Atmospheric neutrinos

Advanced Workshop in Atmospheric Neutrino Physics 2018, ICTP, Trieste

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Zeuthen, Germany

HELMHOLTZ RESEARCH FOR GRAND CHALLENGES

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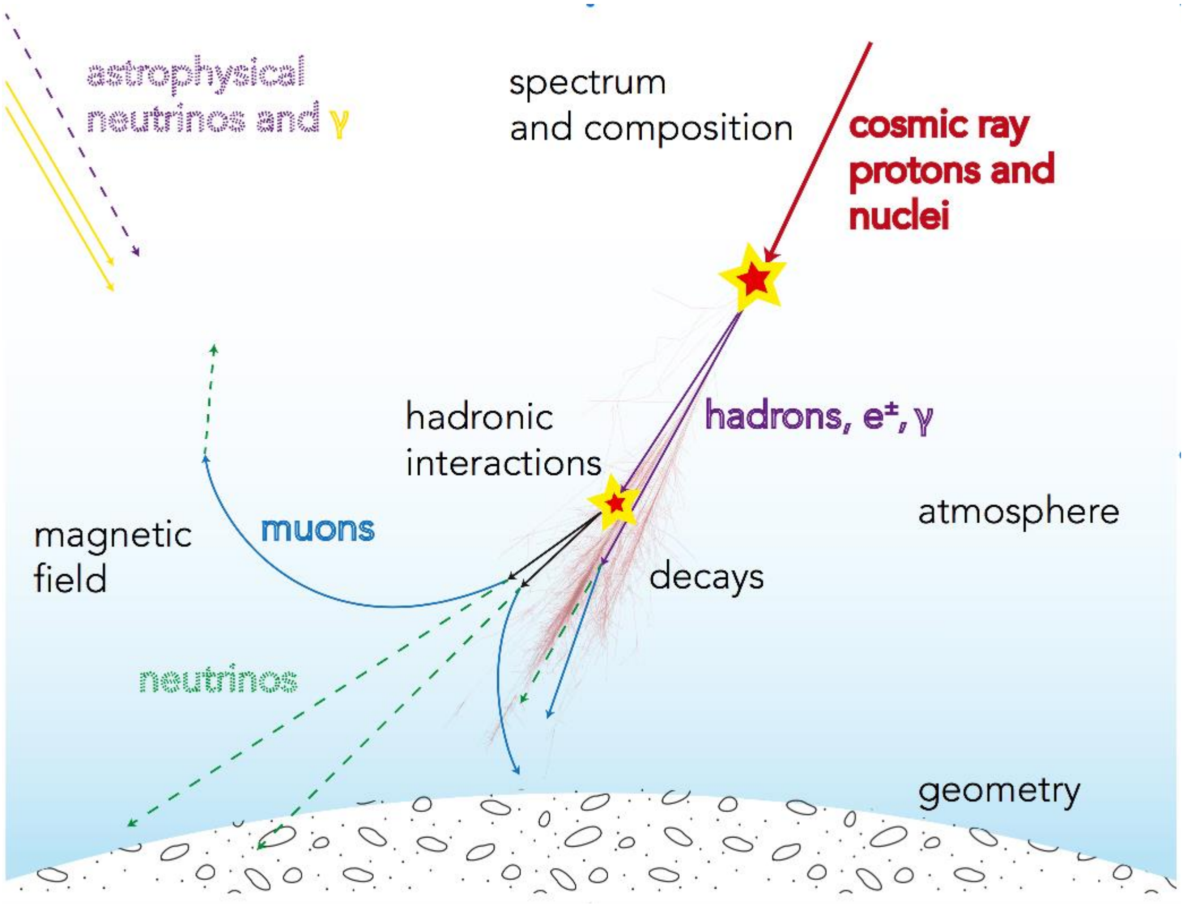
Atmospheric neutrino phenomenology

Ingredients for high-precision atmospheric neutrino flux calculation



- For high precision calculations all phenomena need accurate modeling
- Uncertain “ingredients”:
  - Cosmic ray spectrum and composition
  - Hadronic interactions
  - Atmosphere (dynamic, depends on use case)
  - (Rare) decays
  - Geometry, magnetic fields, solar modulation
- No clear prescription how to handle uncertainties.
- Energy range MeV – EeV!

Production channels

conventional (from longer-lived hadrons)

$$p, A + \mathrm{air} \to \pi^{\pm}, \pi^0, \mathrm{K}^{\pm}, \mathrm{K}^0_{\mathrm{S,L}}$$

muons and muon neutrinos

$$\pi^{\pm}, K^{\pm} \to \mu^{\pm}\nu_{\mu}(\bar{\nu}_{\mu})$$

electron neutrinos

$$K^{\pm}, K^0_L \to [\pi^{\pm}, \pi^0]e^{\pm}\nu_e(\bar{\nu}_e)$$

prompt (from rare short-lived hadrons)

$$p, A + \mathrm{air} \to \mathrm{D}, \Lambda_{\mathrm{C}} \to \nu_{\mu}, \nu_{\mathrm{e}}, \mu$$

Subset of dominant decay channels

decay channel	branching ratio (BR)
$\mu^- \to e^-\bar{\nu}_e\nu_{\mu}$	100 %
$\pi^+ \to \mu^+\nu_{\mu}$	99.9877 %
$K^0_{e3} : K^0_L \to \pi^{\pm}e^{\mp}\nu_e$	40.55 %
$K^0_{\mu 3} : K^0_L \to \pi^{\pm}\mu^{\mp}\nu_{\mu}$	27.04 %
$K^+ \to \mu^+\nu_{\mu}$	63.55 %
$K^+_{e3} : K^+ \to \pi^0 e^+\nu_e$	5.07 %
$K^+_{\mu 3} : K^+ \to \pi^0\mu^+\nu_{\mu}$	3.353 %
$D^+ \to \overline{K}^0\mu^+\nu_{\mu}$	9.2 %
$D^0 \to K^-\mu^+\nu_{\mu}$	3.3 %

+ charge conjugates

http://pdg.lbl.gov

Transport equations (hadronic cascade equations)

System of coupled non-linear PDE for each particle species *h* :

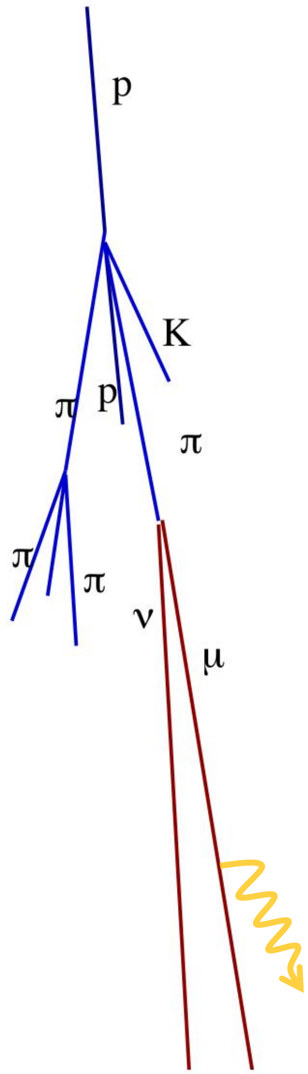
$$\frac{\mathrm{d}\Phi_h(E,X)}{\mathrm{d}X} = -\frac{\Phi_h(E,X)}{\lambda_{\mathrm{int},h}(E)}$$ Interactions with air

$$-\frac{\Phi_h(E,X)}{\lambda_{\mathrm{dec},h}(E,X)}$$ Decays

$$-\frac{\partial}{\partial E}(\mu(E)\Phi_h(E,X))$$ Continuous losses

$$+\sum_k \int_E^\infty \mathrm{d}E_k \, \frac{\mathrm{d}N_{k(E_k)\to h(E)}}{\mathrm{d}E} \frac{\Phi_k(E_k,X)}{\lambda_{\mathrm{int},k}(E_k)}$$ Re-injection from interactions

$$+\sum_k \int_E^\infty \mathrm{d}E_k \, \frac{\mathrm{d}N^{\mathrm{dec}}_{k(E_k)\to h(E)}}{\mathrm{d}E} \frac{\Phi_k(E_k,X)}{\lambda_{\mathrm{dec},k}(E_k,X)}$$ Re-injection from decays



$$X(h_0) = \int_0^{h_0} \mathrm{d}\ell \; \rho_{\mathrm{air}}(\ell)$$

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cosmic ray physics

Interactions with air

Decays

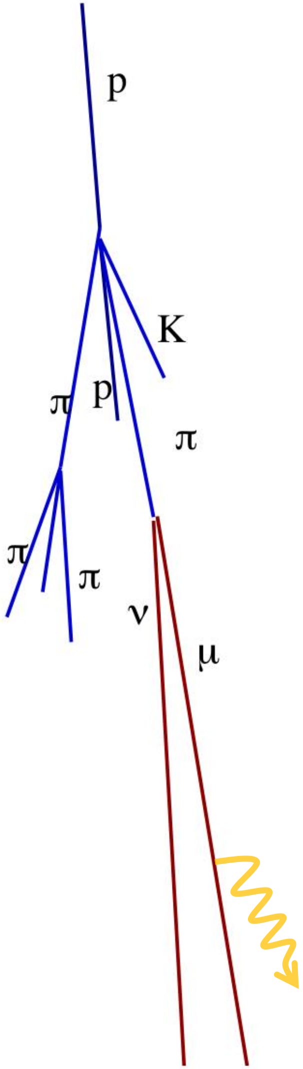
atmospheric physics

Continuous losses

Re-injection from interactions

Re-injection from decays

particle physics



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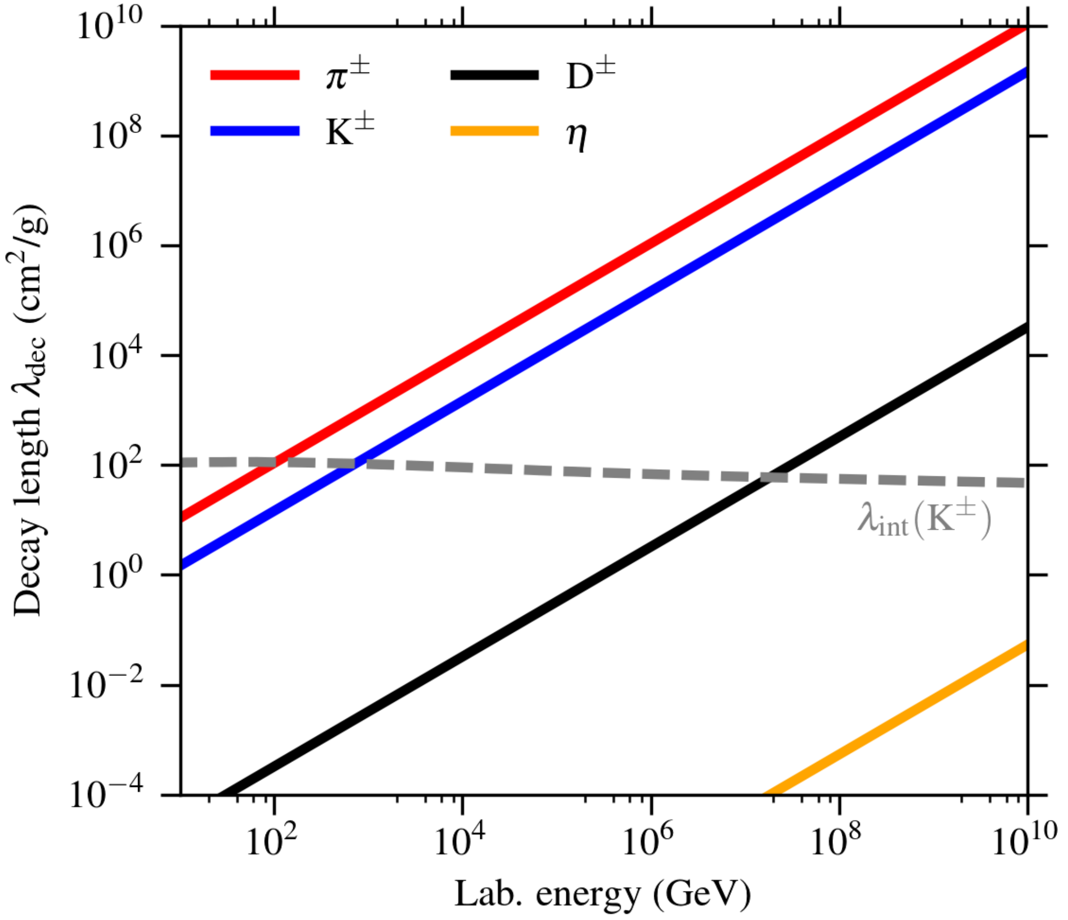

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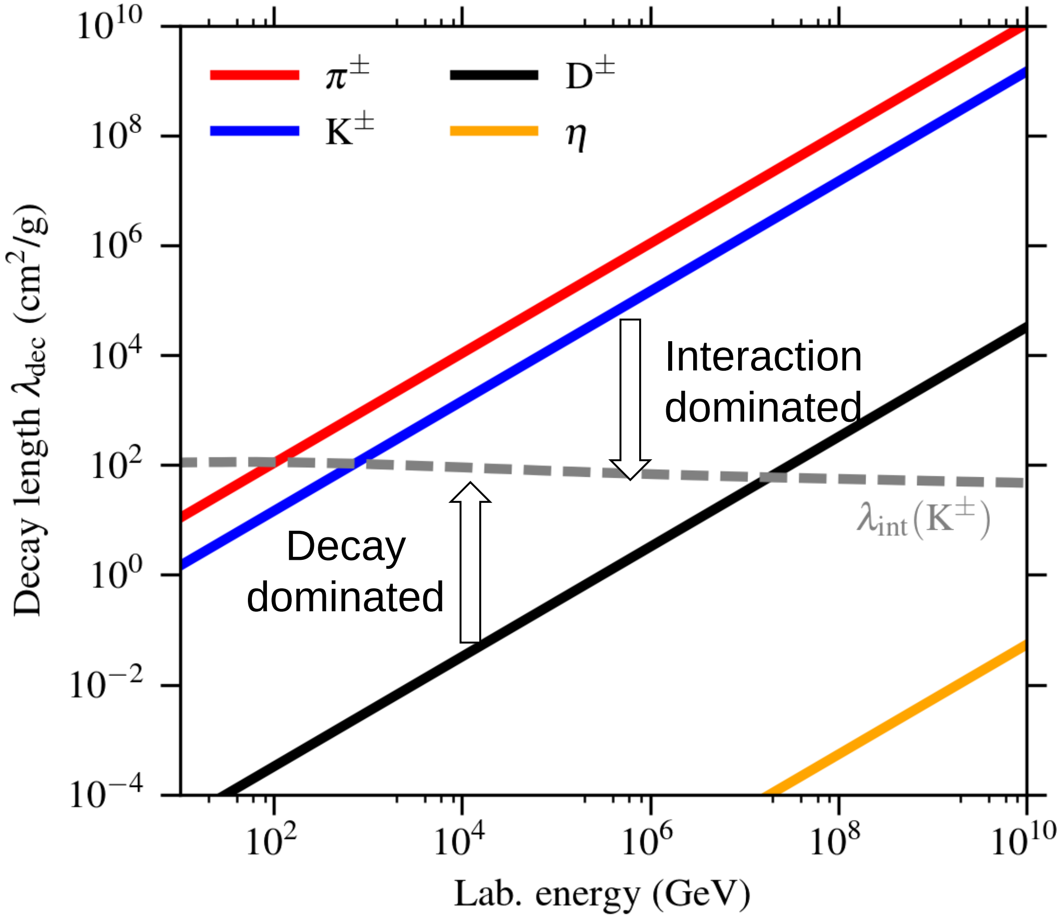
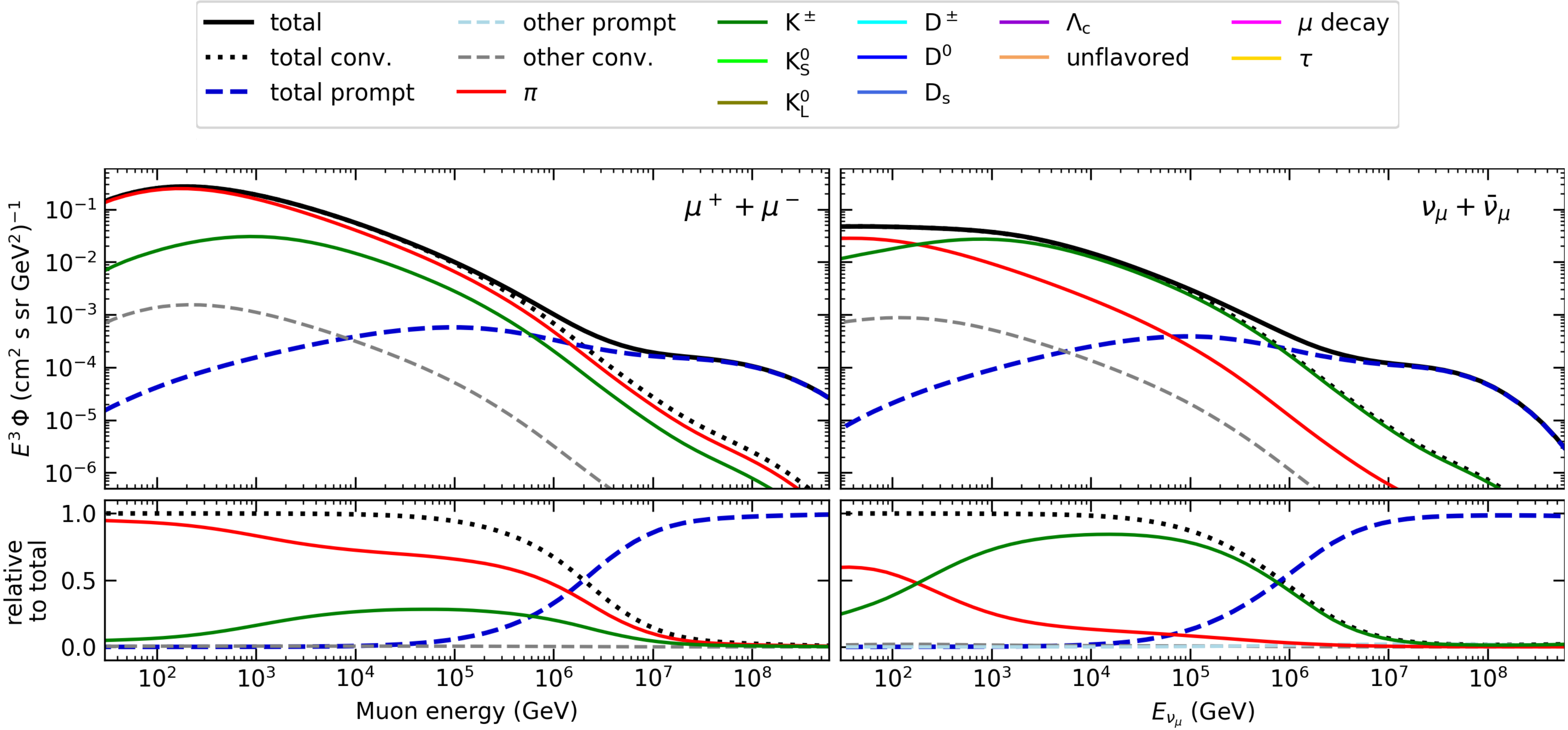


$$X(h_0) = \int_0^{h_0} d\ell\, \rho_{air}(\ell)$$

Hadronic origin of muonic leptons



AF, F. Riehn, R. Engel, T.K. Gaisser, T. Stanev, in prep. (2018)

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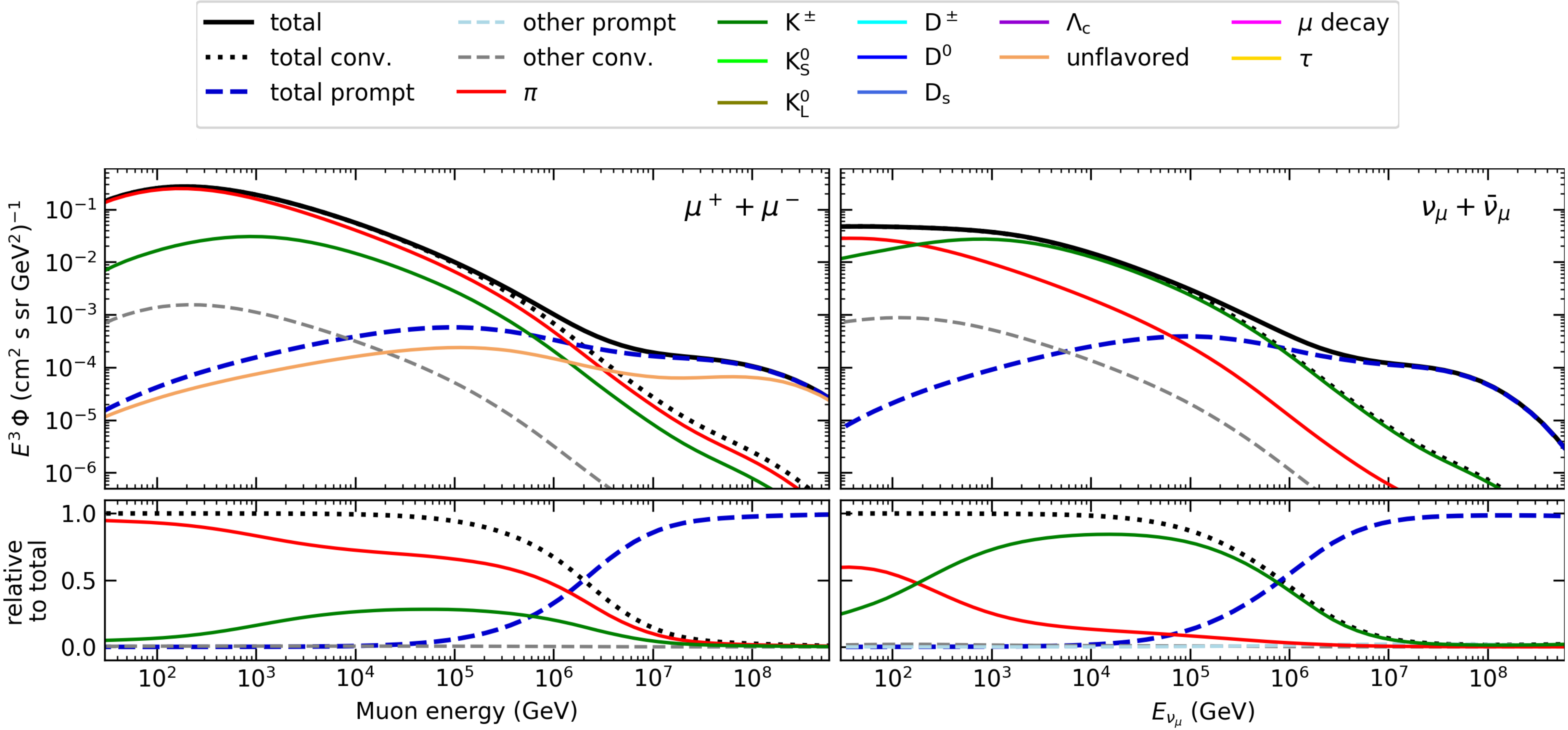

