



# Role of systematic uncertainties in atmospheric neutrino oscillations



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## TALK OUTLINE:

- **Global  $\nu$  data analysis 2018: hints for N.O.**
- **Challenges for future high-statistics expt's**
- **Effect of systematics in PINGU & ORCA**

Mainly based on:

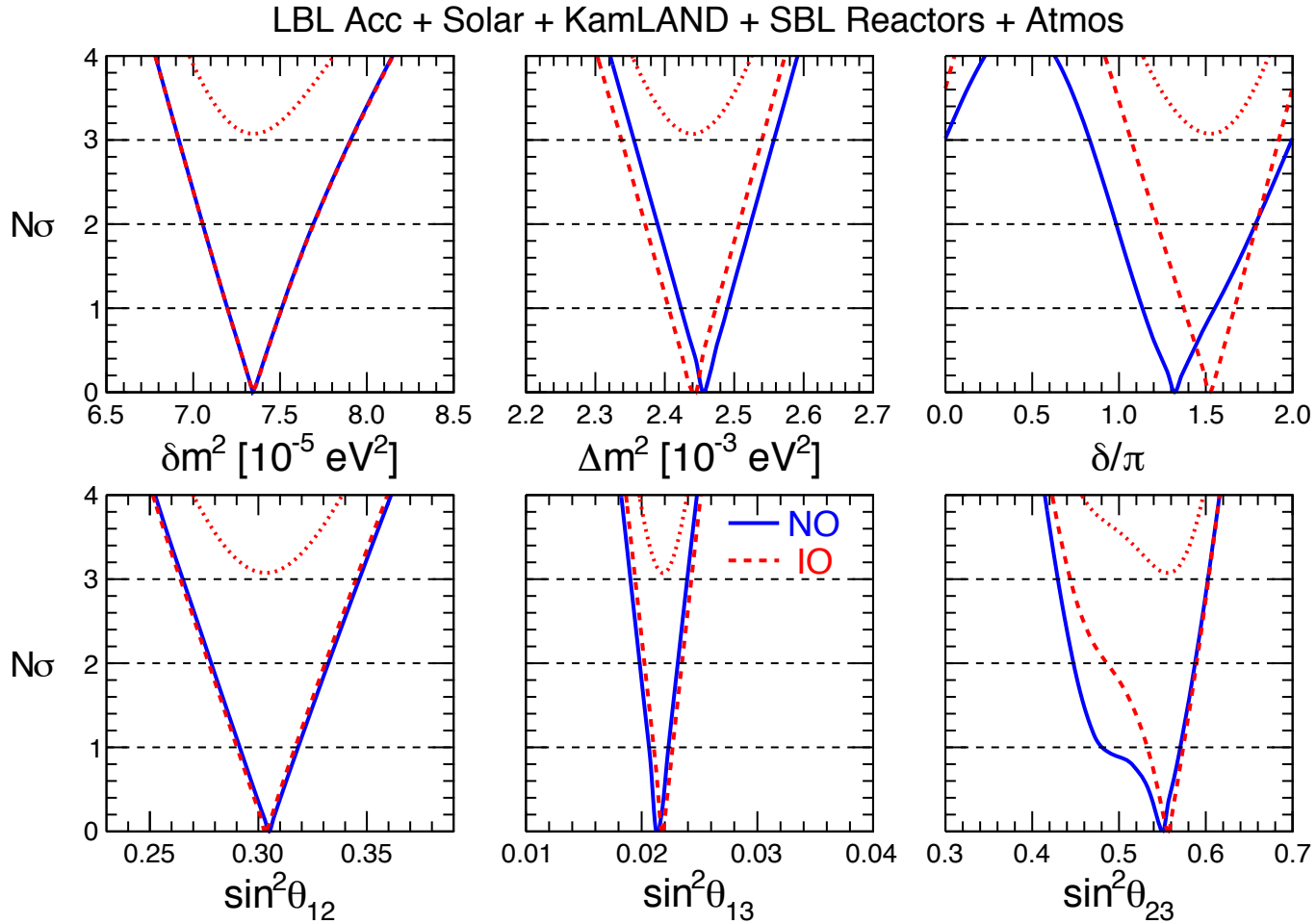
Capozzi, Lisi, Marrone, Palazzo, arXiv:1804:09678 (Global fit)

Capozzi, Lisi, Marrone, arXiv:1708:03022 (ORCA)

Capozzi, Lisi, Marrone, arXiv:1503:01999 (PINGU)

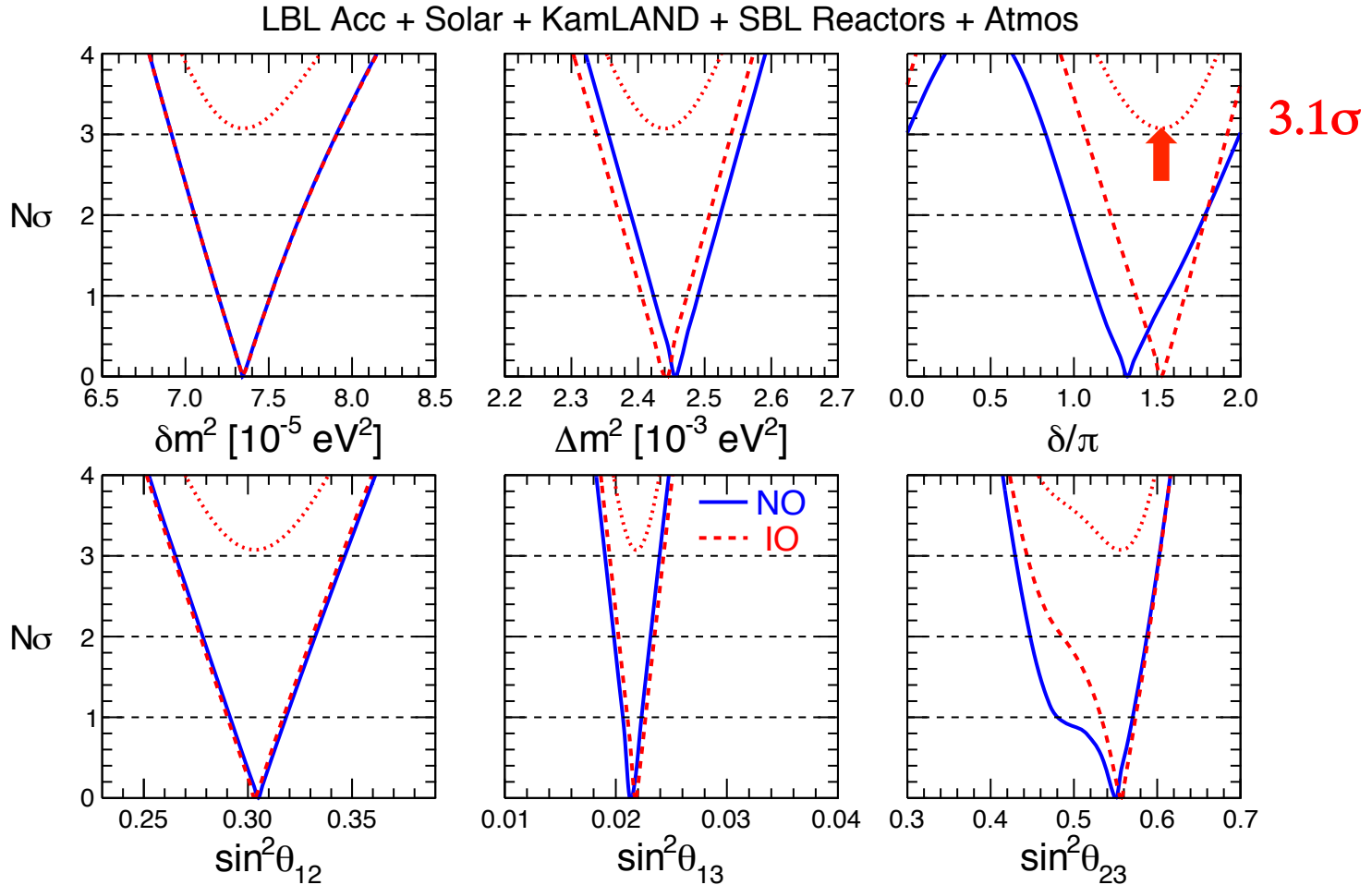
Capozzi, Lisi, Marrone, arXiv:1508:01392 (JUNO)

# Global analysis of neutrino oscillation data, circa 2018 [arXiv:1804.09678]



$$N\sigma = \sqrt{\chi^2}$$

# Global analysis of neutrino oscillation data, circa 2018 [arXiv:1804.09678]

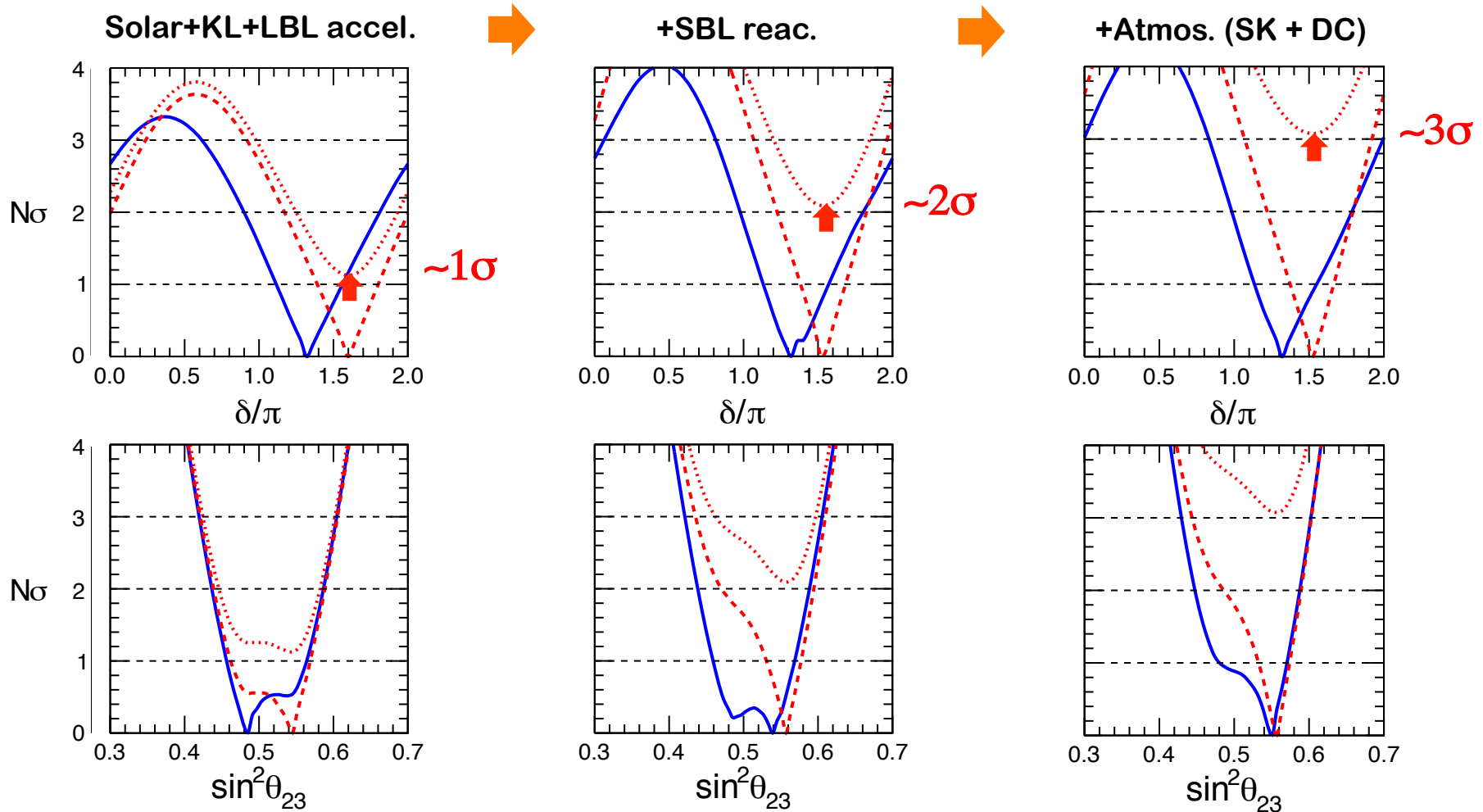


**New:** Inverted Ordering disfavored at  $\sim 3\sigma$

[Also: Improved constraints on  $\delta$ ,  $\theta_{23}$ ]

N.O. + 2<sup>nd</sup> octant hints = good news for atmospheric  $\nu$ !

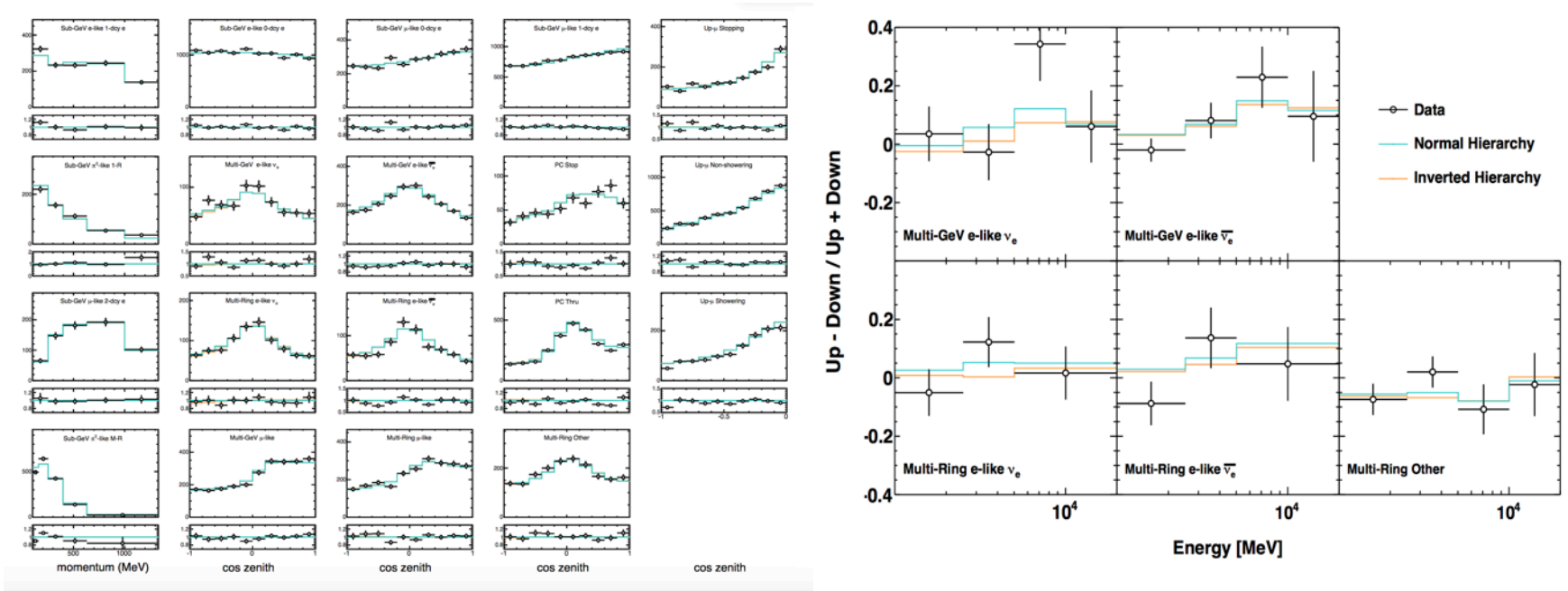
## Progression of bounds on $3\nu$ oscillation unknowns



**Atmospheric  $\nu$  contribute  $\sim 1/2$  of the IO-NO  $\Delta\chi^2$  difference.**

For the 1<sup>st</sup> time, we have used atm.  $\chi^2$  maps directly from SK (& DC). **Why?**

# SK: 520 energy-angle bins, 155 syst's (“pulls”)



Cannot be fully reproduced outside the SK Collaboration. Moreover:

**Mass ordering differences are small (no “bump” or “distortion” by eye!)**

**→ NO – IO sensitivity comes from cumulative  $\chi^2$  effects in the global fit**

External analyses can only use part of the data and of the systematics, and can test only in part these effects [roughly speaking, we saw  $\frac{1}{2}$  of mass-ordering  $\chi^2$  effect in our own atm. analysis].

