Phenomenology of atmospheric neutrino oscillations

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PANE-2018, ICTP, Trieste
May 2018

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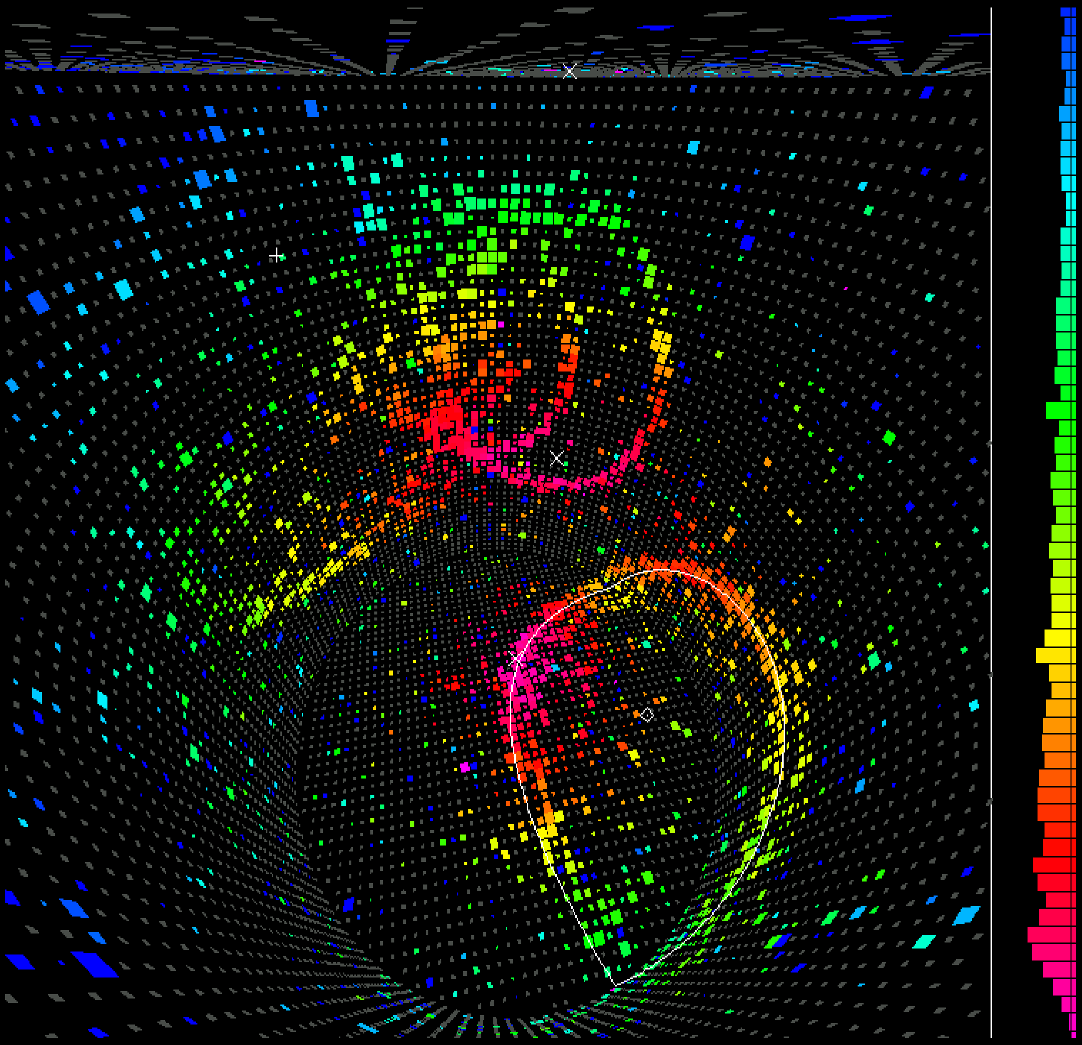
Oscillation Set-up



Set-up

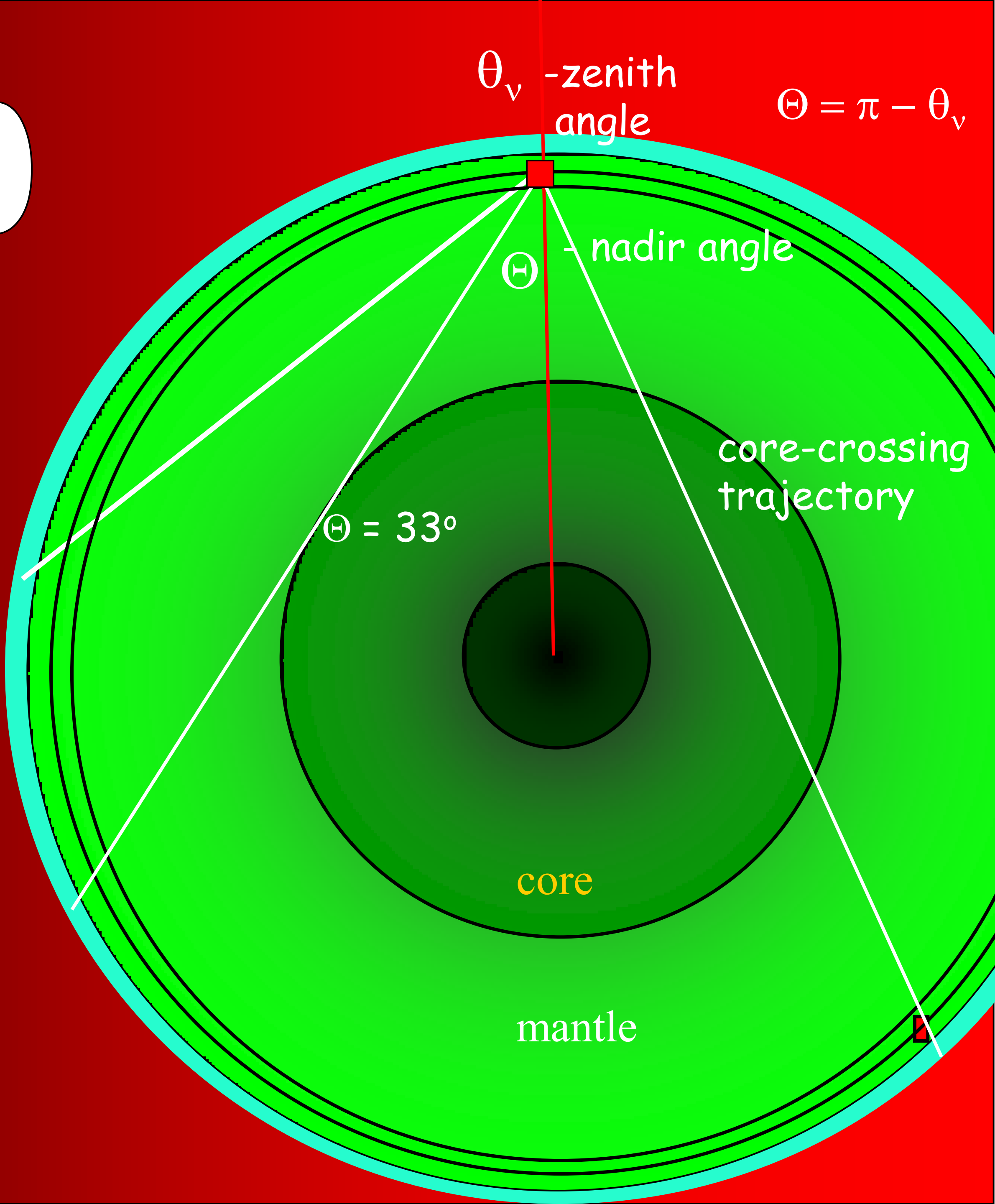
Oscillations in multilayer medium

flavor-to-flavor transitions

Absorption

R_E = 6300 km

R_{core} = 3600 km



Source

π, K, μ - decays free decays ?

