Atmospheric Neutrino Fluxes: The use of muon fluxes to Improve the Accuracy in Low Energies.

May, 28, 2018 M. Honda @ PANE 2018

1. Over view of the calculation of atmospheric neutrino and
   the Muon Calibration of Atmospheric Neutrino.

2. Analytic formalism of the calculation of atmospheric neutrino flux
   and extension of the muon calibration to lower energies.
   a. ONLY with Meson production variation in Hadronic Interactions.
   b. With Nucleus/Nucleon propagation variations

Gaisser Formula for the illustration (by T.K.Gaisser at Takayama, 1998)

$$\Phi_{\nu} = \Phi_{primary} \otimes R_{cut} \otimes Y_{\nu}$$

$$\Phi_{\mu} = \Phi_{primary} \otimes R_{cut} \otimes Y_{\mu}$$

Where

$\Phi_{primary}$: Cosmic Ray Flux

$R_{cut} = R_{cut}(R_{cr}, latt., long., \theta, \phi)$: Geomagnetic field

$Y_{\nu} = Yield_{\nu}(h, \theta)$: Hadronic Interaction Model, Air Profile, and meson-muon decay

$Y_{\mu} = Yield_{\mu}(h, \theta)$: Hadronic Interaction Model, Air Profile, and meson decay

Full 3D-Calculation

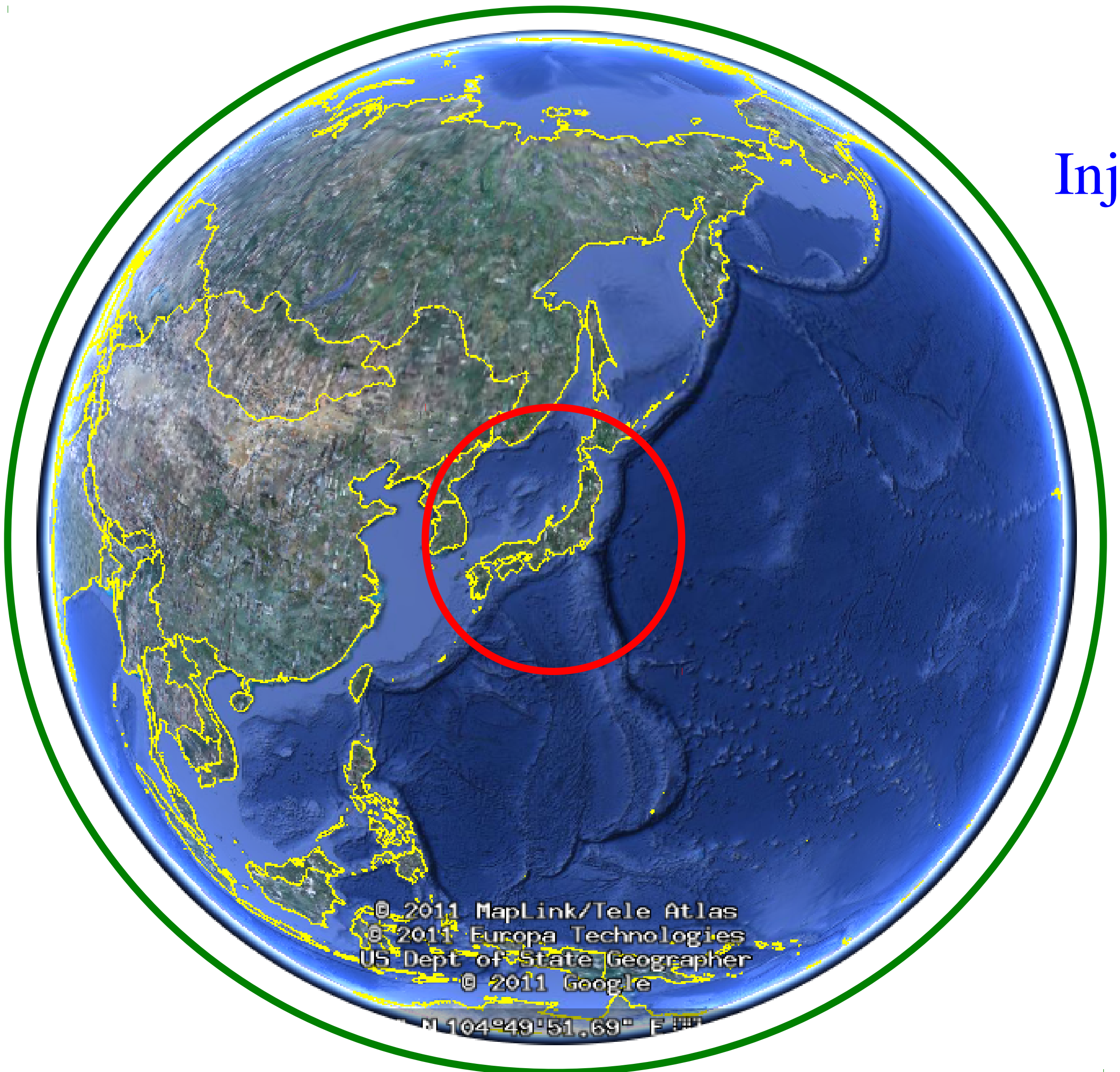
Re = 6378km

Simulation Sphere (Rs = 10 x Re)

Cosmic ray go out this sphere are discarded.
Cosmic rays go beyond are pass the rigidity cutoff test

Injection Sphere (Re +100lm)

Cosmic Rays are sampled and injected here

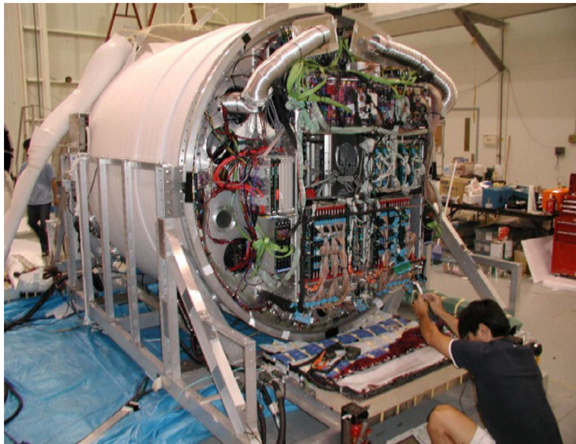


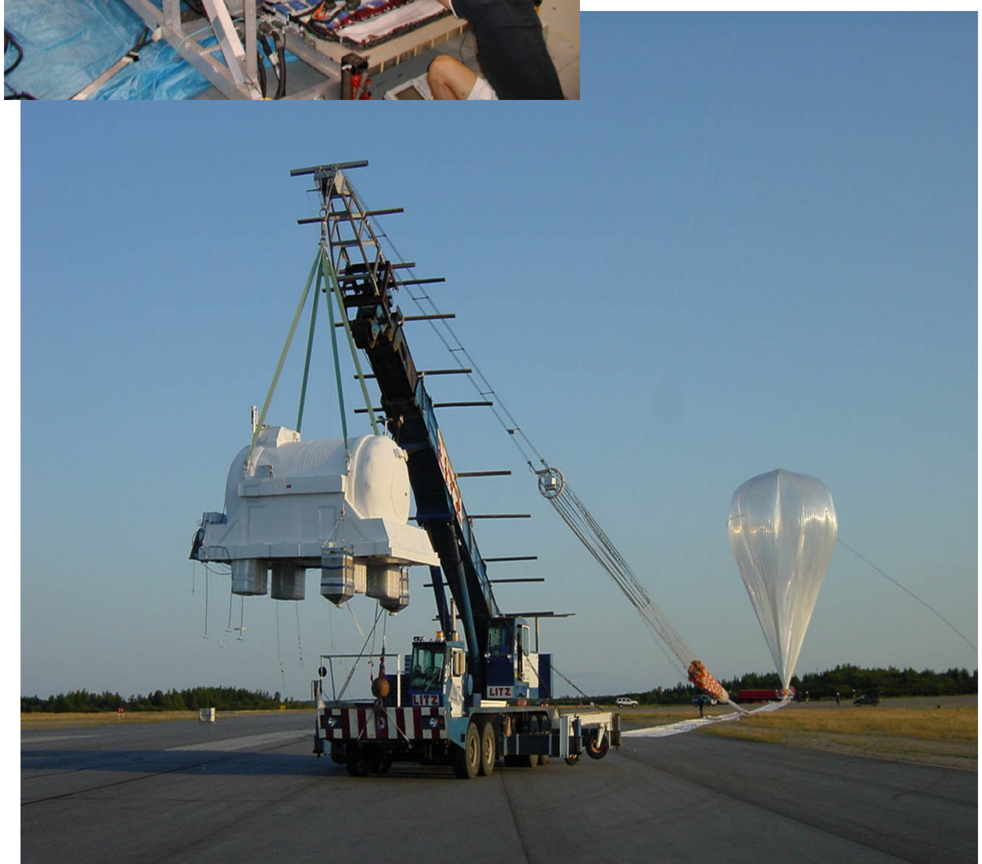
Virtual Detector

The neutrino flux is calculated from the number of neutrinos path through with virtual detector correction.

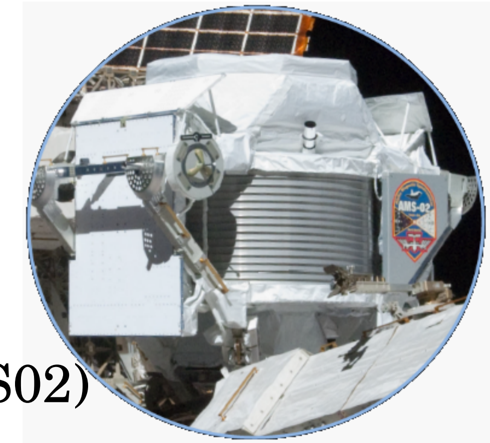
Direct Observation

Balloon Borne (BESS)

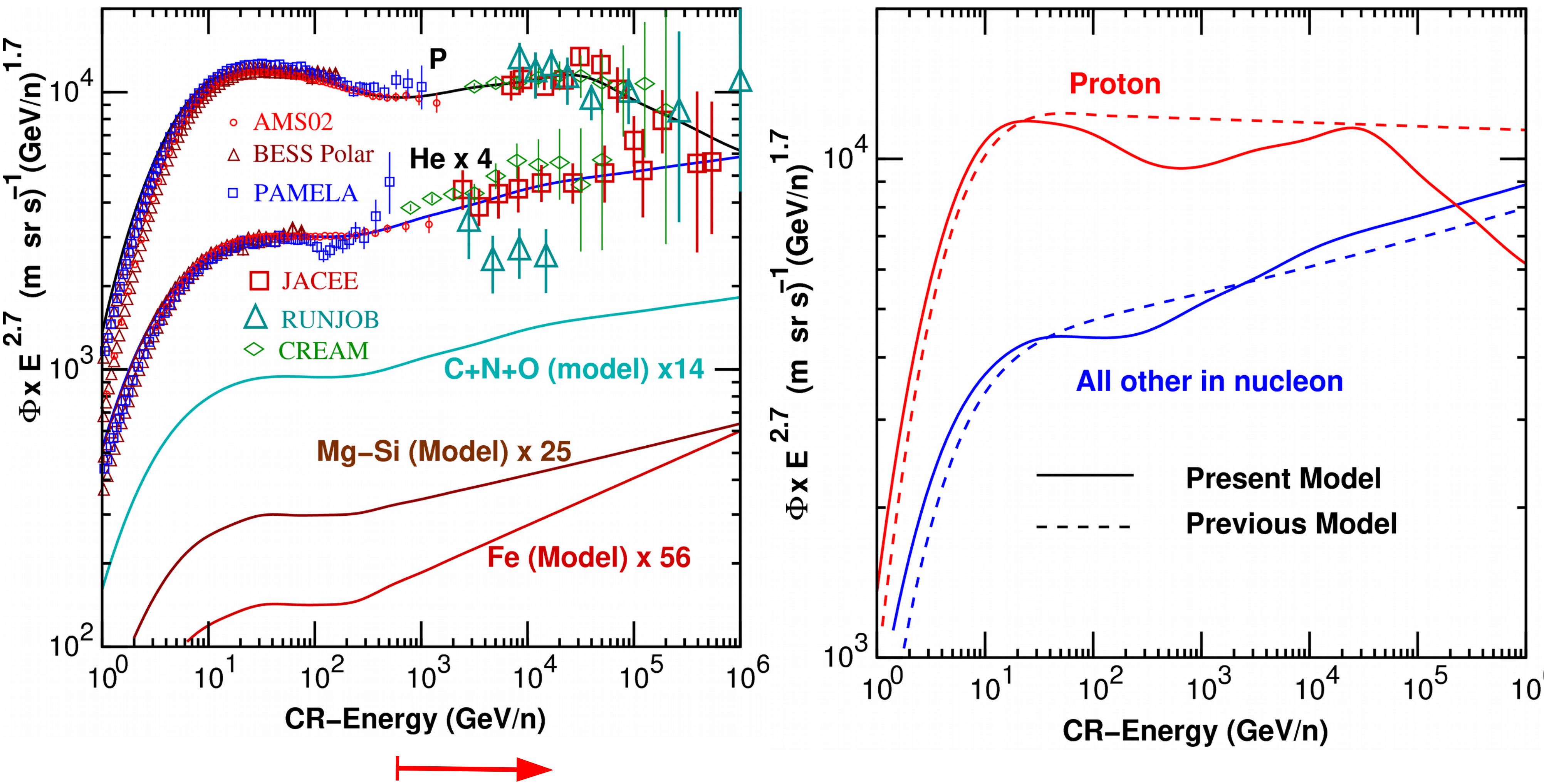




Satellite (ISS, AMS02)

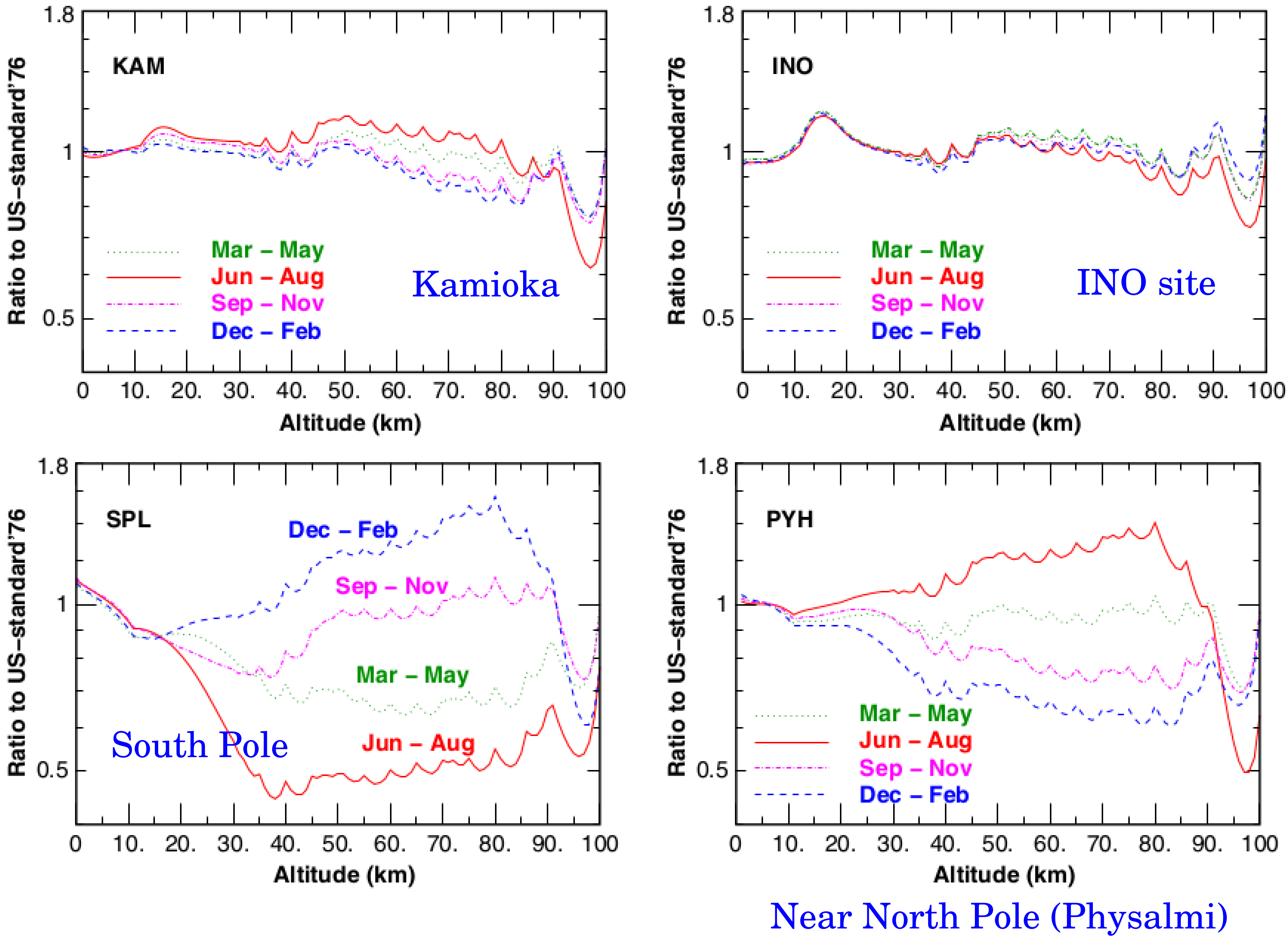


Cosmic Ray Spectra Model Based on AMS02 Observation
(2017 1ry model)

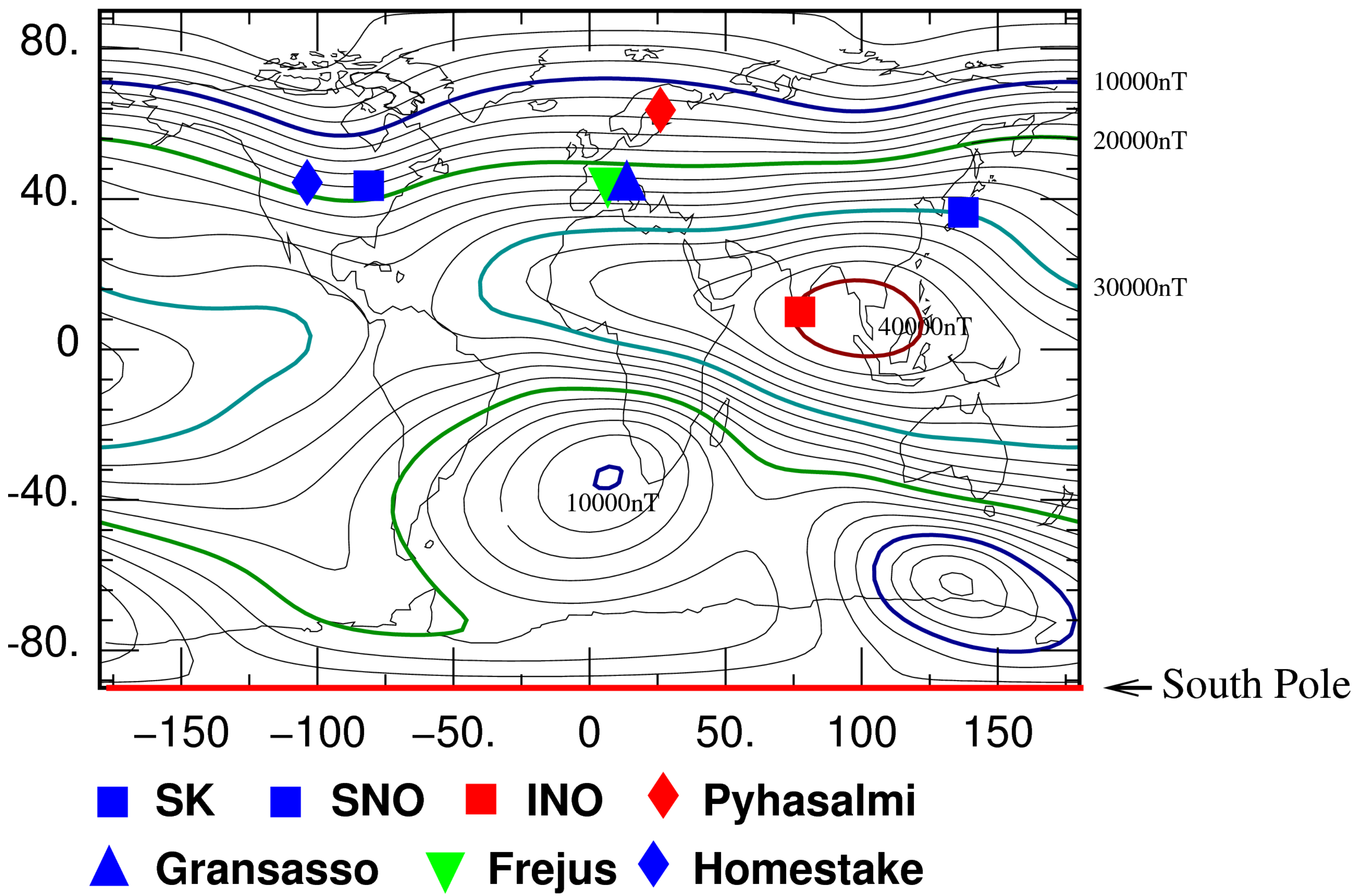


Looking forward to hearing from CALET and ISS-CREAM

Atmosphere model (NRLMSISE-00) and seasonal variations



IGRF10 Geomagnetic Horizontal Field Strength



We use Modefied DPMJET3 as the Hadronic interactio model

Modefied DPMJET3 = parameter fitting of the out put of DMPMJET3

Quick, Easy to modify, but conseration rules are statistical.

Note, we have tried other interaction models, and they give a similar results when they are modified in our method to reproduce the observed muon fluxes.

