Measurement of the Multi-TeV Neutrino-Nucleon Cross Section Via Earth Absorption

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PANE

ICTP, Trieste, May 27, 2018

UNIVERSITY OF COPENHAGEN

Two seemingly unrelated questions –

1. Where are the most energetic particles coming from?

2. What is the structure of matter at the smallest scales?

Symmetry Magazine

Neutrinos interactions are weak ...

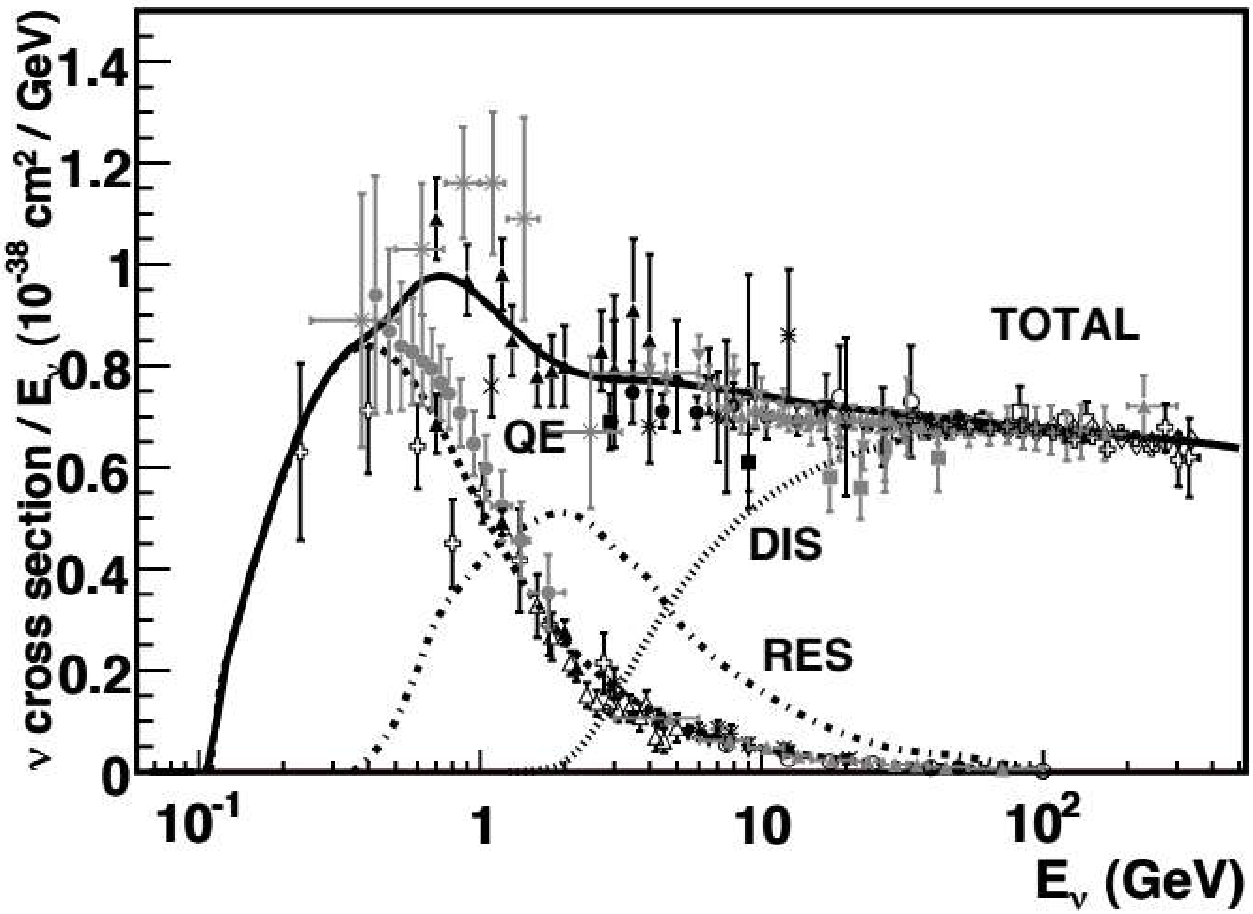
… but we *are* persistent

At center-of-mass energy of 1 GeV:

$$\sigma_{pp} \sim 10^{-28}\ \text{cm}^2$$

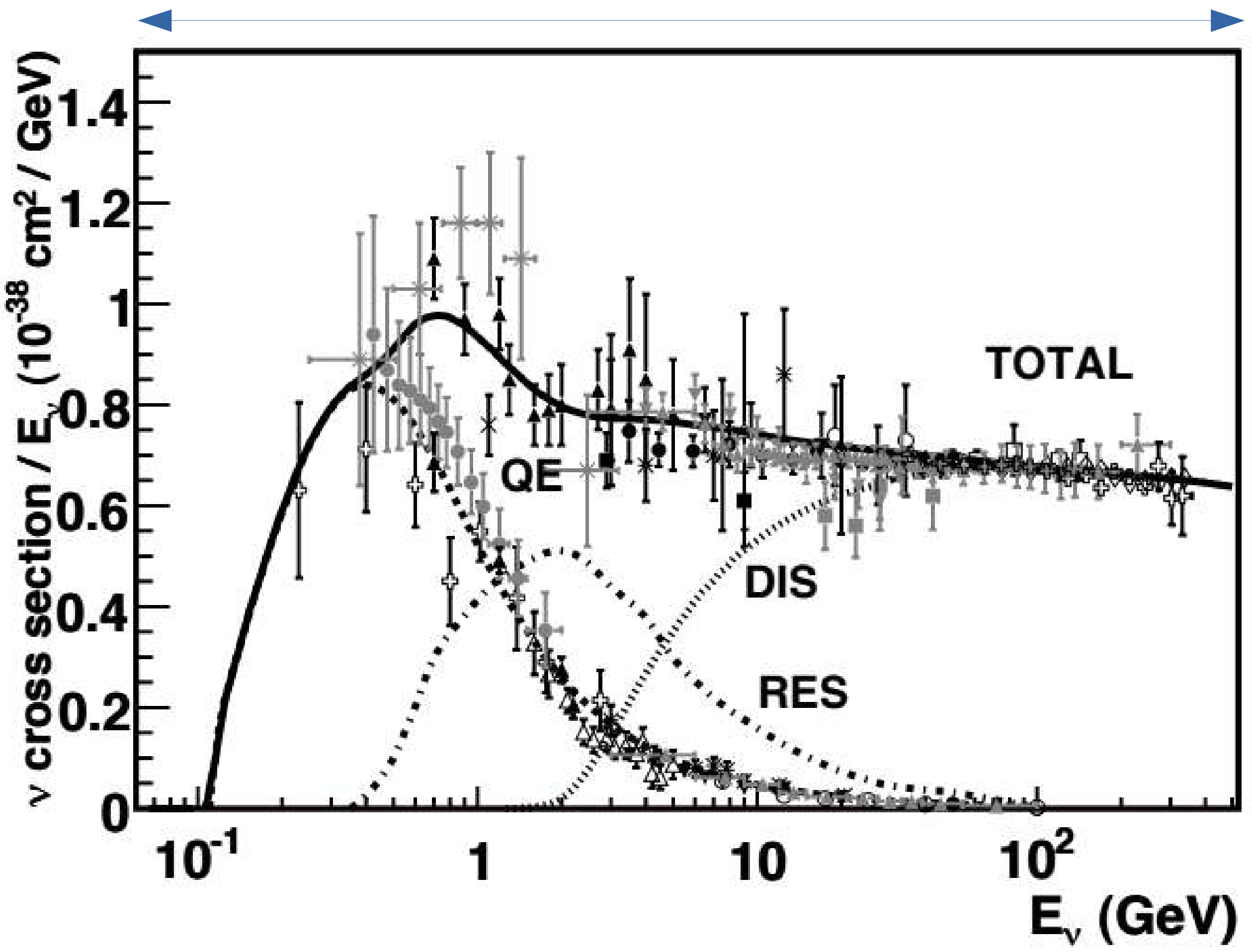
$$\sigma_{\gamma p} \sim 10^{-29}\ \text{cm}^2$$

$$\sigma_{\nu p} \sim 10^{-38}\ \text{cm}^2$$



Particle Data Group

Accelerator experiments



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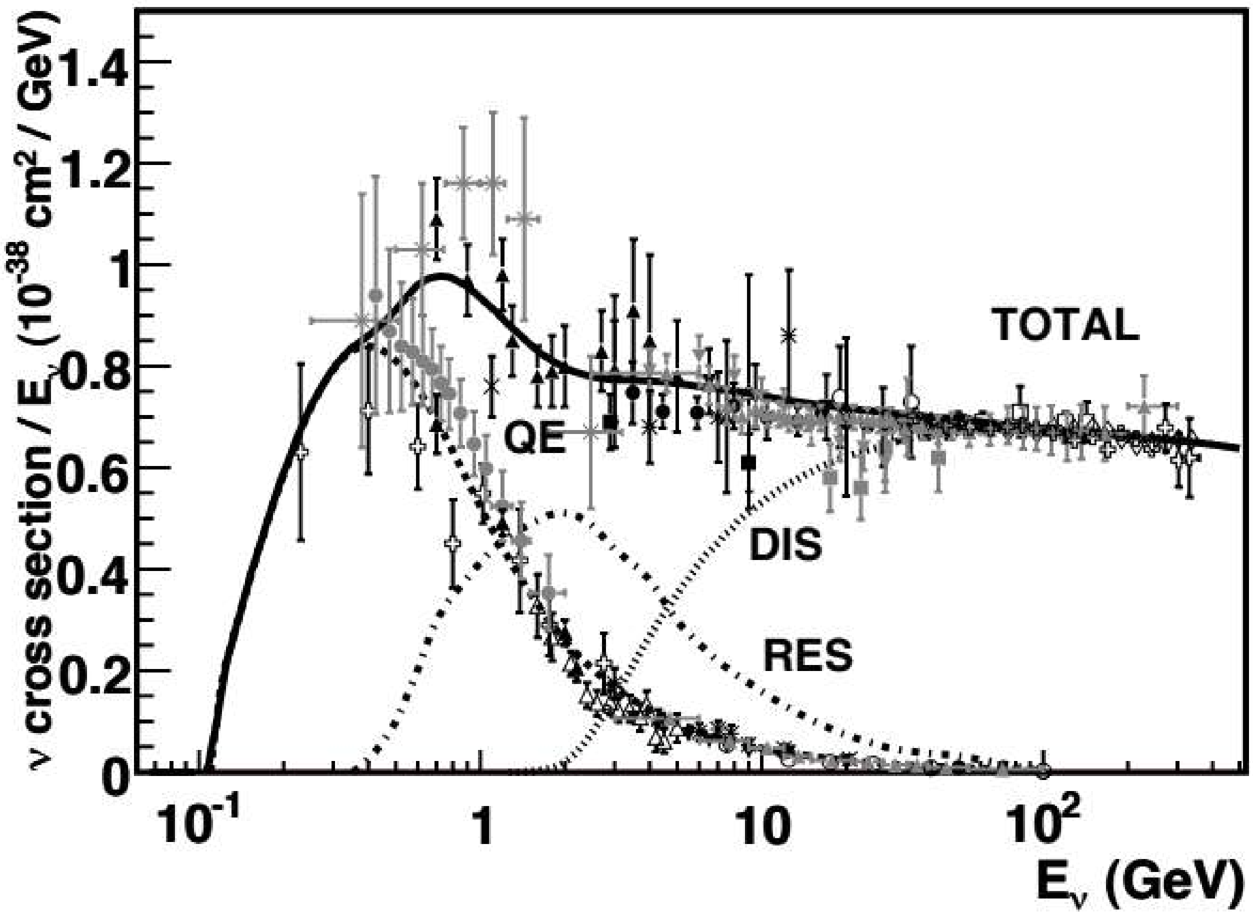

One recent measurement (COHERENT)

Particle Data Group

Accelerator experiments

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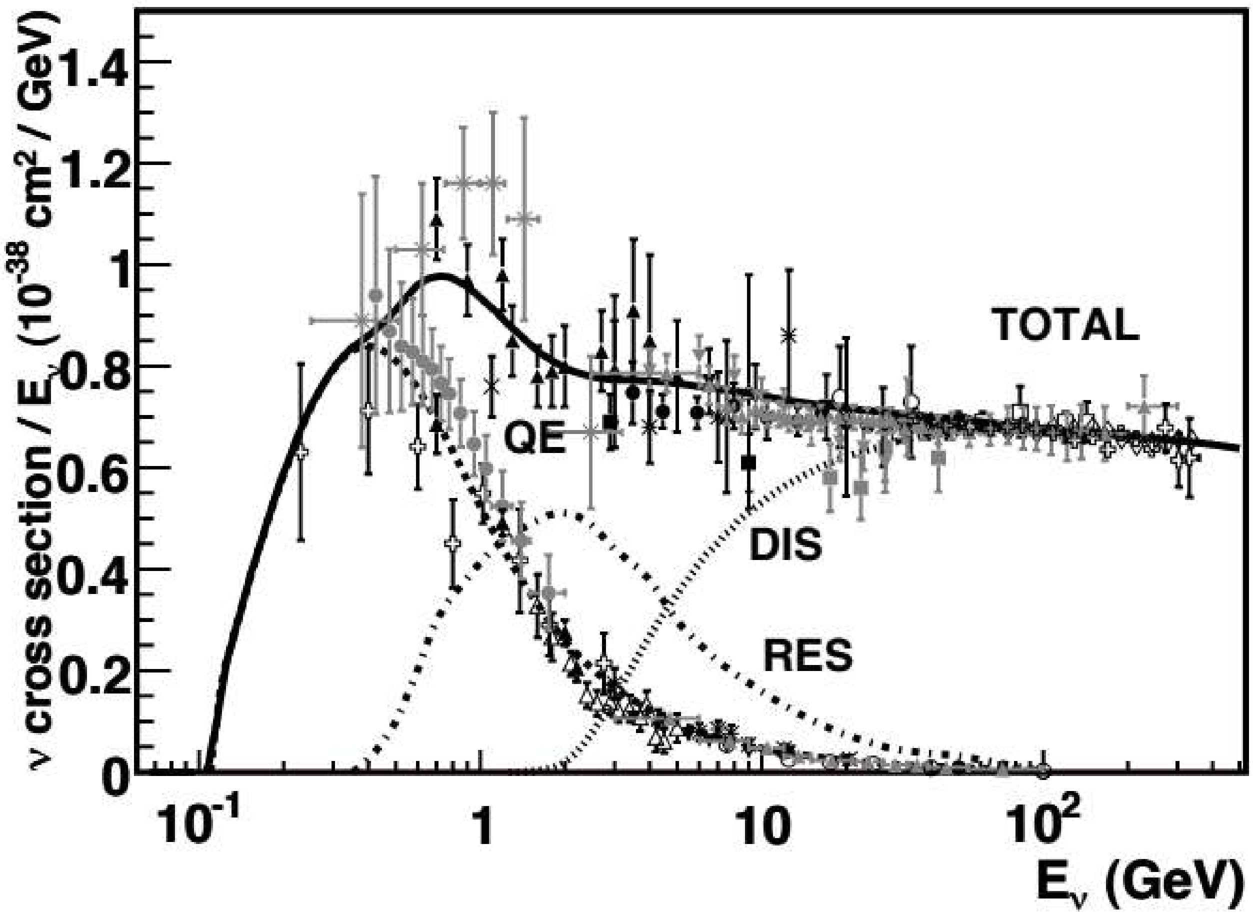


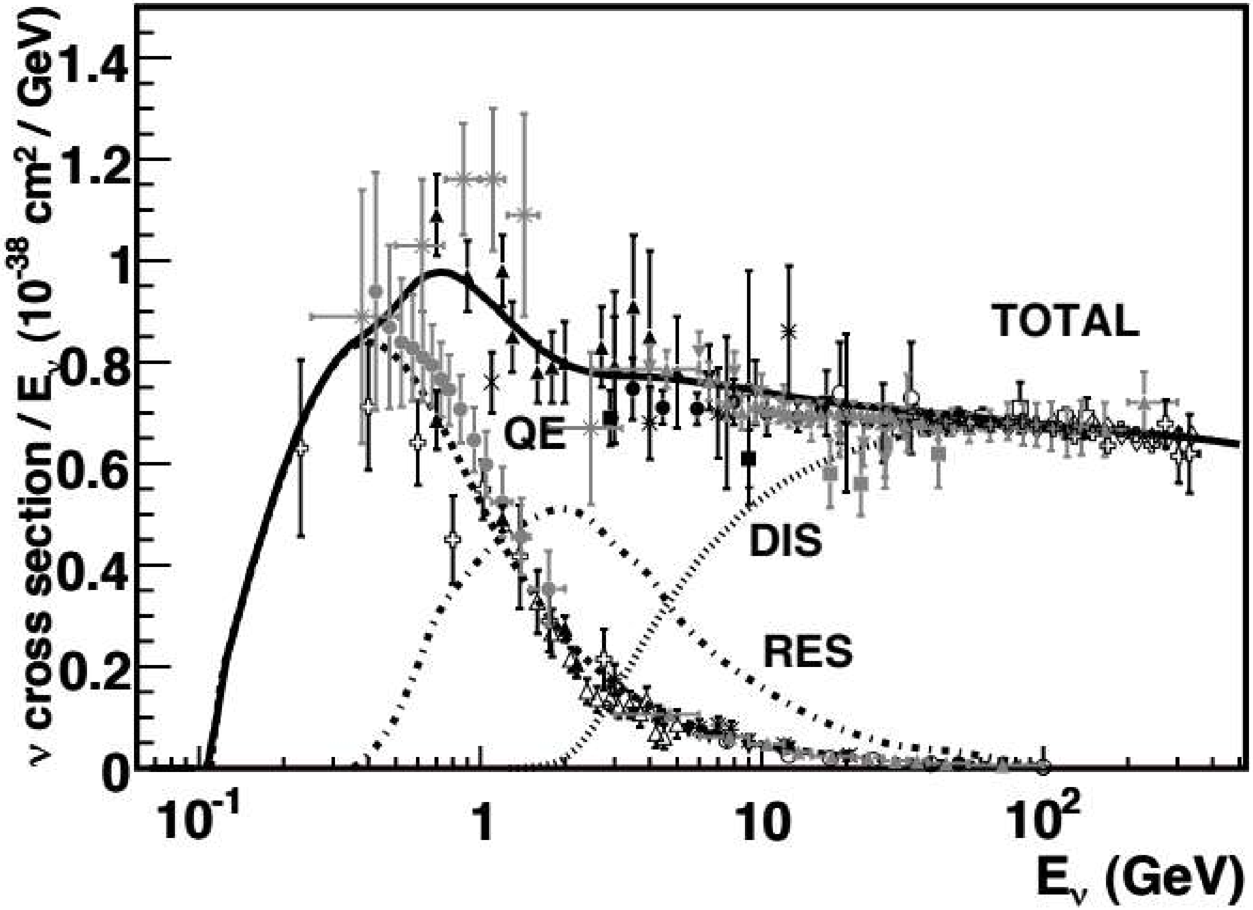
No measurements

… until now!

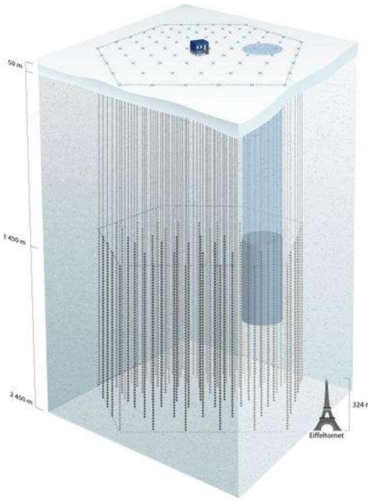
Particle Data Group

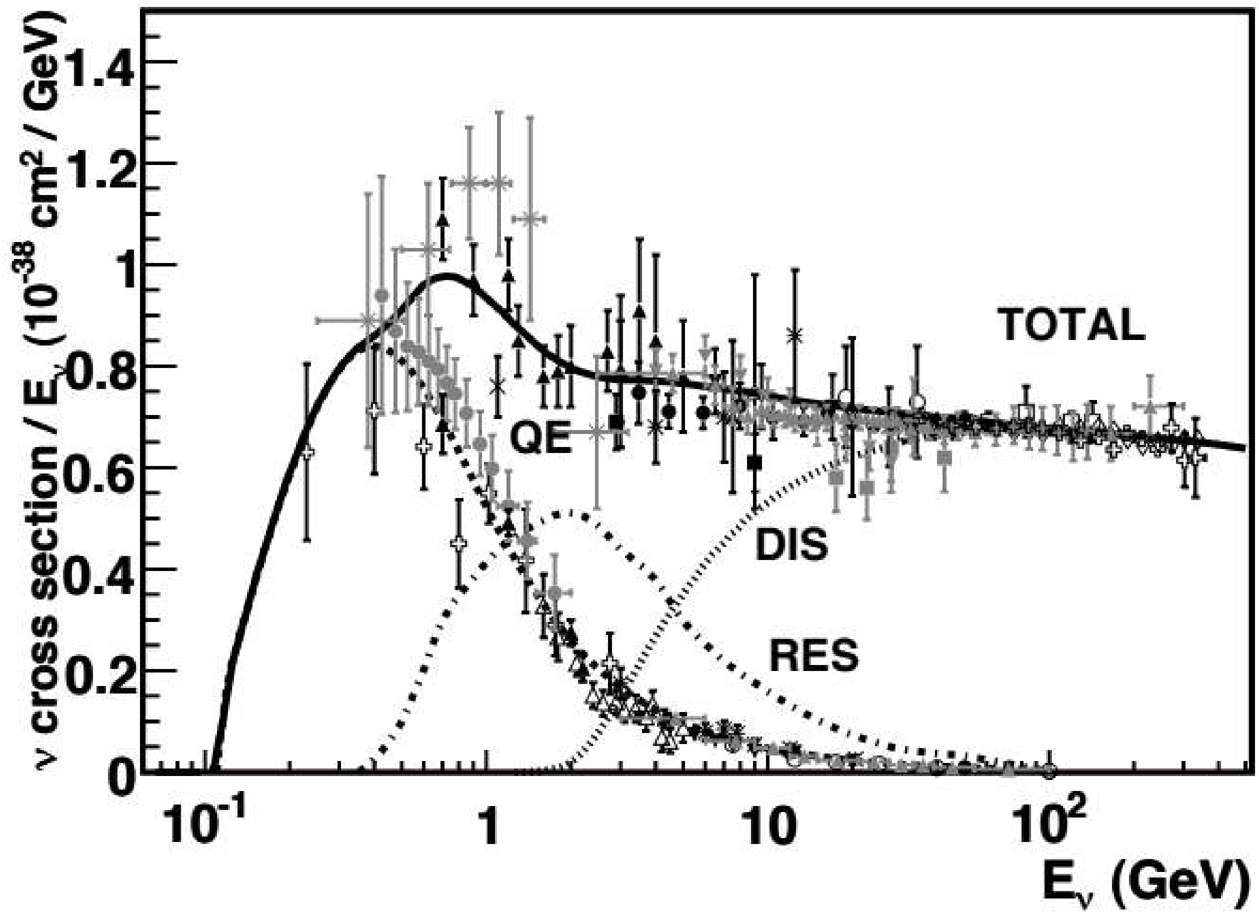
Accelerator experiments

One recent measurement (COHERENT)



Particle Data Group



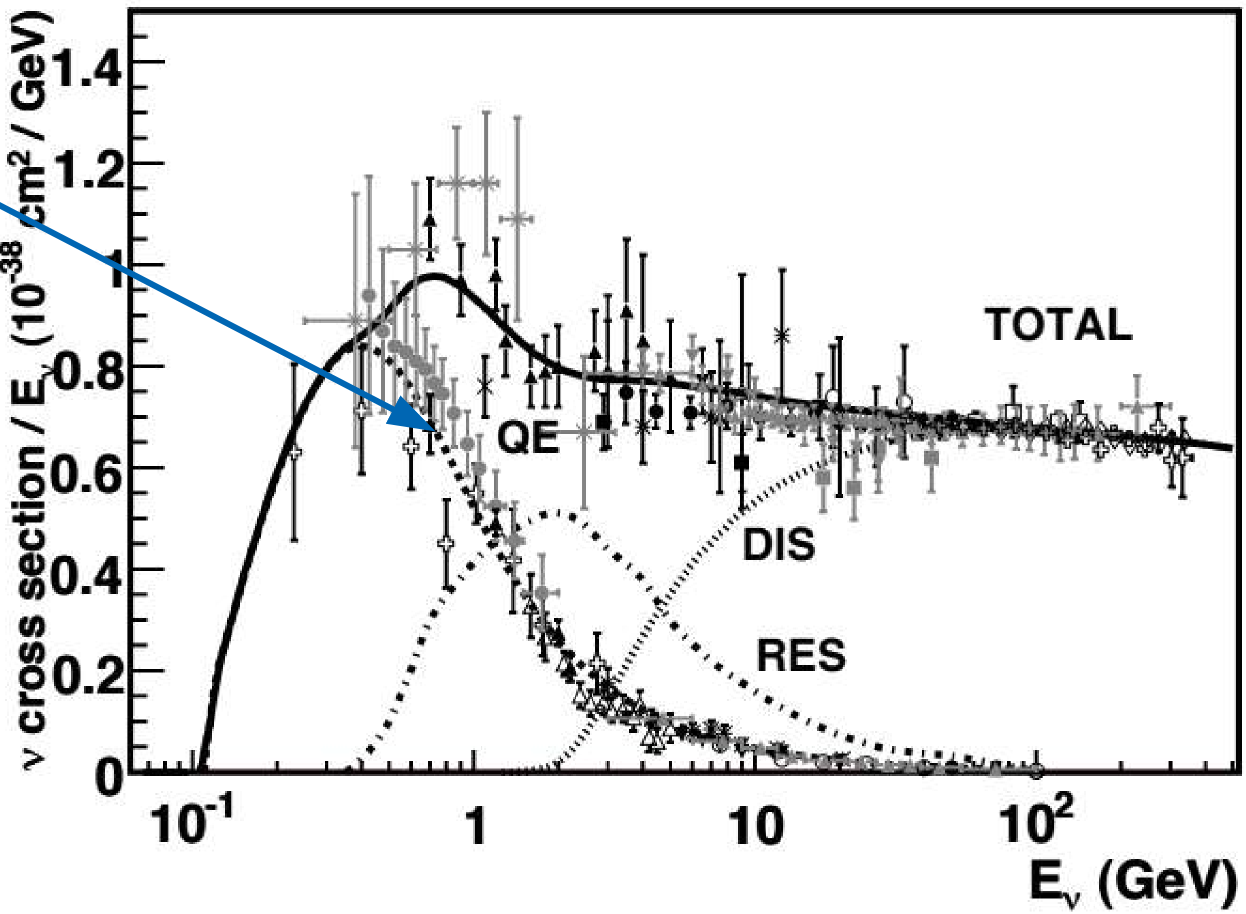


Particle Data Group

Quasi-elastic scattering:

$\nu_l + n \rightarrow l^- + p$

$\bar{\nu}_l + p \rightarrow l^+ + n$



Particle Data Group

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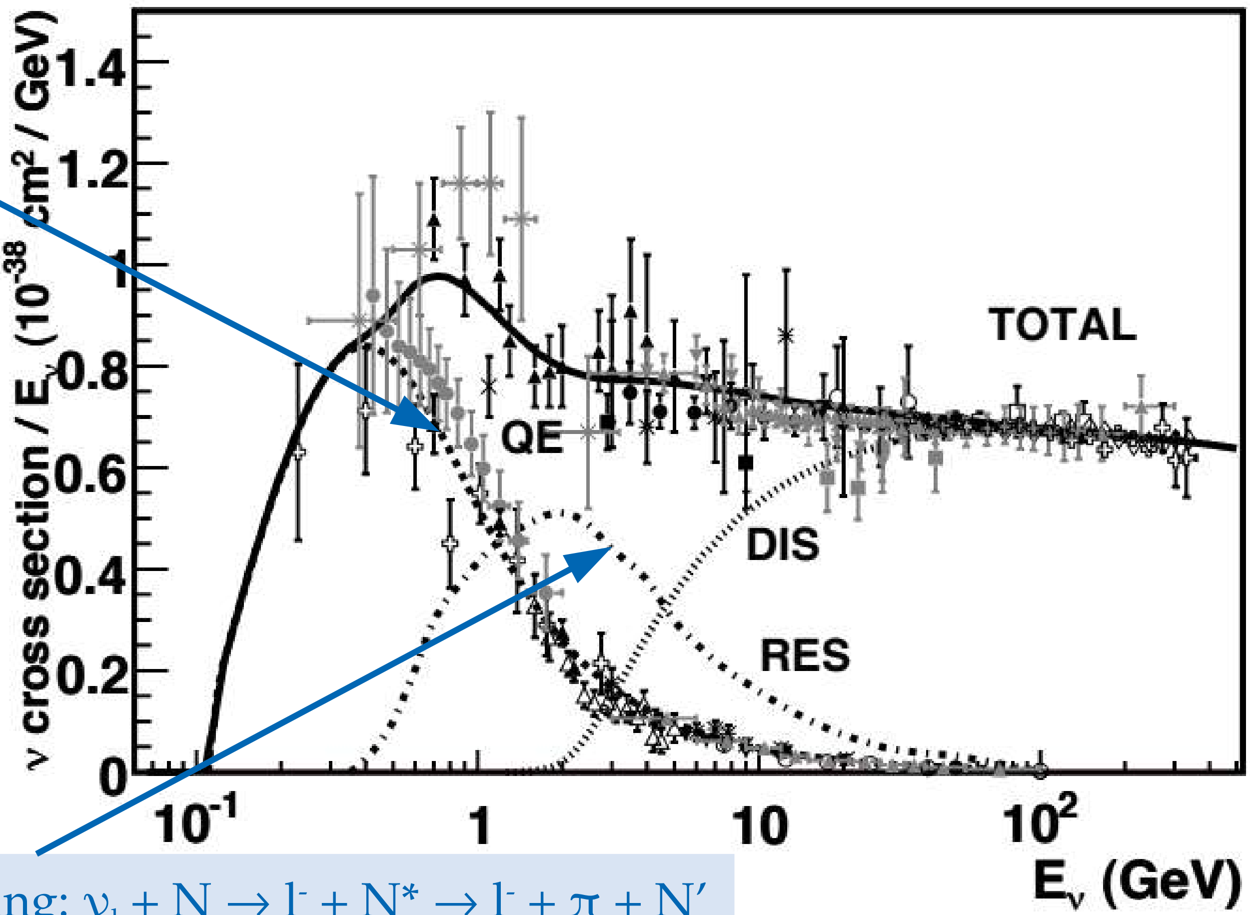