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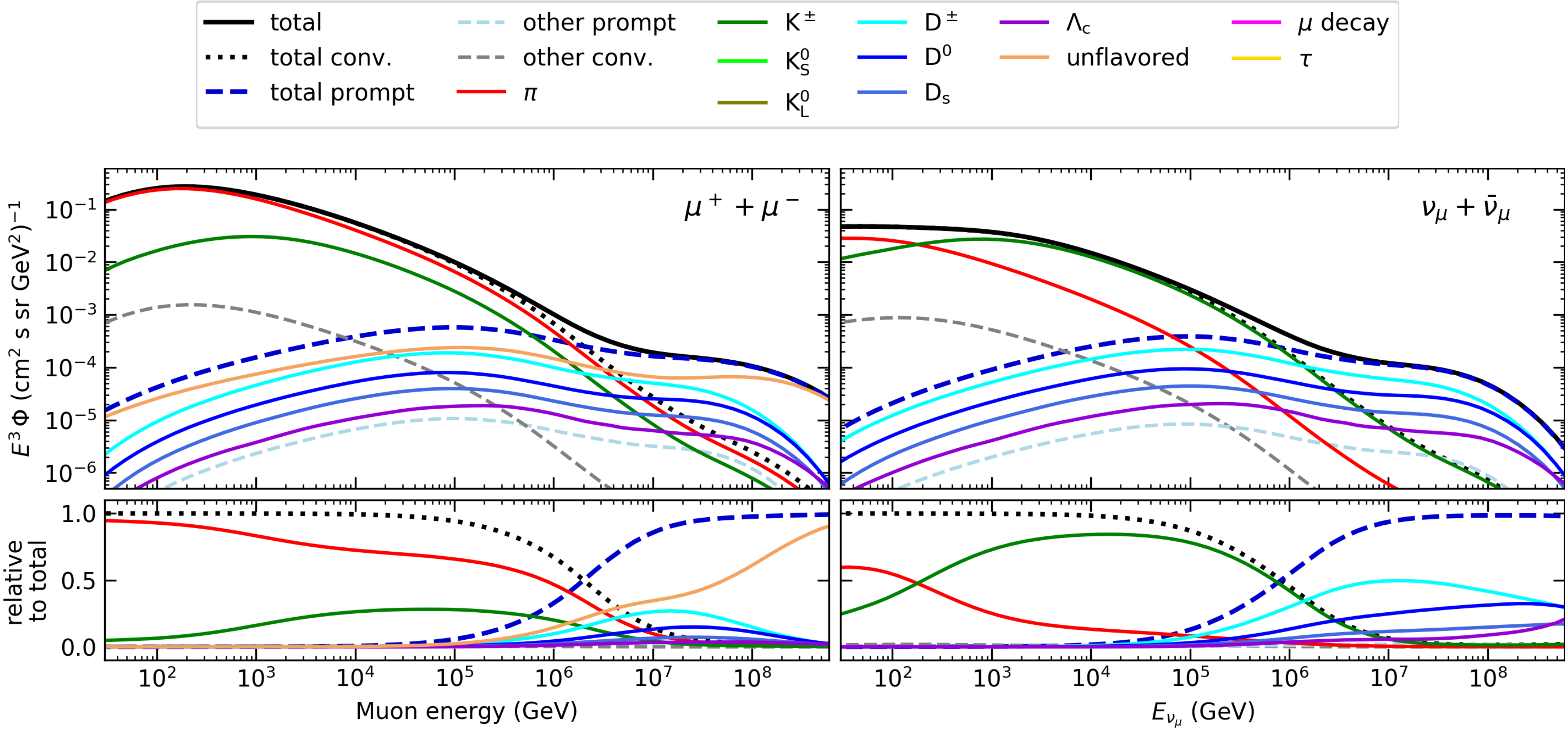

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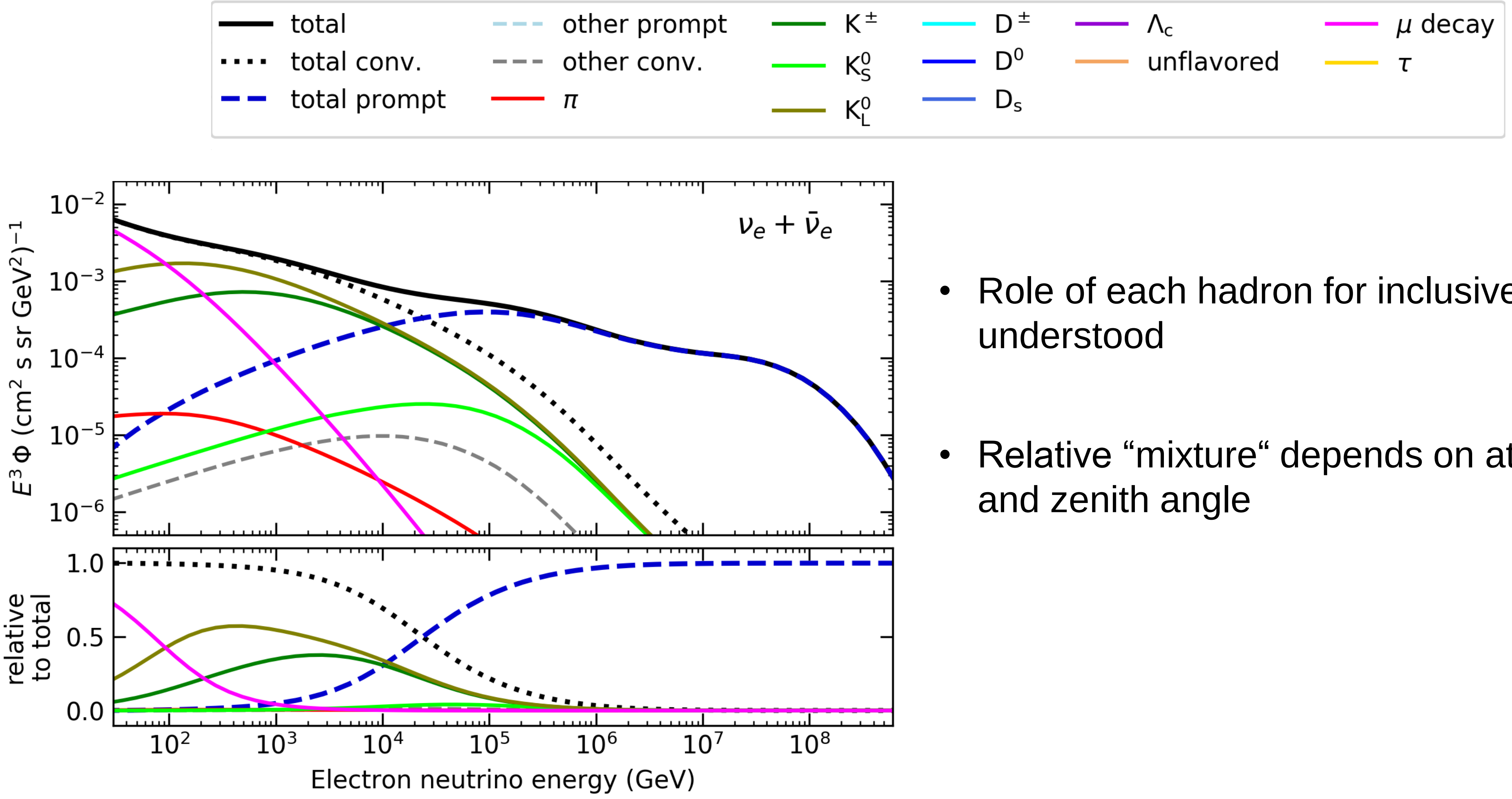
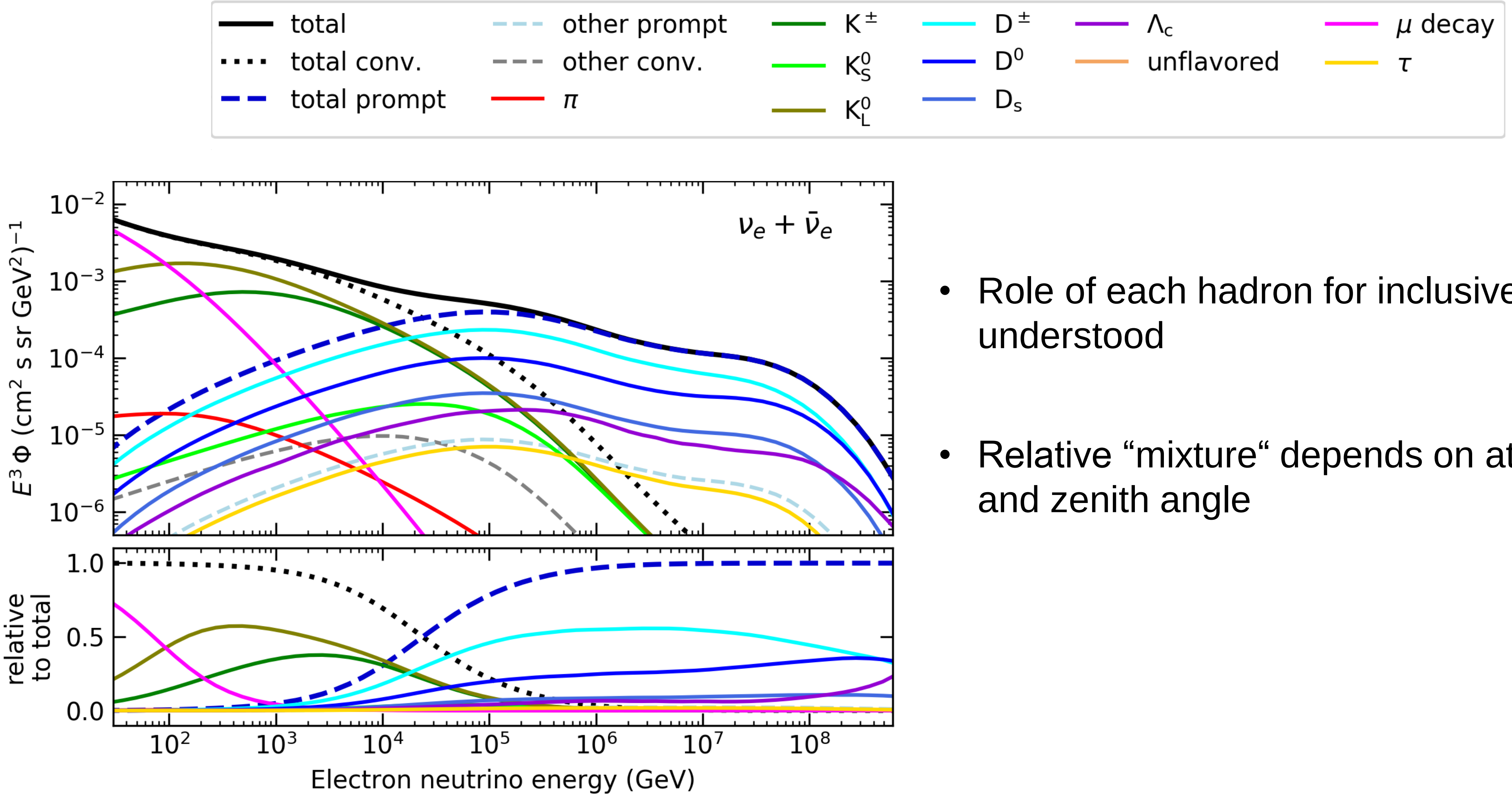


- Role of each hadron for inclusive fluxes understood
- Relative “mixture“ depends on atmosphere and zenith angle

Hadronic origin of muonic leptons



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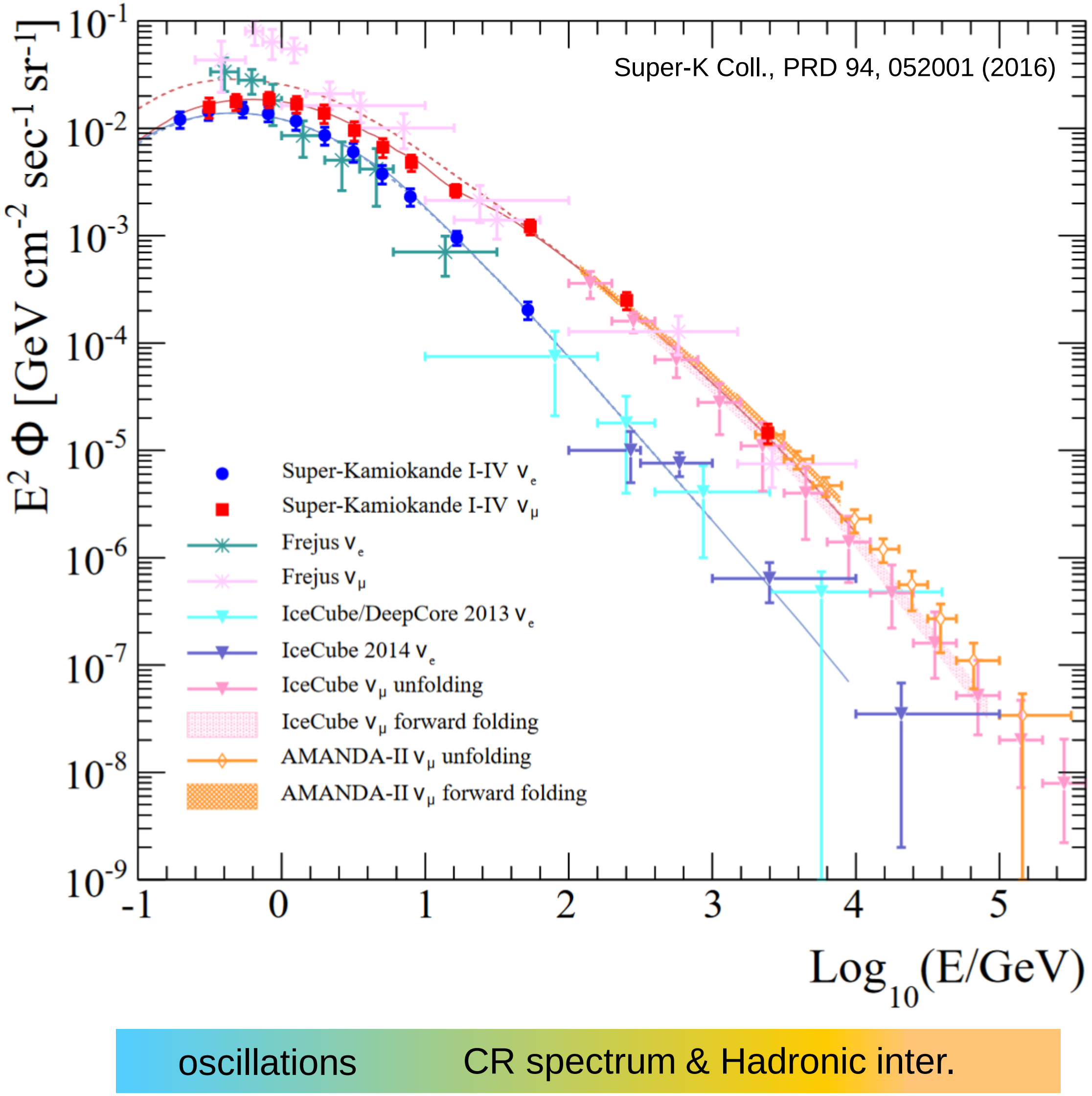
Flux measurements

At lower energies

- Good match between theory and measurement
- Experimental uncertainties dominated by
  - systematical for ν_μ
  - systematical + stat. for ν_e
- Bin-bin correlations due to poor energy resolution for certain event topologies
- Weak constraints on calculation uncertainty from flux measurements



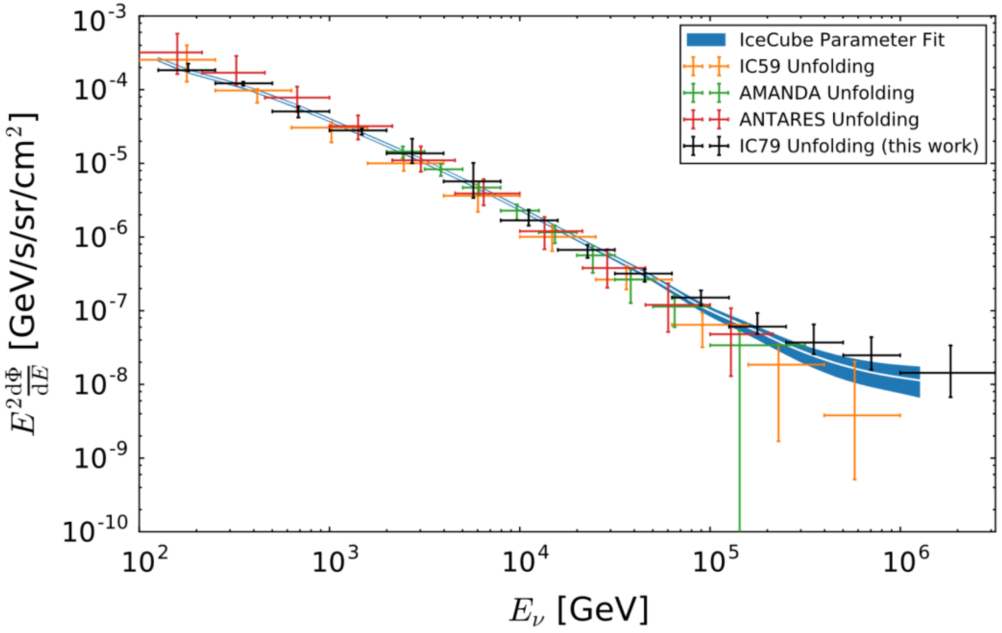
Flux measurements

At high energies

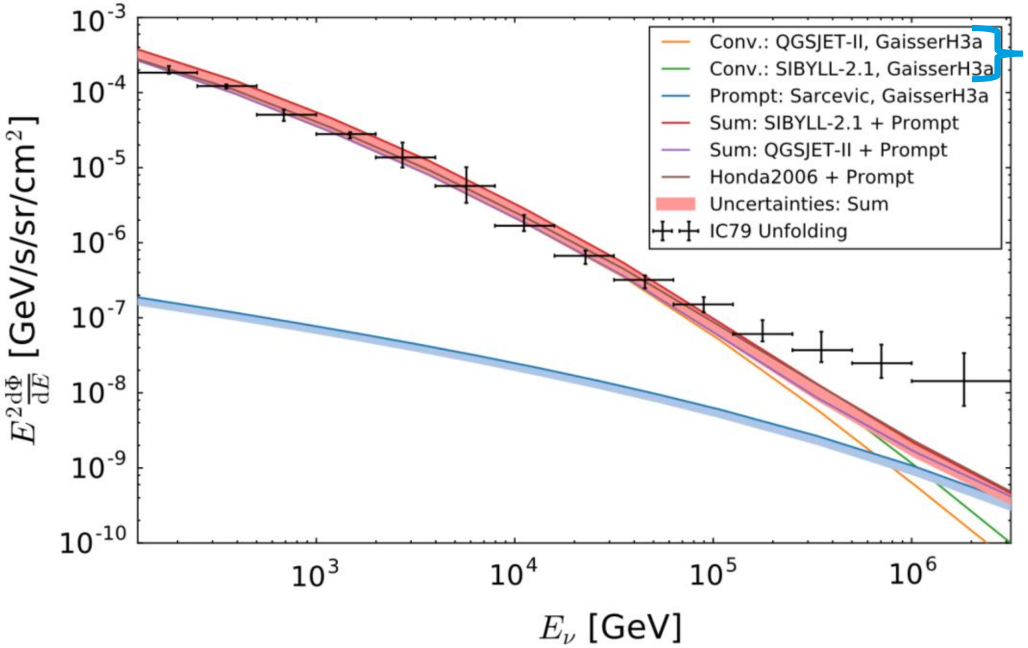
- All measurements agree within uncertainties (20-50%)
- Good agreement between theory and experiment
- Above 100 TeV large bin-bin correlations and astrophysical “background”