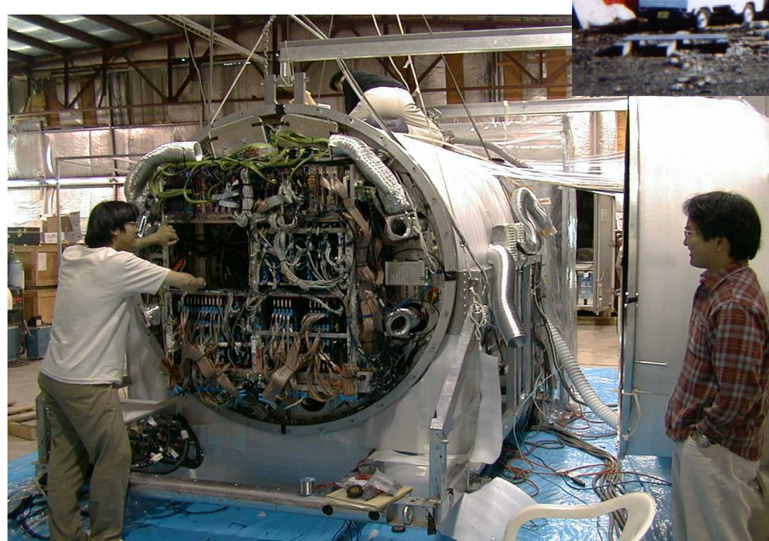
Muon Observations

Balloon
Altitude

L3(+C)

BESS

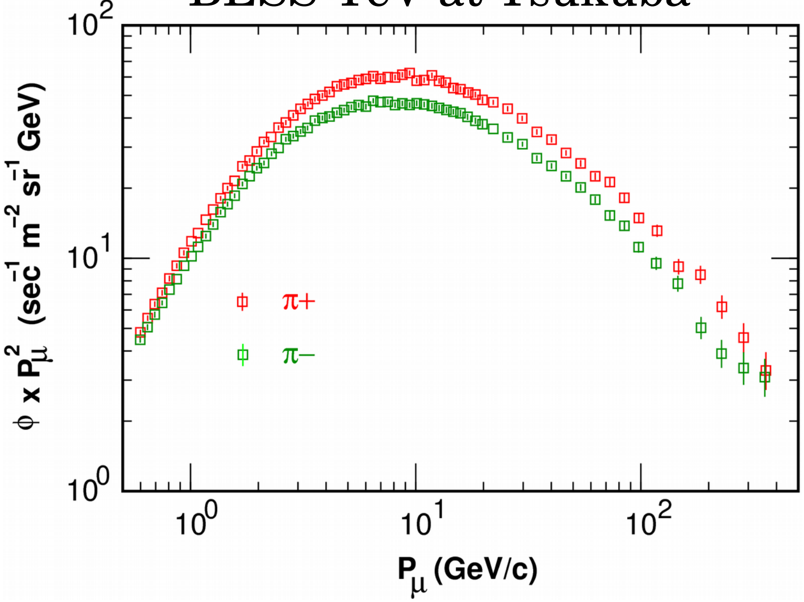
Tsukuba
(KEK)



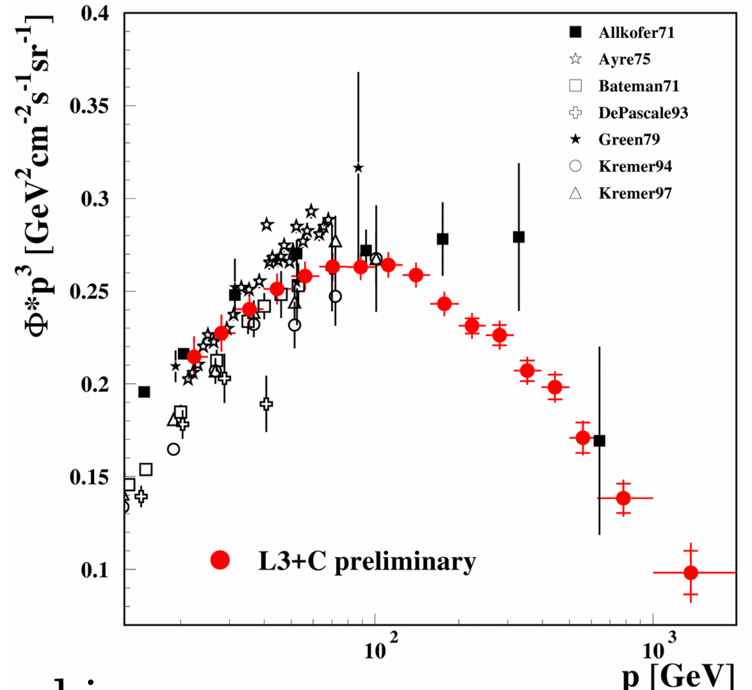
Mt Norikura

Muon data used here

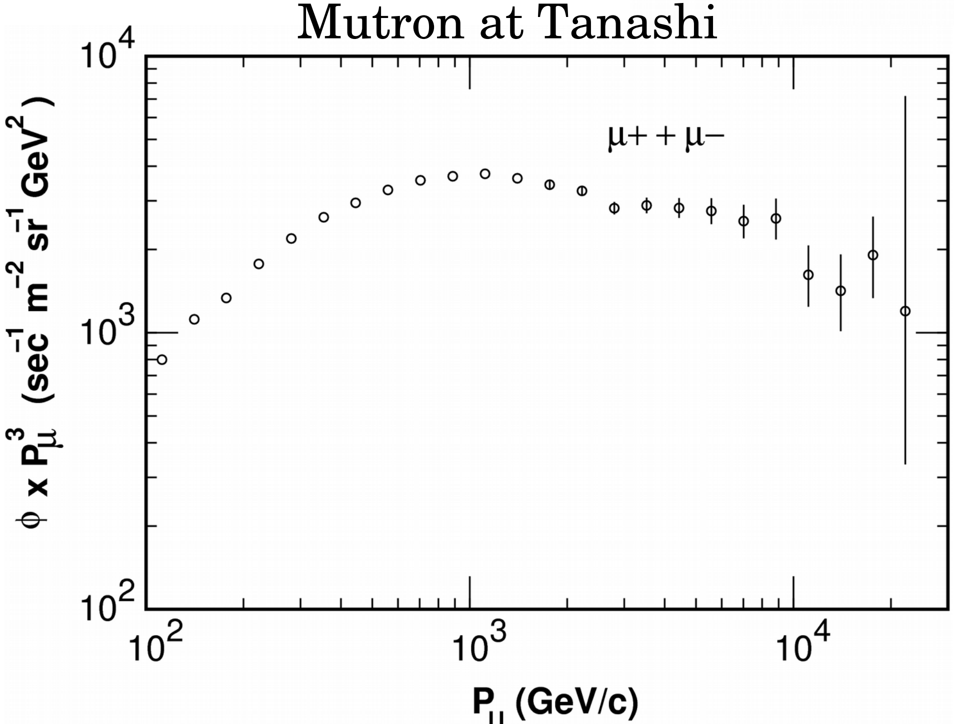
BESS-TeV at Tsukuba

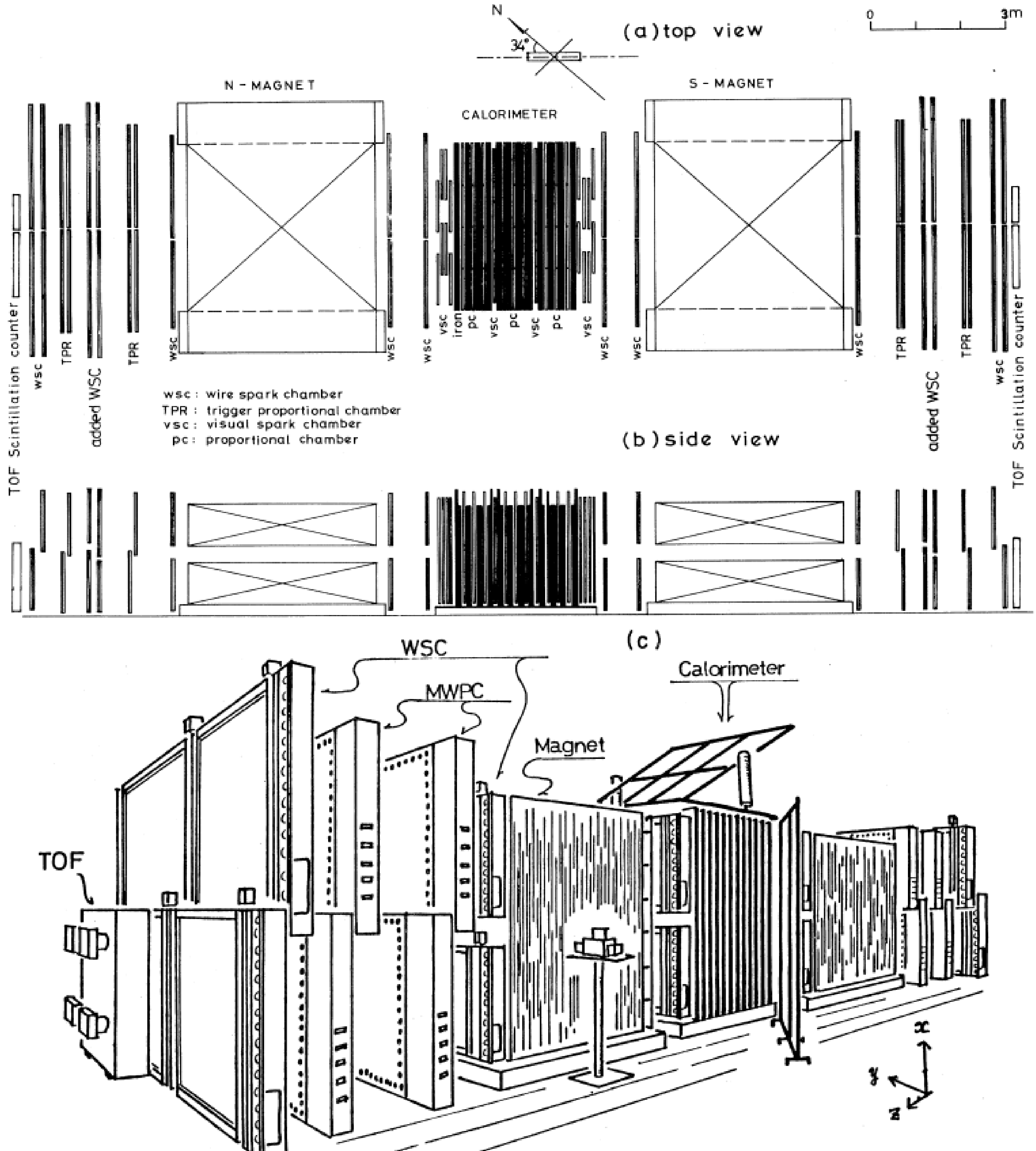


L3+C at Cern



Mutron at Tanashi





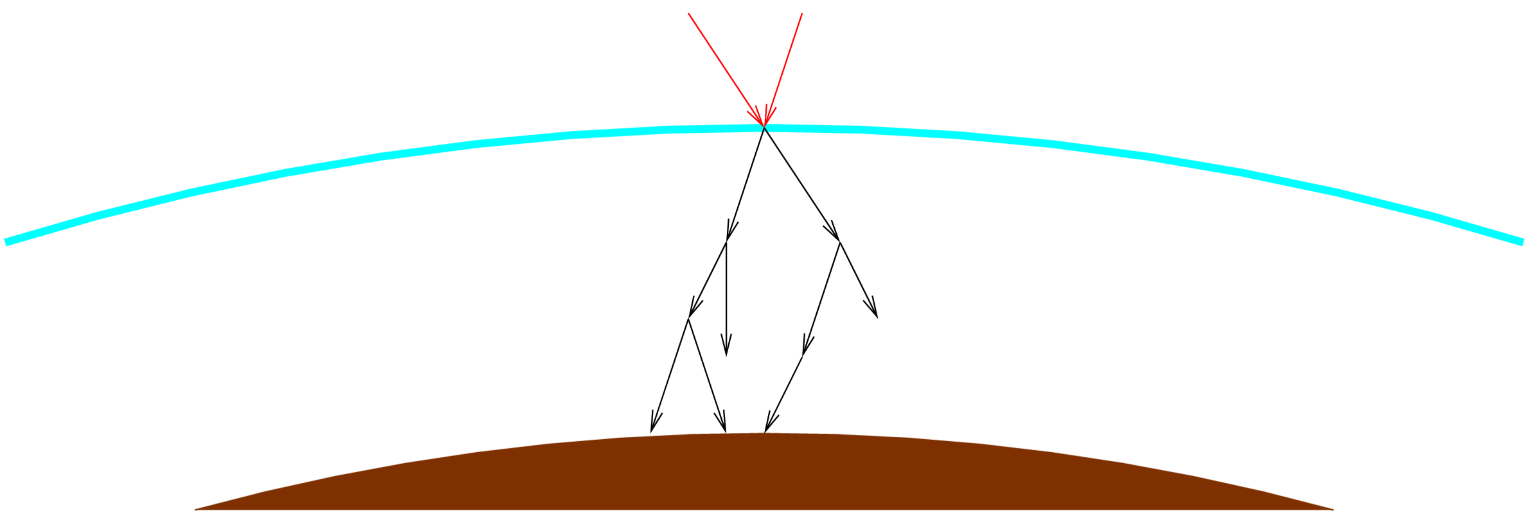
(a) top view
N
34°
0
3m
N - MAGNET
CALORIMETER
S - MAGNET
TOF Scintillation counter
wsc
TPR
added WSC
iron
pc
vsc
wsc : wire spark chamber
TPR : trigger proportional chamber
vsc : visual spark chamber
pc : proportional chamber
(b) side view
(c)
WSC
MWPC
Calorimeter
Magnet
TOF

Muon Calibration of Interaction Model

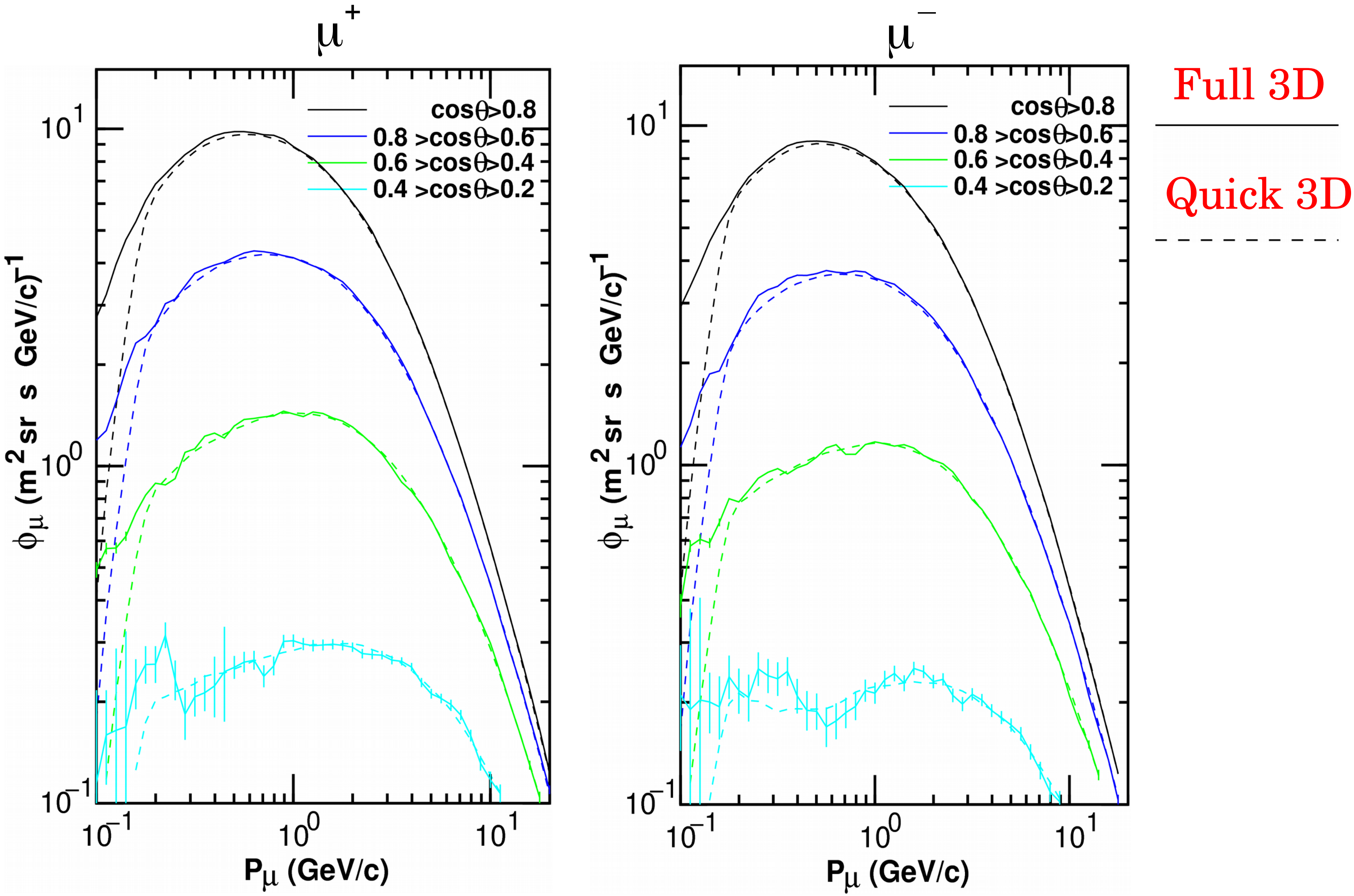
Quick 3D Calculation for Muon flux.

As the muon flux is a “local quantity” ($\gamma ct \sim$ 60km at10 GeV),
We can calculate it in a quick calculation method:

1. Inject cosmic rays just above the observation point,
2. Analyze all muons reach the surface of Earth.



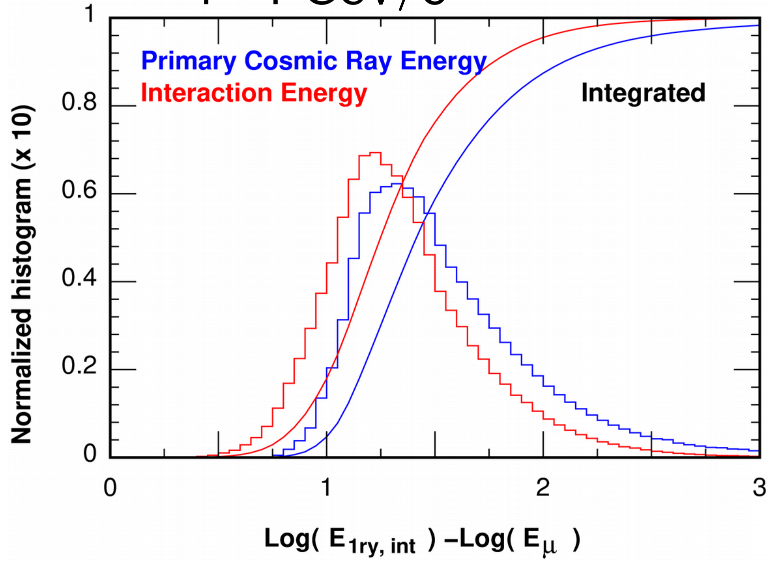
Comparison of Quick 3D and Full 3D calculations



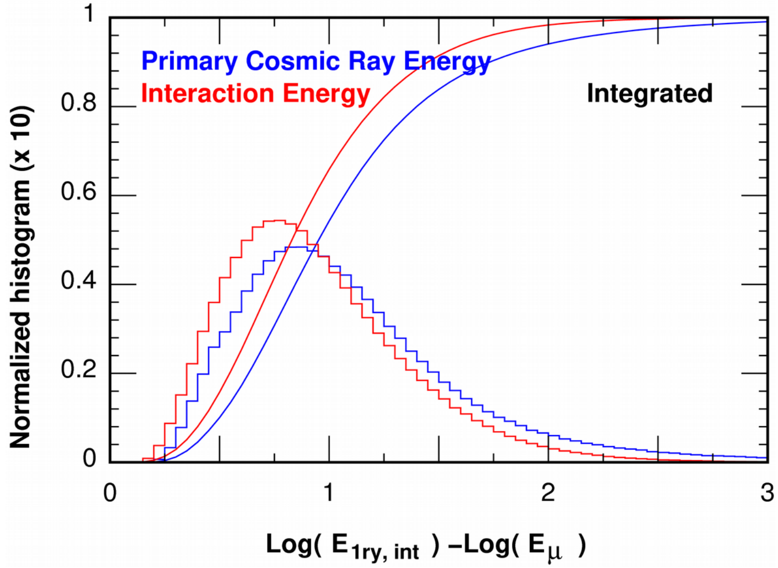
This method works above 0.2 GeV/c.