**Need the best possible calculations of NO-IO templates + error estimates**



**Future: Also ORCA and PINGU will need to use hundreds of bins**

**The NO-IO discrimination will statistically emerge from the sum of many small contributions  $\Delta\chi^2 \ll 1$  in the spectra (histograms) each bin contribution being ~negligible by itself...**

**Need to be sure that, at the same time, the cumulative effect of many small syst's (each one being ~negligible) will not spoil it.**

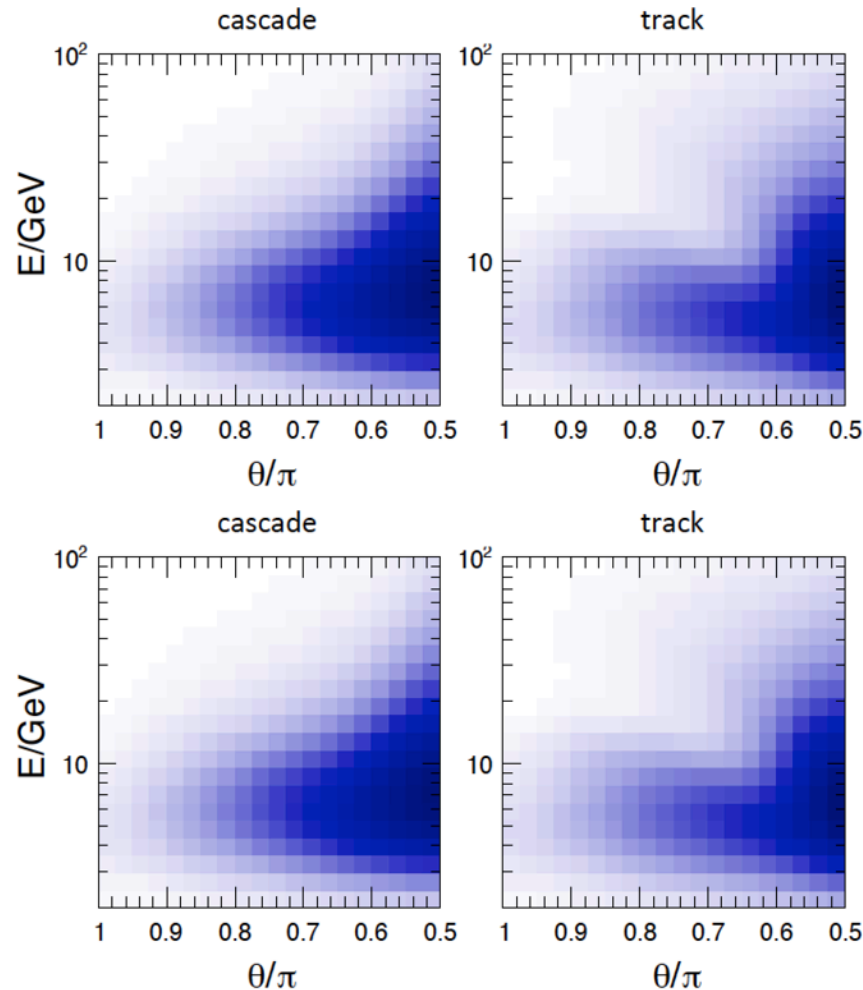
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**Using  $O(10)$  systematics as in current Lol studies is not enough.**

**Learning from SK experience,  $O(10^2)$  is a more realistic estimate. Each new systematic can only \*decrease\* the NO-IO difference. Maybe each decrease is a small fraction of  $1\sigma$ , but... all of them?**

**Even if each syst. is small, their cumulative “damage” may be large!**

Hard to tell any difference “by eye” also in future experiments



E.g., ORCA

NO

IO

Typical NO-IO differences: few % (smaller than the color ladder step)  
Need control of spectral systematics at percent level.



Once upon a time... all neutrino experiments were limited by stat's, and systematics could be treated as numbers (normalization, bias ...)

Now we have as many as  $O(10^6)$  events collected in SBL reactors, and we expect  $O(10^5)$  events in each of JUNO, ORCA, PINGU expt's

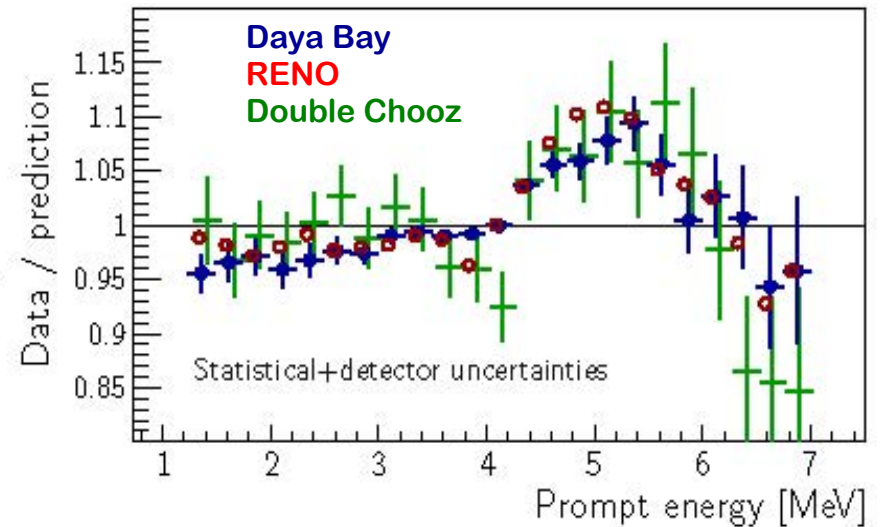
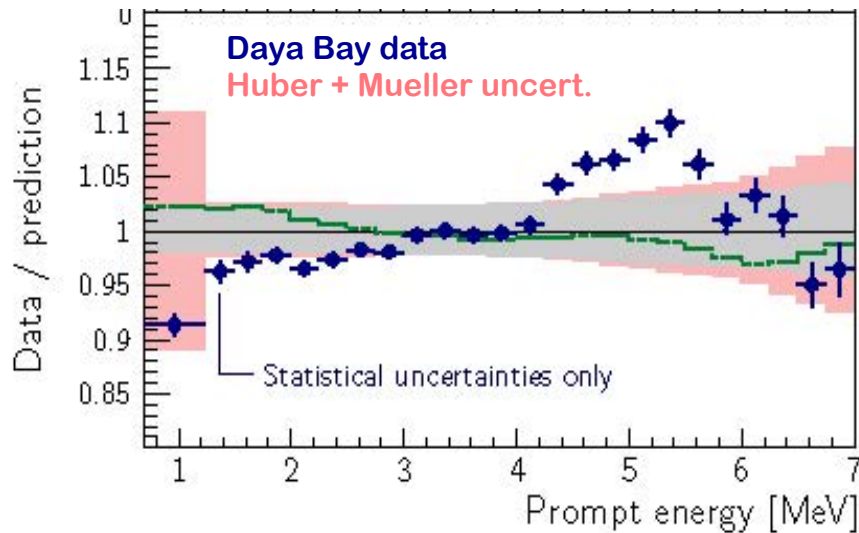
**Systematic errors are no longer “numbers” but become “functions”.  
Dedicated approaches are needed to deal with such uncertainties.**

[This transition has already taken place in other fields, such as in parton distribution function fits and precision cosmology forecasts.]

**Unprecedented challenges are awaiting us in neutrino data analyses:**

We must be prepared to deal with “functions” which *ideally* should be known in size, shape, correlations and probability distributions, but *in practice* may also be partly (or totally) unknown.

Hard lesson learned from SBL reactor experiments:  
**An unknown systematic error source (function)  $\delta\Phi(E)$ ,**  
 well beyond supposedly-known shape uncertainties!



From S. Jetter (TAU 2014) & J. Cao (TAUP 2015)

Now we know its shape, and can correct for it, but residuals do remain:

**energy-scale uncertainties**

$E \rightarrow E'(E)$

(x-axis “stretch”)

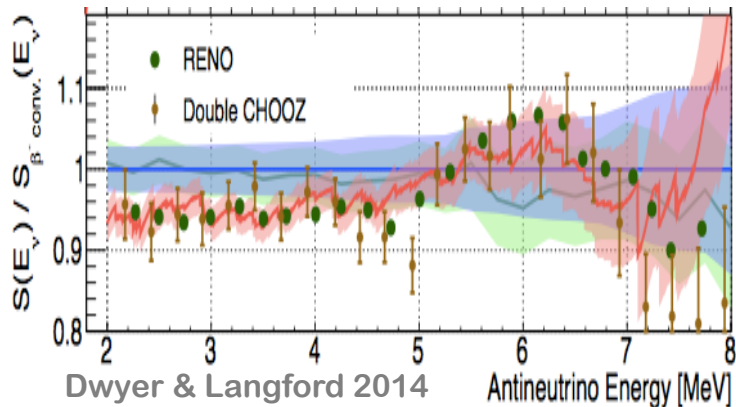
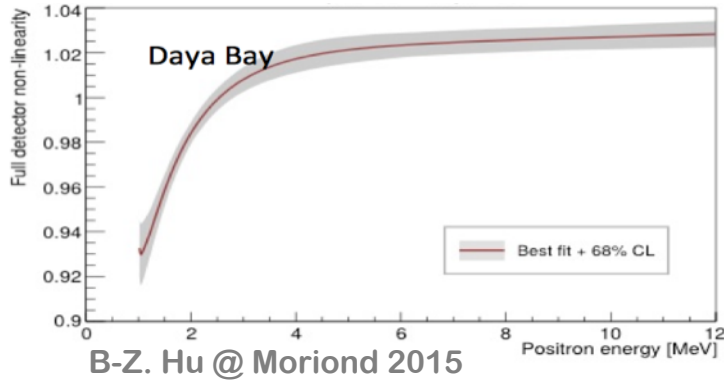
**flux-shape uncertainties**

$\Phi(E) \rightarrow \Phi'(E)$

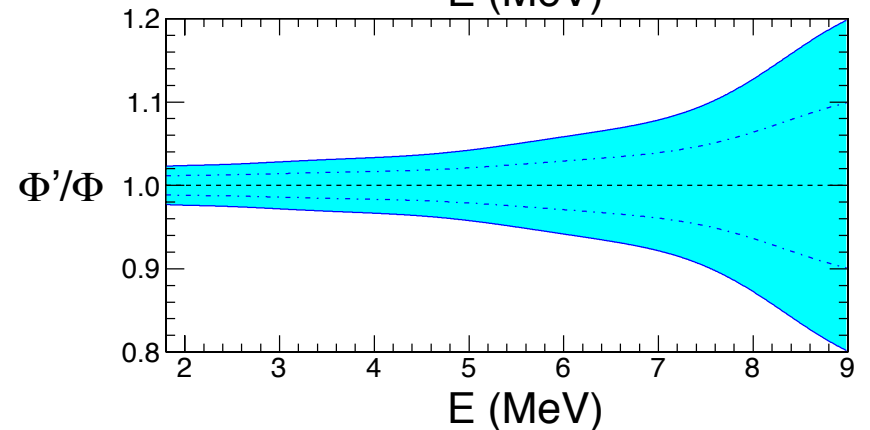
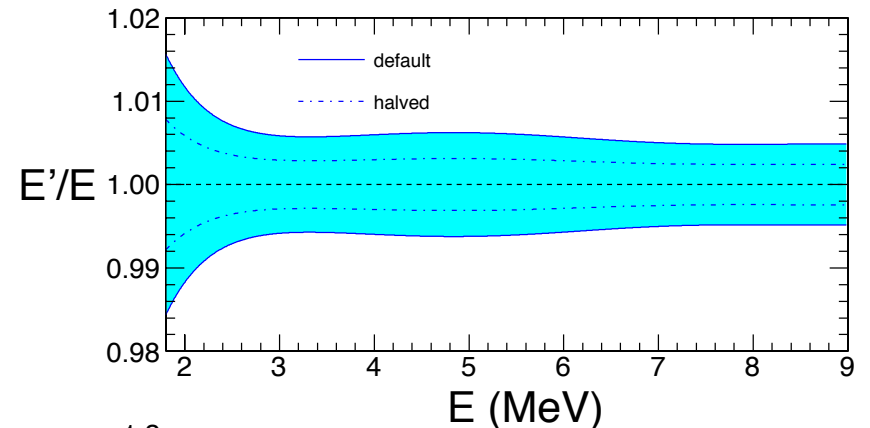
(y-axis “stretch”)

# Typical size of energy-scale and flux-shape errors

## $E'(E)$ and $\Phi'(E)$ models



## Relative $1\sigma$ error bands



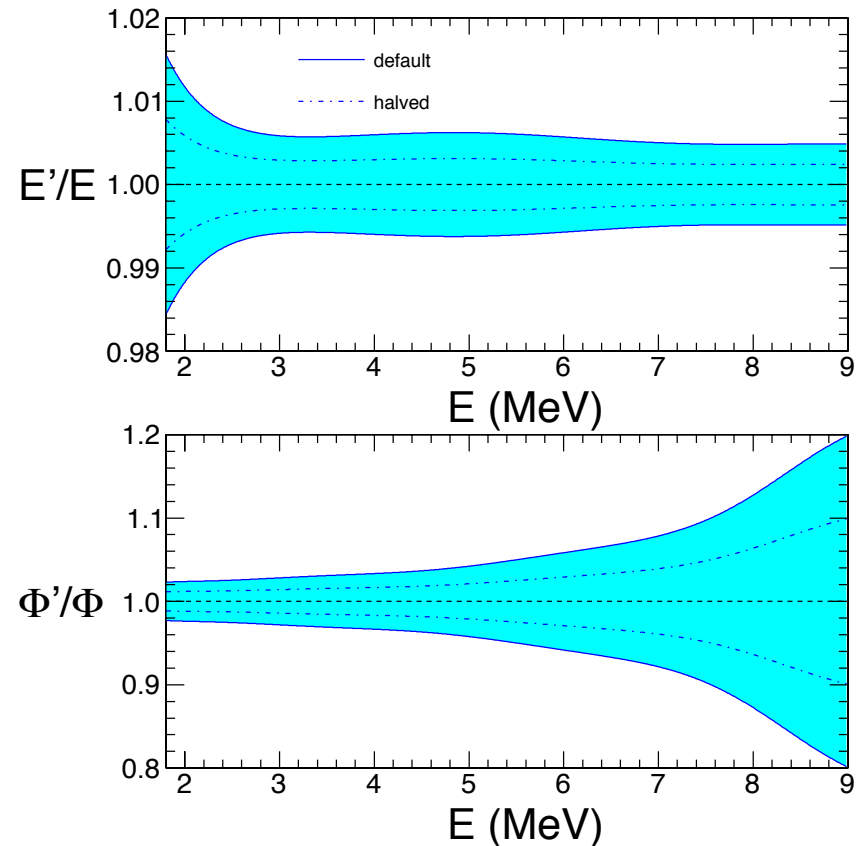
Errors assumed to be linear and symmetric (gaussian)

## Relative $1\sigma$ error bands

Now, allow smooth deviations  
 $E \rightarrow E'(E)$  and  $\Phi(E) \rightarrow \Phi'(E)$   
within the above error bands.