Atmospheric fluxes are sufficiently well known

IceCube, Eur. Phys. J. C (2017) 77 :692

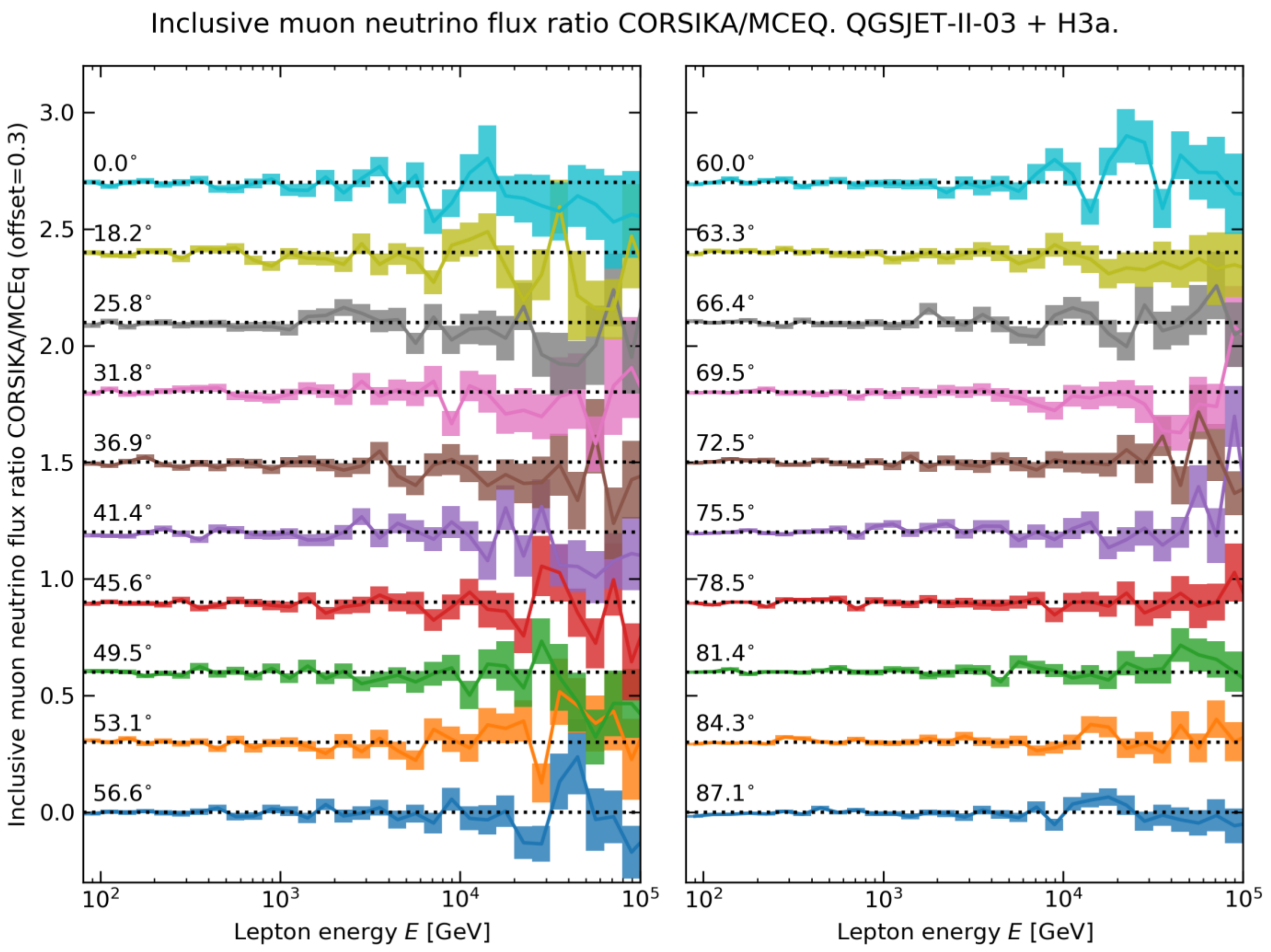


CR spec., hadr. int., BSM   prompt & astrophysical



AF, Becker Tjus, Desiati, PRD (2012)

MCEq: numerical (open source) solver

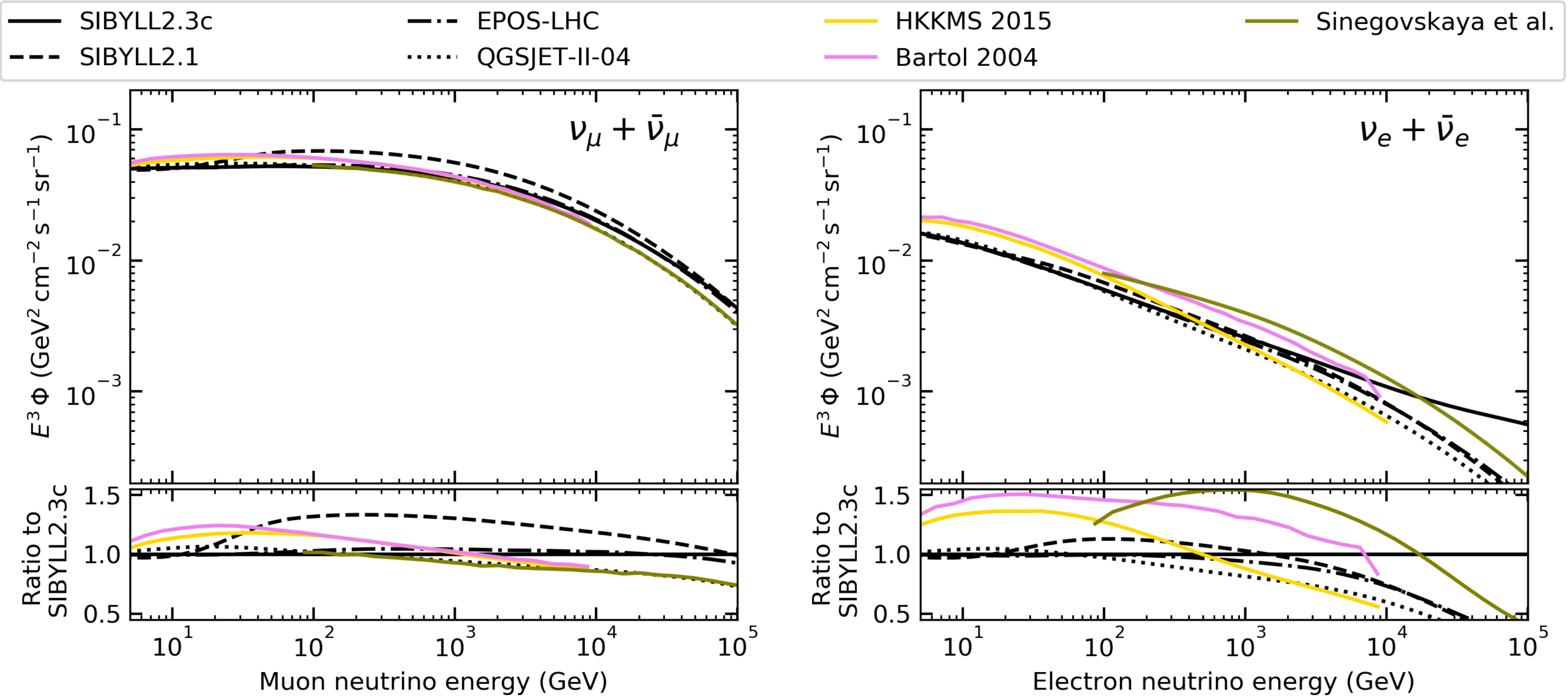


- Simultaneous solution of up to 8000 kinetic equations
- Energy range 1 (5) GeV – 10^{11} GeV
- All models included
- High optimization: multi-core, GPU, … (BLAS, MKL, CUDA) (~milli-seconds)
- MIT licensed @ https://github.com/afedynitch/MCEq

CORSIKA: A. Fedynitch, J. Becker Tjus and P. Desiati, PRD 2012
MCEq: A. Fedynitch, R. Engel, T. K. Gaisser, F. Riehn and S. Todor. PoS ICRC 2015, 1129
MCEq: Code paper, AF, R. Engel, in prep. for submission

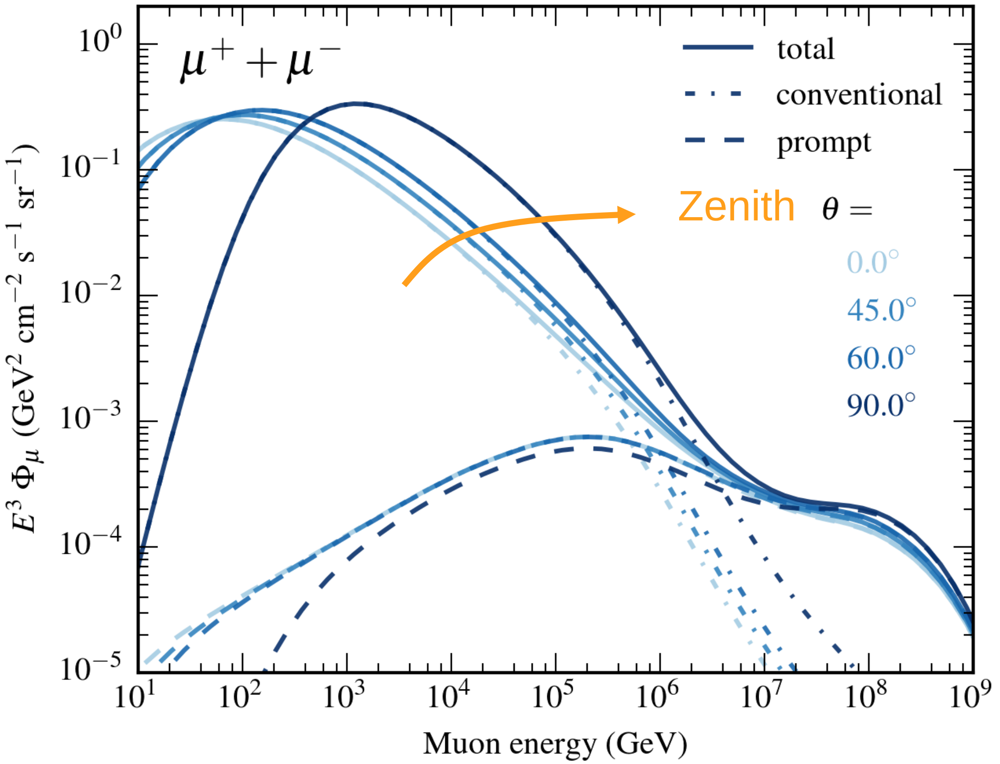
MCEq-based predictions

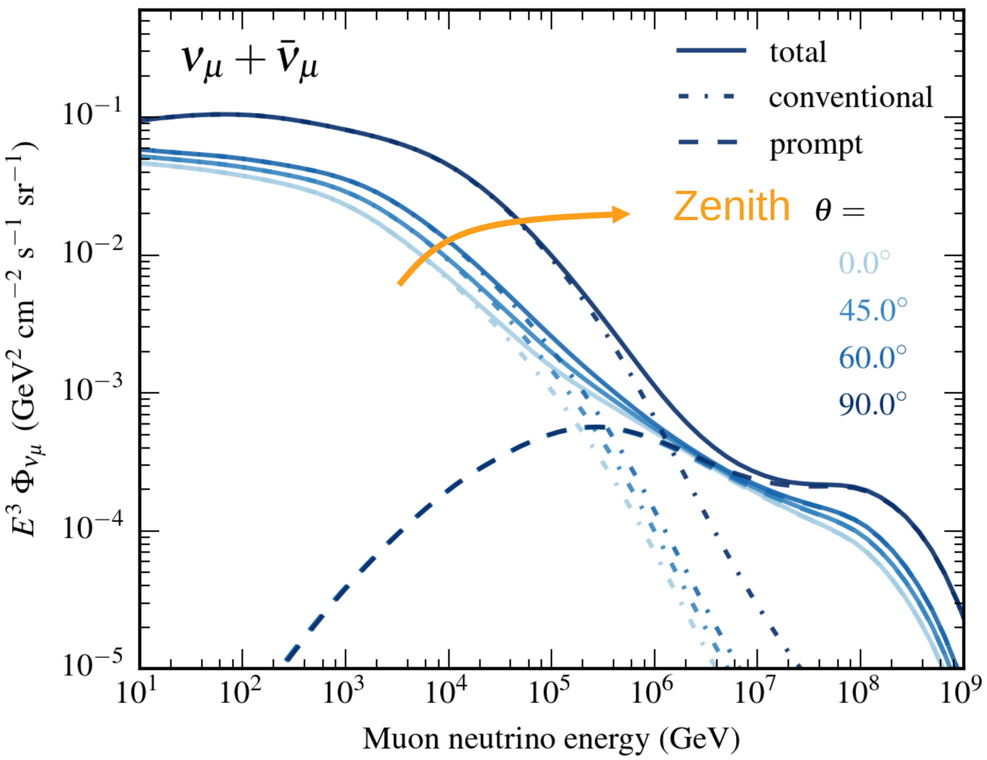
HKKMS: M. Honda et al., PRD 92 (2015)
Bartol: G. Barr et al., PRD 70 (2004)
Sinegovskaya et al. PRD 91 (2015)
MCEq: AF, R. Engel in prep.

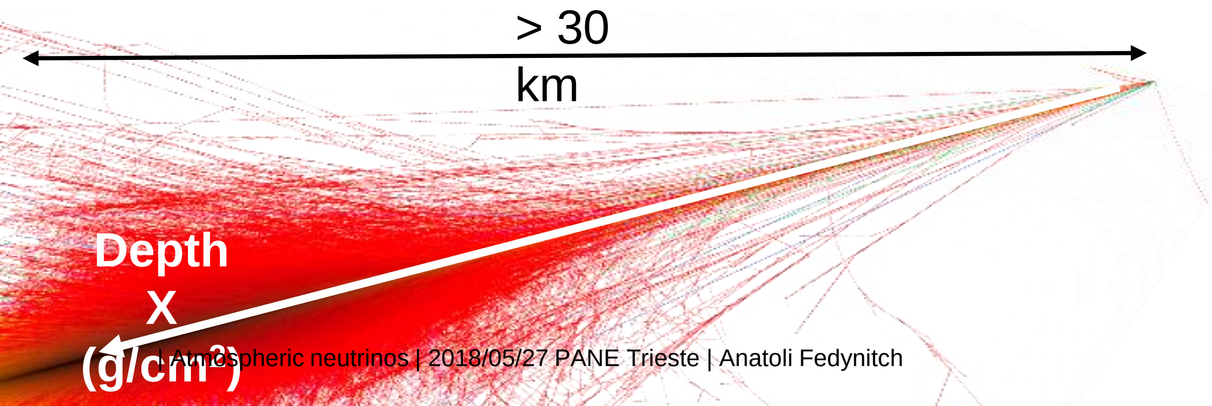


- Old 2002 (GH) primary model for HKKMS and Bartol, H3a for the rest
- Data can not discriminate between calculations
- Shown are zenith and azimuth averages

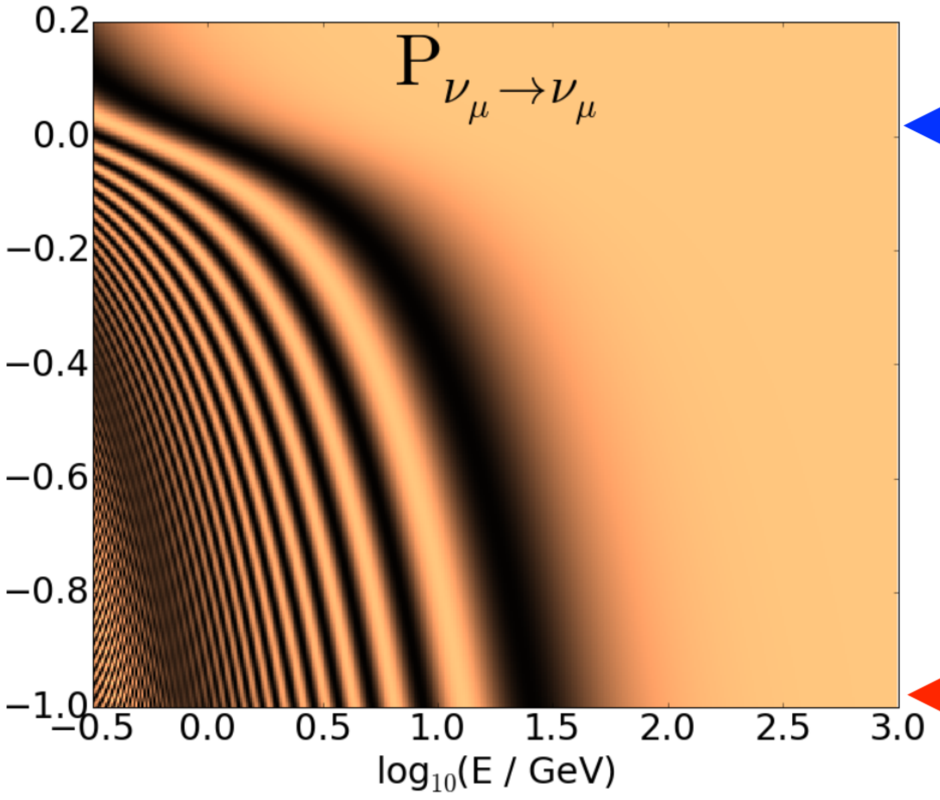
Zenith angle: Modified competition of decay and interactions

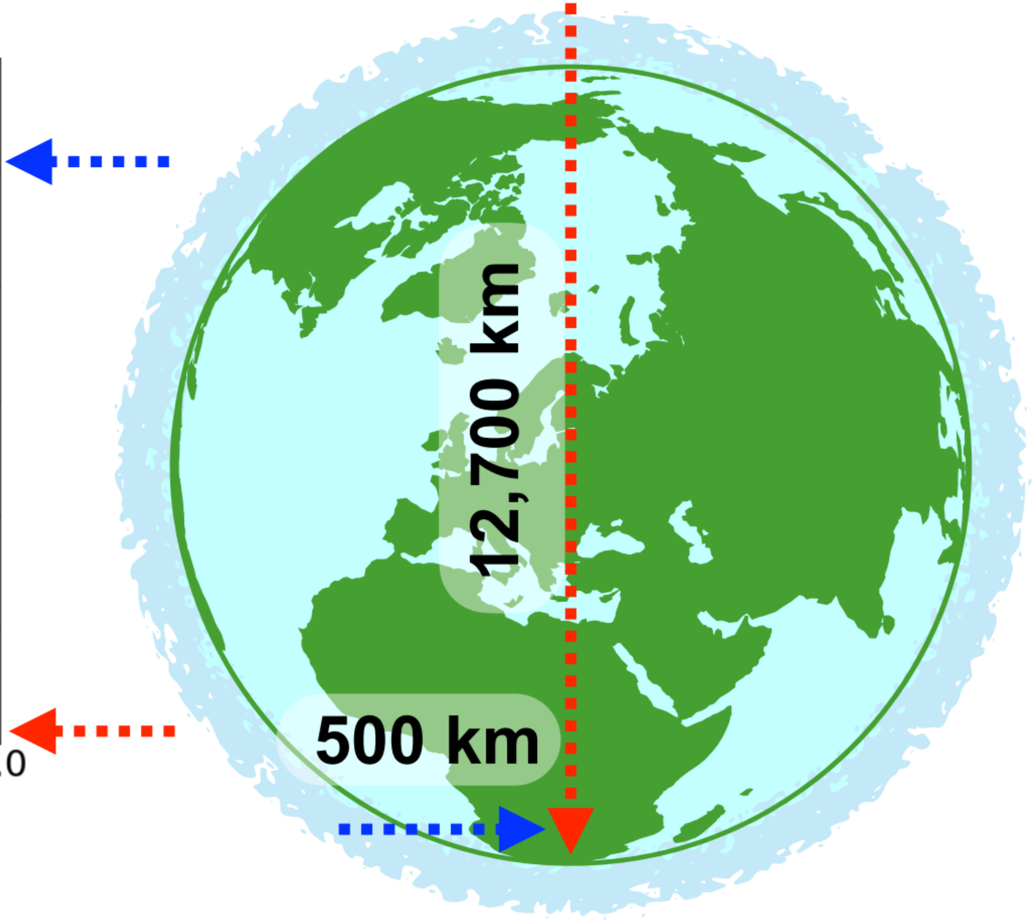






Neutrino properties manifest as pattern in E-θ plane





Normal mass ordering

$\theta_{23}=\pi/2$

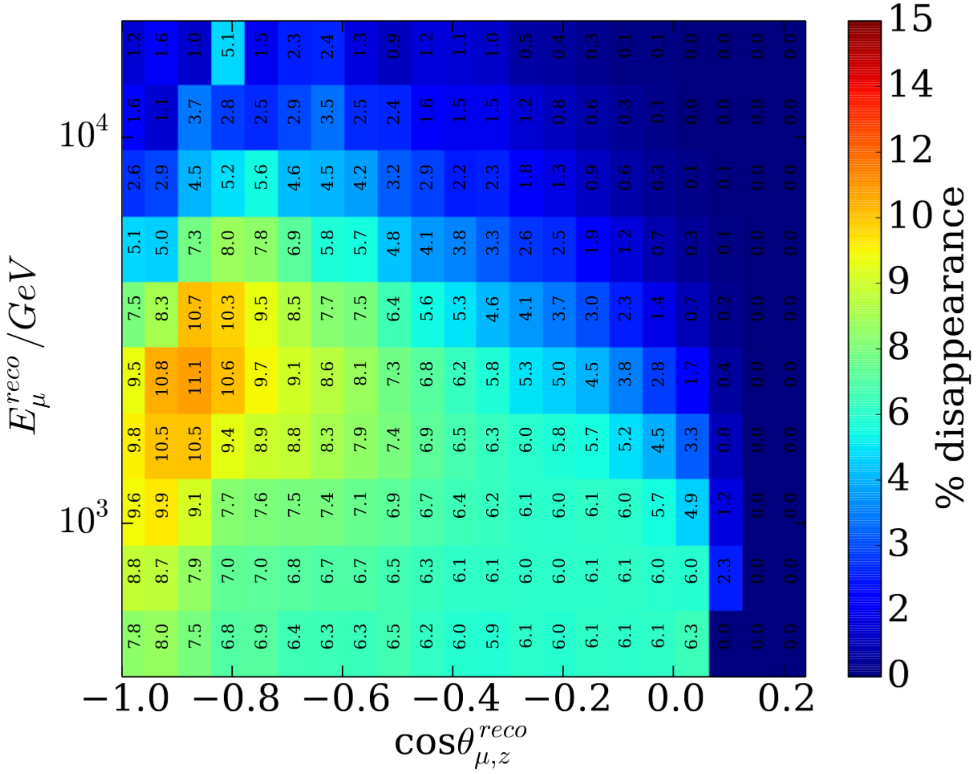
$\Delta m^2_{32}= 2.51 \times 10^{-3}\ \mathrm{eV}^2$