Resonant scattering: $\nu_l + N \rightarrow l^- + N^* \rightarrow l^- + \pi + N'$

Particle Data Group

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$\nu_l + n \rightarrow l^- + p$

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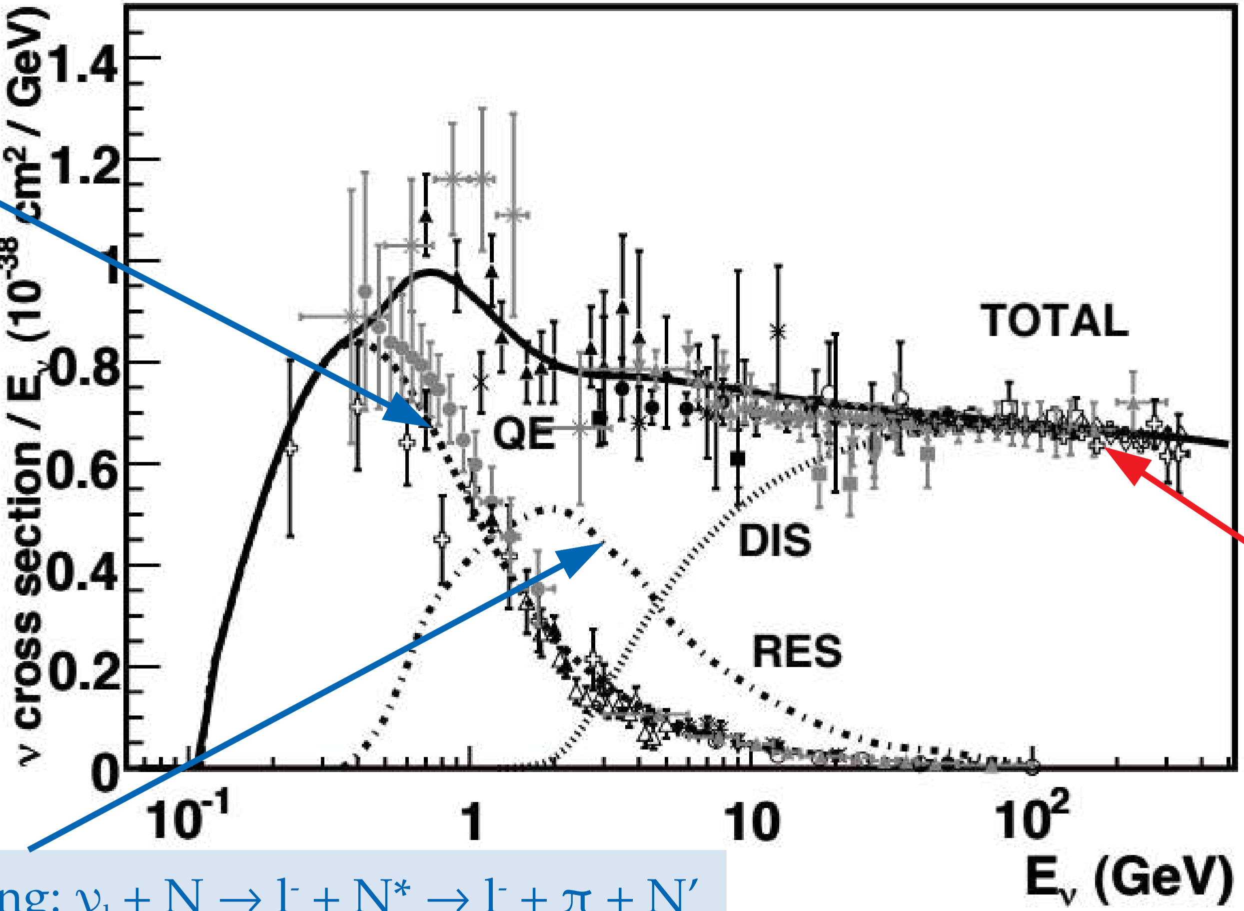


Deep inelastic scattering:

$\nu_l + N \rightarrow l^- + X$

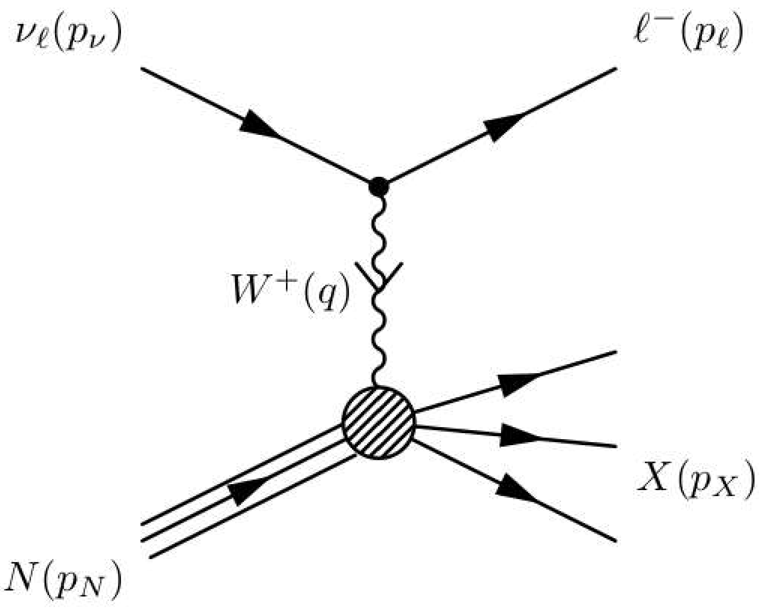
$\bar{\nu}_l + N \rightarrow l^+ + X$

Resonant scattering: $\nu_l + N \rightarrow l^- + N^* \rightarrow l^- + \pi + N'$

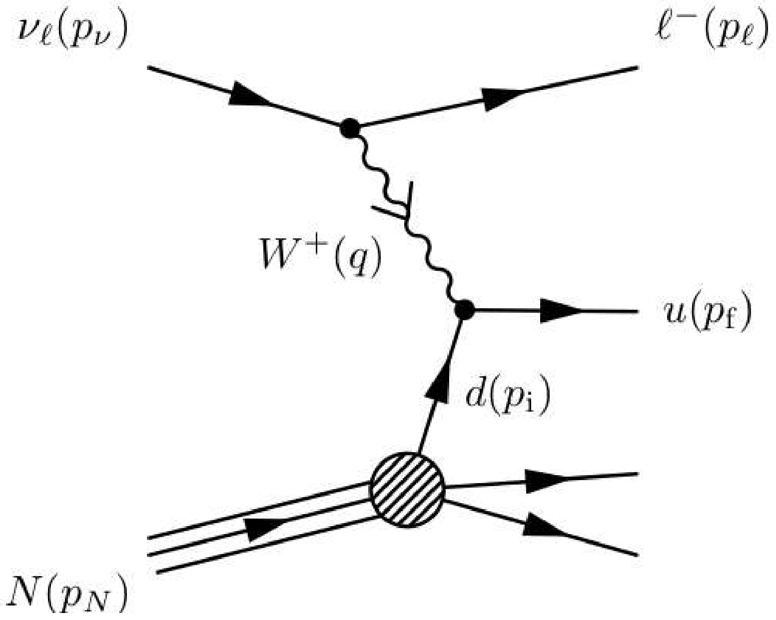
Particle Data Group

How does DIS probe nucleon structure?

What you see

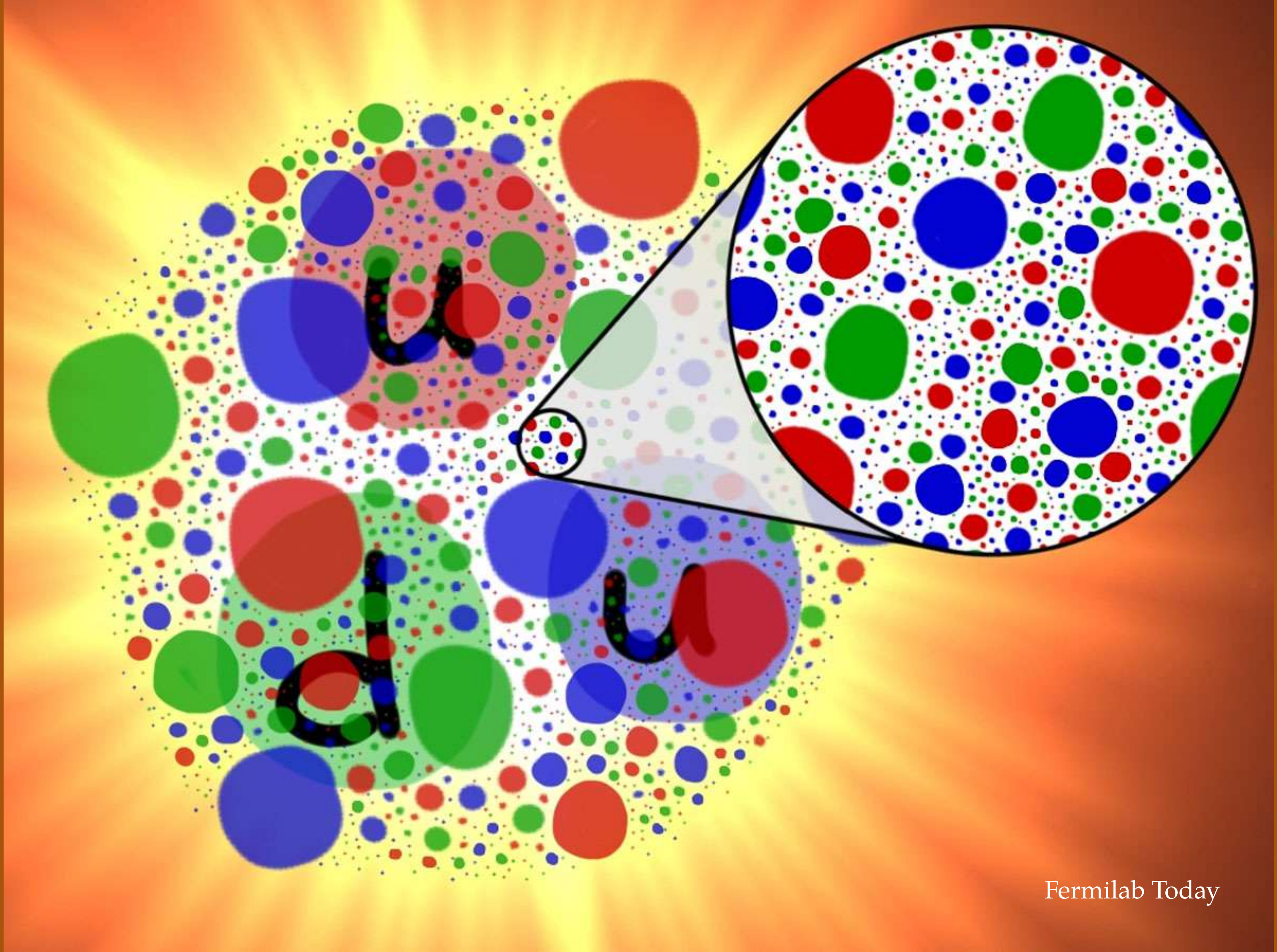


Beneath the hood



(Plus the equivalent neutral-current process (Z-exchange))

Giunti & Kim, *Fundamentals of Neutrino Physics & Astrophysics*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Fermilab Today

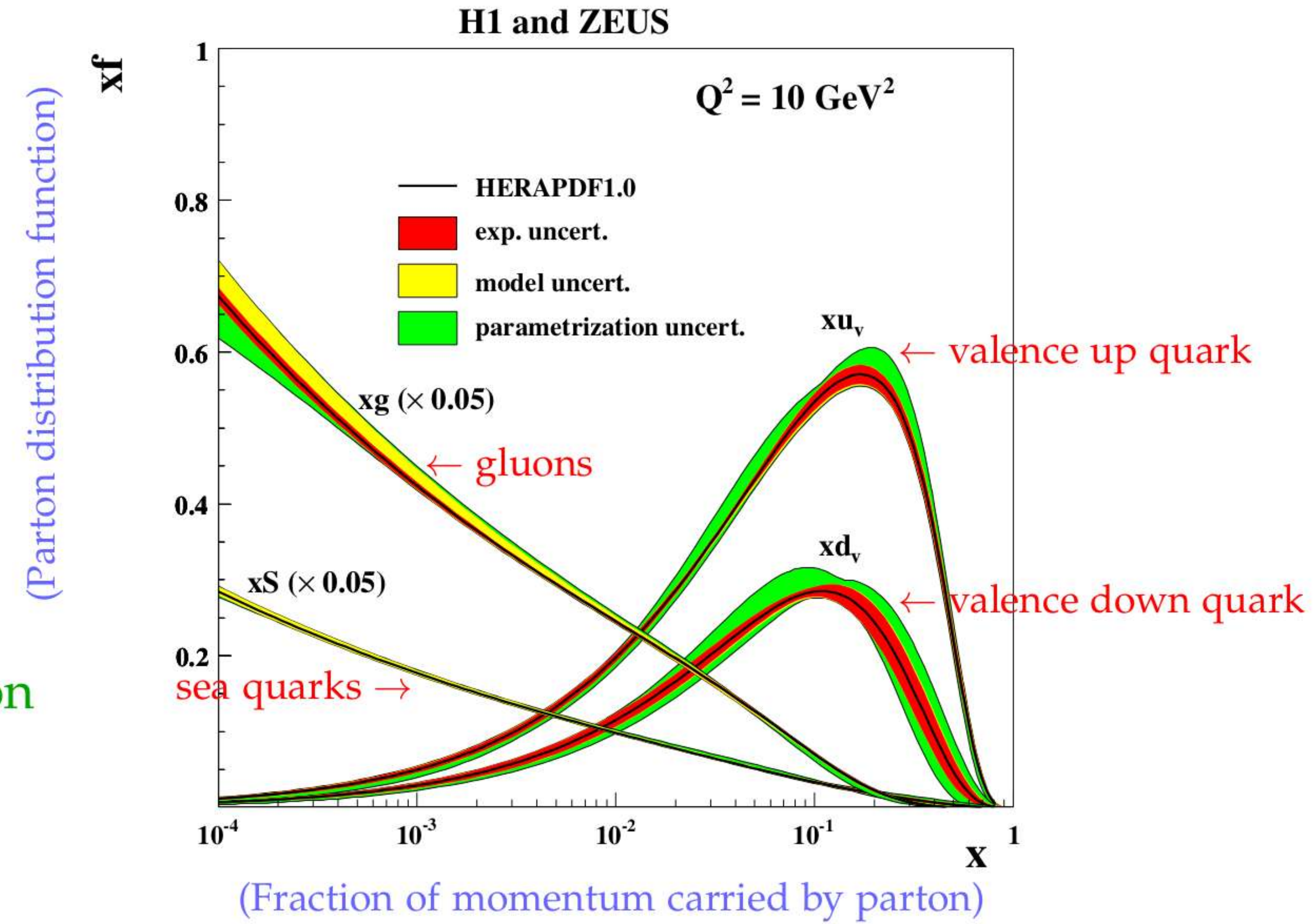
Peeking inside a proton

$$x \sim \frac{m_W}{E_\nu}$$

$\sim 10^{-4}$ for E_ν = PeV

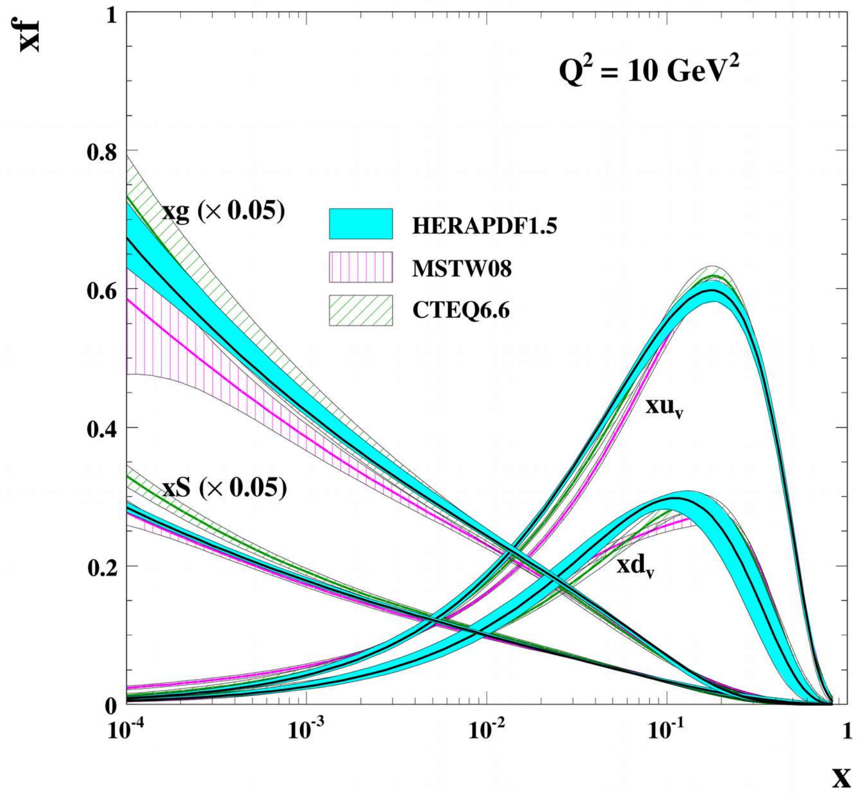
← Extrapolation

A. Cooper-Sarkar 2012

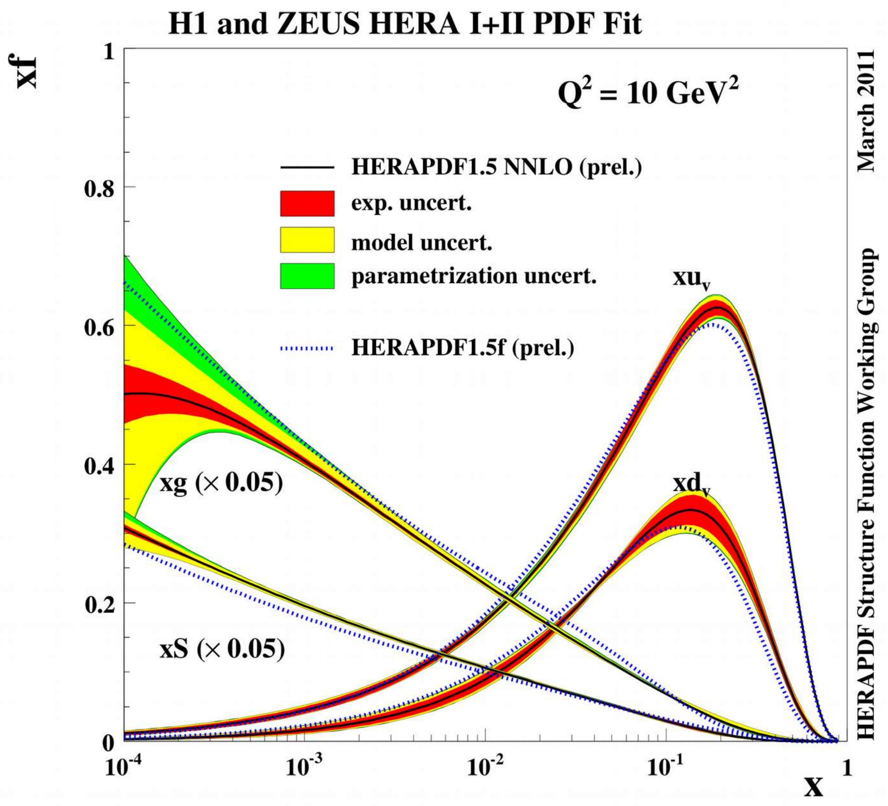


The world of PDFs is complex

Different fitting groups



Different QCD prescriptions



A. Cooper-Sarkar 2012

Extrapolating the cross section to high energies

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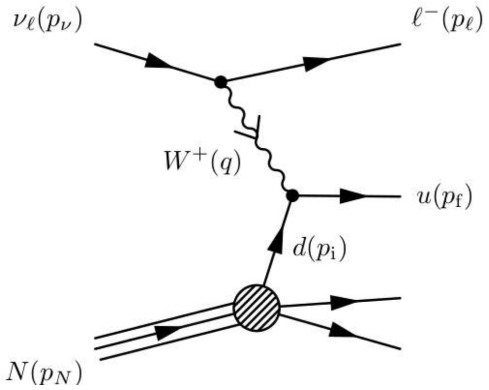
SM



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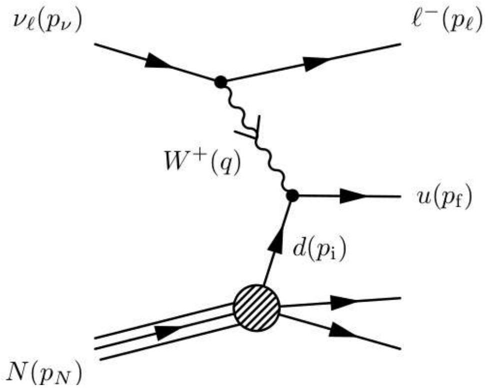

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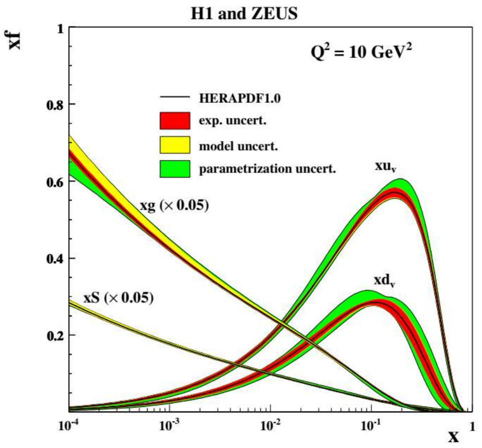
PDFs



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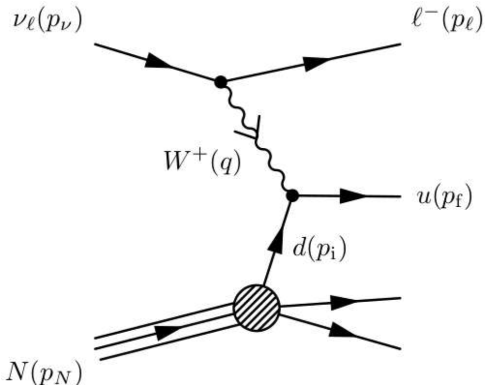

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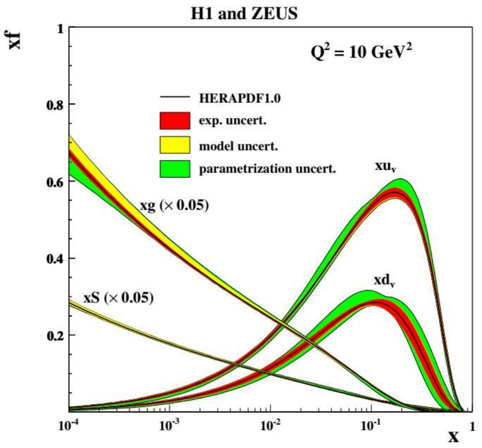

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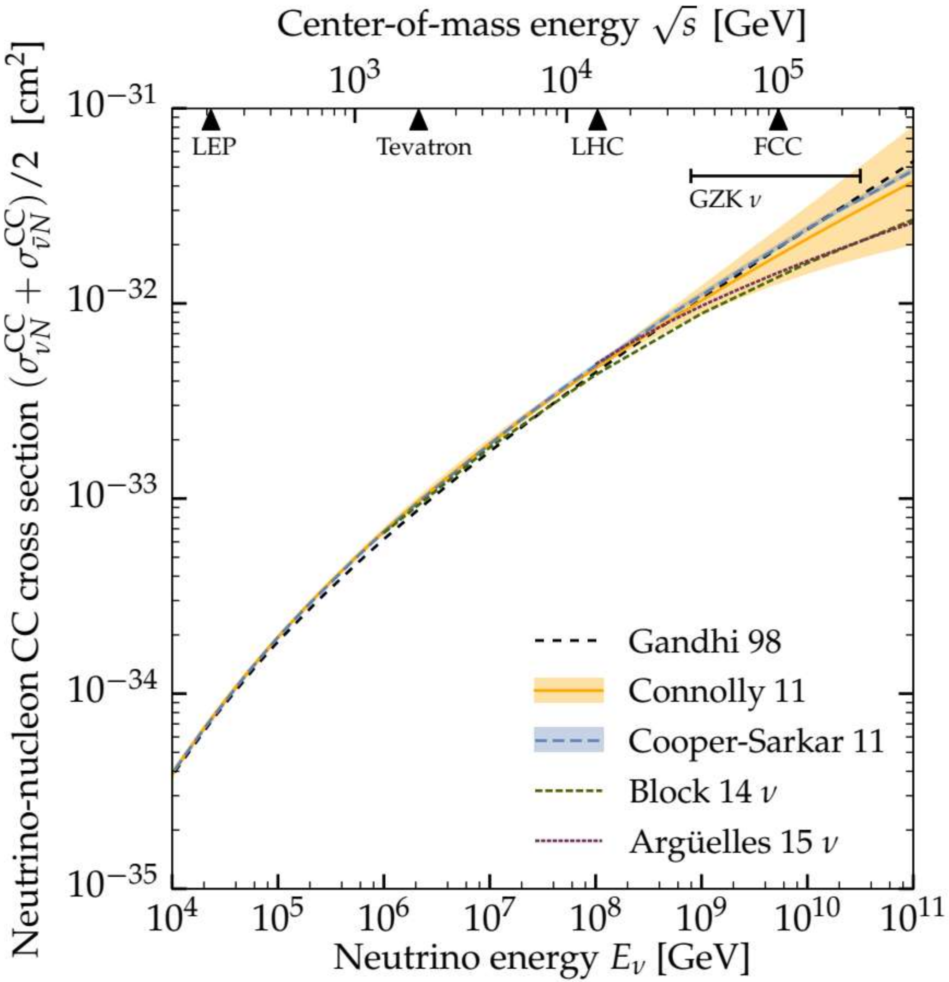
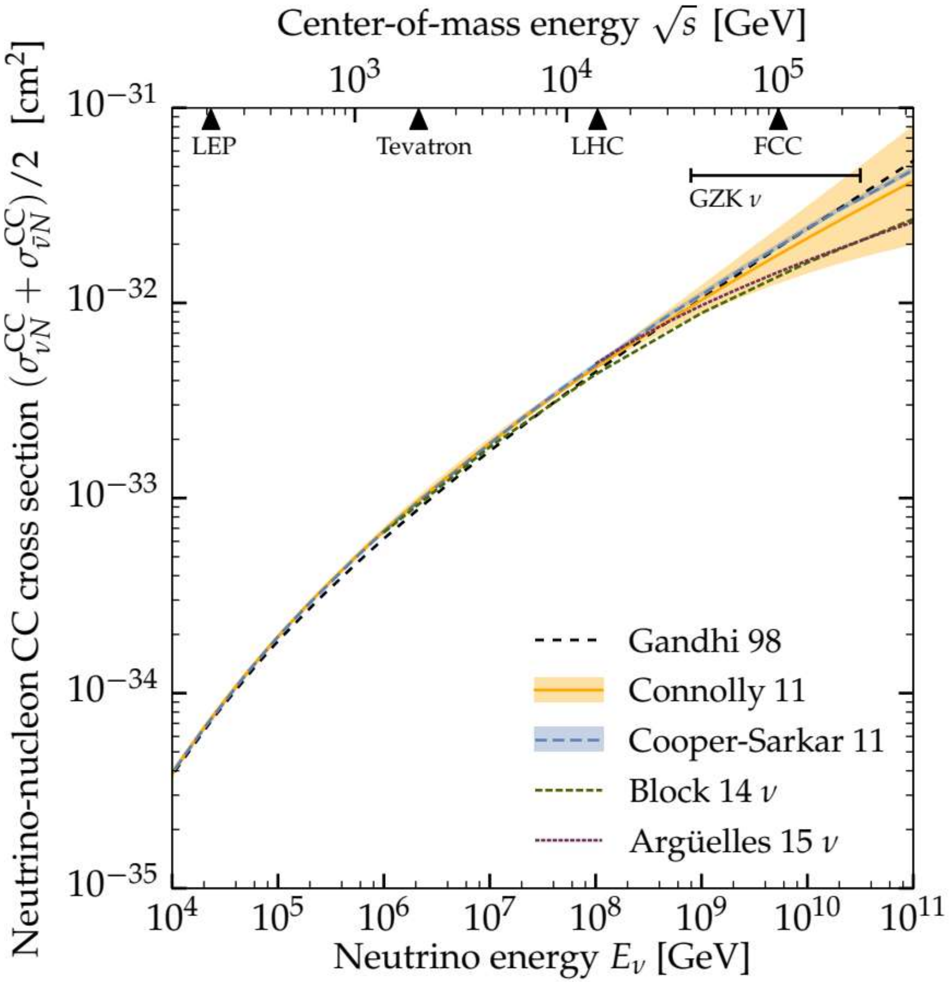


What can we measure *now* and later?



