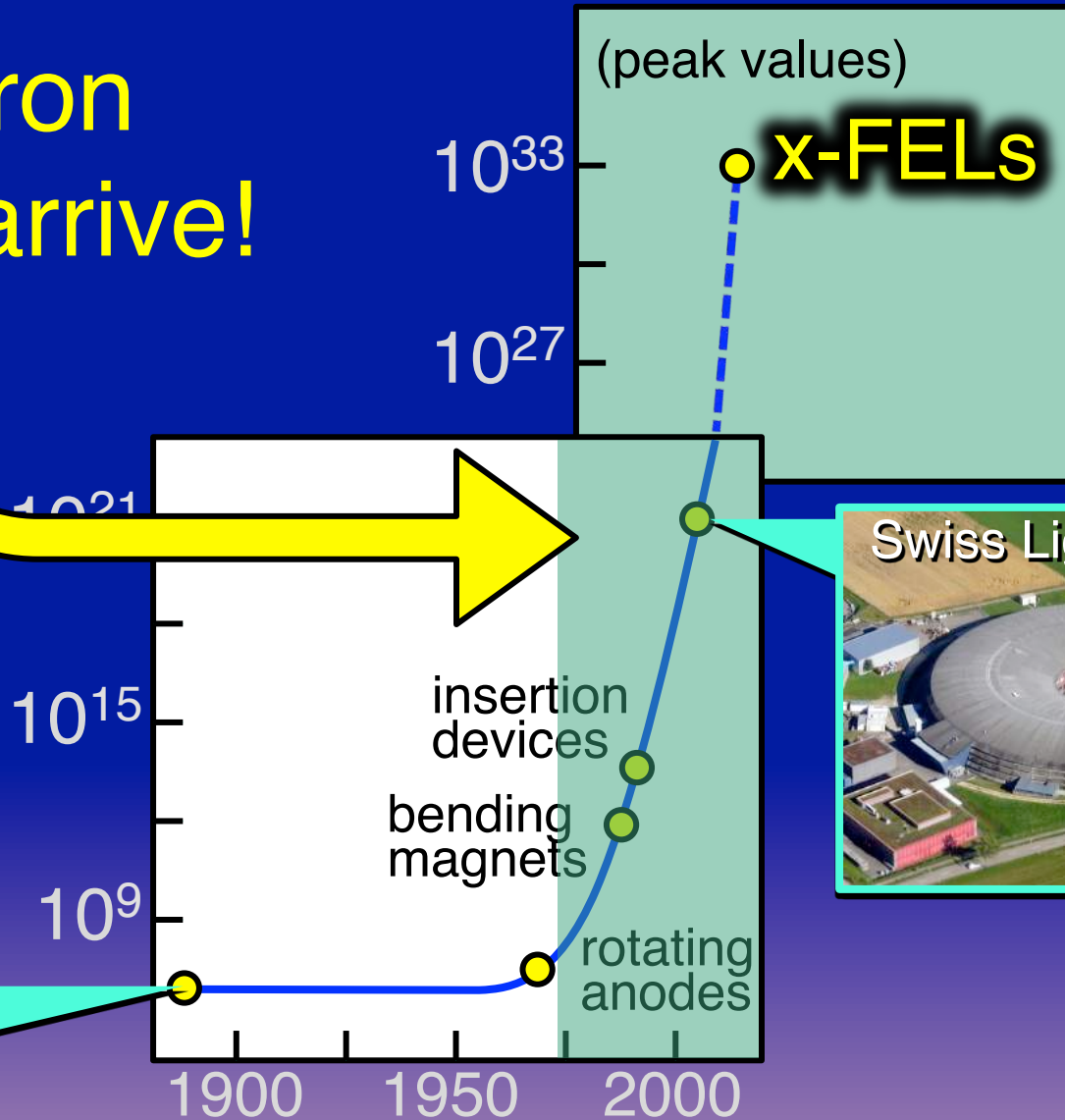


Brightness = constant  $\frac{F}{\xi^2 \Omega}$

# Historical growth in x-ray brightness

(units: photons/mm<sup>2</sup>/s/mrad<sup>2</sup>, 0.1% bandwidth)

## Synchrotron sources arrive!

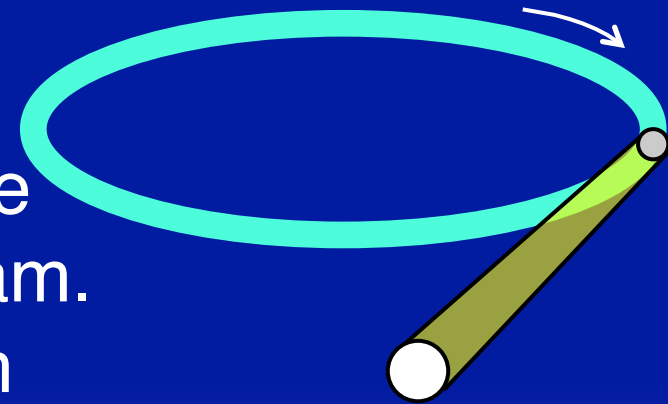


Swiss Light Source



# What allows synchrotron sources to be so bright? Three factors:

1. Electrons in vacuum can emit more power than electrons in a solid because the power does not damage their environment  $\Rightarrow$  **high flux**
2. The **source size** is the transverse cross section of the electron beam. The sophisticated electron beam controls makes it **very small**
3. Relativity drastically reduces the **angular divergence**, collimating the emission

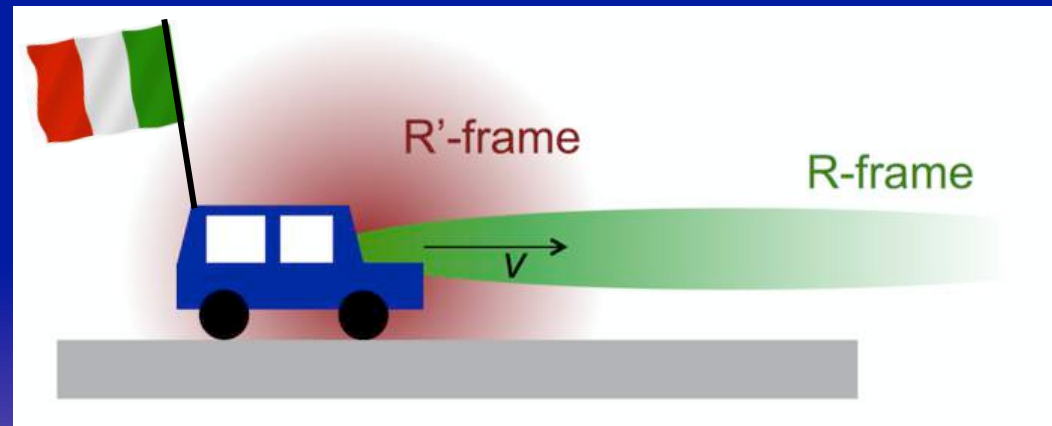




# Relativity at work again: the angular collimation of synchrotron radiation

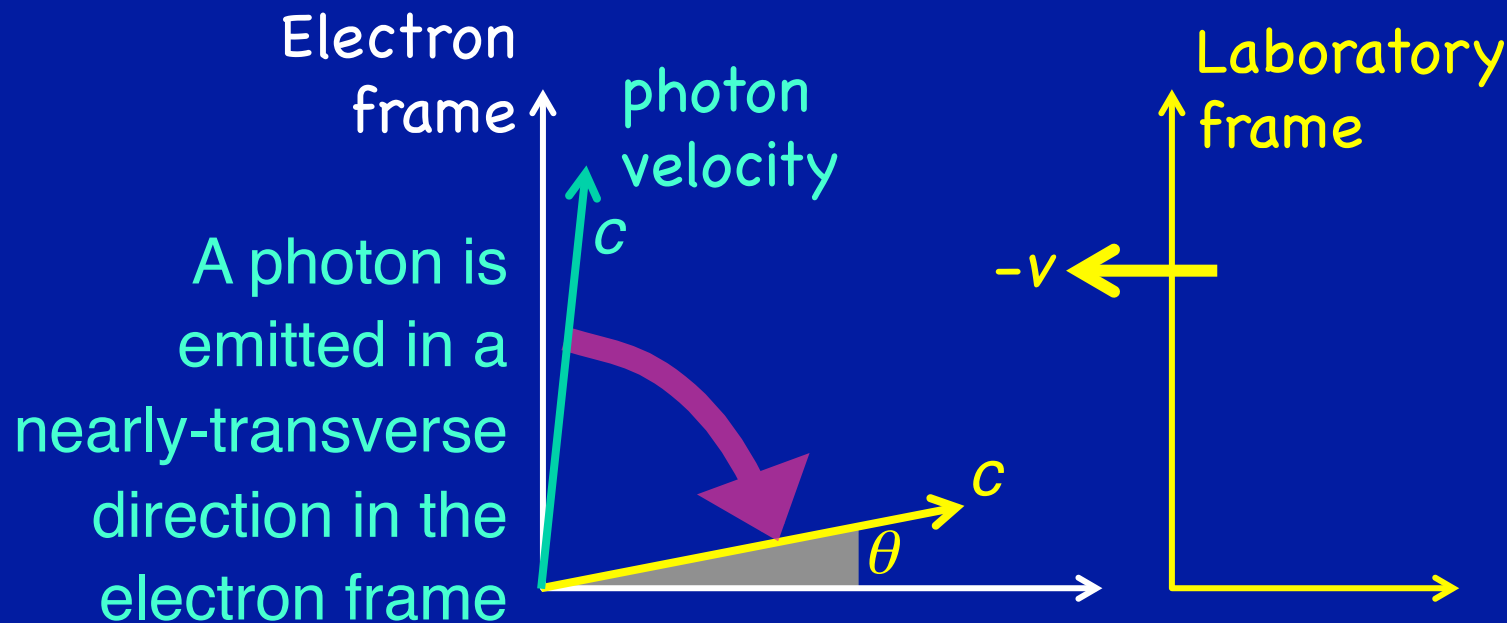
but in the laboratory frame the emission shrinks to a narrow cone

in the electron reference frame, x-rays are emitted in a wide angular range



...like the forward projection of the sound from a car celebrating a victory of the Italian national soccer team --  
but made extreme by relativity

# The relativistic "flashlight" effect:



The Lorentz velocity transformation to the laboratory frame divides by  $\gamma$  the transverse velocity component  $c_y' \approx c$ , giving  $c_y \approx c/\gamma$ . Since  $c$  is invariant, the velocity vector rotates, reaching an angle  $\theta \approx c_y/c \approx 1/\gamma$  from the forward direction:

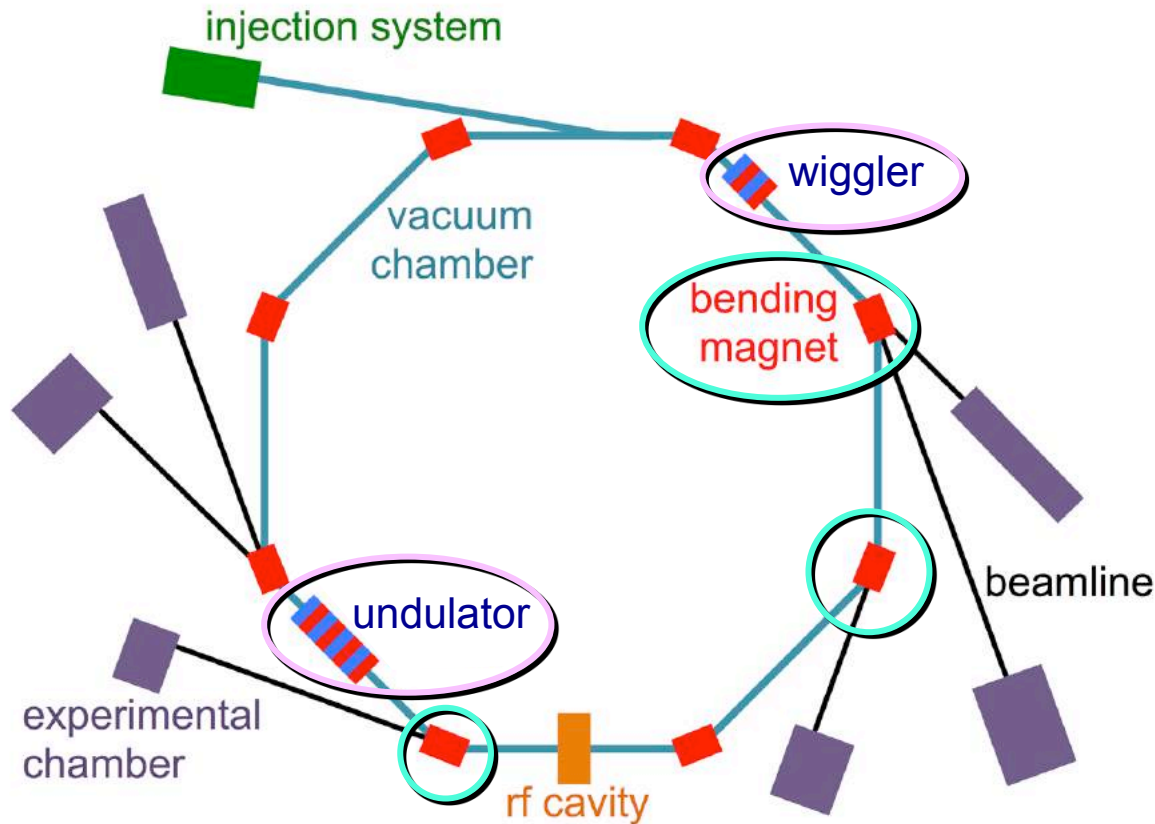
Angular spread:  $\approx 2\theta \approx 2/\gamma$  : narrow!!!

Now it is a good time to "visit"  
a real synchrotron facility



ESRF - Grenoble

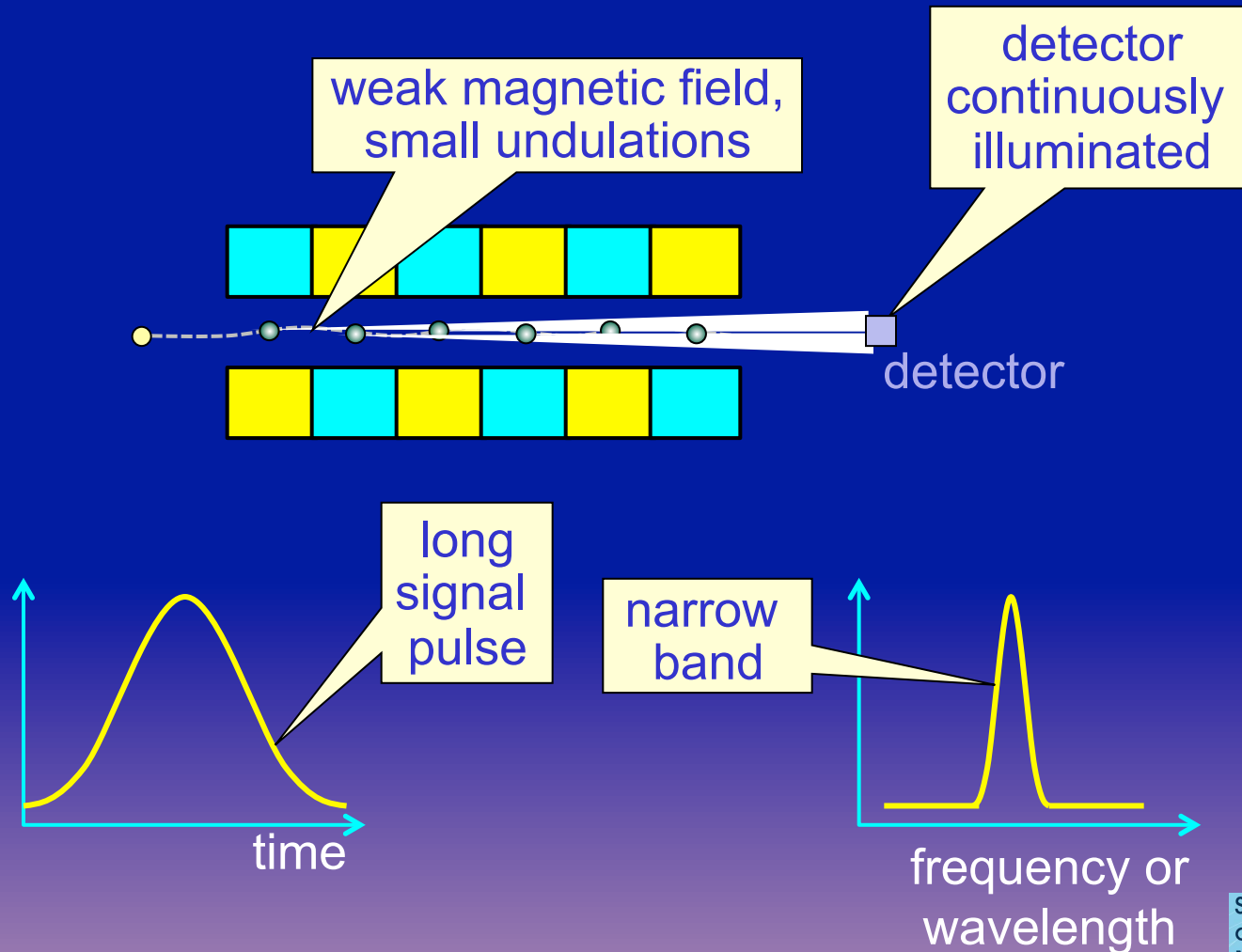
The “flashlight” effect determines the properties of the three main kinds of synchrotron sources:



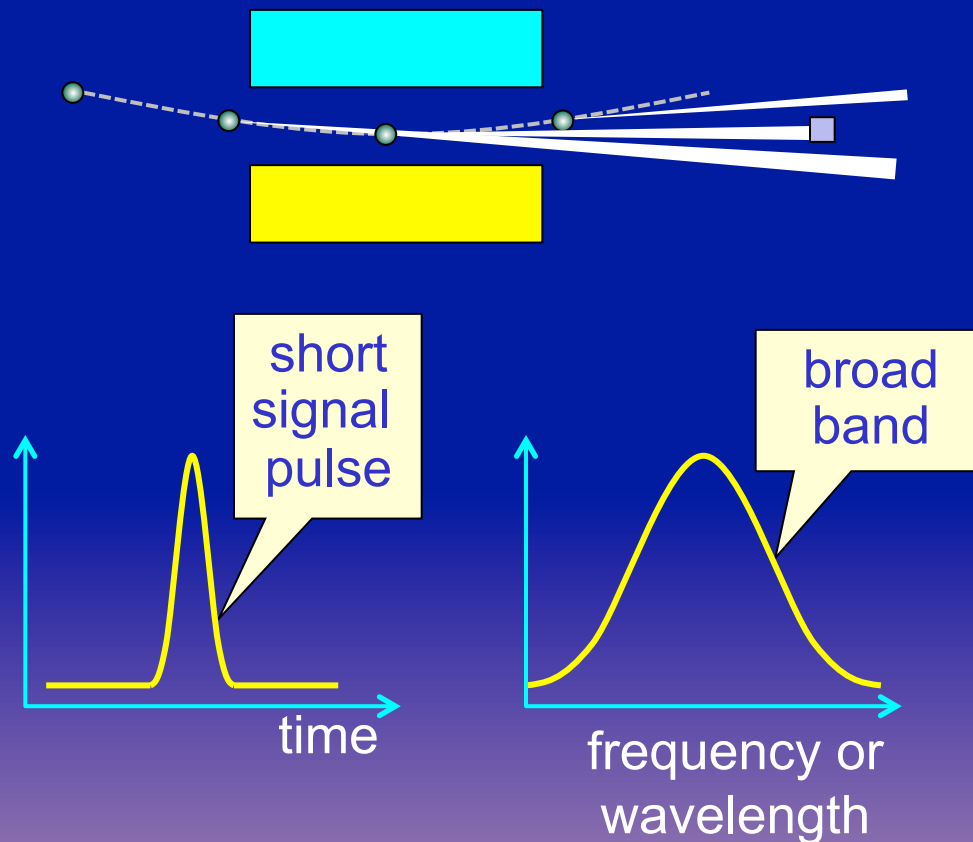
Scheme of synchrotron facility

(1) Bending magnets and two kinds of insertion devices, (2) undulators and (3) wigglers