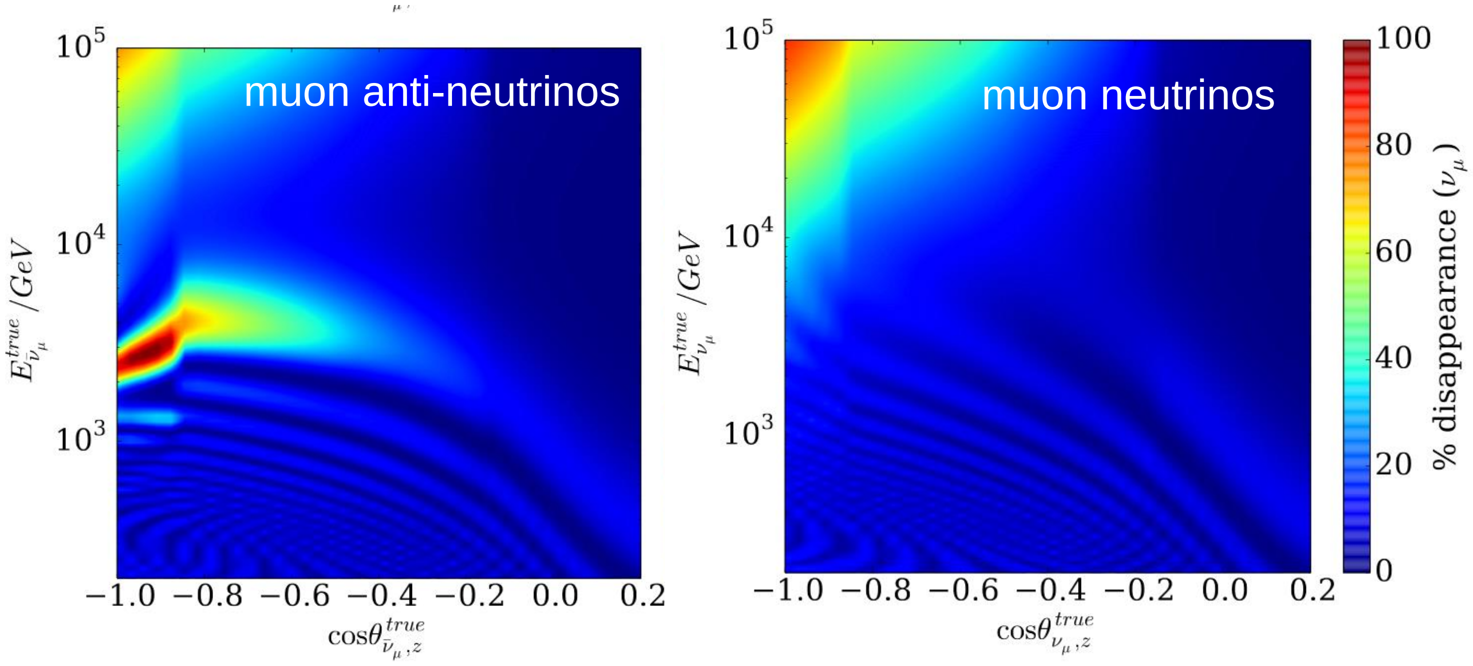
Slide by S. Blot (IceCube)

See talk by Thomas Stuttard tomorrow morning

Non-standard oscillations with high energy neutrinos

Relative rate change due to sterile neutrinos in IceCube > 100 GeV





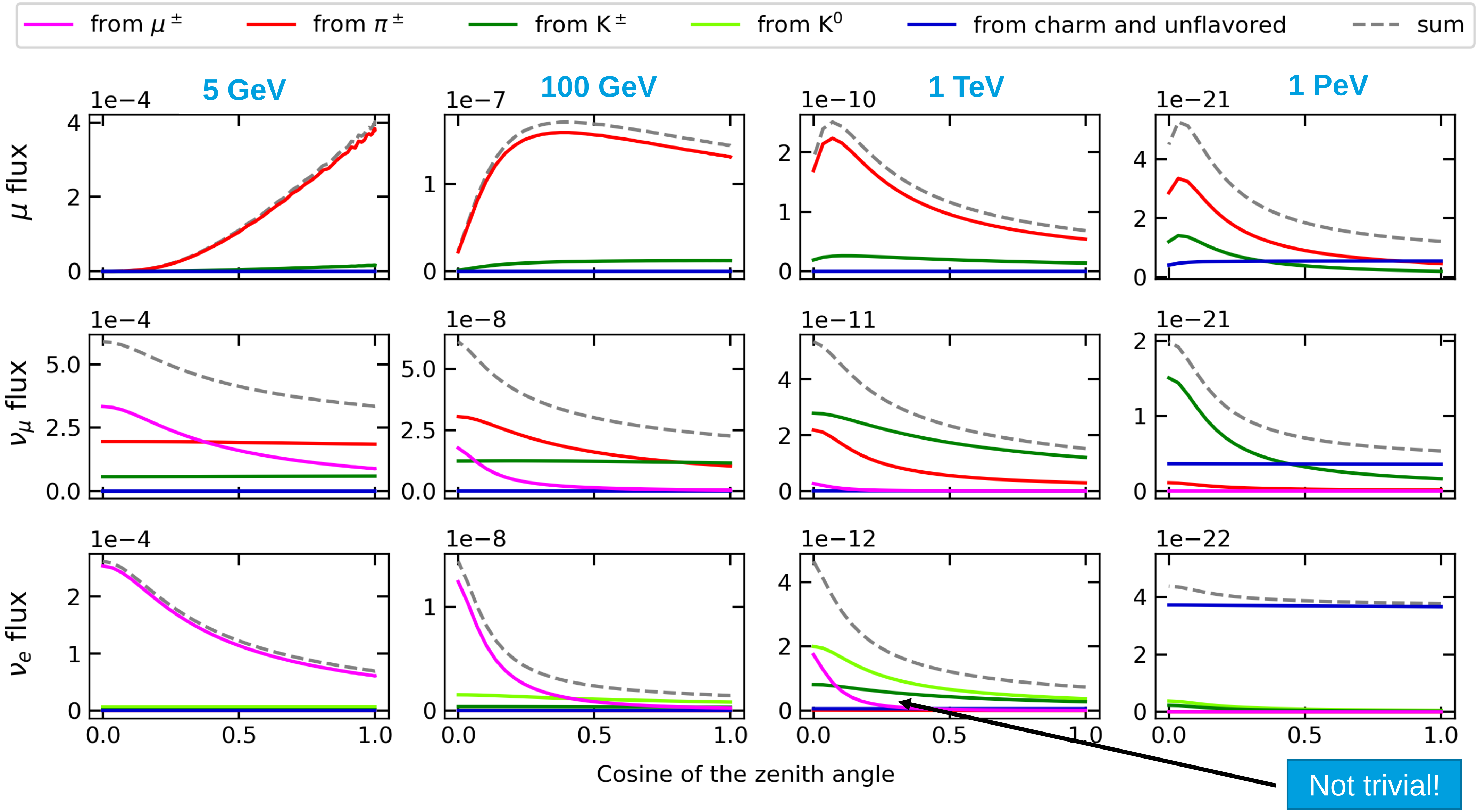
Atmospheric flux		
ν flux template	discrete (7)	
ν / $\bar{\nu}$ ratio	continuous	0.025
π / K ratio	continuous	0.1
Normalization	continuous	none[1]
Cosmic ray spectral index	continuous	0.05
Atmospheric temperature	continuous	model tuned

IceCube, Phys. Rev. Lett. 117, 071801 (2016)

Talk by Thomas Stuttard (& maybe Jordi Salvado)

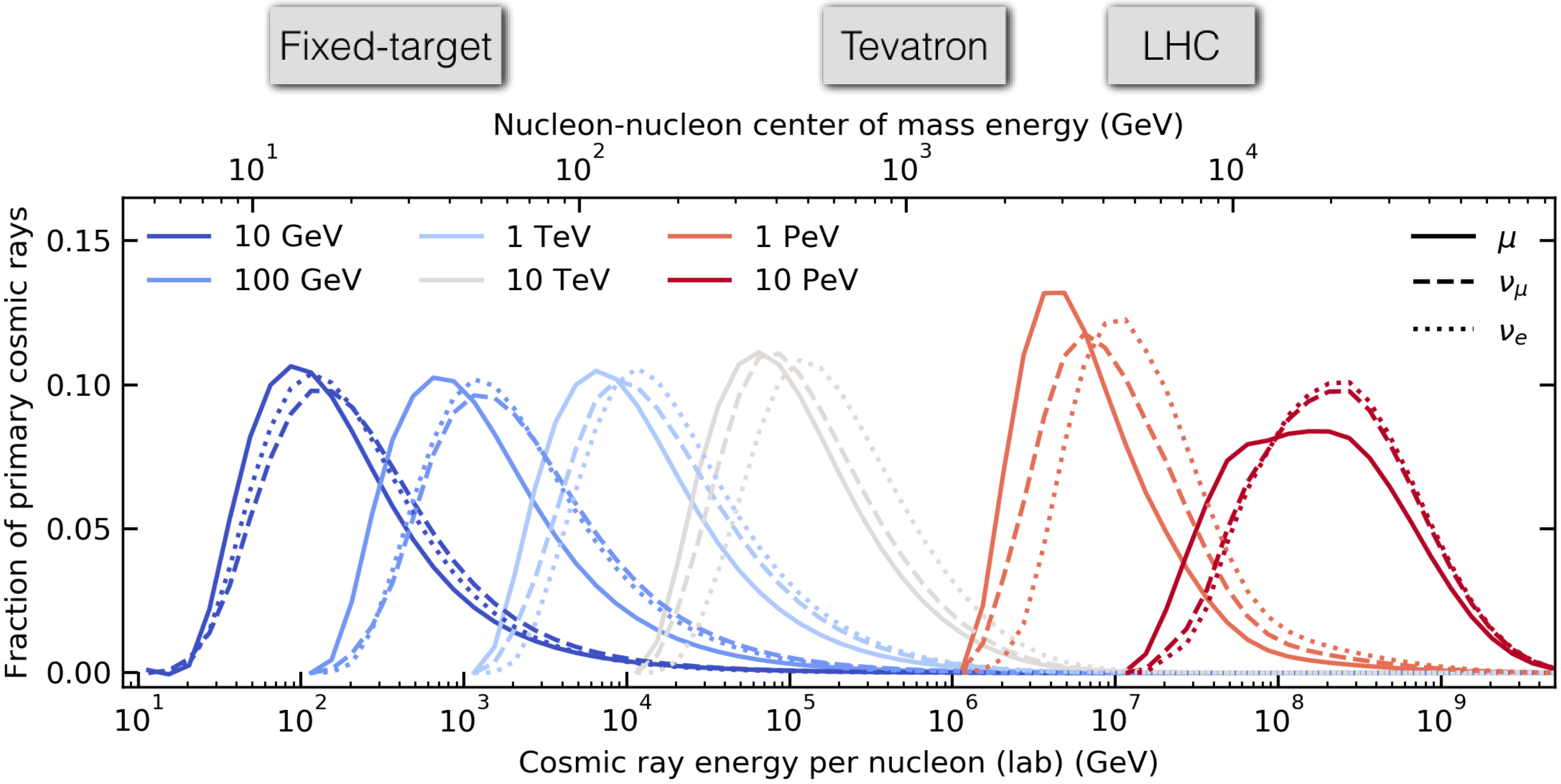
Uncertainties physically correlated and related to different quantities

Different hadronic components shape the zenith distribution

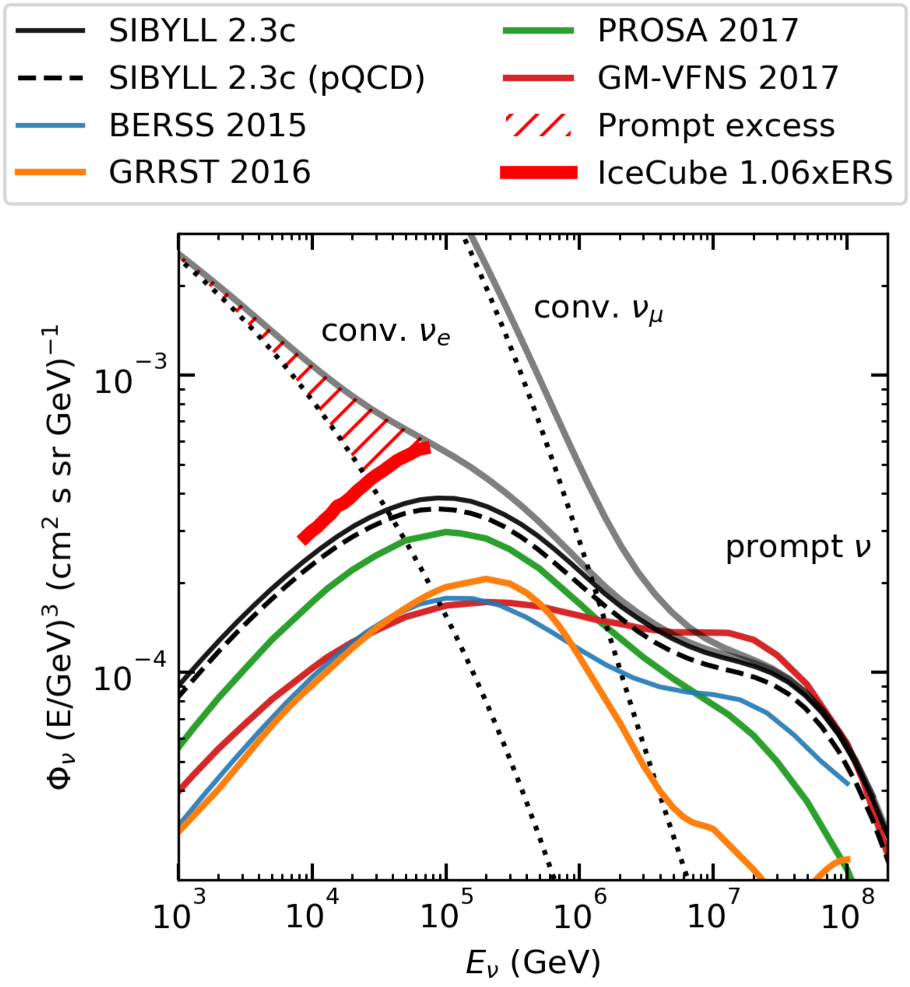


AF, F. Riehn, R. Engel, T.K. Gaisser, T. Stanev, in prep. (2018)

Relation between lepton and cosmic ray energy



Signatures of the prompt neutrino flux



Maria Vittoria G. will talk about prompt in the afternoon

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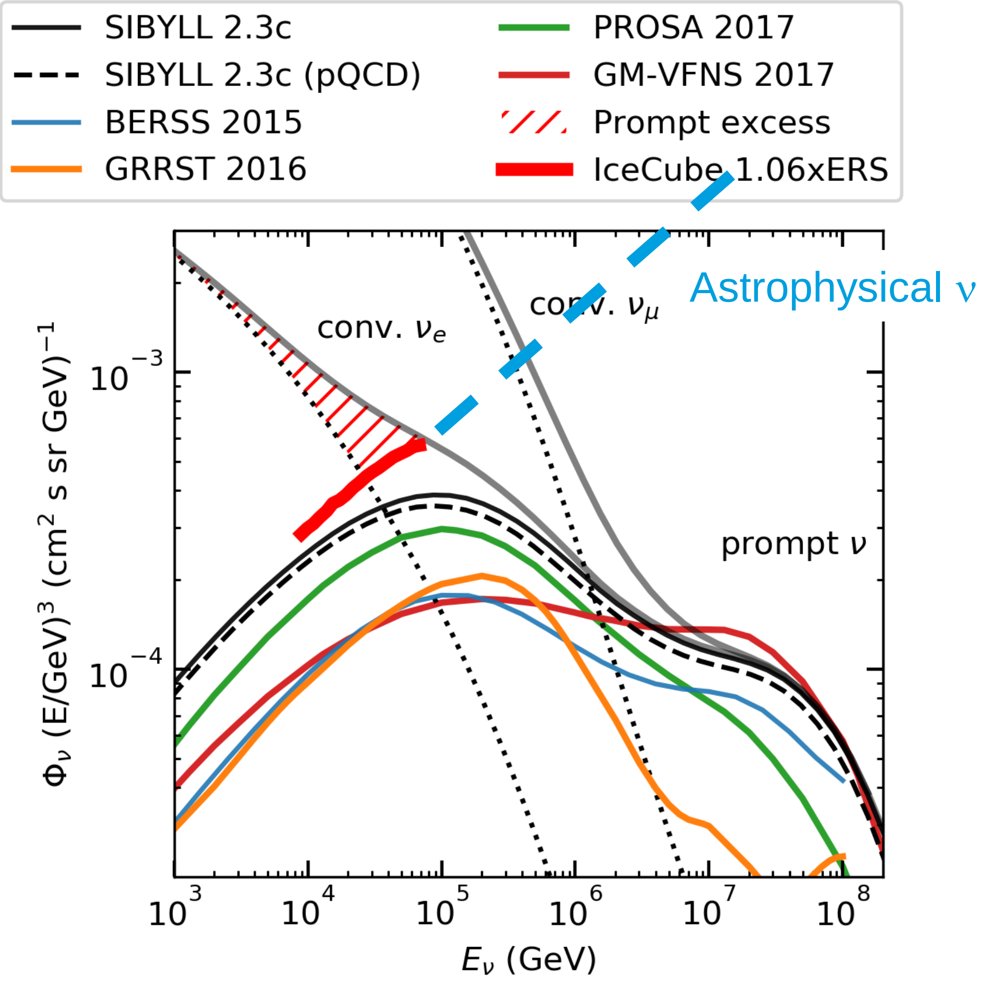

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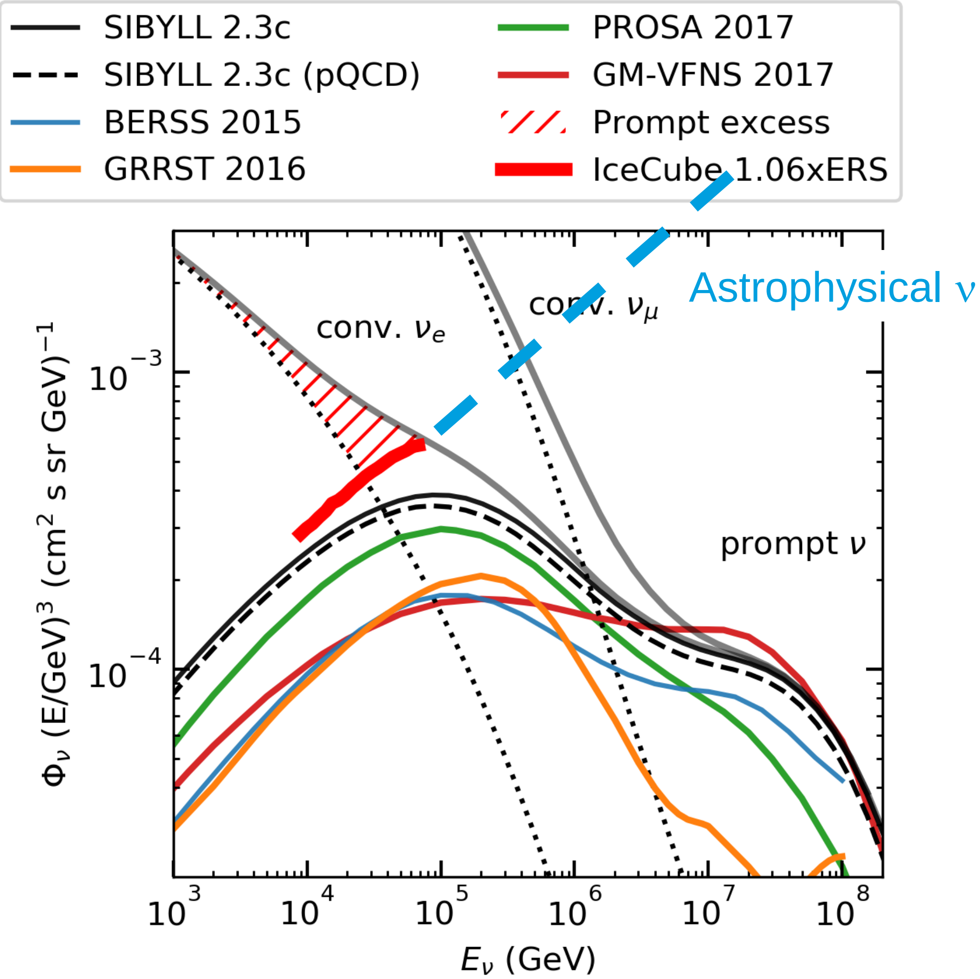