MB & A. Connolly, 1711.11043

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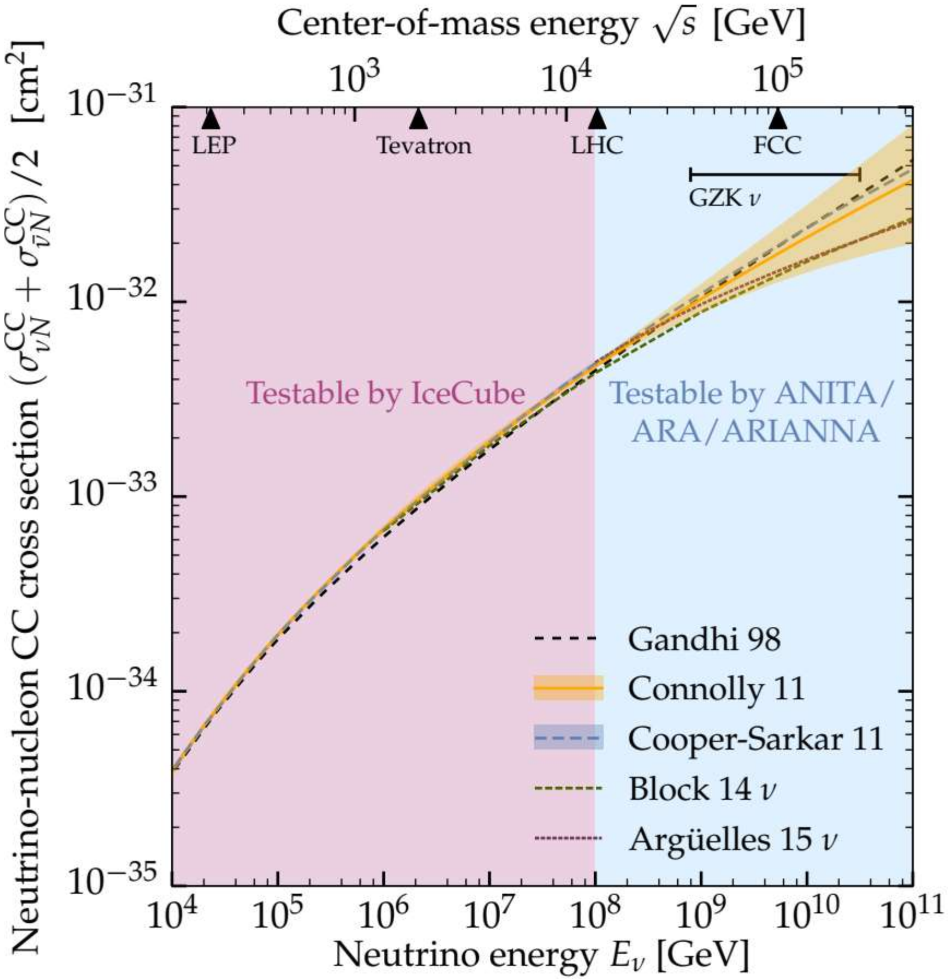

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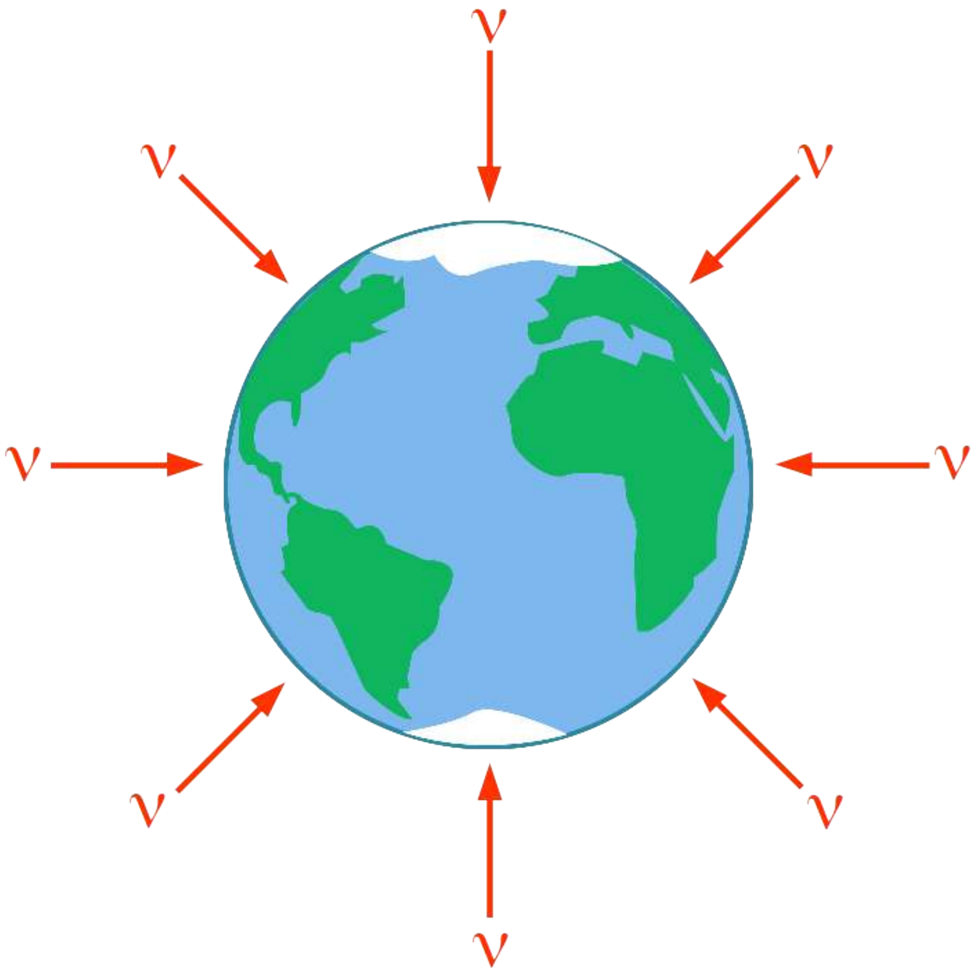
Neutrino, interrupted



Marfatia, McKay, Weiler, *PLB* 2015
Connolly, Thorne, Waters, *PRD* 2011
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Borriello *et al.*, *PRD* 2008
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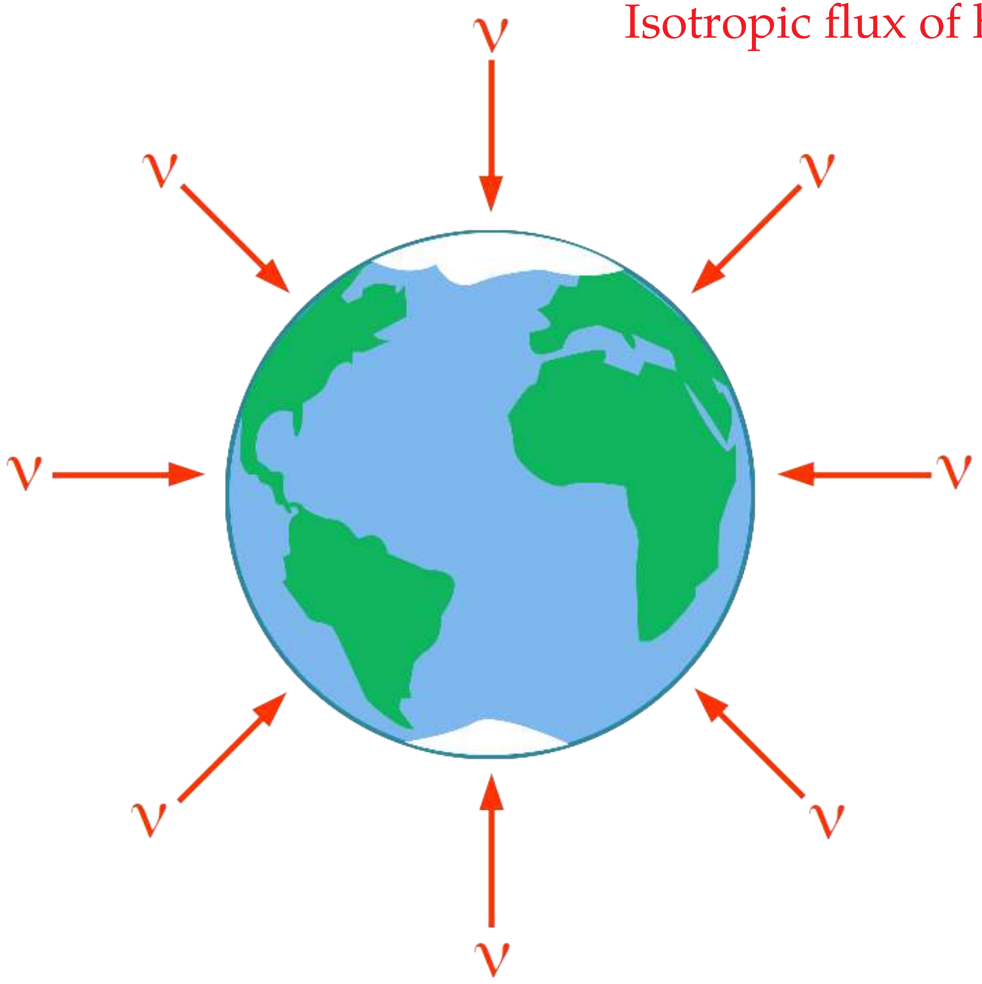
Neutrino, interrupted

Isotropic flux of high-energy neutrinos



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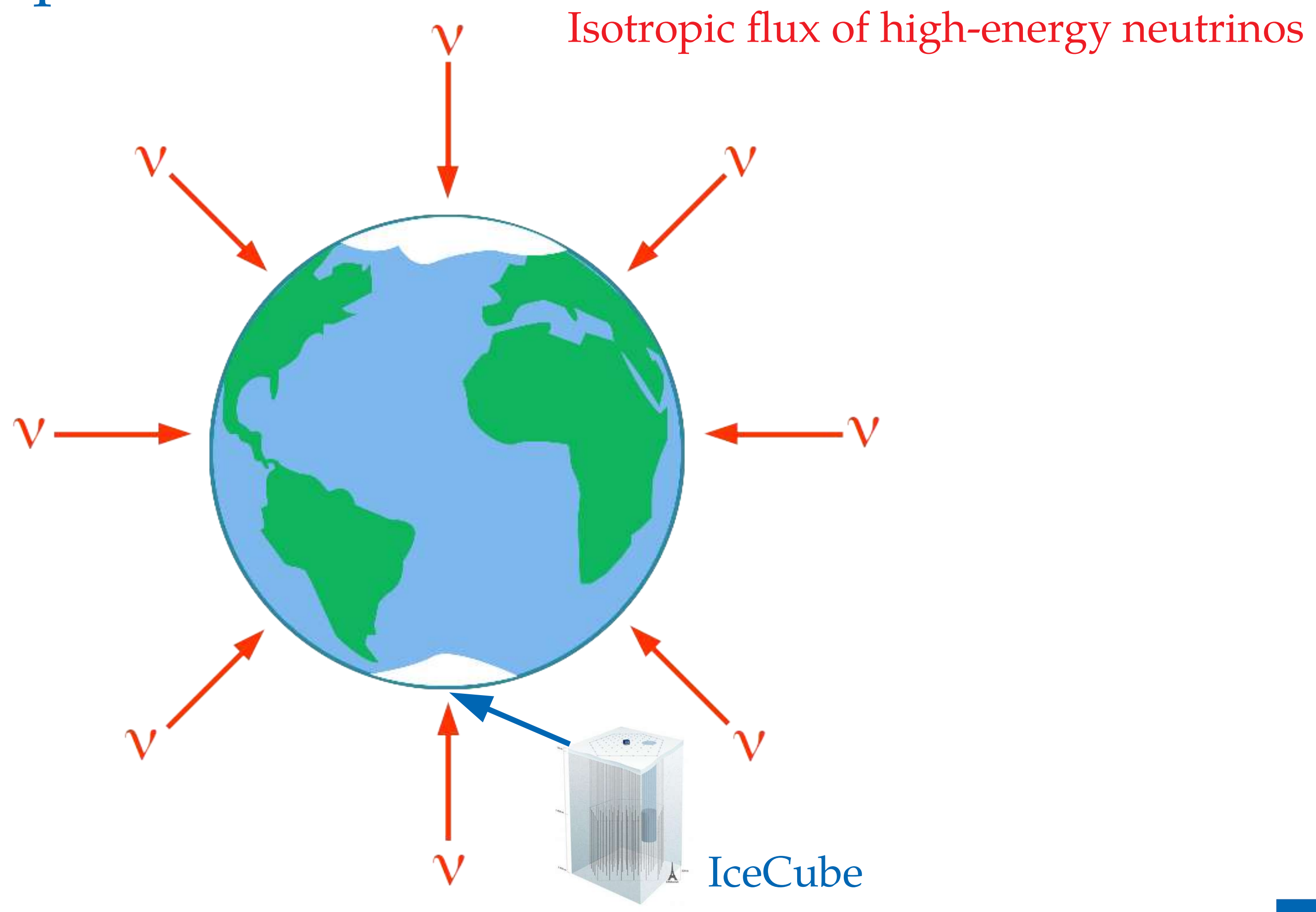

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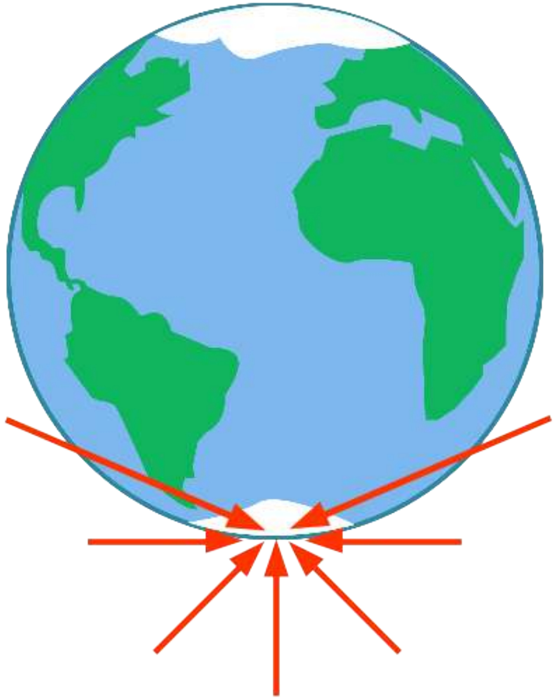

Most of these neutrinos reach IceCube

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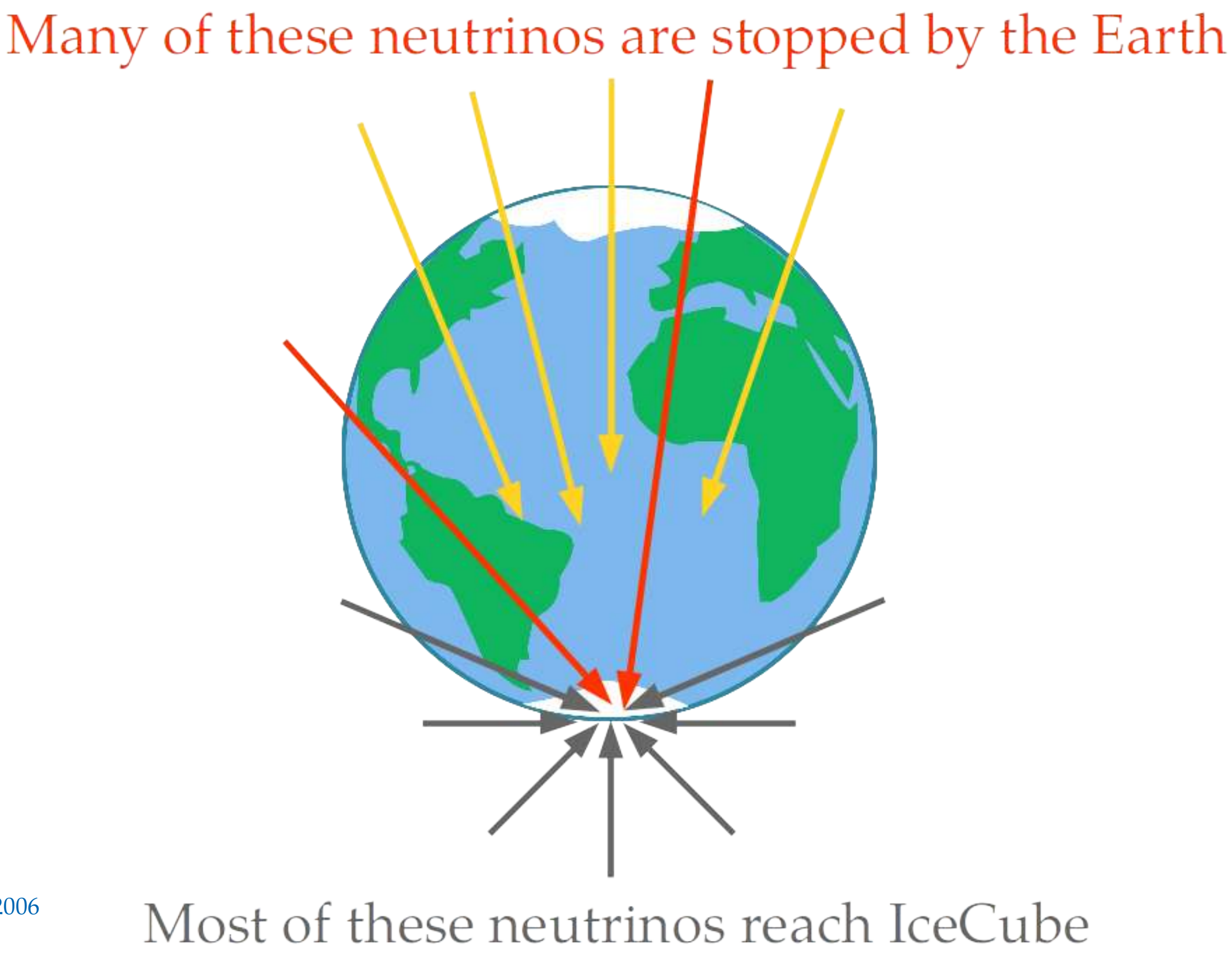

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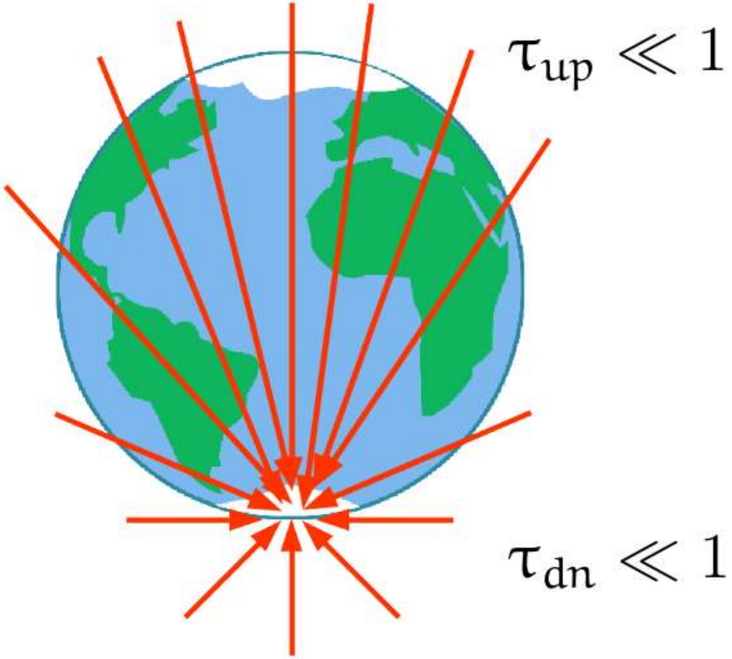
Measuring the high-energy cross section

$$\text{Optical depth to } \nu\text{N int's} = \frac{\text{Distance from Earth's surface to IceCube}}{\text{Mean free path inside Earth}} \equiv \tau(E_\nu, \theta_z) \propto \sigma_{\nu N}$$

Below ~ 10 TeV: Earth is transparent



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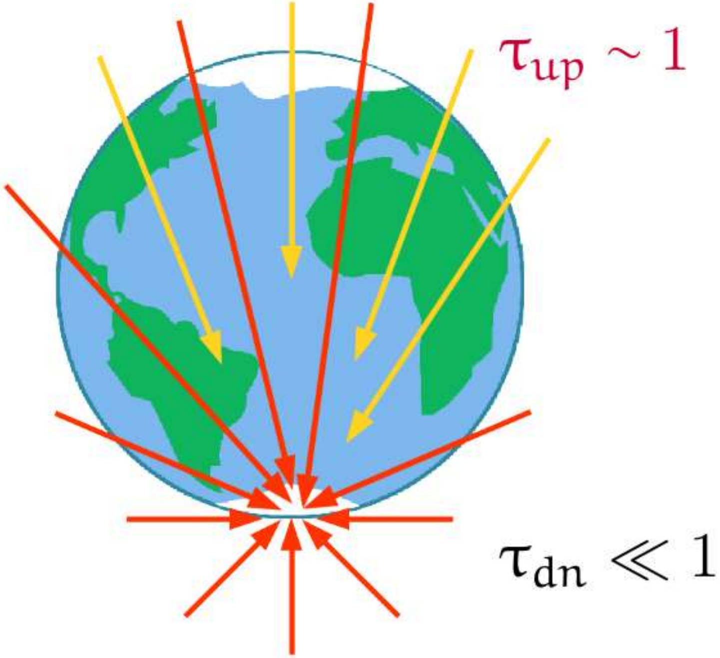

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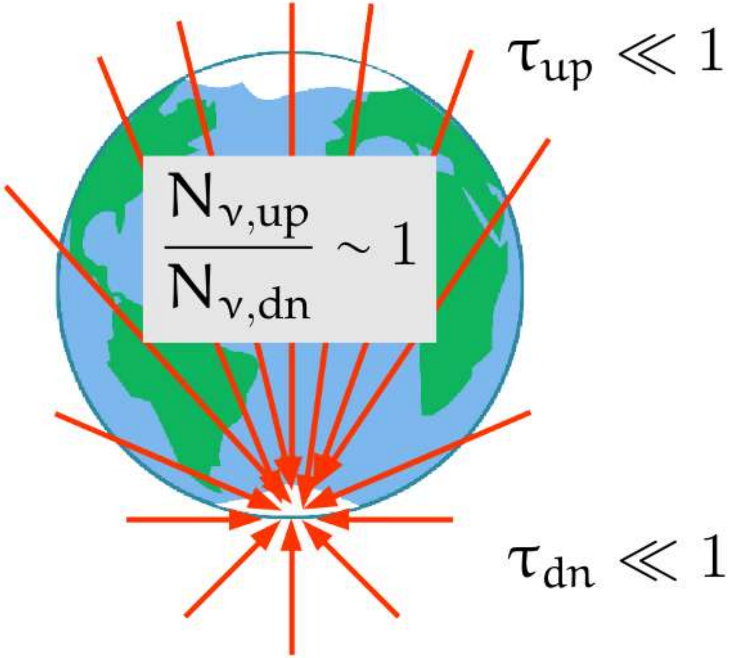

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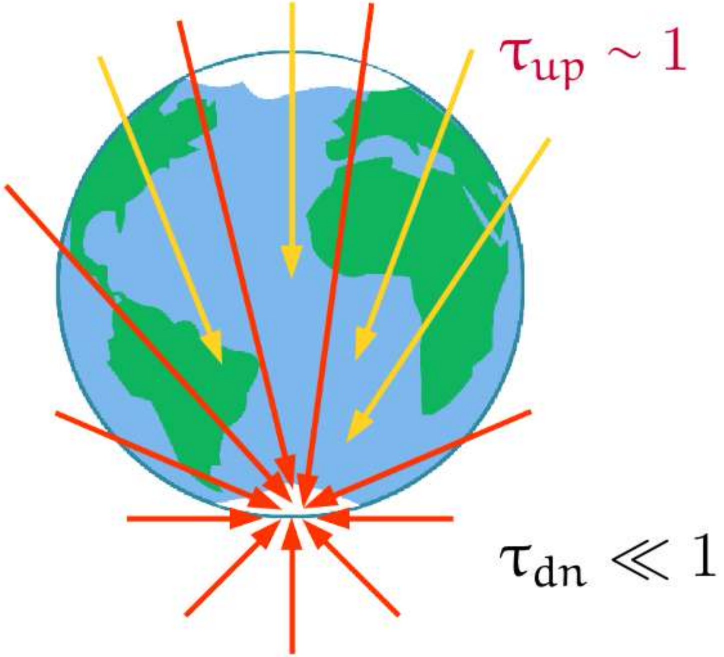

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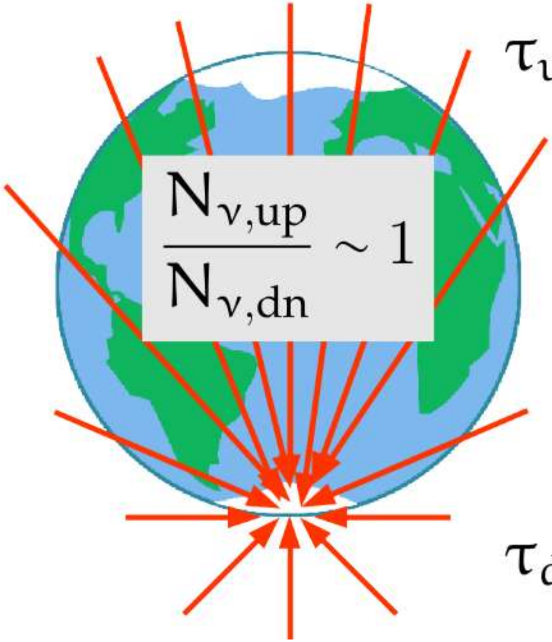

$\tau_{up} \ll 1$

$$\frac{N_{\nu,up}}{N_{\nu,dn}} \sim 1$$

$\tau_{dn} \ll 1$

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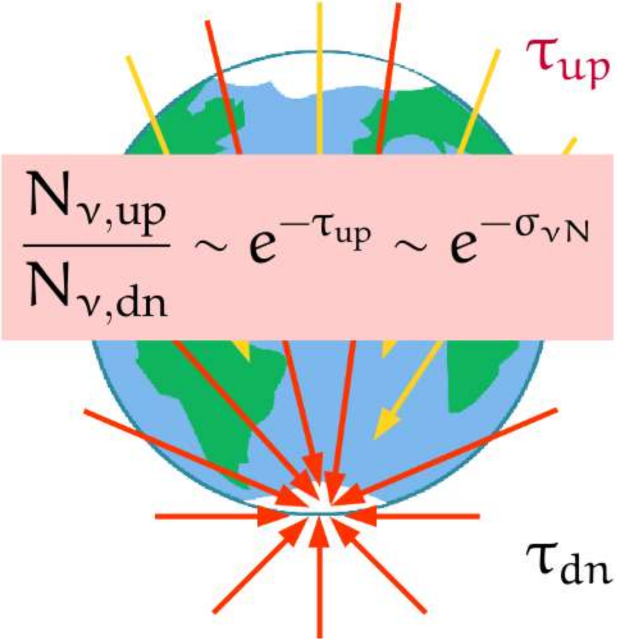


$\tau_{up} \sim 1$

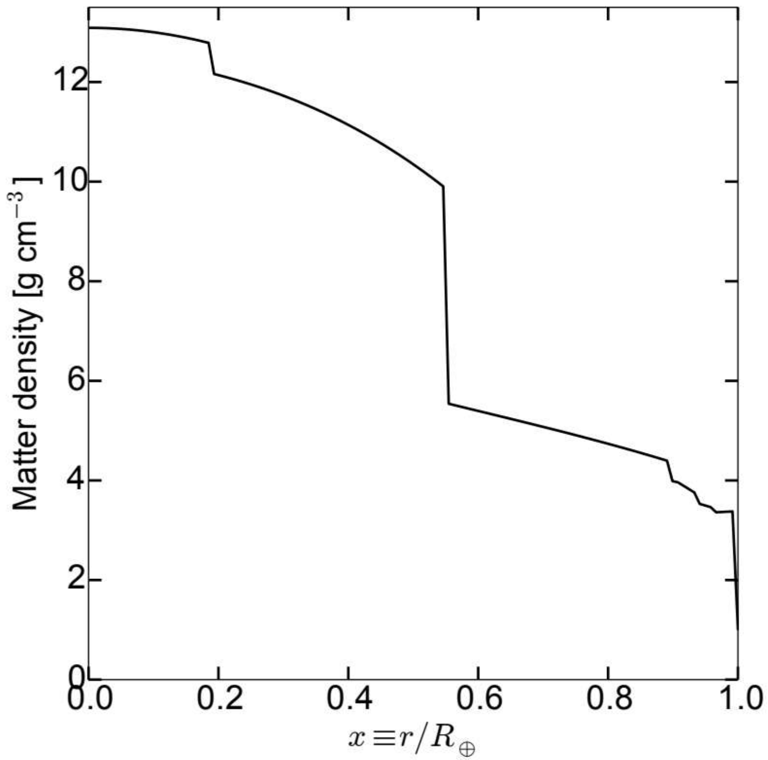
$$\frac{N_{\nu,up}}{N_{\nu,dn}} \sim e^{-\tau_{up}} \sim e^{-\sigma_{\nu N}}$$

$\tau_{dn} \ll 1$

A feel for the in-Earth attenuation

Earth matter density

(Preliminary Reference Earth Model)