Responsible 1ry CR energy and Interaction Energy for Vertical Muon

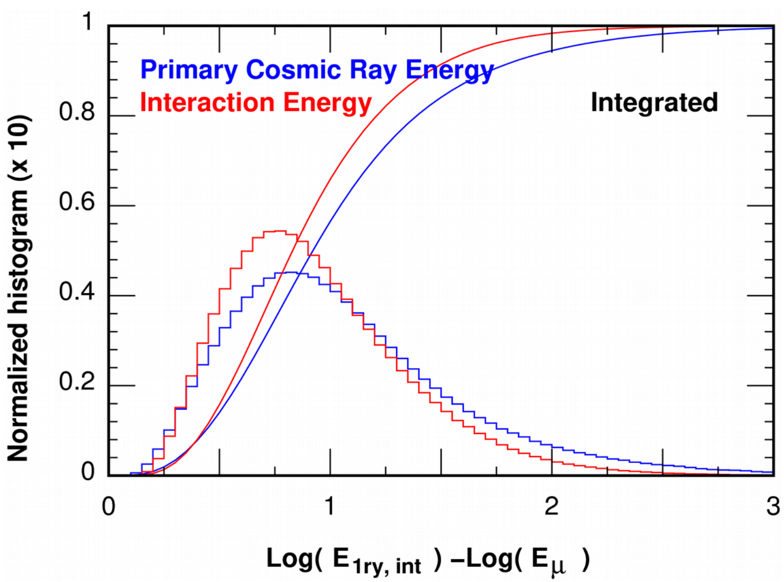
P=1 GeV/c



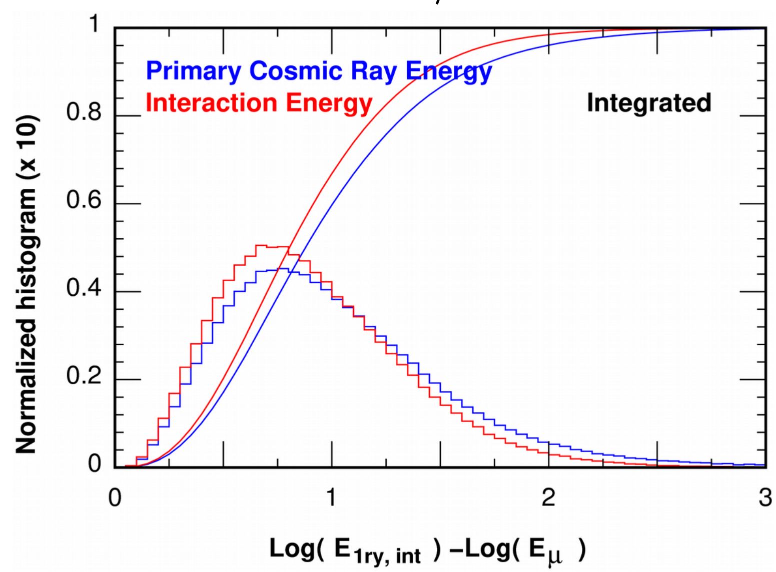
P=10 GeV/c



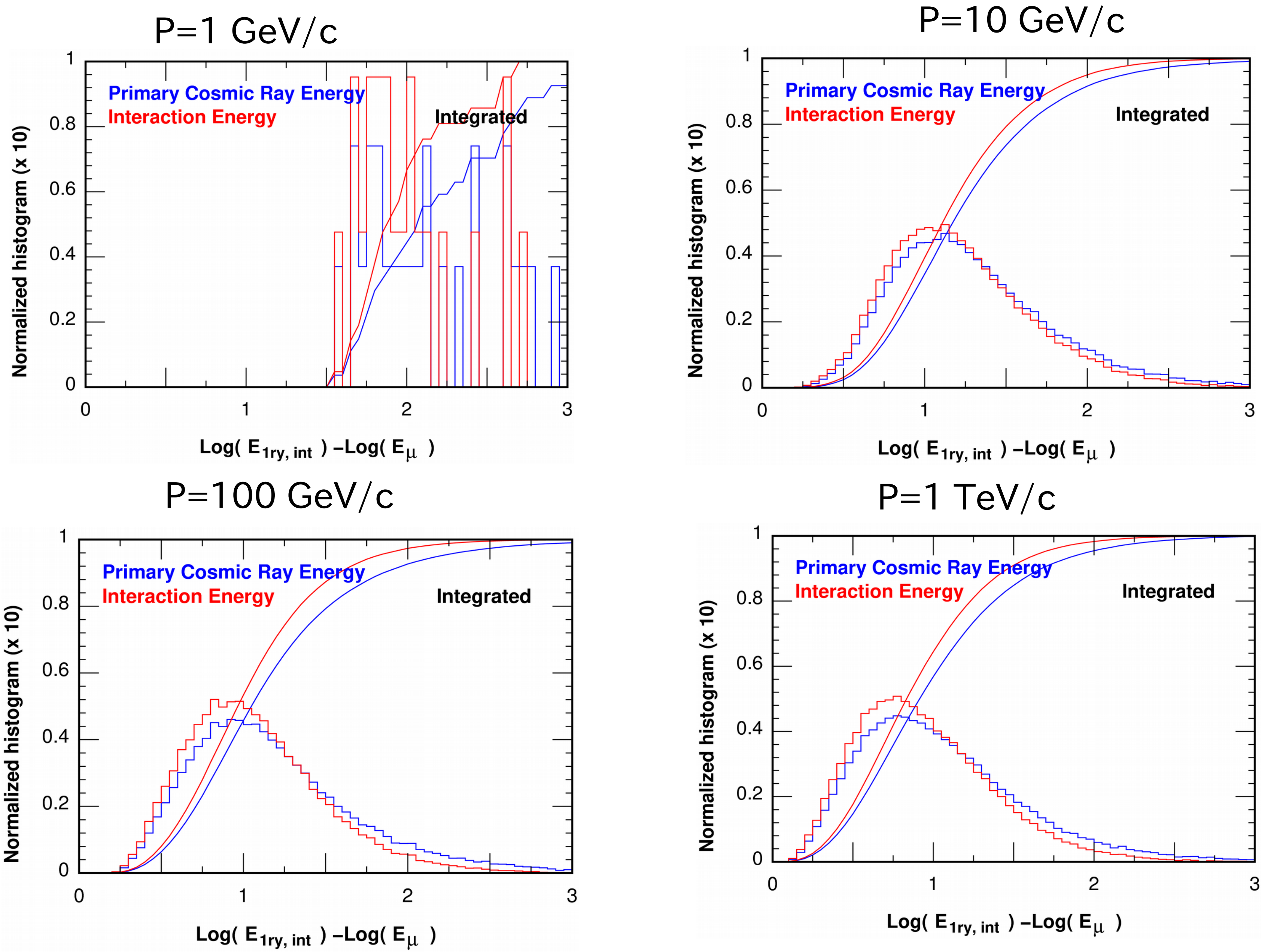
P=100 GeV/c



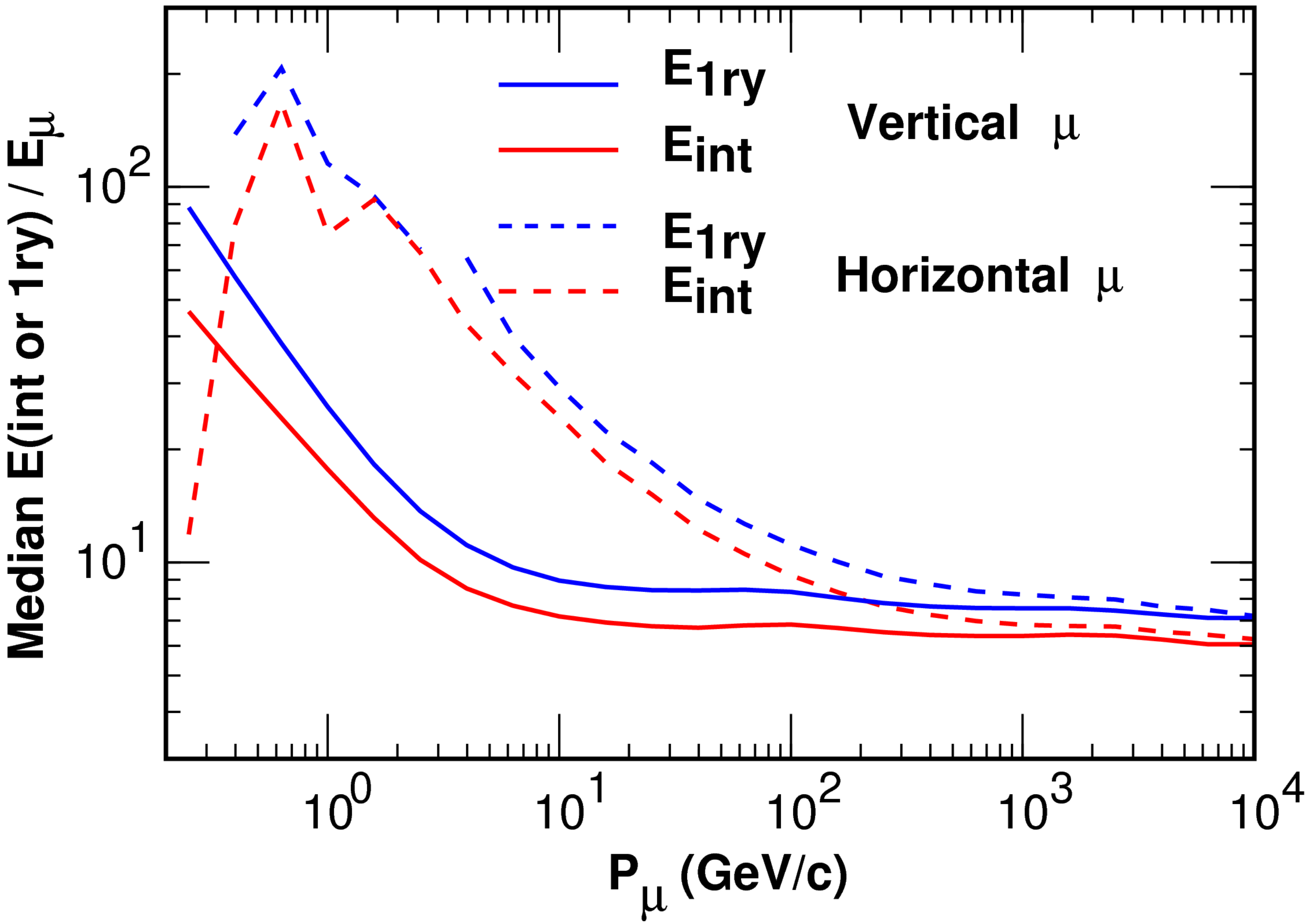
P=1 TeV/c



Responsible 1ry CR energy and Interaction Energy for Horizontal Muon

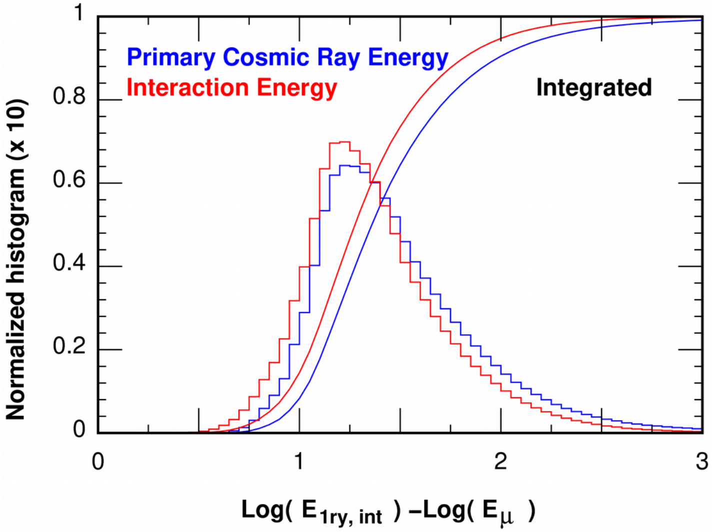


Median Energy of the Responsible 1ry and Interaction Energy for Muons

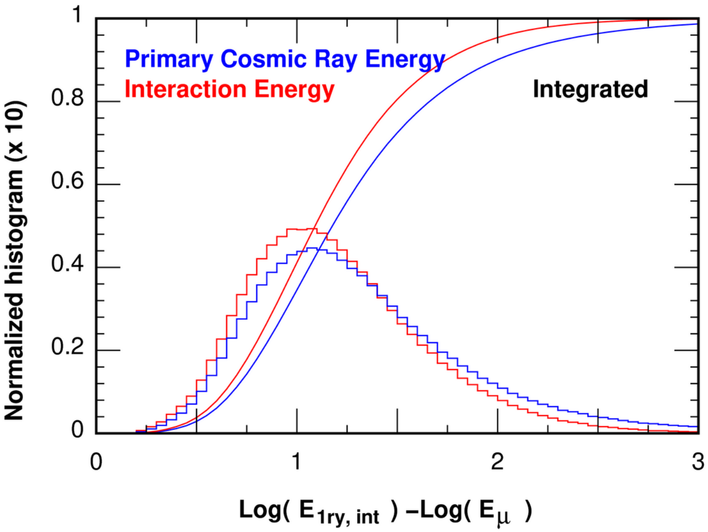


Responsible 1ry CR energy and Interaction Energy for Vertical Neutrino

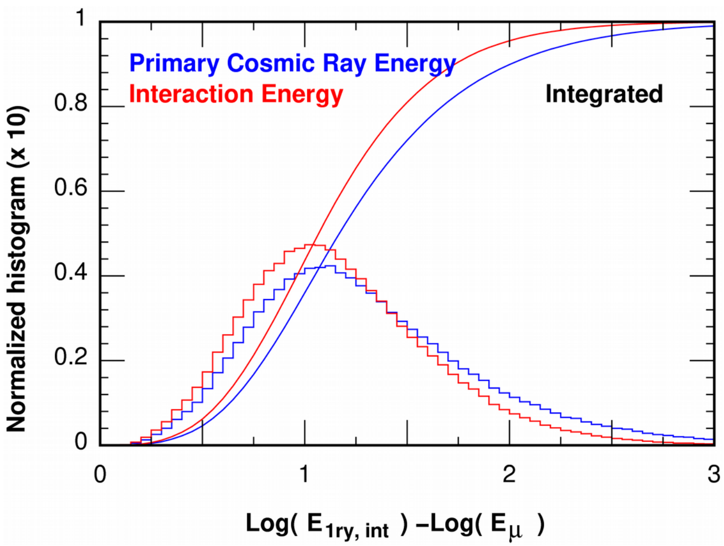
P=1 GeV/c



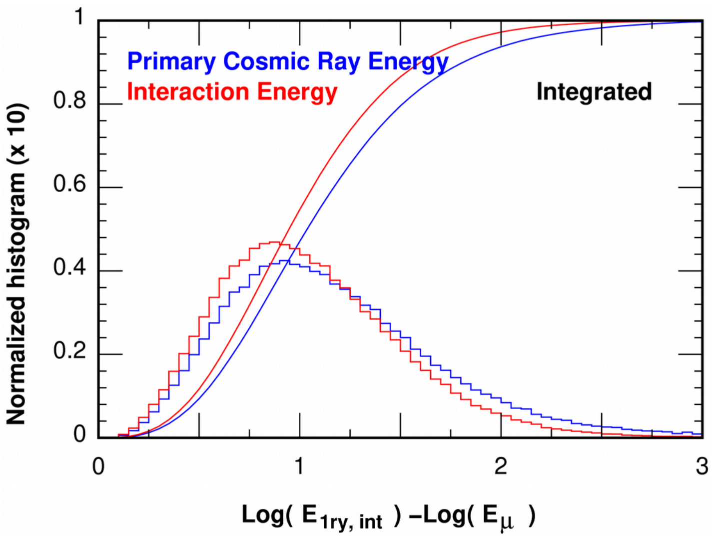
P=10 GeV/c



P=100 GeV/c

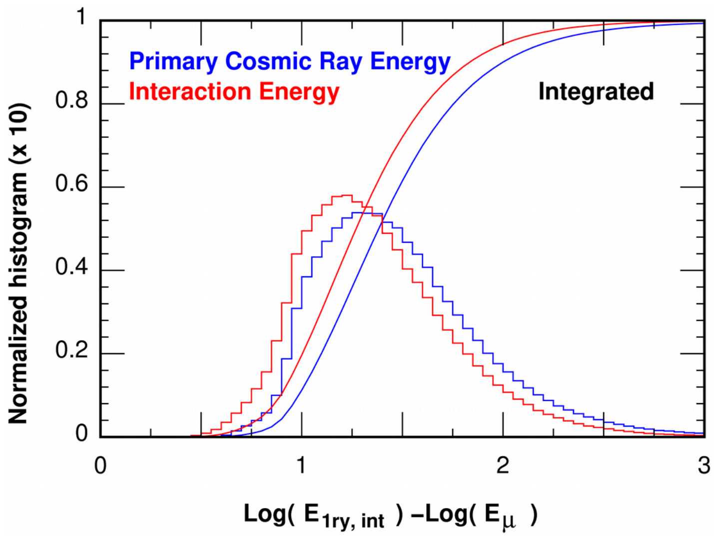


P=1 TeV/c

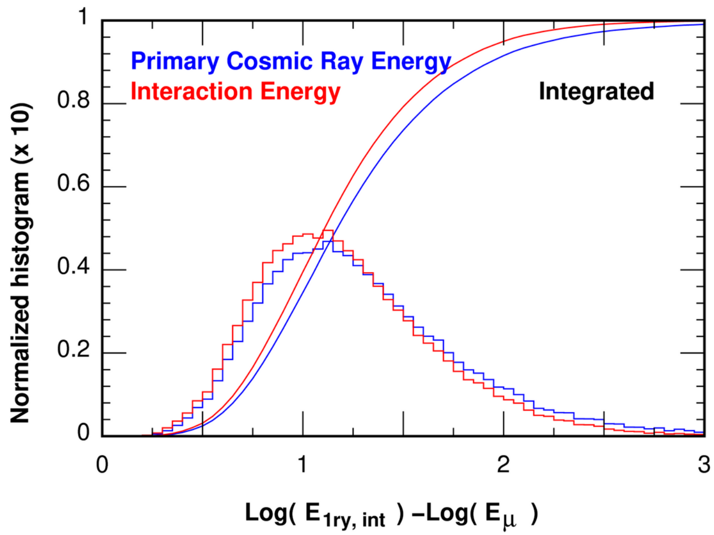