# The flashlight effect for undulators:

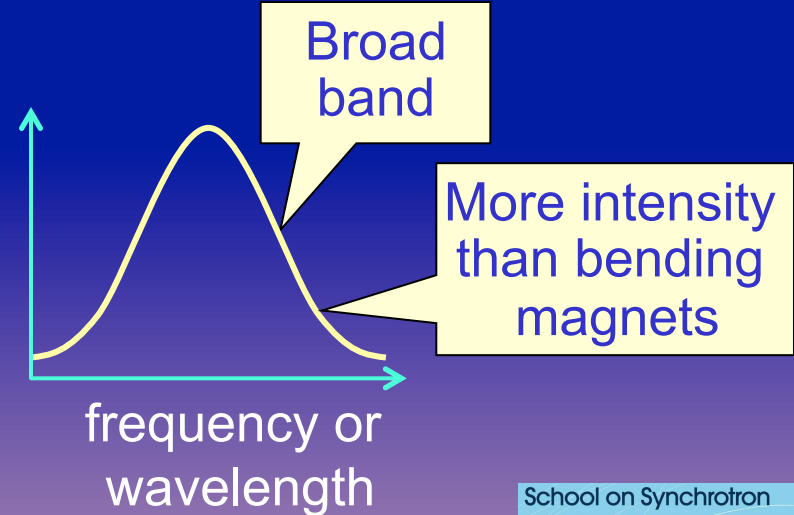
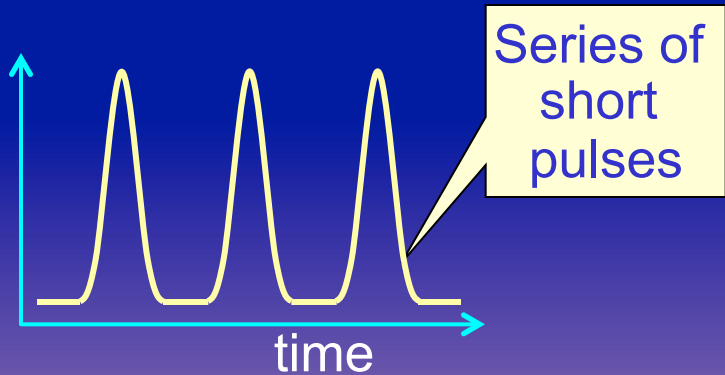
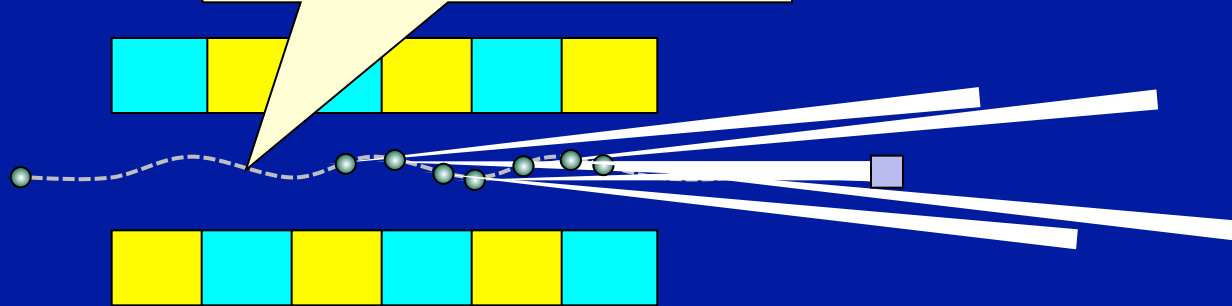


# Second type of sources: bending magnets



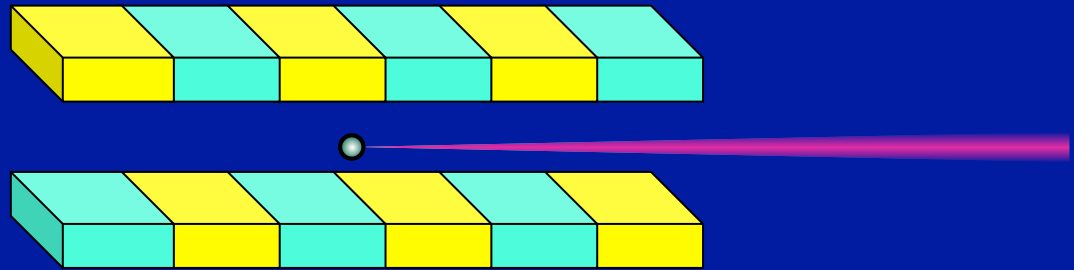
# Third type of sources: wigglers

Strong magnetic field,  
large transverse  
undulations



# Refining our theory of undulators:

What happens when we increase the magnetic field?

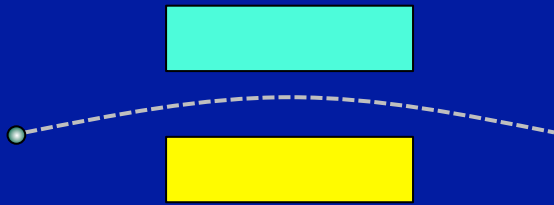


...we increase the transverse electron oscillations and their speed. But the Lorentz force cannot change the kinetic energy, so the longitudinal speed decreases. This modifies the Doppler shift: the detected wavelength changes from  $L/(2\gamma^2)$  to  $[L/(2\gamma^2)](1 + aB^2)$  – and the emission can be tuned by changing  $B$ !

Furthermore: an undulator also emits integer submultiples (harmonics) of the central wavelength. Only odd harmonics along the axis, odd and even harmonics off-axis – extending the spectrum to short wavelengths



# Emission of bending magnets:

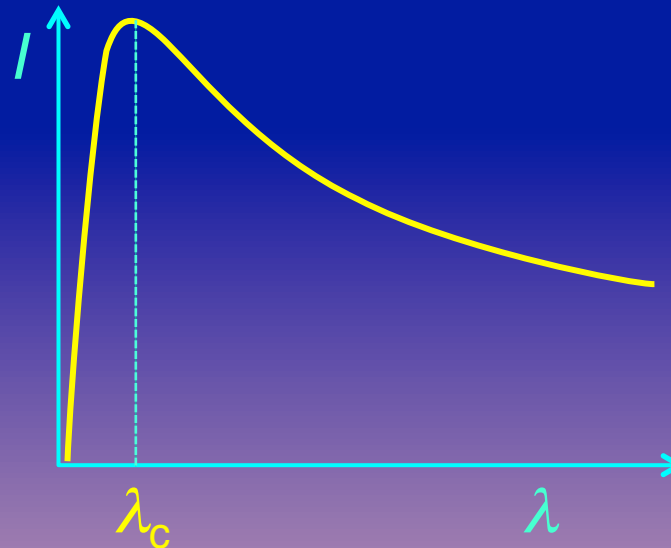


Classical physics: the magnetic force strength is  $e\nu B$ , thus the centripetal acceleration  $\omega_c \nu$  equals  $e\nu B/m_0$  – and the angular speed  $\omega_c$  equals  $eB/m_0$ , the “**cyclotron frequency**”

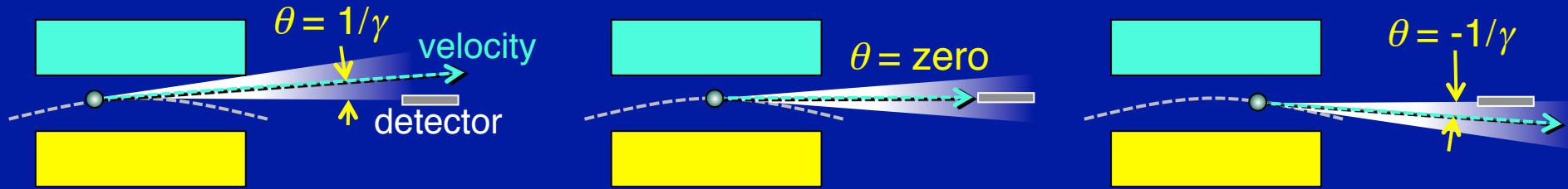
Relativity: in the electron reference frame, the Lorentz transform adds to the magnetic field an electric field of strength  $\gamma \nu B$  -- so the force becomes  $e\gamma \nu B$  and  $\omega_c' = \gamma eB/m_0$

The central emitted wavelength in the electron frame is  $\lambda_c' = 2\pi c/\omega_c' = 2\pi m_0 c/(\gamma eB)$ . Detected in the laboratory, this wavelength is decreased by the Doppler factor  $2\gamma$ , so  $\lambda_c = 2\pi m_0 c/(2\gamma^2 eB)$  -- note the ubiquitous term  $2\gamma^2$ !

BUT:  $\lambda_c$  is not the only emitted wavelength -- around it there is a broad, asymmetric band

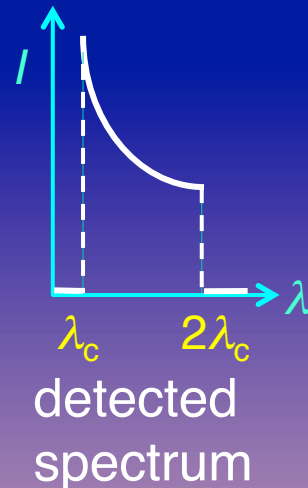
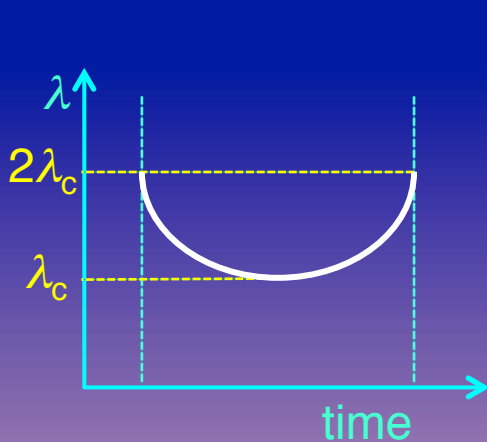


# Why the broad asymmetric band?

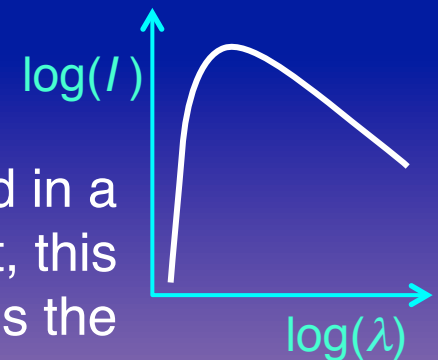


During the detection of the emission cone (half-width  $\approx 1/\gamma$ ), the angle  $\theta$  between the electron velocity and the direction of the detected radiation changes from  $1/\gamma$  to zero, and then to  $-1/\gamma$ .

The Doppler shift changes with the direction: its factor is  $2\gamma$  only in the longitudinal direction ( $\theta = \text{zero}$ ), but becomes  $\approx 2\gamma/(1 + \gamma^2/\theta^2)$  for other directions. For  $\theta = \pm 1/\gamma$  this factor is  $\gamma$  and the detected wavelength is  $2\lambda_c$ . Thus, the wavelength changes with time from  $2\lambda_c$  to  $\lambda_c$  and then again to  $2\lambda_c$  – producing the asymmetric band:



broadened and in a log-log plot, this gives the “standard” bending magnet spectrum



# The emitted power of a synchrotron source:

The classical (Larmor) emitted power is proportional to the square of the (transverse) electron acceleration,  $a^2$

The Lorentz transformation changes the time but not the transverse coordinate, thus it multiplies  $a$  by a factor  $\gamma^2$  – so the emitted power is proportional to  $\gamma^4 = [\text{energy}/(m_0 c^2)]^4$

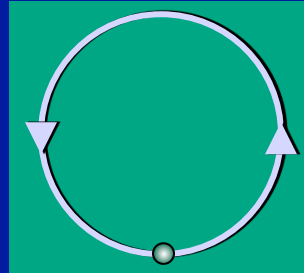
The emission increases with the 4<sup>th</sup> power of the electron energy, to extremely high levels

The emission decreases with the 4<sup>th</sup> power of the mass: electrons emit a lot, protons much less

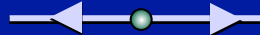
# Synchrotron light polarization:

Electron in a storage ring:

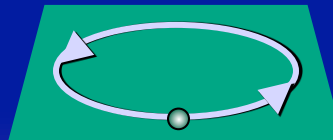
TOP VIEW



SIDE VIEW



TILTED VIEW



Polarization:

**linear** in the plane of the ring,

**elliptical** out of the plane (weak intensity)

Special (elliptical) wigglers and undulators are used to produce elliptically polarized light with high intensity

## New types of sources:

- Ultrabright storage rings (e.g., the upgraded ESRF) reaching the diffraction limit in a large part of the emitted spectrum
- Inverse-Compton-scattering table-top sources
- Energy-recovery machines
- VUV free electron lasers (FEL's)
- **X-ray FEL's**

# Towards FEL's: we start from a normal laser:

Optical cavity: increases the photon beam path and the optical amplification

Result: a collimated, intense, bright and coherent photon beam

Optical pump: puts in the active medium the energy to be converted into photons

Active medium: causes the "optical amplification" of the photon beam

# From a normal laser to an x-FEL:

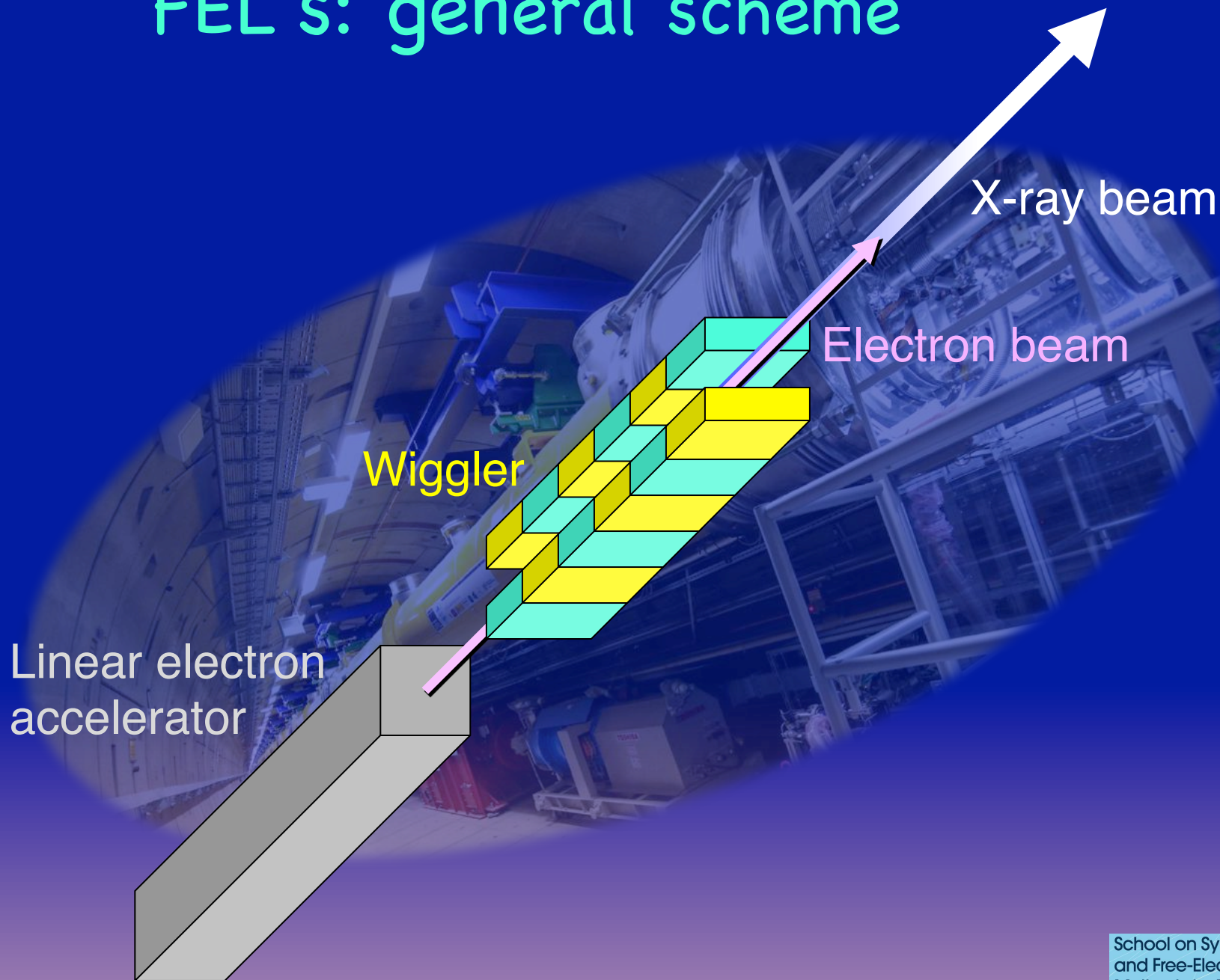
No x-ray mirrors  $\Rightarrow$  no optical cavity  $\Rightarrow$  enough amplification needed for one-pass lasing

Result: a collimated, intense, bright and coherent x-ray beam

Optical pump: the free electrons provide the energy and transfer it to the photons