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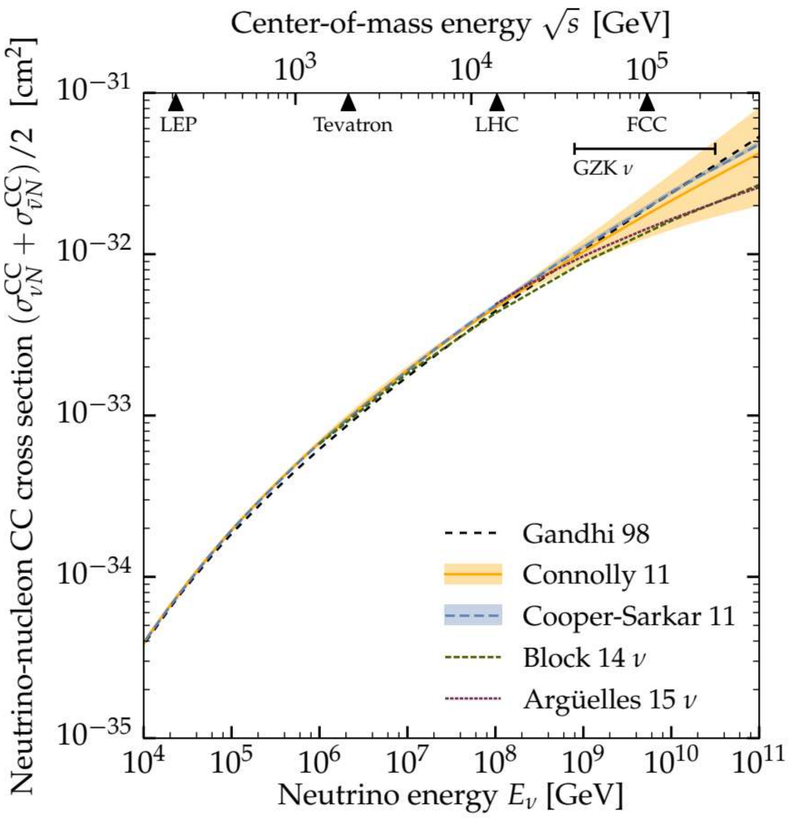
Neutrino-nucleon cross section



A feel for the in-Earth attenuation

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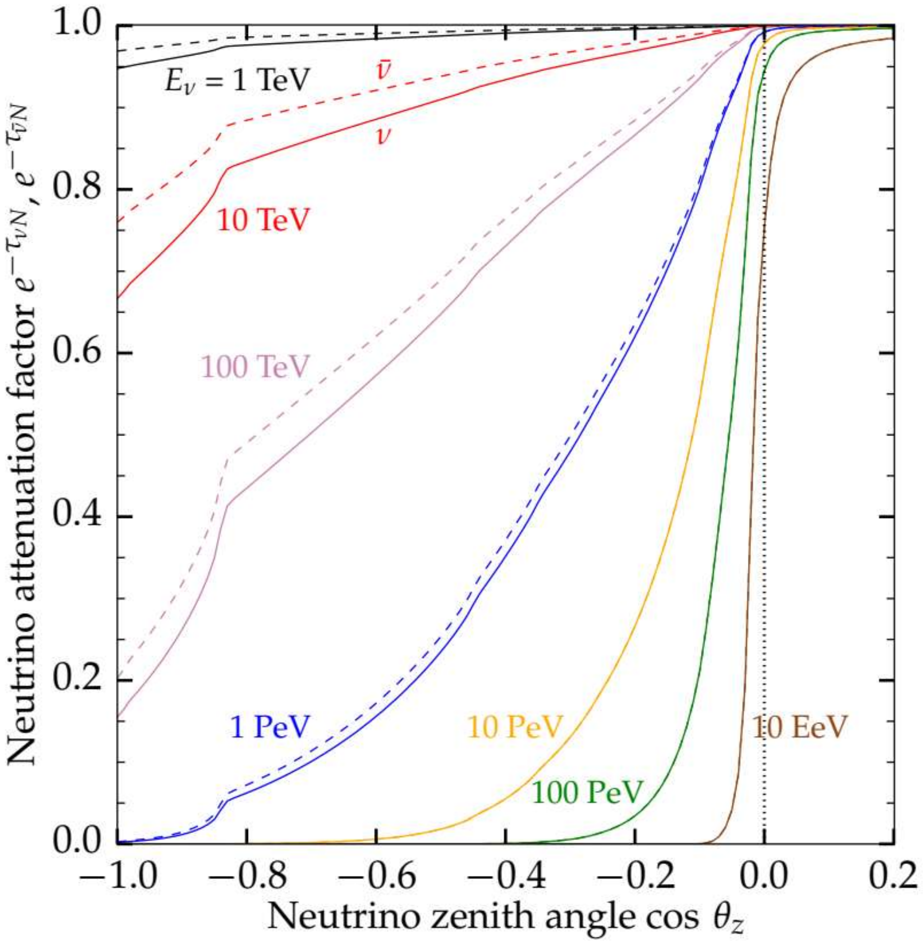
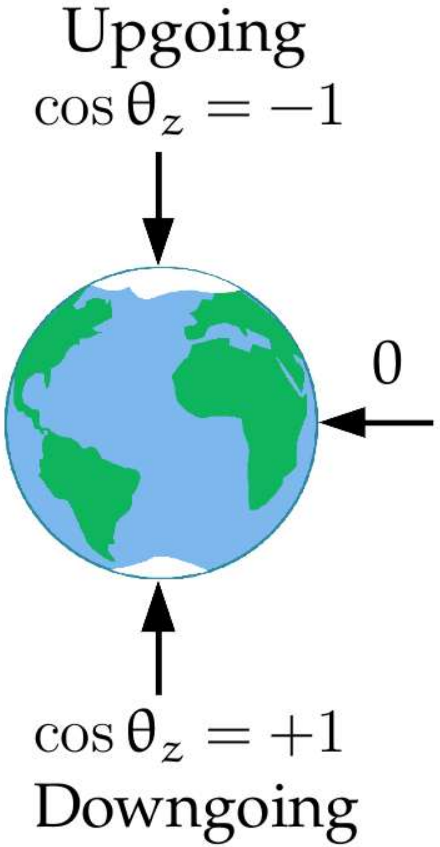
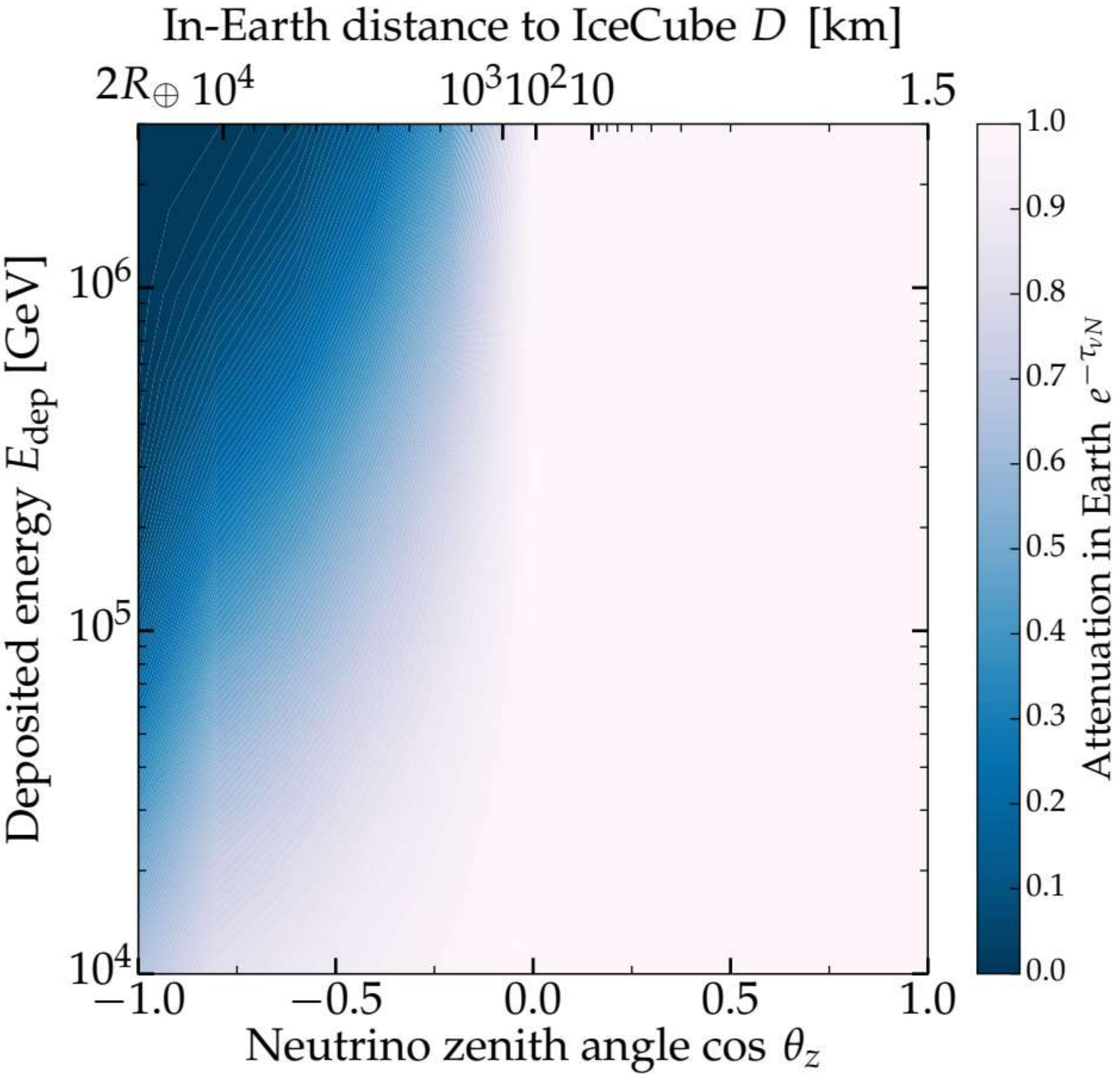
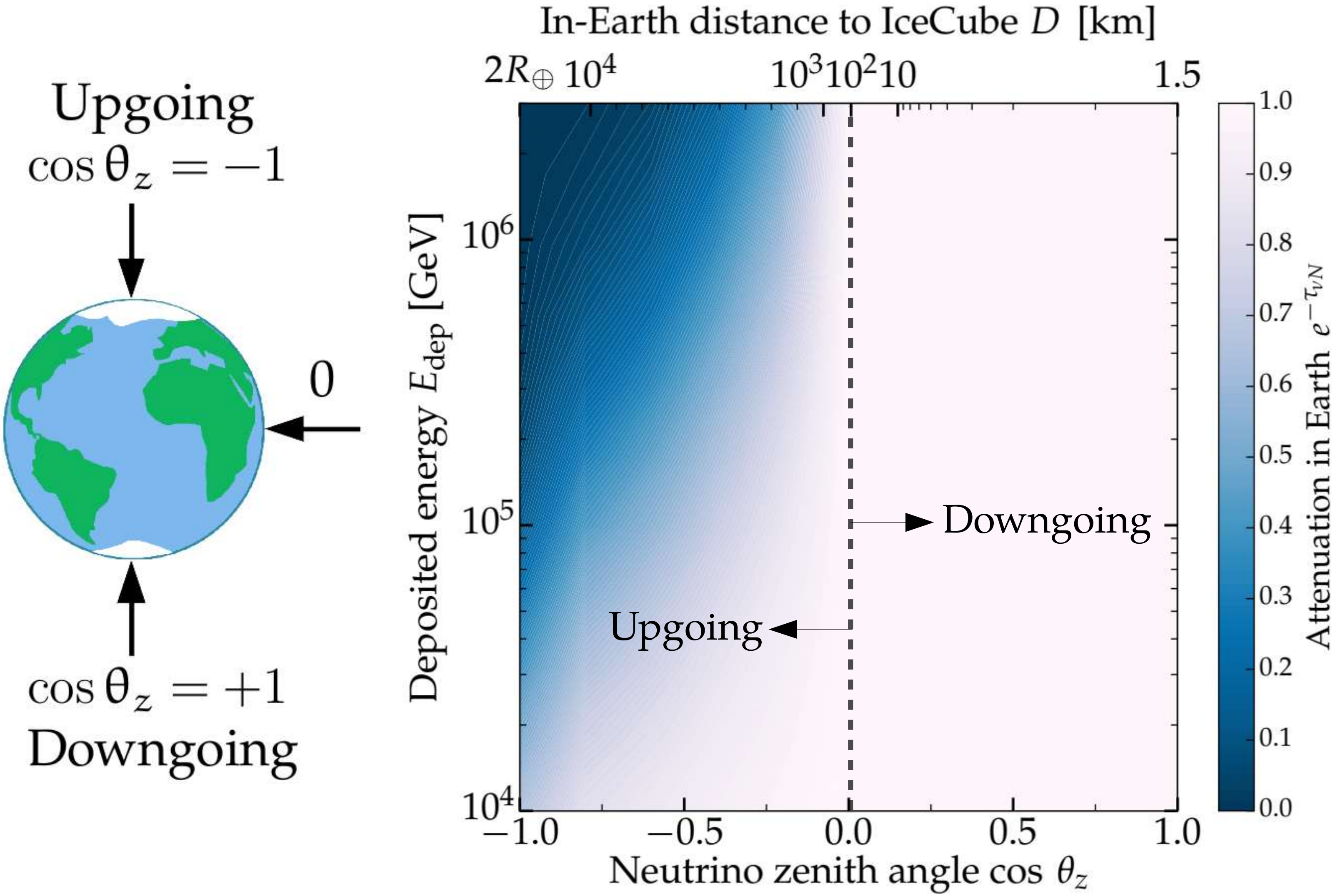
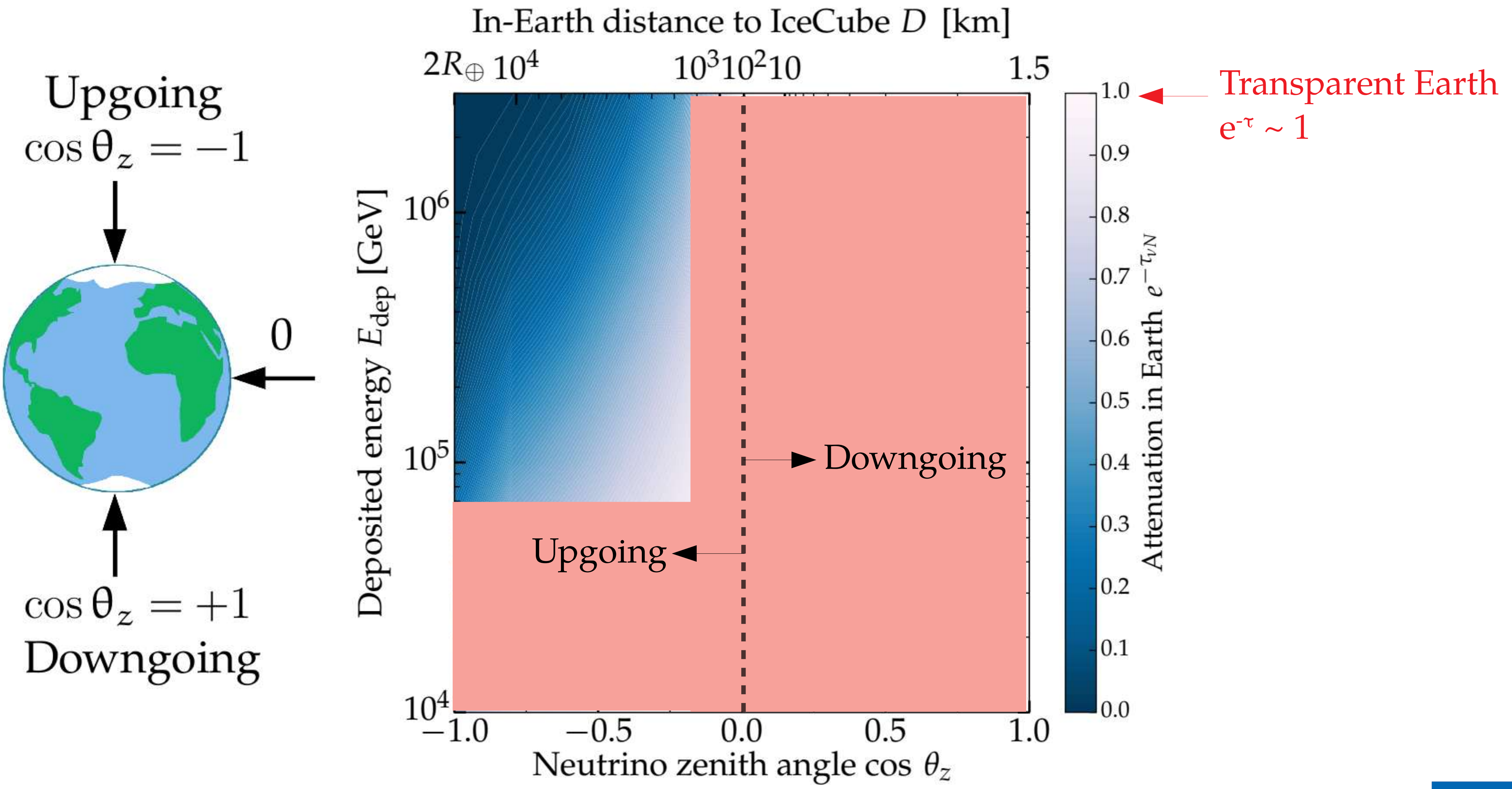


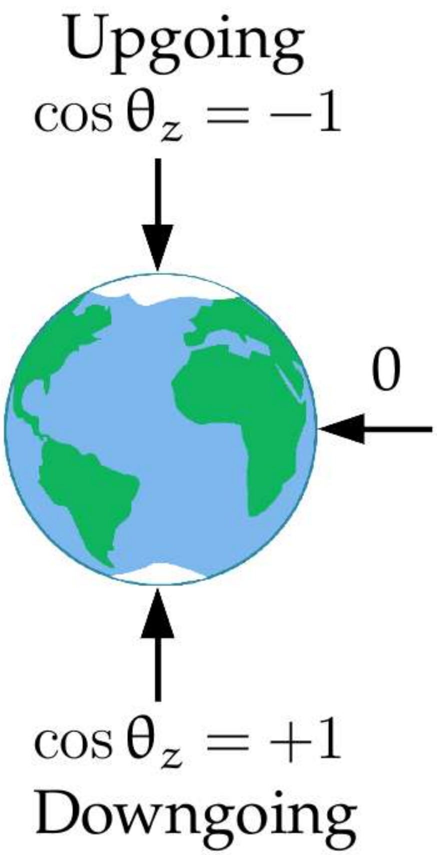


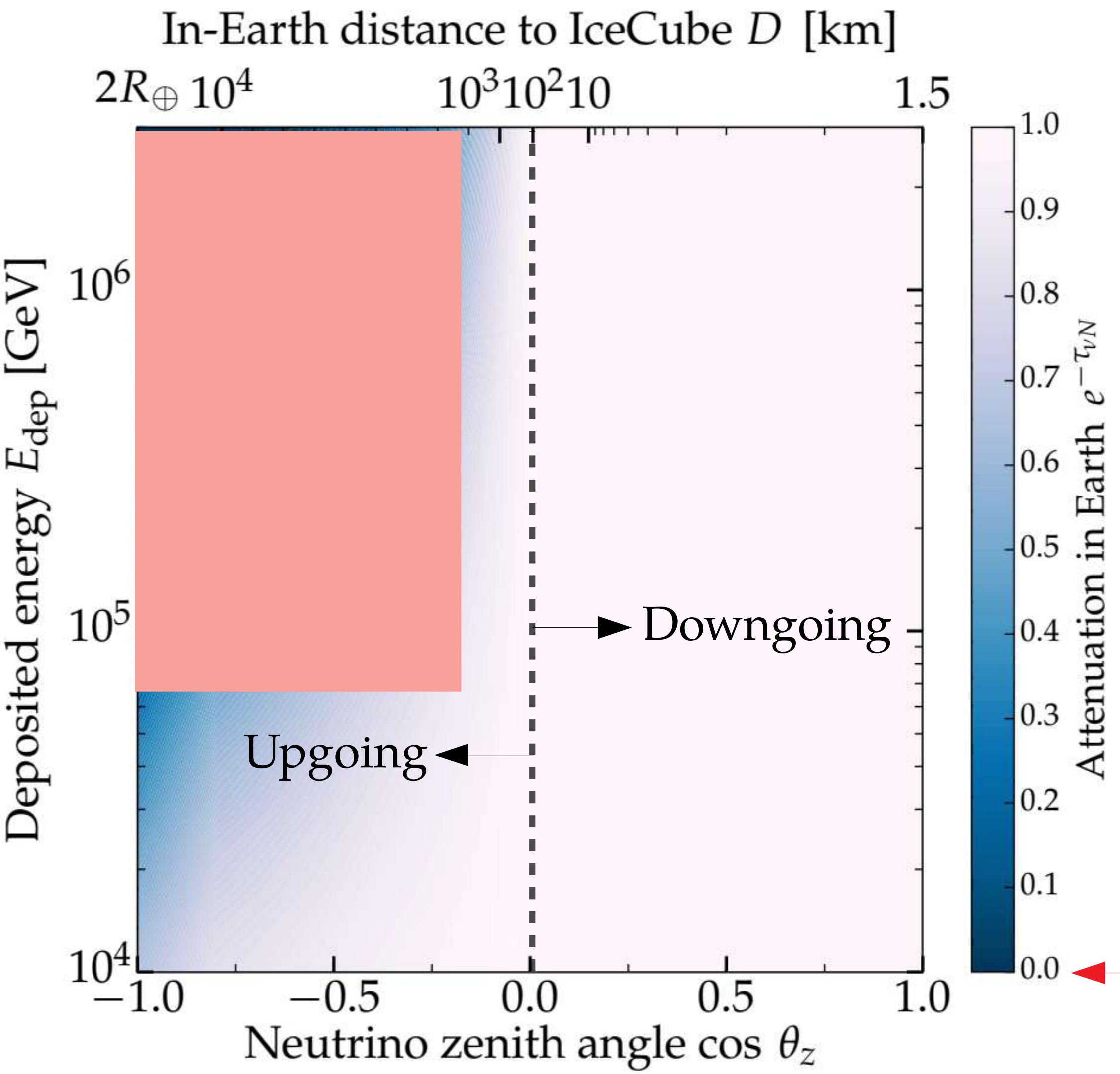


In-Earth distance to IceCube D [km]
$2R_\oplus$ 10^4
10^3 10^2 10
1.5
Deposited energy E_{dep} [GeV]
10^6
10^5
10^4
−1.0
−0.5
0.0
0.5
1.0
Neutrino zenith angle $\cos\theta_z$
Attenuation in Earth $e^{-\tau_{\nu N}}$
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0


Upgoing
$\cos\theta_z = -1$
0
$\cos\theta_z = +1$
Downgoing
In-Earth distance to IceCube D [km]
$2R_\oplus$ 10^4
10^3 10^2 10
1.5
Deposited energy $E_{\rm dep}$ [GeV]
10^6
10^5
10^4
Downgoing
Upgoing
−1.0
−0.5
0.0
0.5
1.0
Neutrino zenith angle $\cos\theta_z$
Attenuation in Earth $e^{-\tau_{\nu N}}$
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0

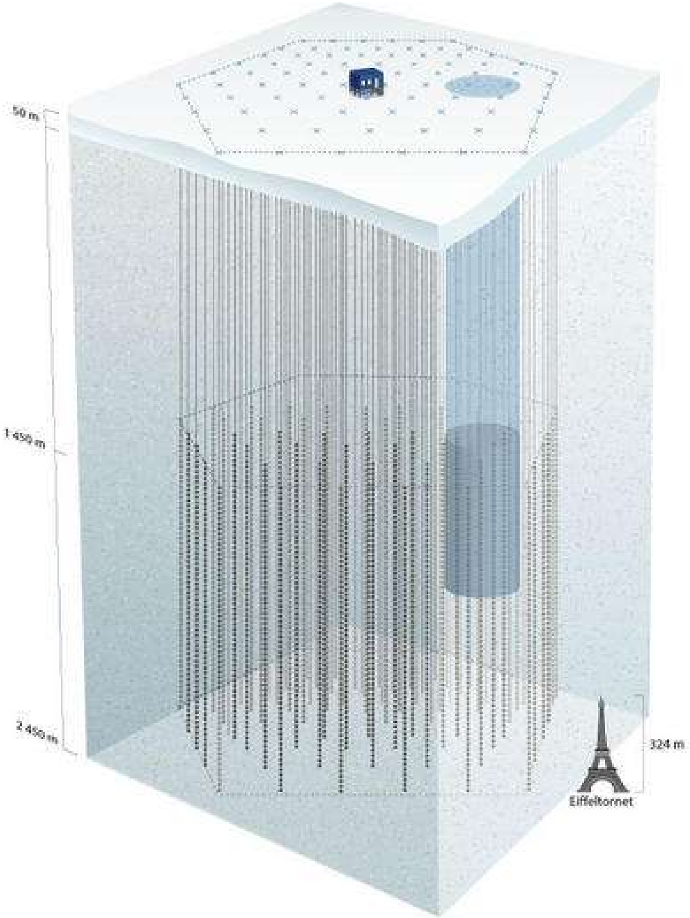

Upgoing
cos θz = −1
0
cos θz = +1
Downgoing
In-Earth distance to IceCube D [km]
2R⊕ 10^4 10^3 10^2 10 1.5
Deposited energy Edep [GeV]
10^6
10^5
10^4
Downgoing
Upgoing
−1.0 −0.5 0.0 0.5 1.0
Neutrino zenith angle cos θz
Attenuation in Earth e^−τνN
1.0 0.9 0.8 0.7 0.6 0.5 0.4 0.3 0.2 0.1 0.0
Transparent Earth
e^-τ ~ 1





Opaque Earth
$e^{-\tau} \sim 0$

IceCube – What is it?



- Km^3 in-ice Cherenkov detector in Antarctica
- >5000 PMTs at 1.5–2.5 km of depth
- Sensitive to neutrino energies > 10 GeV

How does IceCube see neutrinos?