Incoherent fluxes of the flavor states

ν_e, ν_μ and corresponding antineutrinos

Energy range: 0.01 – 10^6 GeV corresponding vacuum oscillation length due 1-3 splitting:

l_ν = 10 – 10^8 km

Flavor ratio $r = \Phi_\mu/\Phi_e$ increases with energy from 2 to 100

Charge asymmetry: $\Phi_\alpha/\overline{\Phi}_\alpha$ increases from 1.2 to 1.5

Size of coherent production region given by decay length l_{dec}
much smaller than oscillation length For : l_{dec} / l_ν - upto 0.05

Oscillations in coherent production region - negligible

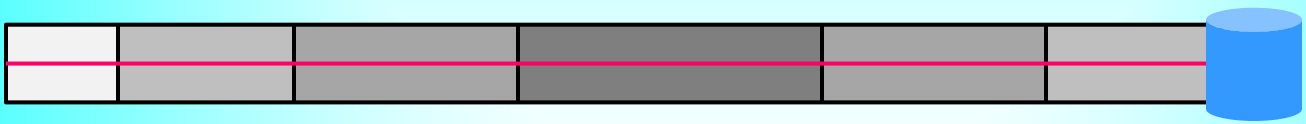
Size of the wave packet from free decay $\sigma_x = l_{dec} \dfrac{m_\pi^2}{E_\pi^2}$

Medium

Baselines: 0 – 13000 km

Matter density 3 – 15 g/cm^3

Density profile depends on the zenith angle



air   approximately symmetric matter profile

For low energies size of external layer in close to horizontal directions can be comparable with oscillation length

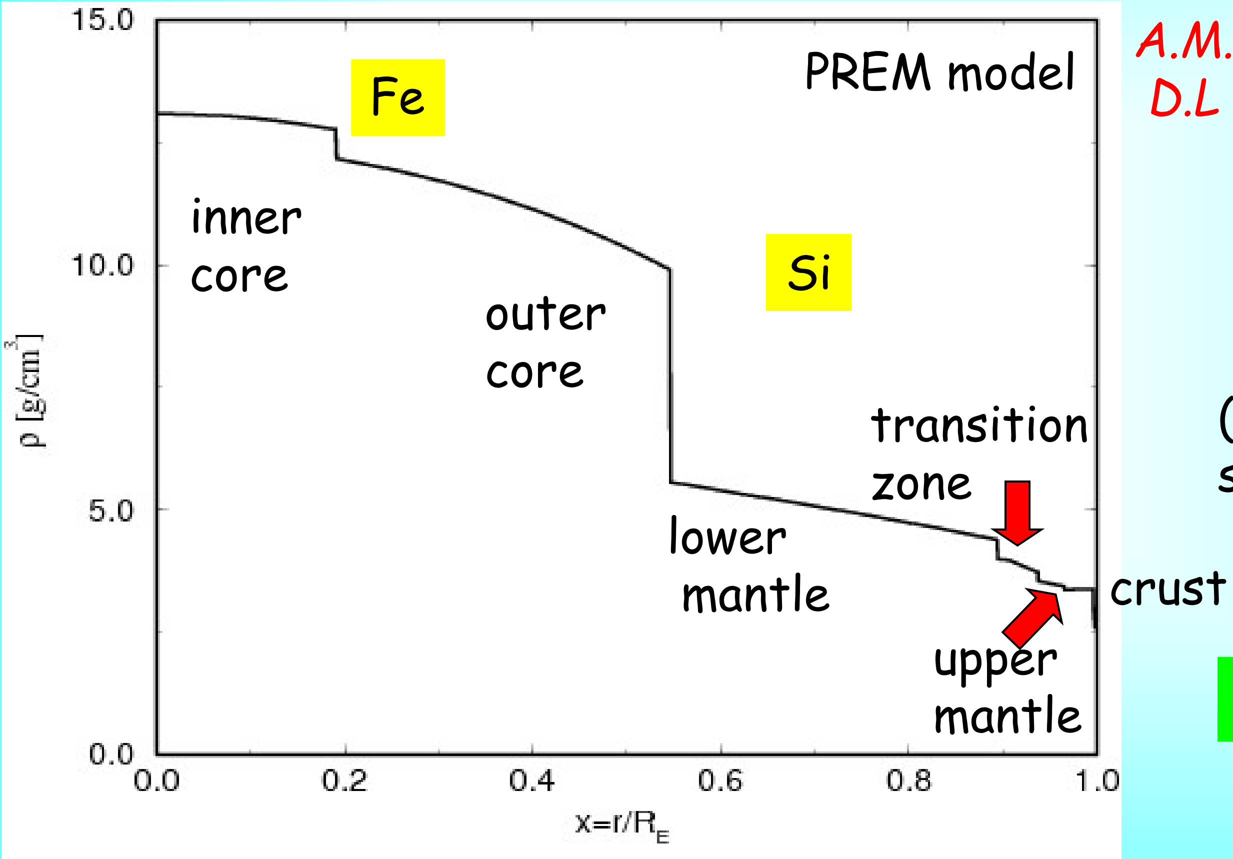
Problem to see effects: neutrino energy and direction reconstruction DUNE?

Absorption can be neglected at energies of interests for standard oscillations: $E < 100$ GeV

The probability of absorption

$$P = 0.005 \, (E / 100 \text{ GeV})$$

The earth density profile



A.M. Dziewonski
D.L Anderson 1981

(phase transitions in silicate minerals)

Alternative models

solid liquid R_e = 6371 km

Detection

Flavor states are detected

Flavor identification

Neutral currents

Scattering on electrons?

Neutrino energy reconstruction σ_E

Neutrino direction (baseline) reconstruction σ_θ

Oscillation amplitudes and probabilities

Propagation basis

$\nu_f = U_{PMNS}\, \nu_{mass}$

Standard parameterization

$U_{PMNS} = U_{23} I_\delta U_{13} I_{-\delta} U_{12}$

$I_\delta = \mathrm{diag}\,(1, 1, e^{i\delta})$

commute

$\nu_f = U_{23} I_\delta \underbrace{U_{13} U_{12}\, \nu_{mass}}$

Propagation basis

$\nu_f = U_{23} I_\delta\, \tilde{\nu}$

$\tilde{\nu} = \begin{pmatrix} \nu_e \\ \tilde{\nu}_2 \\ \tilde{\nu}_3 \end{pmatrix}$