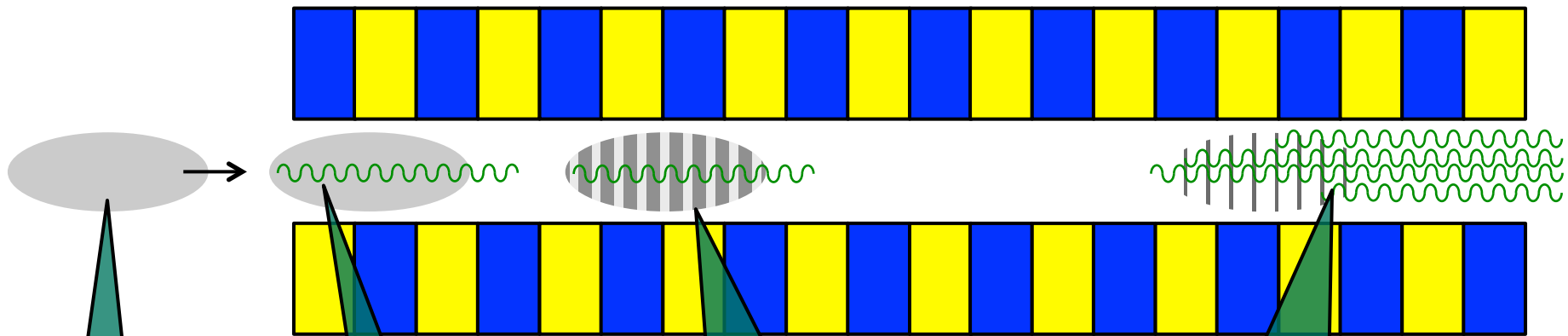
Active medium: no gas, solid or liquid but bunches of "free electrons" in an accelerator: high power possible without damage

# FEL's: general scheme





# "Italian salami" optical amplification mechanism in FEL's:



A bunch of electrons approaches the wiggler

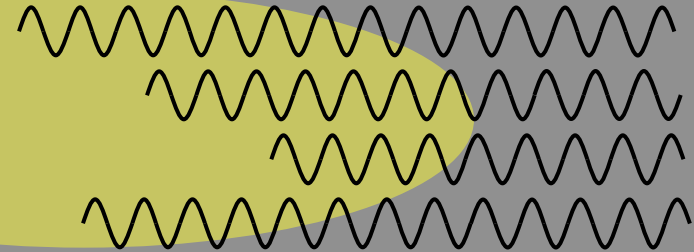
As it enters the wiggler, one of its electrons emits a wave

Traveling with the bunch, the wave reshapes it, creating a microbunch structure with period equal to the wavelength

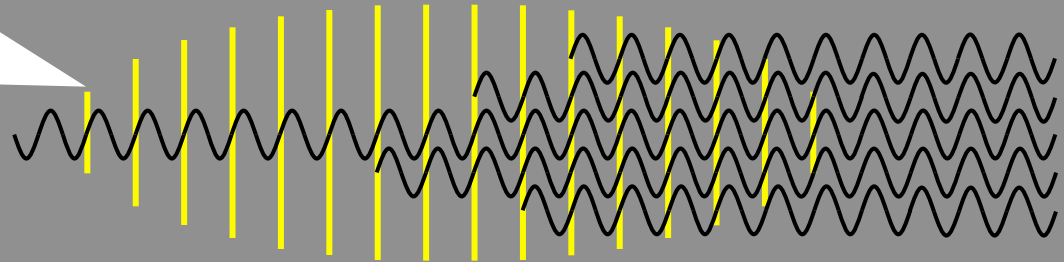
The electrons in the microbunched bunch emit in a coordinated way, amplifying the wave

# The difference made by microbunching:

With no microbunching, the electrons emit in an uncorrelated way



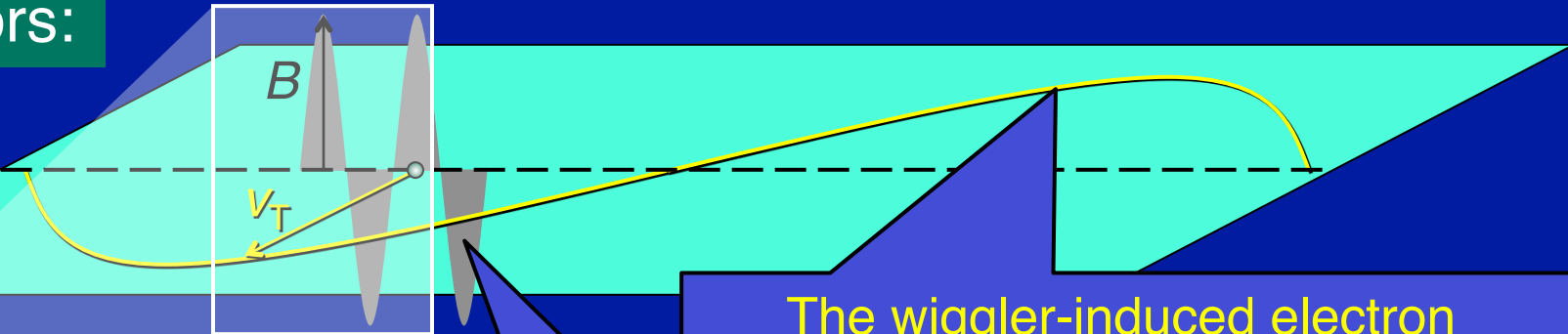
Instead, the electrons confined to the microbunches emit in a correlated way, enhancing previously emitted waves



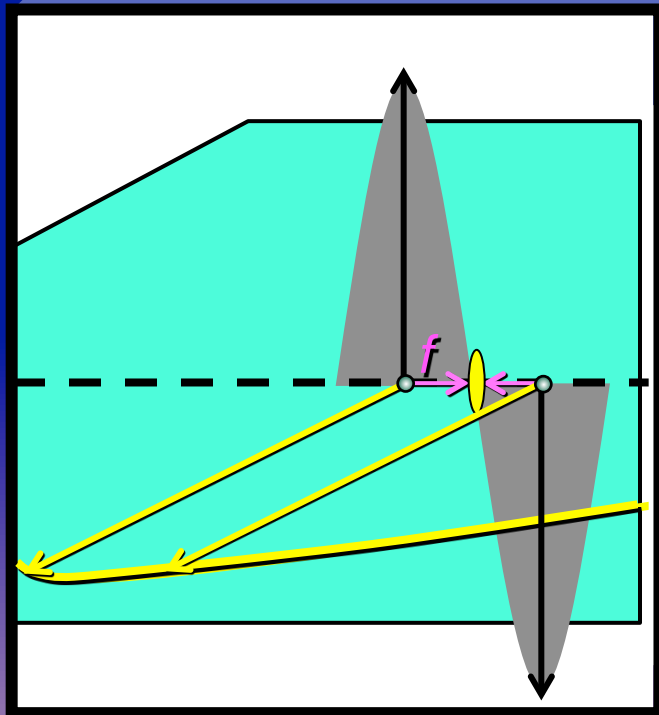
Without microbunching, the wave intensity is proportional to the number of electrons,  $N$ . With microbunching and correlated emission, the wave amplitude is proportional to  $N$ , and the wave intensity is **proportional to  $N^2$** .

# What creates the electron microbunches?

two factors:

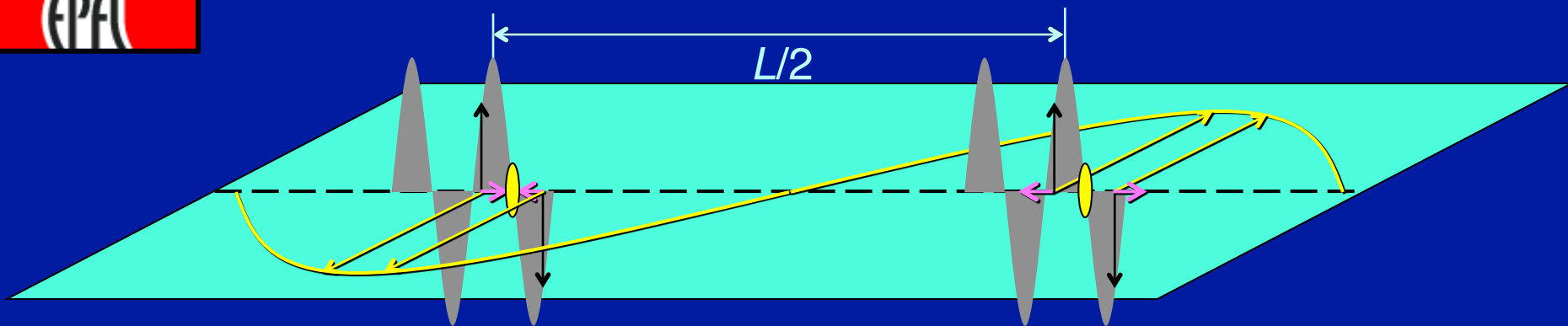


The wiggler-induced electron oscillations ( $v_T$  = transverse velocity)



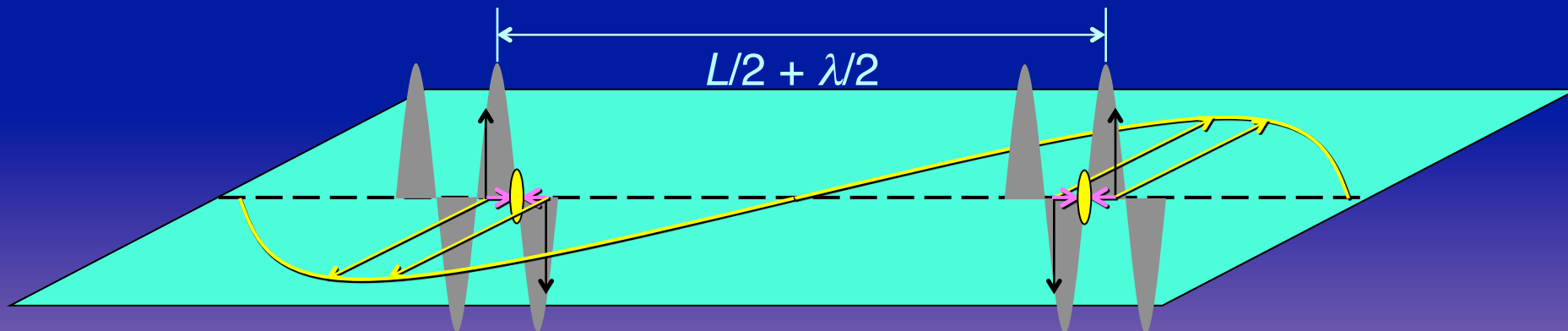
The  $B$ -field of a previously emitted photon wave

The wave  $B$ -field and the electron transverse velocity  $v_T$  produce Lorentz forces  $f$  that push the electrons towards zero-field points: is this what causes microbunching?



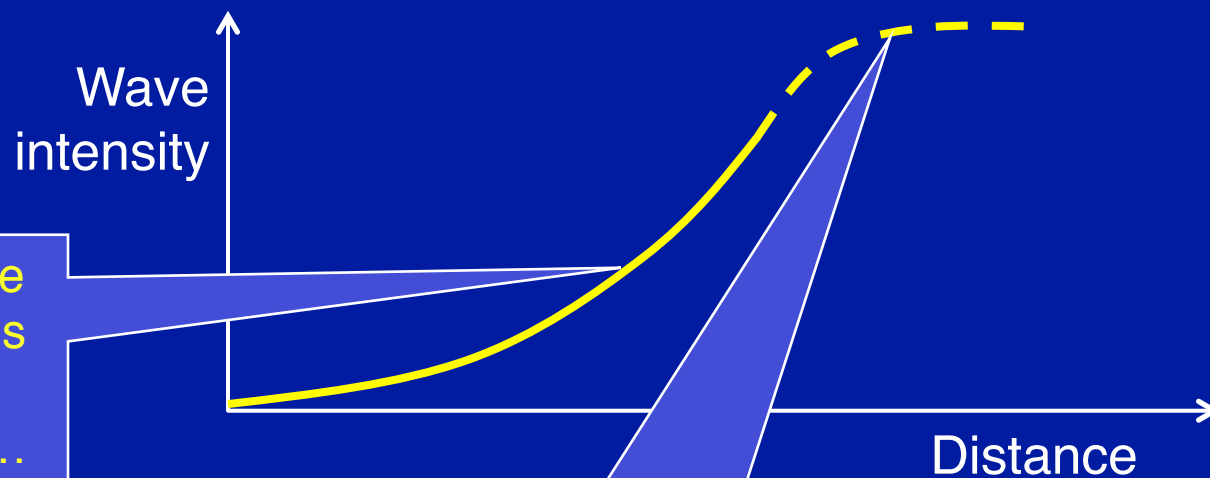
...maybe, but something seems wrong: after  $1/2$  wiggler period, the transverse velocity is reversed. The  $B$ -field is the same, if the wave travels with the electron: are the forces reversed and the microbunching destroyed ?

No! Electron and wave do not travel together: the electron speed is  $v < c$ . As the electron travels over  $L/2$  in a time  $L/(2v)$ , the wave travels over  $[L/(2v)]c$ . The path difference is  $(L/2)(c/v - 1) \approx L/(4\gamma^2) = \underline{\text{half wavelength}}$



Both the  $B$ -fields and the transverse velocities are reversed: the forces are not, and continue to microbunch the electrons

# Microbunching produces a progressive gain of the wave intensity (Self-amplified Spontaneous Emission or SASE)



Because of the gain, the wave intensity increases exponentially with the distance in the wiggler...

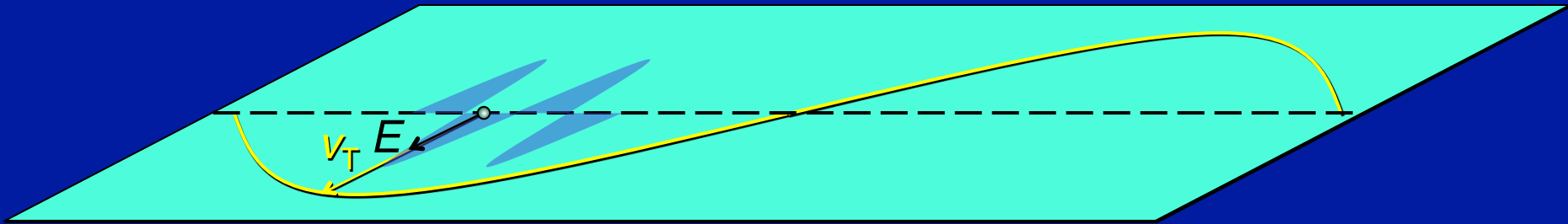
...until maximum microbunching is reached and the gain saturates

For an x-ray FEL (no 2-mirror cavity), gain saturation must be reached before the end of the (very long) wiggler, in a single pass

# Why the exponential intensity increase?

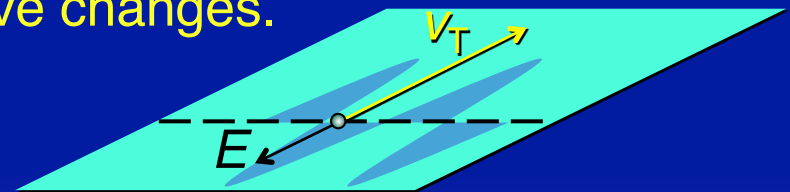
- Define:  $I$  = wave intensity;  $v_{\perp}$  and  $v$  = electron transverse and longitudinal velocities;  $E$  and  $B$  = wave  $E$ -field and  $B$ -field (each proportional to  $I^{1/2}$ )
- $dI/dt$  = energy transfer rate from the electrons to the wave, determined by: (1) the transfer rate for one electron, (2) the microbunching
- The one-electron transfer rate is given by the (negative) work, proportional to  $Ev_{\perp}$  and therefore to  $I^{1/2}$
- Microbunching is caused by the Lorentz force, proportional to  $v_{\perp}B$  and therefore to  $I^{1/2}$
- Overall,  $dI/dt$  is proportional to  $I^{1/2} I^{1/2} = I$ ; this gives an exponential increase of  $I$  vs  $t$  – and vs the distance =  $vt$

# Why does the intensity increase saturate?

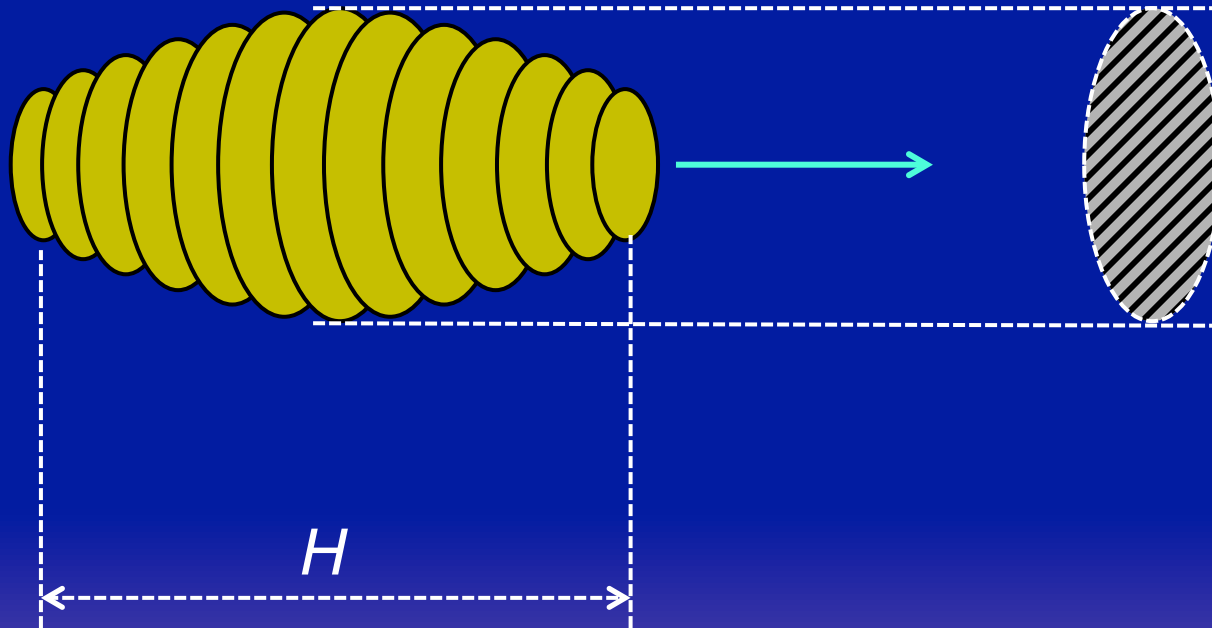


For the electron  $\rightarrow$  wave energy transfer, the directions of  $v_T$  and of the wave  $E$ -field must produce negative work. This is true in the case above

- But as the electron gives energy to the wave, it slows down and the phase of  $v_T$  relative to the wave changes.
- Eventually, the conditions are reversed leading to wave  $\rightarrow$  electron energy transfer
- This accelerates the electrons until the conditions are restored for electron  $\rightarrow$  wave energy transfer
- The mechanism goes on and on, producing an energy oscillation between electrons and wave rather than a continuous increase of the wave intensity: hence, saturation



# Geometry and duration of an FEL pulse:

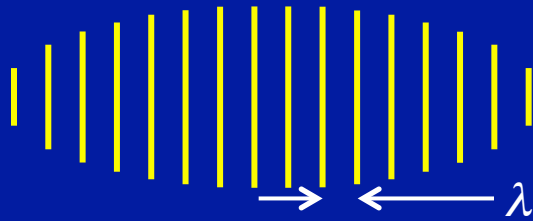


The electron bunch cross section determines the transverse size of the photon pulse. The excellent electron beam control makes it very small

The electron bunch length  $H$  determines the duration  $H/c$  of the photon pulse, which is typically in the femtosecond range



# Why is microbunching (and free electron lasing) more difficult for x-rays than for infrared photons?



At short wavelengths the microbunches are closer to each other, and this should facilitate the microbunching

## BUT:

- Short wavelengths  $L/(2\gamma^2)$  require a large  $\gamma$  (electron energy)
- This makes the electrons “heavy” and very difficult to move to the microbunches, as their longitudinal relativistic mass is  $\gamma^3 m_0$
- Furthermore, the small spacing between microbunches makes the microbunch structure very vulnerable and requires an excellent control system

April 21, 2009

New Era of Research Begins as World's First Hard X-ray Laser Achieves First Light

*X-ray laser pulses of unprecedented energy and brilliance produced at SLAC*



**Claudio Pellegrini, UCLA -- father of the X-FEL theory**

# Some general questions:

- The central emitted photon energy of an undulator in a storage ring with energy  $E = 2$  GeV is  $h\nu = 3$  keV. What is the undulator period  $L$ ?
- The emitted radiation from a bending magnet is confined within a vertical angle  $\approx 2.0$  milliradians. What is the energy in GeV of the storage ring?
- Find the wavelength bandwidth of a bending magnet with  $B = 1.5$  tesla in a 1.2 GeV storage ring
- The pulse duration of an x-FEL is 100 femtoseconds. The central emitted photon energy is 500 eV. How many microbunches are there in each electron bunch?
- Synchrotron-radiation and FEL experiments are always very expensive: true or false?