Responsible 1ry CR energy and Interaction Energy for Horizontal Neutrino

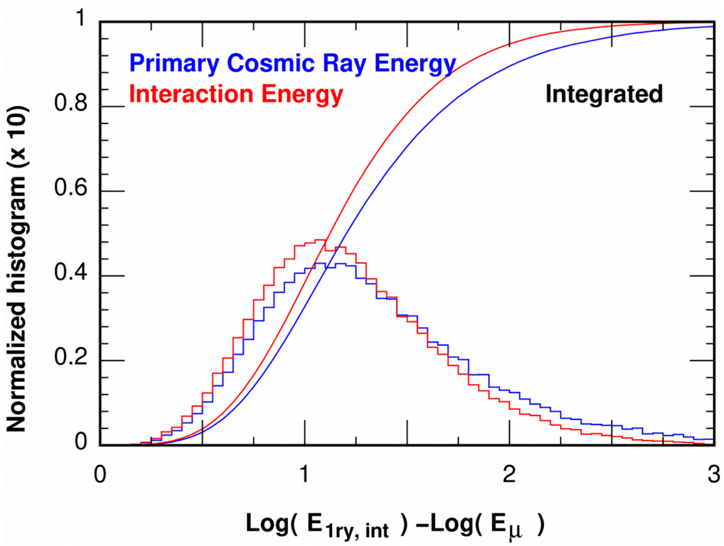
P=1 GeV/c



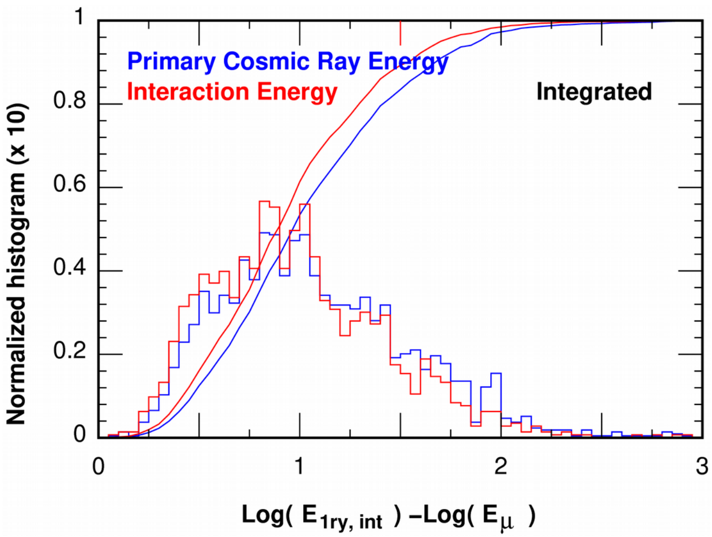
P=10 GeV/c



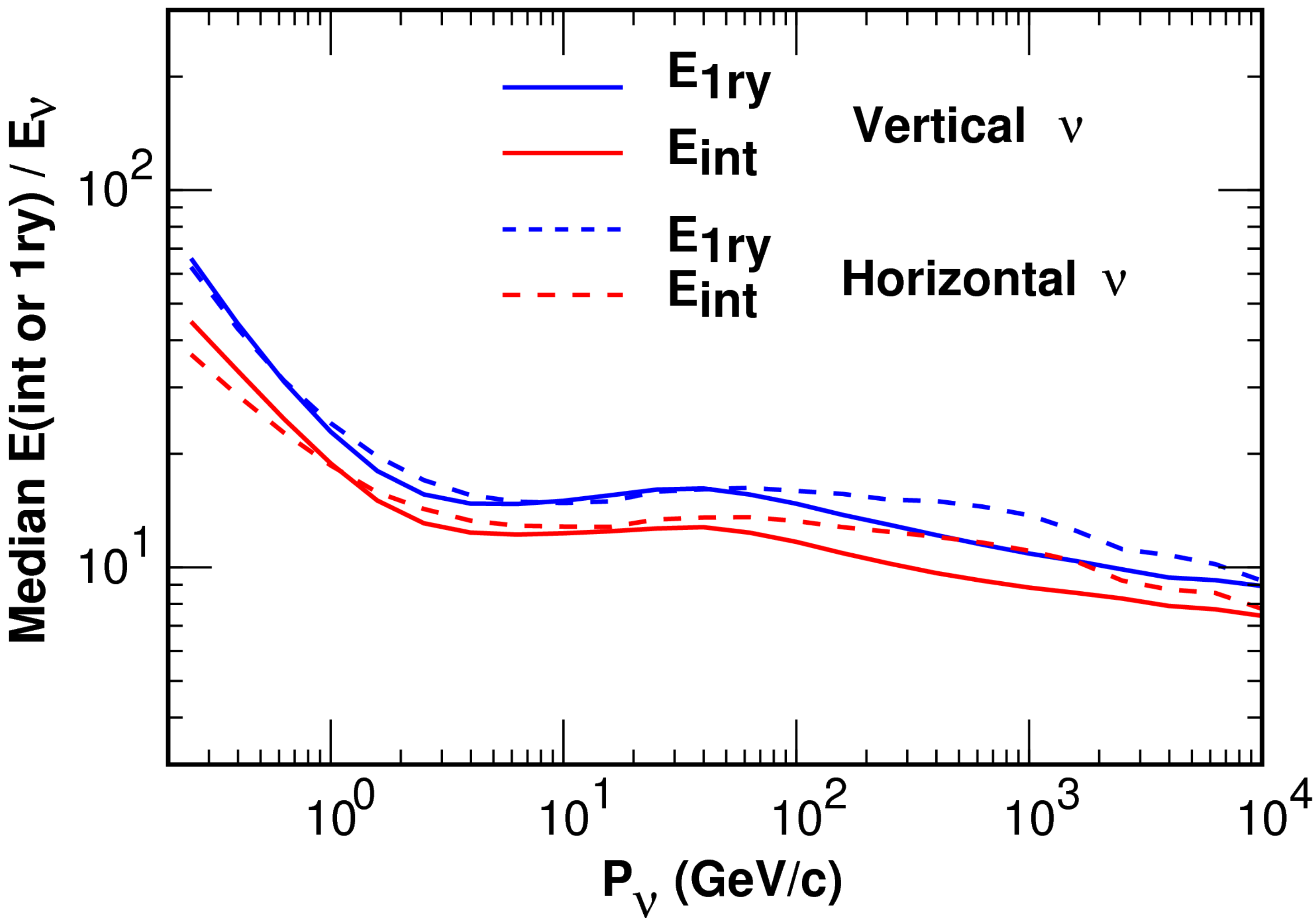
P=100 GeV/c



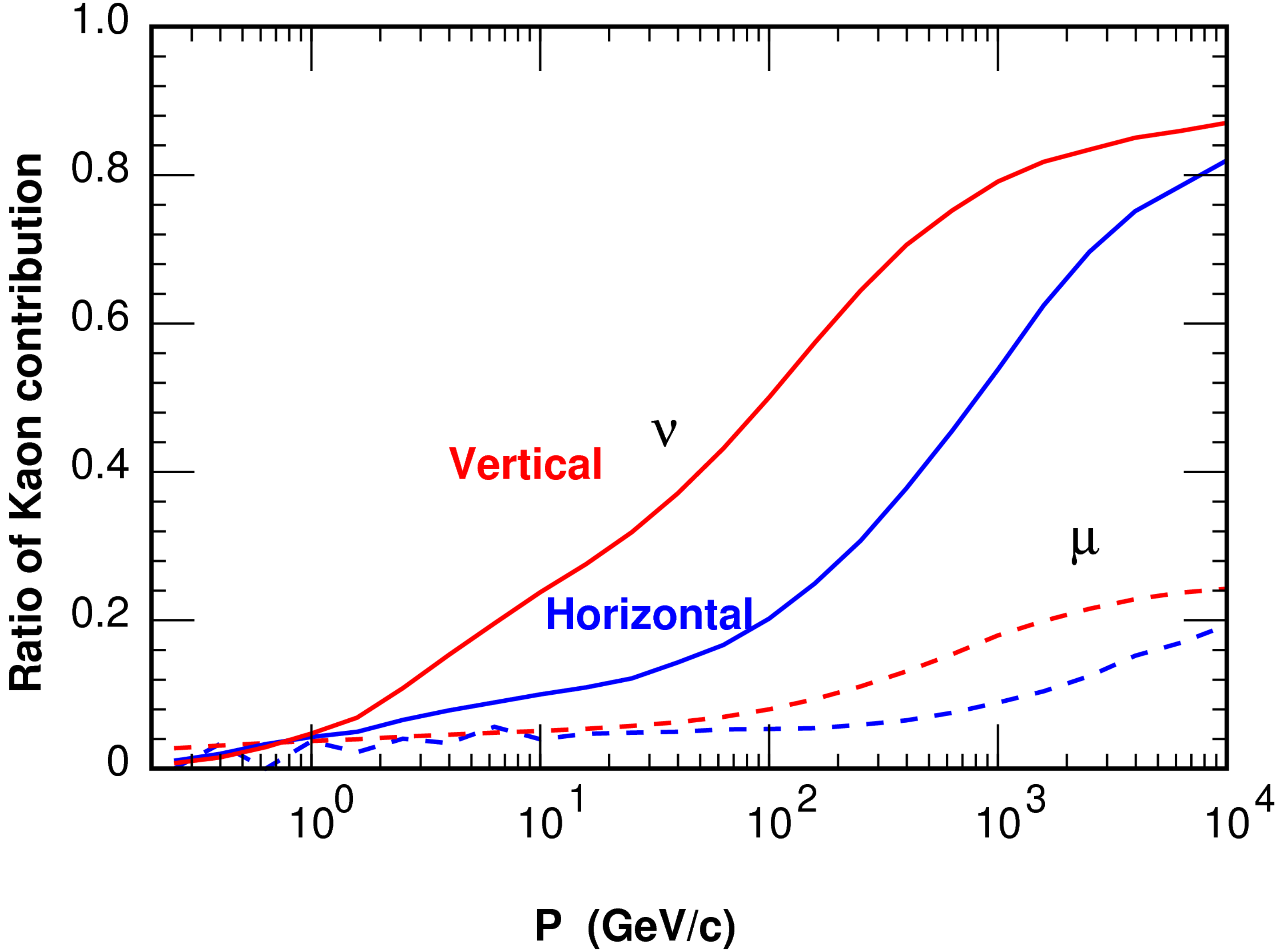
P=1 TeV/c



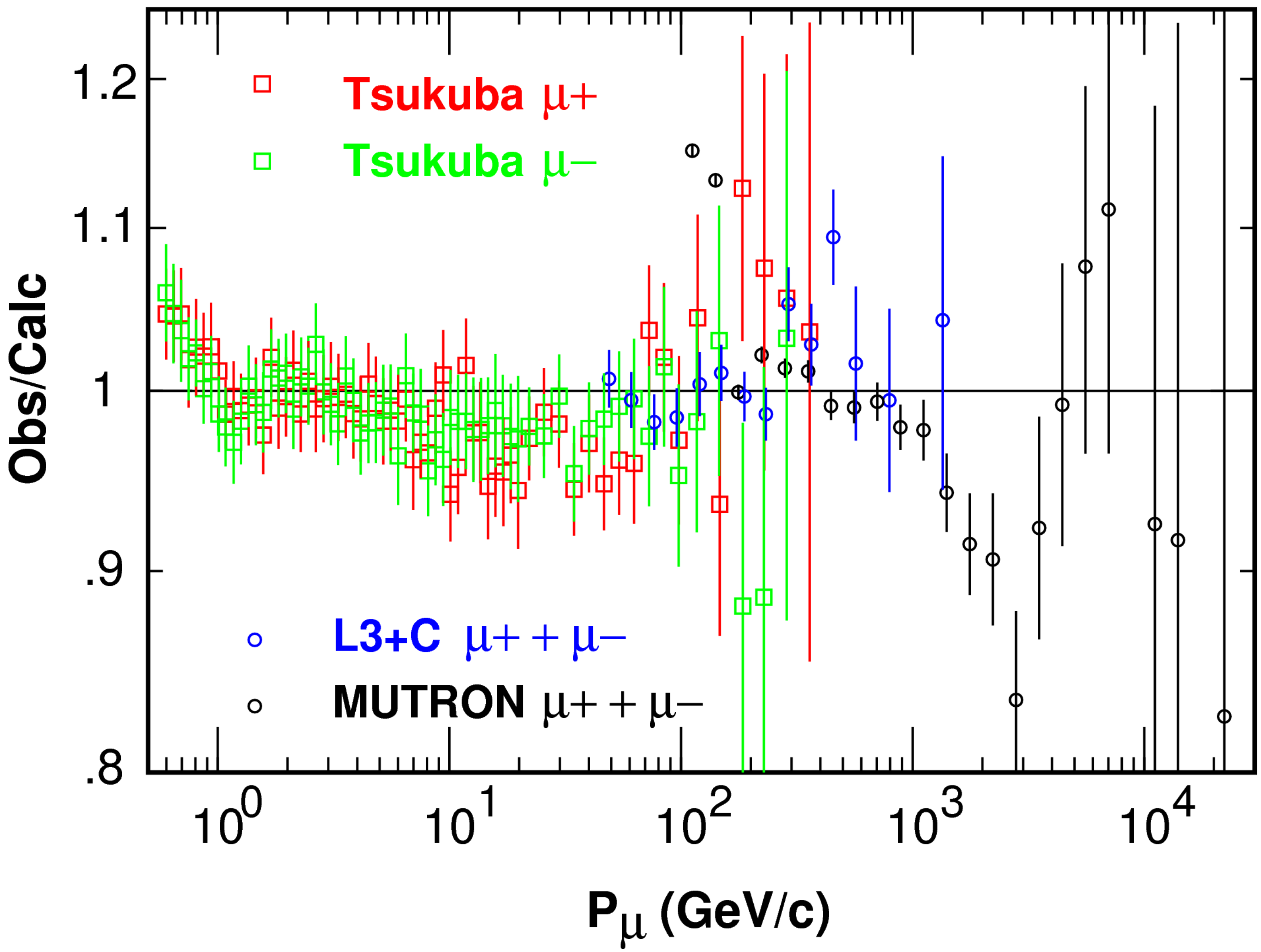
Median Energy of the Responsible 1ry and Interaction Energy for Neutrinos



Contribution of Kaon for atmospheric muons and neutrinos



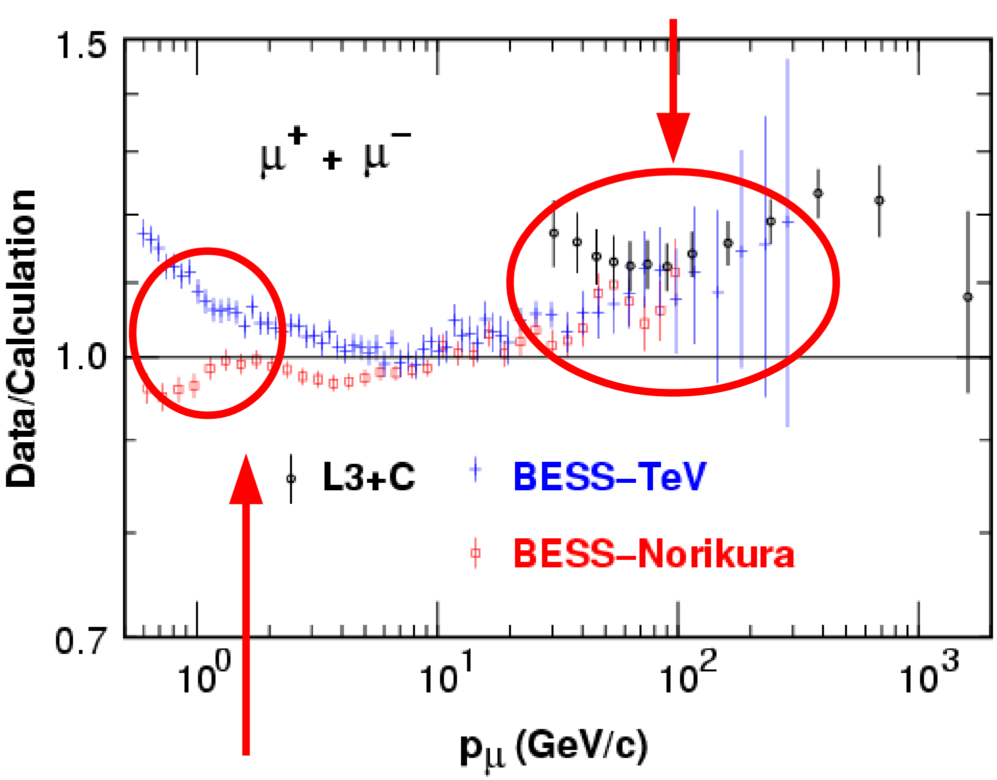
Observation / Calculation ratio with 2004 peimey cosmic ray model and 2006 interaction model

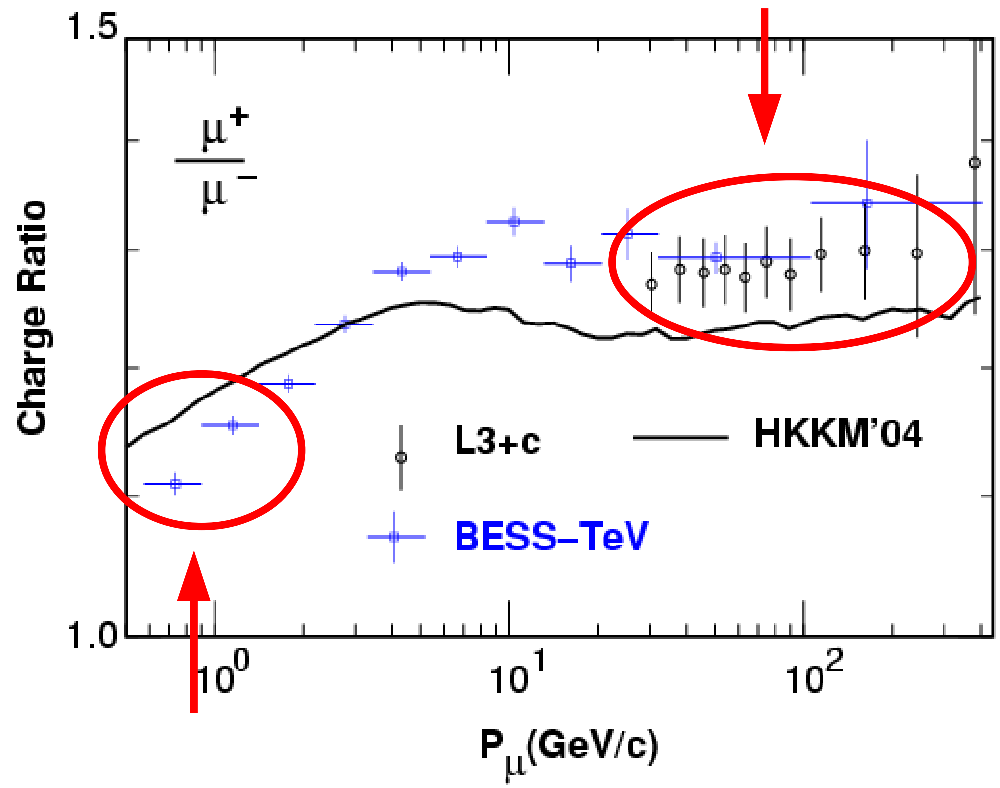


Muon Calibration of inclusive DPMJET-III

Data are larger by ~15%

Data are larger by ~0.05



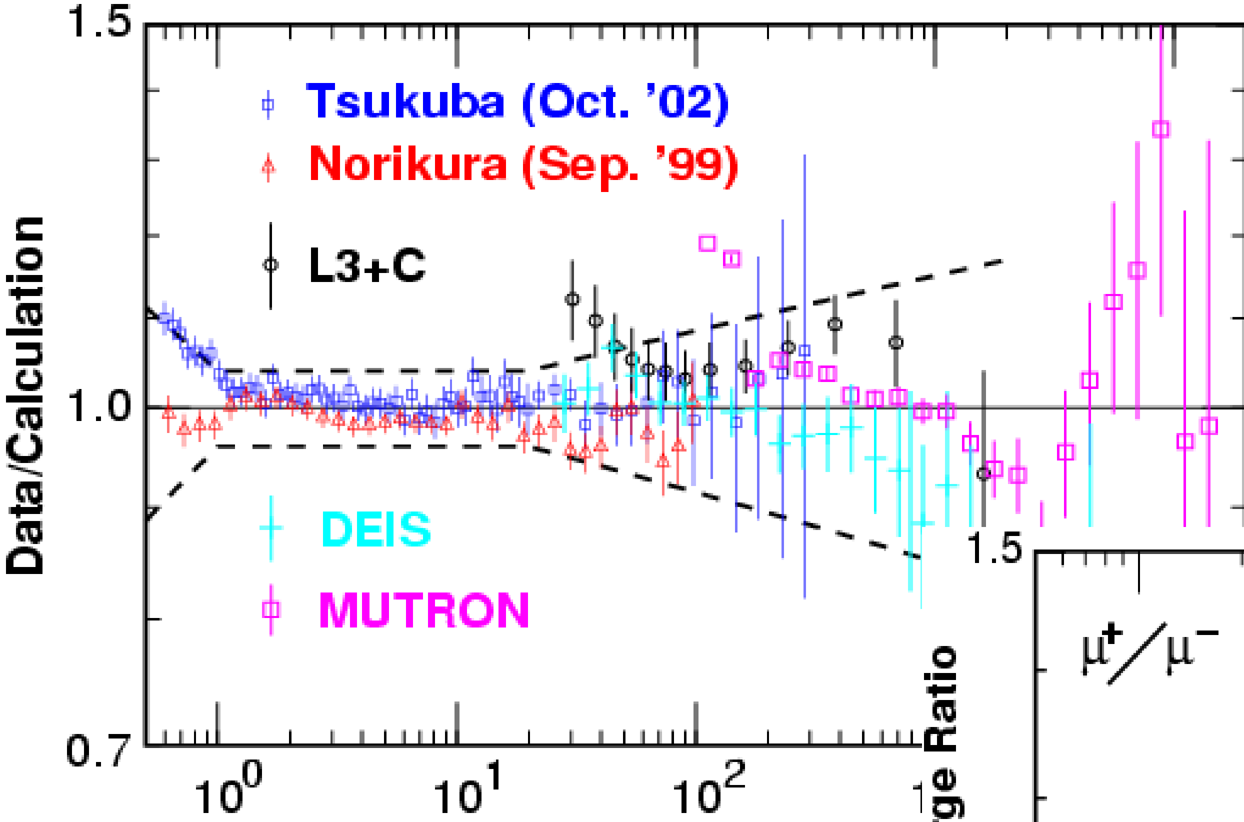


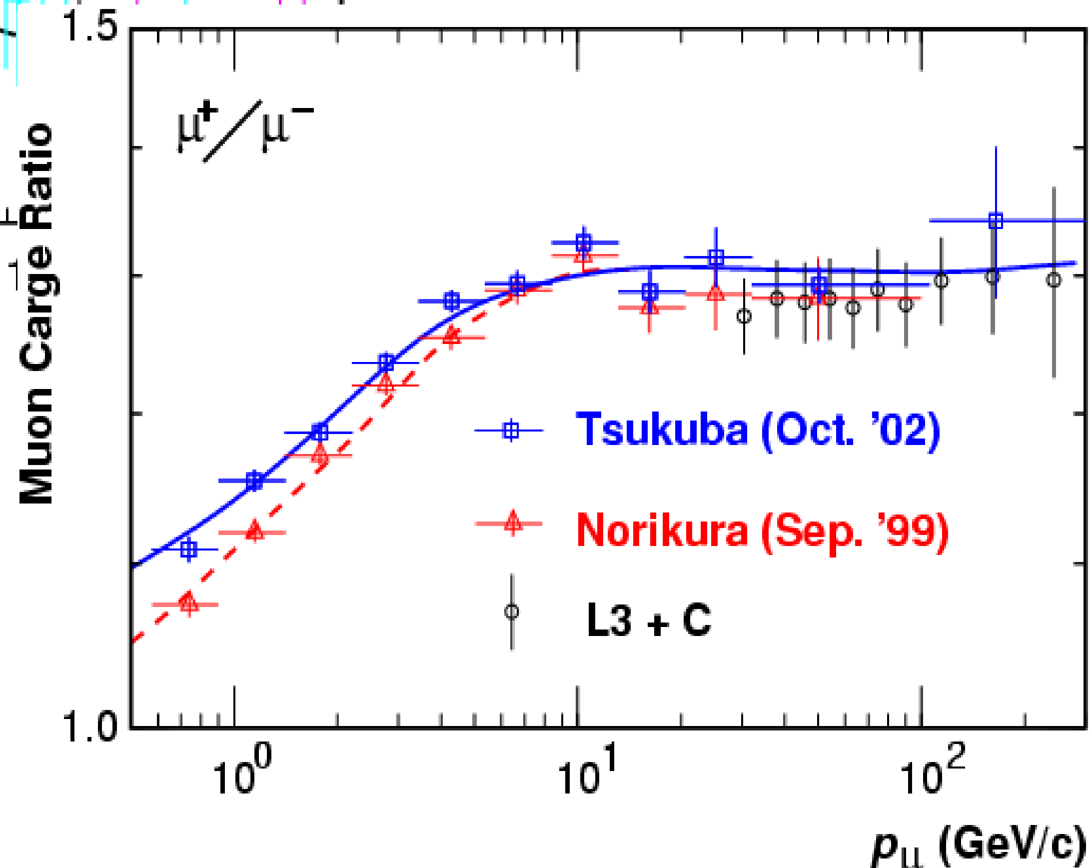
~15% scatter ?