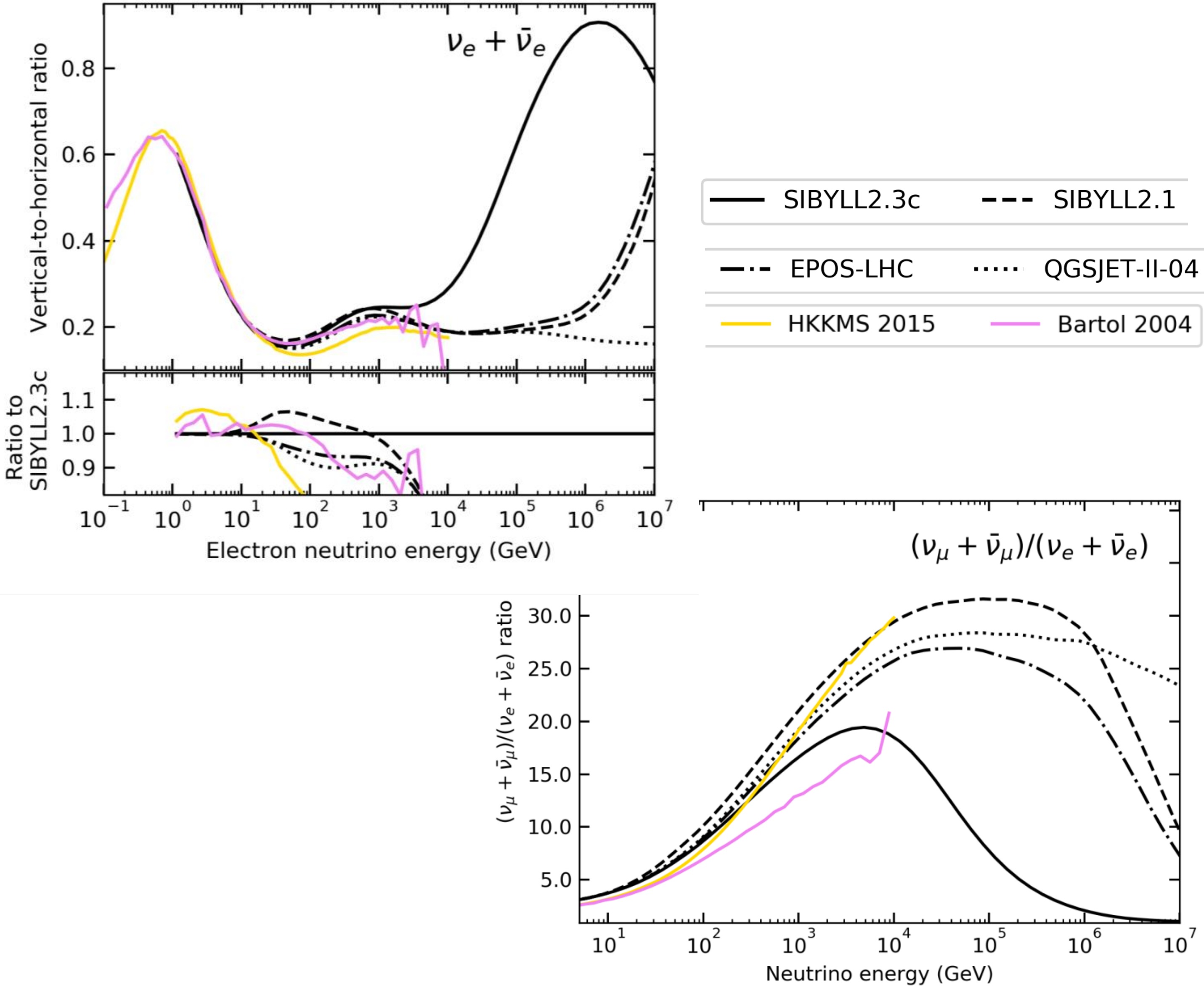

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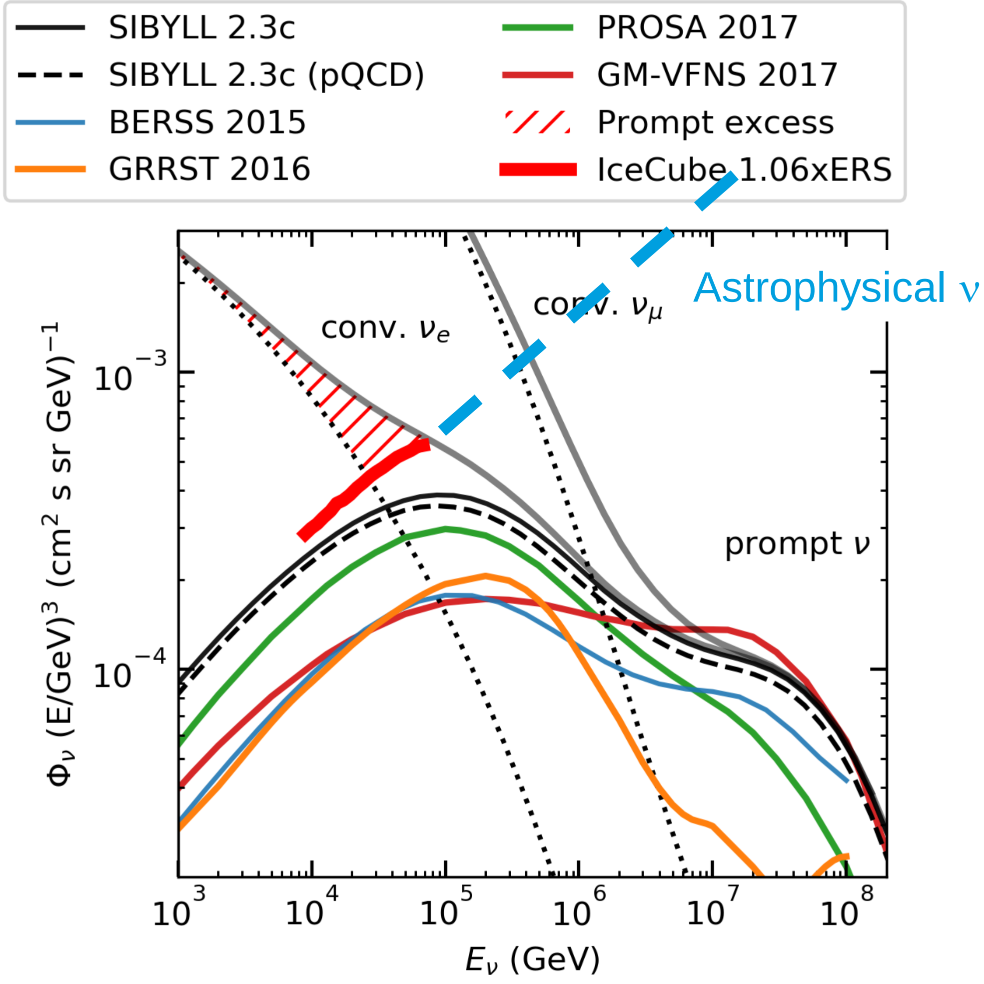

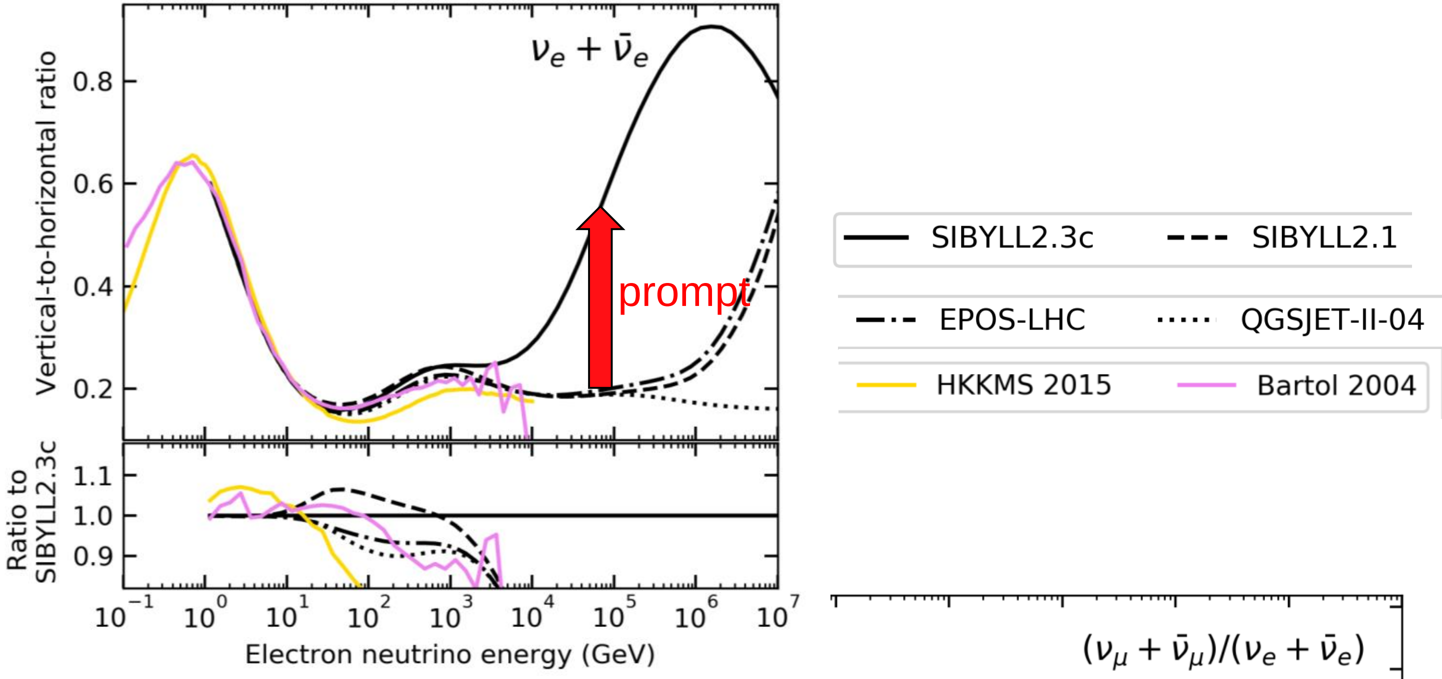

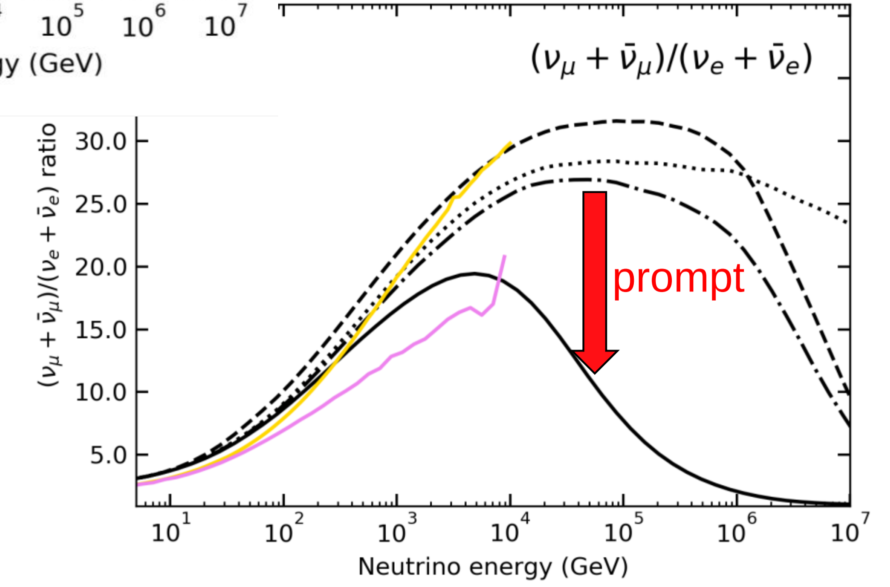

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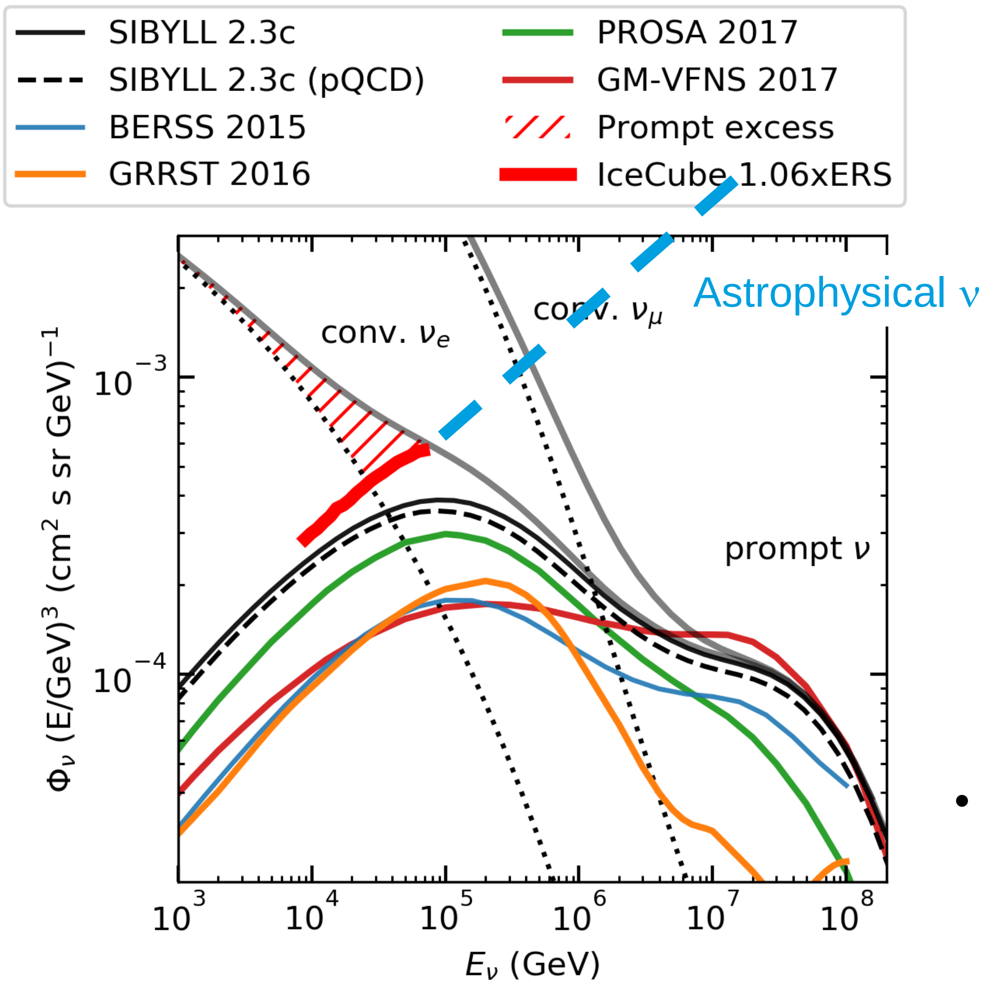

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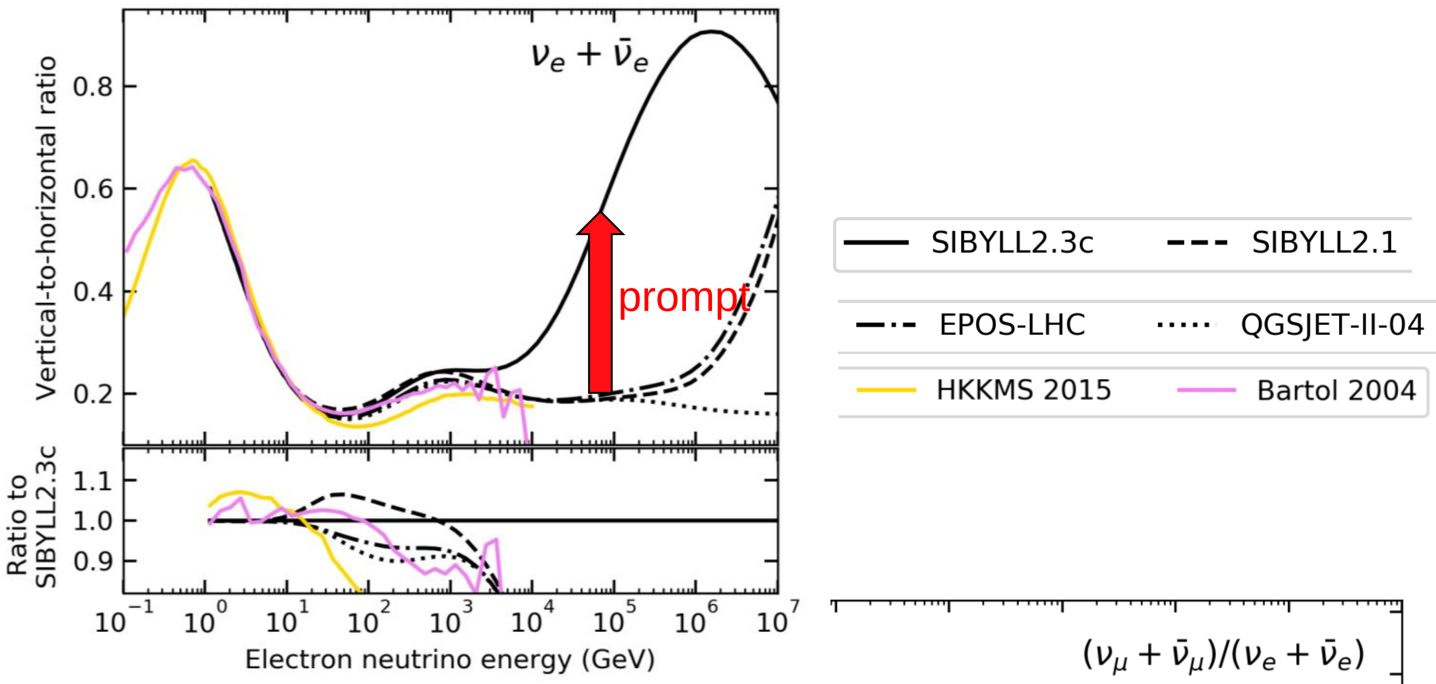
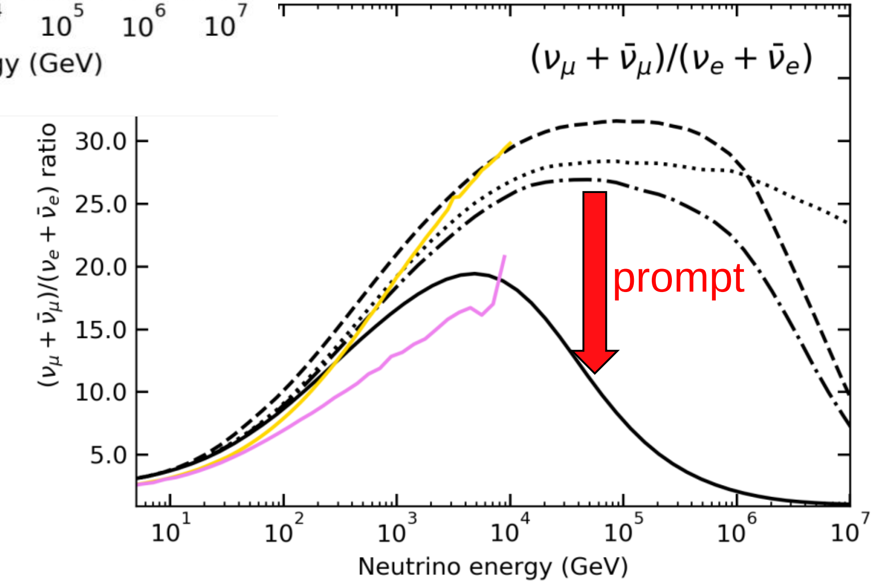
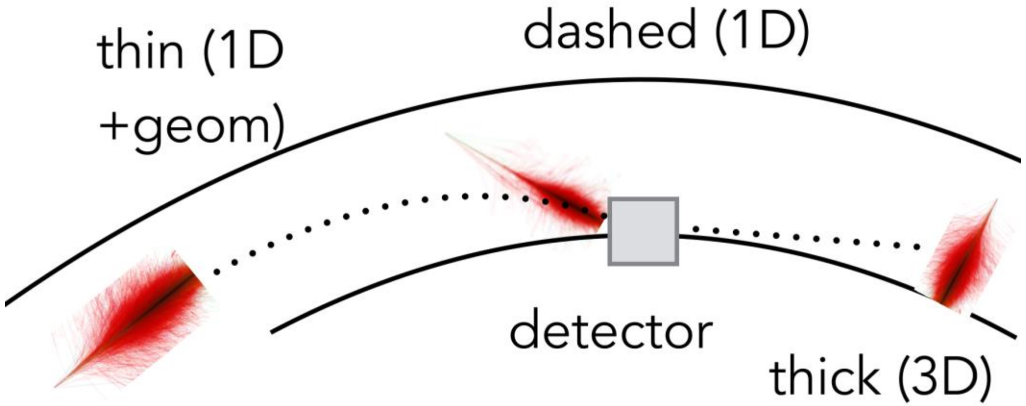
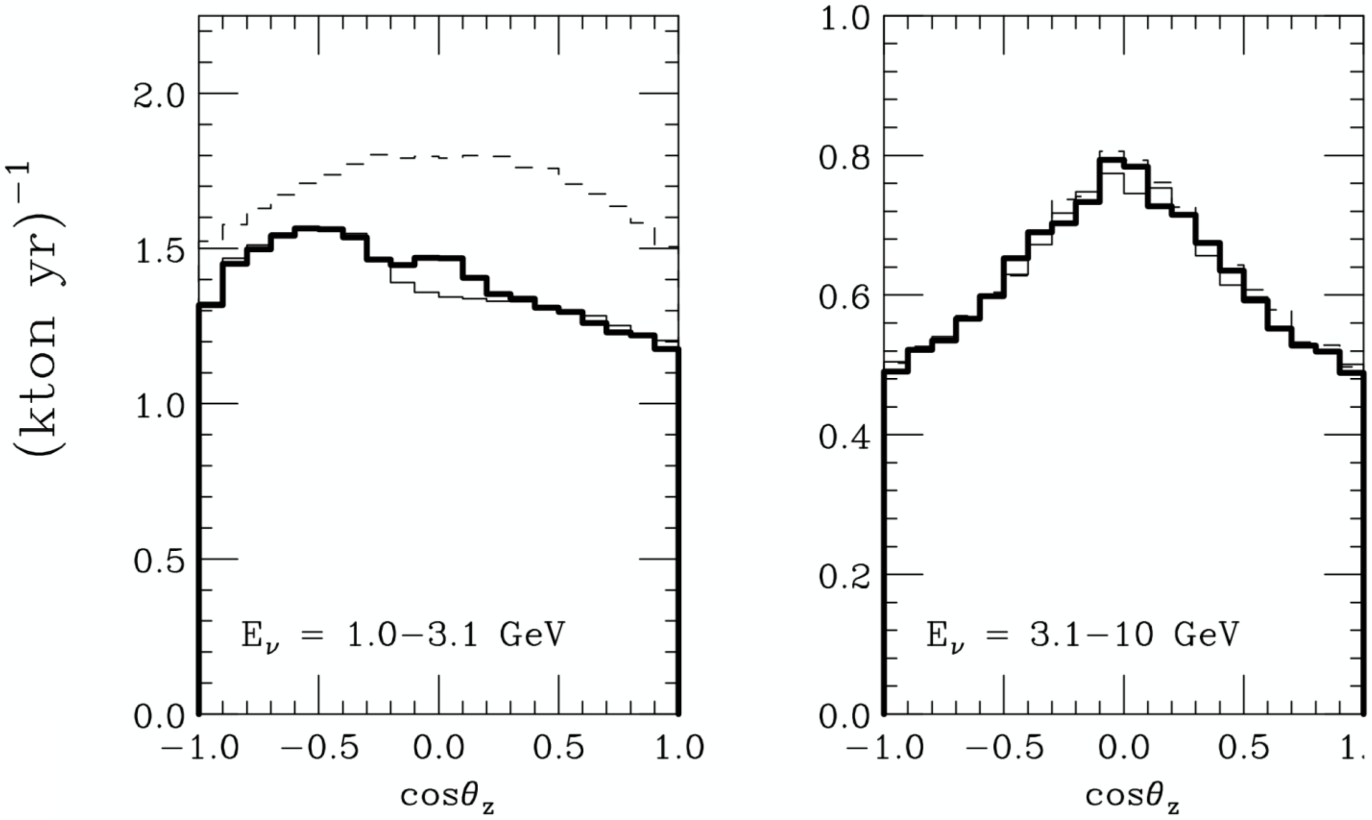


- Sensitive variables
  - Up-horizontal ratio
  - Track/cascade ratio
- Detection of flux excess unlikely



Atmospheric leptons at low energy – 3D vs 1D





A subset of 3D calculations

[1] G. Barr, P. Lipari, S. Robbins, and T. Stanev, International Cosmic Ray Conference 3, 1411 (2003).