Two types of fundamental interactions …

Charged-current (CC)

$\nu_l + N \rightarrow l + \text{hadrons}$

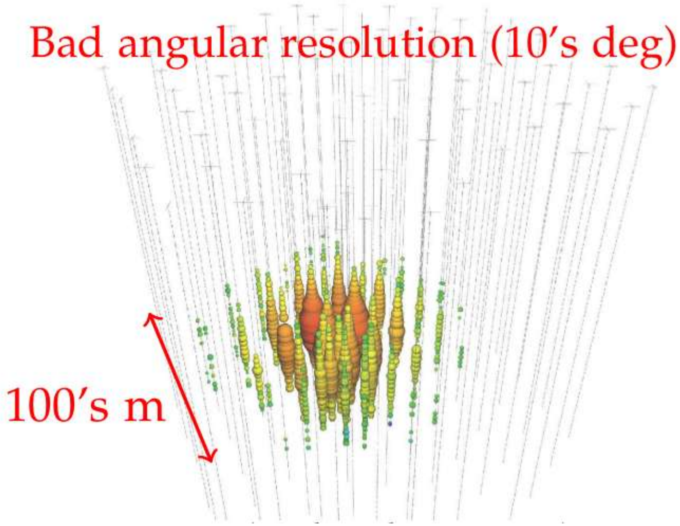
Neutral-current (NC)

$\nu_l + N \rightarrow \nu_l + \text{hadrons}$

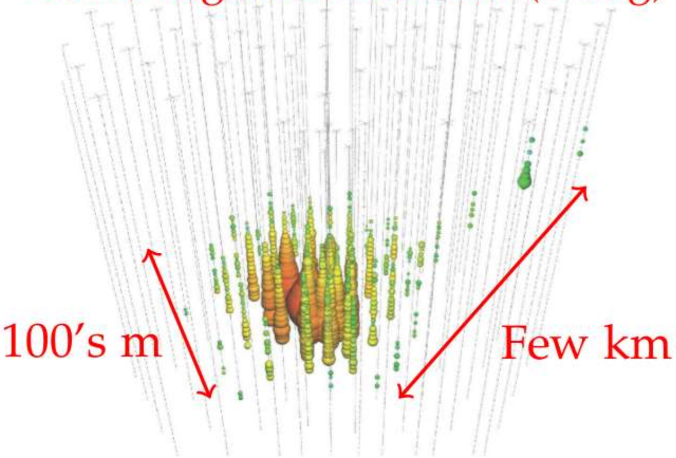
These shower and make light

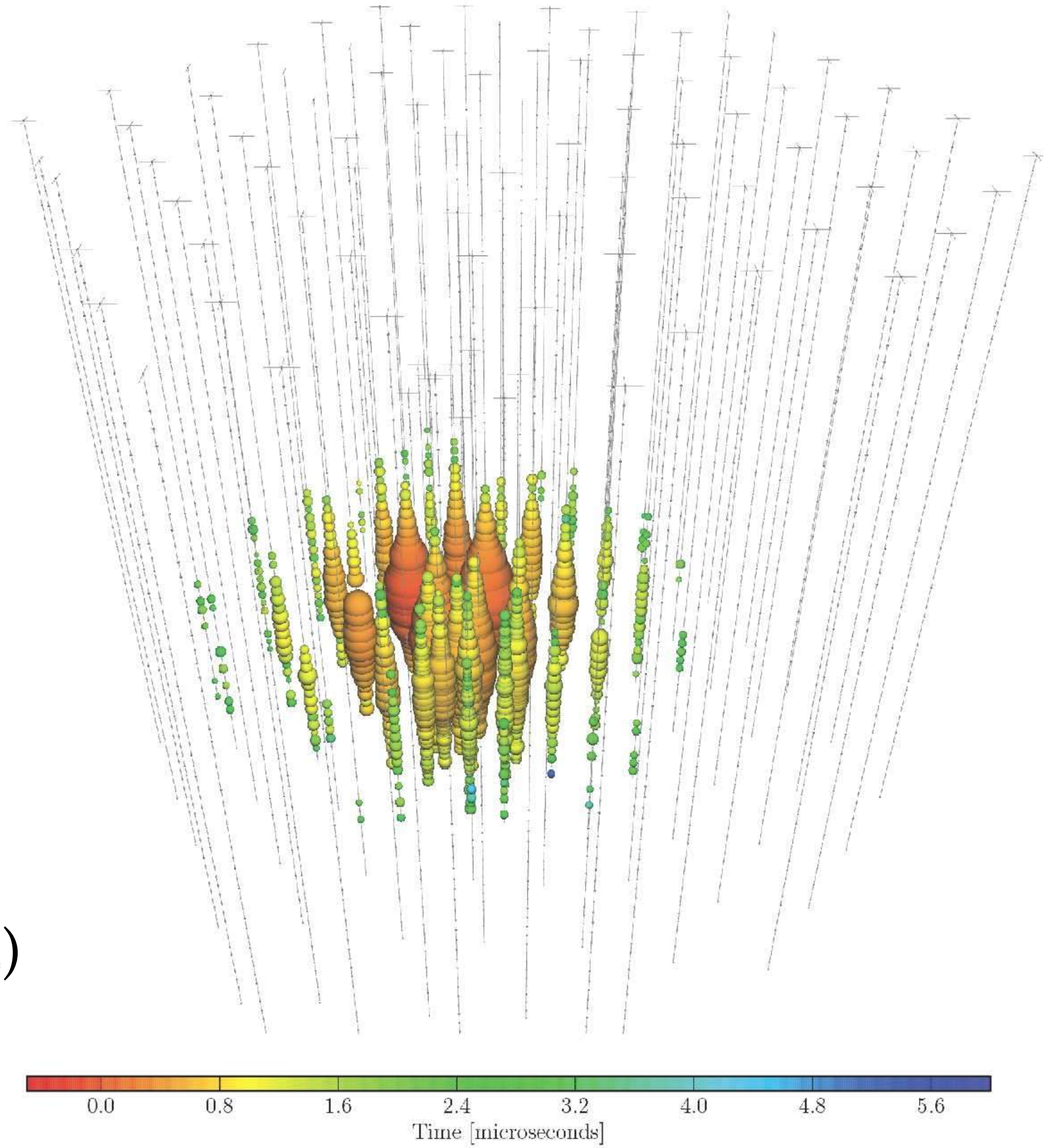
… create two event topologies …

Showers — From CC ν_e or ν_τ, or NC ν_x



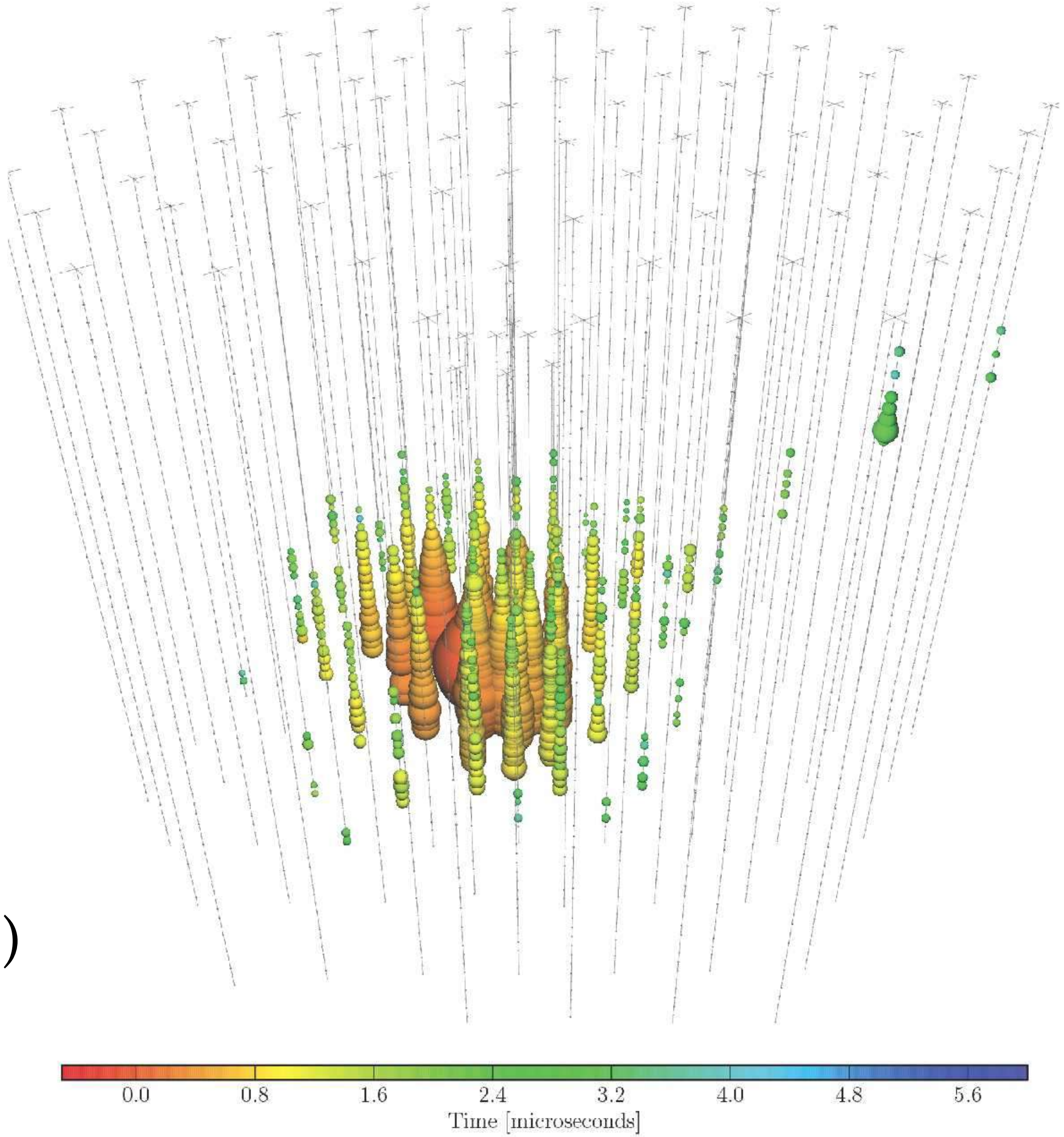
Tracks — From CC ν_μ mainly





Shower
(IceCube event #22)

Track
(IceCube event #15)



What has IceCube found so far (6 years)?

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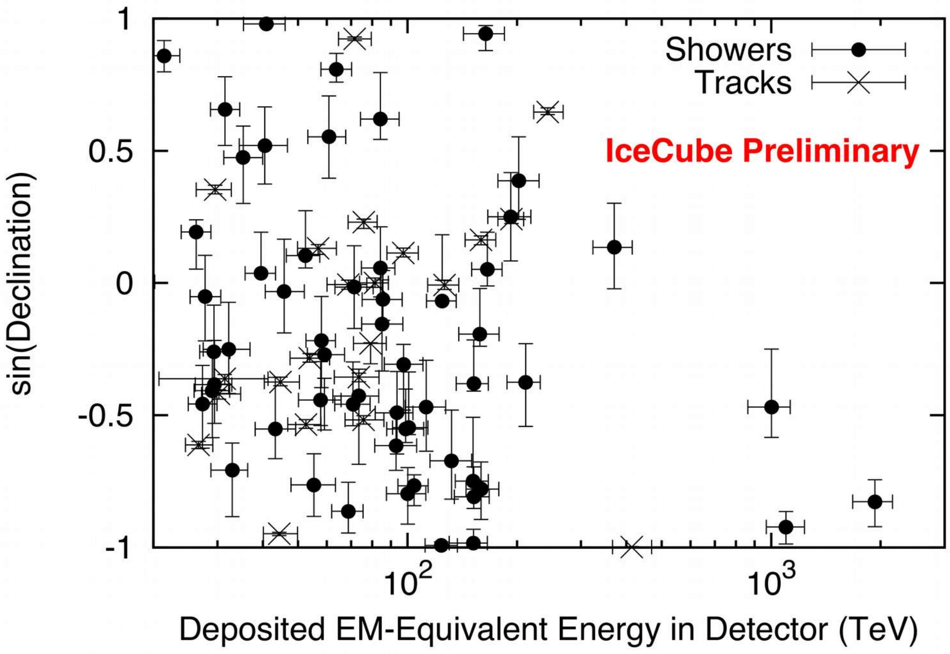
80 contained events between 18 TeV – 2 PeV

(16 atm. neutrinos, 25 atm. muons)



C. Kopper, ICRC 2017

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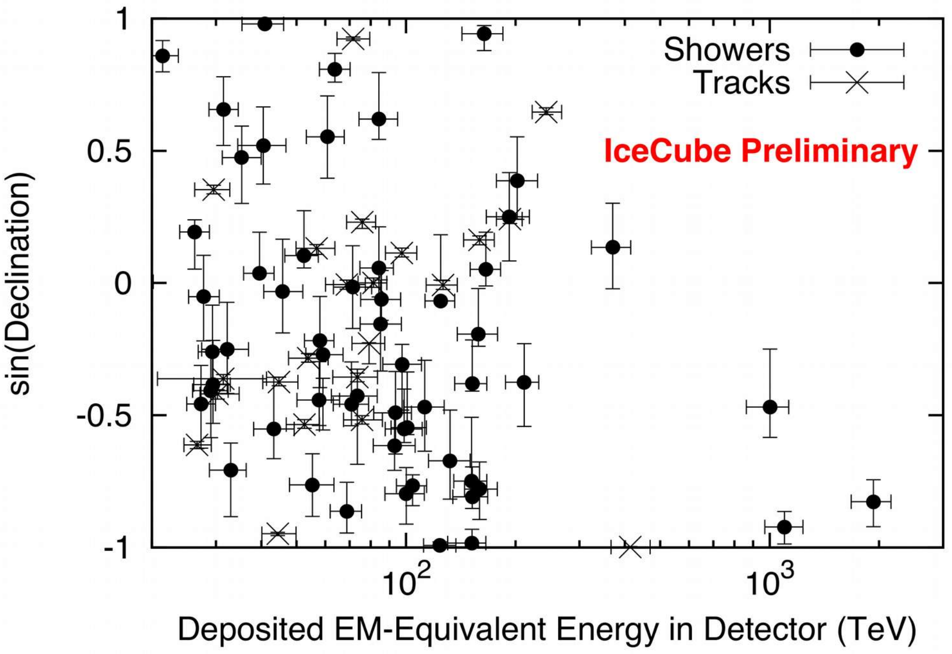
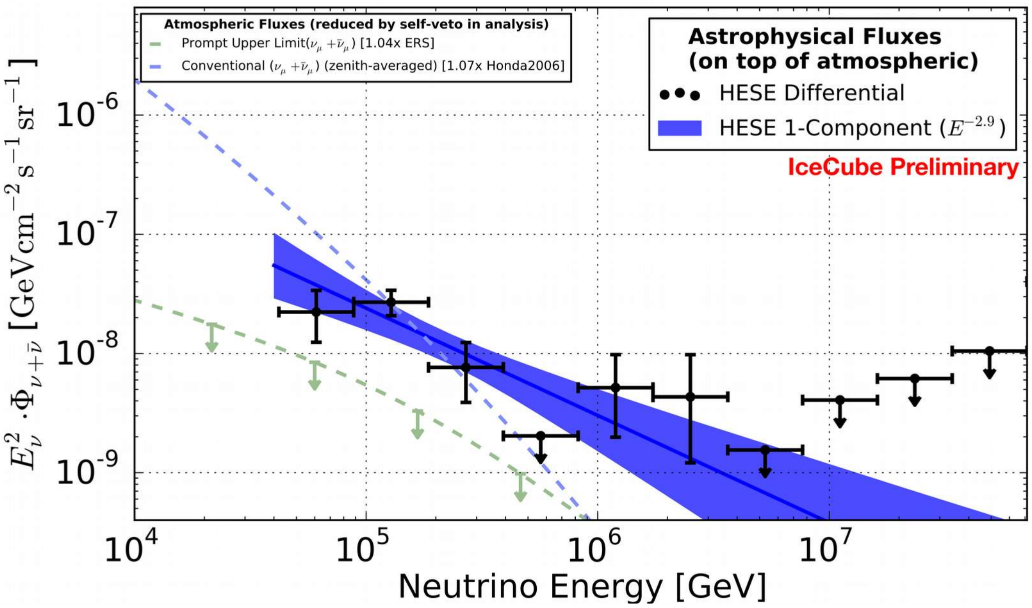


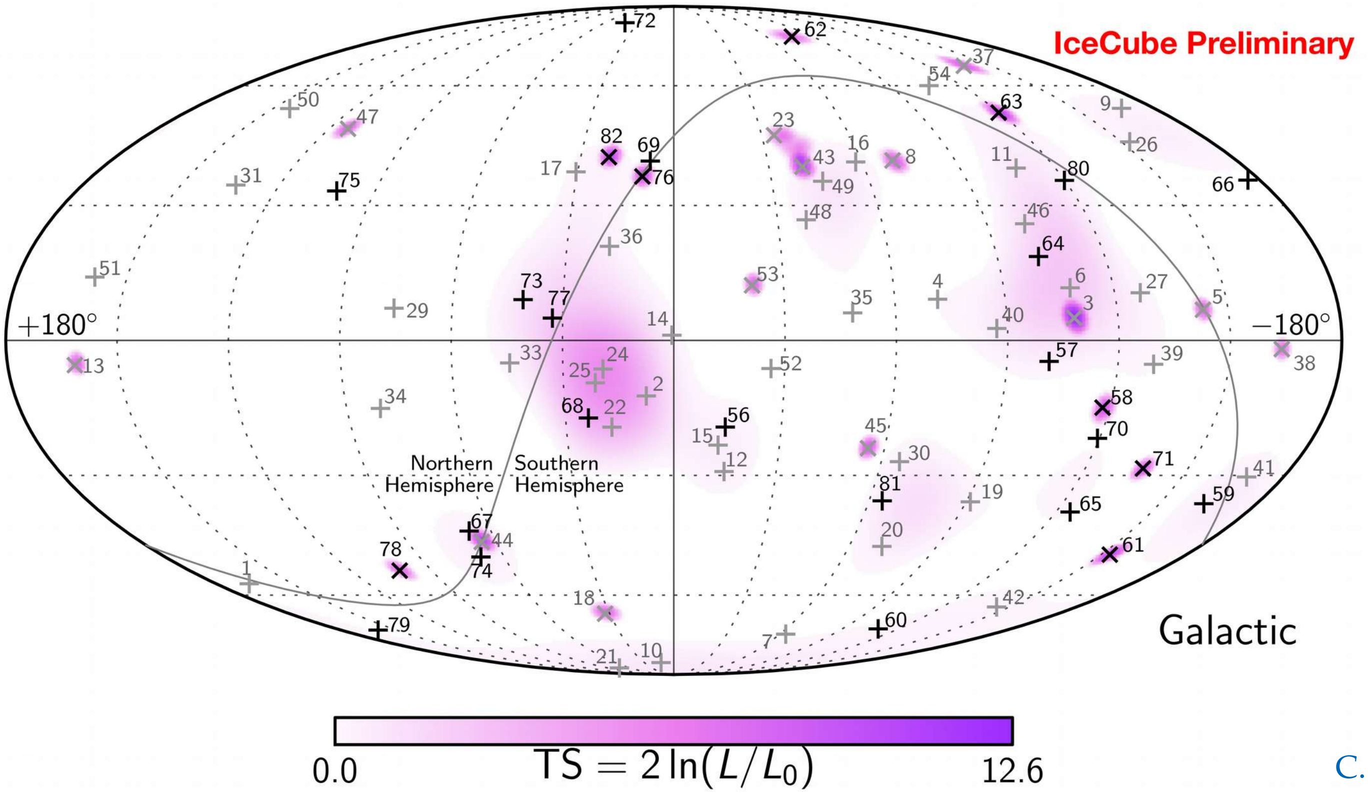
Astrophysical ν flux detected at > 7σ
(Normalization ok, but steep spectrum)



C. Kopper, ICRC 2017

What has IceCube found so far (6 years)?

Arrival directions compatible with isotropy

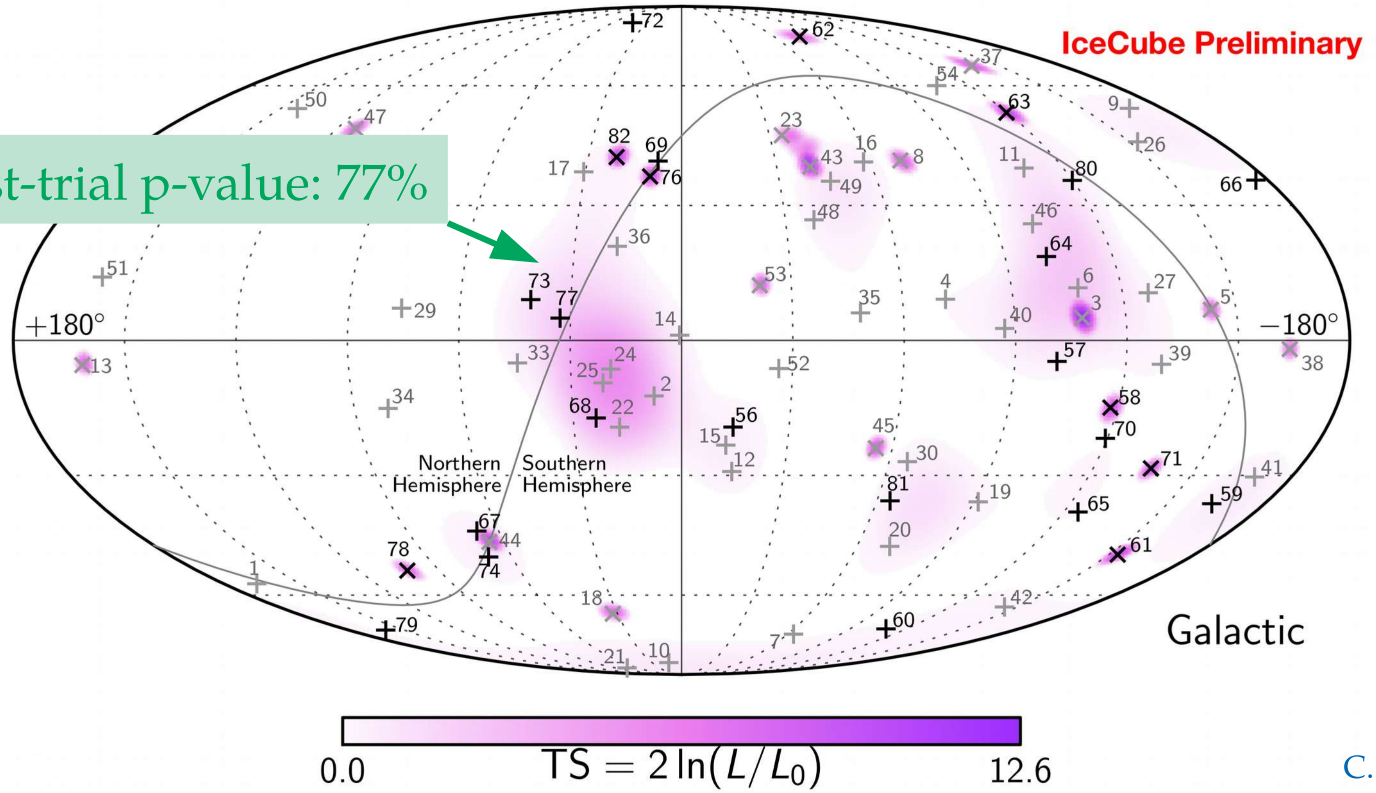


C. Kopper, ICRC 2017

What has IceCube found so far (6 years)?

Arrival directions compatible with isotropy

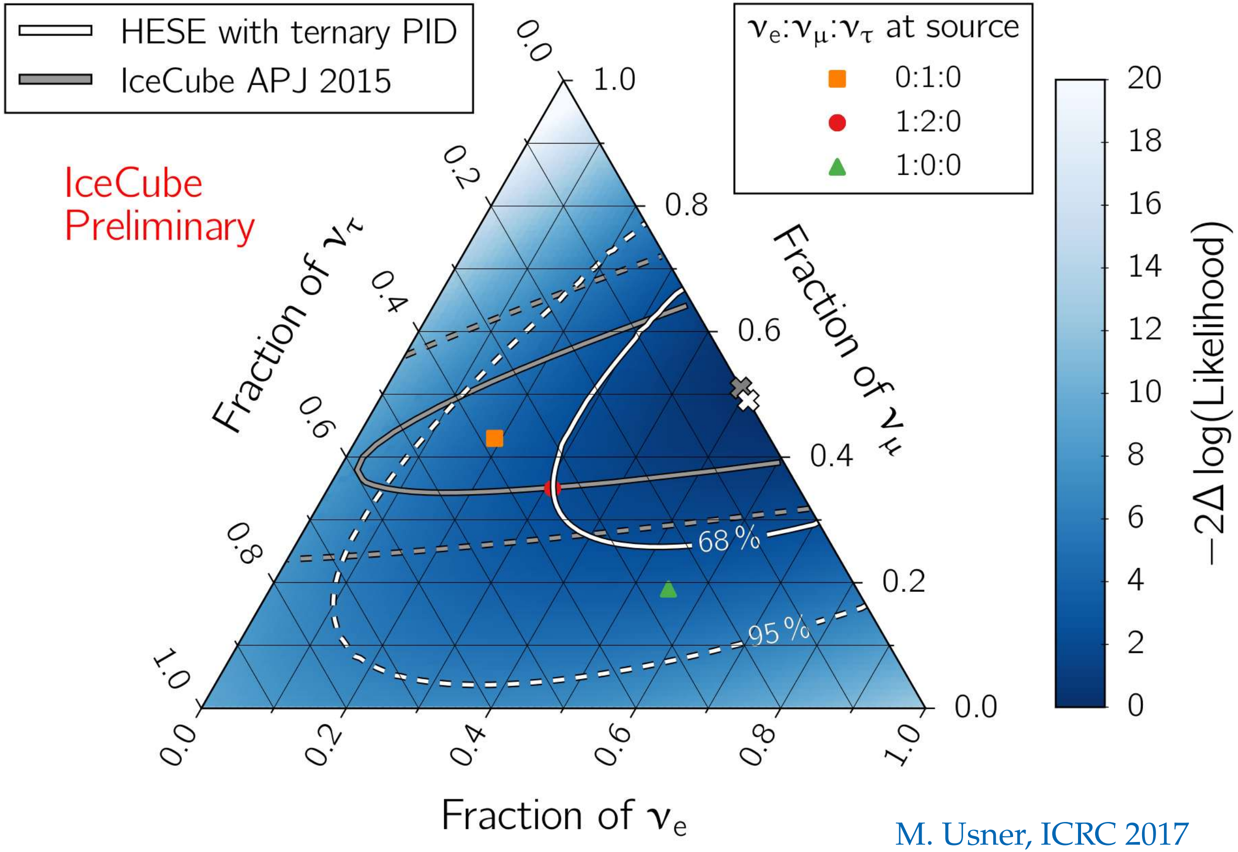
Post-trial p-value: 77%



C. Kopper, ICRC 2017

What has IceCube found so far (6 years)?

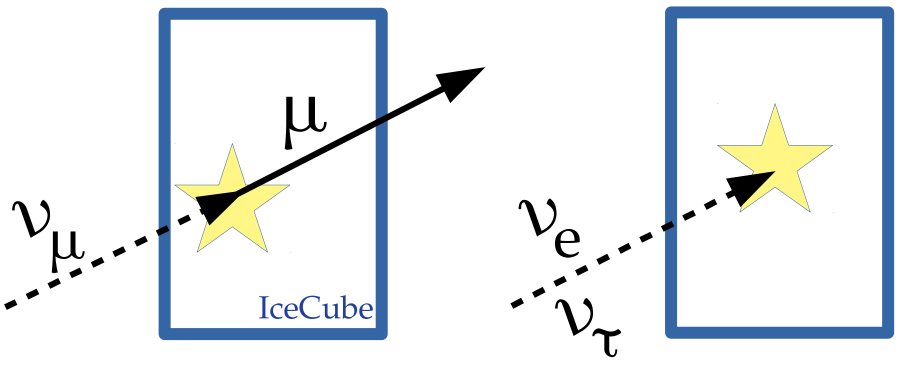
Flavor composition compatible with equal proportion of each flavor



M. Usner, ICRC 2017

Contained vs. uncontained νN interactions

Contained events



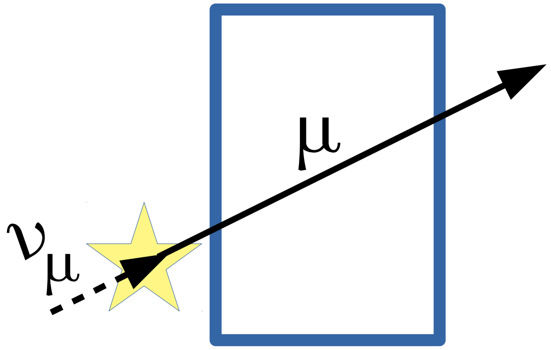
Starting track — Shower

Pro: Clean determination of E_ν

Con: Few events (<100)

Ref.: MB & A. Connolly, 1711.11043

Uncontained events



Through-going muon

Pro: Lots of events (~10k used)

Con: Uncertain estimates of E_ν

Ref.: IceCube, Nature 2017, 1711.08119

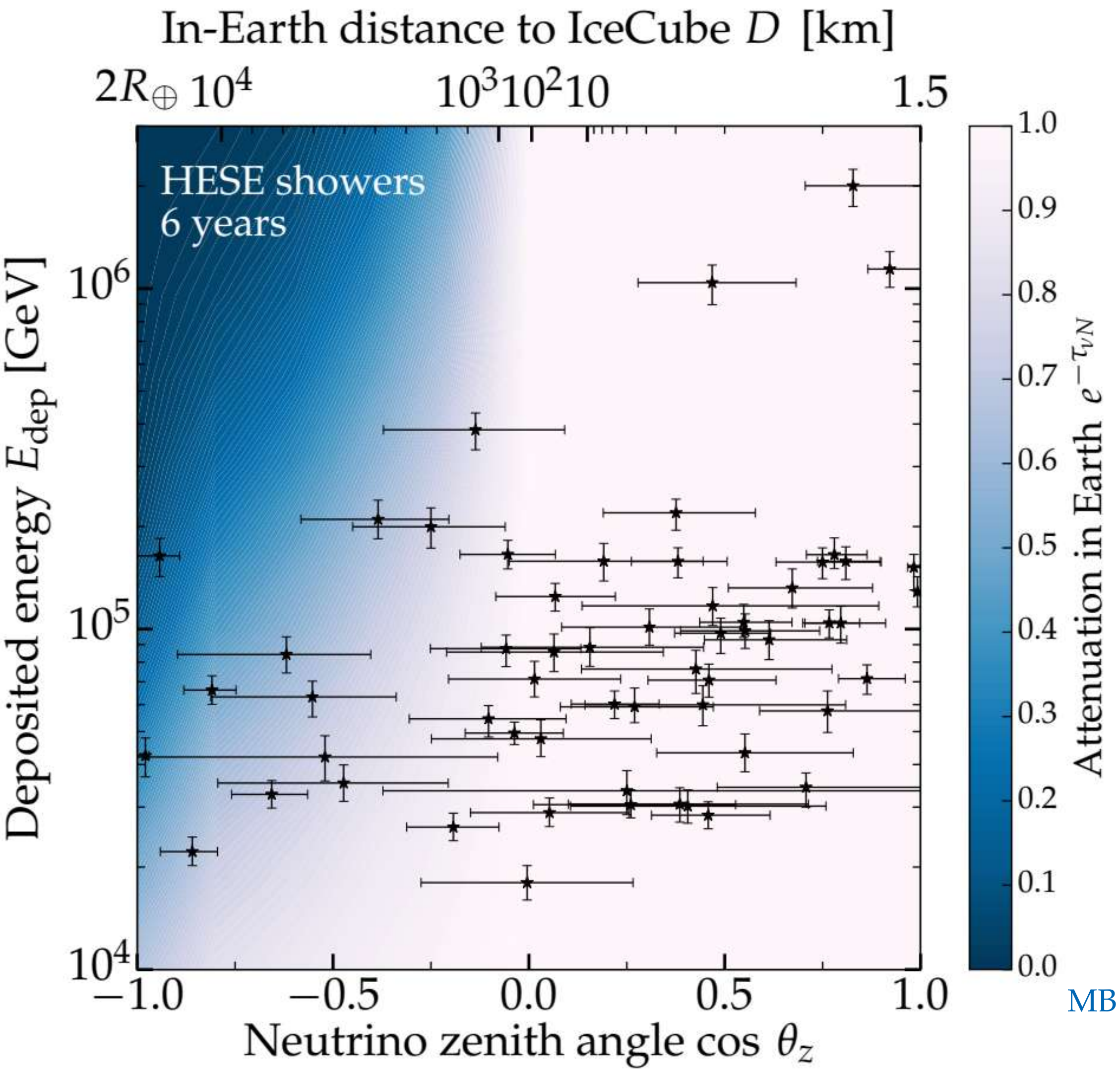
Cross section from contained events

- $\sigma_{\nu N}$ varies with neutrino energy ⇒ use events where E_ν is well-reconstructed