Data are smaller by ~0.05

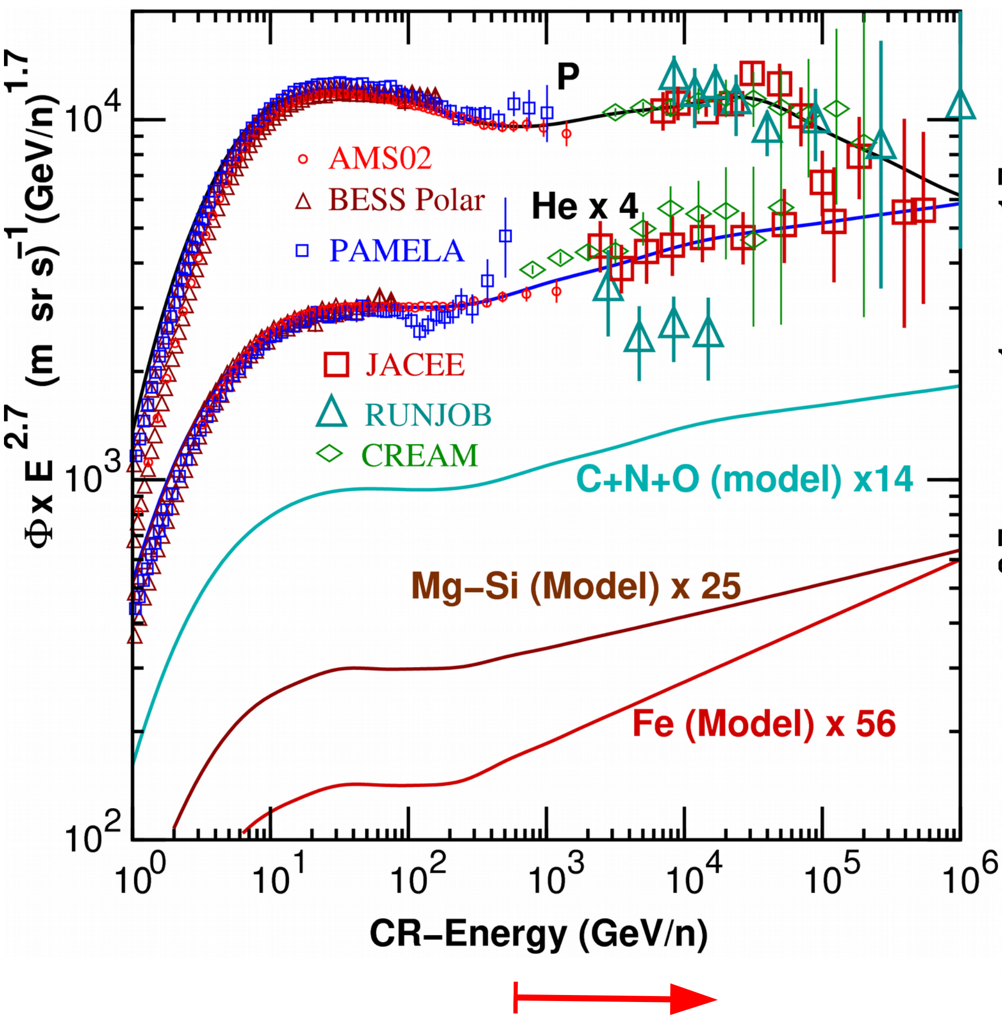
==> DPMJET-III Should be Modified

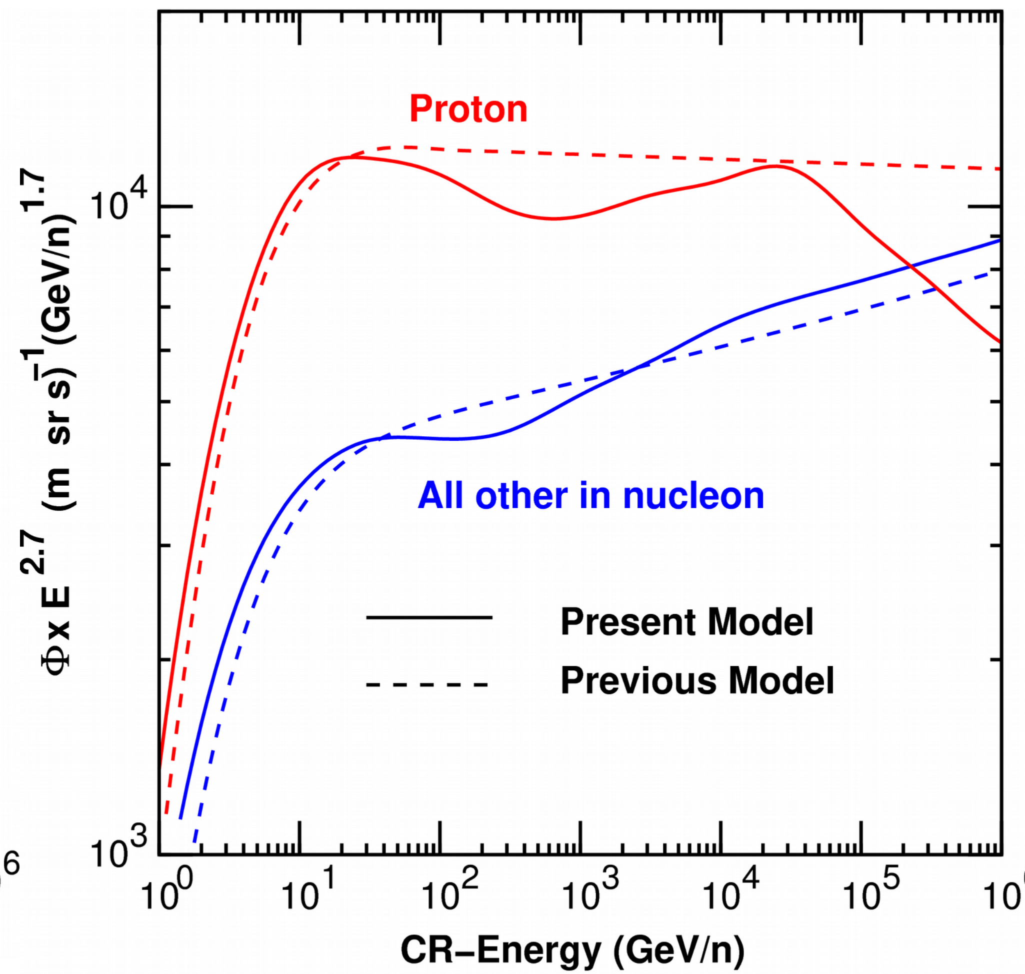
Modification of DPMJET3 in 2006





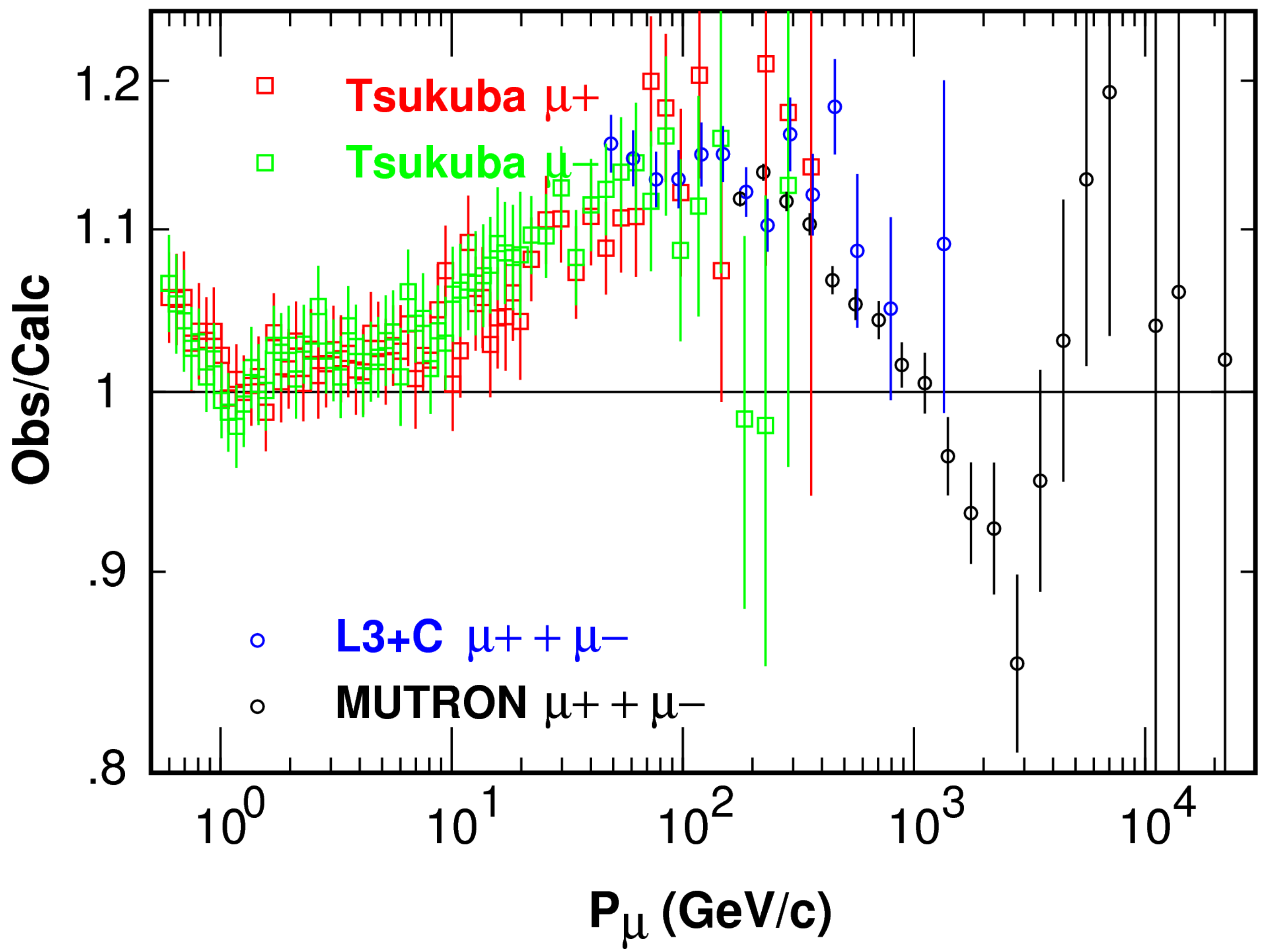
Cosmic Ray Spectra Model Based on AMS02 Observation
(2017 1ry model)



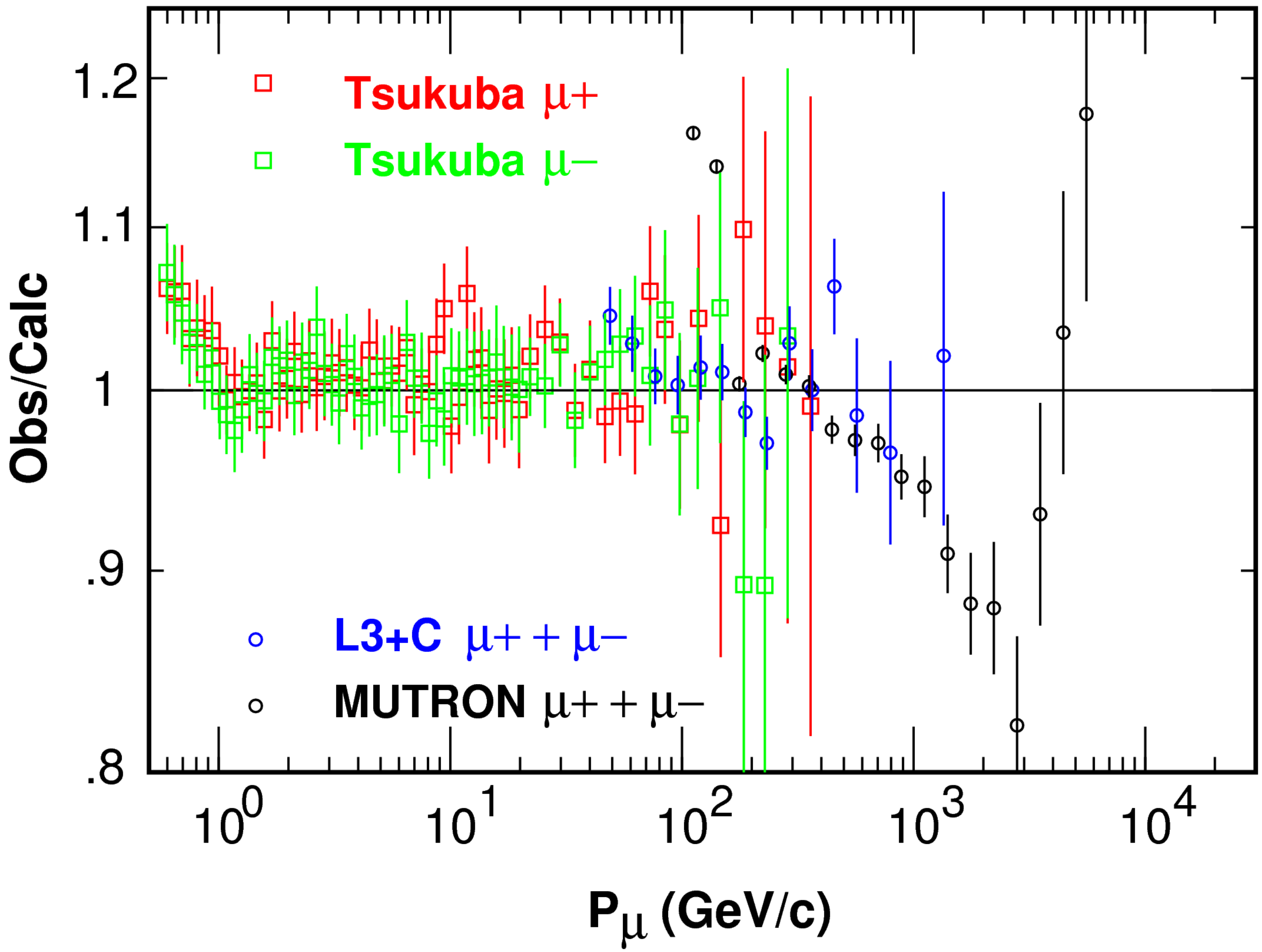


Looking forward to hearing from CALET and ISS-CREAM

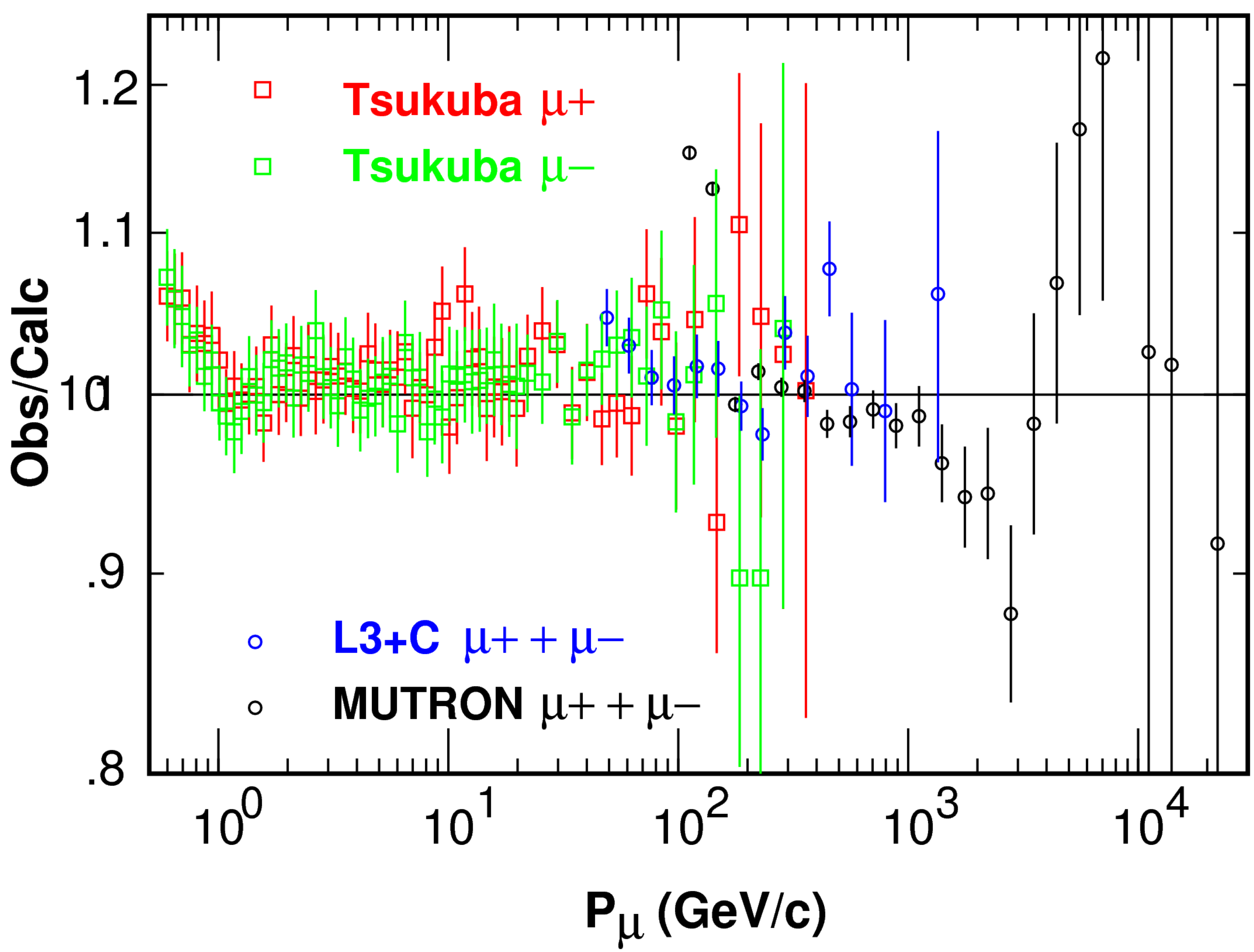
Observation / Calculation ratio
with
2017 primary cosmic ray moddel and 2006 interaction model



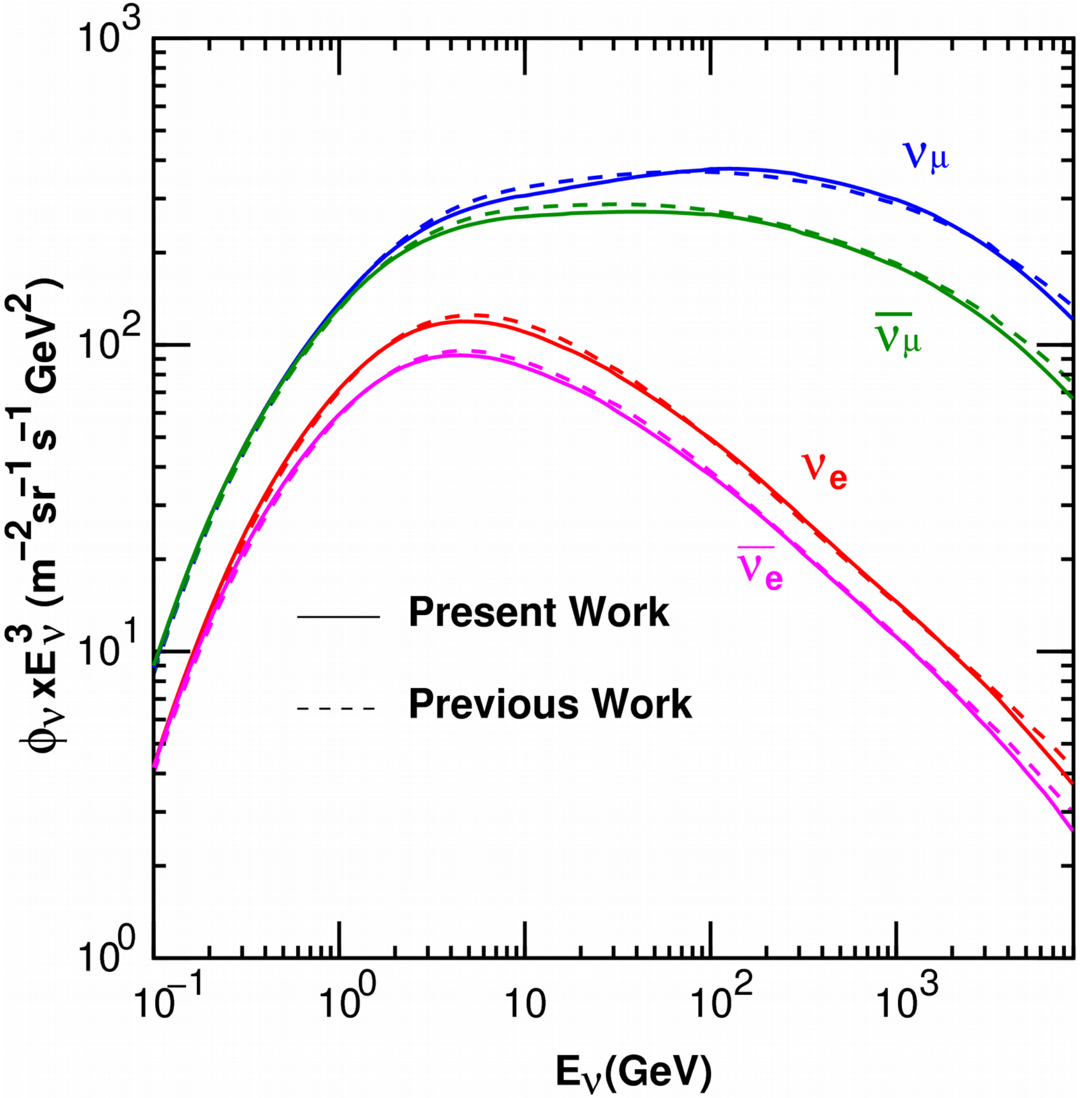
Observation / Calculation ratio
with
2017 primary cosmic ray model and 2017 interaction model A
(Studied without MUTRON)

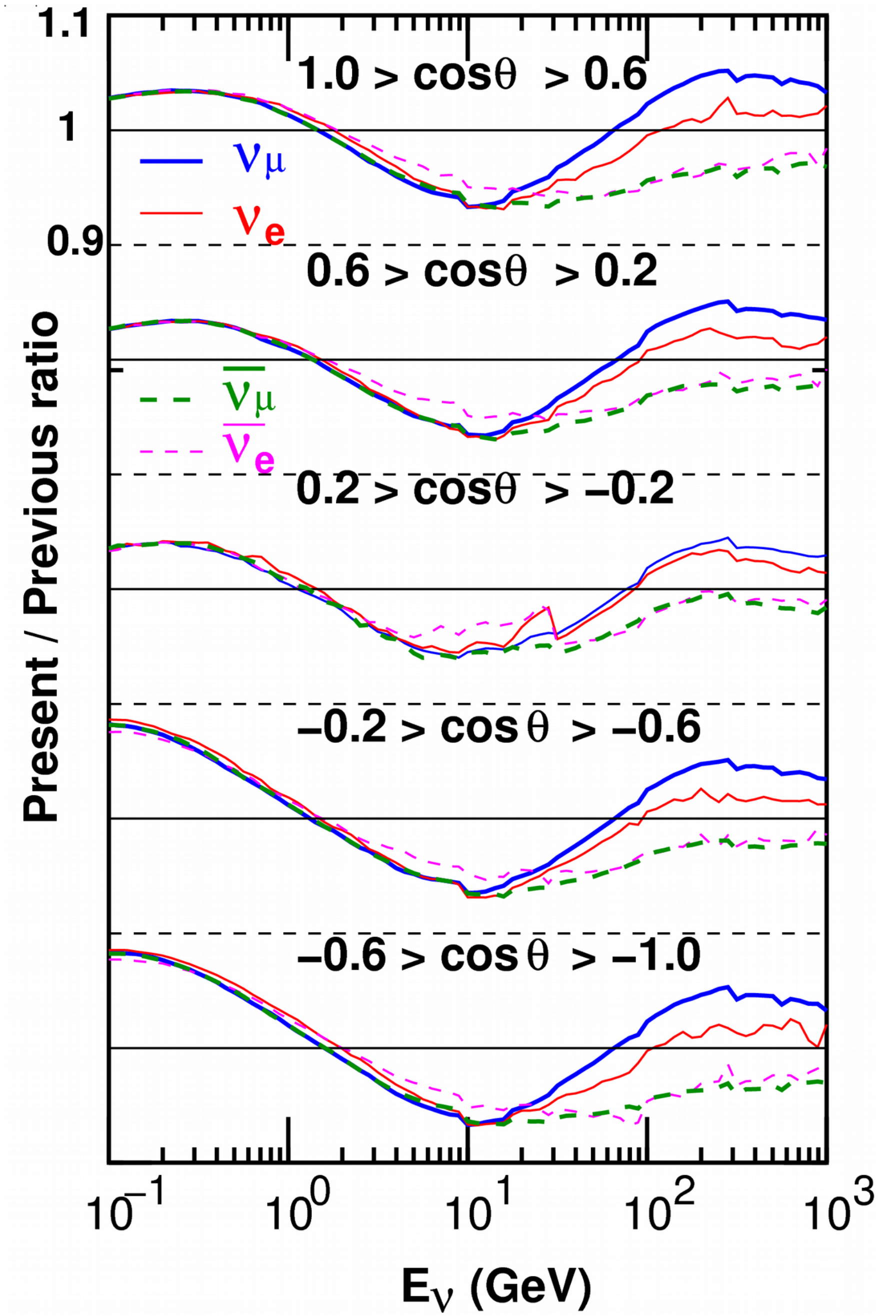


Observation / Calculation ratio
with
2017 primary cosmic ray model and 2017 interaction model B
(Studied with MUTRON)



Based On AMS02 Obervation (Preliminary)





2. Analytic expression of the Calculation of the Atmospheric Lepton Flux

$$\Phi_{L^{obs}}(p_L^{obs}, x^{obs}) = \sum_{N_{CR}} \sum_{N^{int}} \sum_{M^{brn}} \sum_{M^{dcy}} \sum_{L^{brn}} \int \int \cdot\cdot \int$$

$$P_{L\text{-}prp}(L^{brn}, p_L^{brn}, x^{brn} \rightarrow L^{obs}, p_L^{obs}, x^{obs})$$

$$\times P_{M\text{-}dcy}(M^{dcy}, p_M^{dcy} \rightarrow L^{brn}, p_L^{brn})$$

$$\times P_{M\text{-}prp}(M^{brn}, p_M^{brn}, x^{int} \rightarrow M^{dcy}, p_M^{dcy}, x^{dcy})$$

$$\times P_{H\text{-}int}(N^{int}, p_N^{int} \rightarrow M^{brn}, p_M^{brn})$$

$$\times P_{N\text{-}prp}(N_{CR}, p_{CR}^{in}, x^{in} \rightarrow N^{int}, p_N^{int}, x^{int})$$

$$\times \Phi_{CR}(N_{CR}, p_{CR}^{in}, x^{in})$$

$$dp_L^{brn}\, dp_M^{dcy}\, dx^{dcy}\, dp_M^{brn}\, dp_N^{int}\, dx^{int}\, dp_{CR}^{in}\, dx^{in}$$

$P_{\text{L-prp}}(L^0, x^0, p^0 \rightarrow L^1, x^1, p^1)$: The probablility of a L^0-lepton with momentum p^0 at x^0 propagates to x^1 as L^1-lepton with momentum p^1.