# Answers:

- The central emitted photon energy of an undulator in a storage ring with energy  $E = 2$  GeV is  $h\nu = 3$  keV. What is the undulator period  $L$ ?

$$\lambda_c = 1.24 \times 10^4 / (3 \times 10^3) \approx 41 \text{ \AA}; \quad \gamma = 2 \times 10^3 / 0.51 \approx 3.9 \times 10^3;$$
$$L = 2\gamma^2 \lambda_c \approx 2 \times (3.9 \times 10^3)^2 \times 41 \times 10^{-10} \approx 0.12 \text{ m}$$

- The emitted radiation from a bending magnet is confined within a vertical angle  $\approx 2.0$  milliradians. What is the energy in GeV of the storage ring?

$$2/\gamma \approx 2 \times 10^{-3} \text{ rad}; \quad \gamma \approx 10^3; \quad \text{energy} \approx 10^3 \times 0.51 \text{ MeV} = 0.51 \text{ GeV}$$

- Find the wavelength bandwidth of a bending magnet with  $B = 1.5$  tesla in a 1.2 GeV storage ring

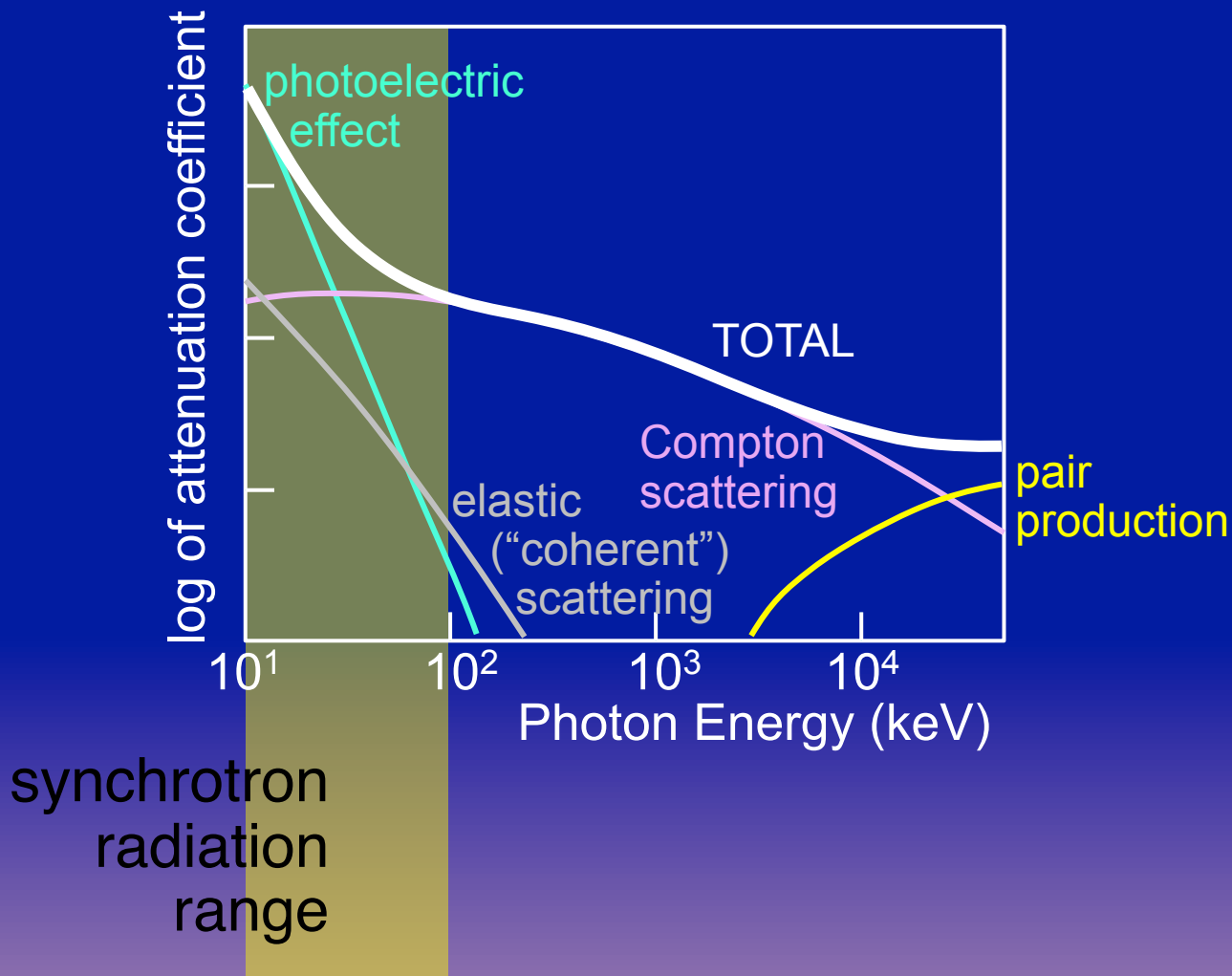
$$\gamma \approx 1.2 \times 10^3 / 0.51 \approx 2.4 \times 10^3; \quad \Delta\lambda = \lambda_c \approx 2\pi m_0 c / (2\gamma^2 e B)$$
$$\approx 2\pi \times 9 \times 10^{-31} \times 3 \times 10^8 / [2 \times (2.4 \times 10^3)^2 \times 1.6 \times 10^{-19} \times 1.5]$$
$$\approx 6.1 \times 10^{-10} \text{ m} = 6.1 \text{ \AA}$$



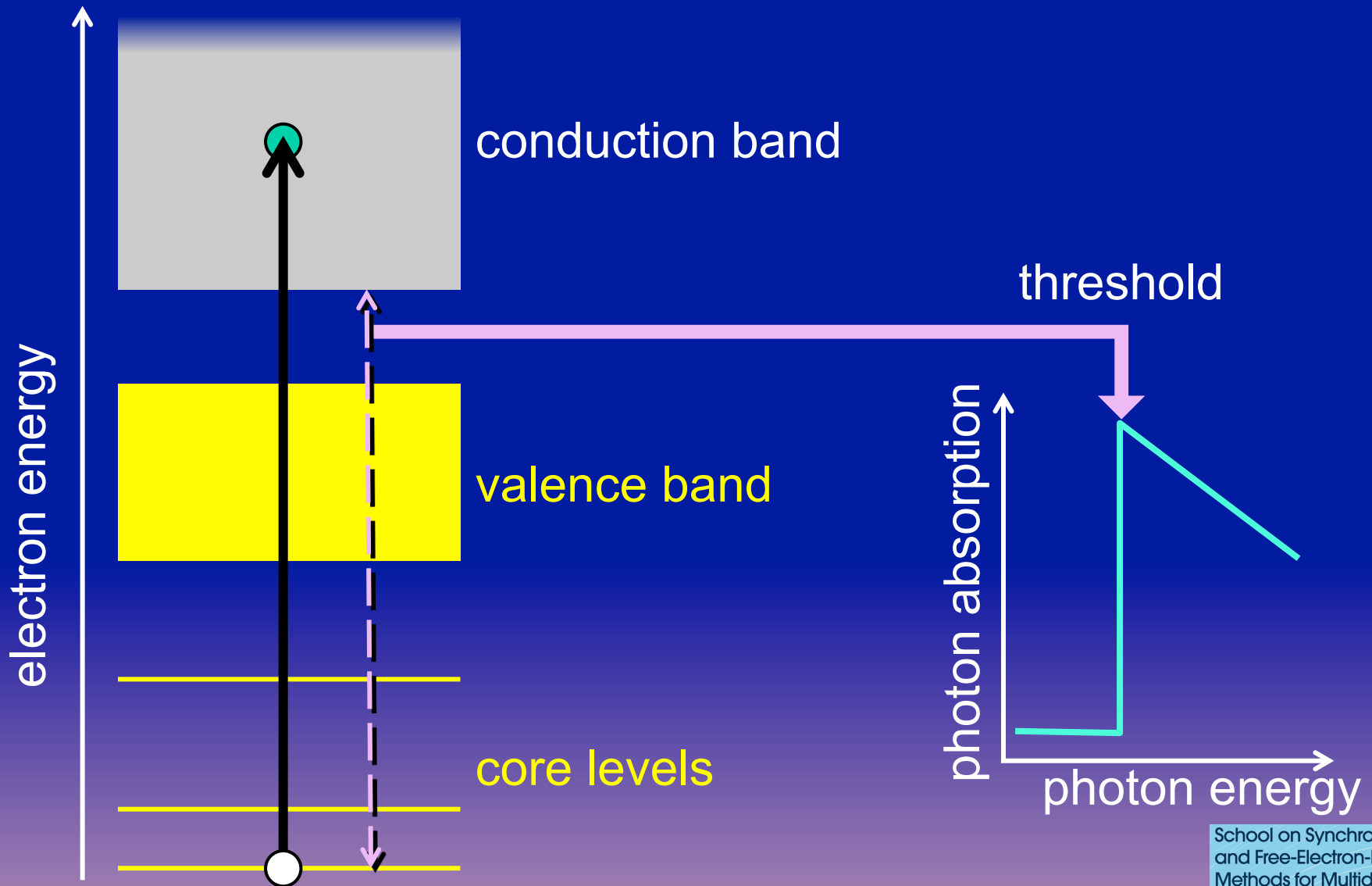
# Fundamentals of the Interactions of X-rays with Matter: Advantages of Coherence

Giorgio Margaritondo  
Ecole Polytechnique Fédérale de Lausanne (EPFL)

# X-rays interact with matter in many different ways:

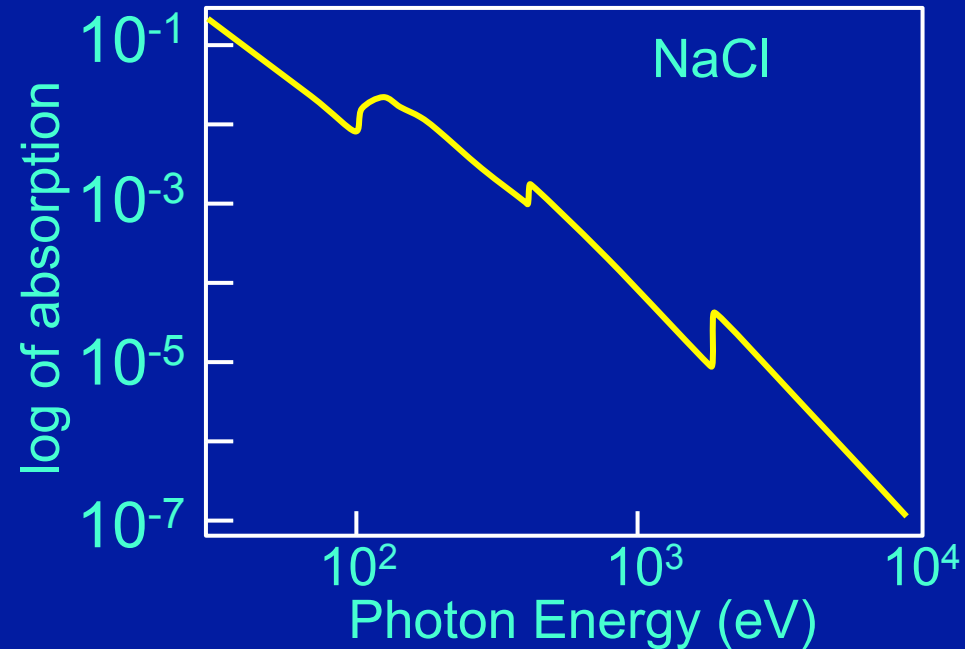
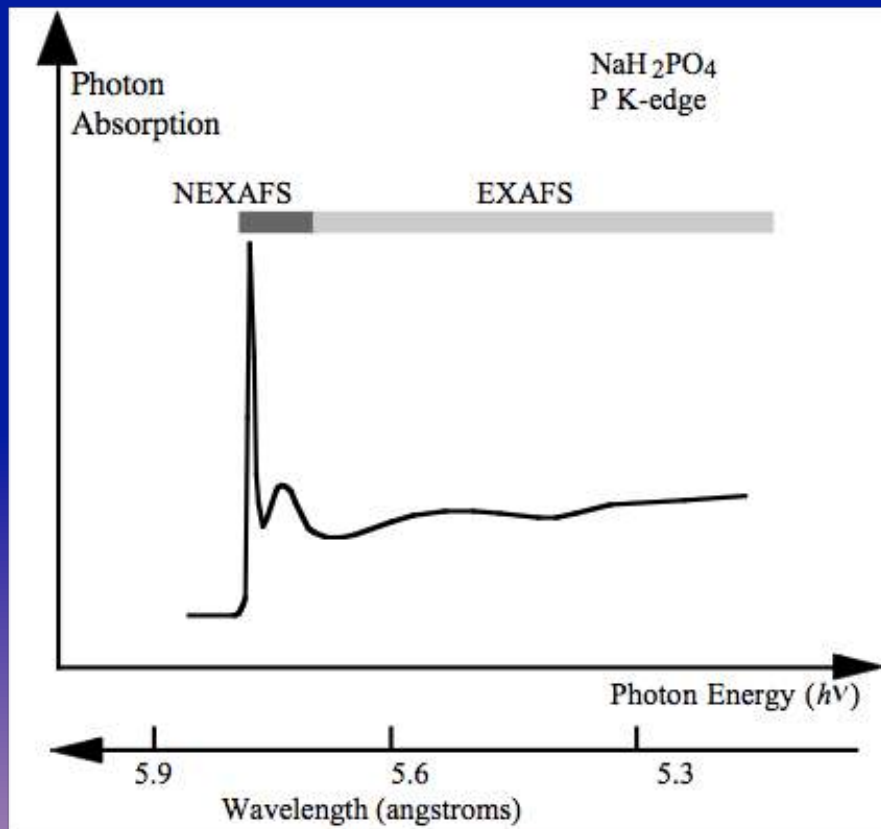


# X-rays absorption: optical transitions from a core level in a solid



# Core-level absorption thresholds: details

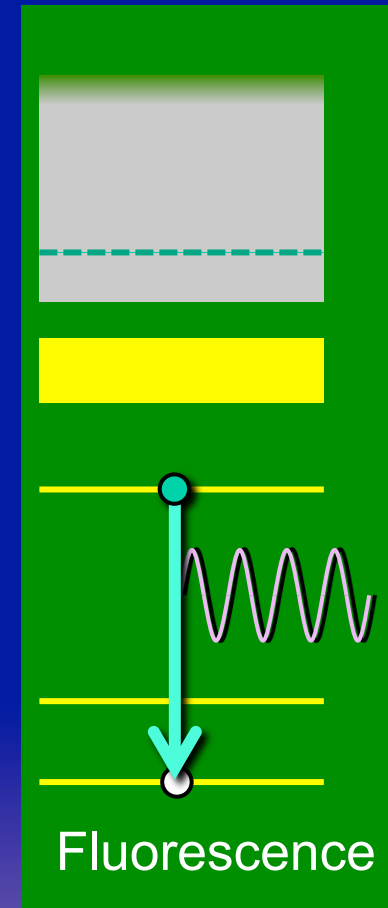
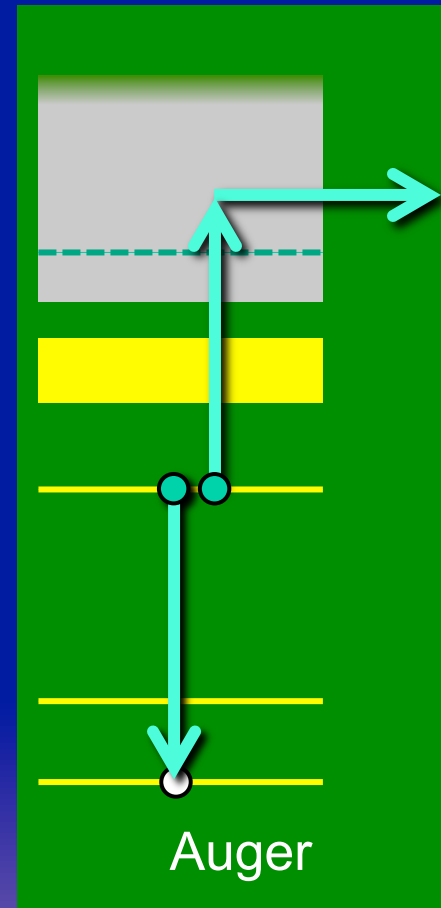
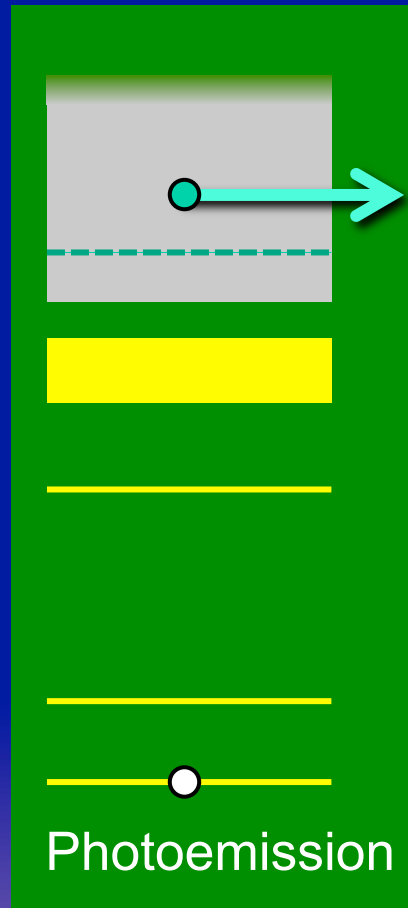
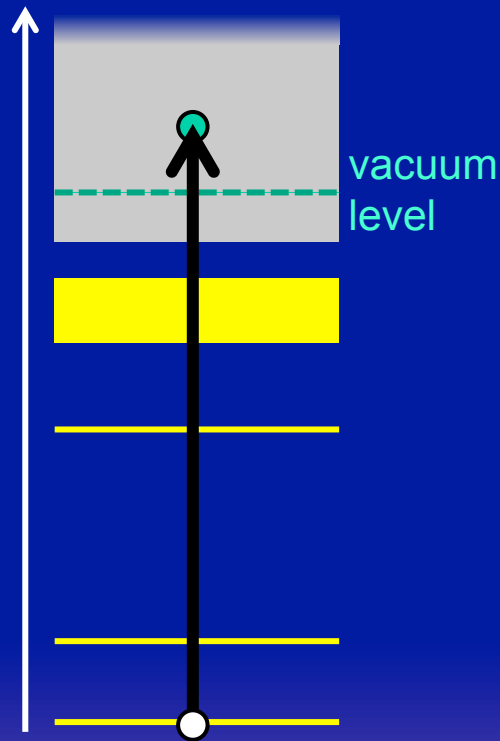
at first glance, in the absorption spectrum (yellow curve) we do see thresholds



but above each threshold there is a modulation carrying very valuable information on the local structure of the solid (EXAFS = extended x-ray absorption fine structure; NEXAFS = near-edge x-ray absorption fine structure)