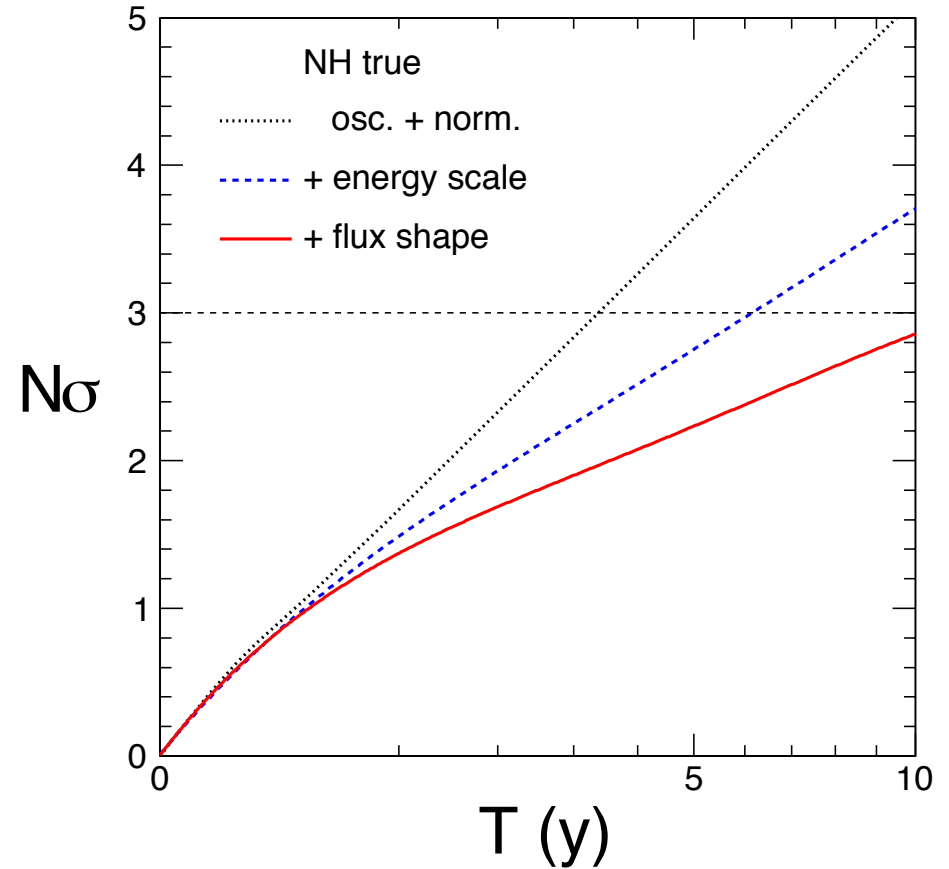
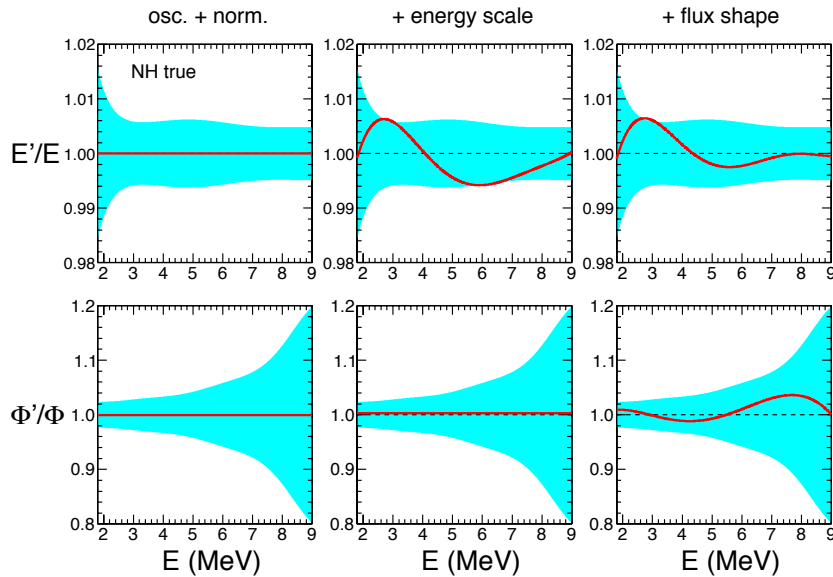
How will these uncertainties  
affect the **hierarchy sensitivity**  
in a JUNO-like experiment?

[in addition to “usual” oscillation  
and normalization uncertainties]



Details in arXiv:1508.01391

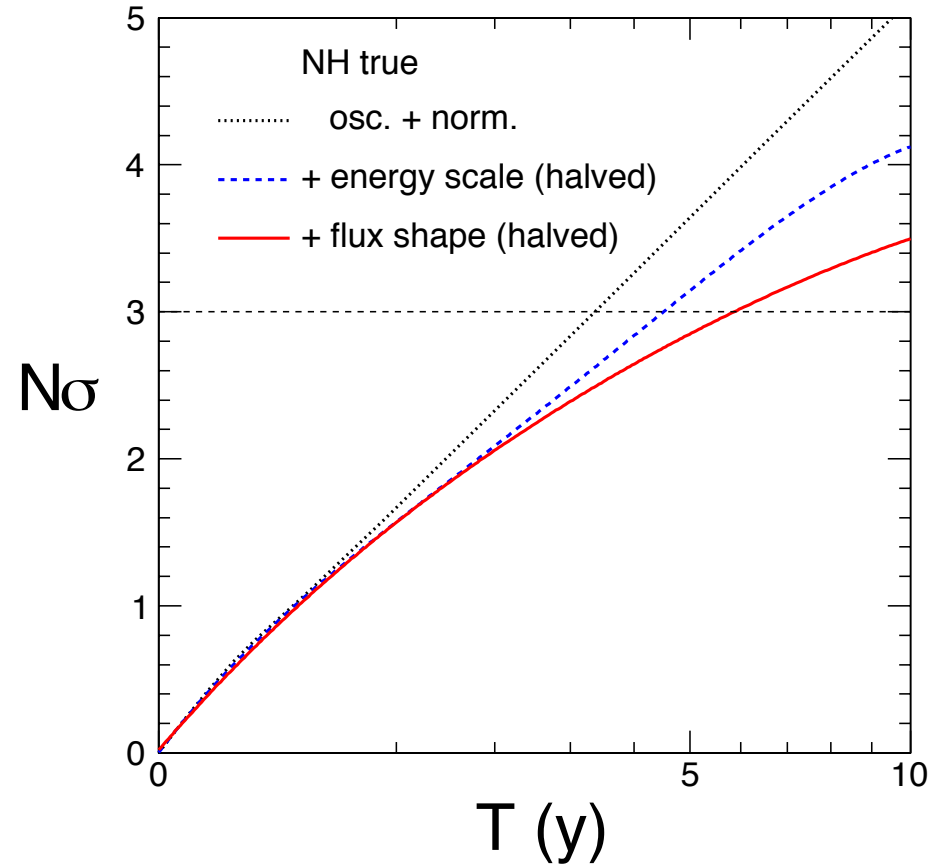
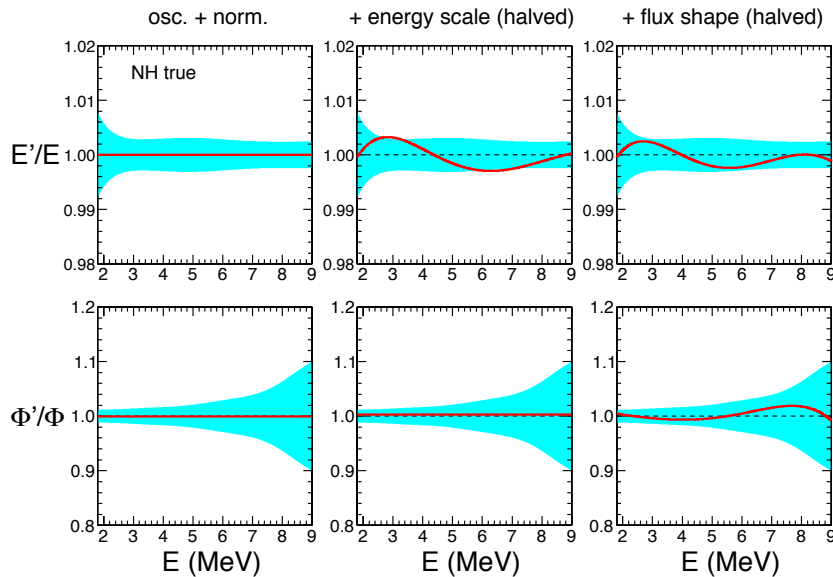
# Energy-scale and flux-shape errors with constrained “size” but unconstrained “shape” can bring the JUNO sensitivity below $3\sigma$



(Note abscissa prop. to  $\sqrt{T}$ )

# Roughly need halving their size to bring JUNO above $3\sigma$ in $\sim 5$ years

[similar results for the case of true IH, see 1508.01391]



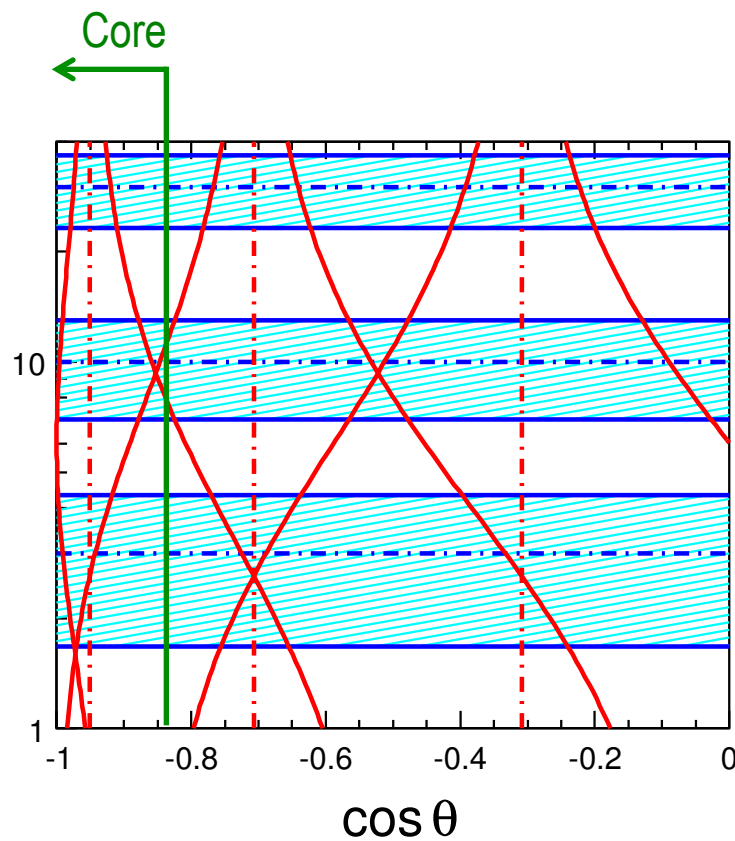
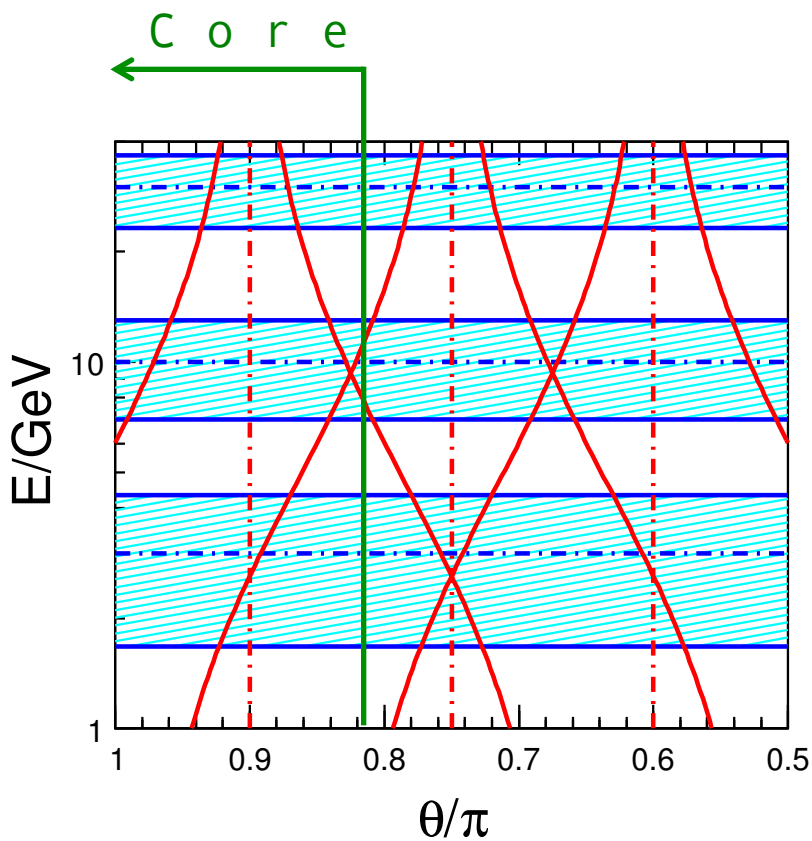
Note that JUNO will involve a 1D spectrum. PINGU and ORCA provide a more challenging 2D spectrum\*, in terms of energy  $E$  and direction  $\theta \rightarrow$

\*2D is already an approximation with respect to real 3D spectra: azimuthal symmetry is never realized...

# Analysis of a PINGU-like experiment [1503.01999]

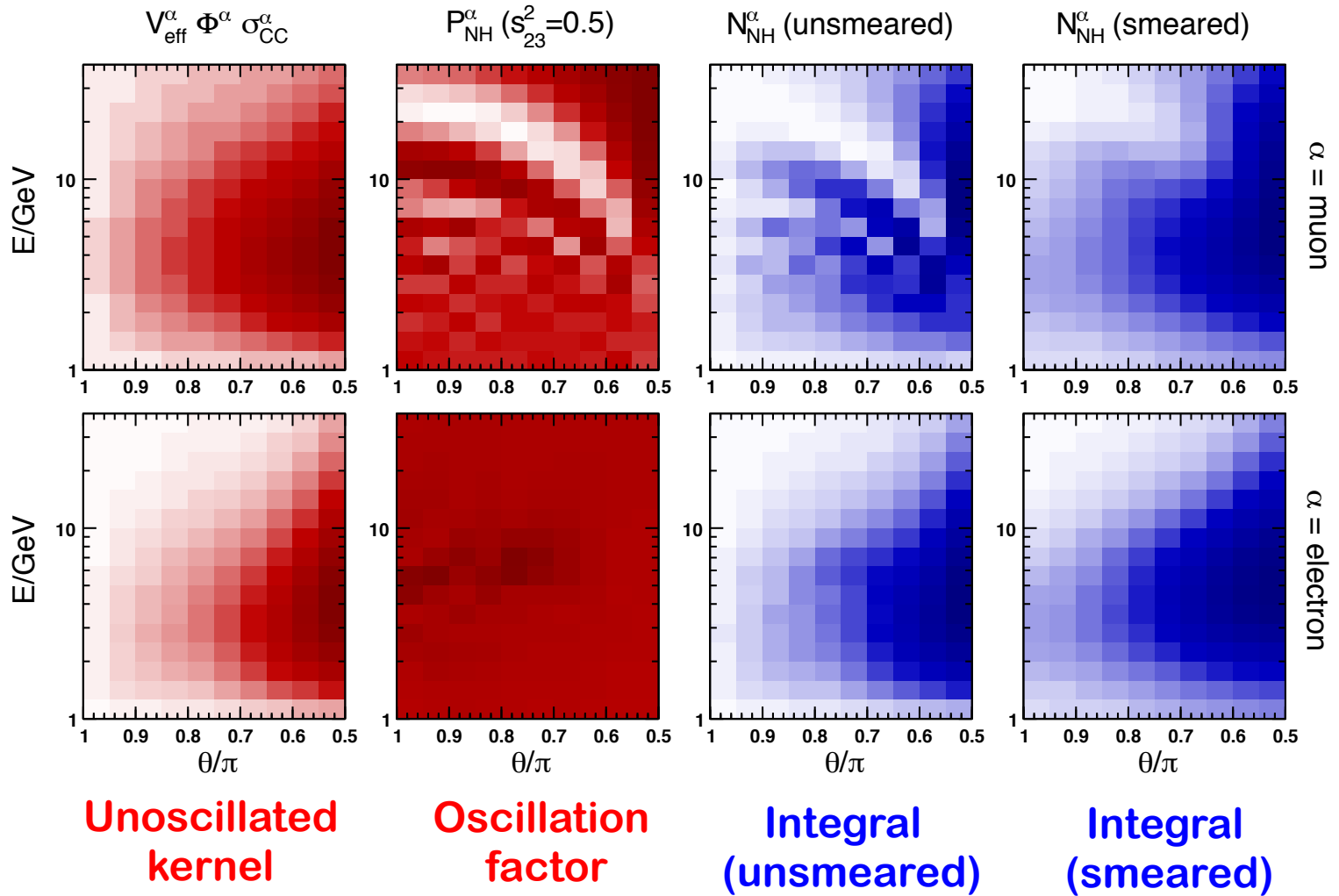
Note: we use  $(E, \theta)$ , not  $(E, \cos\theta)$ . Reasons:

- (1)  $\theta$  resolution width is asymmetric in  $\cos\theta$ ;
- (2)  $\cos\theta$  squeezes the interesting core region.