[2] M. Honda, T. Kajita, K. Kasahara, and S. Midorikawa, Phys. Rev. D 83, (2011).

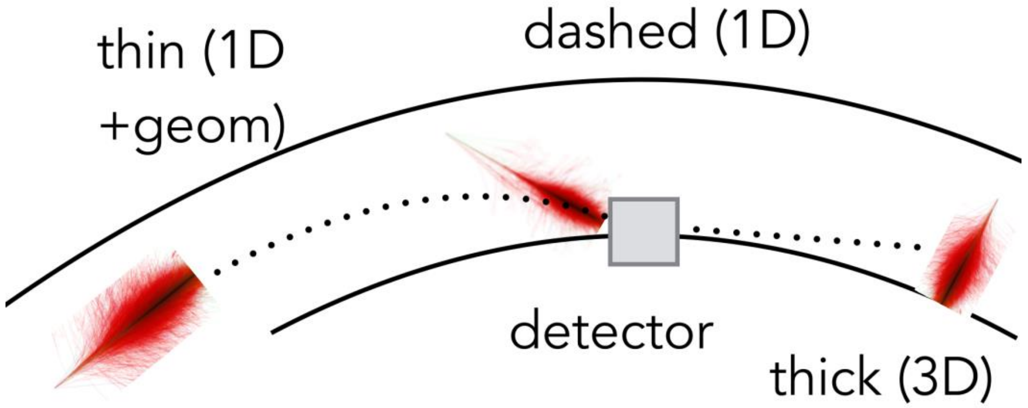
[3] M. Honda, T. Kajita, K. Kasahara, S. Midorikawa, and T. Sanuki, Phys. Rev. D 75, (2007).

*[4] [1] G. Battistoni, A. Ferrari, P. Lipari, T. Montaruli, P. R. Sala, and T. Rancati, Astroparticle Physics **12**, 315 (1999).*

[5] J. Wentz, I. M. Brancus, A. Bercuci, D. Heck, J. Oehlschläger, H. Rebel, and B. Vulpescu, Phys. Rev. D 67, 073020 (2003).

3D needed < ~ 5 - 10 GeV

Atmospheric leptons at low energy – 3D vs 1D



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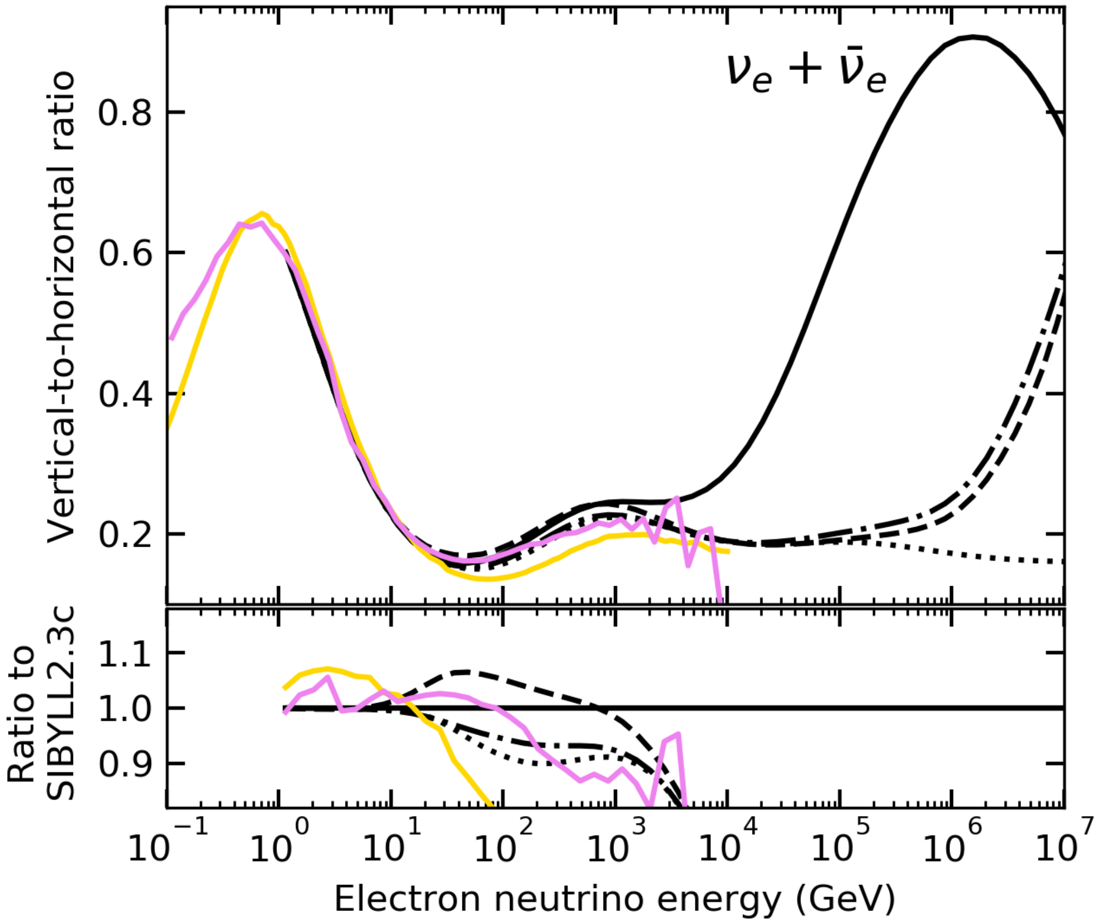

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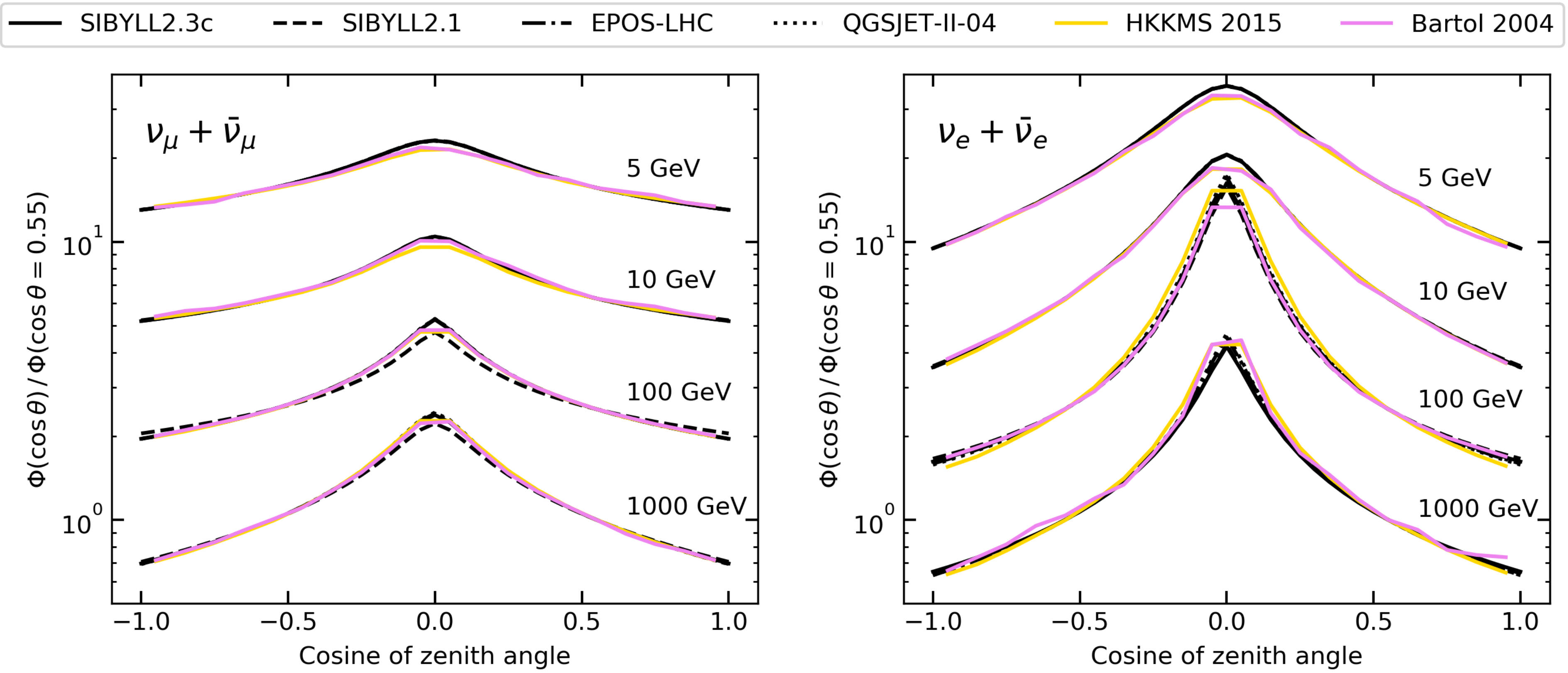
Hadronic model dependence of zenith distributions



- Good agreement above tens of GeV for muon neutrinos
- Some tension between calculations at the horizon in electron neutrinos
- Affected by K/Pi, K^{+-}/K^0_L ratios

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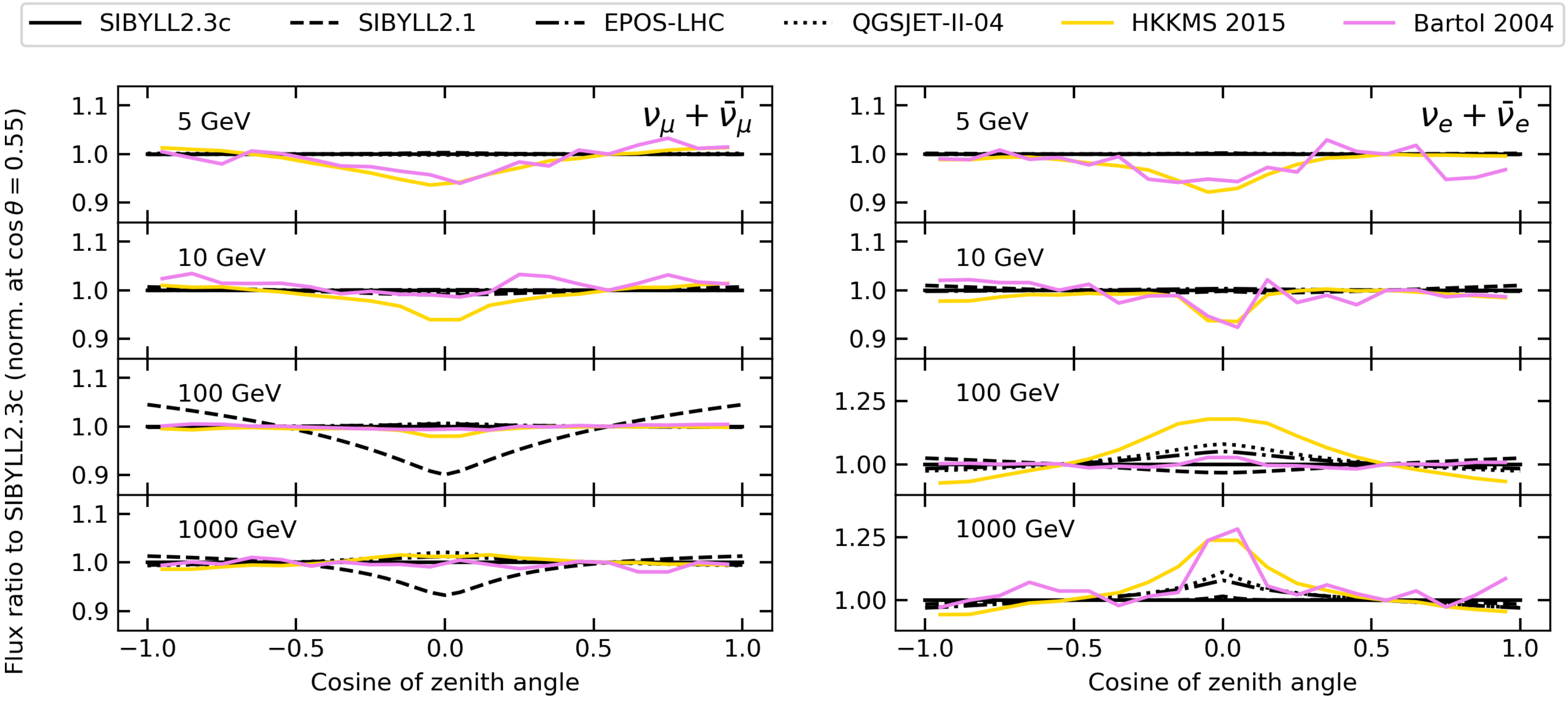

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Hadronic uncertainties: current state of the art

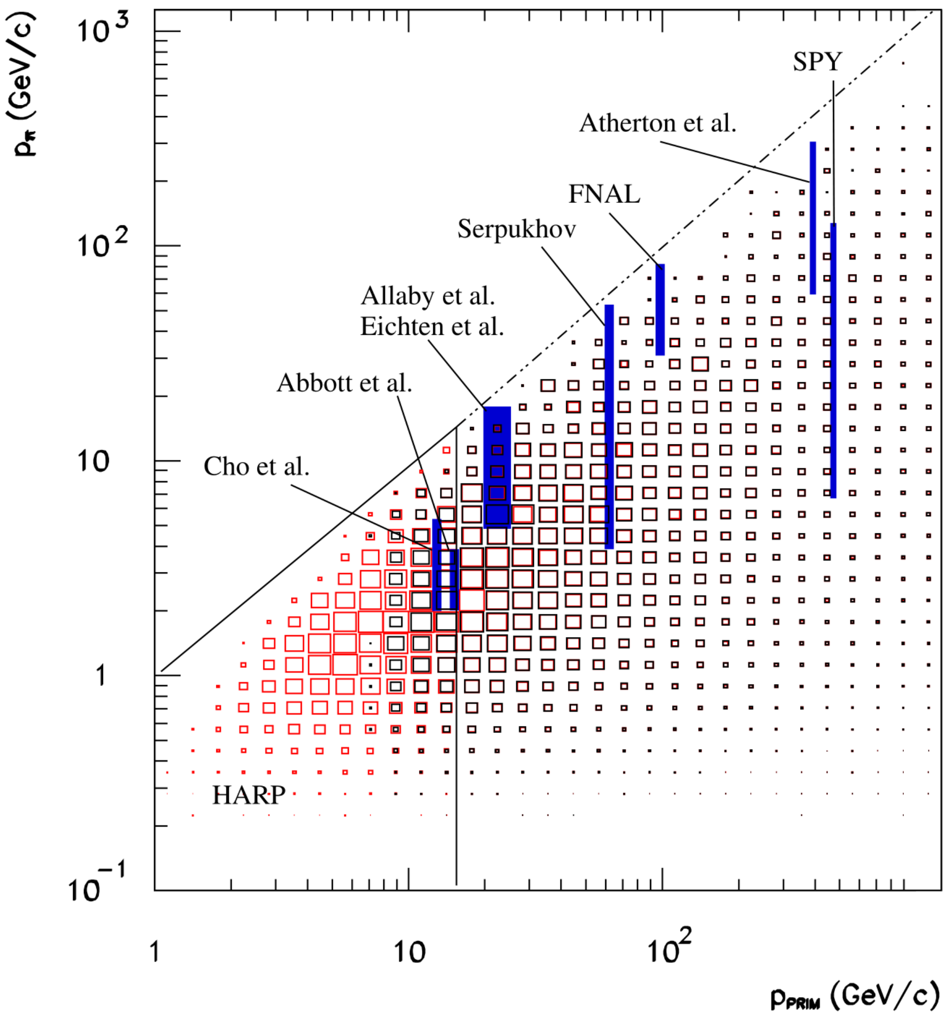
- "*Uncertainties in atmospheric neutrino fluxes*", G. D. Barr, S. Robbins, T. K. Gaisser, and T. Stanev, Phys. Rev. D 74, 094009 (2006) (extensive discussion also in Sanuki et al. PRD 75 (2007)

- Cut phase-space in regions/slices in E_{lab} and x_{lab} and **assign** uncertainty to each slice (uncorrelated)

- **Problem 1**: Uncertainty assigned by hand and judged only from availability of experimental data (not how well the model [TARGET] describes it)

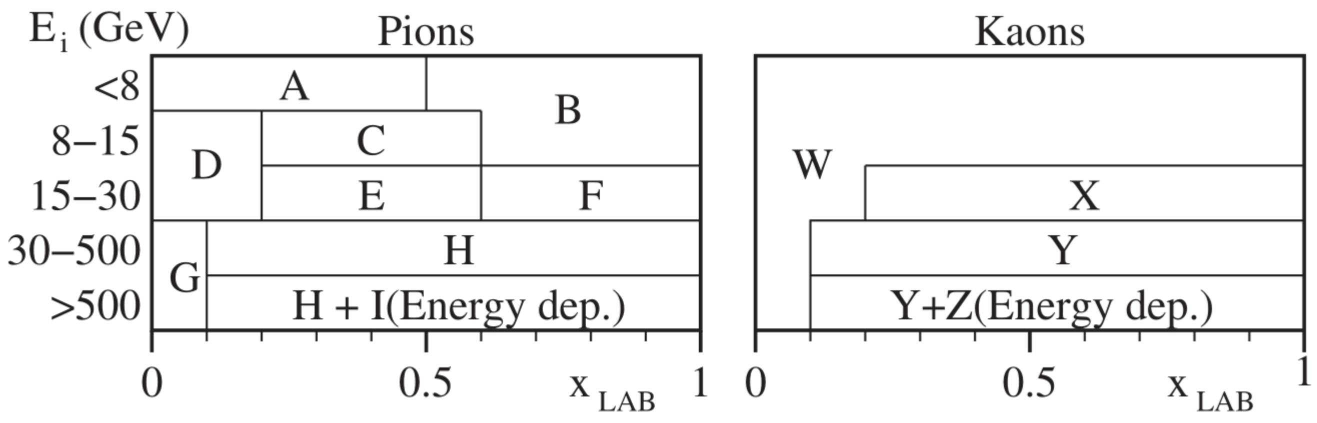
- **Problem 2**: Scheme doesn't tell anything about "best estimate"

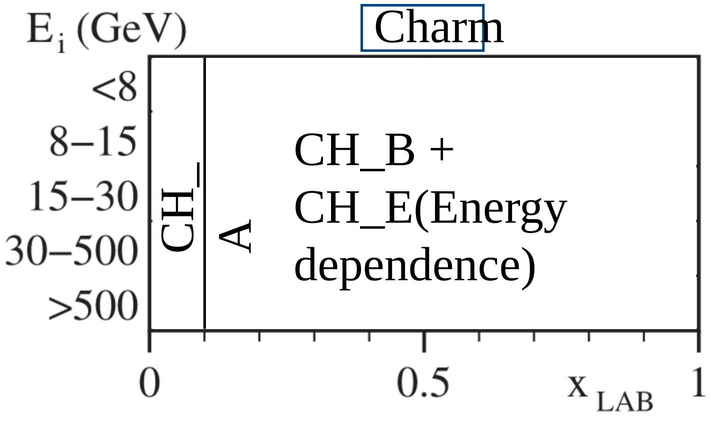
PHYSICAL REVIEW D **74,** 094009 (2006)