$P_{\text{M-prp}}(M^0, x^0, p^0 \rightarrow M^1, x^1, p^1)$: The probablility of a M^0-meson with momentum p^0 at x^0 propagates to x^1 as M^1-meson with momentum p^1.

$P_{\text{N-prp}}(N^{0,} x^0, p^0 \rightarrow N^1, x^1, p^1)$: The probablility of a N^0-nucleus with momentum p^0 at x^0 propagates to x^1 as N^1-nucleus with momentum p^1

$P_{\text{N-int}}(N, p_N \rightarrow M, p_M)$: The probablility of a N-nucleus with momentum p_N produces M-mesion with momentum p_M in a hadronic interaction with air.

$P_{\text{M-dcy}}(M, p_M \rightarrow L, p_L)$: The probablility of a M-meson with momentum p_M produces L-lepton with momentum p_L in its decay.

2. The Variation of Lepton Fluxes caused by the "Variation" of the Nucleus Hadronic Interactions

$$\widetilde{\Phi}_{L^{obs}}(p_L^{obs}, x^{obs}) = \sum_{N_{CR}} \sum_{N^{int}} \sum_{M^{brn}} \sum_{M^{dcy}} \sum_{L^{brn}} \int \int \cdots \int$$

$$P_{L\text{-}prp}(L^{brn}, p_L^{brn}, x^{brn} \rightarrow L^{obs}, p_L^{obs}, x^{obs})$$

$$\times P_{M\text{-}dcy}(M^{dcy}, p_M^{dcy} \rightarrow L^{brn}, p_L^{brn})$$

$$\times P_{M\text{-}prp}(M^{brn}, p_M^{brn}, x^{int} \rightarrow M^{dcy}, p_M^{dcy}, x^{dcy})$$

$$\boxed{\times P_{H\text{-}int}(N^{int}, p_N^{int} \rightarrow M^{brn}, p_M^{brn}) \cdot \left(1+\delta_{H\text{-}int}(N^{int}, p_N^{int}, M^{brn}, p_M^{brn})\right)}$$

$$\boxed{\times P_{N\text{-}prp}(N_{CR}, p_{CR}^{in}, x^{in} \rightarrow N^{int}, p_N^{int}, x^{int}) \cdot \left(1+\delta_{N\text{-}prp}(N_{CR}, p_{CR}, x^{in}, N^{int}, p_N^{int}, x^{int})\right)}$$

$$\times \Phi_{CR}(N_{CR}, p_{CR}^{in}, x^{in})$$

$$dp_L^{brn}\, dp_M^{dcy}\, dx^{dcy}\, dp_M^{brn}\, dp_N^{int}\, dx^{int}\, dp_{CR}^{in}\, dx^{in}$$

$P_{\text{L-prp}}(L^0, x^0, p^0 \rightarrow L^1, x^1, p^1)$: The probablility of a L^0-lepton with momentum p^0 at x^0 propagates to x^1 as L^1-lepton with momentum p^1.

$P_{\text{M-prp}}(M^0, x^0, p^0 \rightarrow M^1, x^1, p^1)$: The probablility of a M^0-meson with momentum p^0 at x^0 propagates to x^1 as M^1-meson with momentum p^1.

$P_{\text{N-prp}}(N^0, x^0, p^0 \rightarrow N^1, x^1, p^1)$: The probablility of a N^0-nucleus with momentum p^0 at x^0 propagates to x^1 as N^1-nucleus with momentum p^1

$P_{\text{N-int}}(N, p_N \rightarrow M, p_M)$: The probablility of a N-nucleus with momentum p_N produces M-mesion with momentum p_M in a hadronic interaction with air.

$P_{\text{M-dcy}}(M, p_M \rightarrow L, p_L)$: The probablility of a M-meson with momentum p_M produces L-lepton with momentum p_L in its decay.

Simplified Expression with the result of Monte Carlo Simulation

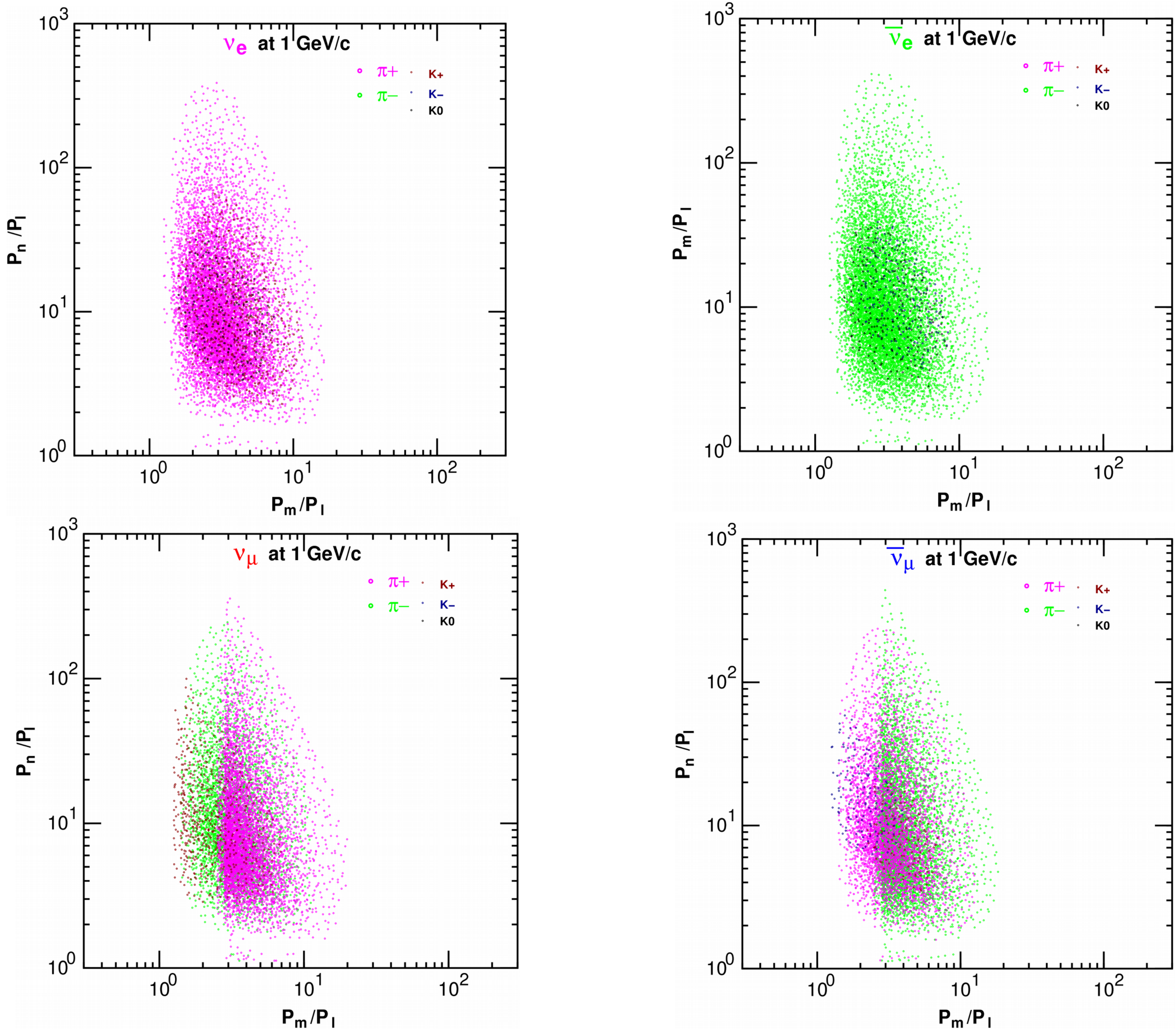
$$\Phi_{L^{obs}}(p_L^{obs}, x^{obs}) = \sum_{N^{int}} \sum_{M^{brn}} \int \int DD(N^{int}, p_N^{int}, M^{brn}, p_M^{brn}, L^{obs}, p_{L^{obs}}, x^{obs}) \; dp_M^{brn} \, dp_N^{int}$$

Where

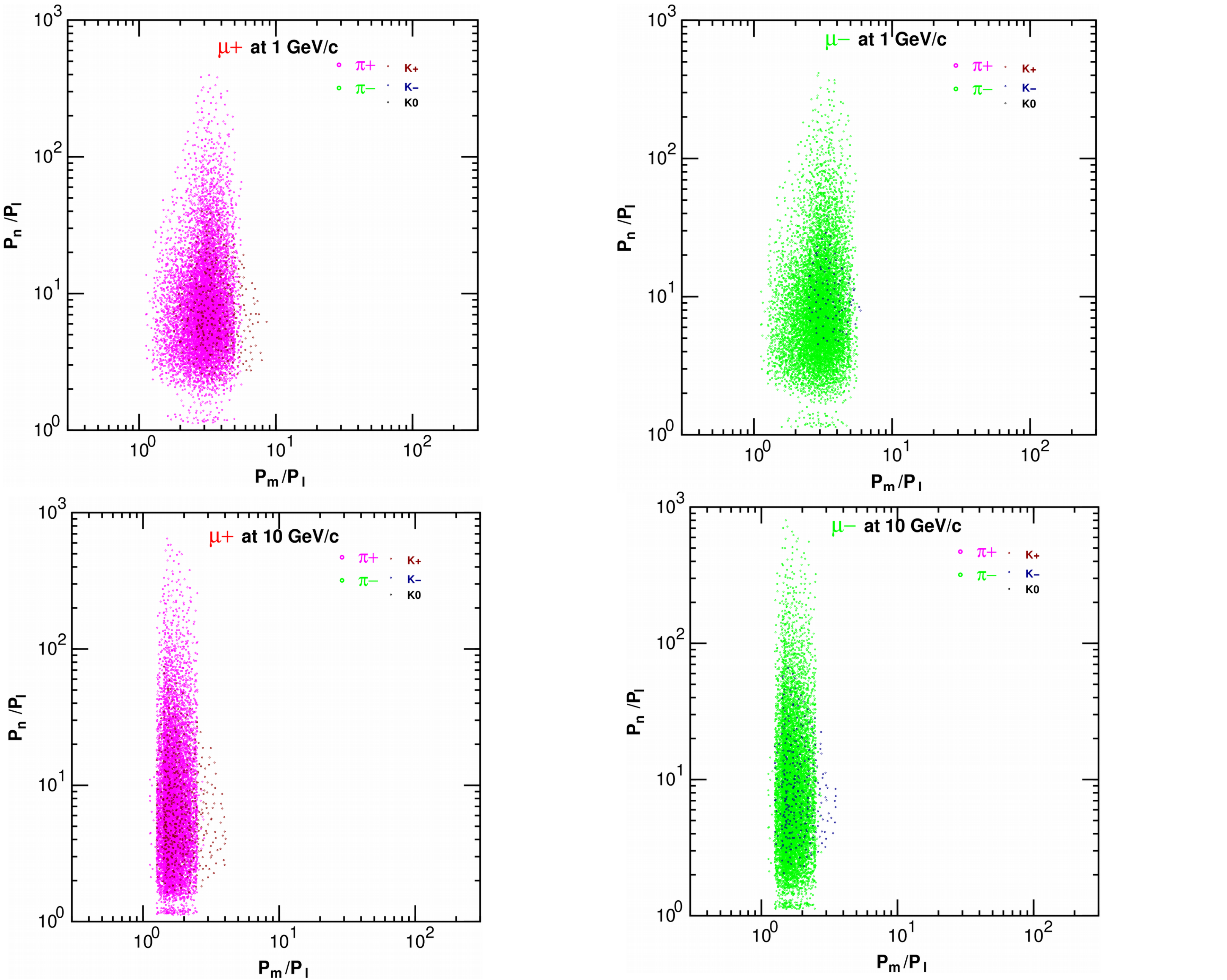
$$\begin{aligned} DD(N, p_N^{int}, M, p_M^{brn}, L^{obs}, p_{L^{obs}}, x^{obs}) \equiv \sum_{N_{CR}} \sum_{M^{dcy}} \sum_{L^{brn}} \int \int \cdot\cdot \int & \\ P_{L\text{-}prp}(L^{brn}, p_L^{brn}, x^{brn} \to L^{obs}, p_L^{obs}, x^{obs}) & \\ \times P_{M\text{-}dcy}(M^{dcy}, p_M^{dcy} \to L^{brn}, p_L^{brn}) & \\ \times P_{M\text{-}prp}(M^{brn}, p_M^{brn}, x^{int} \to M^{dcy}, p_M^{dcy}, x^{dcy}) & \\ \times P_{H\text{-}int}(N^{int}, p_N^{int} \to M^{brn}, p_M^{brn}) & \\ \times P_{N\text{-}prp}(N_{CR}, p_{CR}^{in}, x^{in} \to N^{int}, p_N^{int}, x^{int}) & \\ \times \Phi_{CR}(N_{CR}, p_{CR}^{in}, x^{in}) & \\ dp_L^{brn} \, dp_M^{dcy} \, dx^{dcy} \, dx^{int} \, dp_{CR}^{in} \, dx^{in} & \end{aligned}$$

Note, the DD function is calculated in Monte Carlo Simulation is the usual calculation.

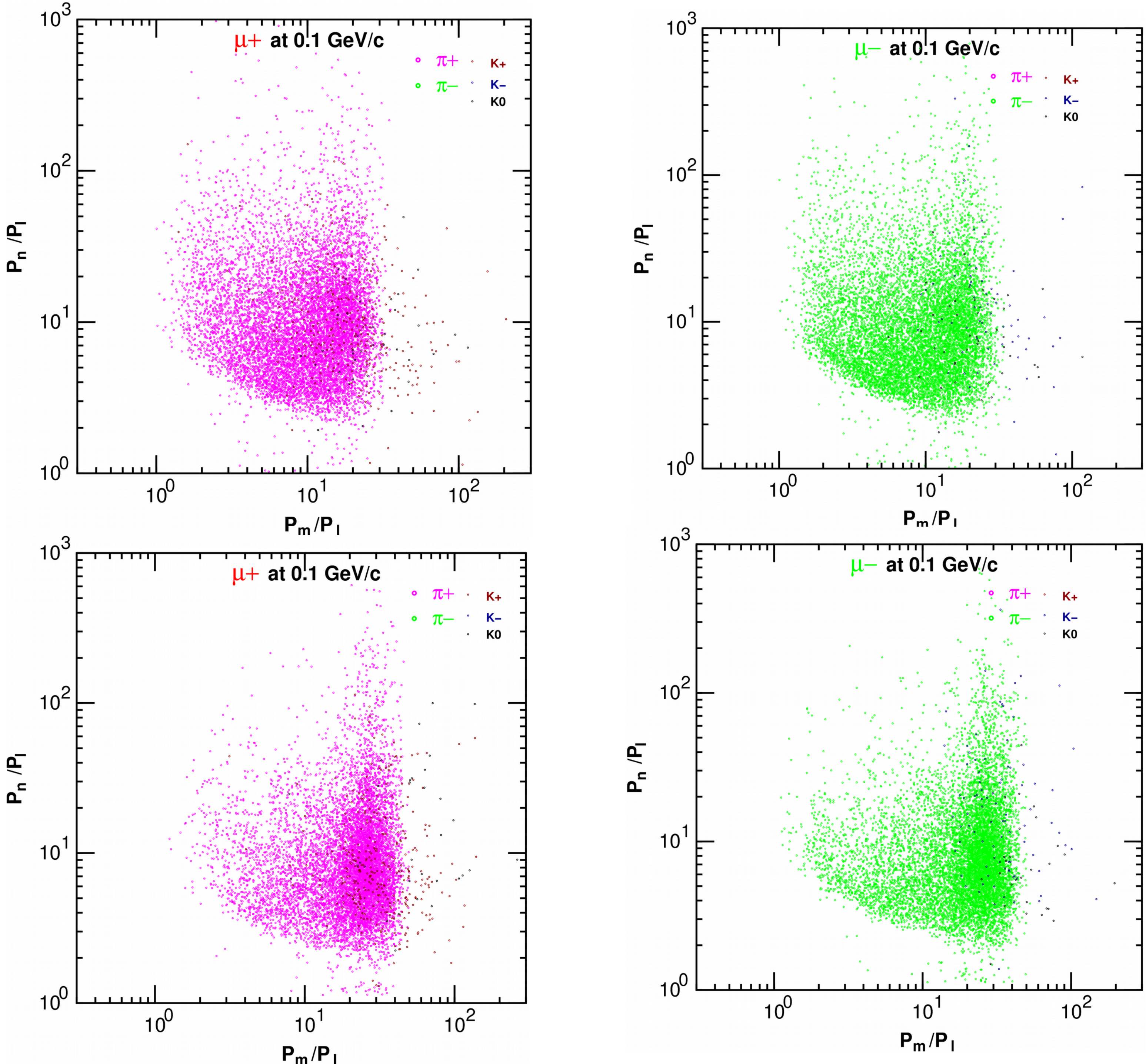
$DD(N^{\text{int}}, p_N^{\text{int}}, M^{brn}, p_M^{brn}, L^{obs}, p_{L^{obs}}, x^{obs})$ in the Simulation for vertical Neutrino at Kamioka at 1 GeV



$DD(N^{int}, p_N^{int}, M^{brn}, p_M^{brn}, L^{obs}, p_{L^{obs}}, x^{obs})$ in the Simulation for vertical Muon at Kamioka at 1 GeV/c and 10 GeV/c



$DD\left(N^{\text{int}}, p_N^{\text{int}}, M^{brn}, p_M^{brn}, L^{obs}, p_{L^{obs}}, x^{obs}\right)$ Site dependence for Muon at 0.1 GeV.