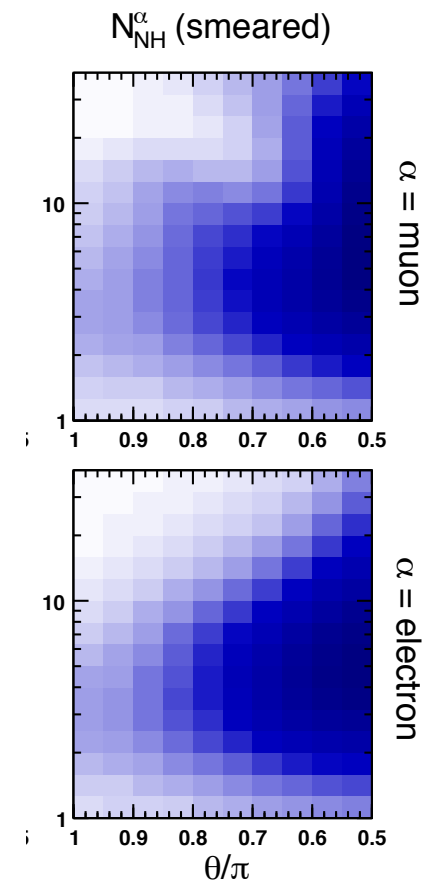
Observable (lepton) spectra come out from multiple integrations over unobservable (neutrino) kernels



Once more: This is what we can experimentally observe.

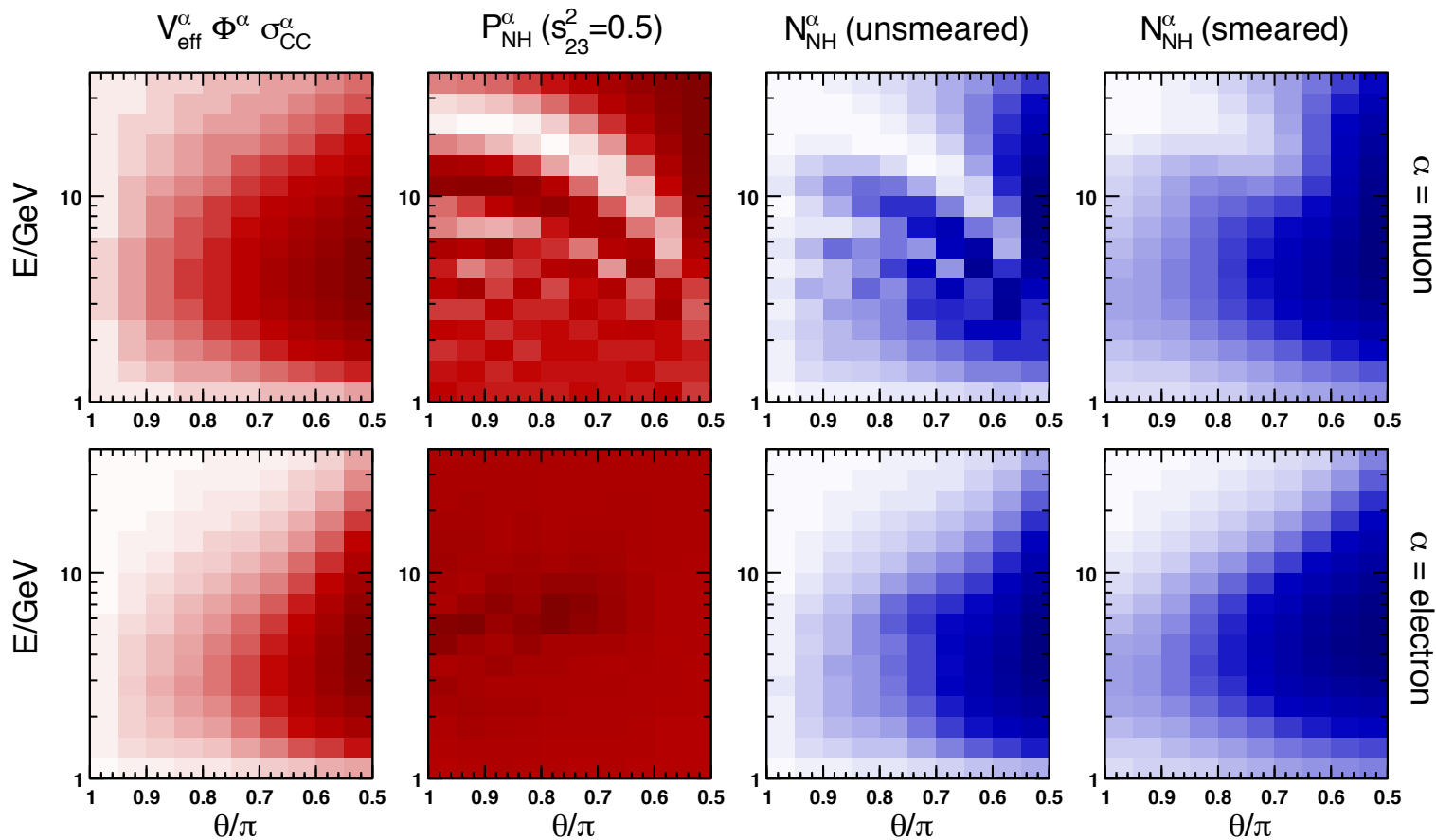
By eye, you would not notice any difference from NO to IO (typically, few % variations in each bin, smaller than color ladder step)

Crucial to control systematic errors at percent level.



Integral  
(smeared)

# Sources of systematic errors (list probably incomplete!)



**Effective volume**  
(normaliz. & shape)

**Atmospheric flux**  
(normaliz. & shape)

**Cross section**  
(normaliz. & shape)

**Earth Matter profile**

**Oscillation param's**

**Simplified kernels**

**Numerical approxim.**

**Finite MC statistics**

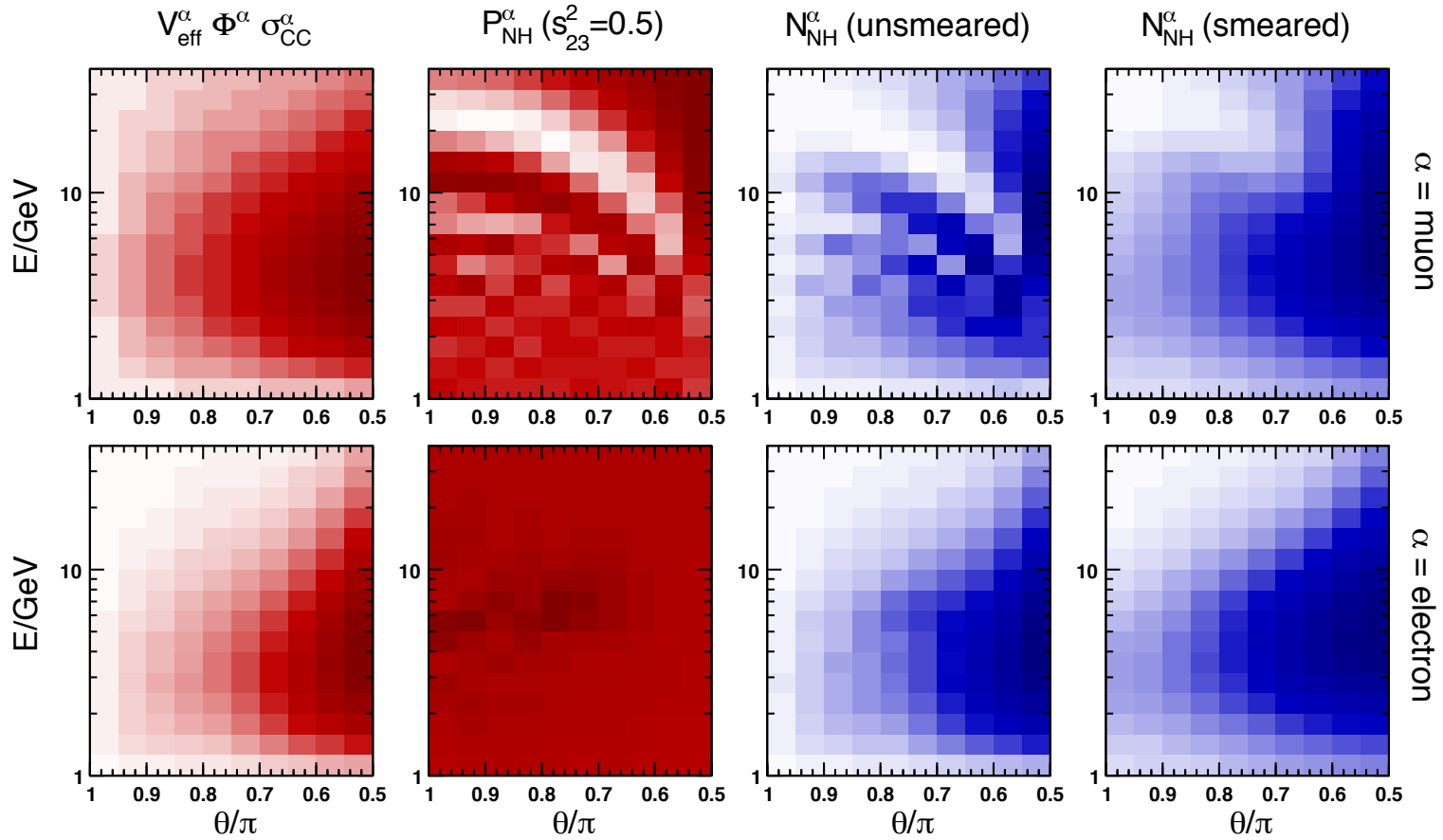
**Unknown residuals**

**Energy scale**

**E-resolution width**

**$\theta$ -resolution width**

# Our implementation of systematic errors



**Effective volume**  
**Atmospheric flux**  
**Cross section**

normalizations:  
 15% overall  
 8% mu/e  
 6% nu/antineu  
 all linearized (pulls)

shape: generic polyn's  
 (up to 28 free param.)

**Earth Matter profile**  
 3% core density error

**Oscillation param's**  
 linearized pulls except  
 for  $\theta$ -23 (scanned) and  
 $\delta$ -CP (marginalized)

**Simplified kernels**  
**Numerical approxim.**  
**Finite MC statistics**  
**Unknown residuals**

uncorrelated syst.  
 errors in each bin

**Energy scale**  
 5% bias

**E-resolution width**  
 10% fractional error  
 **$\theta$ -resolution width**  
 10% fractional error

all linearized pulls

# Comments

- (1) oscillation + normalization errors: most obvious and known sources. Must scan in  $\theta$ -23 and  $\delta$ -CP
- (2) resolution errors: less obvious but quite relevant
- (3) shape errors: poorly known at present
- (4) uncorrelated systematics: mostly unknown “by definition”

## (1) oscillation + normalization errors

Effective volume  
Atmospheric flux  
Cross section

normalizations:  
15% overall  
8%  $\mu/e$   
6%  $\nu/\bar{\nu}$   
all linearized (pulls)

shape:  
generic polynomials

Earth Matter profile  
3% core density

Oscillation param's  
all linearized (pulls)  
but  $\theta$ 23 and  $\delta$ -CP

## (4) uncorr. syst.

Simplified kernels  
Numerical approxim.  
Finite MC statistics  
Unknown residuals

uncorrelated syst.  
errors in each bin

## (2) resolution err.

Energy scale  
5% bias

E-resolution width  
10% fractional error  
 $\theta$ -resolution width  
10% fractional error

all linearized (pulls)

## (3) 2D spectrum shape errors

# 2D-spectrum shape systematics (eff. volume, atmos. flux, Xsection)

Focus on atmospheric flux errors (1D MC) from Barr et al., astro-ph/0611266

Work inspired by a 2004 RCCN Workshop (Kajita & Okumura editors)

Features mostly known to experts in the field, but worth repeating

## Uncertainties in Atmospheric Neutrino Fluxes

G.D. Barr<sup>1</sup> and S. Robbins<sup>1</sup>  
*Department of Physics, University of Oxford,  
Denys Wilkinson Building,  
Keble Road, Oxford, UK, OX1 3RH*