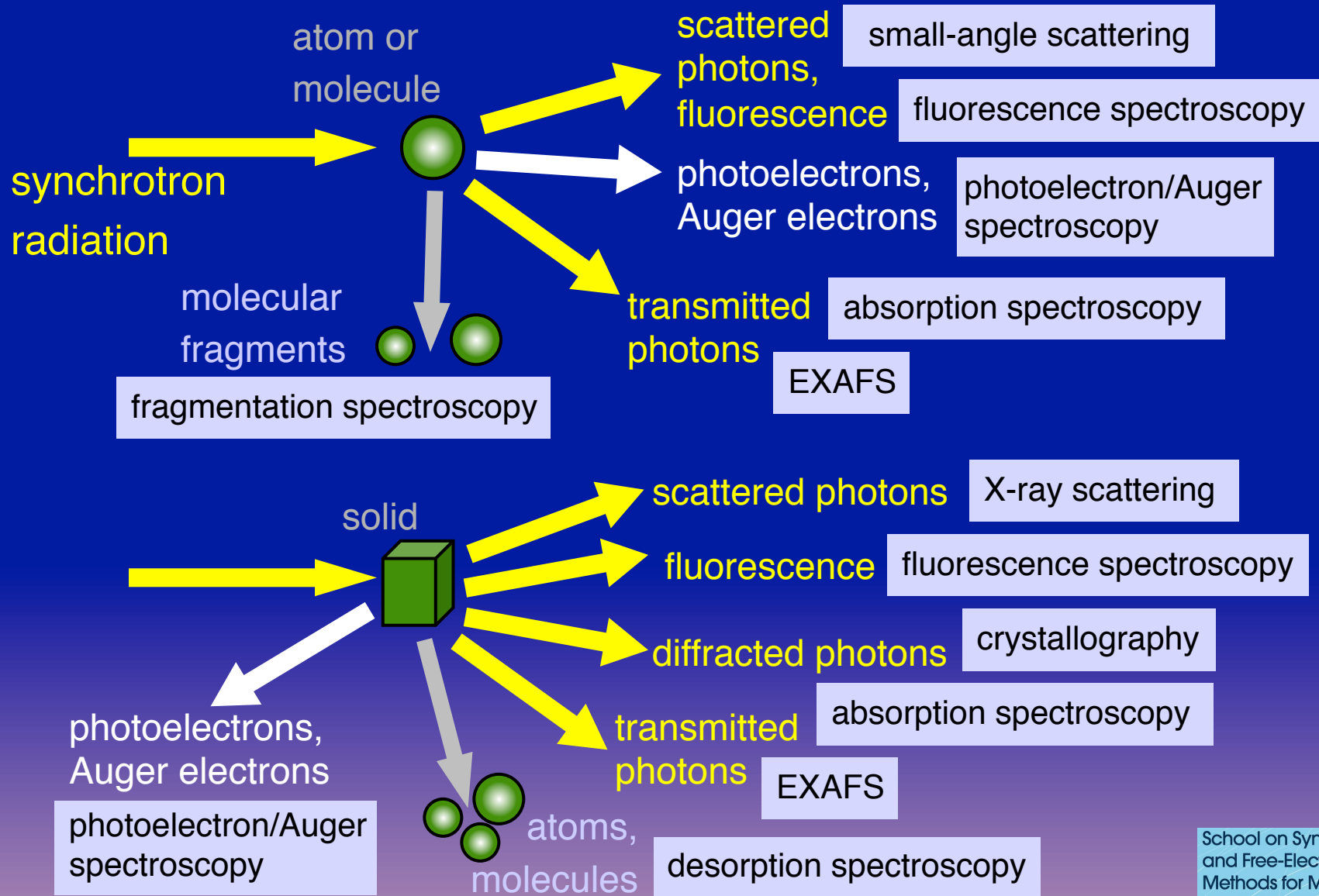


# How a system reacts after a core-level excitation: several different effects



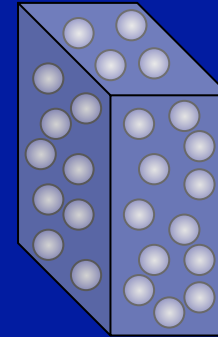
each effect is the foundation of different experimental techniques

# In general: many interactions, many experiments



# Example: photoemission spectroscopy

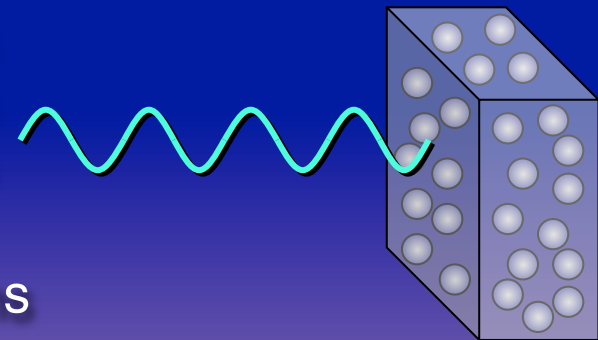
in a solid (or a molecule),  
the electrons cause the  
most important properties



it would be nice to capture these electrons so as to be able to analyze them – but how, if they are confined in the solid?

answer: the  
photoemission  
effect

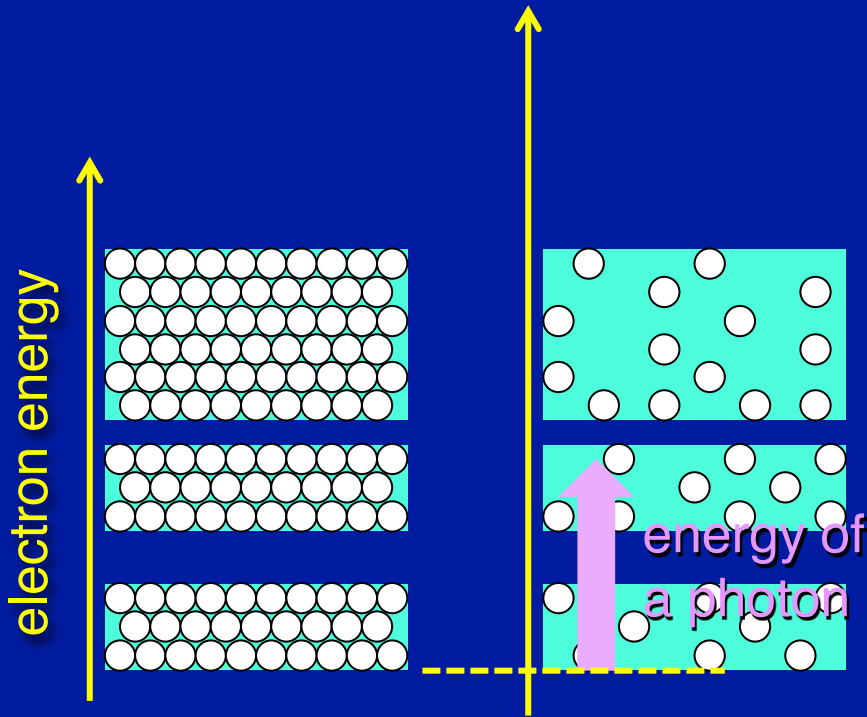
synchrotron  
radiation  
photons



photoelectrons

we can capture and analyze the photoelectrons in vacuum –  
and retrieve the properties of the electrons in the solid

# How to use photoelectrons:



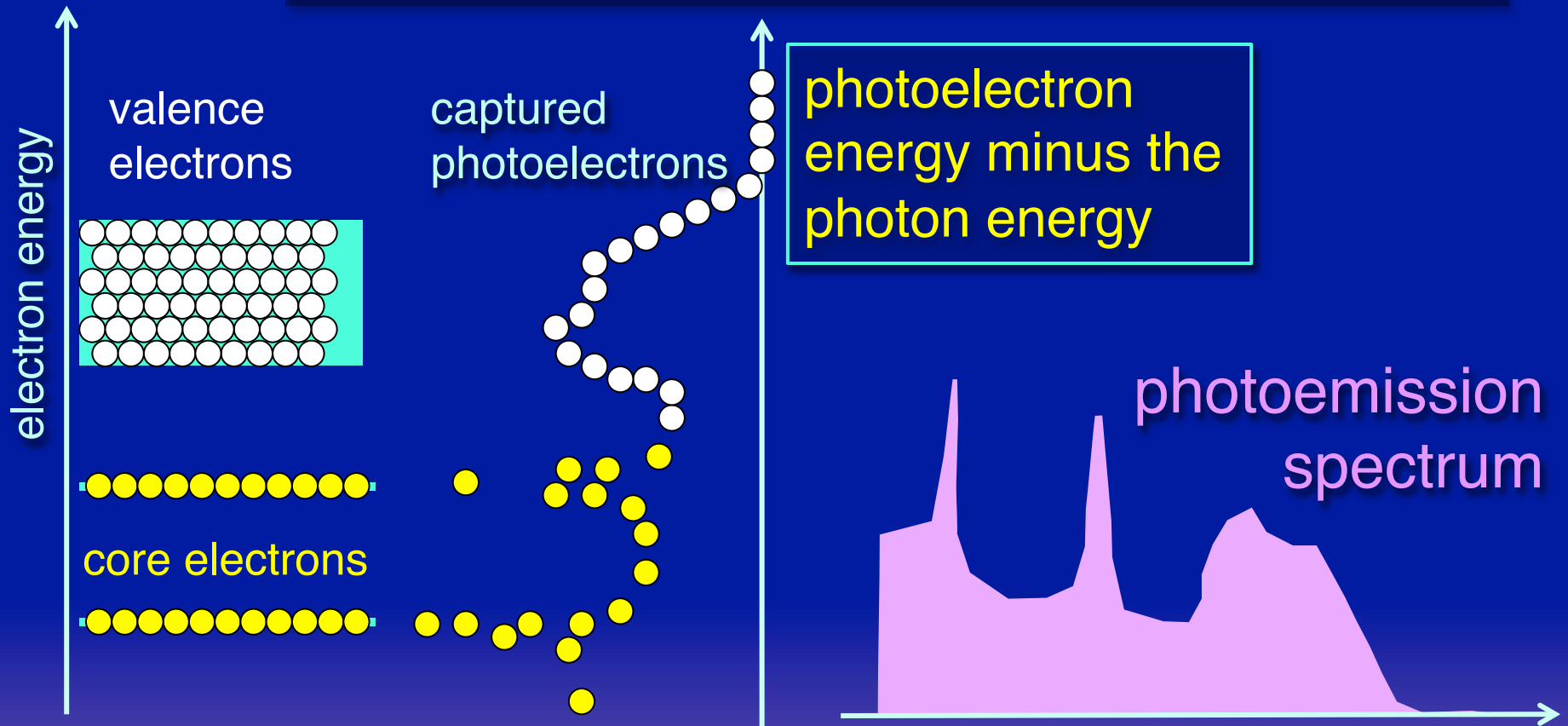
distribution in energy of the electrons inside the solid

photoelectron energy distribution in vacuum

...one measures the energies of the photoelectrons: after subtracting the photon energy, they give the energies of the same electrons when they were inside the solid

...an experimental technique of fundamental importance: 237,000 published articles

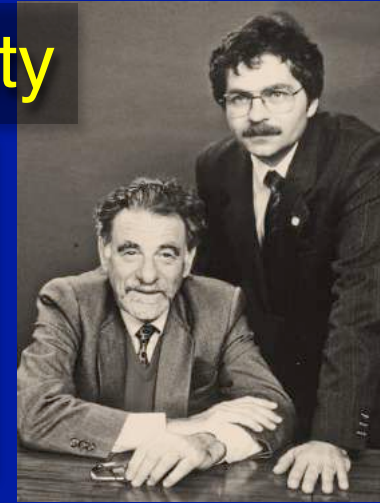
# photoemission detects valence electrons and core electrons:



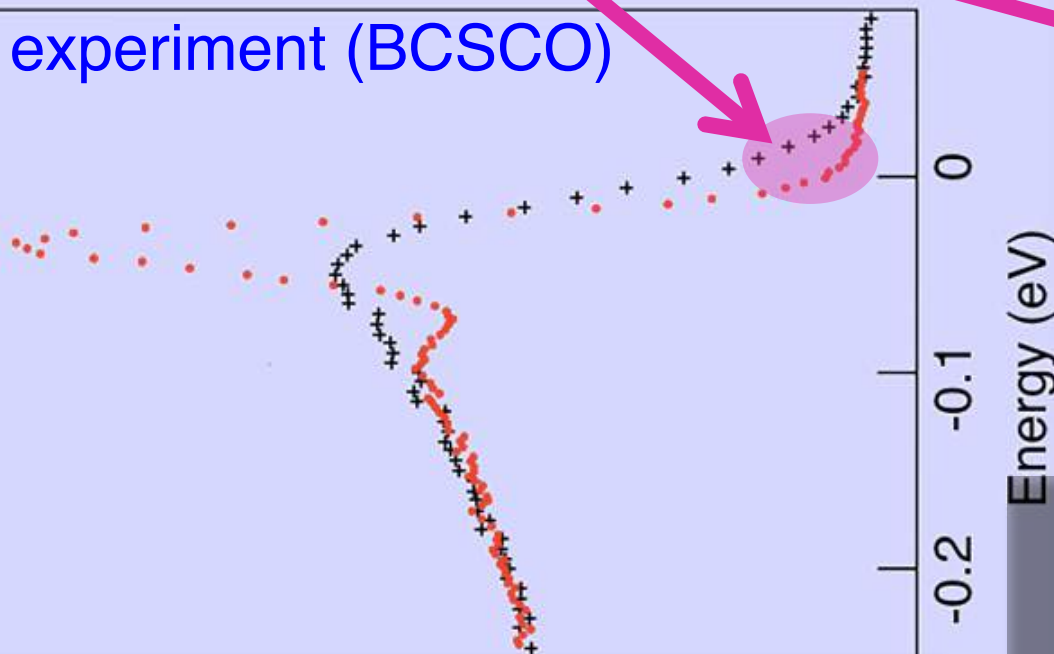
...bookish quantum notions become tangible realities!

# Another example: superconductivity

after Alex Müller and Georg Bednorz discovered in Switzerland high-temperature superconductors, synchrotron photoemission could directly detect the **superconductivity gap**

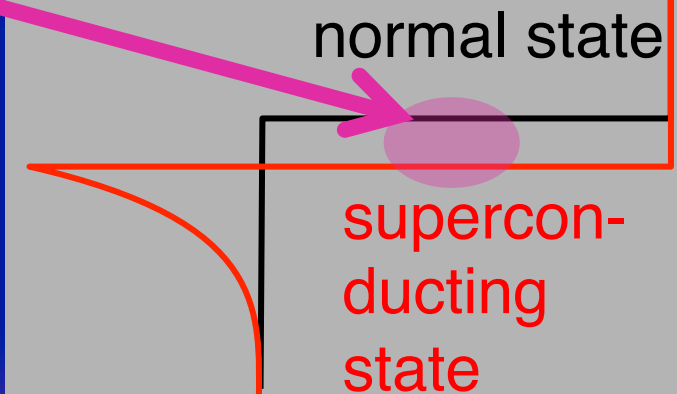


experiment (BCSCO)



[experimental spectra by Y. Hwu et al.]

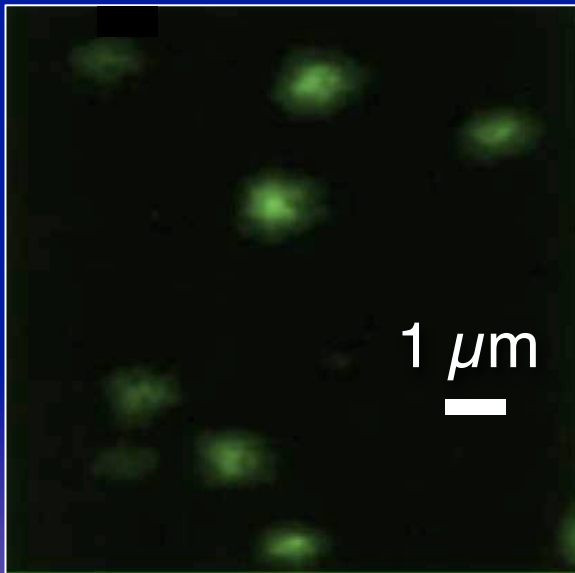
theory



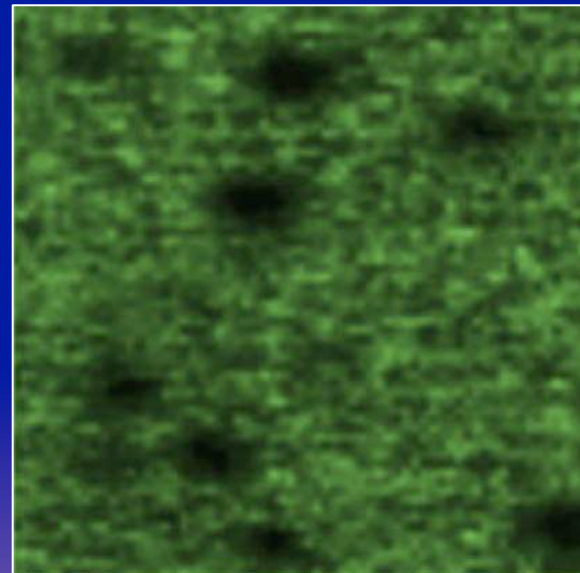
...here again, quantum theories become reality!