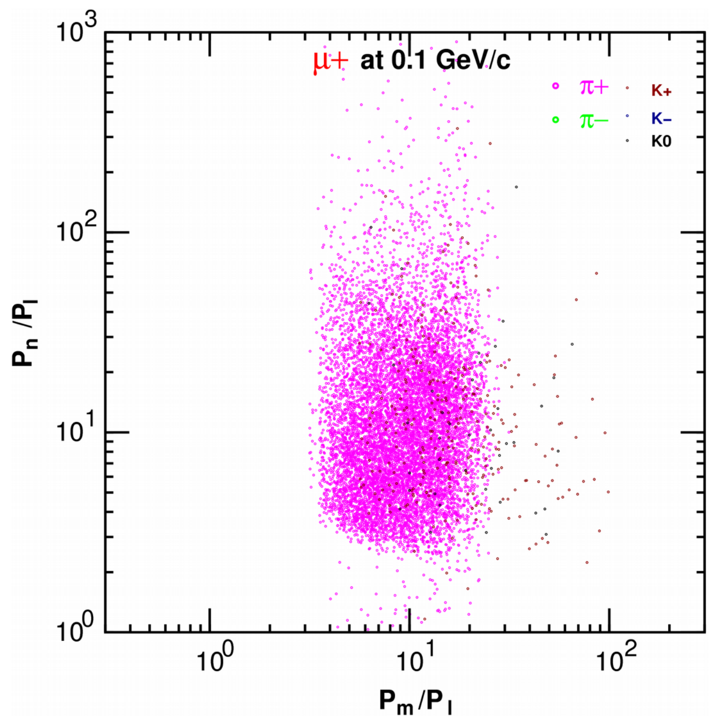


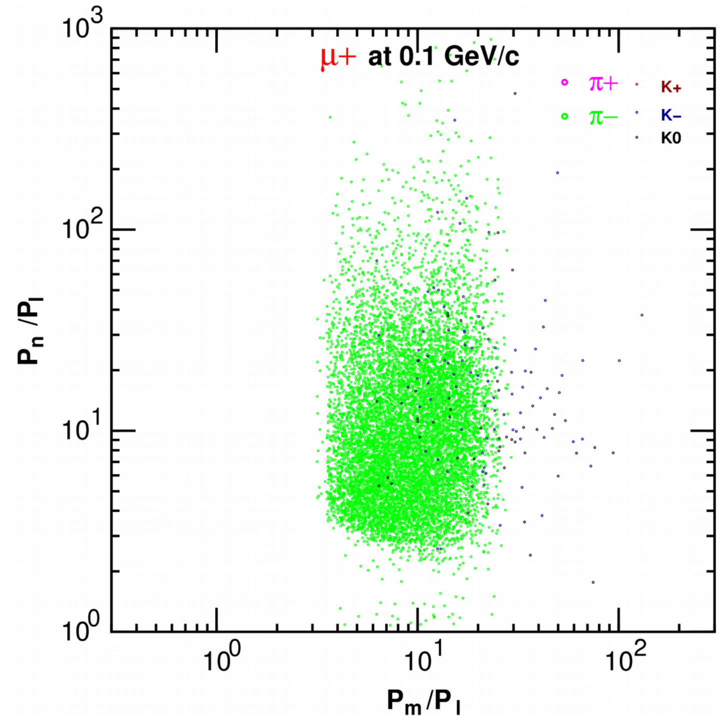
~0 m A.S.L

~3000 m A.S.L

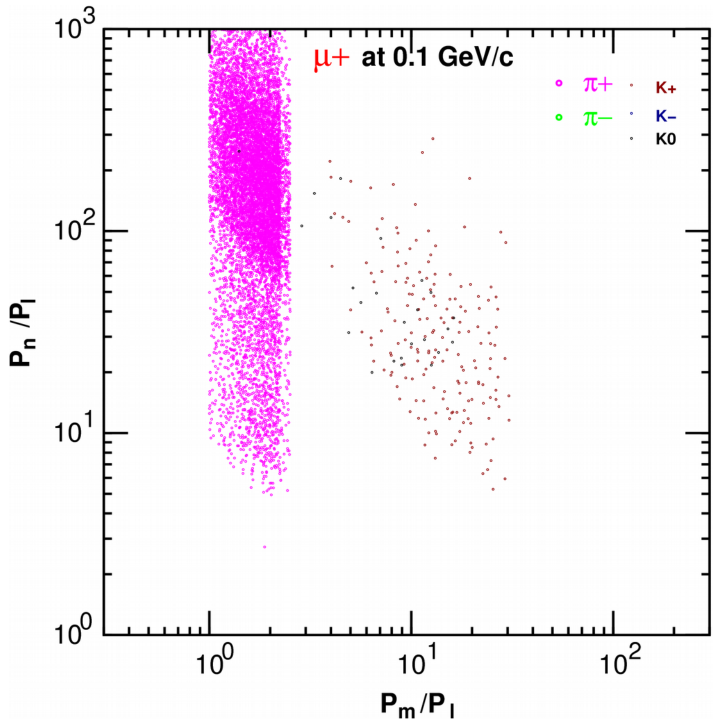
$$DD\left(N^{\text{int}}, p_N^{\text{int}}, M^{brn}, p_M^{brn}, L^{obs}, p_{L^{obs}}, x^{obs}\right)$$

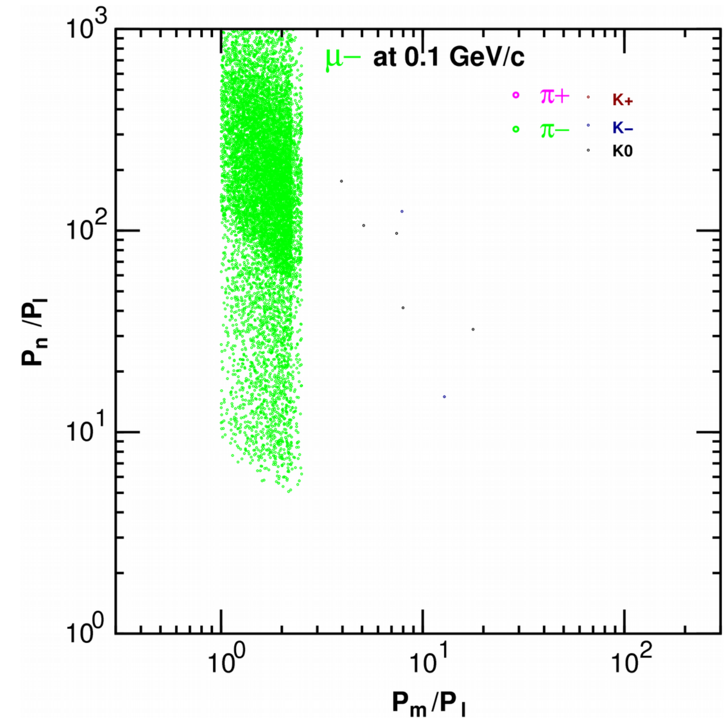
Site dependence for Muon at 0.1 GeV.





~5000 m A.S.L





~30k m A.S.L
(Balloon)

The variation of lepton flux in simplified expression for MC

$$\tilde{\Phi}_{L^{obs}}(p_L^{obs}, x^{obs}) = \sum_{N^{int}} \sum_{M^{brn}} \int\int DD(N^{int}, p_N^{int}, M^{brn}, p_M^{brn}, L^{obs}, p_{L^{obs}}, x^{obs})$$
$$\times\left(1+\delta_{H\text{-}int}(N^{int}, p_N^{int}, M^{brn}, p_M^{brn})\right)$$
$$\times\left(1+\delta_{N\text{-}prp}(N_{CR}, p_{CR}, x^{in}, N^{int}, p_N^{int}, x^{int})\right)\; dp_M^{brn}\, dp_N^{int}$$

Possible variation of the meson-producing Interaction with random numbers as:

$$\delta_{\text{H-int}}(N^{\text{int}}, p_N^{\text{int}}, M^{\text{brn}}, p_M^{\text{brn}}) = \lambda \cdot \sum_{i,j} r_{i,j} B_i(\log p_m) B_j(\log p_n)$$

where

$B_i(\log p)$ is the B-spline function of $\log p$ with constant grid separation of $\Delta \log p = 0.5$

$\{r_{i,j}\}$ is the set of Random Numbers with Normal Distribution for each grid point.

For $DD(N^{\text{int}}, p_N^{\text{int}}, M^{brn}, p_M^{brn}, L^{obs}, p_{L^{obs}}, x^{obs})$, the ones calculated by our Simulation is used.

The estimation of possible errors in the the Propagation of Nucleus is rather difficult. We just assume

$$\delta_{\text{N-prp}}\left(N_{CR}, p_{CR}, x^{\text{in}}, N^{\text{int}}, p_N^{\text{int}}, x^{\text{int}}\right)=\kappa \cdot r_{N^{\text{int}}}$$

for the possible variation of propagation process of Nucleus, where $r_{N^{\text{int}}}$ is the Random Number with Normal Distribution for each kind of projectile nucleus.

We consider those variation parameters λ, κ are small, or our Interaction Model is already a Good Approximations for the real Cosmic Ray Interaction and Propagation in Air. The lepton flux variation is expanded with λ, κ and we study only the lowest order of them as;

$$\begin{aligned}\Delta \Phi_{L^{obs}}\left(p_L^{obs}, x^{obs}\right)=&\sum_{N^{int}} \sum_{M^{brn}} \int \int DD\left(N^{int}, p_N^{int}, M^{brn}, p_M^{brn}, L^{obs}, p_{L^{obs}}, x^{obs}\right) \\ &\times\left(\lambda \cdot \sum_{i,j} r_{i,j} B_i\left(\log p_m\right) B_j\left(\log p_n\right) + \kappa \cdot r_{N^{int}}\right) dp_M^{brn} dp_N^{int}\end{aligned}$$

Now we can study the distribution of $\Delta \Phi_L$ in MC.

2.a

We normally consider the error due to the propagation of nucleus is small. Therefore, we first assume $\lambda \gg \kappa$, and just

$$\Delta \Phi_{L^{obs}}(p_L^{obs}, x^{obs}) = \sum_{N^{int}} \sum_{M^{brn}} \int\int DD(N^{int}, p_N^{int}, M^{brn}, p_M^{brn}, L^{obs}, p_{L^{obs}}, x^{obs}) \times \left(\lambda \cdot \sum_{i,j} r_{i,j} B_i(\log p_m) B_j(\log p_n)\right) \; dp_M^{brn} \, dp_N^{int}$$

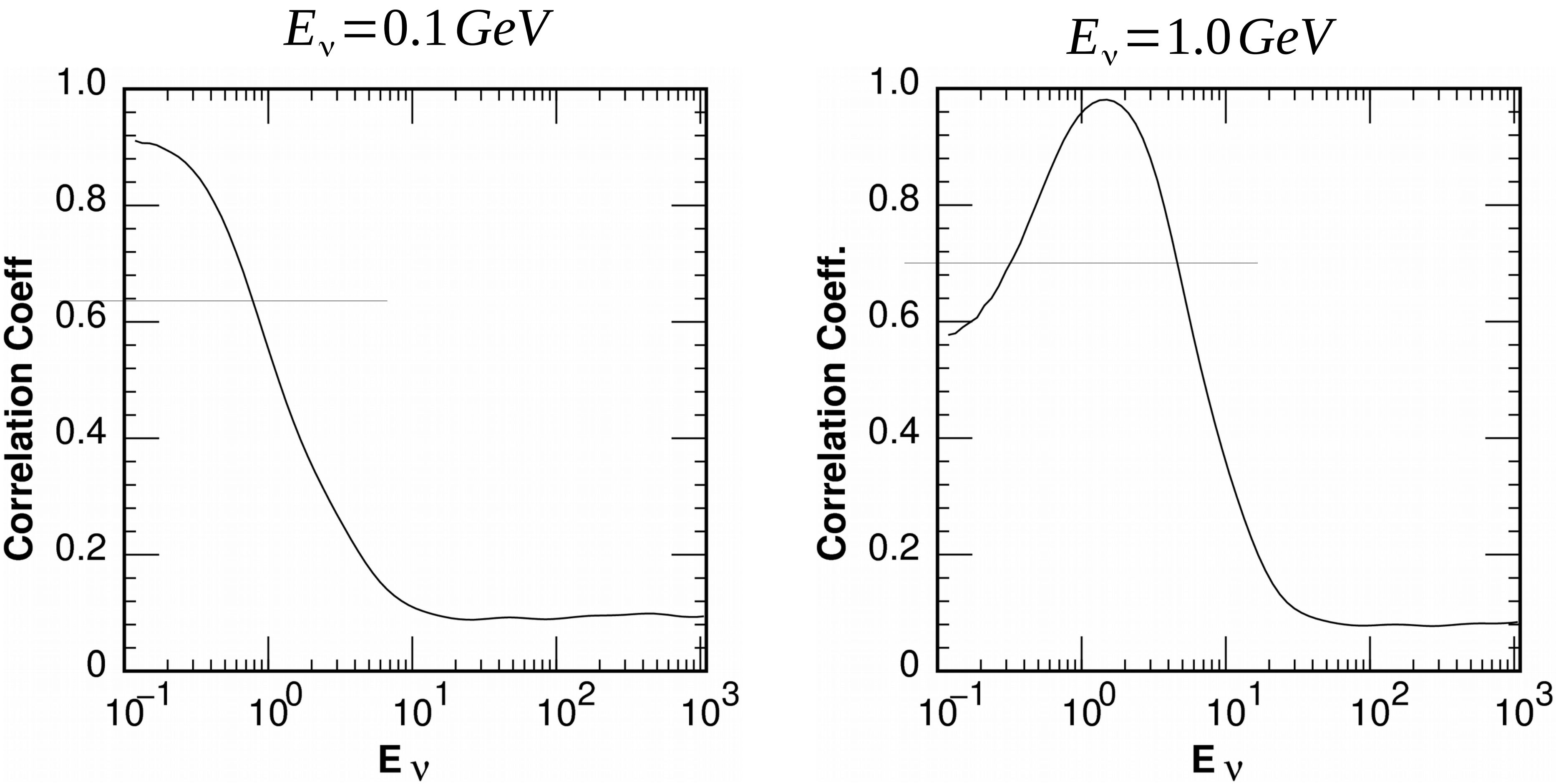
The correlation coefficient of variation between neutrino flux and muon fluxes is calculated as,

$$c(p_\nu, x_\nu^{obs}, p_\mu, x_\mu^{obs}) \equiv \frac{< \Delta \Phi_\nu(p_\nu, x_\nu^{obs}) \cdot \Delta \Phi_\mu(p_\mu, x_\mu^{obs}) >}{|\Delta \Phi_\nu(p_\nu, x_\nu^{obs})| |\Delta \Phi_\mu(p_\mu, x_\mu^{obs})|}$$

Note, this correlation coefficient has different value even for the same p_ν and p_μ, when the observation sites for muon or neutrino is different.

For a combination of muon and neutrino observation sites and a neutrino momentum p_ν, a maximum correlation coefficient c_{max} and a muon momentum region with $c > 0.7\, c_{max}$ is determined.

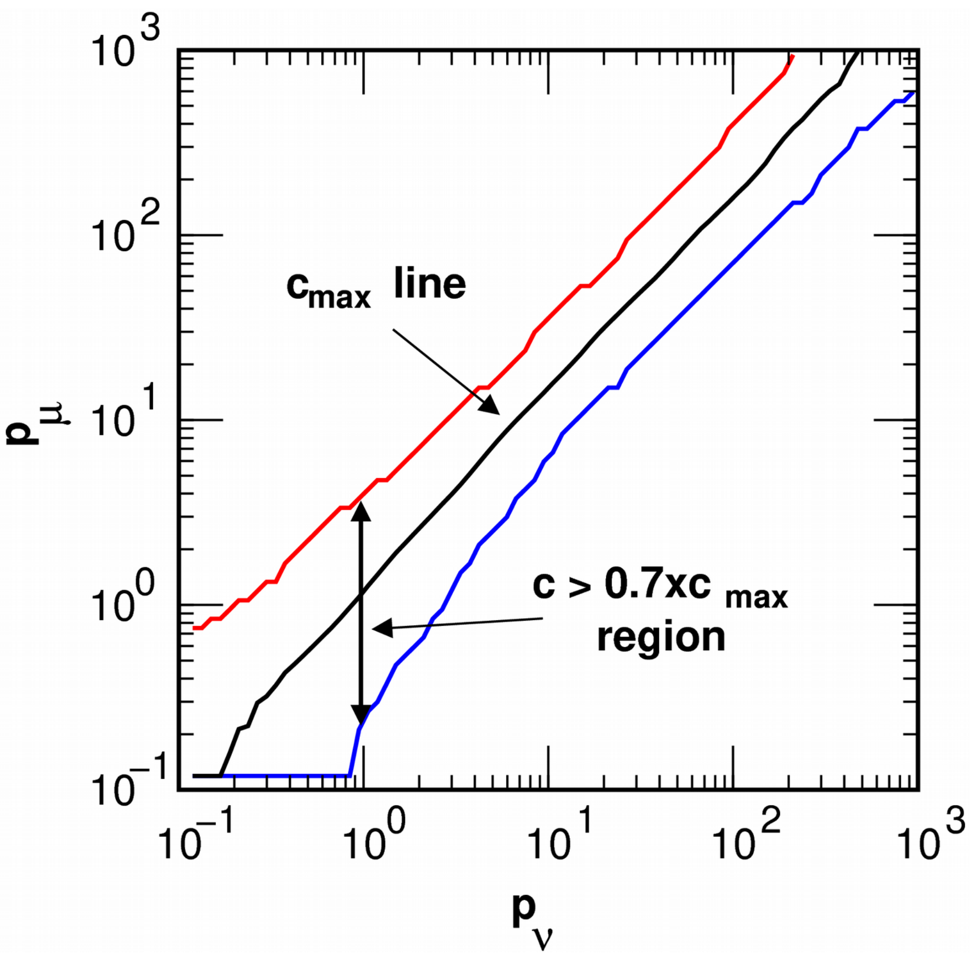
Correlation Coefficient calculate for ν_e
With muon flux at Kamioka



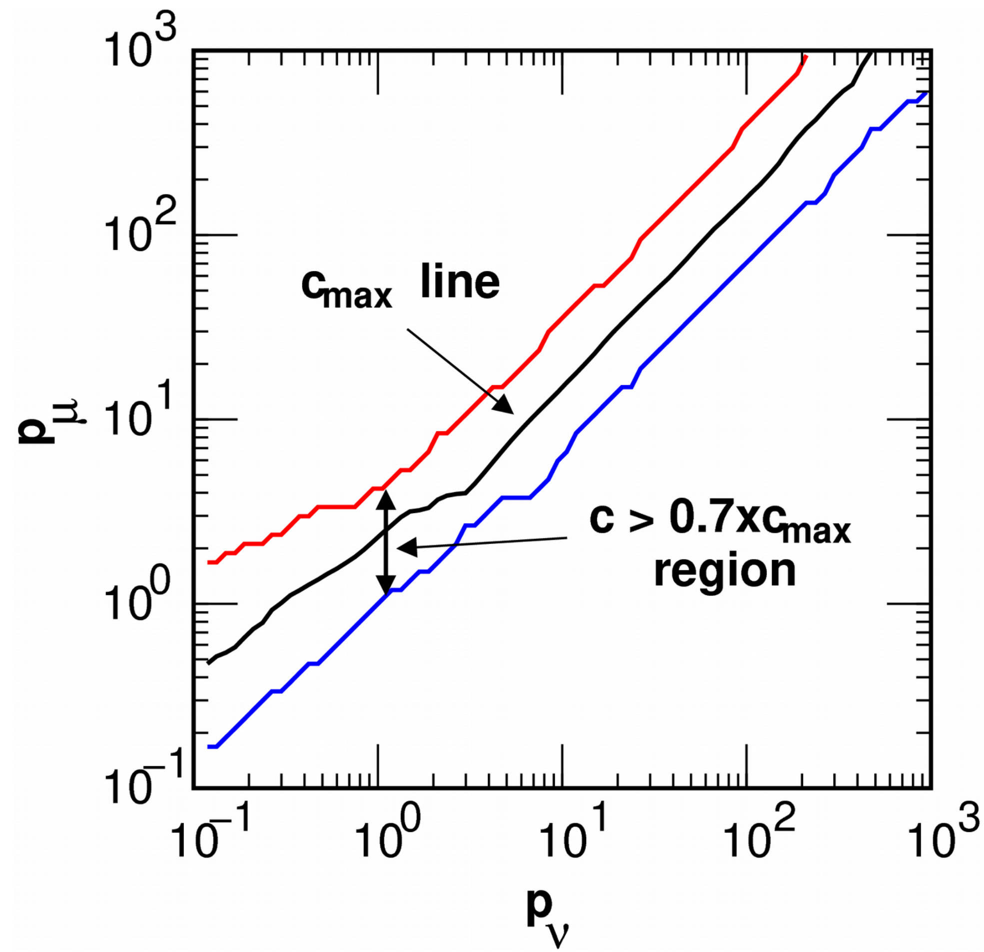
For a combination of muon and neutrino observation sites and a neutrino momentum p_ν, a maximum correlation coefficient c_{max} and a muon momentum region with $c > 0.7\, c_{max}$ is determined.