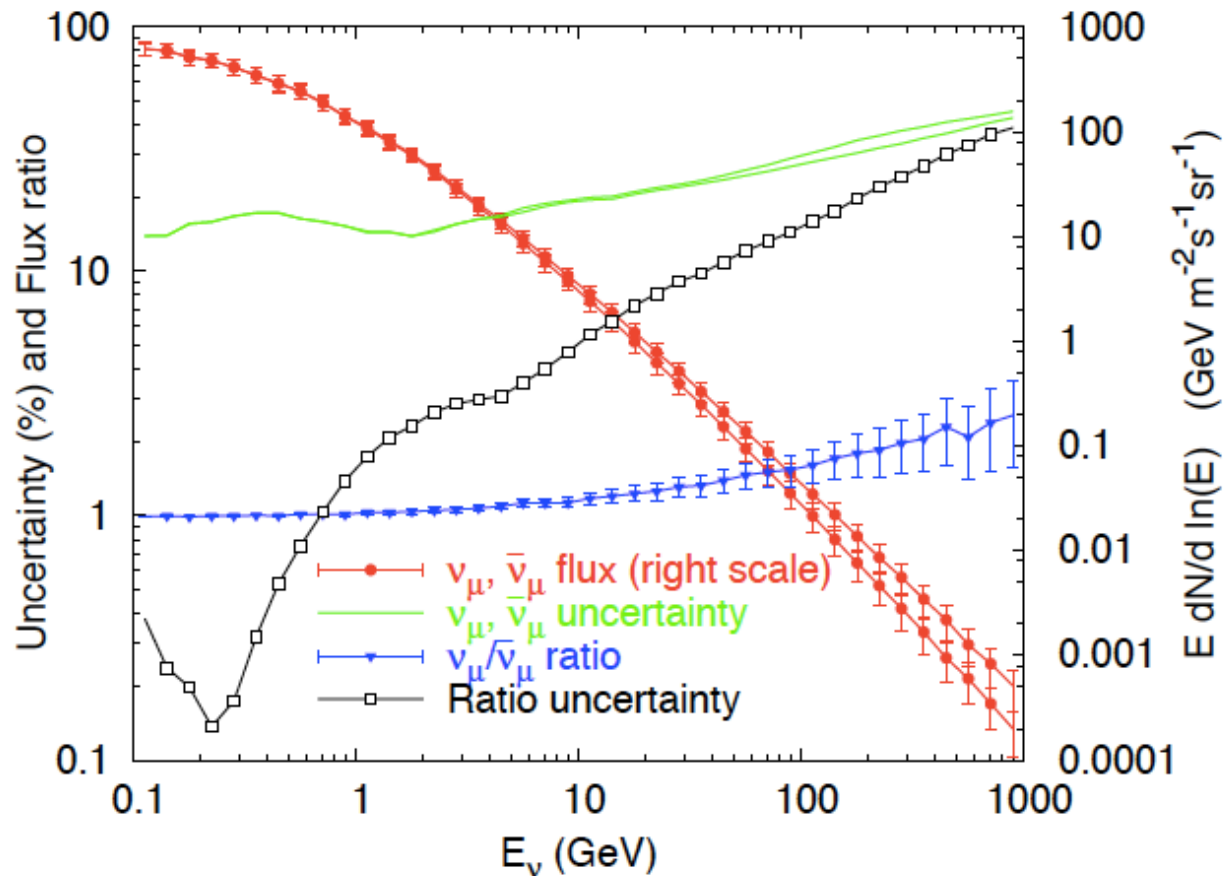
T.K. Gaisser and T. Stanev  
*Bartol Research Institute and Department of Physics and Astronomy,  
University of Delaware, Newark, Delaware, USA 19716*  
(Dated: 21 June 2006)

An evaluation of the principal uncertainties in the computation of neutrino fluxes produced in cosmic ray showers in the atmosphere is presented. The neutrino flux predictions are needed for comparison with experiment to perform neutrino oscillation studies. The paper concentrates on the main limitations which are due to hadron production uncertainties. It also treats primary cosmic ray flux uncertainties, which are at a lower level. The absolute neutrino fluxes are found to have errors of around 15% in the neutrino energy region important for contained events underground. Large cancellations of these errors occur when ratios of fluxes are considered, in particular, the  $\nu_\mu/\bar{\nu}_\mu$  ratio below  $E_\nu = 1$  GeV, the  $(\nu_\mu + \bar{\nu}_\mu)/(\nu_e + \bar{\nu}_e)$  ratio below  $E_\nu = 10$  GeV and the up/down ratios above  $E_\nu = 1$  GeV are at the 1% level. A detailed breakdown of the origin of these errors and cancellations is presented.

[Qualitatively, similar arguments apply also to effective volume and Xsection uncertainties.]

Recent related work by Fedynitch+, ICRC 2017 + here @ PANE 2018

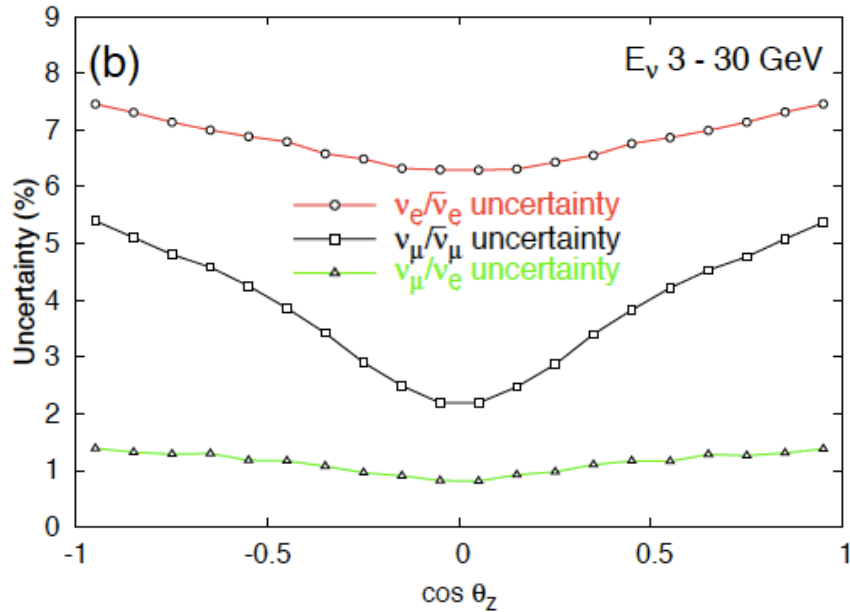
Absolute flux errors are large and energy-dependent  
Flux ratio errors are smaller but still energy-dependent



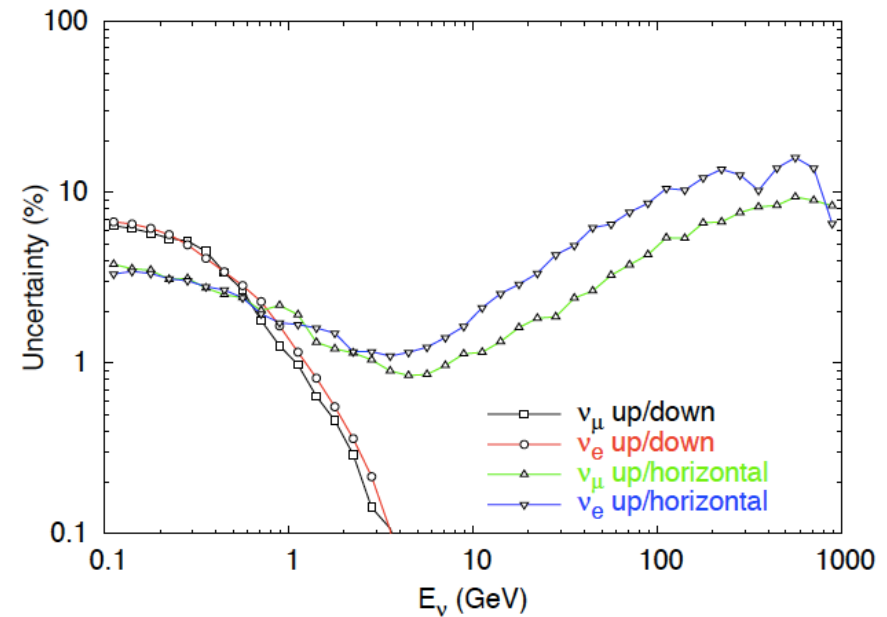
Actually they depend \*at the same time\* on energy and direction



relative errors depend on direction at given energy...



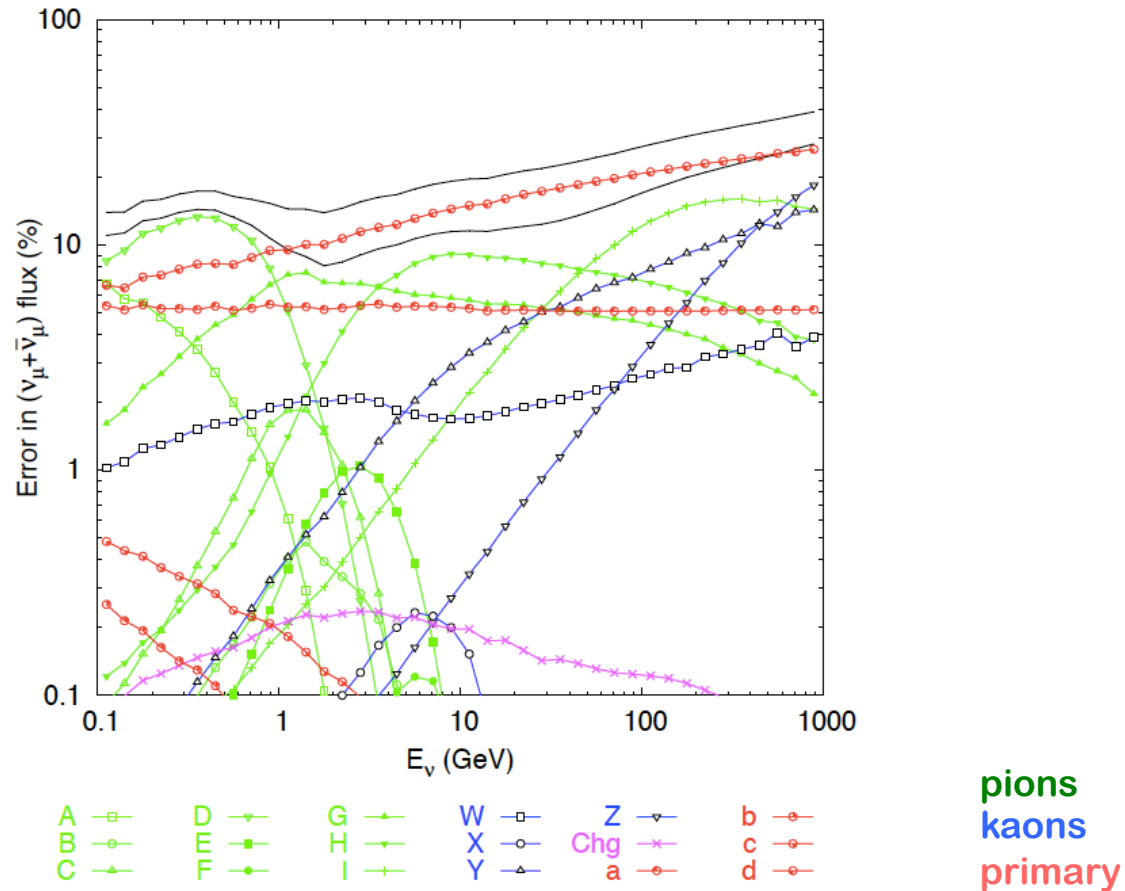
and on energy for given directions...



In general, 2D dependence cannot be factorized

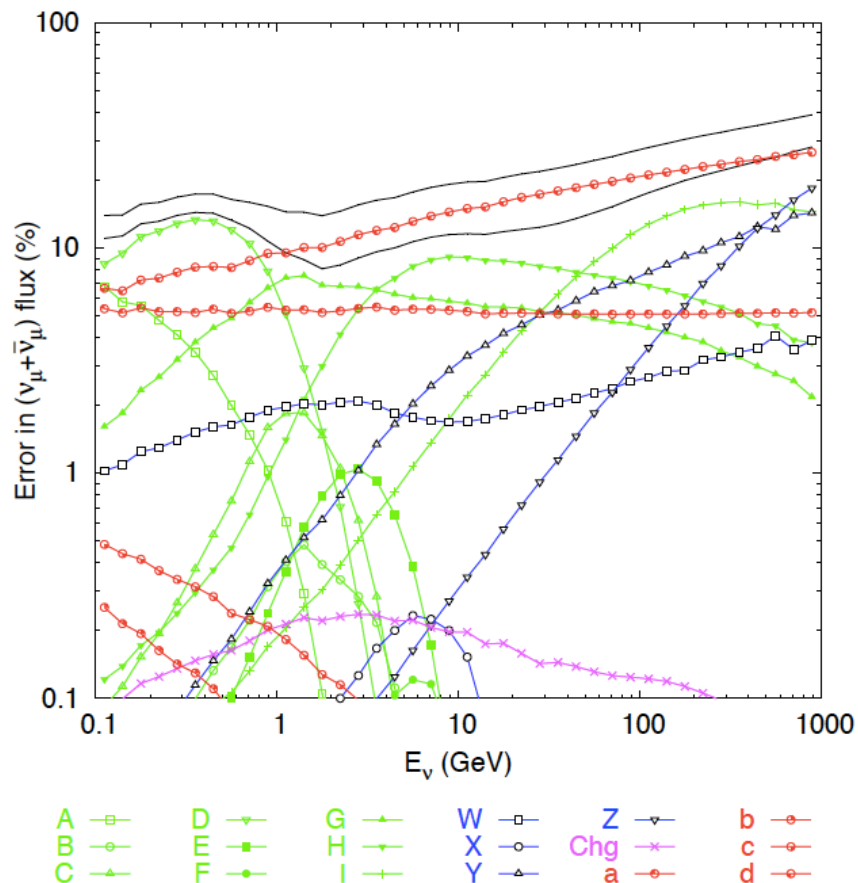
Note typical size from O(1) to O(10) percent

## Breakdown to individual error sources (26 in Barr et al.)



Treating each source as a “pull” function with penalties in a  $\chi^2$  approach, the overall 2D flux spectrum would be rather “flexible” at the few % level  
**Not captured by a handful of normalization or tilt uncertainties!**

Ideally, this breakdown should be repeated and specialized for PINGU and ORCA sites, possibly within up-to-date, 3D atmospheric MC calculations.

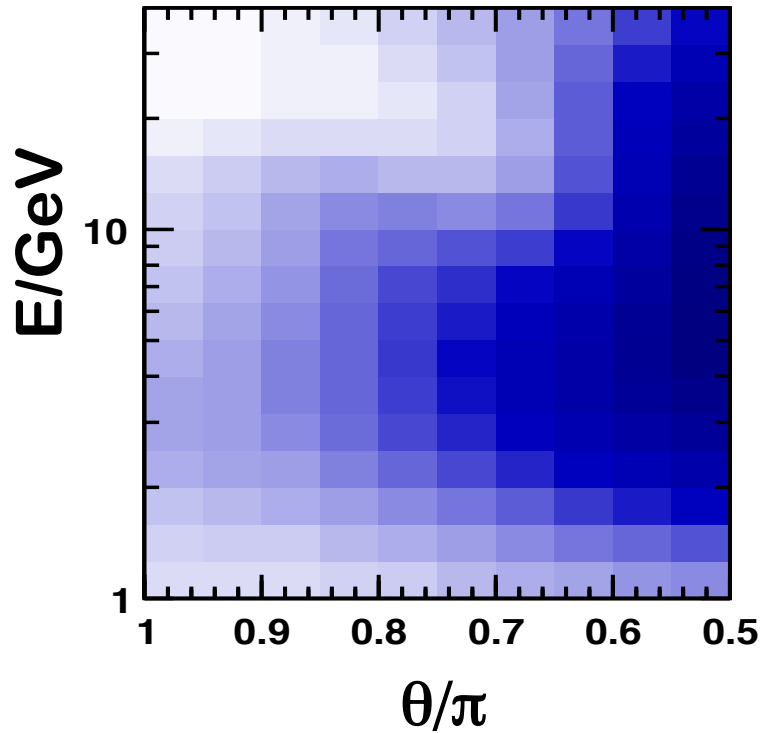


Similarly, one expects **effective volume** and **Xsec.** errors at few percent level affecting the 2D spectrum shape: should be broken down into components

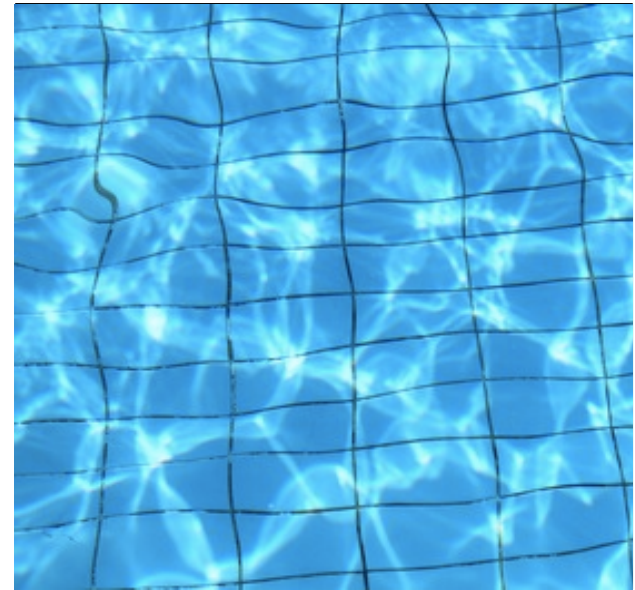
[But: can one really know the 2D functional form of each of these components?]