# Photoemission spectromicroscopy: chemical pictures on a microscopic scale

photoemission micrographs of the  
Au + Ag/Si interface ( $h\nu = 495$  eV)  
[M. Marsi et al., J. Electron Spectroscopy **84**, 73 (1997)]



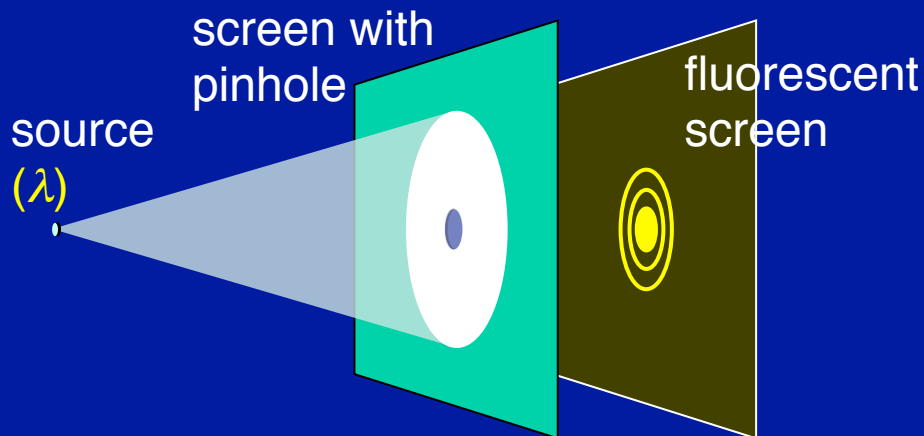
Ag3d photoelectrons



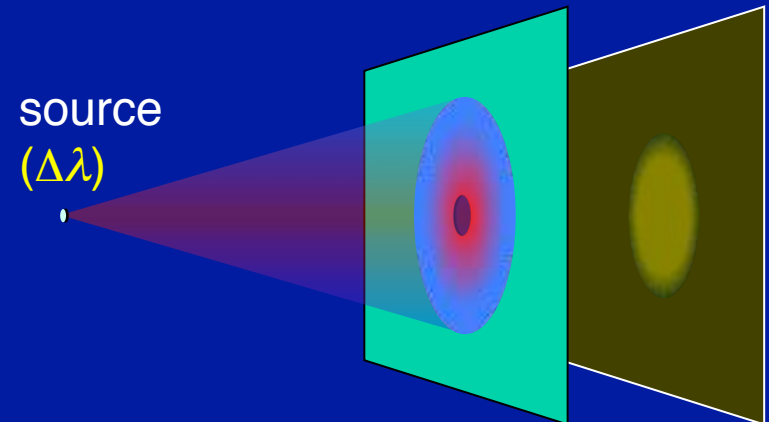
Si2p photoelectrons

THE CHEMICAL CONTRAST IS REVERSED!

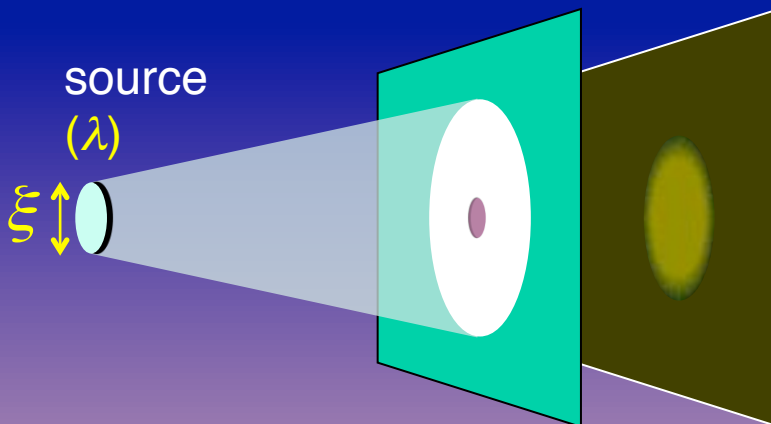
**Coherence:** "the property that enables a wave to produce **visible** diffraction and interference effects" (such as pinhole diffraction)



A **point** source emitting only **one wavelength** always produces a visible diffraction pattern



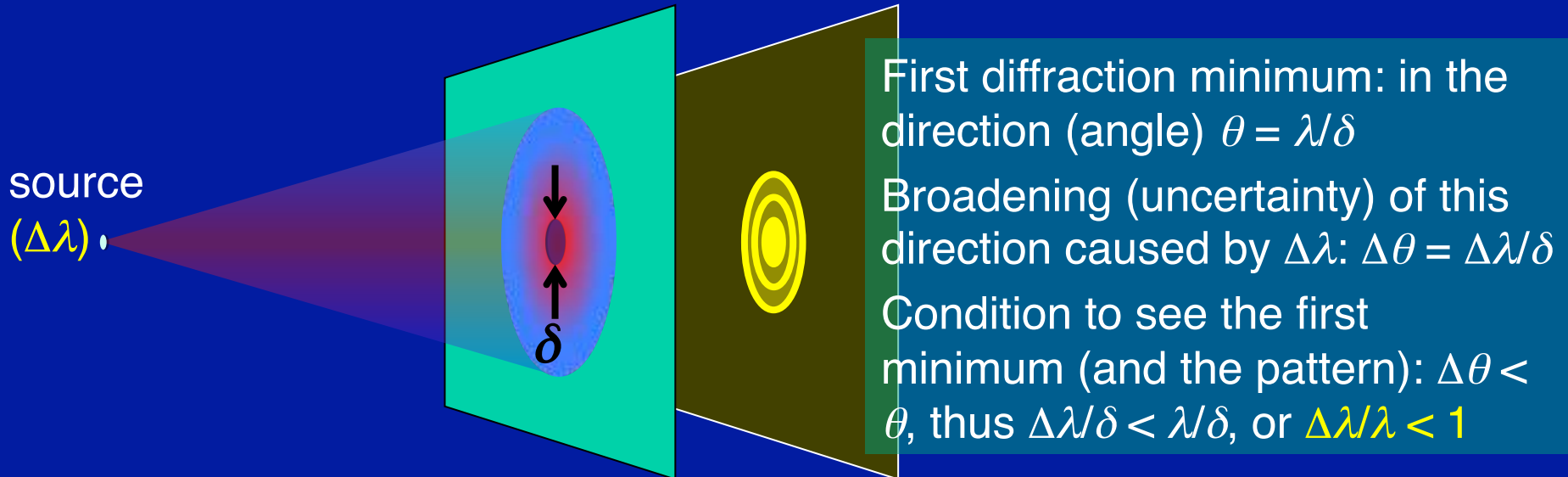
...but if the source emits a **band** of wavelengths, the pattern may no longer be visible



Likewise, if the source has a **finite size** the pattern may become impossible to see

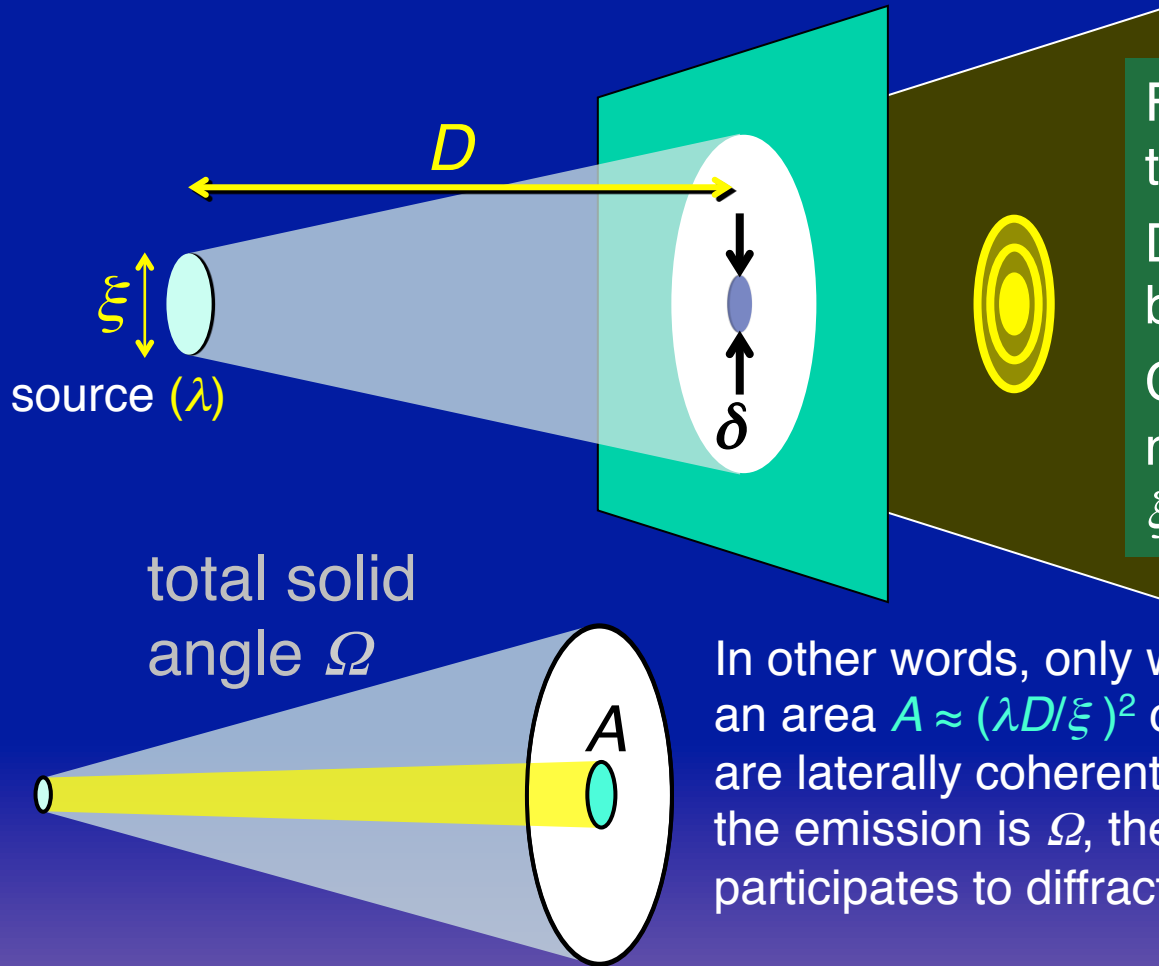


# Consequences of the wavelength bandwidth: longitudinal (time) coherence:



- Parameter that characterizes the longitudinal coherence: the “coherence length”  $L_c = \lambda^2/\Delta\lambda$
- Minimum condition of longitudinal coherence:  $L_c > \lambda$

# Role of the source geometry: lateral (spatial) coherence:



First diffraction minimum: in the direction (angle)  $\theta = \lambda/\delta$   
 Direction uncertainty caused by the source size:  $\Delta\theta \approx \xi/D$   
 Condition to see the first minimum (and the pattern):  
 $\xi/D \leq \lambda/\delta$ , or  $\delta \leq \lambda D/\xi$

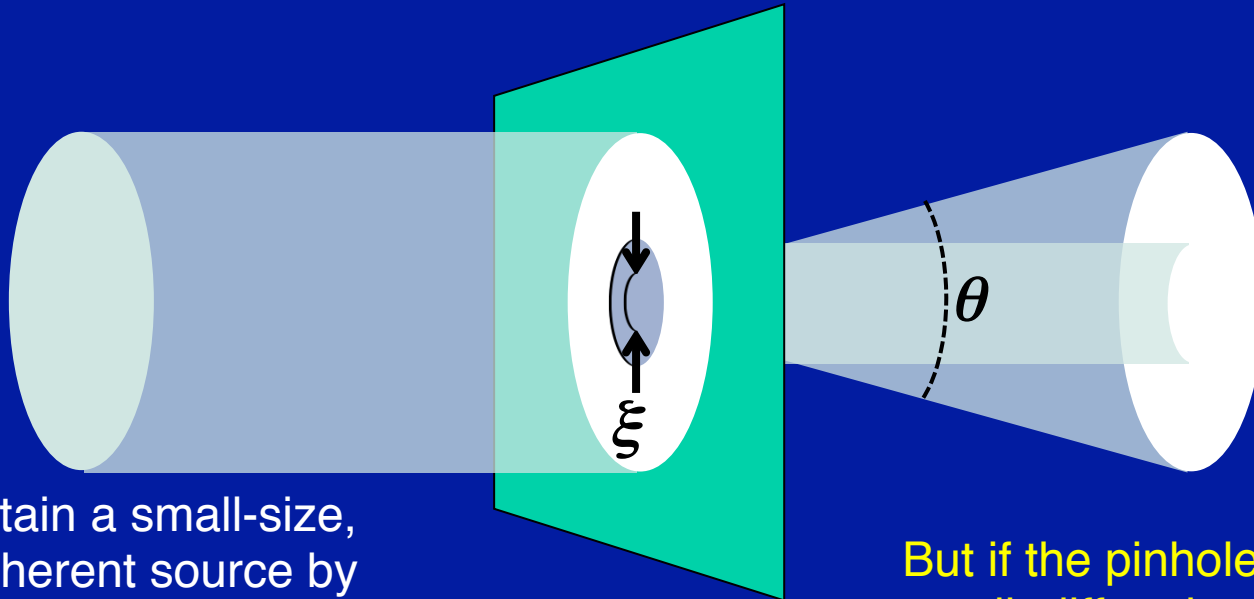
In other words, only waves reaching the screen within an area  $A \approx (\lambda D/\xi)^2$  can contribute to diffraction and are laterally coherent. If the (solid) angular spread of the emission is  $\Omega$ , the portion of the emission that participates to diffraction is  $(\lambda D/\xi)^2 / (\Omega D^2) = \lambda^2 / (\xi^2 \Omega)$

this defines the “coherent power”  $\approx \text{constant} \frac{\lambda^2}{\xi^2 \Omega}$

# Coherence — summary:

- Longitudinal (time) coherence requires a large enough coherence length,  $\lambda^2/\Delta\lambda > \lambda$  or  $\lambda/\Delta\lambda > 1$
- Lateral (space) coherence requires a large coherent power factor  $\lambda^2/(\xi^2\Omega)$
- Both are difficult to achieve for small wavelengths (x-rays)
- The conditions for large coherent power are **equivalent** to the geometric **conditions for high brightness**: small  $\xi^2$  and  $\Omega$

# Full lateral coherence:



We can obtain a small-size, laterally coherent source by passing a wave through a pinhole

But if the pinhole becomes too small, diffraction increases the angular divergence

The diffraction theory gives  $\xi\theta \approx \lambda$ . The solid-angle divergence is  $\Omega \approx \theta^2$ , so  $\xi^2\Omega \approx \lambda^2$

This is the “diffraction limit” for the coherent power factor:

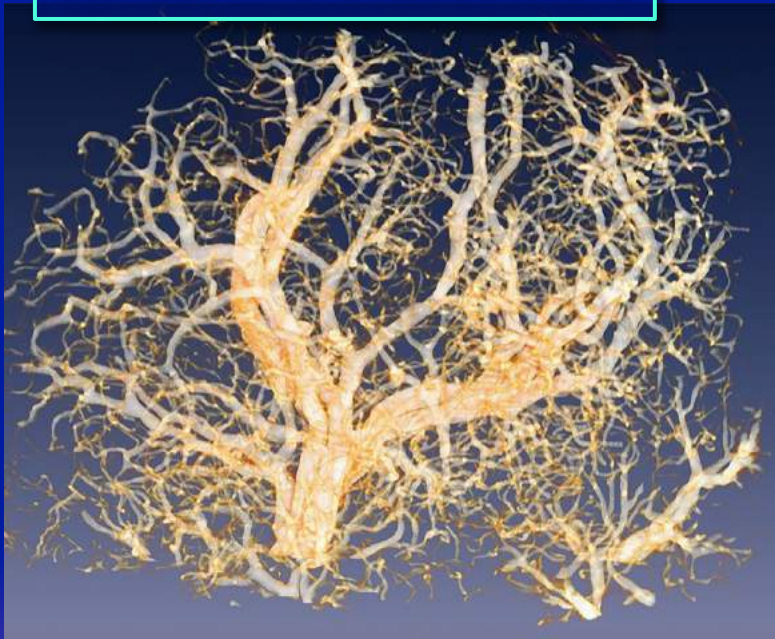
$$\frac{\lambda^2}{\xi^2\Omega} \approx 1$$

...corresponding to full lateral coherence

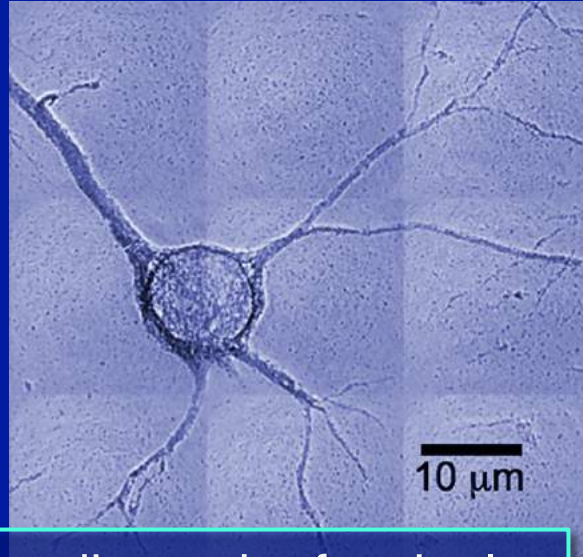
x-FELs and some synchrotrons now reach this limit!

# How to use coherence for radiology -- the main application of x-rays

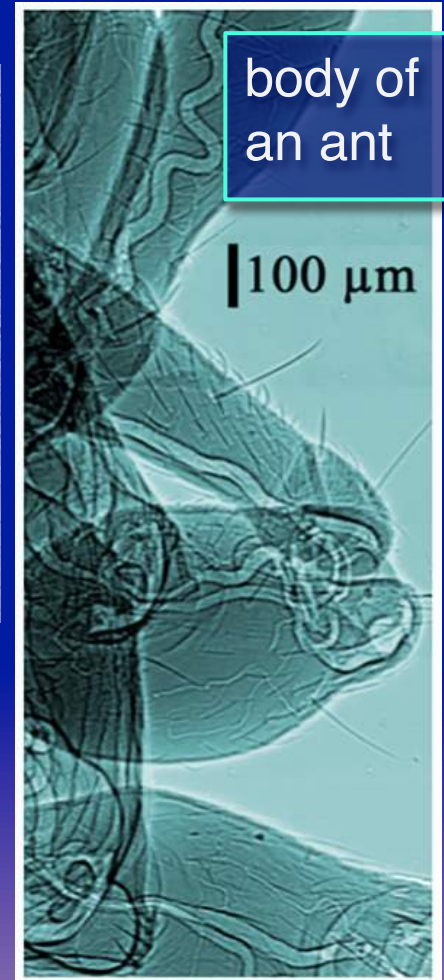
cancer microvasculature



[Y. Hwu et al.]



radiograph of a single neuron: world record of spatial resolution



body of an ant

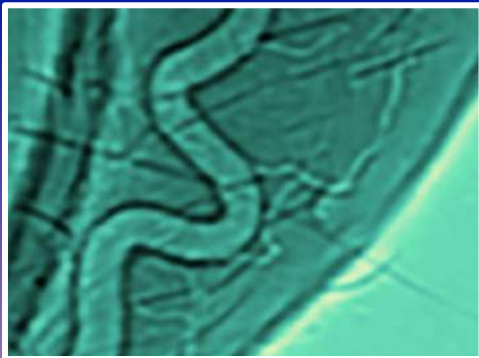
excellent contrast, detection of very small details: how is it possible?