$c > 0.7\, c_{max}$ Regions

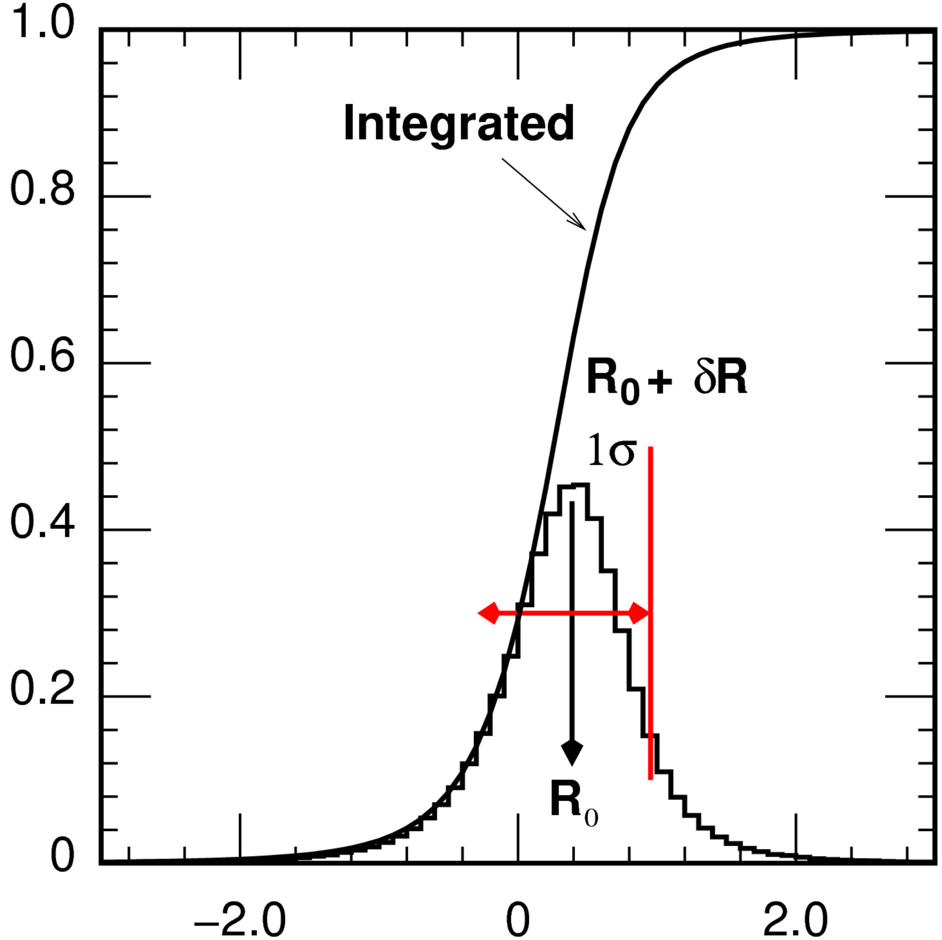
Kamioka vertical neutrino
Kamioka muon



Kamioka vertical neutrino
Balloon altitude muon + Kamioka (>3GeV)



$$R \equiv \frac{\Delta \Phi_\nu(p_\nu^{obs}, x^{obs}) / \Phi_\nu(p_\nu^{obs}, x^{obs})}{max\left|\Delta \Phi_\mu(p_\mu^{obs}, x^{obs}) / \Phi_\mu(p_\mu^{obs}, x^{obs})_{\text{in } c>0.7c_{max}}\right|}$$ distribution and Error Factor



Estimating the distributiob region of R as

$$|R - R_0| \lesssim \delta R \quad \text{and} \quad \delta R = \sqrt{<(R - R_0)>^2}$$

We may write

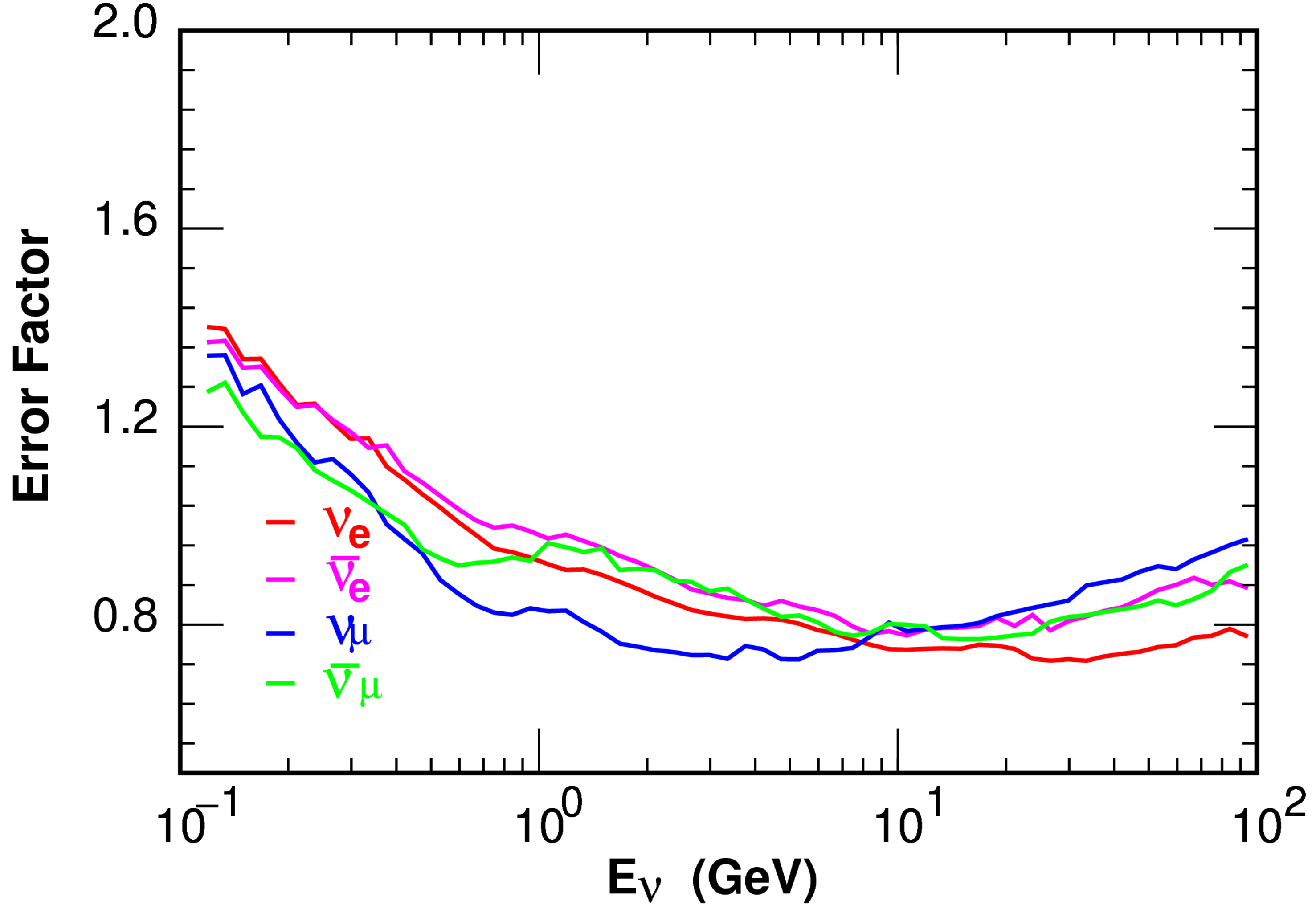
$$\frac{|\Delta \Phi_\nu(p_\nu^{obs}, x^{obs})|}{\Phi_\nu(p_\nu^{obs}, x^{obs})} \lesssim (R_0 + \delta R) \times \frac{max\left|\Delta \Phi_\mu(p_\mu^{obs}, x^{obs})_{\text{in } c>0.7c_{max}}\right|}{\Phi_\mu(p_\mu^{obs}, x^{obs})}$$

Substituting the **residual** of muon flux reconstruction and **experimental error** to

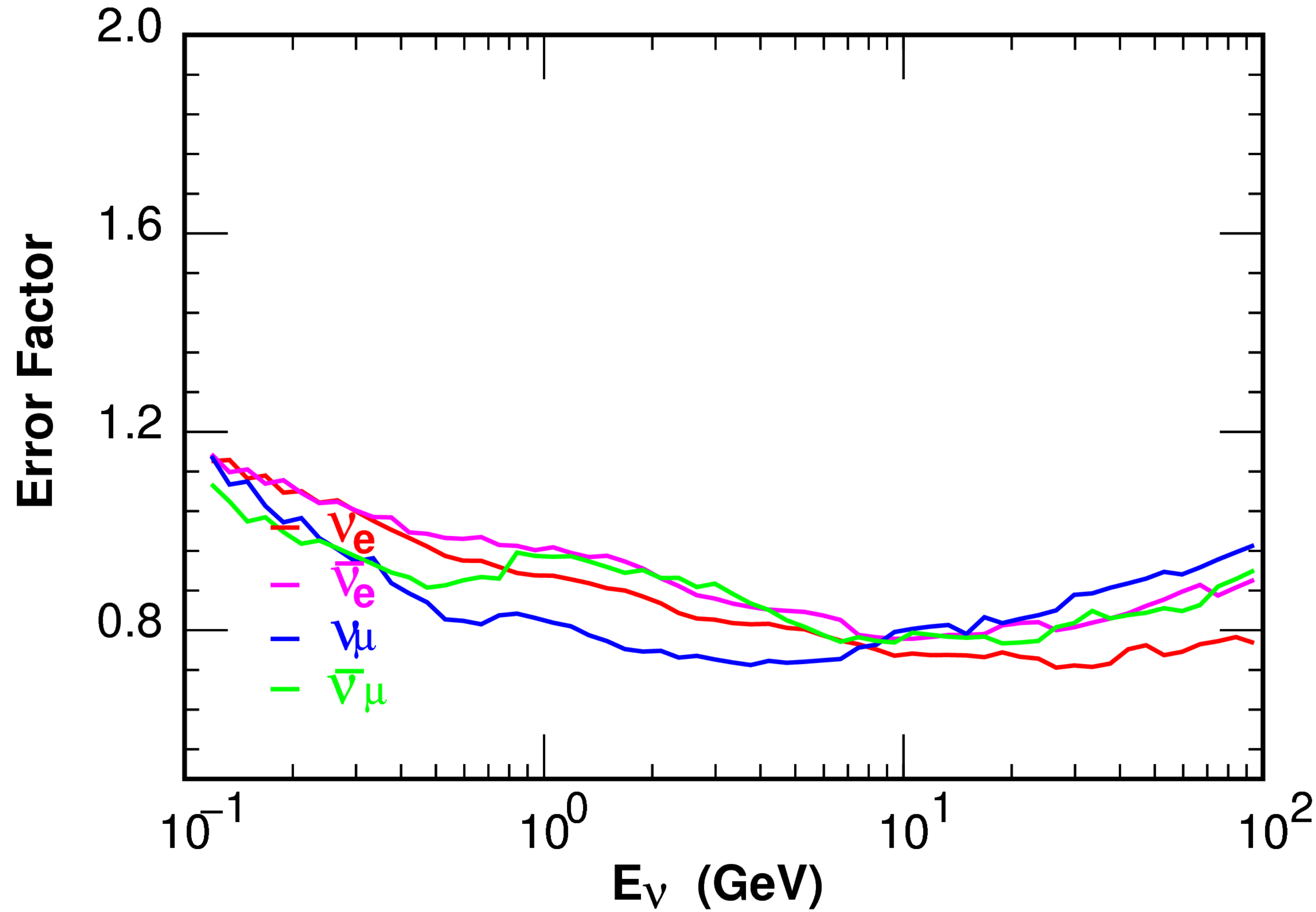
$$max\left|\Delta \Phi_\mu(p_\mu^{obs}, x^{obs})_{\text{in } c>0.7c_{max}}\right|,$$

we can estimate the **calculation error** of neutrino fluxc, and we call $R_0 + \delta R$ as Error Factor.

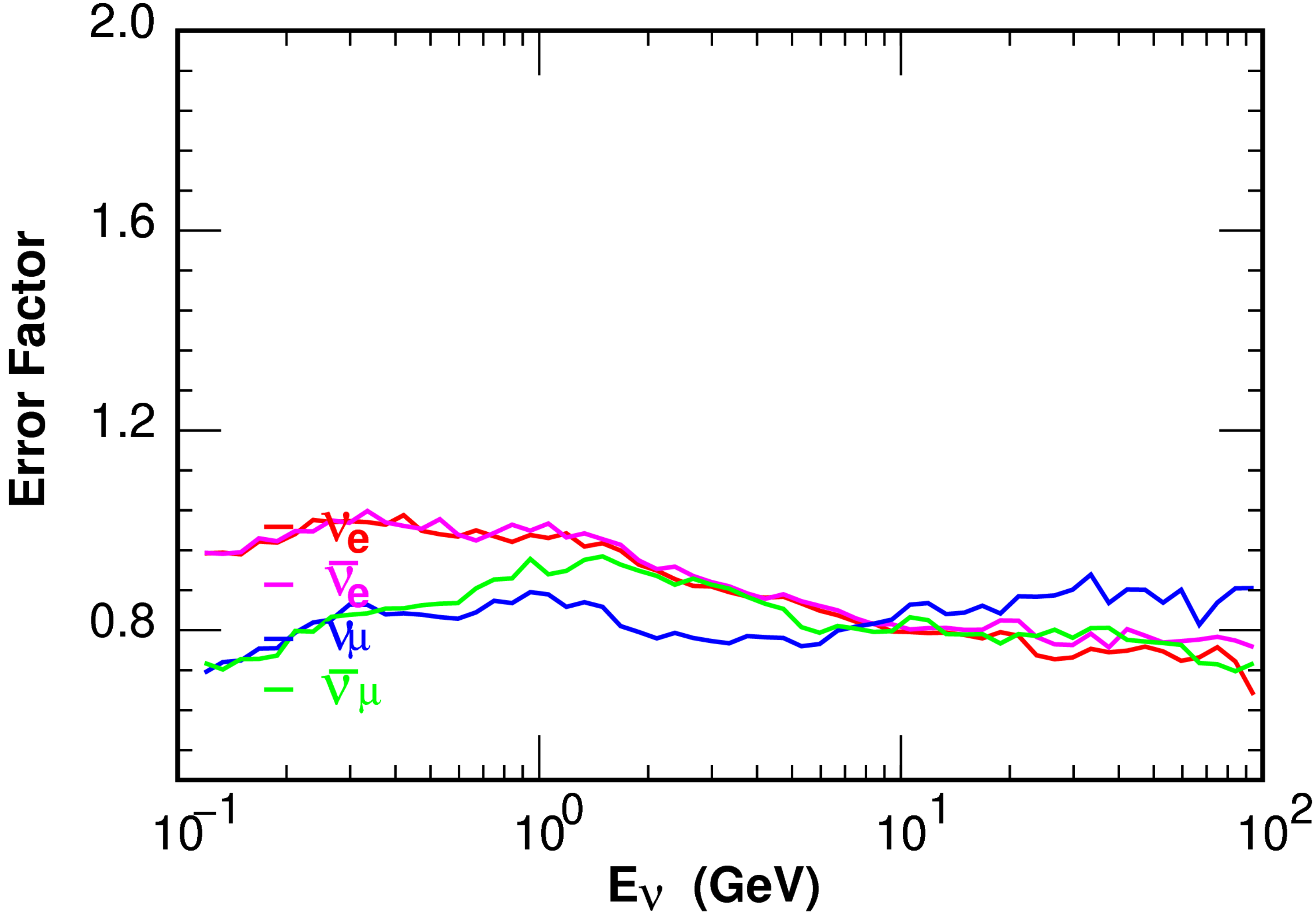
The error factor for vertical neutrino at Kamioka, using the muon observed at Kamioka



The error factor for vertical neutrino at Kamioka, using the muon observed at Hanle, India (4500 A.S.L)



The error factor for vertical neutrino at Kamioka, using the muon observed at Balloon Altitude (30km A.S.L)



2.b With the variation of Nucleus/Nucleon propagation in air

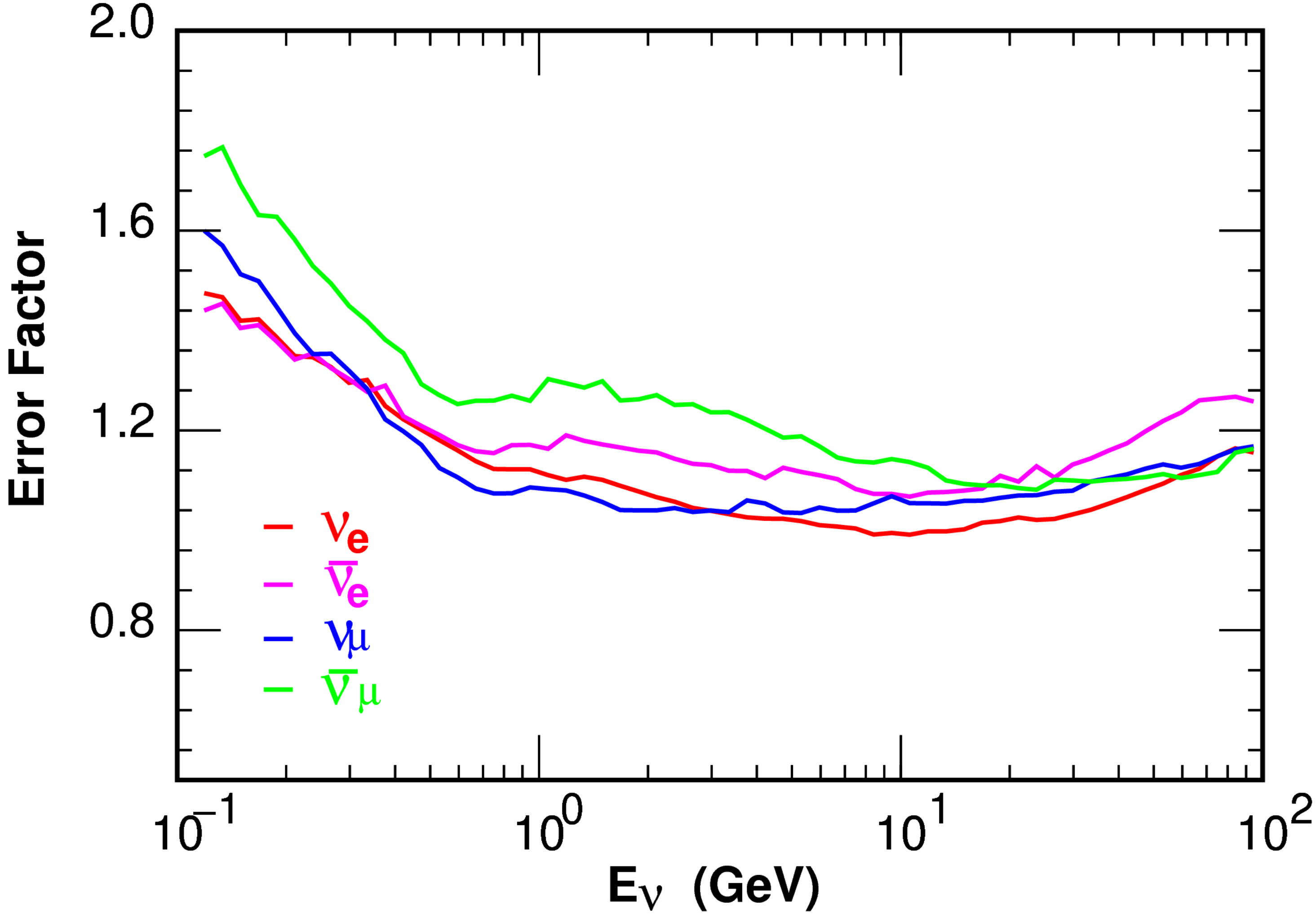
In $\kappa \sim \lambda$ case, we need to come back to the expression

$$\Delta \Phi_{L^{obs}}(p_L^{obs}, x^{obs}) = \sum_{N^{int}} \sum_{M^{brn}} \int \int DD(N^{int}, p_N^{int}, M^{brn}, p_M^{brn}, L^{obs}, p_{L^{obs}}, x^{obs})$$
$$\times \left(\lambda \cdot \sum_{i,j} r_{i,j} B_i(\log p_m) B_j(\log p_n) \; + \; \kappa \cdot r_{N^{int}} \right) dp_M^{brn} dp_N^{int}$$

The quantity $R \equiv \dfrac{\Delta \Phi_\nu(p_\nu^{obs}, x^{obs})}{max\left|\Delta \Phi_\mu(p_\mu^{obs}, x^{obs})\right|}$ is now dependent on the ratio $\dfrac{\kappa}{\lambda}$.

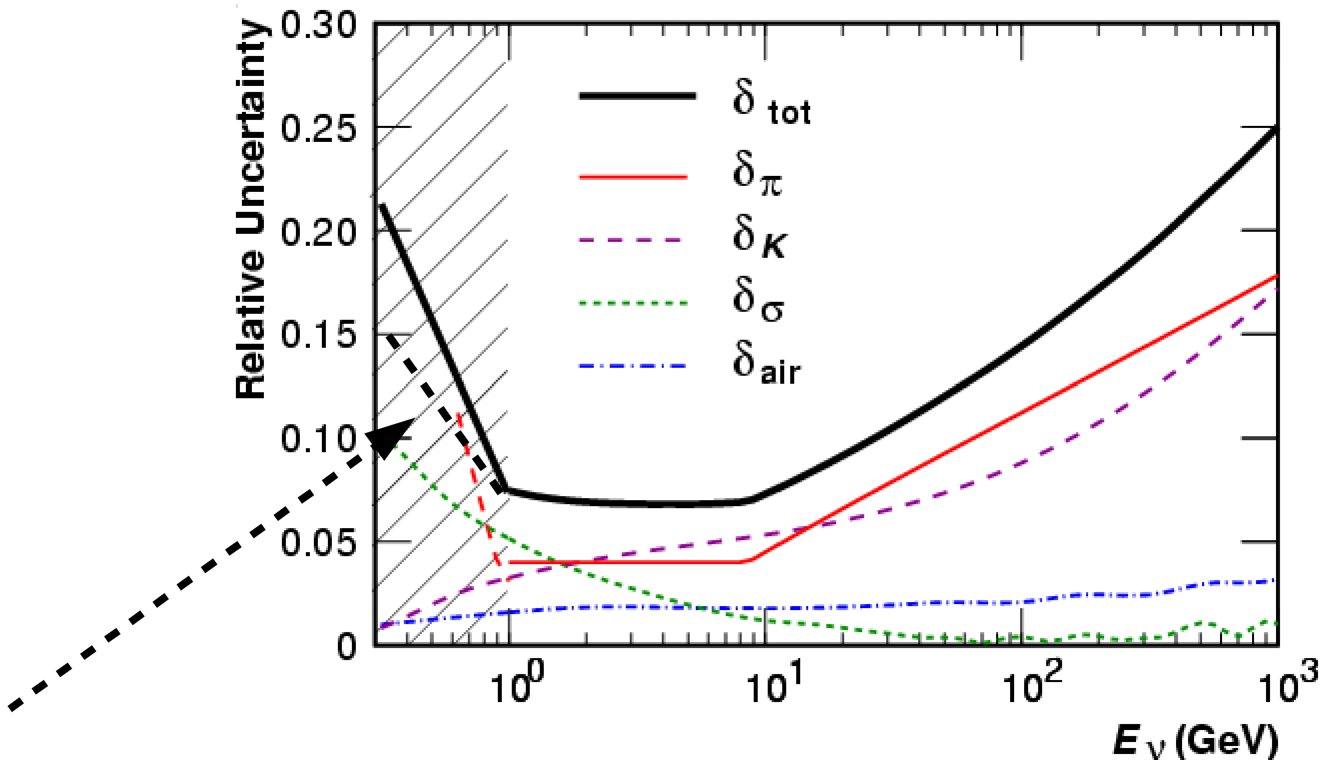
The random numbers $\{ r_{N^{int}} \}$ for the expression of muon flux and neutrino flux have relation to each other, but not the same in general.
Here we simply assume the same random number for muon flux and neutrino flux calculations and calculate the Error Factor.

The error factor for vertical neutrino at Kamioka, using the muon observed at Kamioka with $\frac{\kappa}{\lambda}=1$



~20 % increase in all the energy region, but this is maximum for $\kappa < \lambda$

Estimated Error in Atmospheric ν-flux Calculation (HKKMS07)



Possible Error with JAM (HKKM11)

δ_π μ -observation error + Residual of reconstruction

δ_K Kaon production uncertainty

δ_σ Mean free path (interaction crossection) uncertainty

δ_{air} Atmosphere density profule uncertainty

Summary

1. An overview of the calculation of atmospheric neutrino flux and the muon calibration are presented.

2. To extend the muon calibratio to lower energy of neutrinos, we study the analytic formulation of the atmospheric neutrino flux calculation, then apply the artificial random variations to the Hadronic Interactions.

2.a The possible error due to the uncertainty of Meson production in Hadronic Interactions is well studied by the comparison of the calculated muon flux with the observed ones especially that at high altitulde.

2.b The possible error due to Nucleus/Nucleon probagation is a little difficulat. However when it is not too large compared to that of Meson productons, the error is also estimated by the observed muon fluxes.