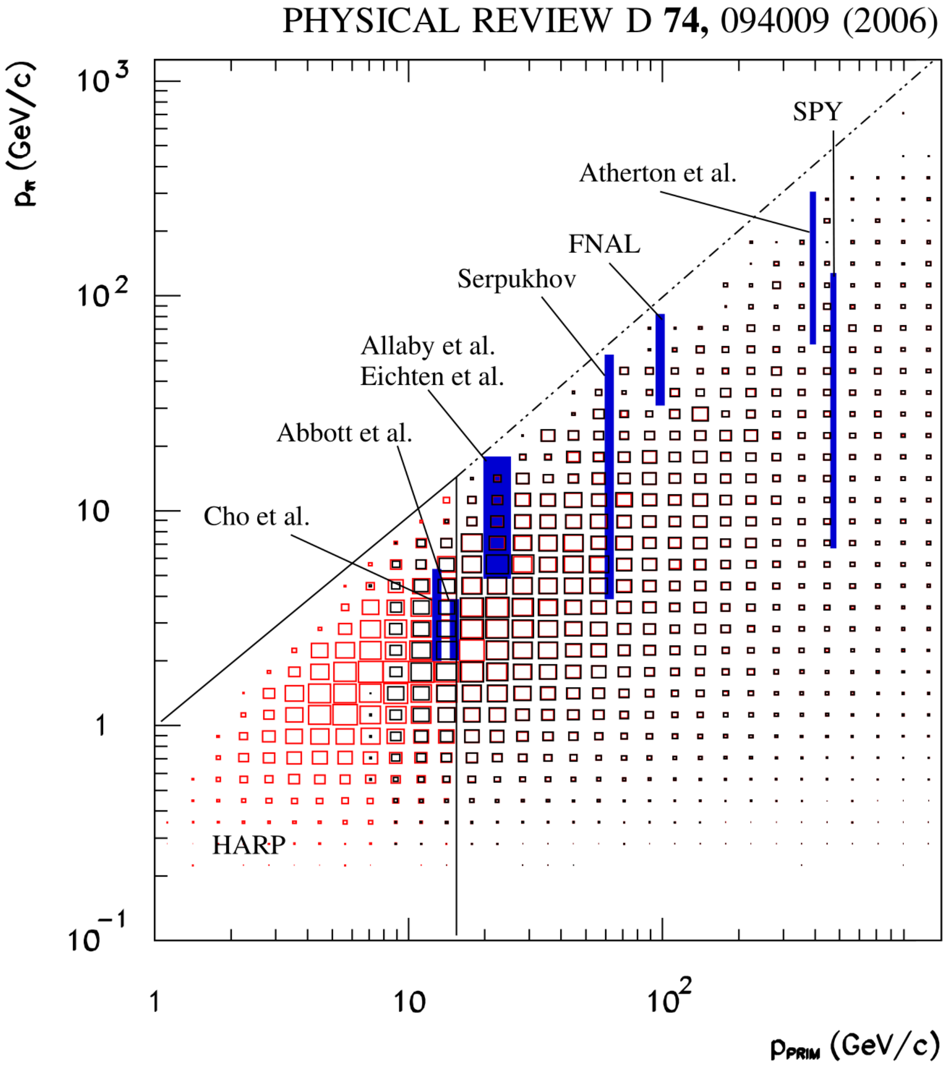


Phase space regions

The







Phase space regions

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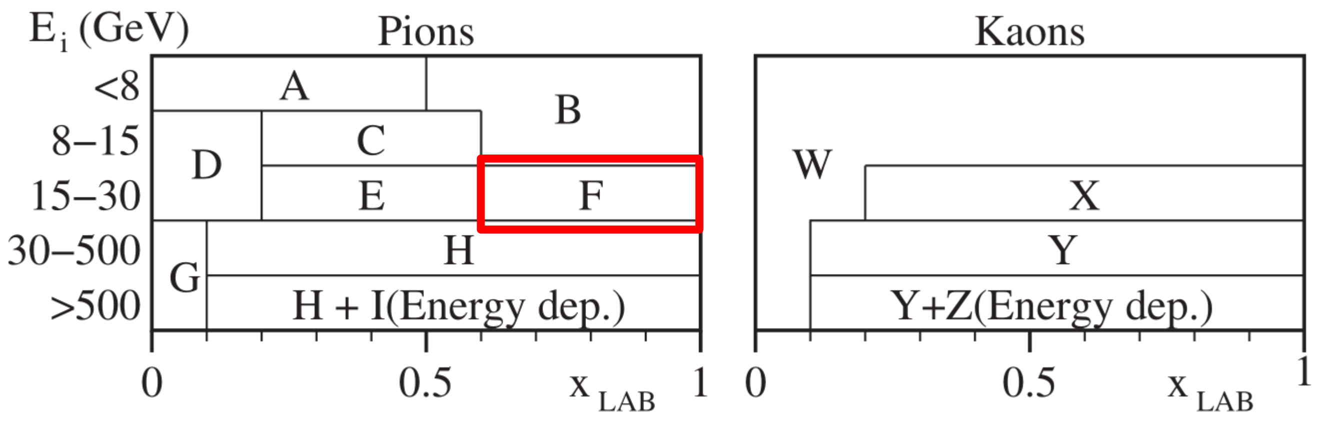
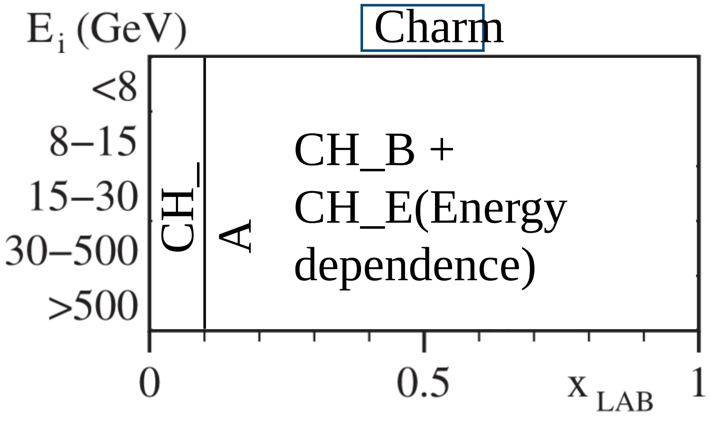
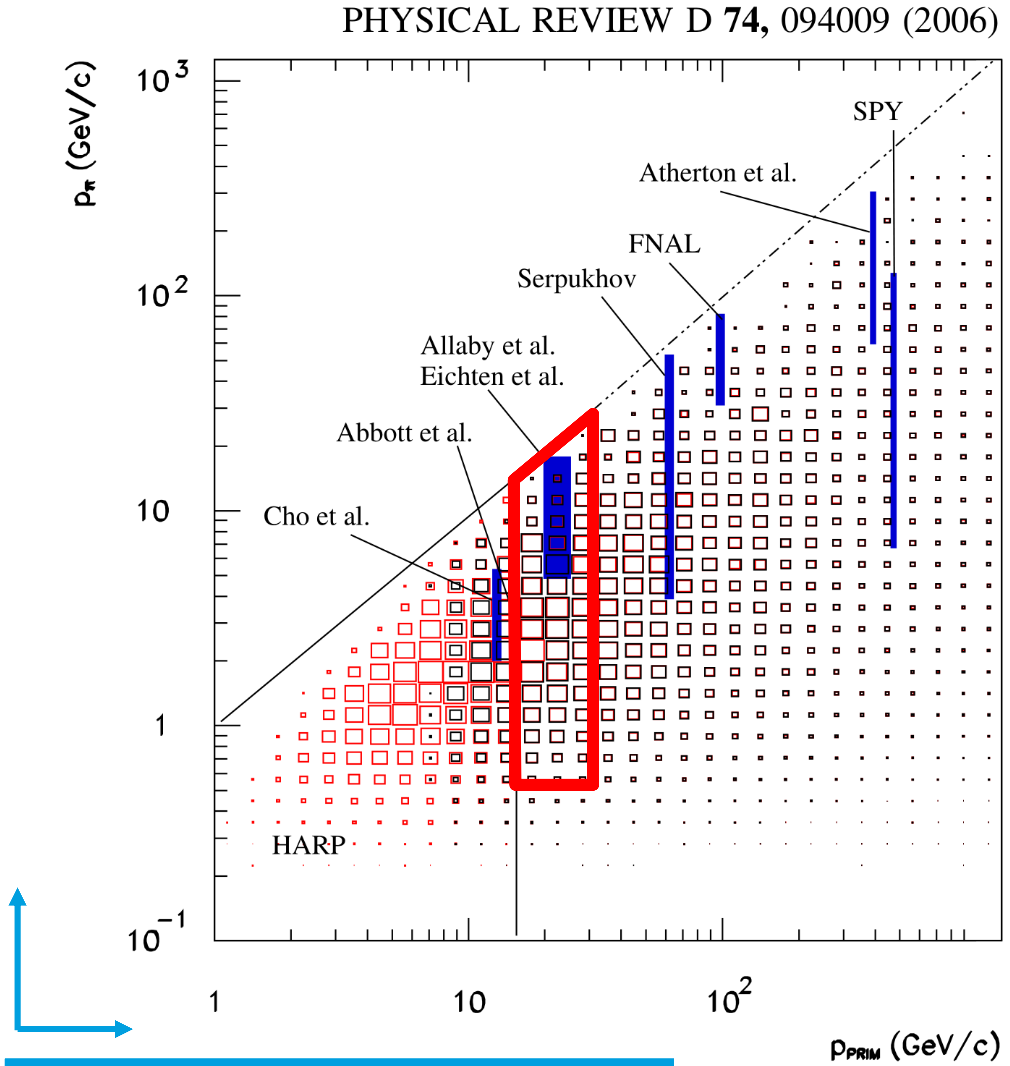


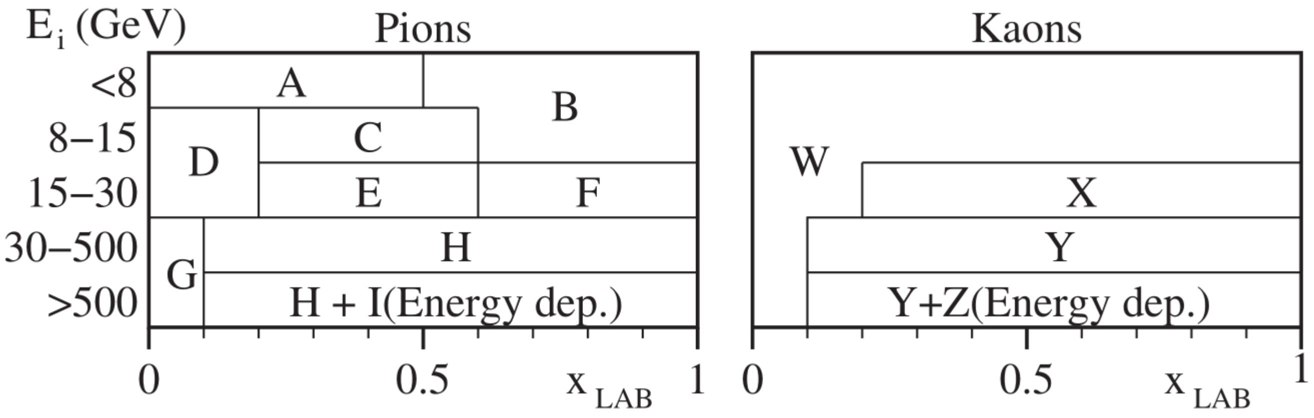


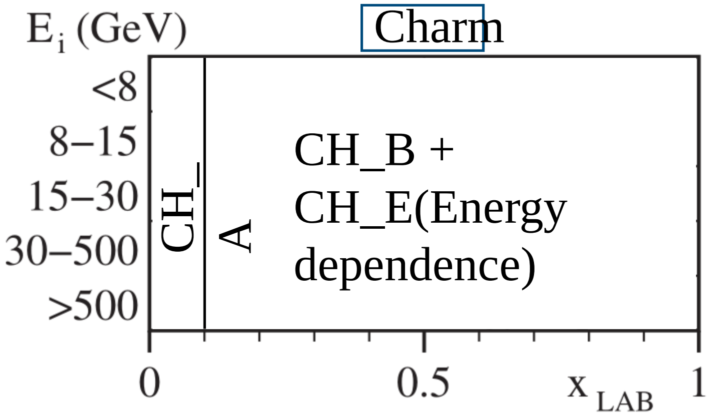


Same axes as matrices in

MCEq-based method

The regions





- Compute partial derivatives wrt. phase-space regions, i.e. $\frac{\partial \Phi_\nu}{\partial \mathrm{W}}$
- No correlations between phase-space regions (as in Barr et al.) or add. correlations

Elements of Jacobian (numerical)

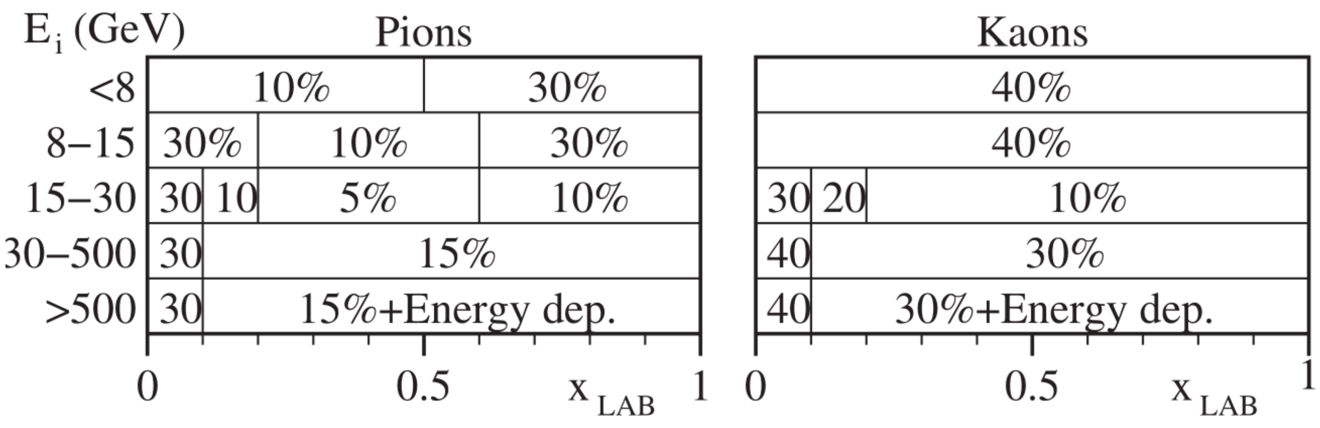
$$J_{E_i j} = \frac{\partial \Phi_\nu(E_i)}{\partial p} = \frac{\Phi_\nu(\delta p_j +) - \Phi_\nu(\delta p_j -)}{2\delta p_j}$$

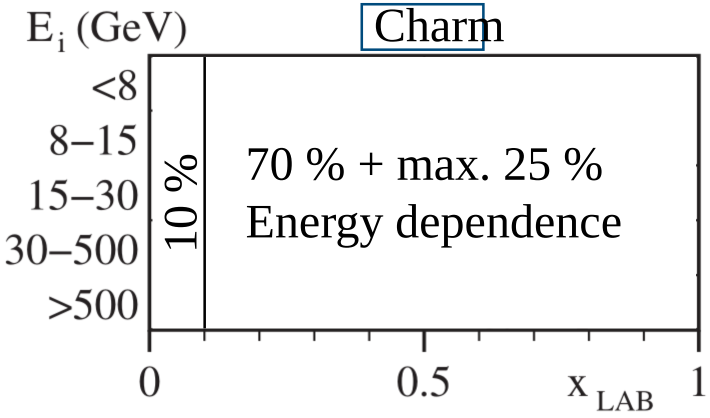
Error propagation

$$\mathrm{cov}[\Phi_\nu(E_i), \Phi_\nu(E_j)] = \sum_{mn} J_{E_i m} J_{E_j n} \mathrm{cov}[p_m, p_l]$$