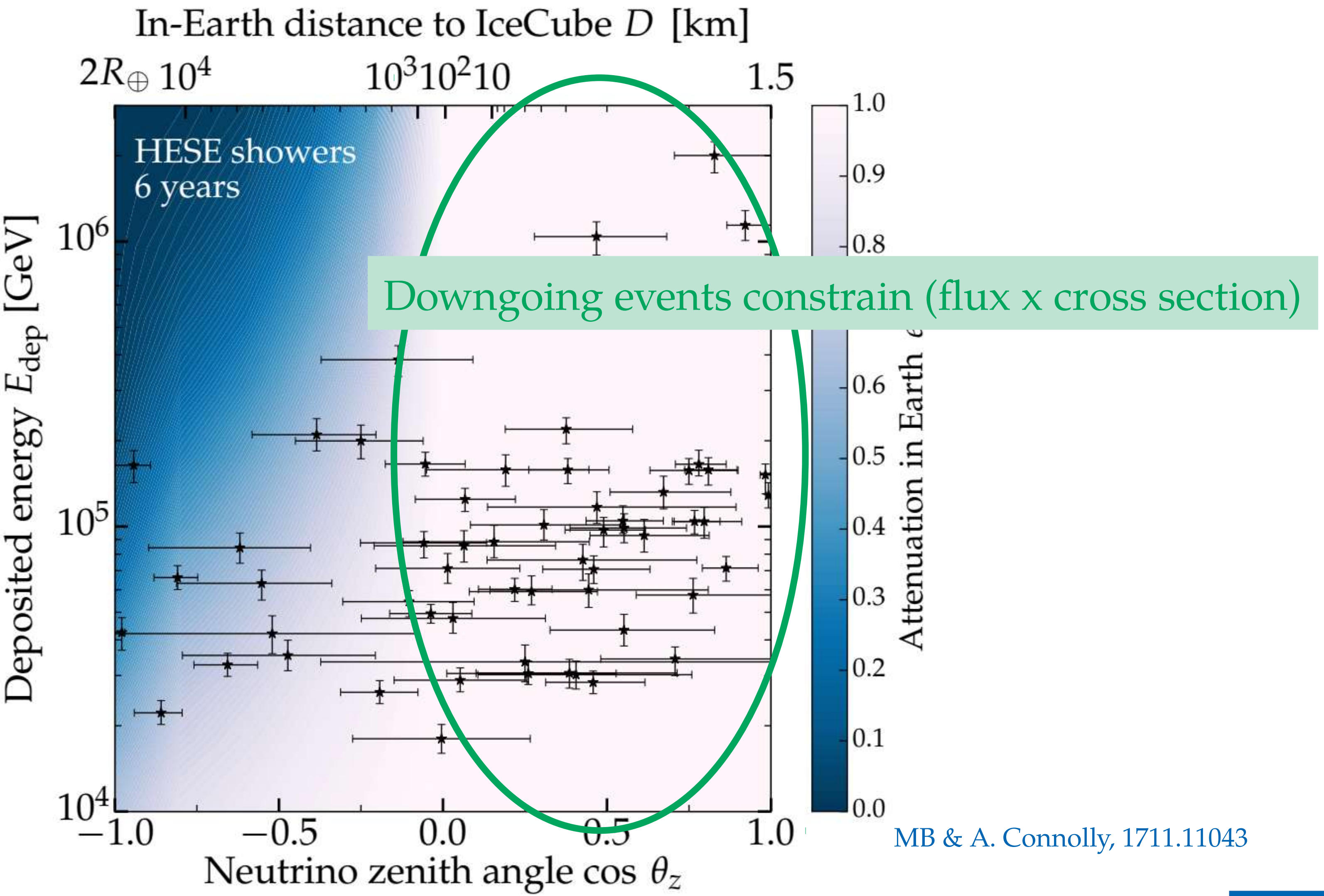
- These are IceCube High-Energy Starting Events (HESE):
  - νN interaction occurs inside the detector
  - ☑ **Showers:** completely contained in the detector ($E_{dep} \approx E_\nu$)
  - ☒ **Tracks:** partially contained ($E_{dep} < E_\nu$)

- We use the 58 publicly available HESE showers (6-year sample)

- HESE tracks *could* be used
  – but we would need non-public data to reconstruct E_ν without bias



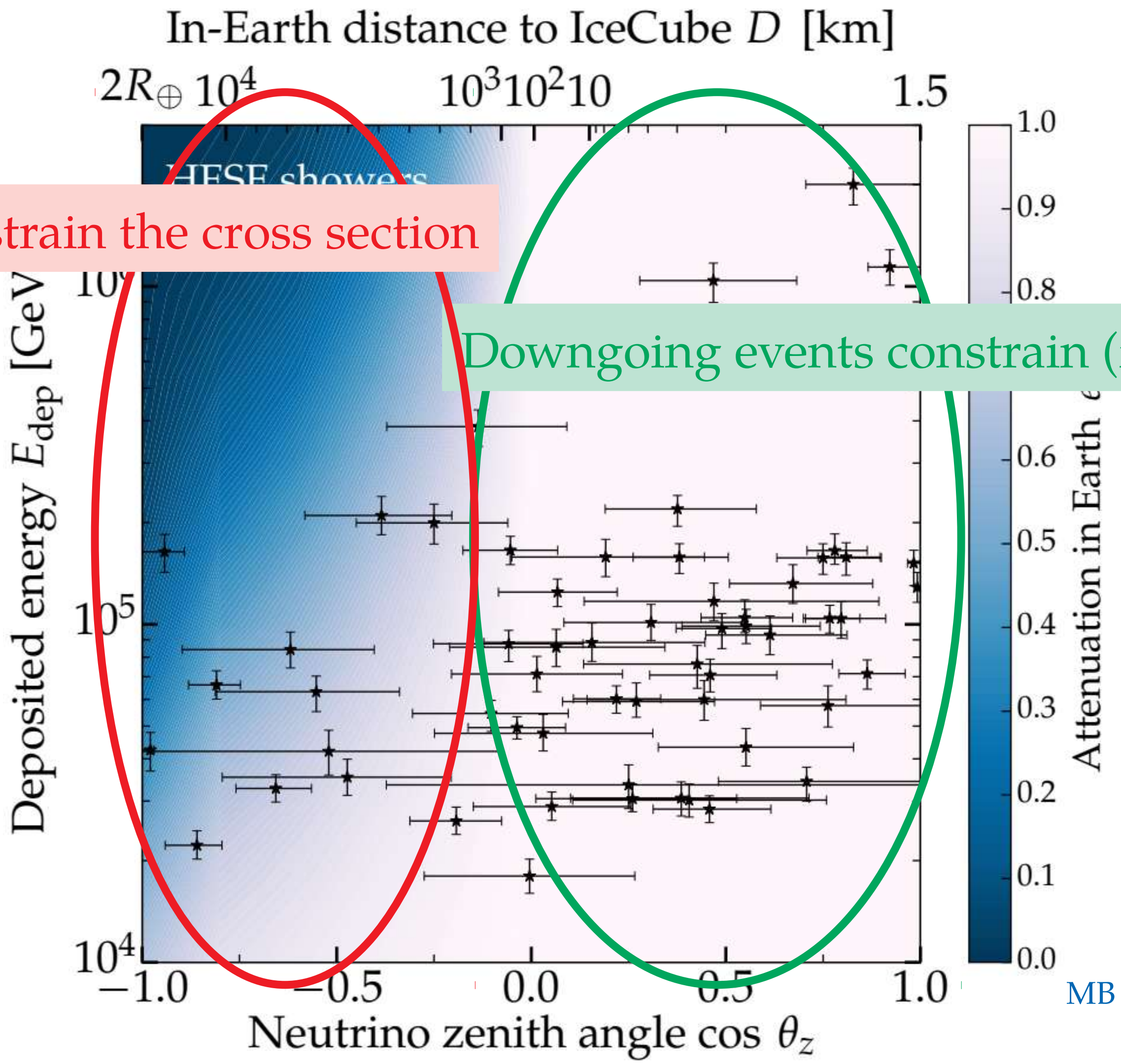
MB & A. Connolly, 1711.11043



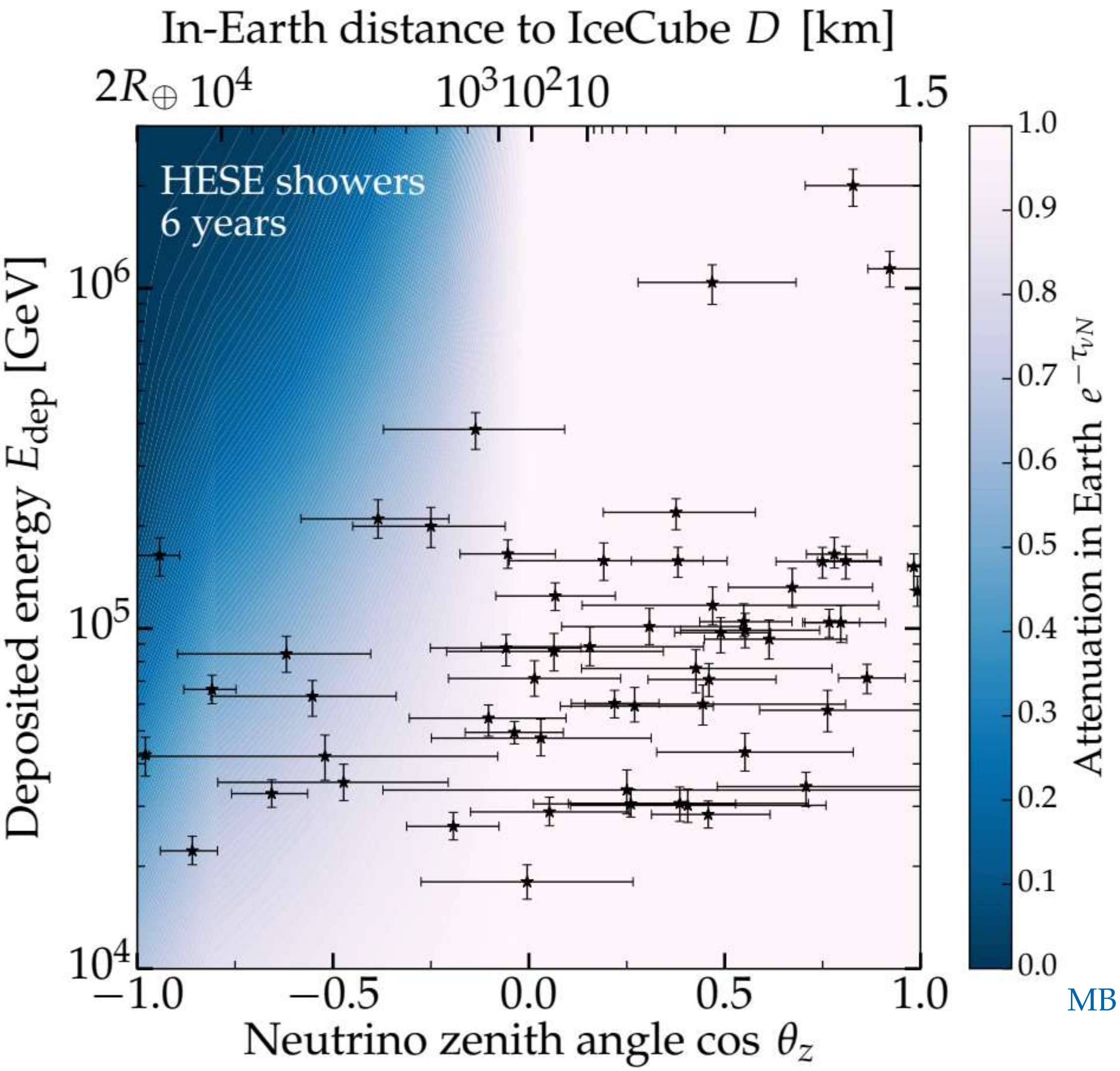
MB & A. Connolly, 1711.11043

Upgoing events constrain the cross section

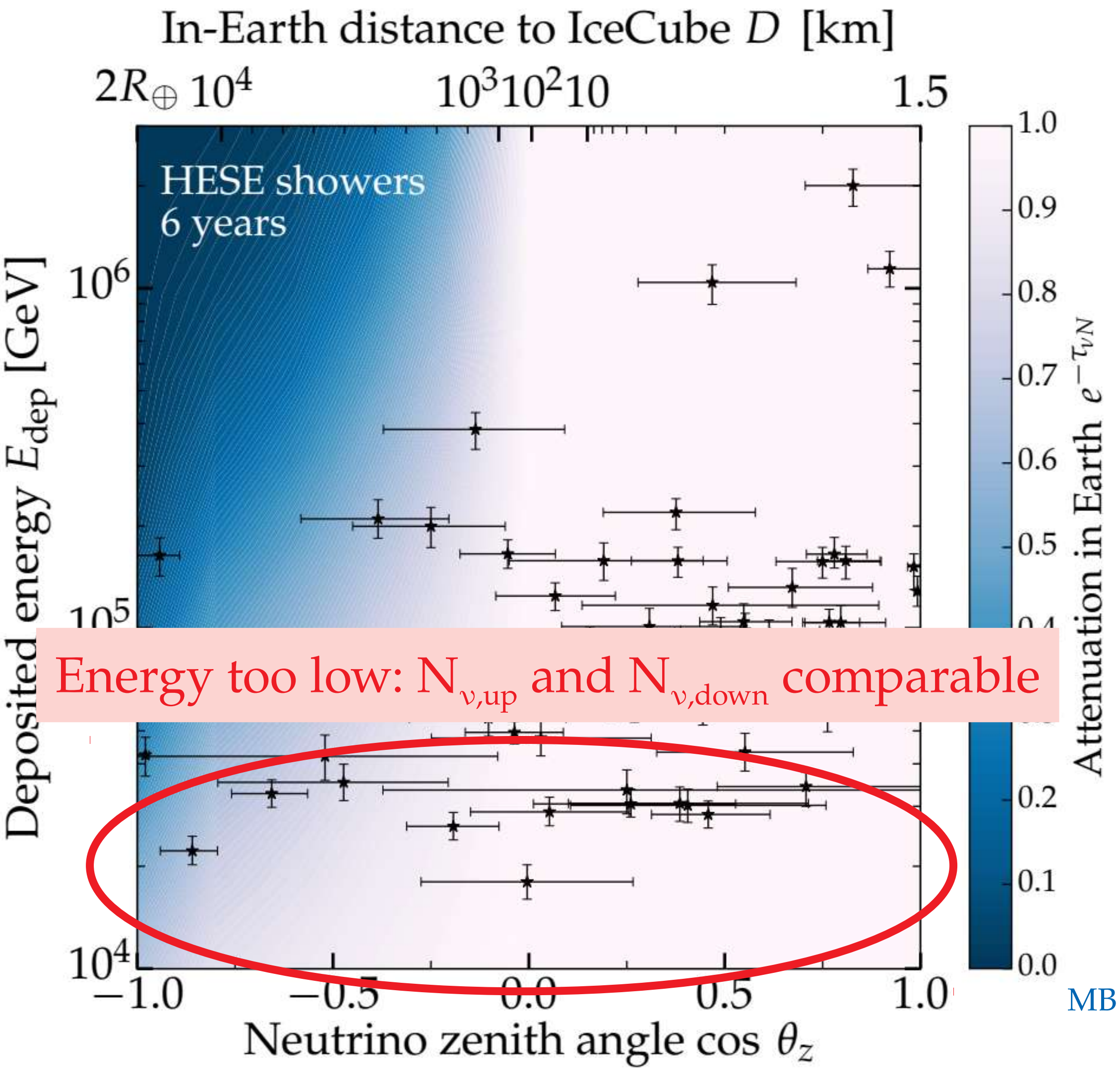
Downgoing events constrain (flux x cross section)



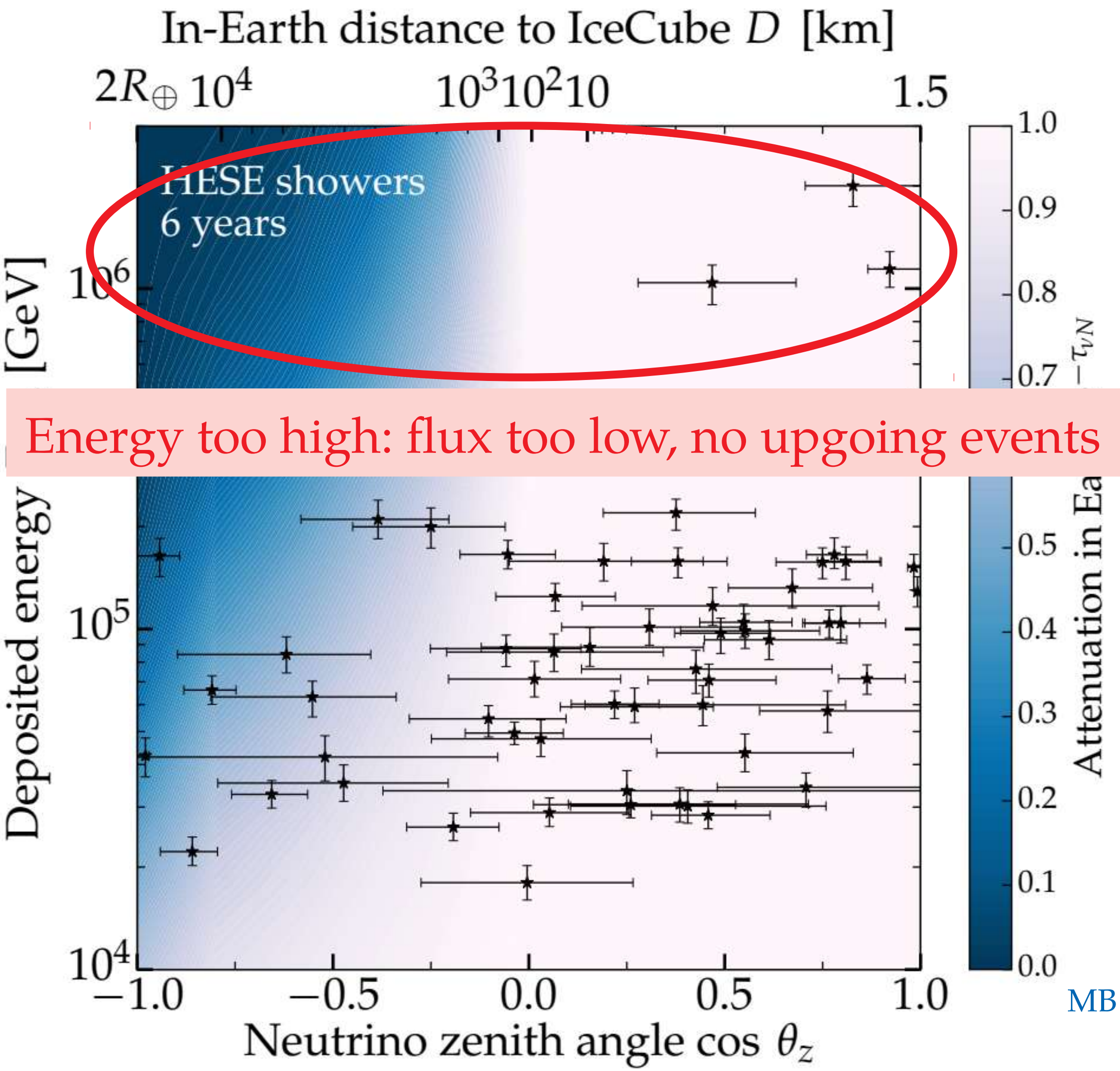
MB & A. Connolly, 1711.11043



MB & A. Connolly, 1711.11043

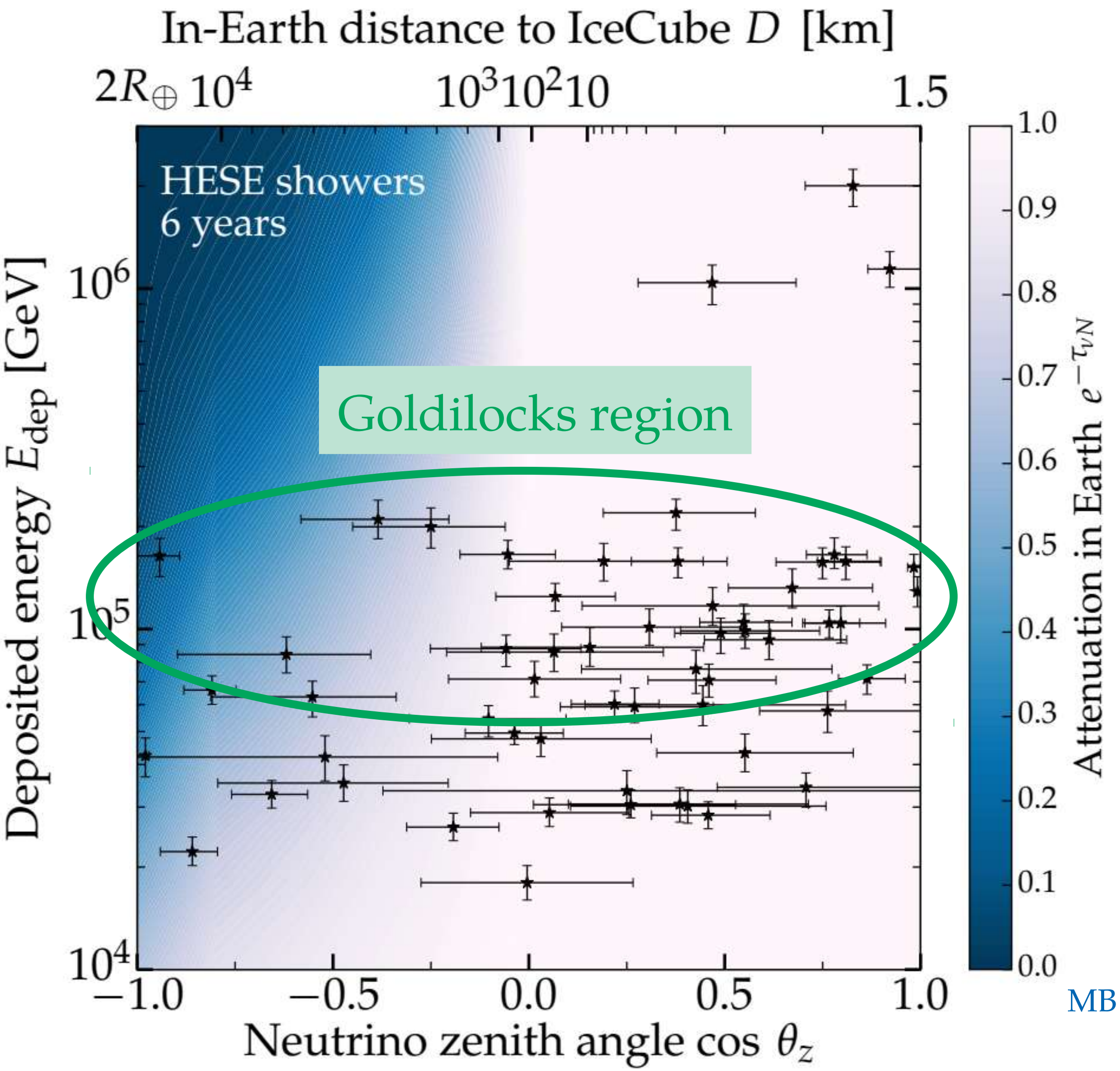


MB & A. Connolly, 1711.11043



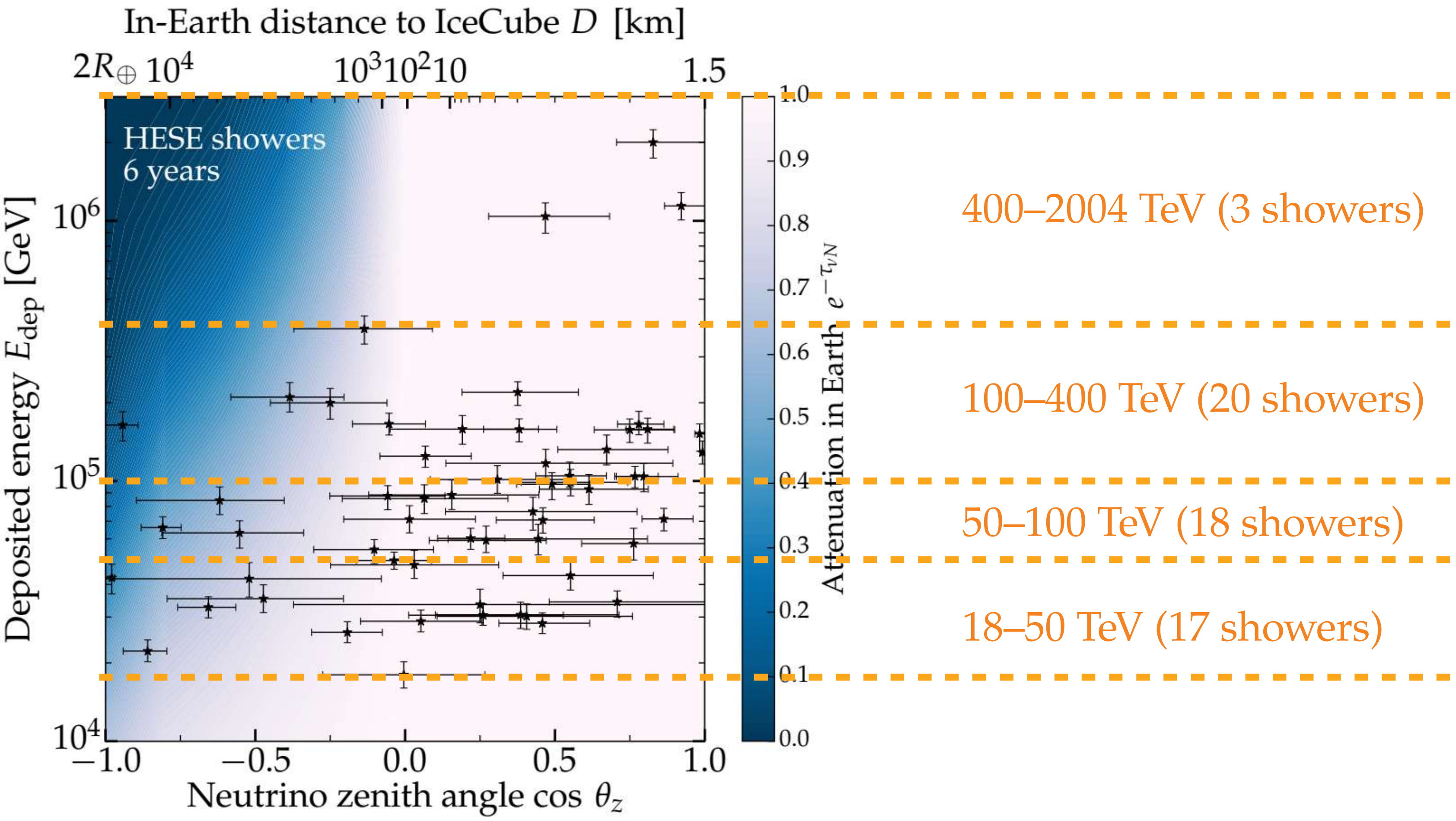
Energy too high: flux too low, no upgoing events

MB & A. Connolly, 1711.11043



MB & A. Connolly, 1711.11043

Bin-by-bin analysis



Sensitivity to σ in each bin

Number of contained events in an energy bin:

$$N_\nu \sim \Phi_\nu \cdot \sigma_{\nu N} \cdot e^{-\tau} = \Phi_\nu \cdot \sigma_{\nu N} \cdot e^{-L\sigma_{\nu N} n_N}$$

Downgoing (no matter)

$$N_{\nu,\mathrm{dn}} \sim \Phi_\nu \cdot \sigma_{\nu N}$$

Downgoing events fix the product $\Phi_\nu \cdot \sigma_{\nu N}$

Upgoing (lots of matter)

$$N_{\nu,\mathrm{up}} \sim N_{\nu,\mathrm{dn}} \cdot e^{-\tau}$$

Upgoing events measure $\sigma_{\nu N}$ via τ

Reality check:
Few events (per energy bin), so we are statistics-limited

The fine print

- High-energy ν's: astrophysical (isotropic) + atmospheric (**anisotropic**)
  ↦ We take into account the shape of the atmospheric contribution

- The shape of the astrophysical ν **energy spectrum** is still uncertain
  ↦ We take a $E^{-\gamma}$ spectrum in *narrow* energy bins

- **NC showers** are sub-dominant to **CC showers**, but they are indistinguishable
  ↦ Following Standard-Model predictions, we take $\sigma_{NC} = \sigma_{CC}/3$

- IceCube does not **distinguish ν from $\bar{\nu}$**, and their cross-sections are different
  ↦ We assume equal fluxes, expected from production via *pp* collisions
  ↦ We assume the avg. ratio $\langle\sigma_{\bar{\nu}N}/\sigma_{\nu N}\rangle$ in each bin known, from SM predictions

- The **flavor composition** of astrophysical neutrinos is still uncertain
  ↦ We assume equal flux of each flavor, compatible with theory and observations

What goes into the (likelihood) mix?