# Think about "seeing" a wine glass:

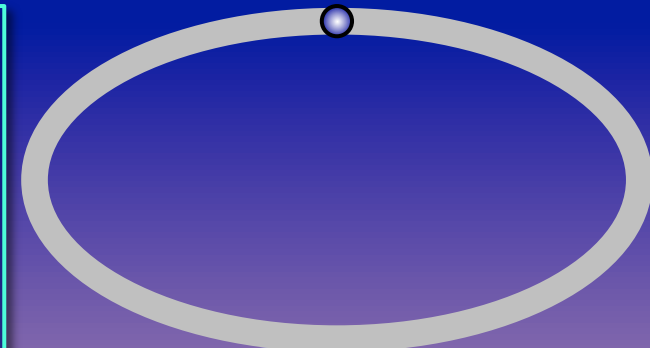
you detect the wine because it absorbs certain colors and looks red

but you also see the edges of the (transparent) glass because they deviate the light by refraction/scattering

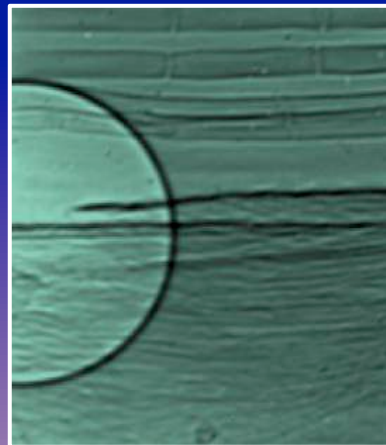
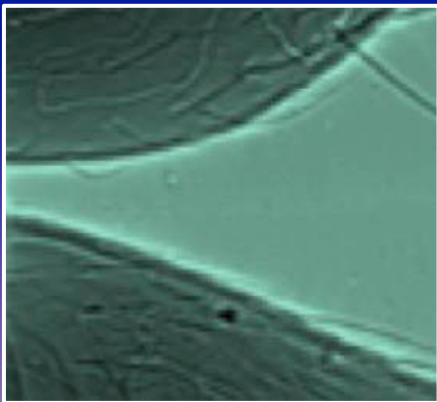
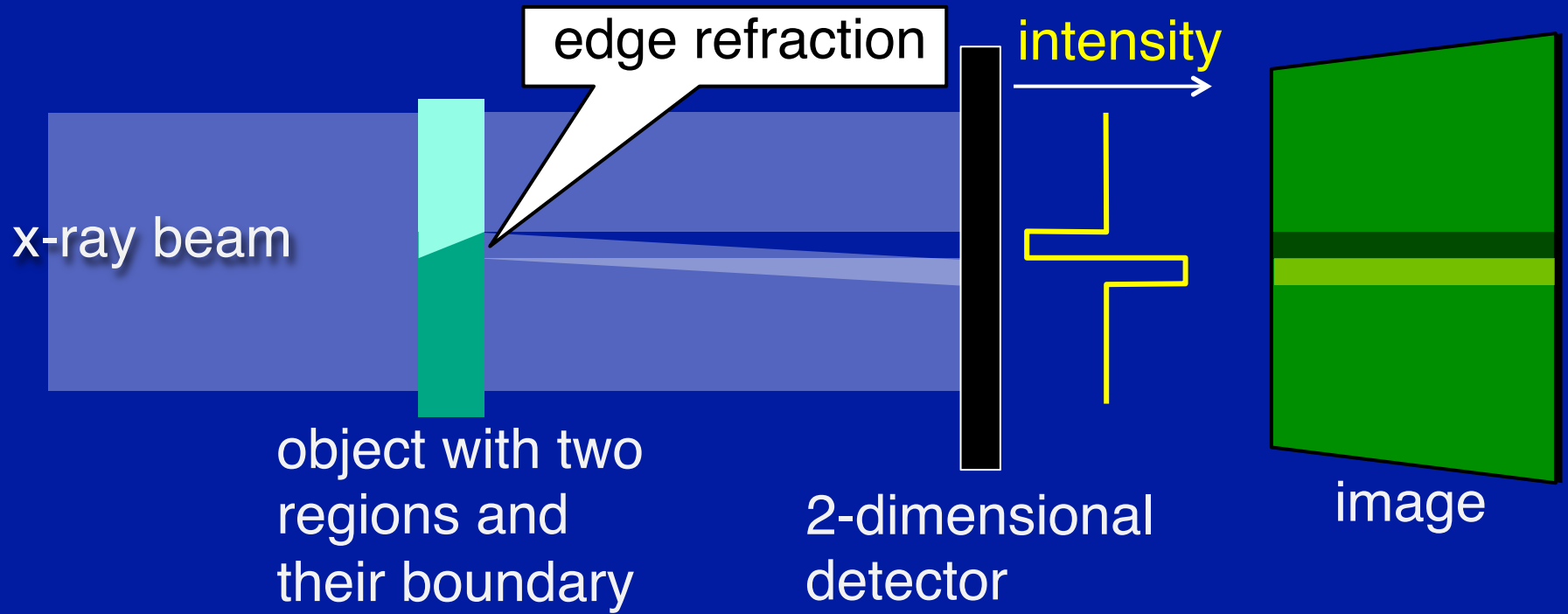
likewise, "phase contrast" (refraction/scattering) causes sharp, highly visible edges in synchrotron radiographs; however, this mechanism requires an x-ray beam with a well-defined direction



...which is true, since lateral coherence implies a strong angular collimation

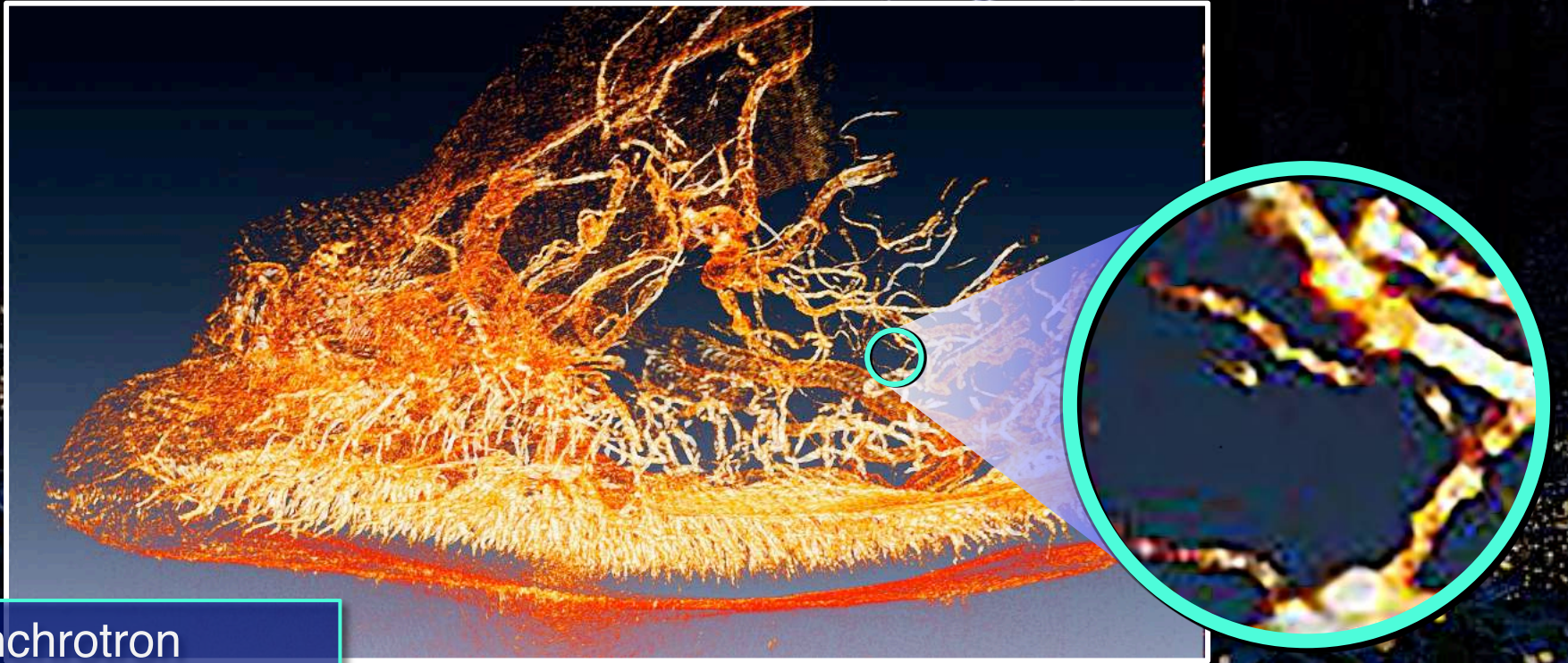


# Coherent radiology: a simple model



real coherent radiographs do exhibit the characteristic bright-dark fringes caused by edge refraction

# Coherent synchrotron radiology: the magic light of fireflies



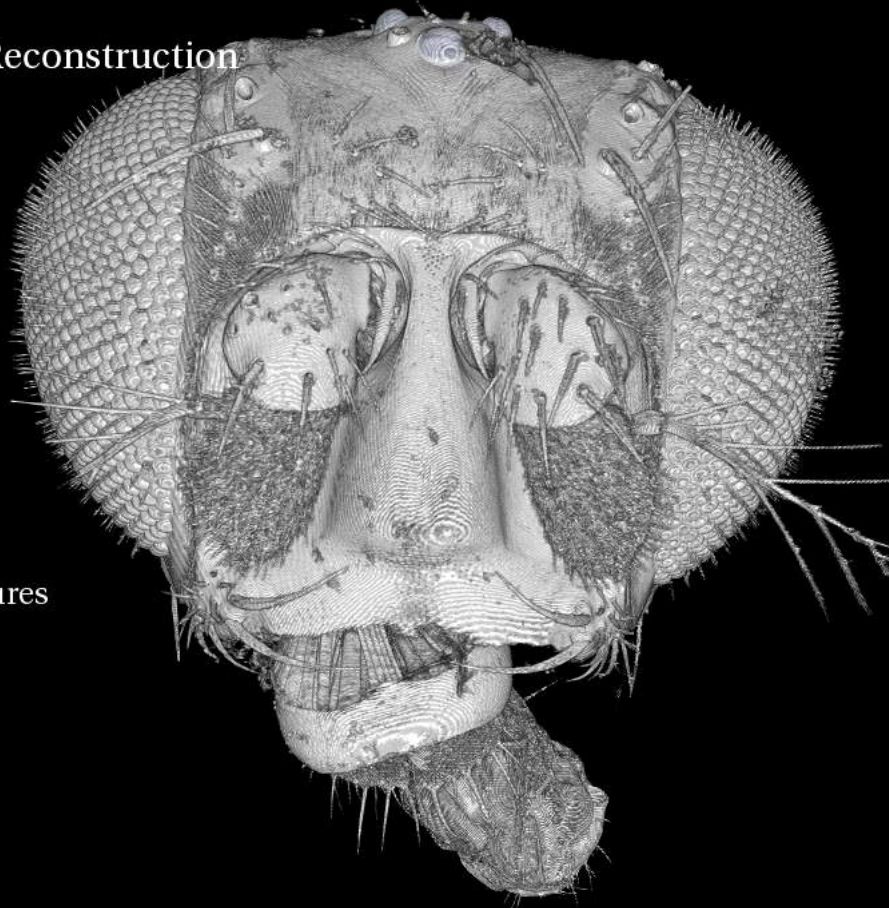
synchrotron  
microtomograph  
of a firefly  
“lantern” [Y. L.  
Tsai, Y. Hwu et al.]

...after detecting all vessels, including the smallest ones,  
we clarified the extremely effective emission mechanism



# CT-scans of today: drosophila

X-ray Tomographic Reconstruction



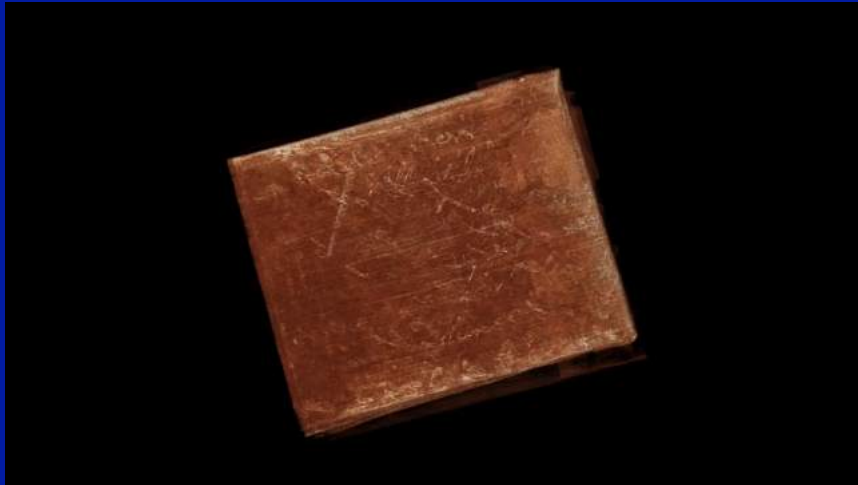
Fly Head General Structures

Ocelli

we can map one by one all the neurons of the insect brain!

[A.-S. Chiang, Y. Hwu et al.]

# Synchrotron tomography reads ancient manuscripts even under seal:



visible-light picture

x-ray image



so, for example, Lady Catarina Savonarolo of Venice could speak to us after 7 centuries



...all this, thanks to another remarkable Italian lady: Fauzia Albertin

# X-ray FELs are now reality: what can we do with them?



**The European  
x-FEL in  
Hamburg**



**The FERMI x-FEL  
at Elettra, Trieste**



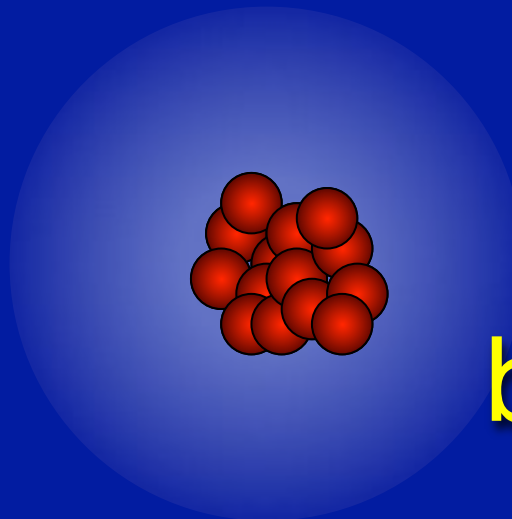
**The Swiss x-FEL at the  
Paul-Scherrer Institut**

# Operating X-ray FELs – parameters:

- FLASH at DESY (Germany):  
 $h\nu = 28\text{-}295$  eV, 30-300 fs pulses, max pulse energy  $500 \mu\text{J}$
- FERMI at ELETTRA (Italy):  
 $h\nu = 9\text{-}308$  eV, 40-100 fs pulses, max pulse energy  $100 \mu\text{J}$
- SACLA at SPRING-8 (Japan):  
 $h\nu = 5\text{-}19$  keV, down to 10 fs pulses, max pulse energy  $500 \mu\text{J}$
- LCLS at SLAC (USA):  
 $h\nu = 0.28\text{-}11.2$  keV, 10-300 fs pulses, max pulse energy  $3000 \mu\text{J}$

We are talking about several-gigawatt femtosecond pulses: what can we do with all this power?

...sent into a molecule or a nanoparticle,  
it causes an explosion:

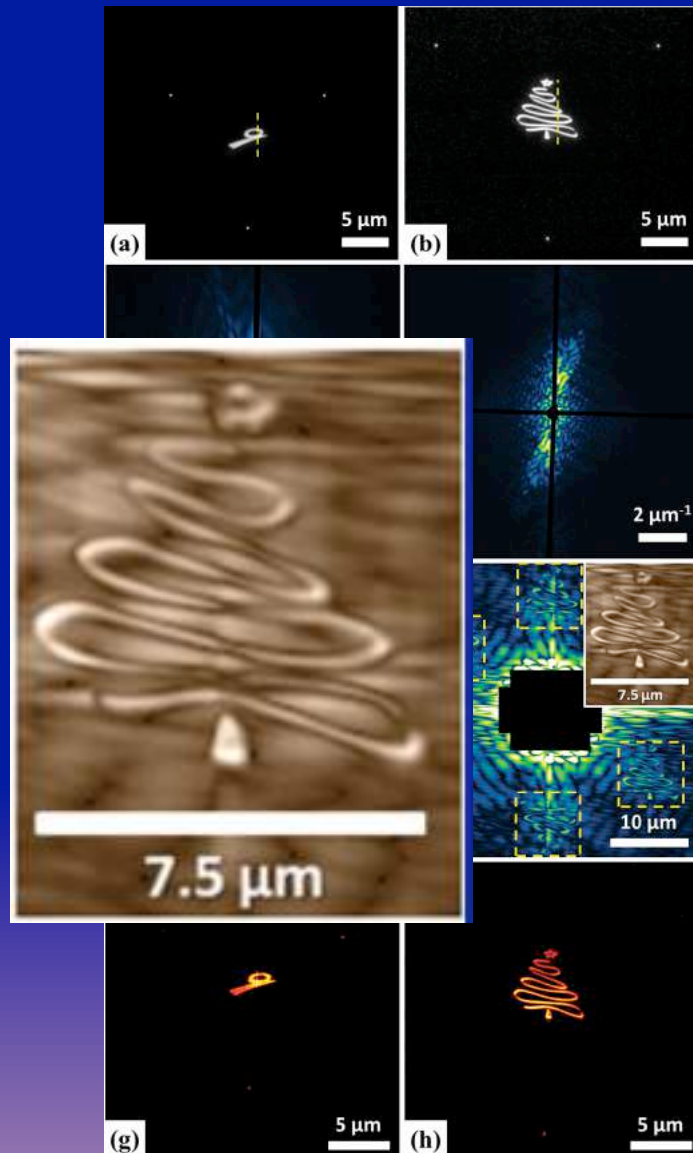


**boom!**

...but, as the pulse is ultrashort,  
one can derive from diffraction data  
the structure before the explosion

# Pioneering tests at the FERMI FEL (Trieste)

(F. Capotondi et al., Rev. Sci. Instruments 84, 051301 (2013):