ν_e and the potential are not affected

Hamiltonian in the propagation basis

$H = U_{13} U_{12}\; H_0^{diag}\, U_{12}^+\, U_{13}^+ + V$

$H_0^{diag} = \Delta m_{13}^2\; \mathrm{diag}\,(0,\; r_\Delta,\; 1)$

$r_\Delta = \dfrac{\Delta m^2_{21}}{\Delta m^2_{31}}$

$H = H\,(\theta_{12}, \theta_{13}, V)$ does not depend on θ_{23}, δ

Evolution in the propagation basis does not depend on CP violation phase

Scheme of transitions

Propagation basis

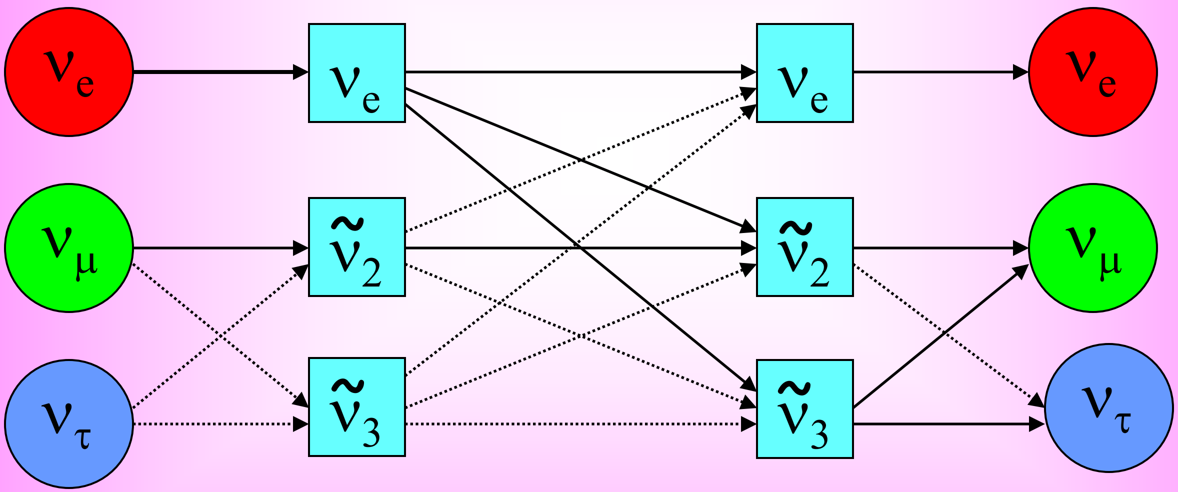
$\tilde{\nu} = U_{13} U_{12} \nu_{mass}$

$\tilde{\nu}_f = U_{23} I_\delta \nu$ $\quad S_{prop} = ||A_{ij}||$

$\sin\theta_{23}$

$\cos\theta_{23}$

$\cos\theta_{23}\, e^{-i\delta}$



projection propagation projection back

CP appears in projection only

Amplitudes in the propagation basis

$$S_{prop} = \begin{pmatrix} A_{ee} & A_{e2} & A_{e3} \\ A_{2e} & A_{22} & A_{23} \\ A_{3e} & A_{32} & A_{33} \end{pmatrix}$$

For symmetric density profile due to T-invariance:

$$A_{ij} = A_{ji}$$

Hierarchy of amplitudes

$A_{e3} \sim \sin\theta_{13}$ $\quad A_{e2} \sim r_\Delta \sim \sin^2\theta_{13}$ $\quad A_{23} \sim \sin\theta_{13}\, r_\Delta \sim \sin^3\theta_{13}$

Matrix of transitions in the flavor basis

$$S_f = U_{23} I_\delta\, S_{prop}\, I_{-\delta}\, U_{23}^+$$

A_{22} A_{23} A_{33} can be expressed via A_{e2} A_{e3}

Constant density

$|A_{e2}| = \cos\theta_{13}^m \ \sin 2\theta_{12}^m \ \sin\phi_{12}^m$

Solar amplitude

$|A_{e3}| = \sin 2\theta_{13}^m \ |\sin\phi_{13}^m \, e^{-i\phi_{13}^m} + \sin 2\theta_{12}^m \ \sin\phi_{12}^m|$

$\sim \sin 2\theta_{13}^m \ \sin\phi_{13}^m$

Atmospheric 2nu amplitude

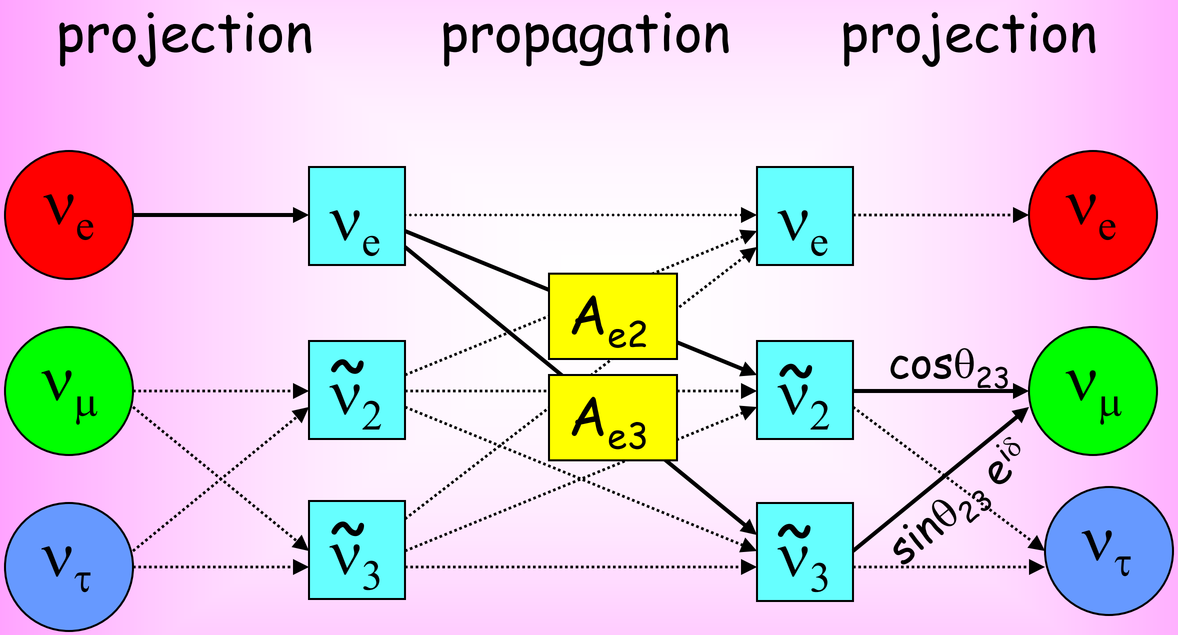
$|A_{23}| = \sin\theta_{13}^m \sin 2\theta_{12}^m \ \sin\phi_{12}^m$