Lacking such information, we used a pragmatic approach to shape errors →

$N_{\text{NH}}^\alpha$  (smeared)

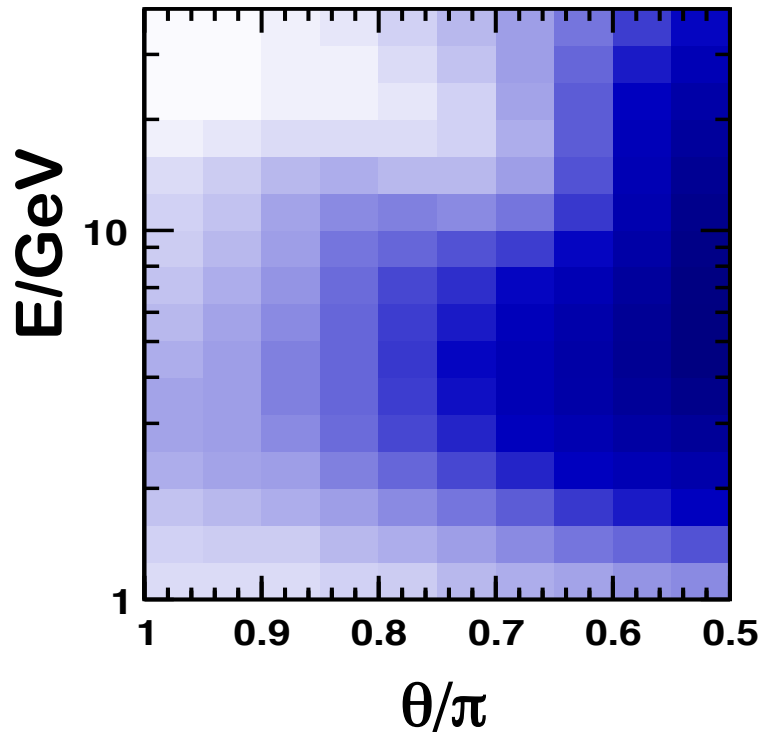


+ smooth 2D variations



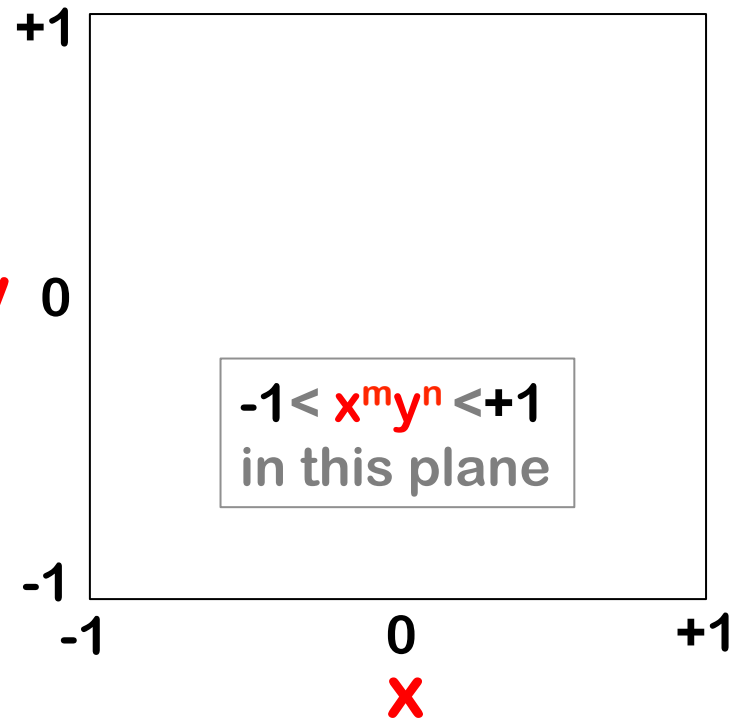
**Add smooth 2D deviations with constrained size  
on top of estimated templates**

$N_{\text{NH}}^\alpha$  (smeared)



$y$

change coordinates



Introduce generic 2D polynomial factor:  $(1 + \sum c_{nm} x^m y^n)$

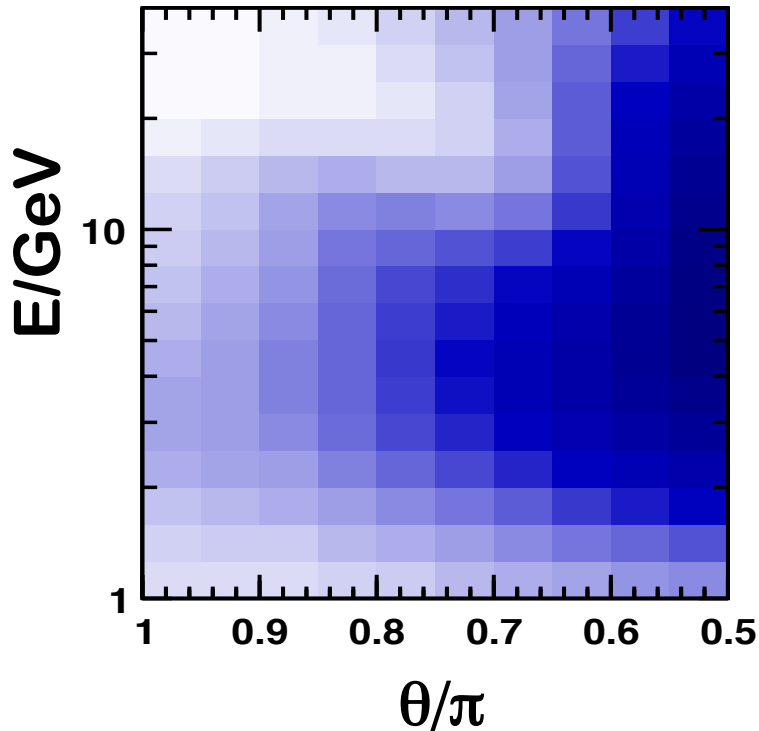
$n=0=m$ : recover normalization errors

$n+m=1$ : recover tilt error on  $x$  or  $y$

$n+m>1$ : nonlinear 2D shape errors (quadratic, cubic, quartic ...)

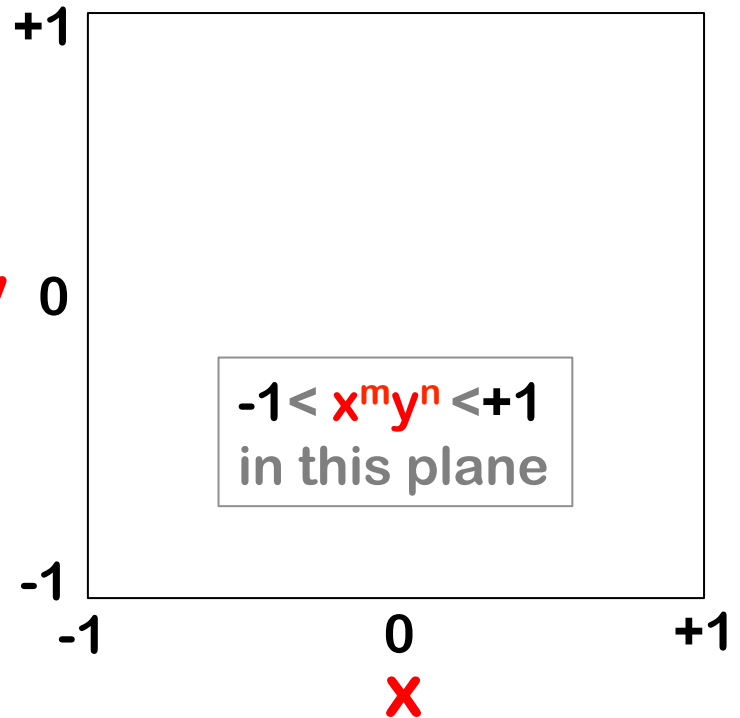
[Stopping at  $n+m=4$  is enough: we find stable results for  $n+m>4$ ]

$N_{\text{NH}}^\alpha$  (smeared)



$y$

change coordinates



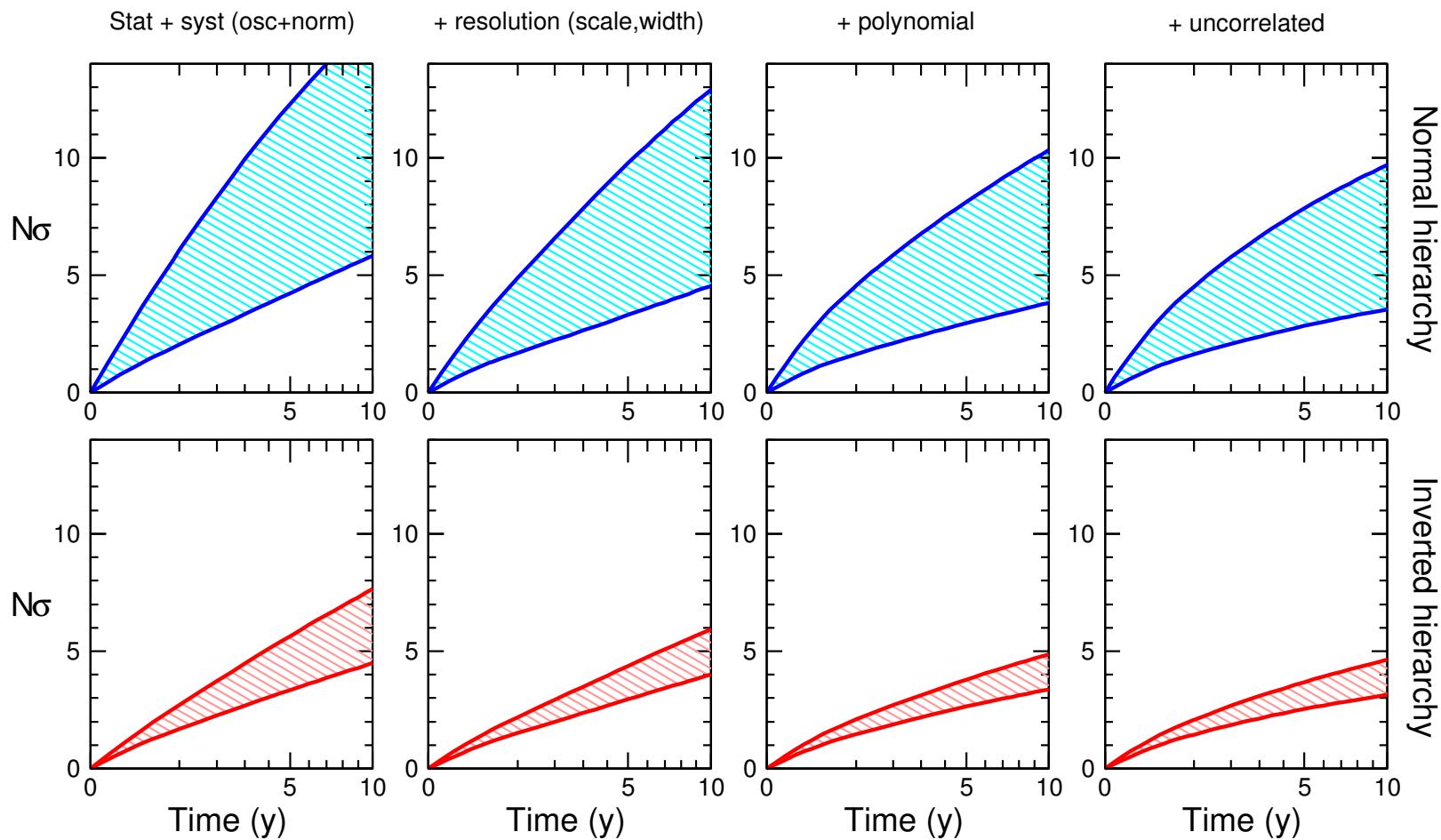
Add quadratic penalties in  $\chi^2$ , corresponding to  $\sigma(c_{nm}) = p\%$

Then, at  $1\sigma$ , each polynomial term is bounded by  $|c_{nm} x^m y^n| < p\%$

3 cases studied:  $p=1.5$  (default),  $p=3.0$  (doubled),  $p=0.75$  (halved)

Finally, we also add **uncorrelated syst. errors** at the same  $p\%$  level

# Results for default case: nonnegligible reduction of sensitivity



**Bands correspond to  $\sin^2\theta_{23}$  spanning the range [0.4, 0.6]**

**Note abscissa scaling as  $\sqrt{T}$**

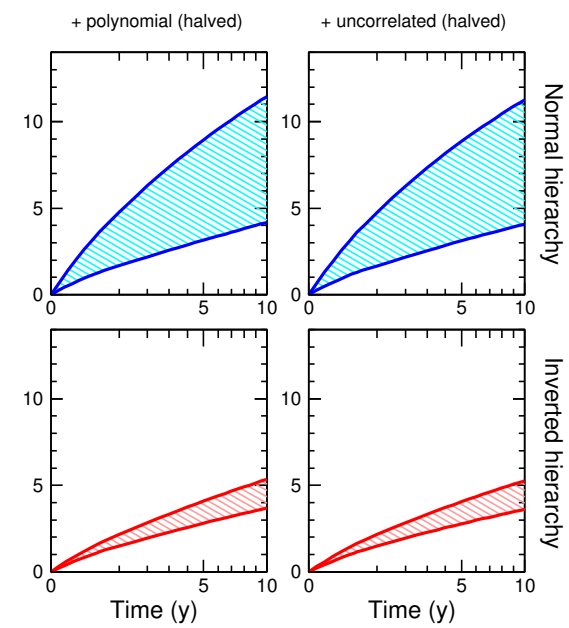
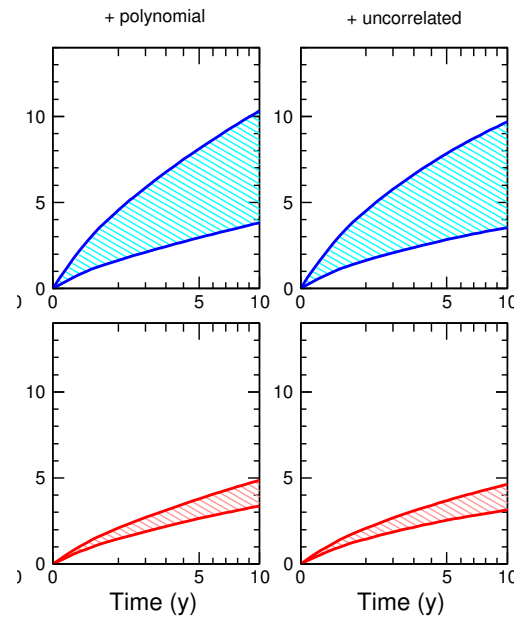
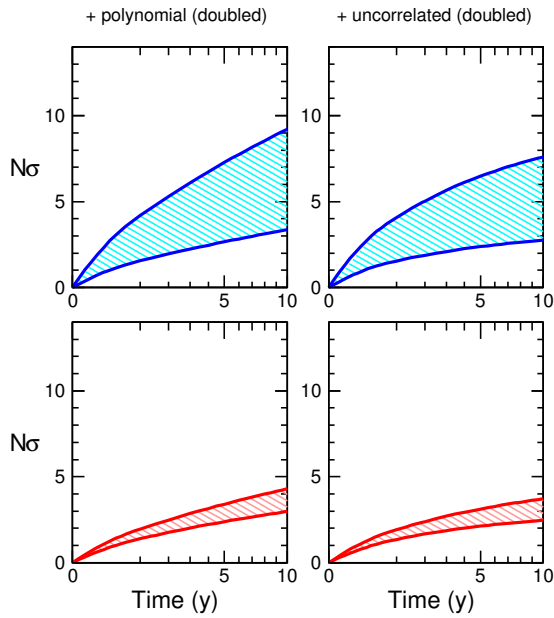
[Details in 1503.01999]