- Inside each energy bin, we freely vary
  - N_{ast} (showers from astrophysical neutrinos)
  - N_{atm} (showers from atmospheric neutrinos)
  - γ (astrophysical spectral index)
  - σ_{CC} (neutrino-nucleon charged-current cross section)
- For each combination, we generate the angular and energy shower spectrum…
- … and compare it to the observed HESE spectrum via a likelihood
- Maximum likelihood yields σ_{CC} (marginalized over nuisance parameters)
- Bins are independent of each other – there are no (significant) cross-bin correlations

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Including detector resolution
(10% in energy, 15° in direction)

- For each combination, we generate the angular and energy shower spectrum…
- … and compare it to the observed HESE spectrum via a likelihood
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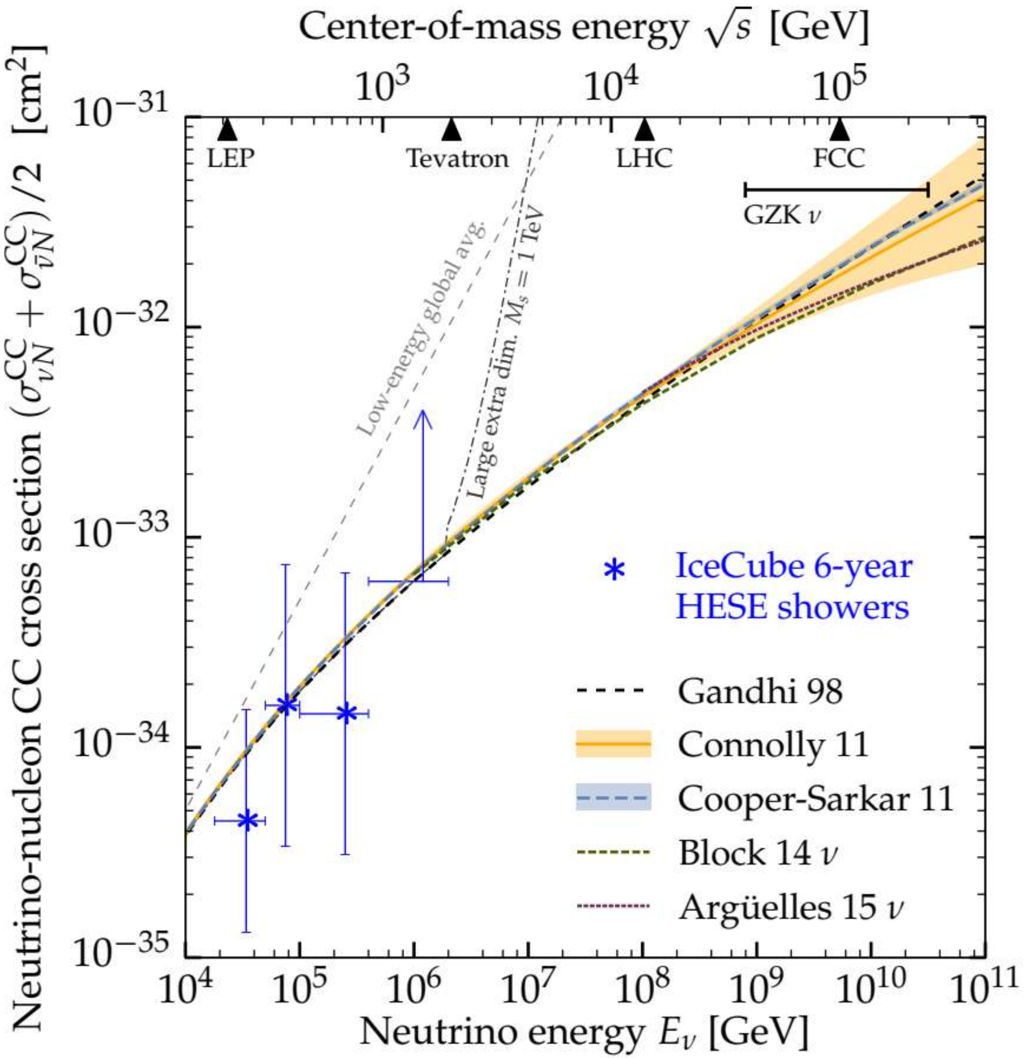
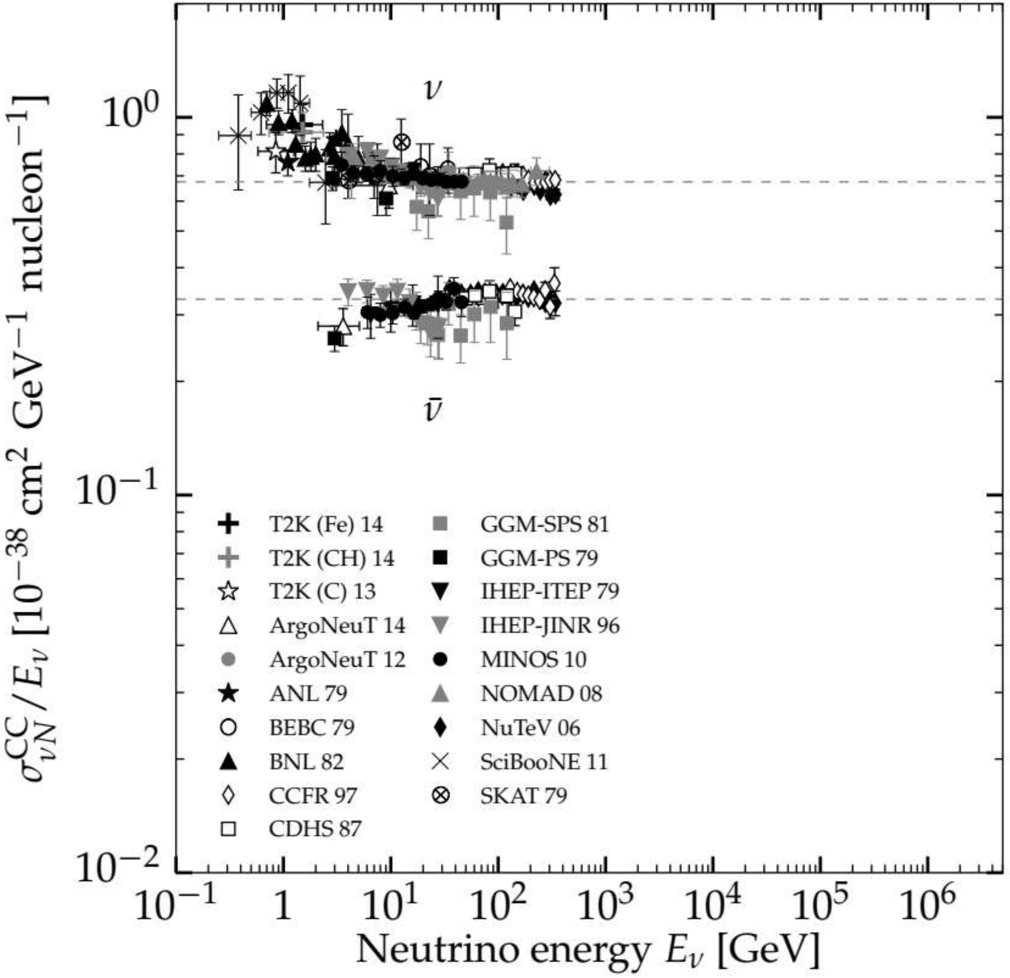
Our result



MB & A. Connolly, 1711.11043

Extending cross section measurements



MB & A. Connolly, 1711.11043

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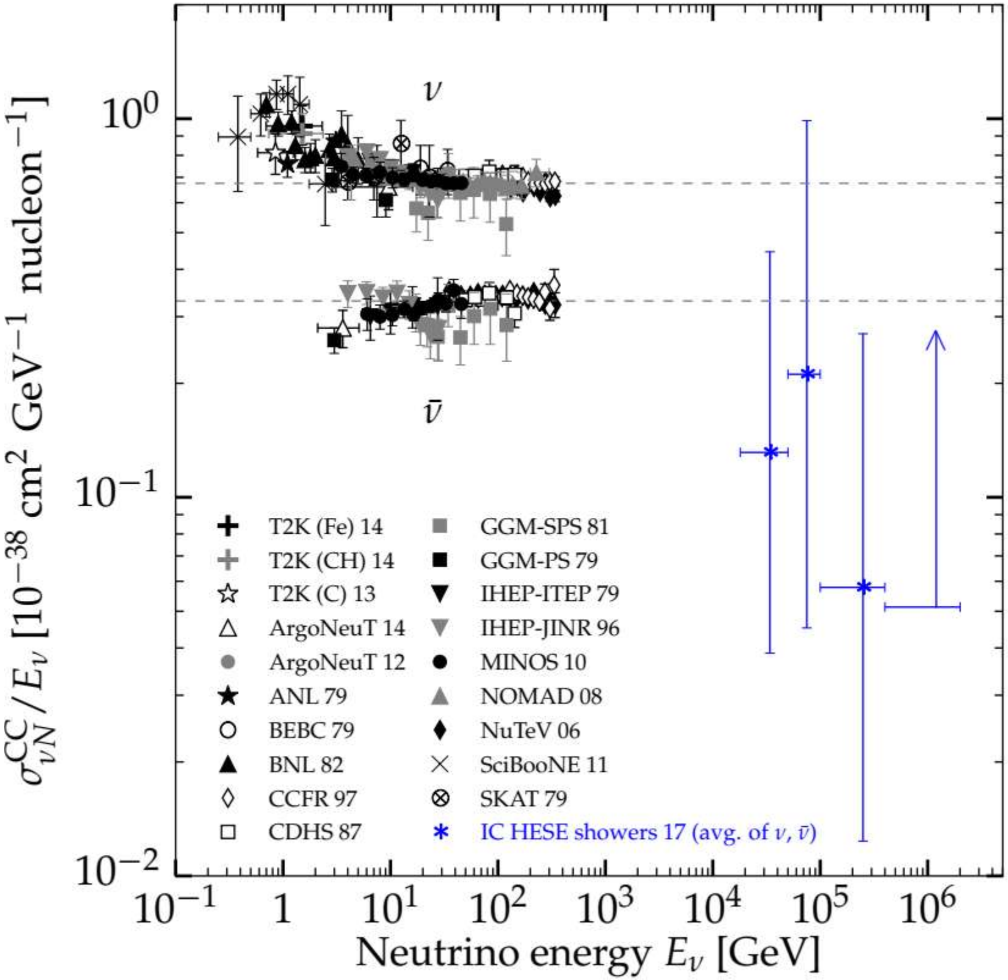

MB & A. Connolly, 1711.11043

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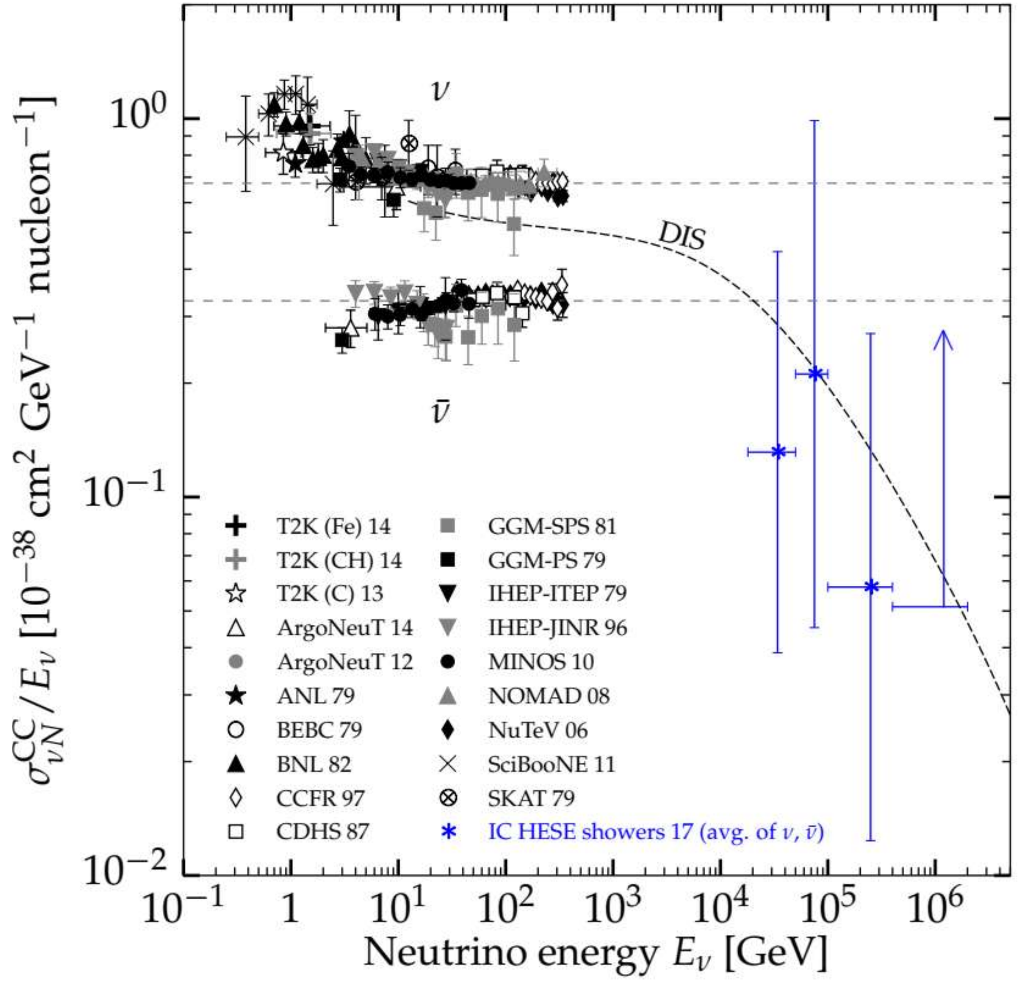


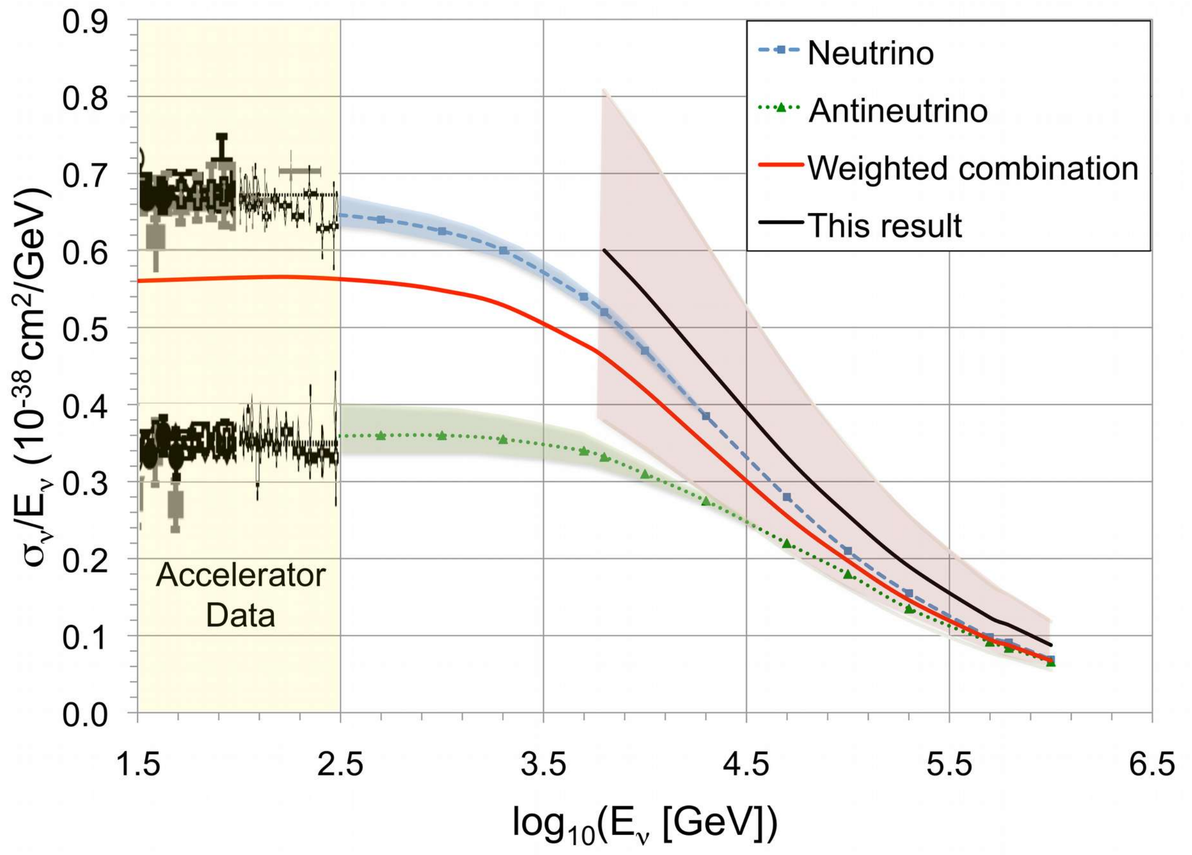
MB & A. Connolly, 1711.11043

How to do better / more?

- Currently, we are statistics-limited
  ↦ Solvable with more data from IceCube, IceCube-Gen2, KM3NeT
- Large errors in arrival direction (~10°) give errors in attenuation
  ↦ Solvable with ongoing IceCube improvements + KM3NeT
- Charged-current + neutral-current cross sections are indistinguishable
  ↦ Solvable (?) with muon and neutron echoes (Li, MB, Beacom 16)
- Cannot separate ν from $\bar{\nu}$
  ↦ Wait to detect Glashow resonance (~6.3 PeV), sensitive only to $\bar{\nu}_e$
- Use starting tracks / through-going muons
  ↦ Doable / done by IceCube (more next)

Using through-going muons instead

- Use $\sim 10^4$ through-going muons
- Measured: $\mathrm{d}E_\mu / \mathrm{d}x$
- Inferred: $E_\mu \approx \mathrm{d}E_\mu / \mathrm{d}x$
- From simulations (uncertain): most likely E_ν given E_μ
- Fit the ratio $\sigma_{\rm obs} / \sigma_{\rm SM}$
  $1.30^{+0.21}_{-0.19}\,(\text{stat.})^{+0.39}_{-0.43}\,(\text{syst.})$
- All events grouped in a single energy bin 6–980 TeV



IceCube, *Nature* 2017

Bonus: Measuring the inelasticity $\langle y \rangle$

- Inelasticity in CC ν_μ interaction $\nu_\mu + N \to \mu + X$:

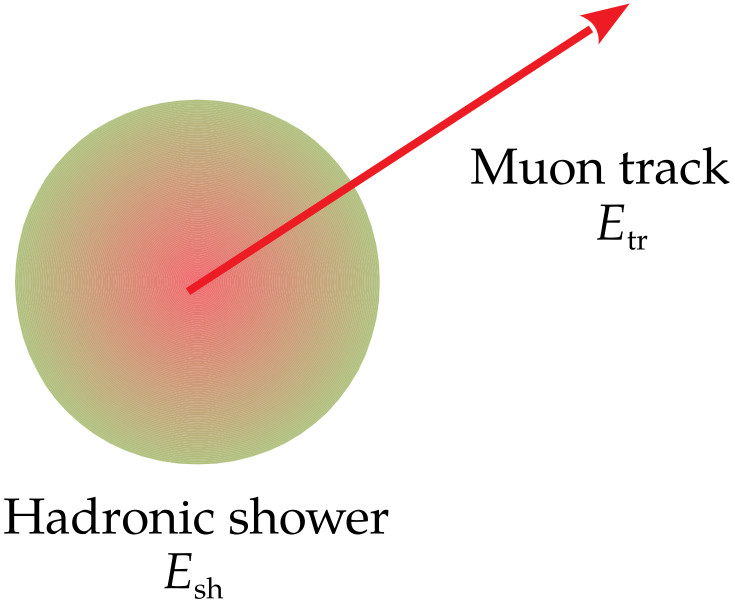
  $E_X = y\, E_\nu$ and $E_\mu = (1-y)\, E_\nu \;\Rightarrow\; y = (1 + E_\mu / E_X)^{-1}$

- The value of y follows a distribution $d\sigma/dy$

- In a HESE starting track:

  $E_X = E_{sh}$ (energy of shower)
  $E_\mu = E_{tr}$ (energy of track)
  $\Big\}\; y = (1 + E_{tr}/E_{sh})^{-1}$

- New IceCube analysis:
  - 5 years of starting-track data (2650 tracks)
  - Machine learning separates shower from track
  - Different y distributions for ν and $\bar{\nu}$



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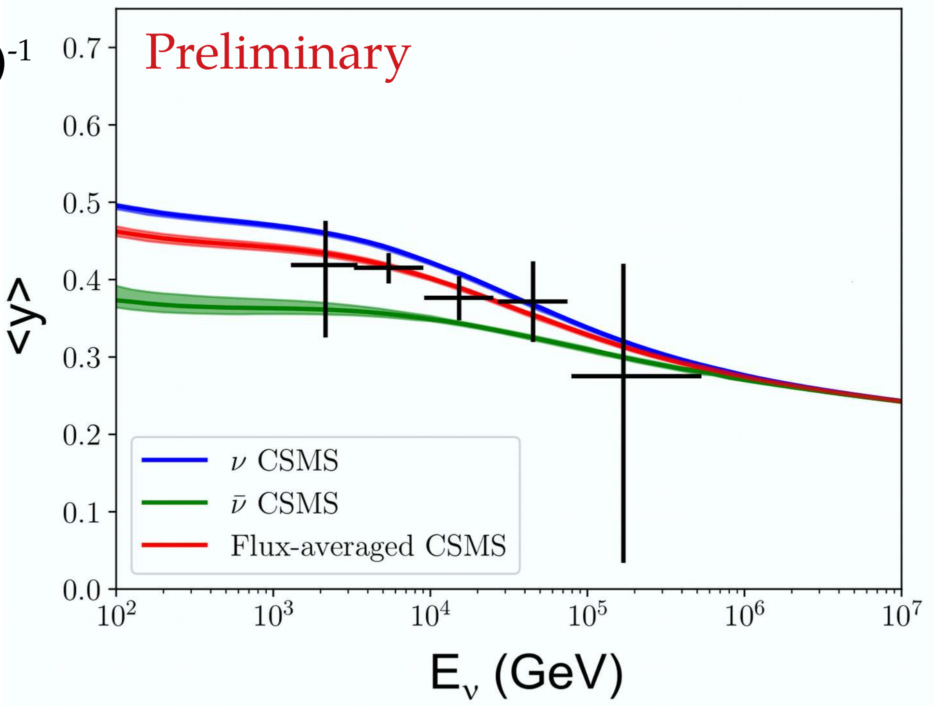

See presentation by Spencer Klein, ISVHECRI 2018

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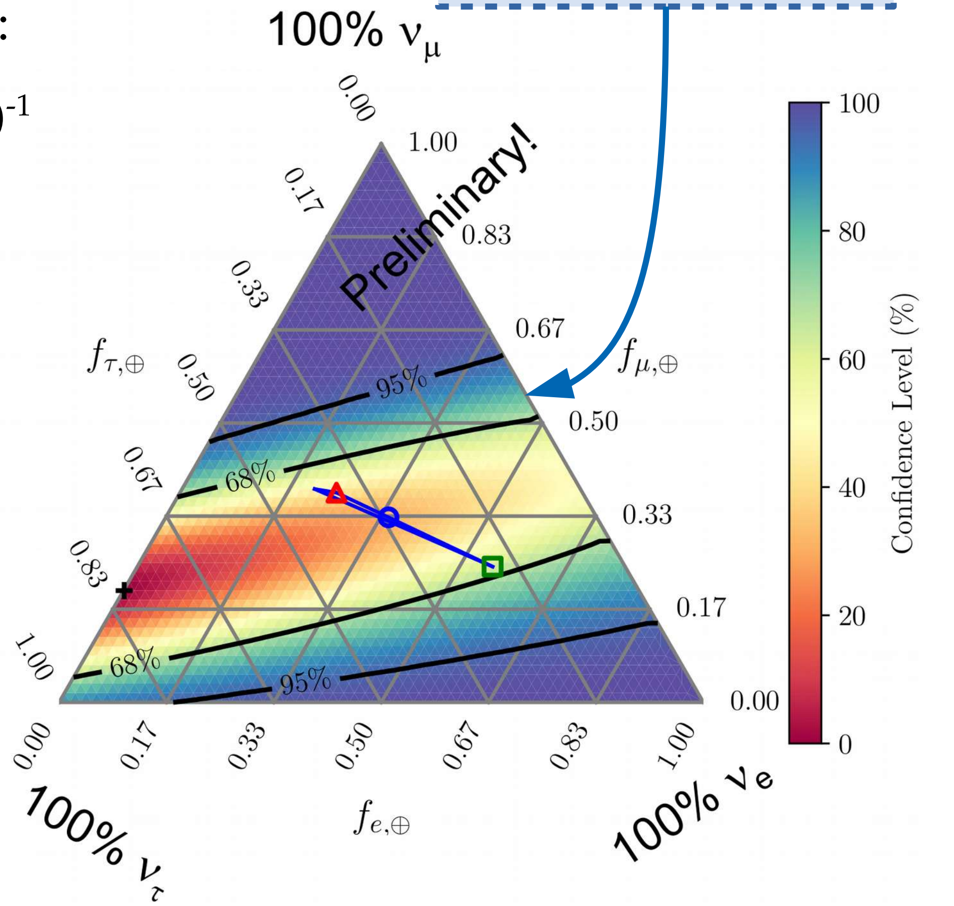
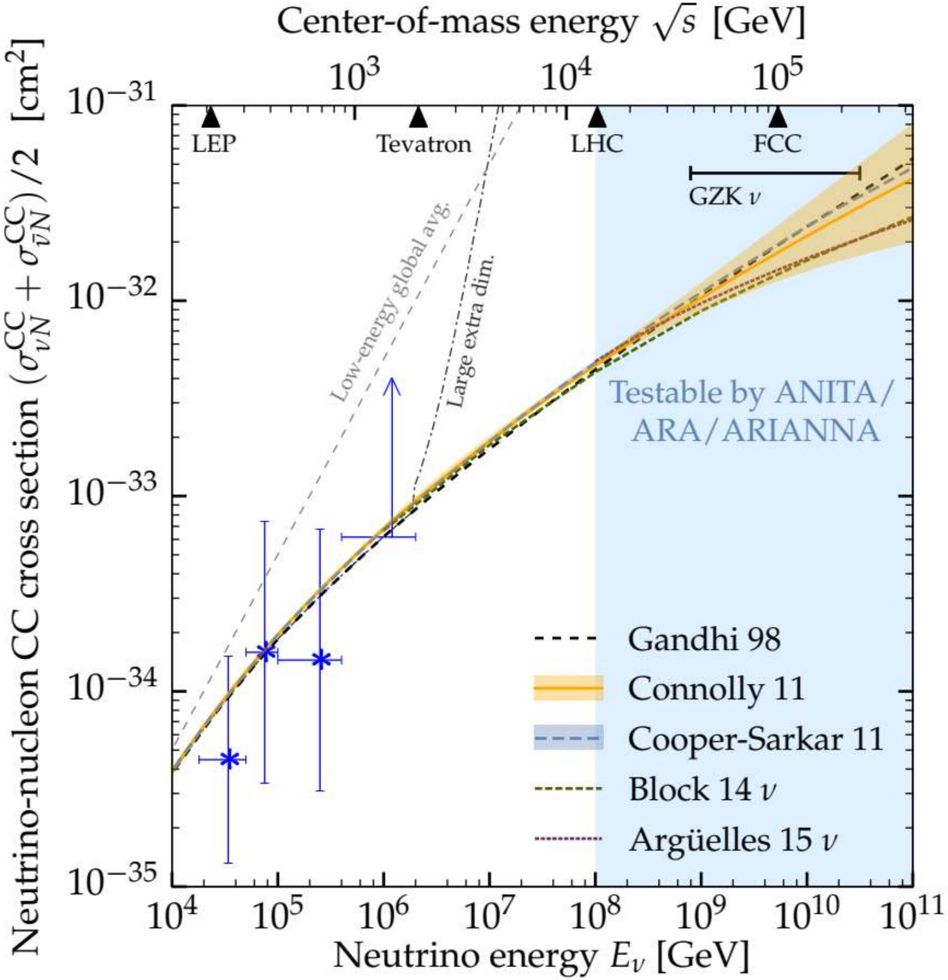
  - Machine learning separates shower from track
  - Different y distributions for ν and $\bar{\nu}$

Tighter contours, closer to combined fit



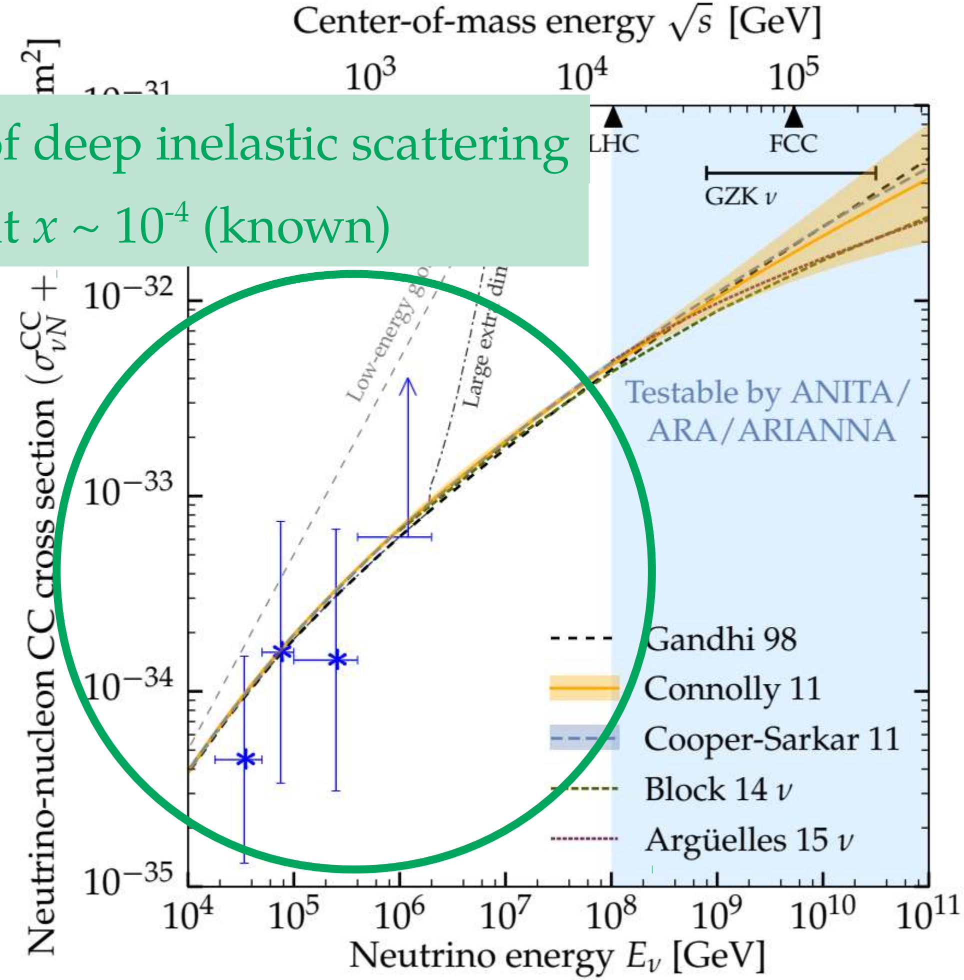
See presentation by Spencer Klein, ISVHECRI 2018