$\sin 2\theta_{12}^m \sim \sin 2\theta_{12} / \xi_{12}$

$\phi_{12}^m \sim \phi_{12}\, \xi_{12} = VL/2$

matter dominated limit

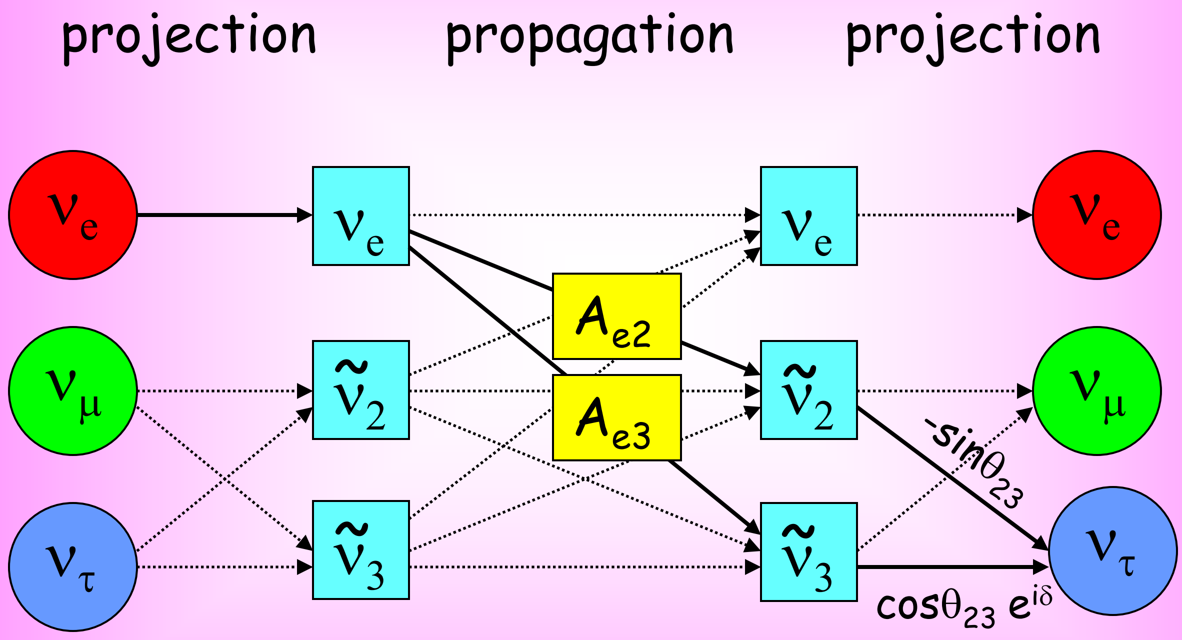
$\nu_e - \nu_\mu$



$$P(\nu_e \rightarrow \nu_\mu) = |\cos\theta_{23} A_{e2} + e^{i\delta} \sin\theta_{23} A_{e3}|^2$$

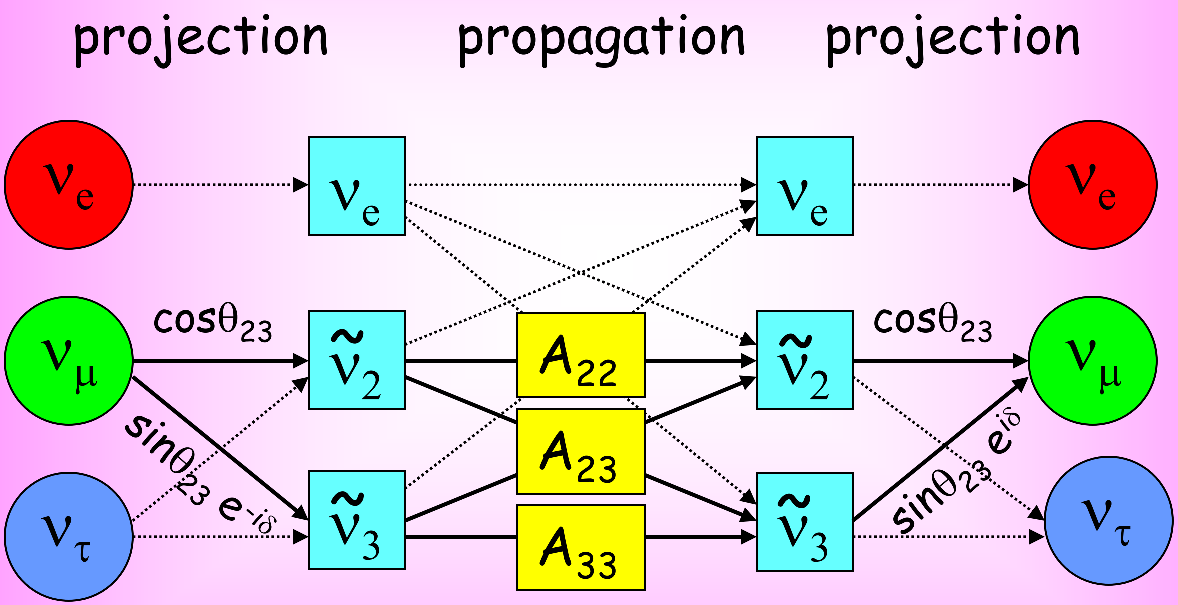
$$P(\nu_\mu \rightarrow \nu_e) = P(\nu_e \rightarrow \nu_\mu)\ (\delta \rightarrow -\delta)$$

$\nu_e - \nu_\tau$



$$P(\nu_e \rightarrow \nu_\tau) = |\sin\theta_{23} A_{e2} - e^{i\delta} \sin\theta_{23} A_{e3}|^2$$

$\nu_\mu - \nu_\mu$



4 channels

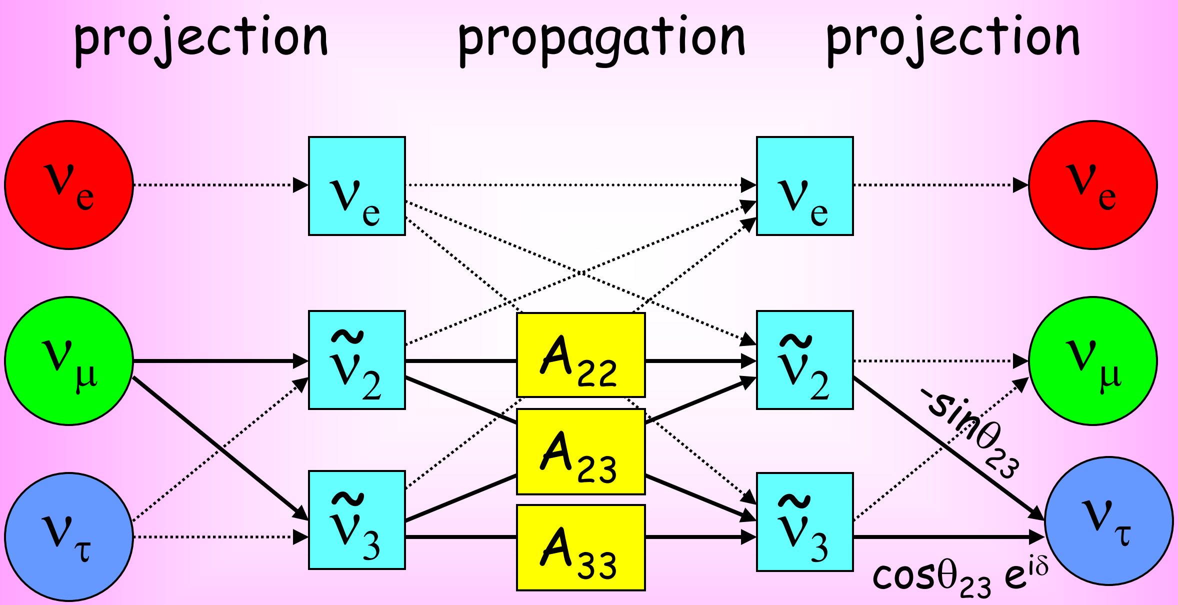
$A_{32} = A_{23}$

interference of 3 terms:

$$P(\nu_\mu \rightarrow \nu_\mu) = |\cos^2\theta_{23}\, A_{22} + \sin^2\theta_{23} A_{33} + \cos\delta\, \sin 2\theta_{23}\, A_{23}|^2$$

even function of δ

$\nu_\mu - \nu_\tau$



4 channels

$$P(\nu_\mu \rightarrow \nu_\tau) = \left| \tfrac{1}{2} \sin 2\theta_{23} (A_{33} - A_{22}) + (\cos 2\theta_{23} \cos\delta + i \sin\delta) A_{23} \right|^2$$

Other probabilities

$P(\nu_\alpha \to \nu_\beta) = P(\nu_\beta \to \nu_\alpha)\ (\delta \to -\delta)$

For antineutrinos:

$\delta \to -\delta$ $\quad$ $V \to -V$

$A_{ij} \to \overline{A_{ij}} = A_{ij}(V \to -V)$

Unitarity...