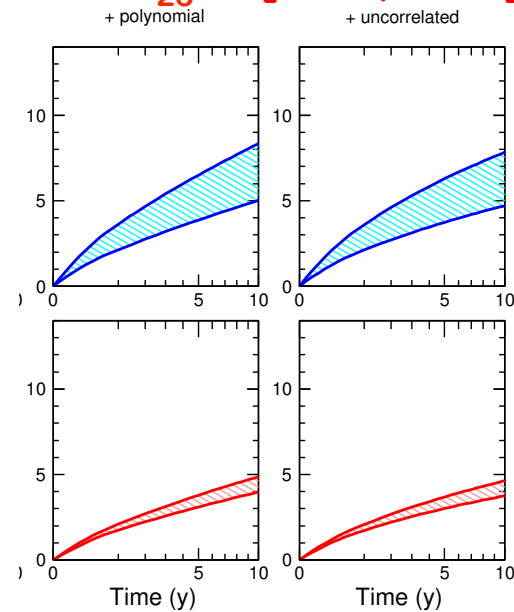
# doubled (3%)

# default (1.5%)

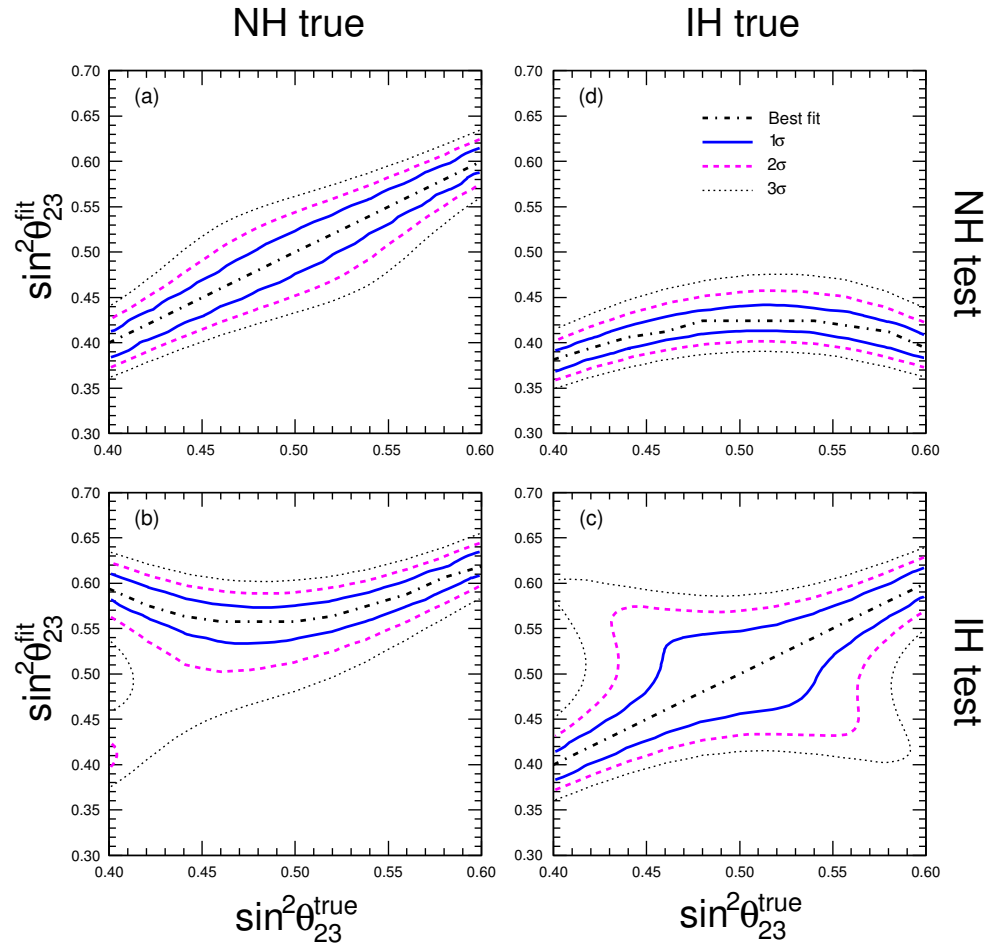
# halved (0.75%)



# $\sin^2\theta_{23}$ in [0.45, 0.55]



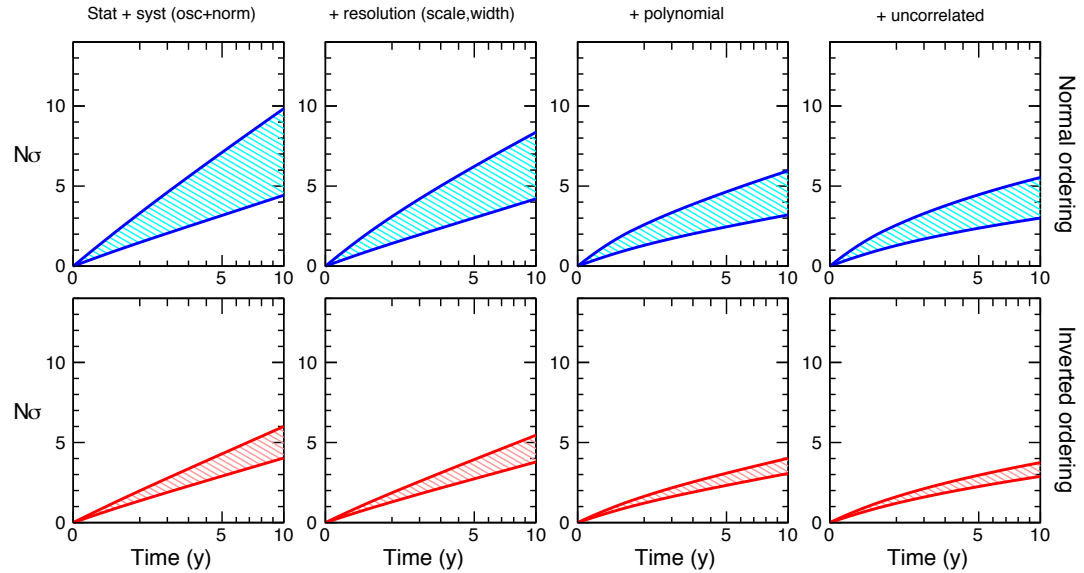
PINGU itself can better constrain  $\theta_{23}$ , but with **strong bias** if hierarchy is unknown (so  $\theta_{23}$  and hierarchy must be determined at the same time)



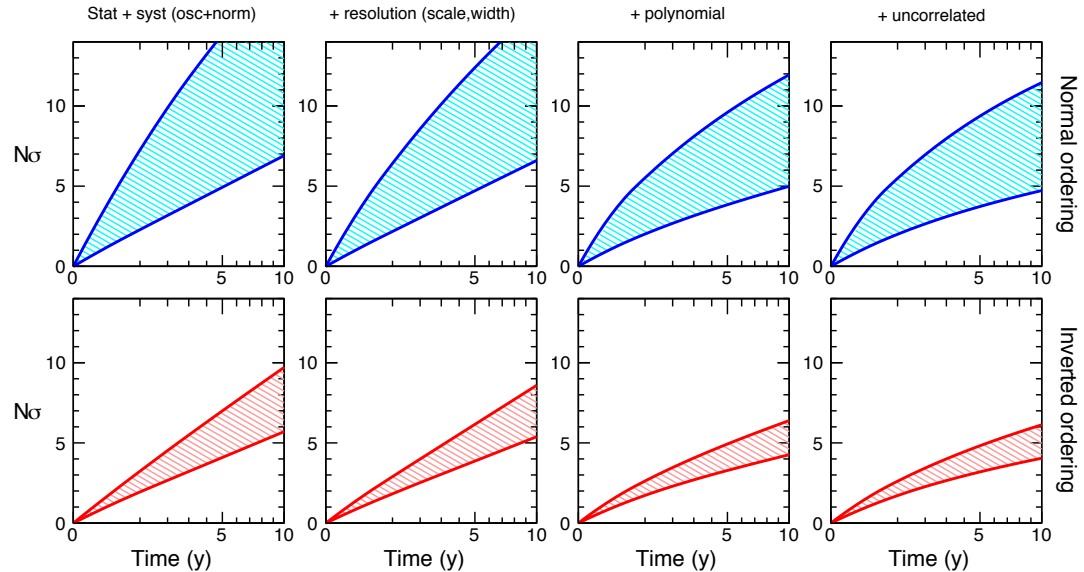
Note: Most of the previous comments apply also to ORCA (and HK, INO, ...)

For **ORCA** we also studied two cases:

Including flavor mis-identification →

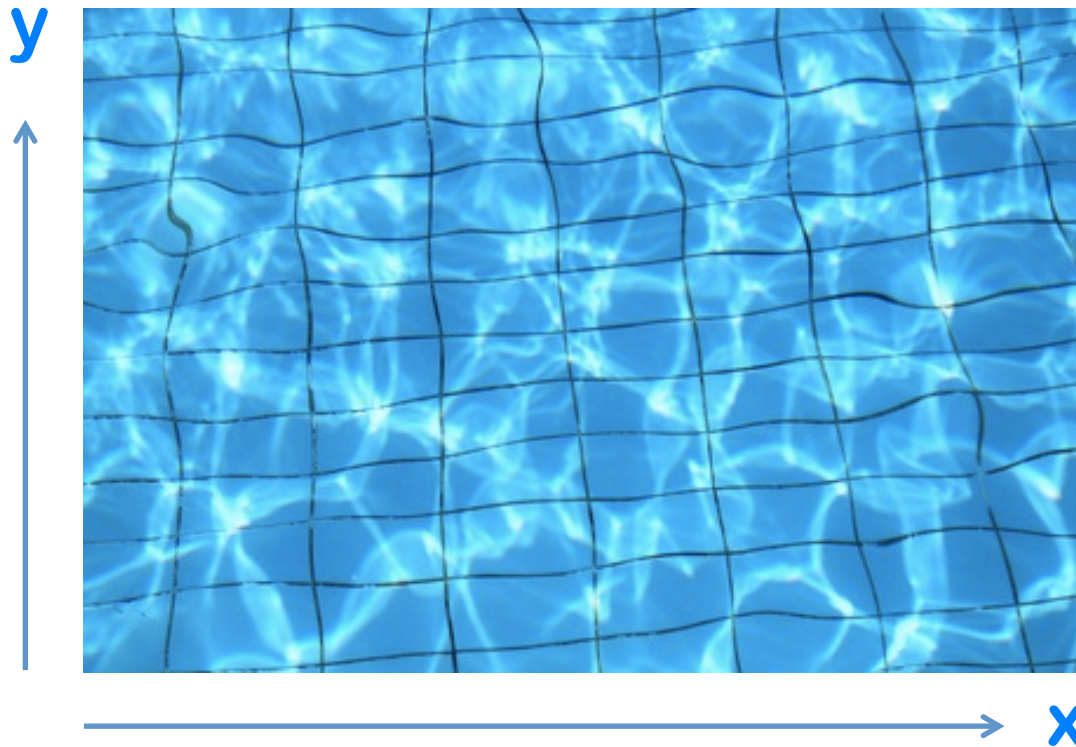


Assuming perfect flavor identification →



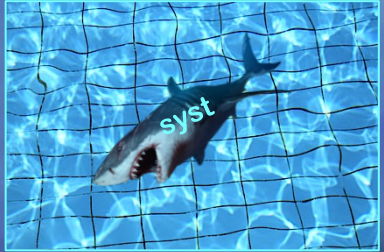
Improving flavor ID is very rewarding!

**Message: nonlinear deformations of 1D and 2D spectra (3D?) will be relevant in future high-statistics experiments.**



**Must find ways to break down, estimate and include (calibrate?) them, allowing extra room for poorly known effects. Challenging, but necessary to understand & control percent-level systematics!**