Quo vadis: IceCube vs. ANITA / ARA / ARIANNA



Quo vadis: IceCube vs. ANITA/ARA/ARIANNA

Test predictions of deep inelastic scattering – down to PDFs at $x \sim 10^{-4}$ (known)



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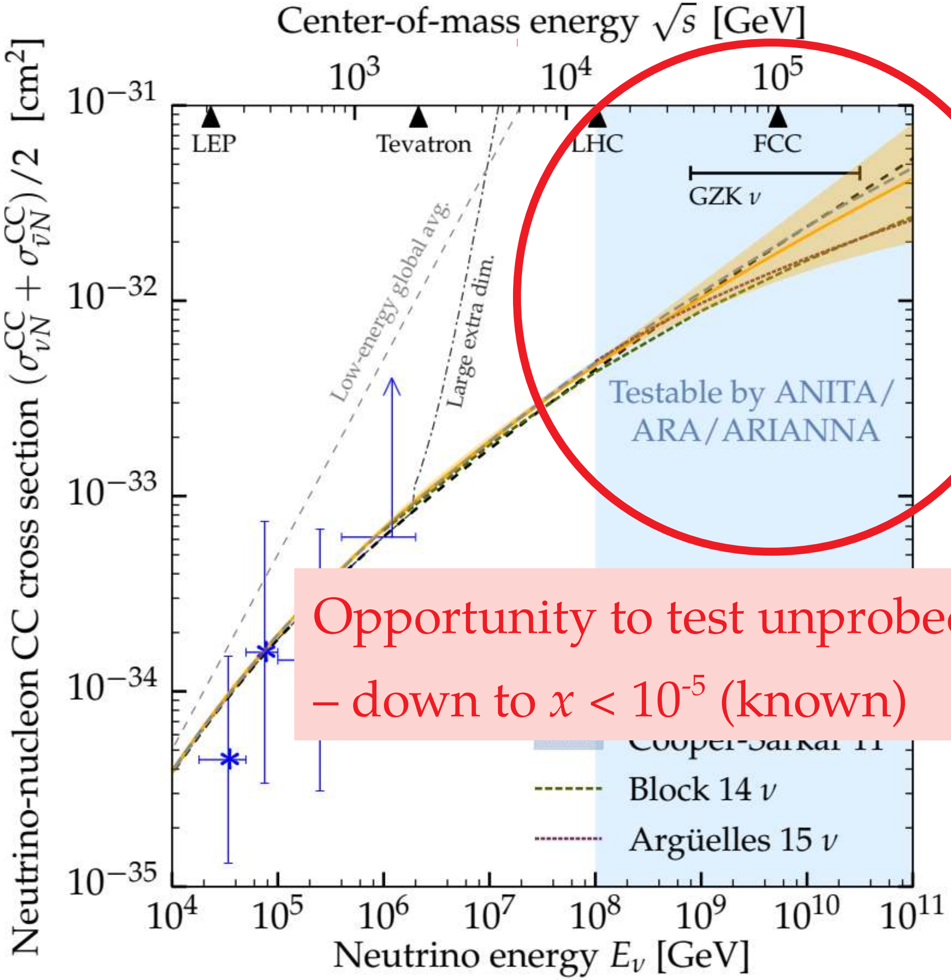


Opportunity to test unprobed nucleon structure – down to $x < 10^{-5}$ (known)

Summary

- We extracted the neutrino-nucleon cross section from 18 TeV to 2 PeV
  ↦ Previously known up to 350 GeV
- Found consistency with Standard-Model predictions
- Errors are still large due to statistics and astrophysical unknowns
- But both will be improved in the future

Already today, neutrino telescopes
are probing fundamental particle physics

Backup slides

Marginalized cross section in each bin

TABLE I. Neutrino-nucleon charged-current inclusive cross sections, averaged between neutrinos ($\sigma_{\nu N}^{\rm CC}$) and anti-neutrinos ($\sigma_{\bar{\nu} N}^{\rm CC}$), extracted from 6 years of IceCube HESE showers. To obtain these results, we fixed $\sigma_{\bar{\nu} N}^{\rm CC} = \langle \sigma_{\bar{\nu} N}^{\rm CC} / \sigma_{\nu N}^{\rm CC} \rangle \cdot \sigma_{\nu N}^{\rm CC}$ — where $\langle \sigma_{\bar{\nu} N}^{\rm CC} / \sigma_{\nu N}^{\rm CC} \rangle$ is the average ratio of $\bar{\nu}$ to ν cross sections calculated using the standard prediction from Ref. [60] — and $\sigma_{\nu N}^{\rm NC} = \sigma_{\nu N}^{\rm CC}/3$, $\sigma_{\bar{\nu} N}^{\rm NC} = \sigma_{\bar{\nu} N}^{\rm CC}/3$. Uncertainties are statistical plus systematic, added in quadrature.

E_ν [TeV]	$\langle E_\nu \rangle$ [TeV]	$\langle \sigma_{\bar{\nu} N}^{\rm CC} / \sigma_{\nu N}^{\rm CC} \rangle$	$\log_{10}[\frac{1}{2}(\sigma_{\nu N}^{\rm CC} + \sigma_{\bar{\nu} N}^{\rm CC})/{\rm cm}^2]$
18–50	32	0.752	-34.35 ± 0.53
50–100	75	0.825	-33.80 ± 0.67
100–400	250	0.888	-33.84 ± 0.67
400–2004	1202	0.957	> -33.21 (1σ)

MB & A. Connolly, 1711.11043

Neutrino zenith angle distribution

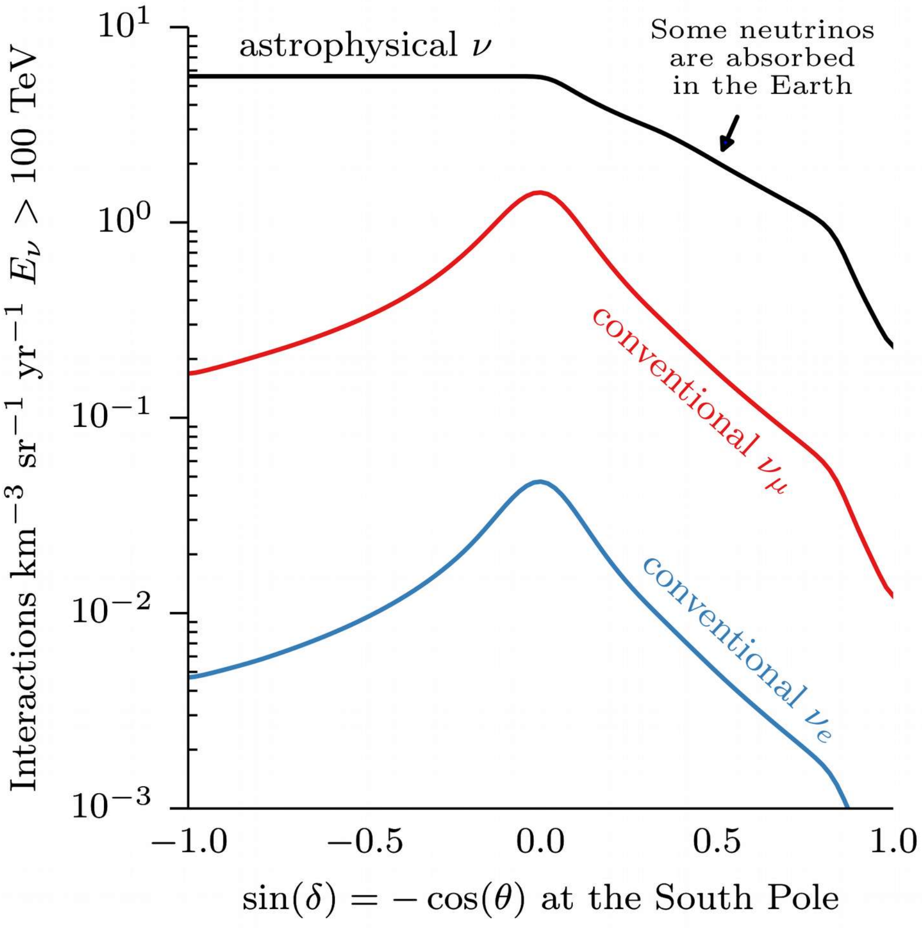
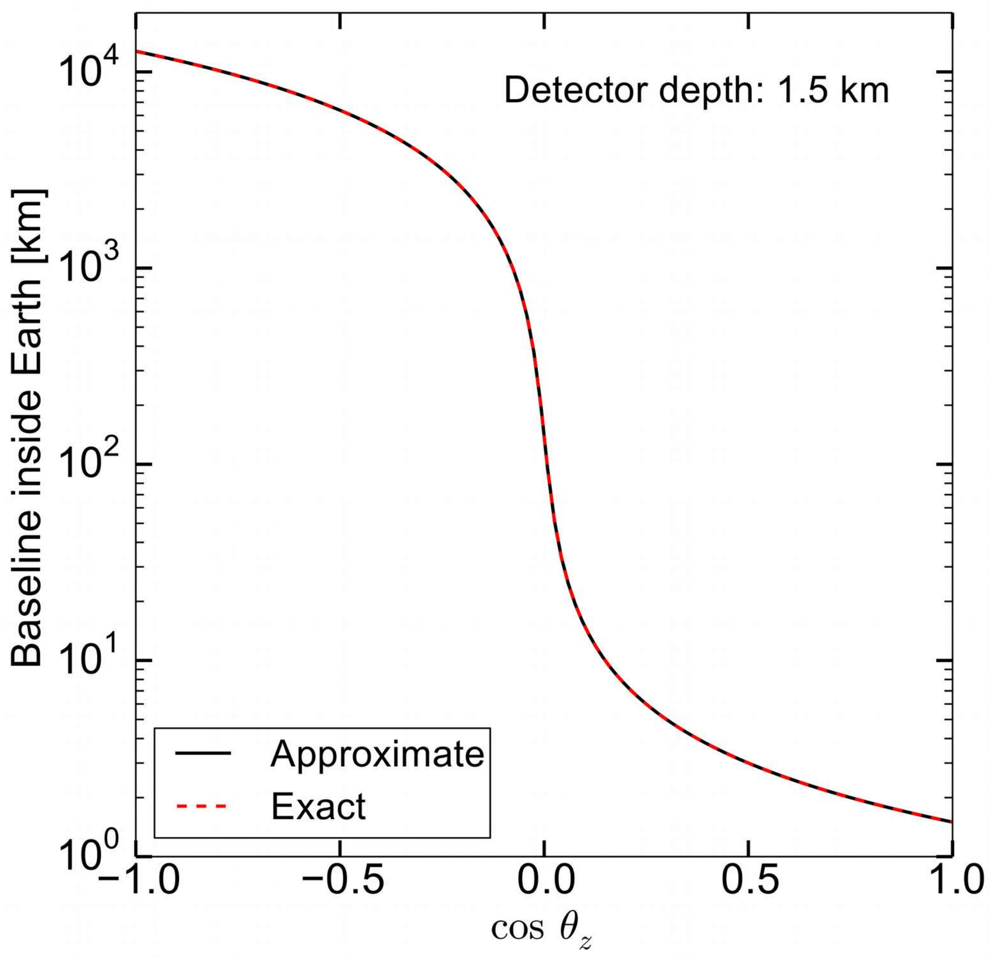
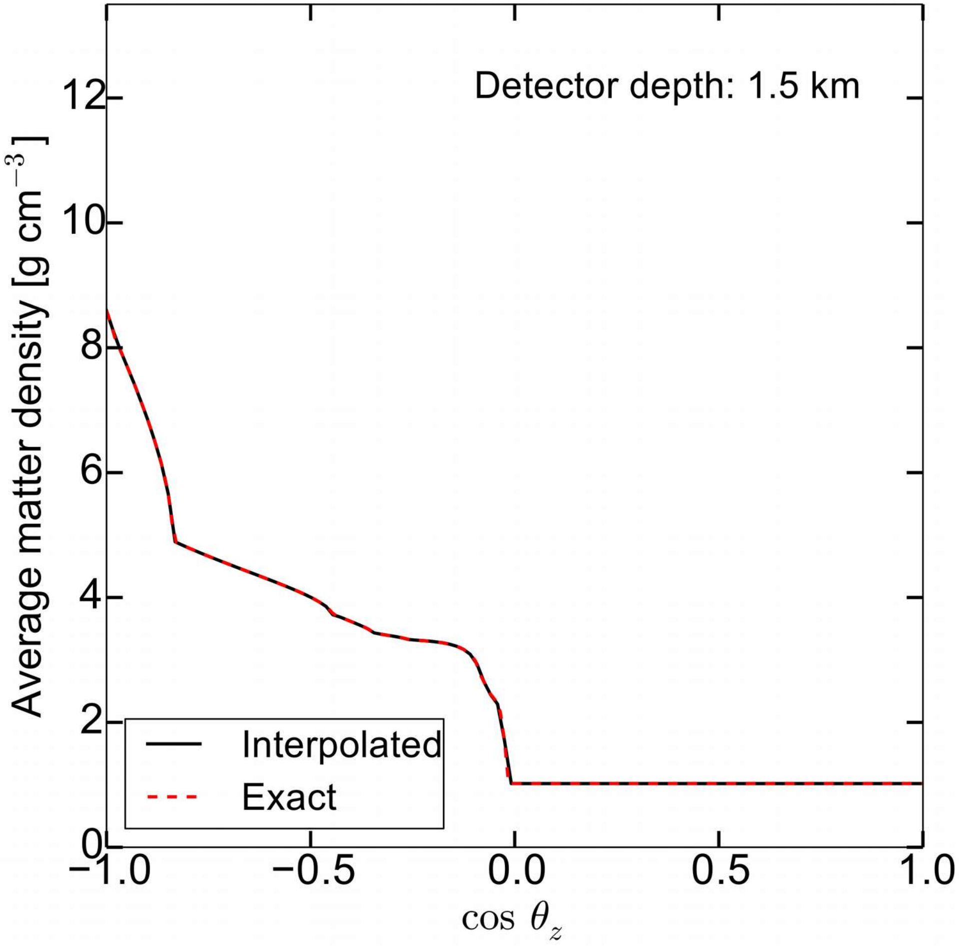


Figure by
Jakob Van Santen
ICRC 2017

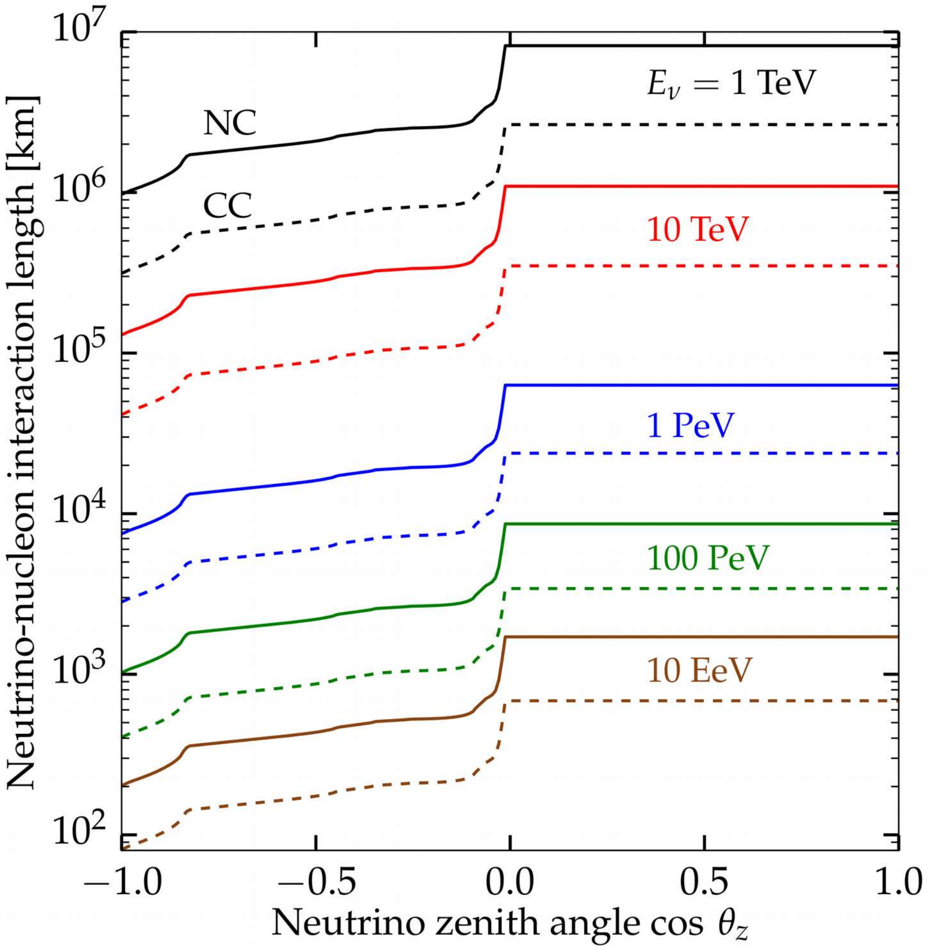
Neutrino baseline inside the Earth



Average Earth matter density



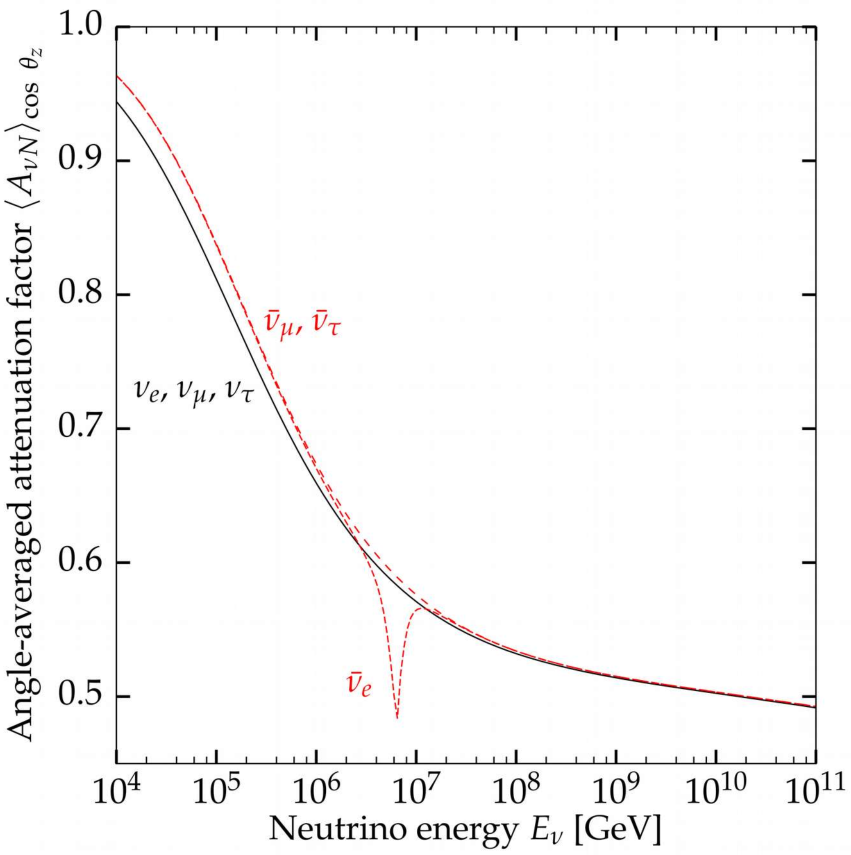
νN interaction length inside the Earth

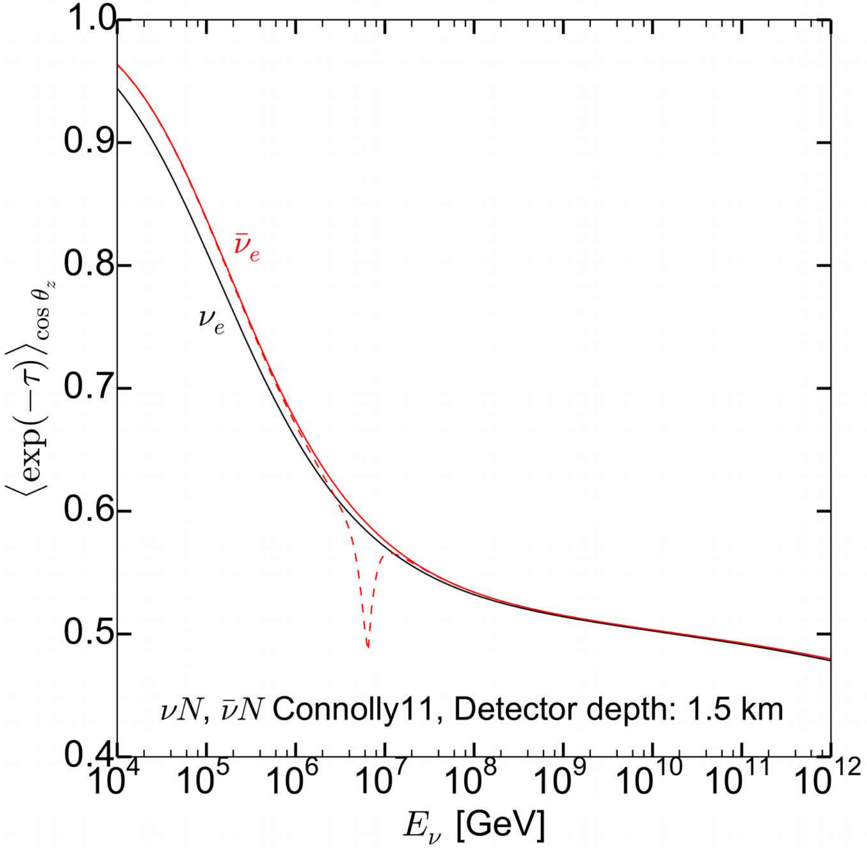


The $\overline{\nu}N$ interaction length is ~60% larger

MB & A. Connolly, 1711.11043

Angle-averaged absorption inside the Earth





Energy and angular shower spectra

Rate from all flavors, CC + NC:

$$\frac{d^2 N_{\rm sh}}{dE_{\rm sh}\, d\cos\theta_z} = \frac{d^2 N^{\rm CC}_{{\rm sh},e}}{dE_{\rm sh}\, d\cos\theta_z} + \underbrace{{\rm Br}_{\tau\to{\rm sh}}}_{=\,0.83} \frac{d^2 N^{\rm CC}_{{\rm sh},\tau}}{dE_{\rm sh}\, d\cos\theta_z} + \sum_{l=e,\mu,\tau} \frac{d^2 N^{\rm NC}_{{\rm sh},l}}{dE_{\rm sh}\, d\cos\theta_z}$$

Contribution from one flavor CC:

$$\frac{d^2 N^{\rm CC}_{{\rm sh},l}}{dE_{\rm sh}\, d\cos\theta_z}(E_{\rm sh}, \cos\theta_z) \simeq -2\pi \rho_{\rm ice} N_A V T \left\{ \Phi_l(E_\nu) \sigma^{\rm CC}_{\nu N}(E_\nu) e^{-\tau_{\nu N}(E_\nu,\theta_z)} + \Phi_{\bar{l}}(E_\nu) \sigma^{\rm CC}_{\bar{\nu} N}(E_\nu) e^{-\tau_{\bar{\nu} N}(E_\nu,\theta_z)} \right\} \Big|_{E_\nu = E_{\rm sh}/f_{l,{\rm CC}}}$$

Conversion between shower energy and neutrino energy:

$$f_{l,t} \equiv \frac{E_{\rm sh}}{E_\nu} \simeq \begin{cases} 1 & \text{for } l = e \;\text{ and }\; t = {\rm CC} \\ [\langle y \rangle + 0.7\,(1 - \langle y \rangle)] \simeq 0.8 & \text{for } l = \tau \;\text{ and }\; t = {\rm CC} \\ \langle y \rangle \simeq 0.25 & \text{for } l = e, \mu, \tau \;\text{ and }\; t = {\rm NC} \end{cases}$$

MB & A. Connolly, 1711.11043

Detector resolution

Number of contained showers:

$$\frac{d^2 N_{\rm sh}}{dE_{\rm dep} d\cos\theta_z} = \int dE_{\rm sh} \int d\cos\theta_z' \frac{d^2 N_{\rm sh}}{dE_{\rm sh} d\cos\theta_z'} R_E(E_{\rm sh}, E_{\rm dep}, \sigma_E(E_{\rm sh})) R_\theta(\cos\theta_z', \cos\theta_z, \sigma_{\cos\theta_z})$$

Energy resolution: [Palomares-Ruiz, Vincent, Mena *PRD* 2015; Vincent, Palomares-Ruiz, Mena *PRD* 2016; MB, Beacom. Murase, *PRD* 2016]

$$R_E(E_{\rm sh}, E_{\rm dep}, \sigma_E(E_{\rm sh})) = \frac{1}{\sqrt{2\pi\sigma_E^2(E_{\rm sh})}} \exp\left[-\frac{(E_{\rm sh} - E_{\rm dep})^2}{2\sigma_E^2(E_{\rm sh})}\right] \quad \text{with} \quad \sigma_E(E_{\rm sh}) = 0.1 E_{\rm sh}$$

IceCube, *JINST* 2014

Angular resolution:

$$R_\theta(\cos\theta_z', \cos\theta_z, \sigma_{\cos\theta_z}) = \frac{1}{\sqrt{2\pi\sigma_{\cos\theta_z}^2}} \exp\left[-\frac{(\cos\theta_z' - \cos\theta_z)^2}{2\sigma_{\cos\theta_z}^2}\right]$$

$$\text{with} \quad \sigma_{\cos\theta_z} \equiv \frac{1}{2}\left[|\cos(\theta_z + \sigma_{\theta_z}) - \cos\theta_z| + |\cos(\theta_z - \sigma_{\theta_z}) - \cos\theta_z|\right] \quad \text{and} \quad \sigma_{\theta_z} = 15^\circ$$

MB & A. Connolly, 1711.11043

Likelihood

In an energy bin containing $N_{\rm sh}^{\rm obs}$ observed showers, the likelihood is

Each energy bin is independent

$$\mathcal{L} = \frac{e^{-(N_{\rm sh}^{\rm atm}+N_{\rm sh}^{\rm ast})}}{N_{\rm sh}^{\rm obs}!} \prod_{i=1}^{N_{\rm sh}^{\rm obs}} \mathcal{L}_i$$

Partial likelihood, *i.e.*, relative probability of the *i*-th shower being from an atmospheric neutrino or an astrophysical neutrino:

Depends on $\sigma_{\nu N}$

$$\mathcal{L}_i = N_{\rm sh}^{\rm atm}\,\mathcal{P}_i^{\rm atm} + N_{\rm sh}^{\rm ast}\,\mathcal{P}_i^{\rm ast}$$

$$\mathcal{P}_i^{\rm atm} = \left(\int_{E_{\rm dep}^{\rm min}}^{E_{\rm dep}^{\rm max}} dE_{\rm dep} \int_{-1}^{1} d\cos\theta_z \frac{d^2 N_{\rm sh}^{\rm atm}}{dE_{\rm dep}d\cos\theta_z} \right)^{-1} \left(\left. \frac{d^2 N_{\rm sh}^{\rm atm}}{dE_{\rm dep}d\cos\theta_z} \right|_{E_{{\rm dep},i},\cos\theta_{z,i}} \right)$$

PDF for this shower to be made by an atmospheric ν

$$\mathcal{P}_i^{\rm ast} = \left(\int_{E_{\rm dep}^{\rm min}}^{E_{\rm dep}^{\rm max}} dE_{\rm dep} \int_{-1}^{1} d\cos\theta_z \frac{d^2 N_{\rm sh}^{\rm ast}}{dE_{\rm dep}d\cos\theta_z} \right)^{-1} \left(\left. \frac{d^2 N_{\rm sh}^{\rm ast}}{dE_{\rm dep}d\cos\theta_z} \right|_{E_{{\rm dep},i},\cos\theta_{z,i}} \right)$$

PDF for this shower to be made by an astrophysical ν

Depends on γ and $\sigma_{\nu N}$

MB & A. Connolly, 1711.11043

See also: Palomares-Ruiz, Vincent, Mena *PRD* 2015; Vincent, Palomares-Ruiz, Mena *PRD* 2016

Best-fit values and uncertainties

TABLE II. Best-fit values and 1σ uncertainties of the nuisance parameters in each energy bin: number of showers due to atmospheric neutrinos $N_{\rm sh}^{\rm atm}$, number of showers due to astrophysical neutrinos $N_{\rm sh}^{\rm ast}$, and astrophysical spectral index γ.

E_ν [TeV]	$N_{\rm sh}^{\rm atm}$	$N_{\rm sh}^{\rm ast}$	γ
18–50	4.2 ± 4.9	11.4 ± 3.5	2.38 ± 0.31
50–100	6.3 ± 5.3	11.7 ± 4.5	2.43 ± 0.31
100–400	6.4 ± 6.0	12.9 ± 5.2	2.49 ± 0.31
400–2004	1.2 ± 1.0	1.73 ± 0.89	2.37 ± 0.32

MB & A. Connolly, 1711.11043