Approximations: $\sin\theta_{13} = 0$

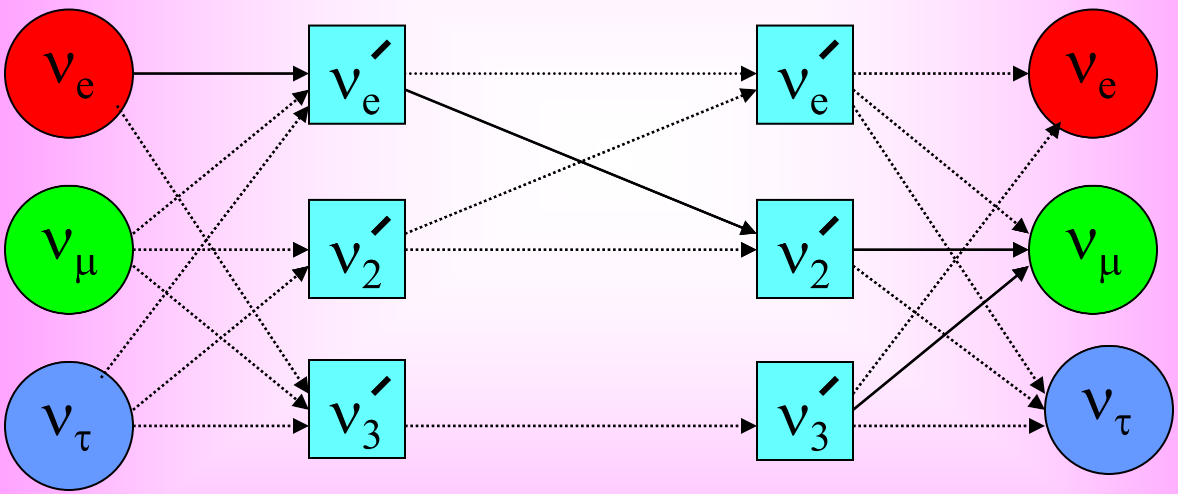
$\Delta m^2_{21} = 0$

Another propagation basis

$$\nu_f = U_{23} I_\delta U_{13} \nu'$$

Convenient for
for sub-sub GeV events

additional 1-3 (vacuum or matter) rotation



projection propagation projection

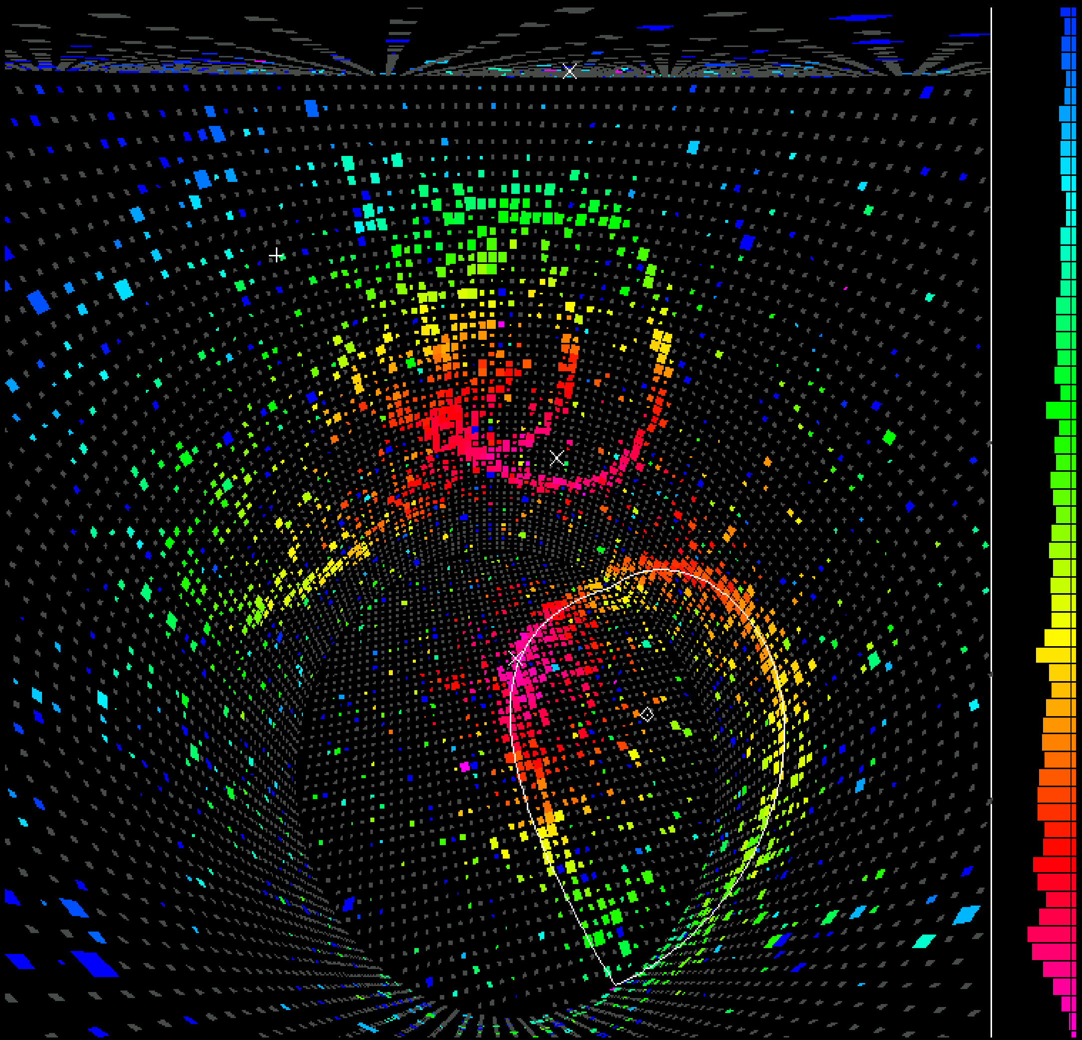
Decoupling of the third
state frim evolution