MCEq-based method

The regions





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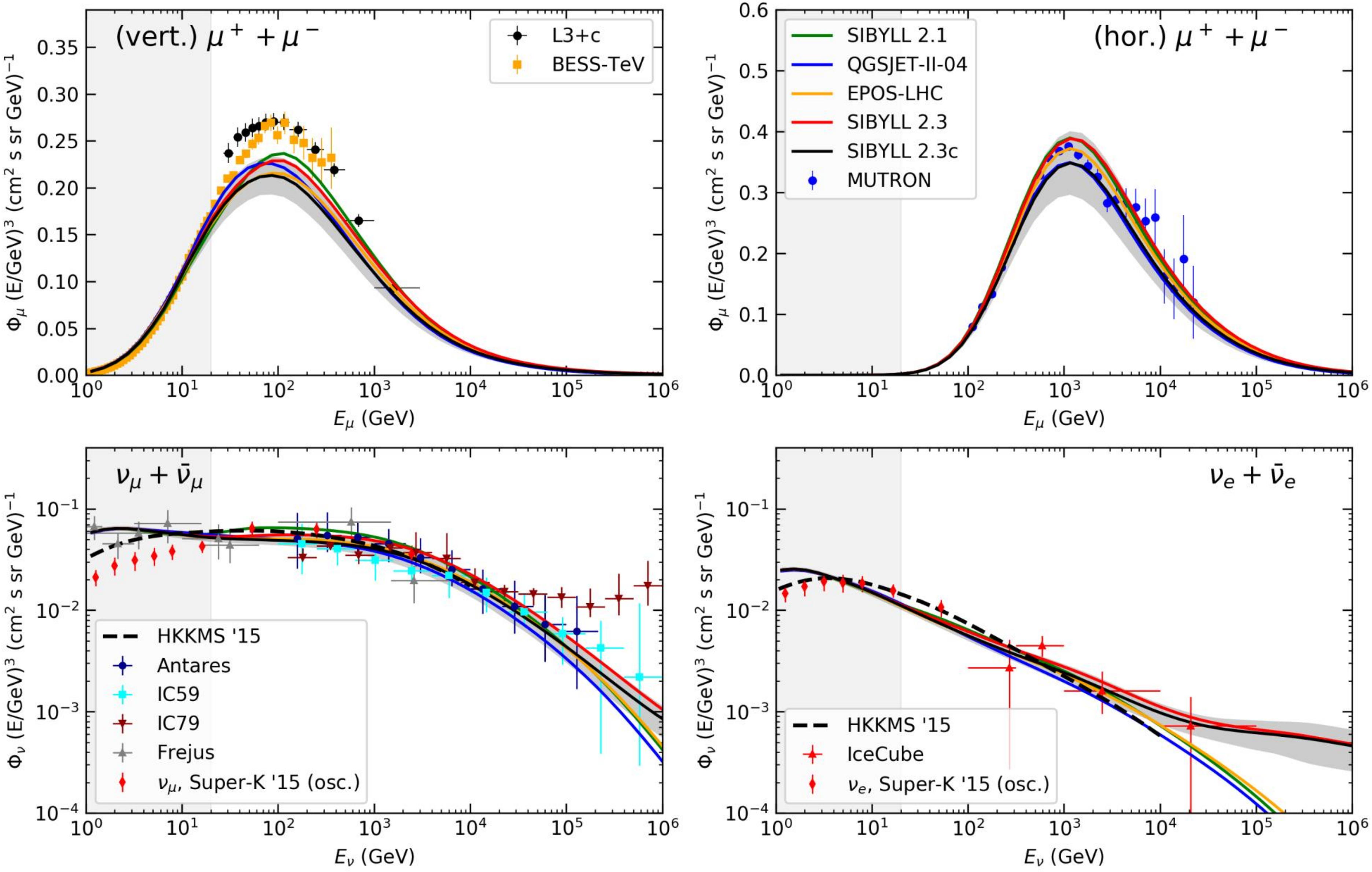
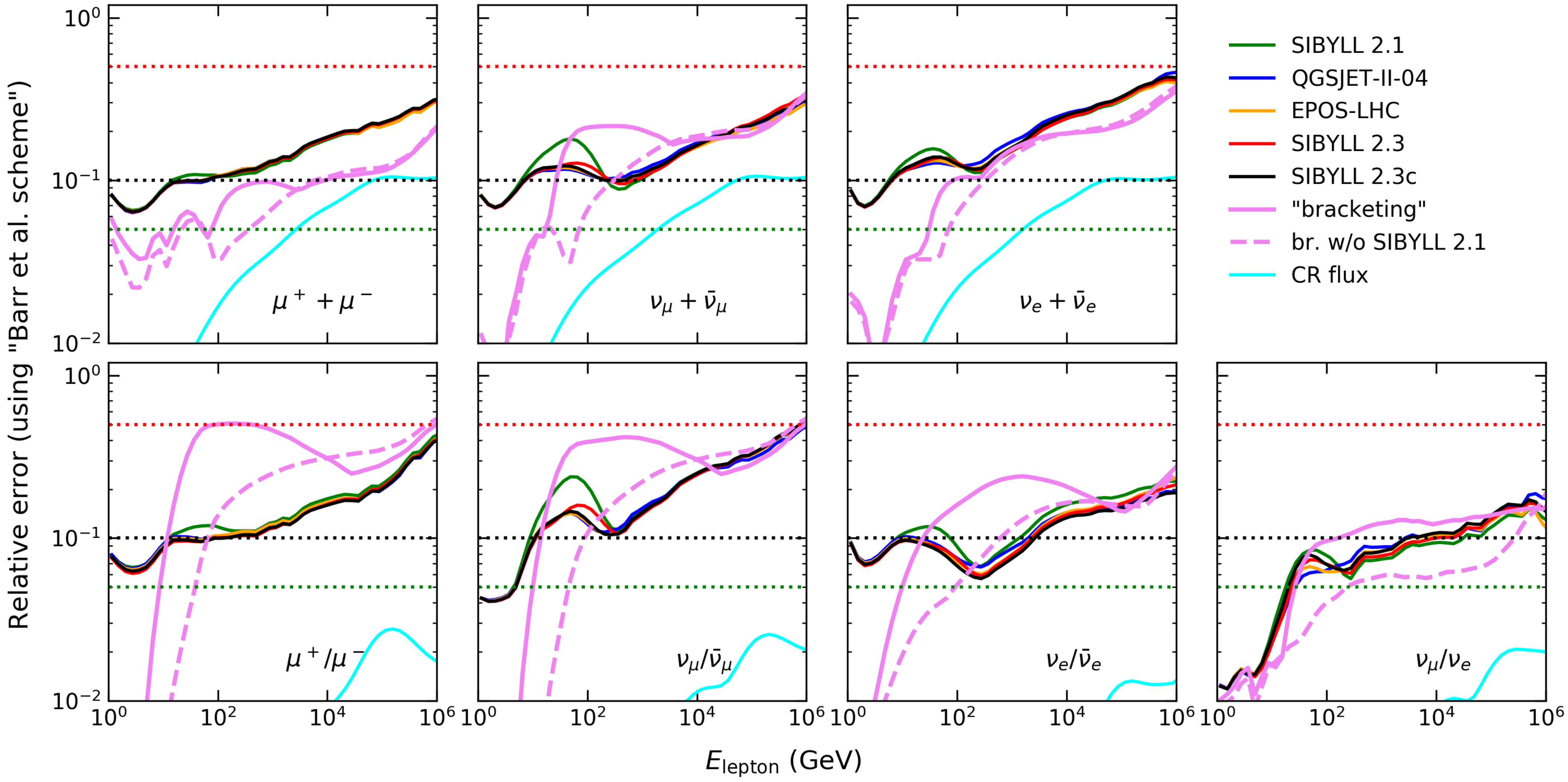
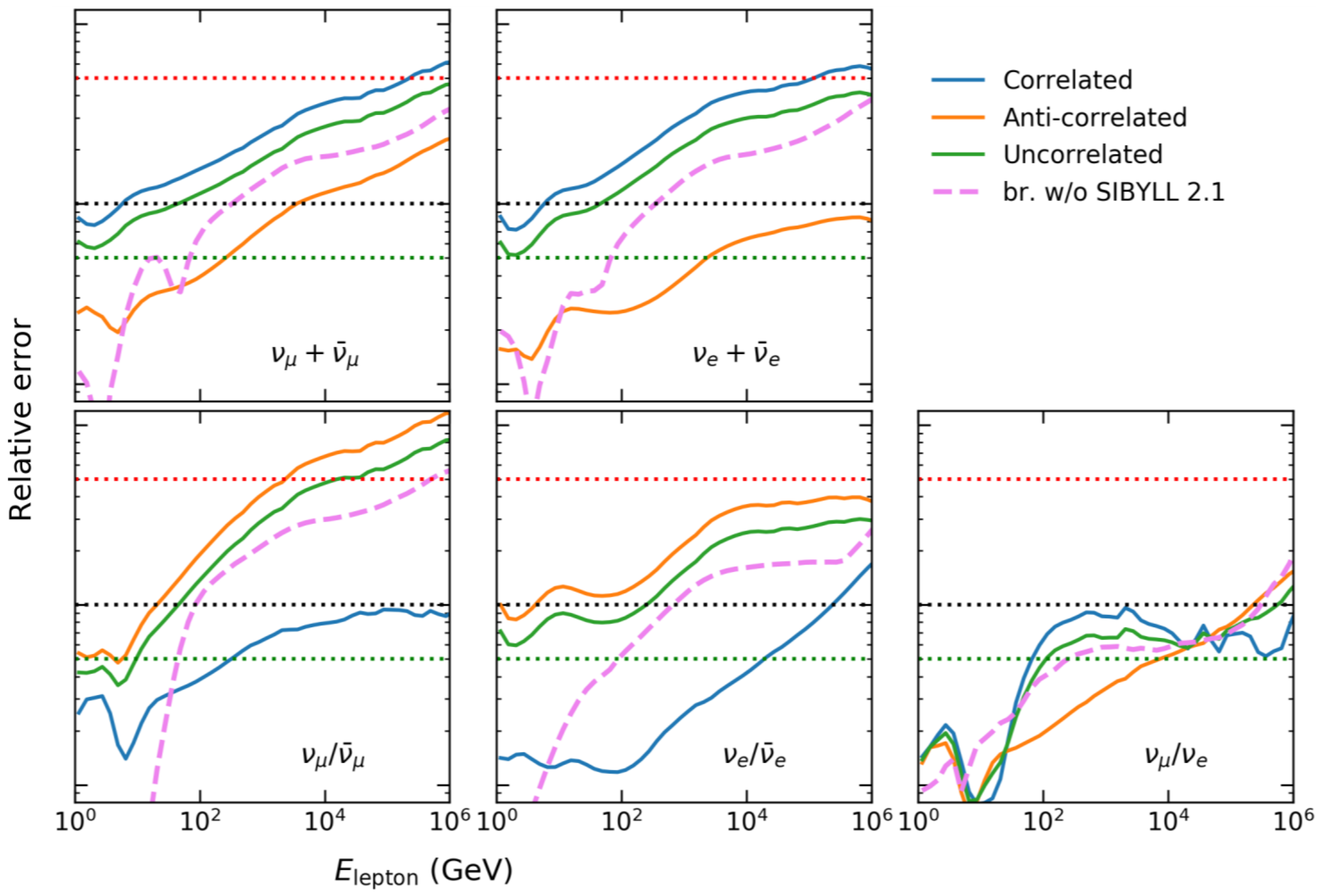
Computation of error bands through error propagation



Computation of error bands through error propagation



Correlations between phase-space patches unclear



Examples	For one "Barr" - parameters
symmetric	$p\pi^+\uparrow$ $n\pi^+\uparrow$ $p\pi^-\uparrow$ $n\pi^-\uparrow$
asymmetric	$p\pi^+\uparrow$ $n\pi^+\uparrow$ $p\pi^-\downarrow$ $n\pi^-\downarrow$
uncorrelated	$p\pi^+\uparrow$ $n\pi^+0$ $p\pi^-0$ $n\pi^-0$

- The production of charged secondaries is physically not independent
- It is very difficult to extract this information from hadronic interaction models directly

Conclusions and future path

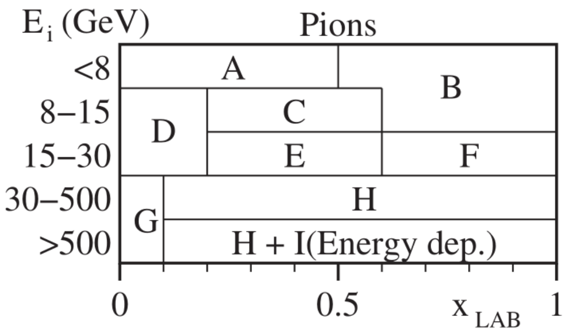
- Current atmospheric neutrino detectors cover **9 orders of magnitude** in energy (MeV-PeV) → **challenge** for flux calculations!

- Theoretical flux calculations keep improving. **Unsolved problems remain**, in particular hadronic interactions

- **New high-precision** (and high-performance) calculations available (MCEq) that well match full Monte Carlo

- This allows for a **new approach towards flux systematics** by replacing effective parameters with physical parameters (see next generation IceCube analyses)

Atmospheric flux		
ν flux template	discrete (7)	
ν / $\overline{\nu}$ ratio	continuous	0.025
π / K ratio	continuous	0.1
Normalization	continuous	none[1]
Cosmic ray spectral index	continuous	0.05
Atmospheric temperature	continuous	model tuned



MCEq: Matrix Cascade Equations

$$
\begin{aligned}
\frac{\mathrm{d}\Phi_h(E,X)}{\mathrm{d}X} = & -\frac{\Phi_h(E,X)}{\lambda_{\mathrm{int},h}(E)} \\
& -\frac{\Phi_h(E,X)}{\lambda_{\mathrm{dec},h}(E,X)} \\
& + \sum_k \int_E^\infty \mathrm{d}E_k \, \frac{\mathrm{d}N_{k(E_k)\to h(E)}}{\mathrm{d}E} \frac{\Phi_k(E_k,X)}{\lambda_{\mathrm{int},k}(E_k)} \\
& + \sum_k \int_E^\infty \mathrm{d}E_k \, \frac{\mathrm{d}N^{\mathrm{dec}}_{k(E_k)\to h(E)}}{\mathrm{d}E} \frac{\Phi_k(E_k,X)}{\lambda_{\mathrm{dec},k}(E_k,X)}
\end{aligned}
$$

A. Fedynitch, R. Engel, T. K. Gaisser, F. Riehn and S. Todor
PoS ICRC 2015, 1129 (2015), EPJ Web Conf. 99, 08001
(2015) and EPJ Web Conf. 116, 11010 (2016)

MCEq: Matrix Cascade Equations

$$\frac{\mathrm{d}\Phi^h_{E_i}}{\mathrm{d}X} = -\frac{\Phi^h_{E_i}}{\lambda^h_{\mathrm{int},E_i}} - \frac{\Phi^h_{E_i}}{\lambda^h_{\mathrm{dec},E_i}(X)} + \sum_{E_k \geq E_i}^{E_N} \sum_k \frac{c_{k(E_k)\to h(E_i)}}{\lambda^k_{\mathrm{int},E_k}} \Phi^k_{E_k} + \sum_{E_k \geq E_i}^{E_N} \sum_k \frac{d_{k(E_k)\to h(E_i)}}{\lambda^k_{\mathrm{dec},E_k}(X)} \Phi^k_{E_k}$$

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$$\frac{d\Phi^h_{E_i}}{dX} = -\frac{\Phi^h_{E_i}}{\lambda^h_{\mathrm{int},E_i}} - \frac{\Phi^h_{E_i}}{\lambda^h_{\mathrm{dec},E_i}(X)} + \sum_{E_k \geq E_i}^{E_N} \sum_k \frac{c_{k(E_k)\to h(E_i)}}{\lambda^k_{\mathrm{int},E_k}} \Phi^k_{E_k} + \sum_{E_k \geq E_i}^{E_N} \sum_k \frac{d_{k(E_k)\to h(E_i)}}{\lambda^k_{\mathrm{dec},E_k}(X)} \Phi^k_{E_k}$$

Matrix-form

$$\frac{d}{dX}\boldsymbol{\phi} = \left[(-\mathbf{1} + \boldsymbol{C})\boldsymbol{\Lambda}_{int} + \frac{1}{\rho(X)}(-\mathbf{1} + \boldsymbol{D})\boldsymbol{\Lambda}_{dec}\right]\boldsymbol{\phi}.$$

geometry & atmosphere

flux vector

$$\vec{\phi} = \begin{pmatrix} \phi_p(E_0) \\ \phi_p(E_1) \\ \ldots \\ \phi_p(E_N) \\ \phi_n(E_0) \\ \ldots \\ \phi_n(E_N) \\ \phi^+_\pi(E_0) \\ \ldots \\ \phi_{\bar{\nu}_e}(E_N) \end{pmatrix}$$

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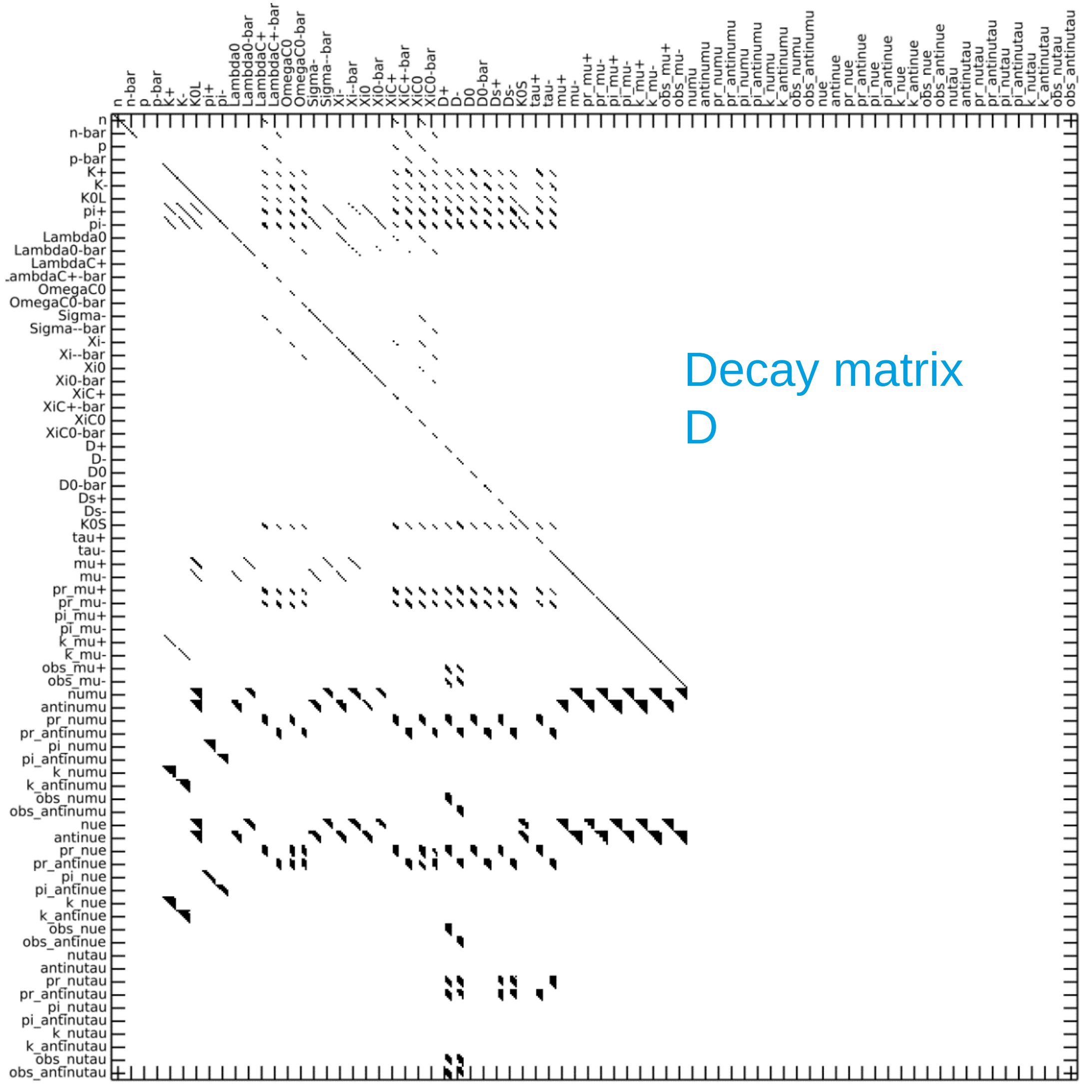
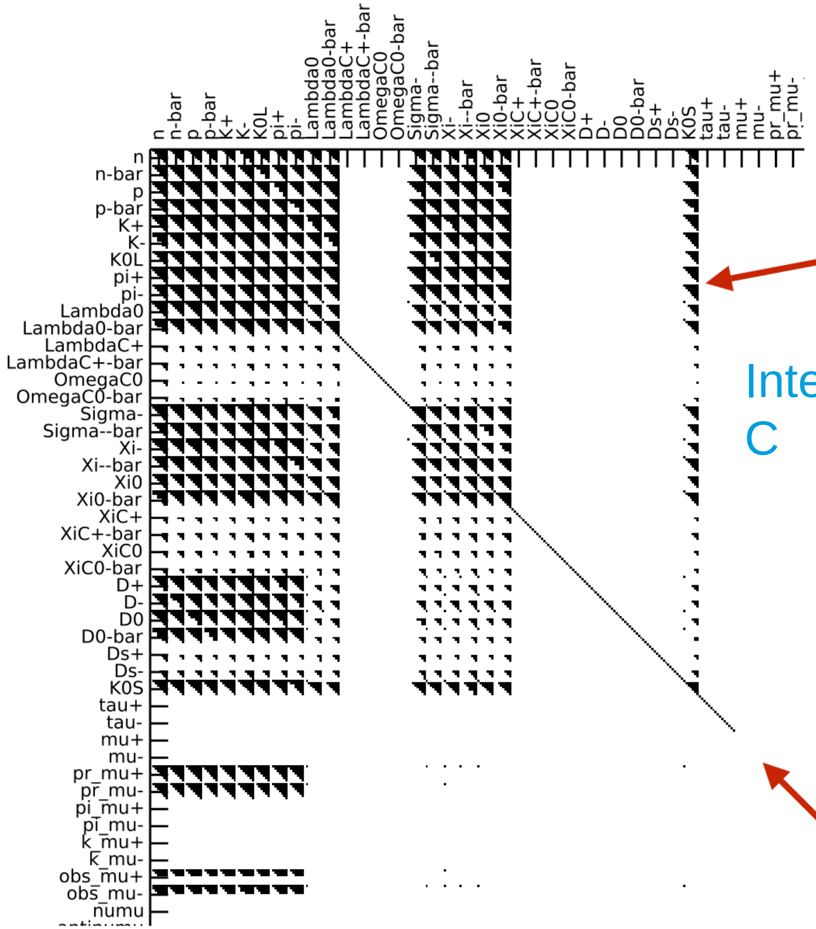
Matrix forms



Decay matrix
D



$$\frac{c_{k\to h}(E_i, E_k)}{\lambda_{int}^{(k)}(E_k)}$$

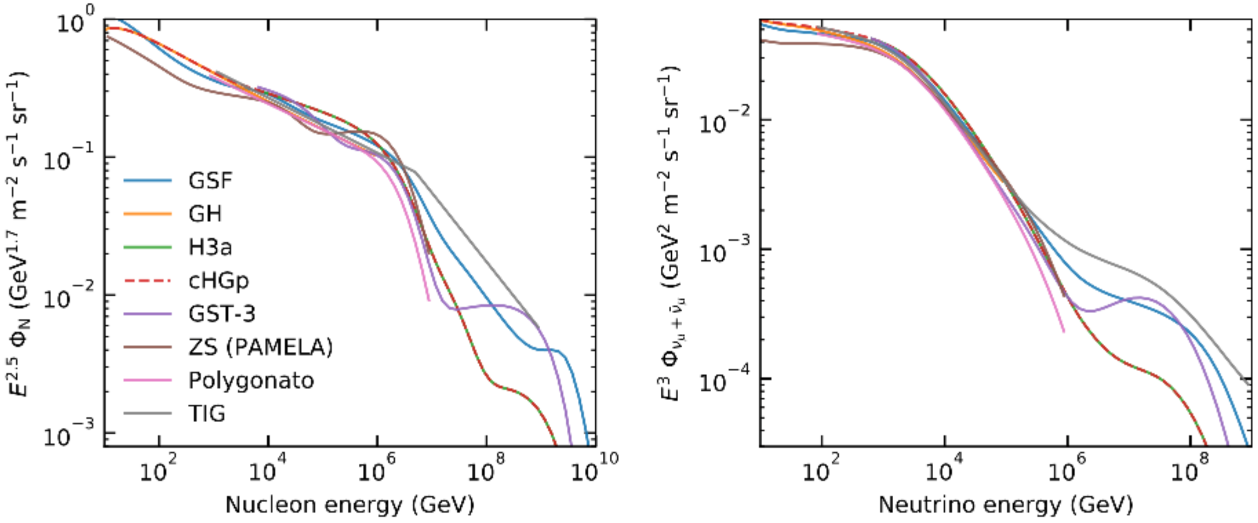
Interaction matrix
C

$$-\ \frac{\phi_h(E_i)}{\lambda_{int}^{(h)}(E_i)}$$

matrices are sparse → high performance

MCEq: models included in the package

Cosmic ray flux at the top of the atmosphere



Short name	Reference	Description	Valid range [GeV]
H3a	[35]	three astrophysical populations, broken power laws, five mass groups, heavier composition at ultra-high energies (UHE)	$10^3 - 10^{11}$ GeV
H4a	[35]	same as H3a but with proton composition at UHE	$10^3 - 10^{11}$ GeV
GST-3	[36]	three population, broken power-law fit heavier composition between knee and ankle (second knee)	$10^3 - 10^{11}$ GeV
GST-4	[36]	like GST-3 but with an fourth extragalactic proton component at UHE	$10^3 - 10^{11}$ GeV
GH	[37]	power-law model with five mass groups, often used in atmospheric neutrino flux calculations below knee energies [7, 38, 6]	< PeV
cHGp	[8, 37, 35]	combination of GH at low energy and H4a above	tens -10^{11} GeV
cHGm	[8, 37, 35]	like cHGp but with H3a instead of H4a	tens -10^{11} GeV
Polygonato	[39]	broken power-law fit, based on renormalization of various cosmic ray measurements up to knee energies	few TeV – PeV
ZS	[40, 41]	original model by Zatsepin and Sokolskaya, also including re-fitted parameters by the PAMELA collaboration	tens GeV – PeV
TIG	[17]	simple broken power law spectrum of nucleons (protons)	TeV – PeV
GSF	add..	Global Spline Fit to recent cosmic ray observations with errors	10 GeV -10^{12} GeV

Hadronic interactions

Name	Reference
SIBYLL 2.1	[26]
SIBYLL 2.3	[27, 28]
SIBYLL 2.3c	[29]
SIBYLL 2.3c (pp)	[29]
QGSJET 01-c, II-03, II-04	[30, 31]
EPOS LHC	[32]
DPMJET-III	[14]
DPMJET-III 2017.1	[15]