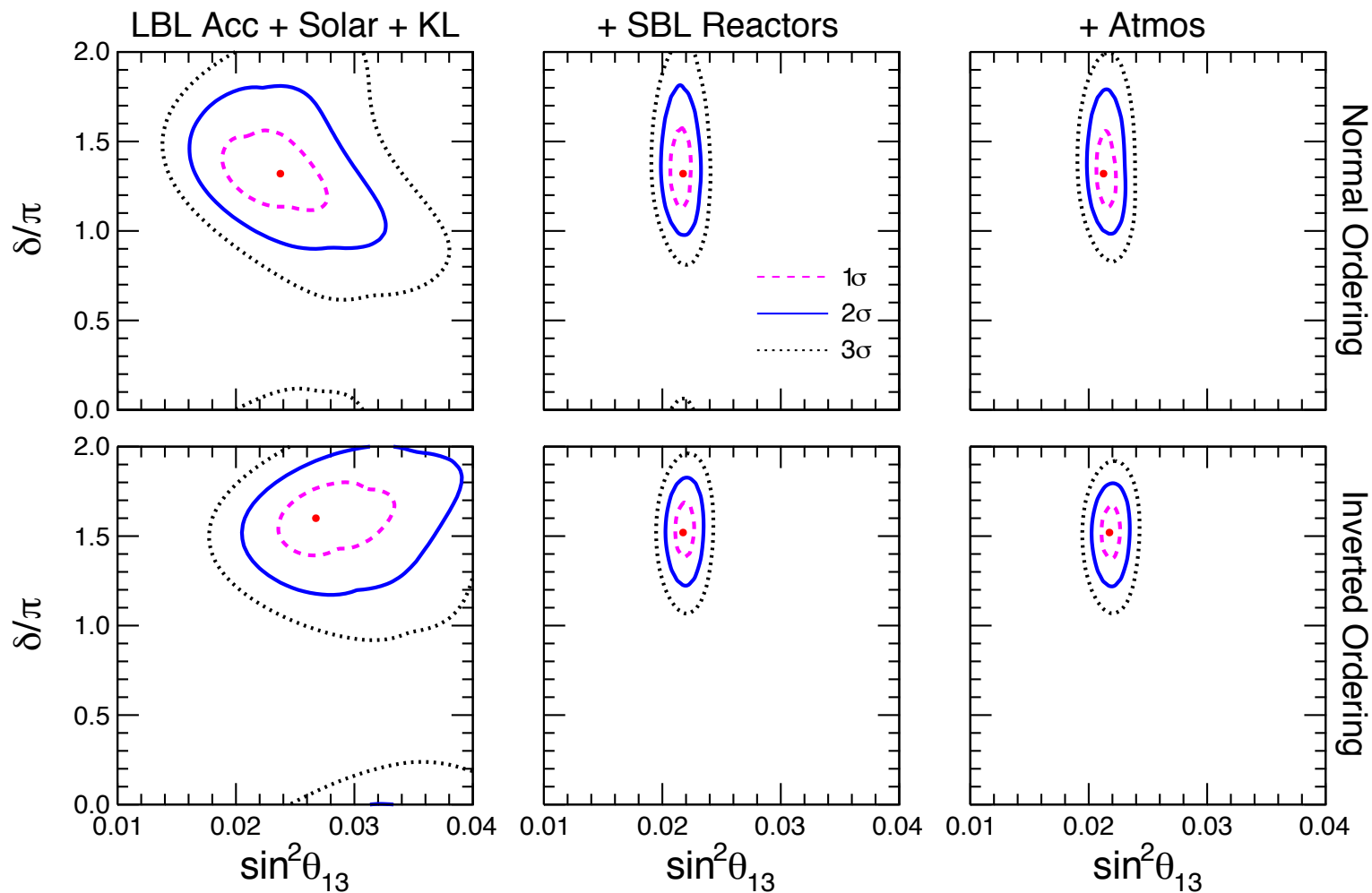


$V_3$   
 $V_2$   
 $V_1$

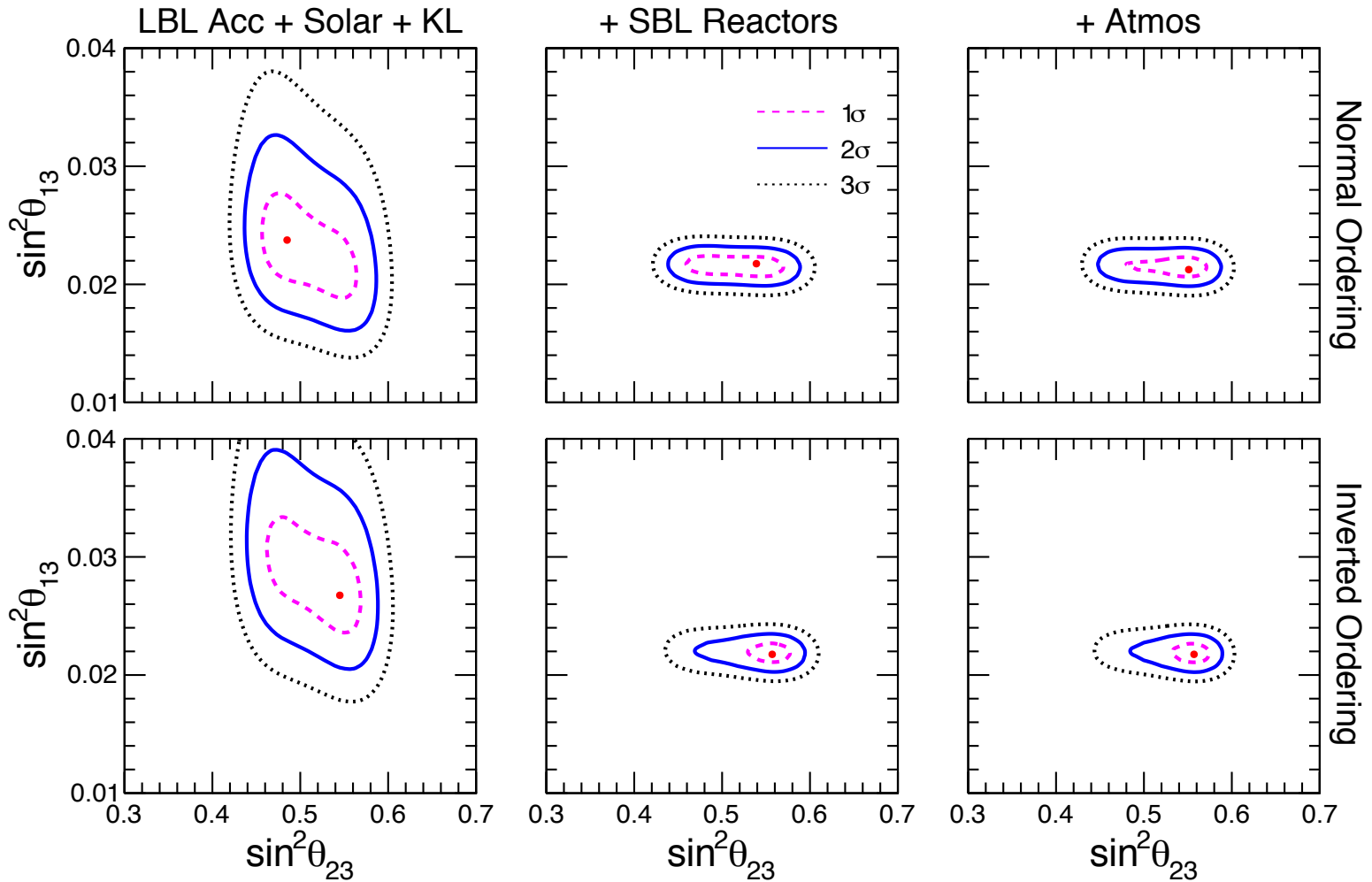
Thank you for your attention

# Extra slides

# $\delta - \theta_{13}$ correlation

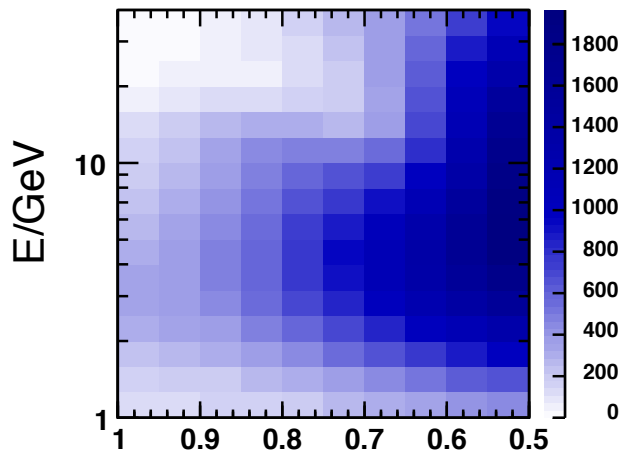


# $\theta_{23} - \theta_{13}$ correlation

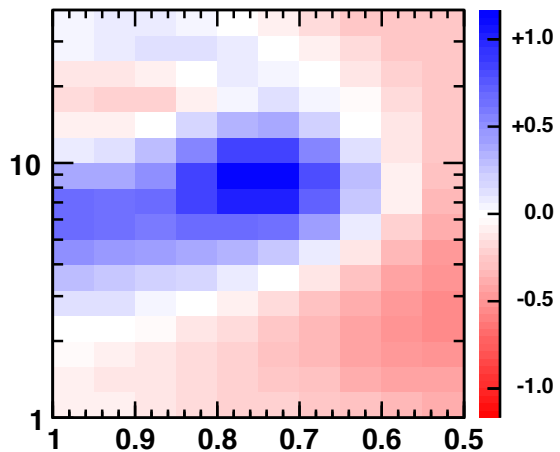




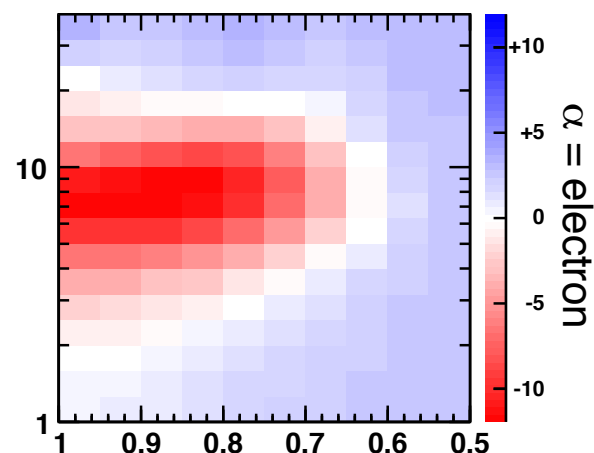
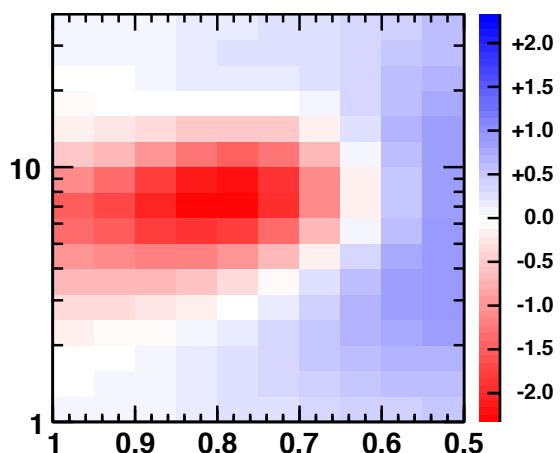
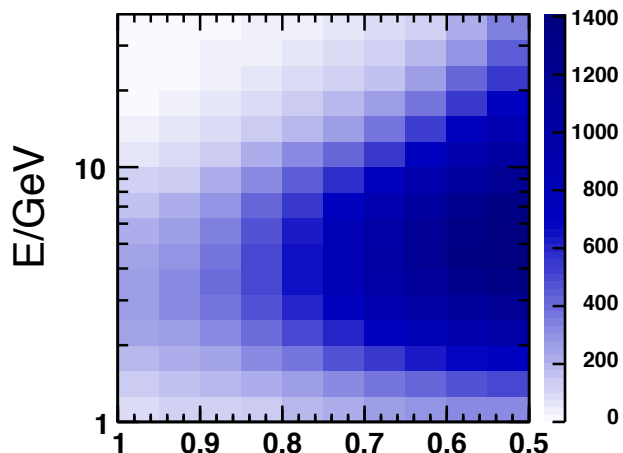
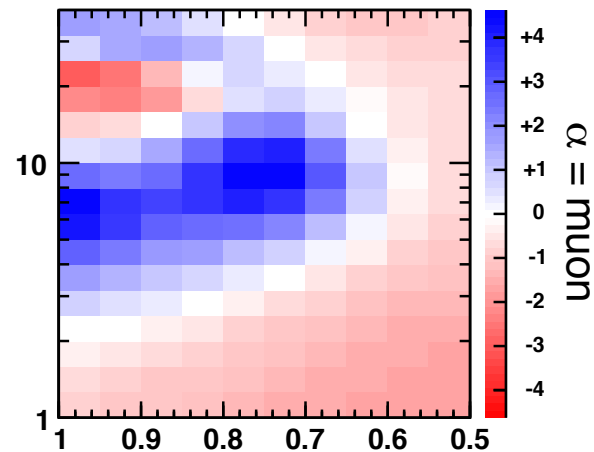
$$N_{\text{NH}}^\alpha (s_{23}^2 = 0.6)$$



$$(N_{\text{IH}}^\alpha - N_{\text{NH}}^\alpha) / \sqrt{N_{\text{NH}}^\alpha}$$



$$100 \times (N_{\text{IH}}^\alpha - N_{\text{NH}}^\alpha) / N_{\text{NH}}^\alpha$$


 $\theta/\pi$ 
 $\theta/\pi$ 
 $\theta/\pi$

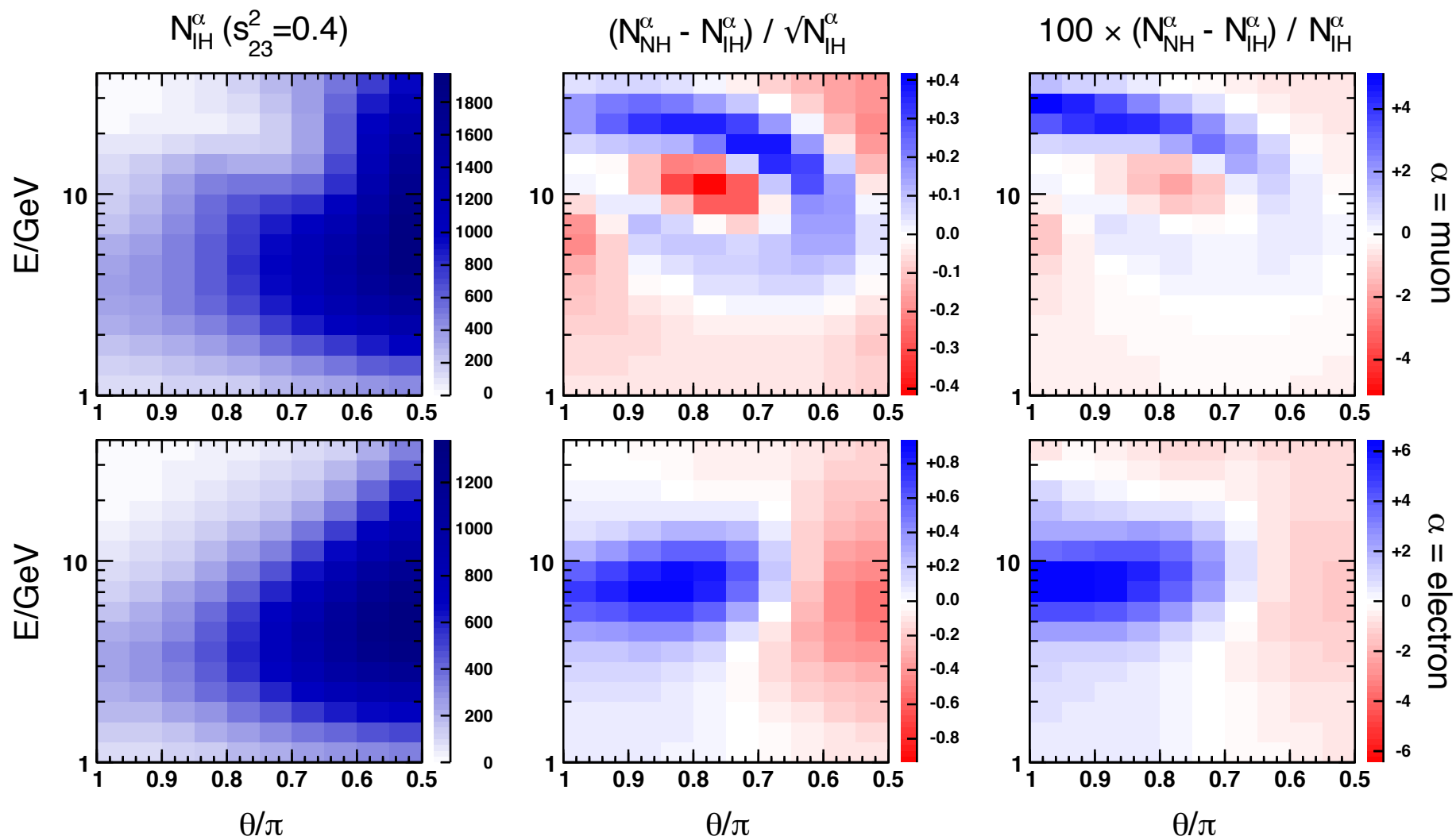


TABLE I: Reduction of the PINGU sensitivity to the hierarchy (expressed in terms of  $N_\sigma$  range for  $\sin^2 \theta_{23} \in [0.4, 0.6]$ ) due to the progressive inclusion of various shape systematics, for 5 and 10 years of exposure. Correlated polynomial and uncorrelated systematic uncertainties are taken at the default level of 1.5%. See the text for details.

Errors included in the fit	5-year sensitivity $N_\sigma$		10-year sensitivity $N_\sigma$	
	True NH	True IH	True NH	True IH
Stat. + syst (osc.+norm.)	4.23–12.3	3.34–5.64	5.82–16.1	4.49–7.64
+ resolution (scale, width)	3.31–9.76	2.95–4.37	4.54–12.9	4.00–5.94
+ polynomial (linear)	3.14–9.17	2.86–4.16	4.23–11.9	3.81–5.49
+ polynomial (quadratic)	3.01–8.29	2.69–3.88	3.93–10.6	3.47–5.05
+ polynomial (cubic)	2.98–8.26	2.67–3.84	3.87–10.5	3.42–4.94
+ polynomial (quartic)	2.95–8.12	2.64–3.79	3.82–10.3	3.37–4.87
+ uncorrelated systematics	2.84–7.84	2.54–3.68	3.55–9.69	3.14–4.63
Total $N_\sigma$ reduction from 1st row	33–36%	24–35%	39–40%	30–39%

# ORCA resolutions