$3\nu \rightarrow 2\nu$

Single amplitude matters

$A_{e'2}$

Dynamics Oscillograms



Effects

Resonance enhancement of oscillations

Parametric enhancement of oscillations

Interference of 1-2 and 1-3 modes of oscillations

CP-violation

Resonance (MSW) peaks

Mantle – core – mantle trajectories

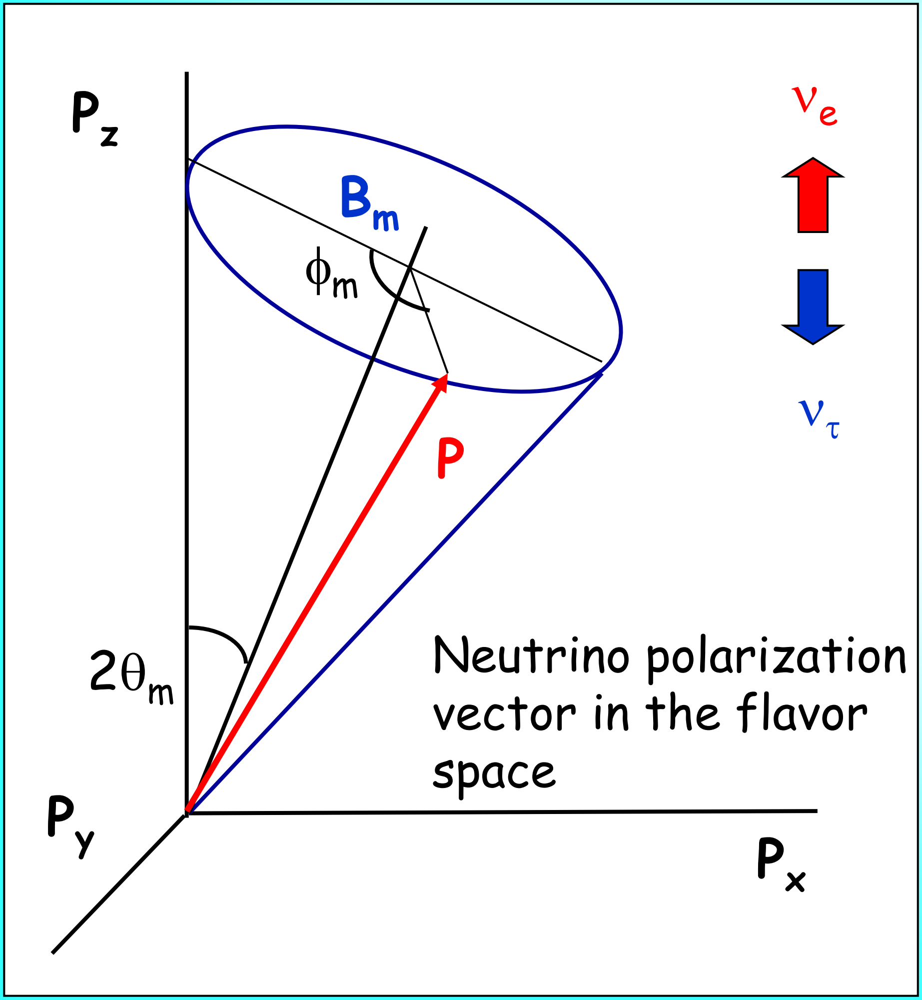
Parametric ridges

Magic baselines, CP-violation domains

In oscillation probabilities and distributions of events

Graphic representation

Electron spin precession in the magnetic field



$| P | = \frac{1}{2}$ polarization vector

$$B_m = \frac{2\pi}{l_m} (\sin 2\theta_m, \ 0, \ \cos 2\theta_m)$$

$$P_{ee} = \nu_e^+ \nu_e = P_Z + 1/2$$

$\phi_m = 2\pi t / l_m$ oscillation phase

Degrees of freedom:

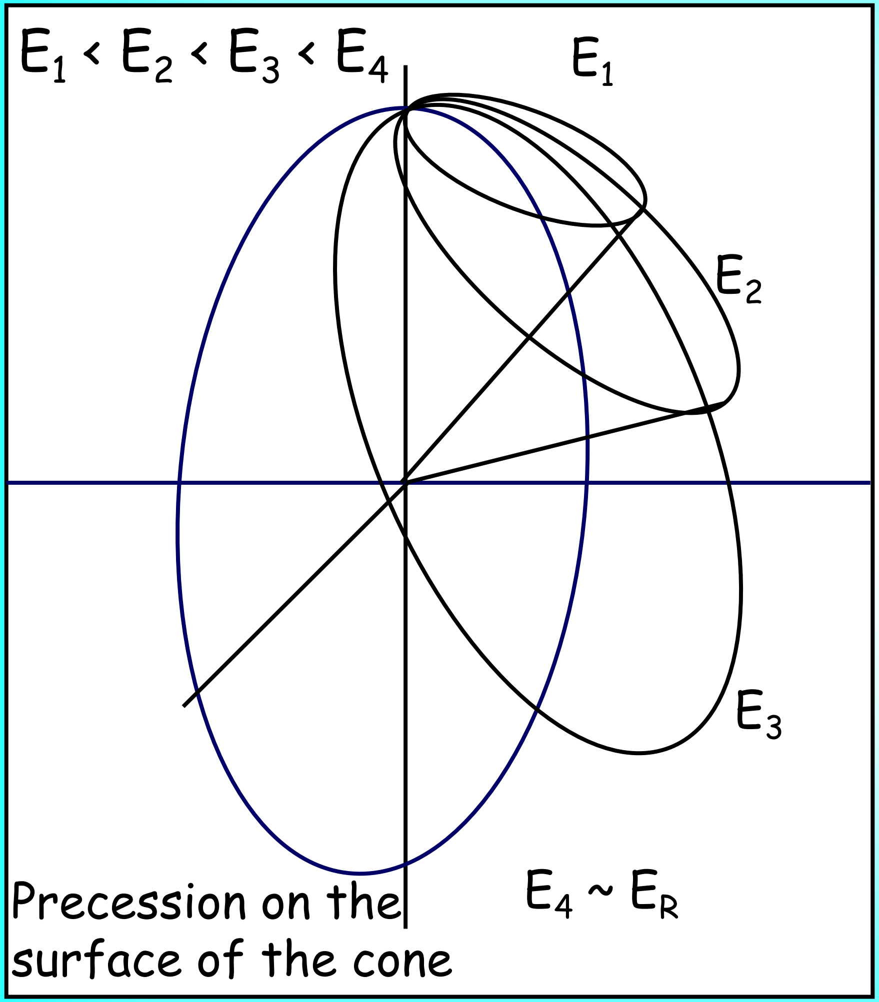
$\theta_m (n)$ - mixing angle

$\phi_m (n)$ - phase

$\theta_{cone} (dn/dx)$ - cone angle

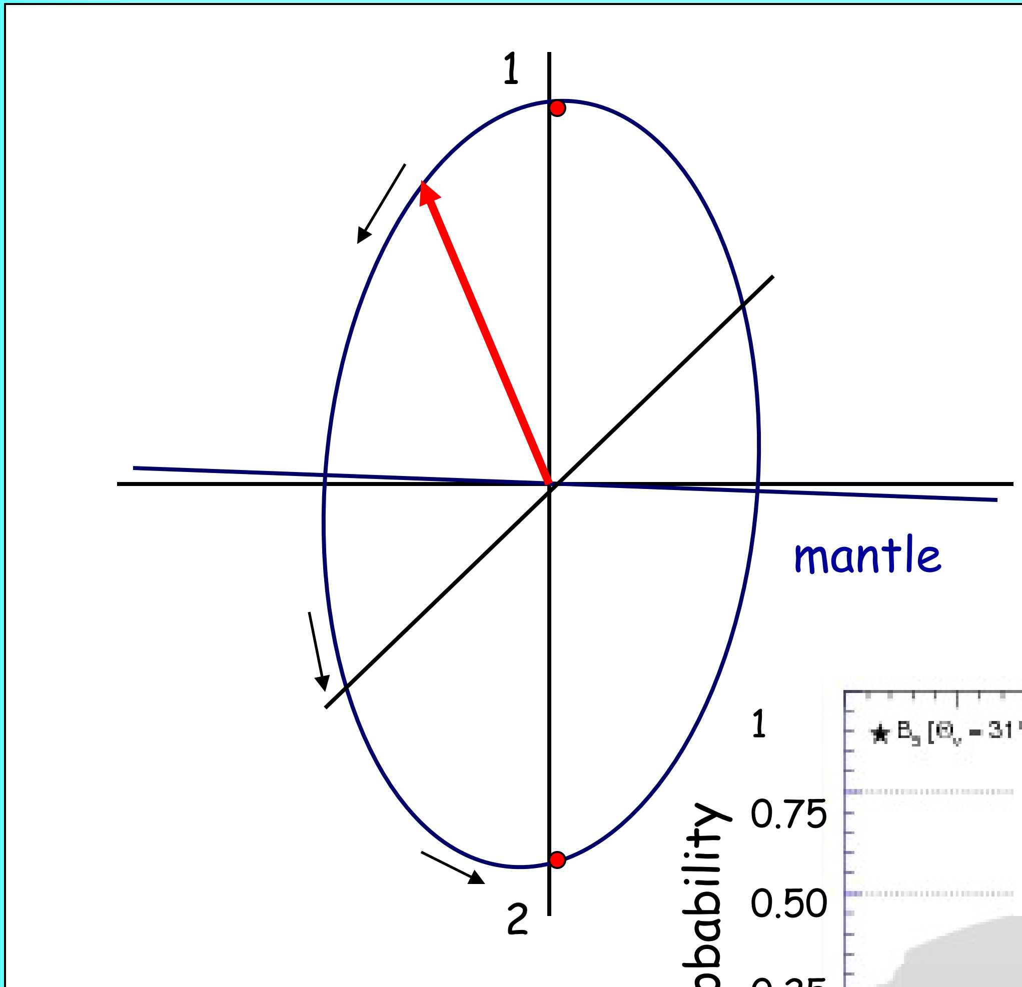
Using various density profiles $n(x)$ one can do "engineering" of new effects