← visible micrographs of the objects

← single-shot coherent diffraction patterns obtained with 32.5 nm FEL pulses

← holographic reconstructions

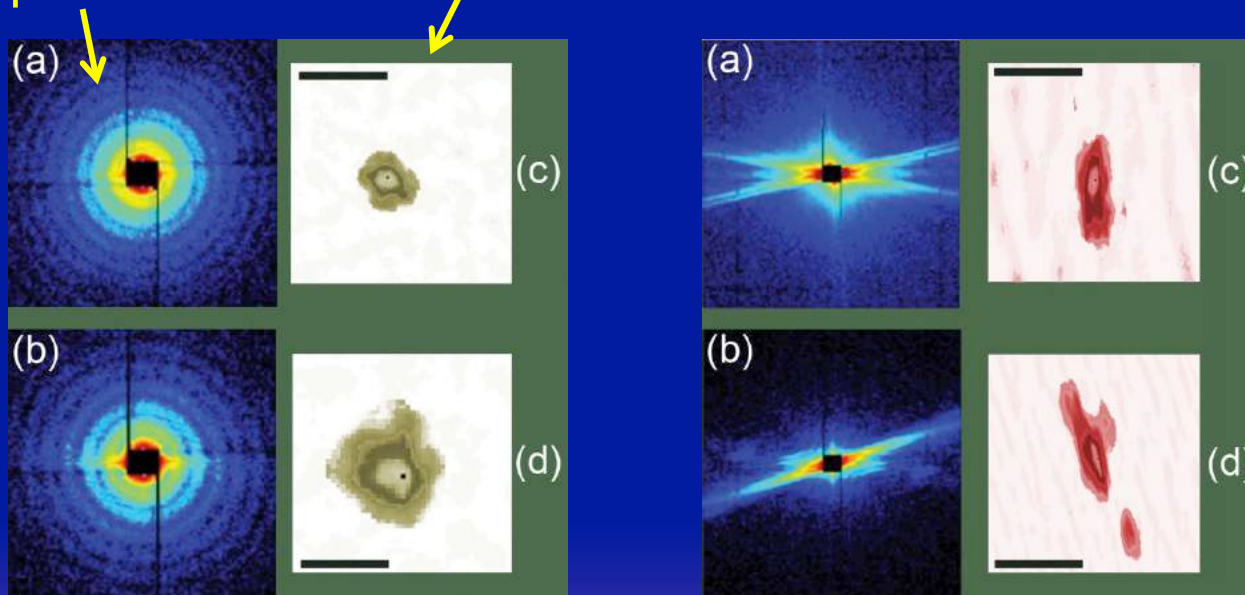
← phase-retrieval reconstructions

## A recent example:

Dynamic coherent diffraction imaging in water of individual liposomes (micro-bags made of phospholipid molecules) carrying drugs (doxorubicin nanorods, 100–200 nm)

Diffraction pattern

Reconstruction



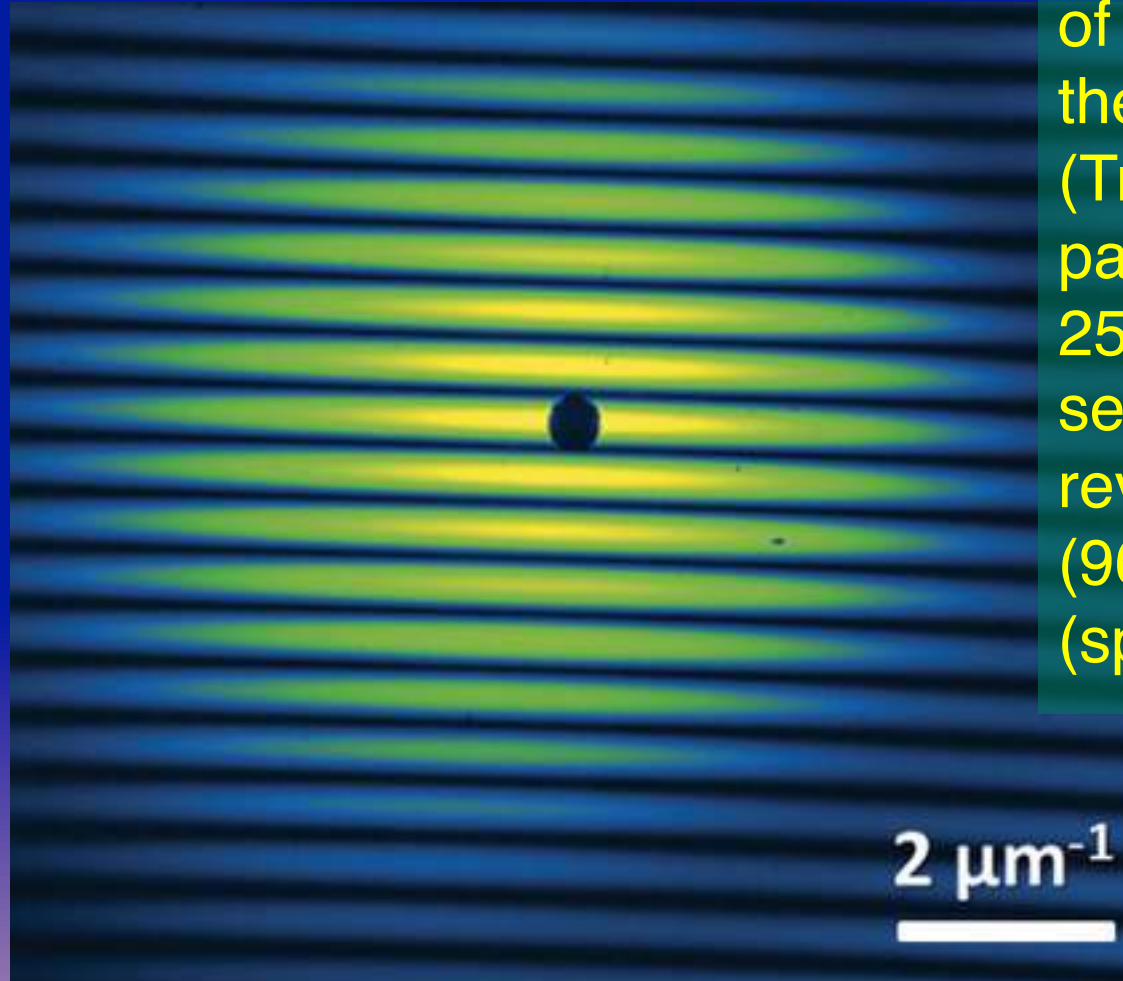
Micro-bags with no drug

Drug-carrying micro-bags

From the data, we were able to extract the quantitative structural parameters of the nanorods [results obtained by Yeukuang Hwu et al. at SACLA]

# The high coherence of x-ray FEL's :

testing the coherence of 32.5 nm pulses from the Fermi FEL (Trieste): the diffraction pattern created by two 250 nm pinholes separated by  $2\ \mu\text{m}$  reveals a very high (96%) level of lateral (space) coherence

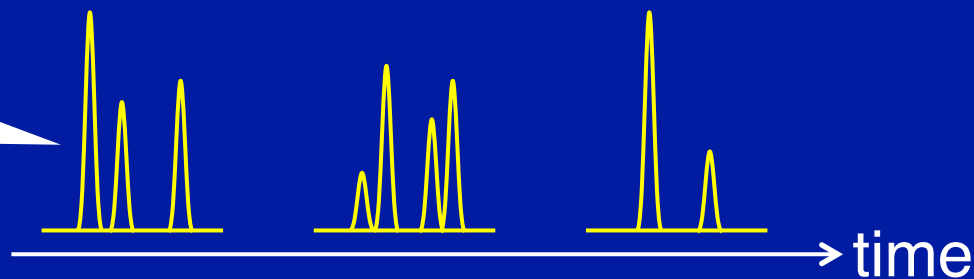




The X-FEL longitudinal (time) coherence is affected by a problem:

SASE amplifies waves that are stochastically emitted when the electron bunch enters the wiggler

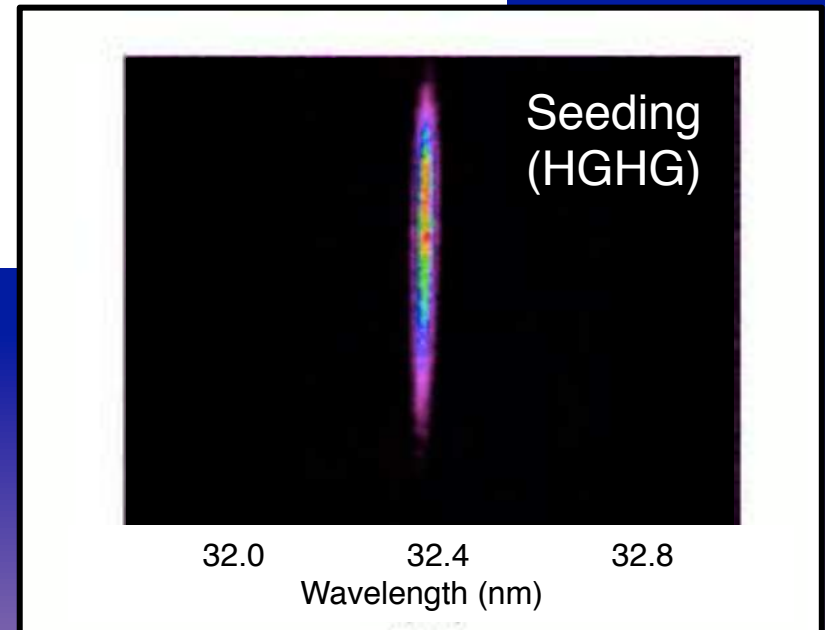
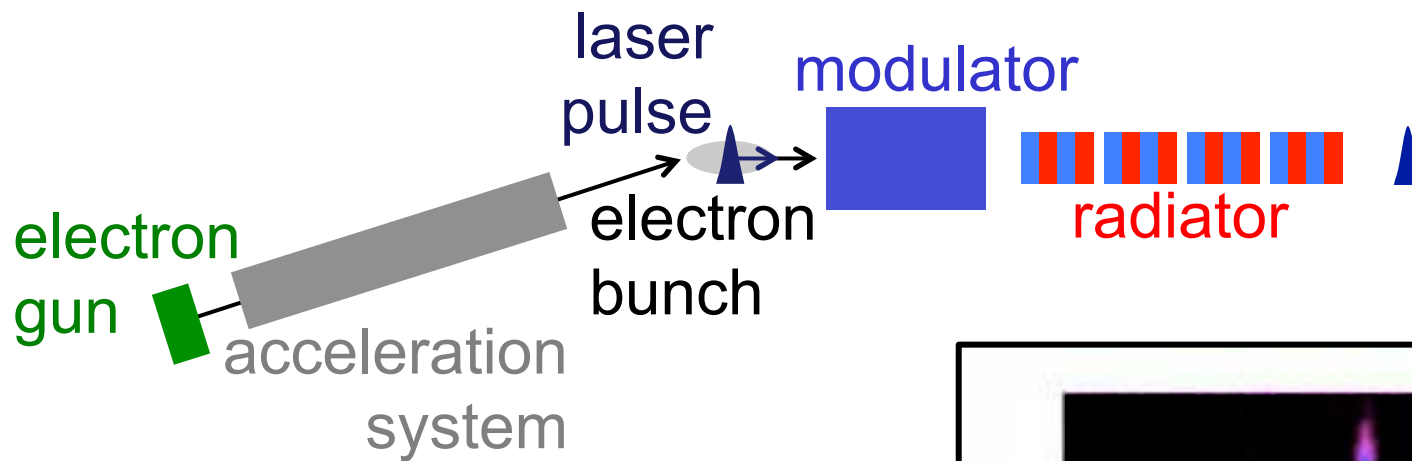
The pulse time structure changes from bunch to bunch, limiting the time coherence



Solution: “seeding” – amplification of a longitudinally coherent wave emitted by an external source and injected into the x-FEL

A complicated technology, recently implemented

# FERMI: Seeding SASE-FEL using an optical laser



E. Allaria et al.,  
 Nature Photonics 6  
 (2012), 7 (2013)

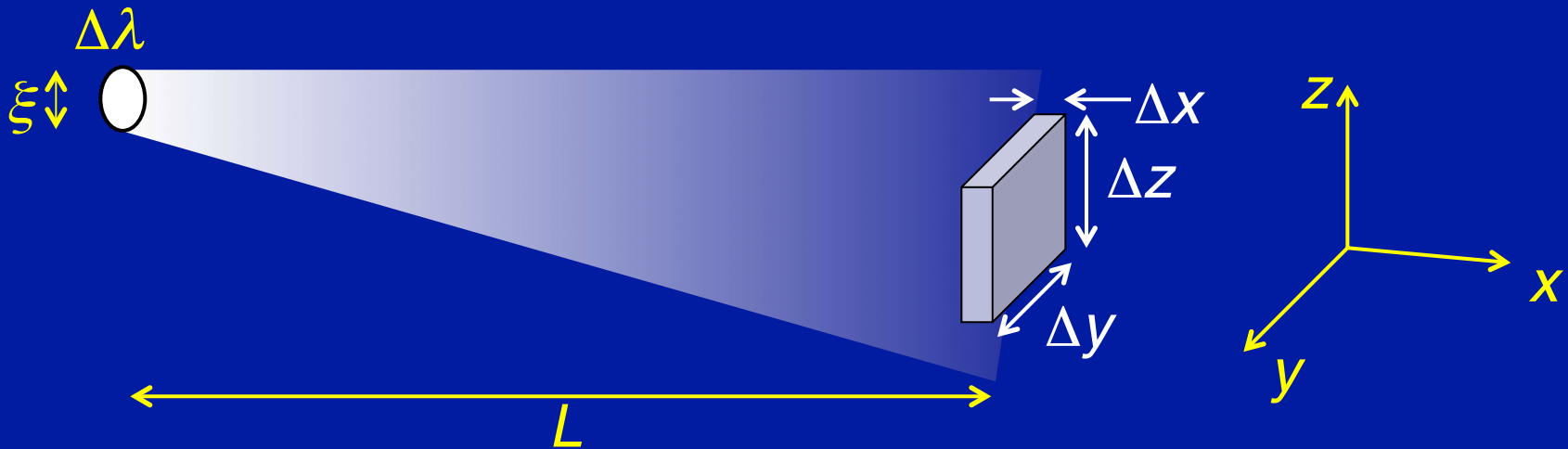
# Thanks to:

- Primoz Rebernik for his key contributions to our FEL theory
- Maya Kiskinova, Yeukuang Hwu and their coworkers for communicating very recent experimental results



**your future looks  
so very bright!**

# Refinements: a closer look at coherence



A source of size  $\xi$  and bandwidth  $\Delta\lambda$  **coherently illuminates** a volume  $\Delta x\Delta y\Delta z$  at the distance  $L$ . This is this **coherence volume**.

Along  $x$ : if two waves  $\lambda$  and  $\lambda + \Delta\lambda$  are in phase at a certain time, they will be out of phase after  $\Delta t$  such that  $\Delta v\Delta t = 1$  or  $\Delta t = 1/\Delta v = \lambda^2/(c\Delta\lambda)$ .

Thus,  $\Delta x = c\Delta t = \lambda^2/\Delta\lambda = L_c$ .

Along  $y$ : the uncertainty of the  $k$ -vector is  $\Delta k \approx k(\xi/L) = 2\pi\xi/(L\lambda)$ .

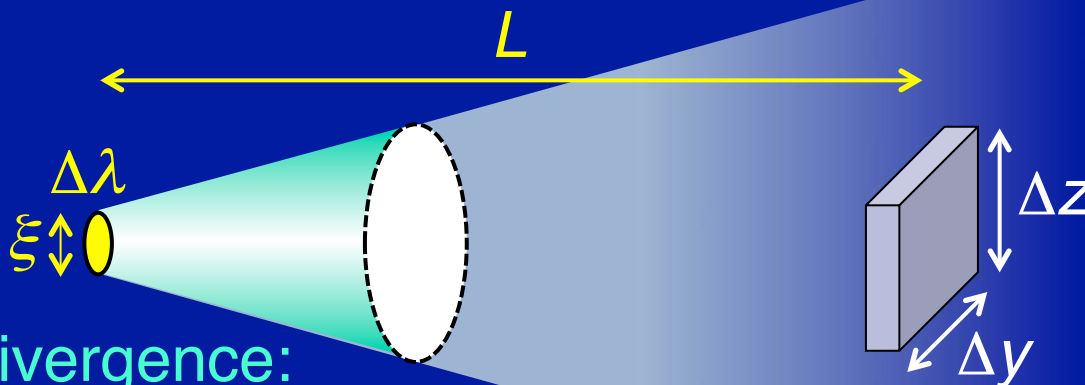
If two waves with  $k$ -vectors  $0$  and  $\Delta k$  along  $y$  are in phase at a certain point, they will be out of phase at a distance  $\Delta y$  such that  $\Delta k\Delta y = 2\pi$ ; thus,  $\Delta y = 2\pi/\Delta k_y = L\lambda/\xi$ .

Along  $z$ : same as along  $y$ .

**Coherence volume:**  $\Delta x\Delta y\Delta z = L^2\lambda^4/(\xi^2\Delta\lambda)$

Behind this: Heisenberg! Photons in the coherence volume cannot be distinguished from each other

# Refinements: the notion of “coherent power”



Angular divergence:  
solid angle  $\Omega \propto \theta^2$

The solid angle corresponding to the area  $\Delta y \Delta z$  is  $\Delta y \Delta z / L^2$ .

If the solid angle of the emitted light is  $\approx \theta^2$ , then only a portion  $(\Delta y \Delta z / L^2) / \theta^2$  of the total emitted power illuminates the coherence volume.

This is the **coherent power**.

Since  $\Delta y \Delta z = (L \lambda \xi)^2$ , the **coherent power factor** is  $\approx [\lambda / (\xi \theta)]^2$ .

# Refinements: the number of photons $n_c$ in the “coherence volume” for an x-FEL with full transverse coherence

Full transverse coherence means that all emitted photons are within the “coherence volume”. Thus, their number  $n_c$  is given by the flux  $F$  times  $L_c/c$ , which gives  $F\lambda^2/(c\Delta\lambda)$ .

The brightness  $B$  is proportional to  $F/(\xi\theta)^2$ ; for full transverse coherence, the coherent power factor  $[\lambda/(\xi\theta)]^2$  is  $\approx 1$ , therefore  $F/(\xi\theta)^2 \approx F/\lambda^2$  and  $F$  is proportional to  $\lambda^2 B$ .

The  $F$ - $B$  proportionality factor contains the relative bandwidth  $\Delta\lambda/\lambda$ , i.e.,  $F$  is proportional to  $\lambda^2 B(\Delta\lambda/\lambda)$ .

Thus,  $n_c = F\lambda^2/(c\Delta\lambda)$  is proportional to  $(\lambda^2 B)(\Delta\lambda/\lambda)[\lambda^2/(c\Delta\lambda)]$

Overall,  $n_c$  proportional to  $B\lambda^3$ : to increase it, high brightness helps, but short wavelengths are a problem!

# Some general questions:

- The central emitted photon energy of an undulator in a storage ring with energy  $E = 2$  GeV is  $h\nu = 3$  keV. What is the undulator period  $L$ ?
- The emitted radiation from a bending magnet is confined within a vertical angle  $\approx 2.0$  milliradians. What is the energy in GeV of the storage ring?
- Find the wavelength bandwidth of a bending magnet with  $B = 1.5$  tesla in a 1.2 GeV storage ring
- The pulse duration of an x-FEL is 100 femtoseconds. The central emitted photon energy is 500 eV. How many microbunches are there in each electron bunch?
- Synchrotron-radiation and FEL experiments are always very expensive: true or false?

# Answers:

- The pulse duration of an x-FEL is 100 femtoseconds. The central emitted photon energy is 500 eV. How many microbunches are there in each electron bunch?

The central wavelength is  $1.24 \times 10^4 / 500 \approx 24.8 \text{ \AA}$ . This is also the microbunching period. The bunch length is  $c \times 10^2 \times 10^{-15} = 3 \times 10^8 \times 10^{-13} \approx 3 \times 10^{-5} \text{ m}$ . The number of microbunches is  $3 \times 10^{-5} / (24.8 \times 10^{-10}) \approx 1.2 \times 10^4$

- Synchrotron-radiation and FEL experiments are always very expensive: true or false?

False for storage ring and infrared FELs. Maybe true for x-FELs. Try to calculate the cost per experiment in each case!