Resonance enhancement



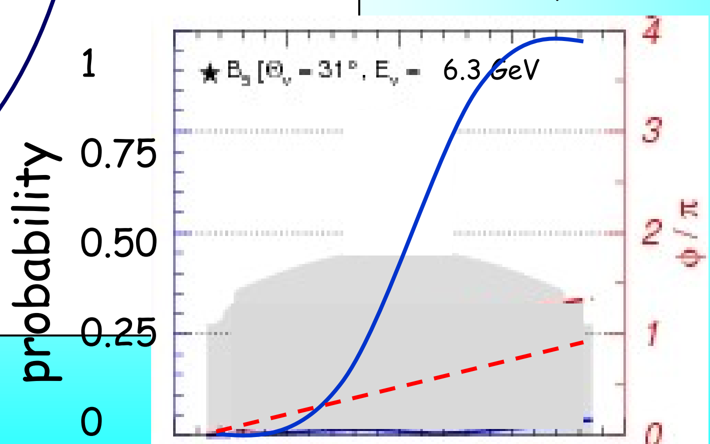
Constant, nearly constant density

Resonance enhancement in mantle



Conditions for resonance peak

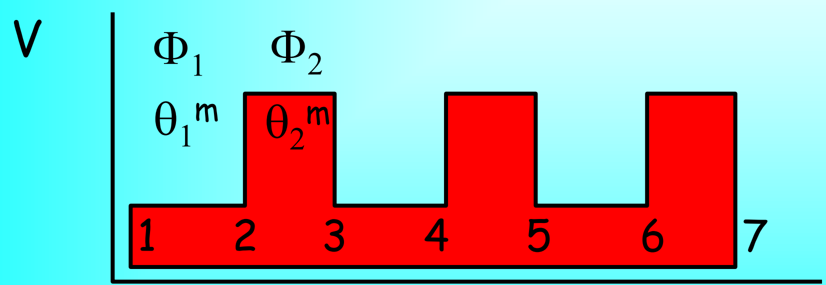
$\nu_e \rightarrow \nu_\mu \, , \, \nu_\tau$



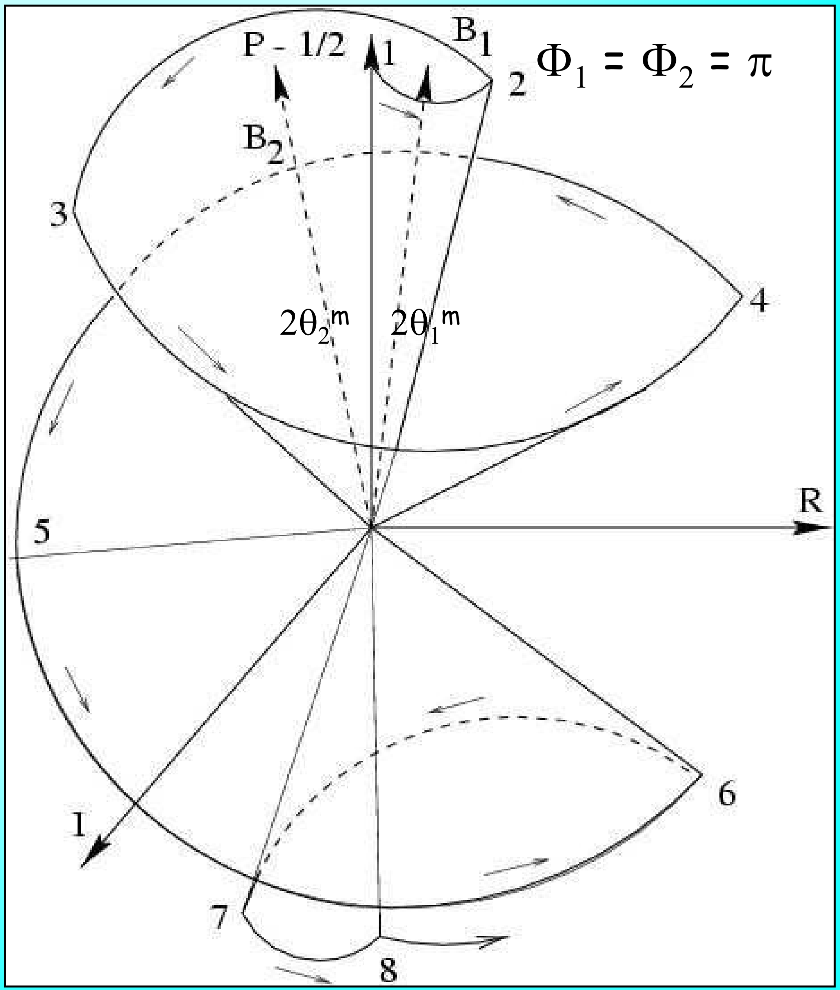
Parametric resonance

Enhancement is associated to certain conditions for the phase of oscillations

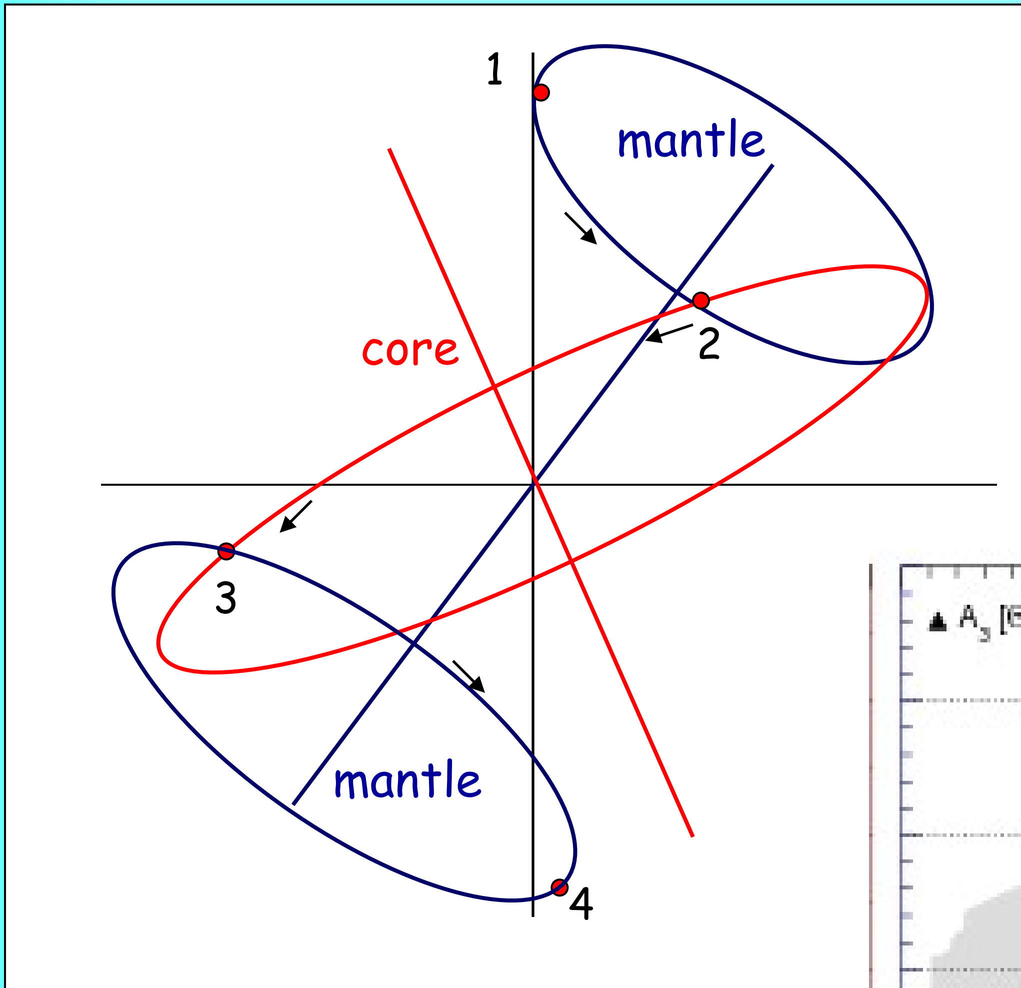
V. Ermilova V. Tsarev, V. Chechin
E. Akhmedov
P. Krastev, A.S., Q. Y. Liu,
S.T. Petcov, M. Chizhov

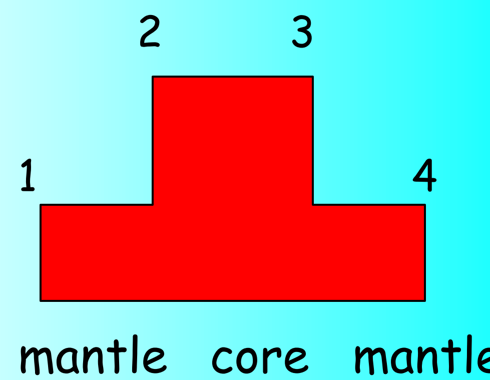


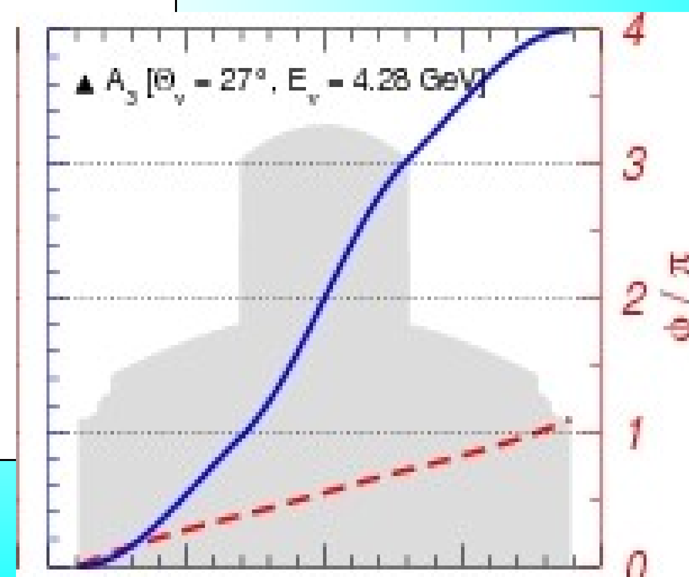
``Castle wall profile"



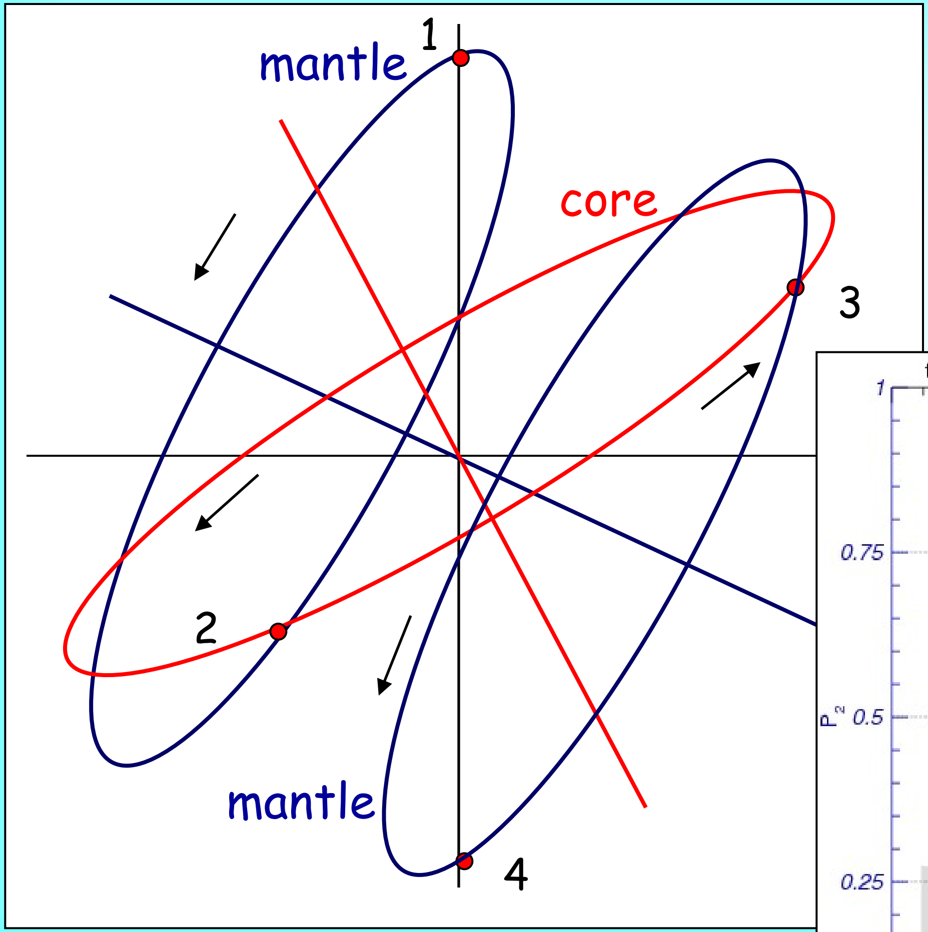
Parametric enhancement

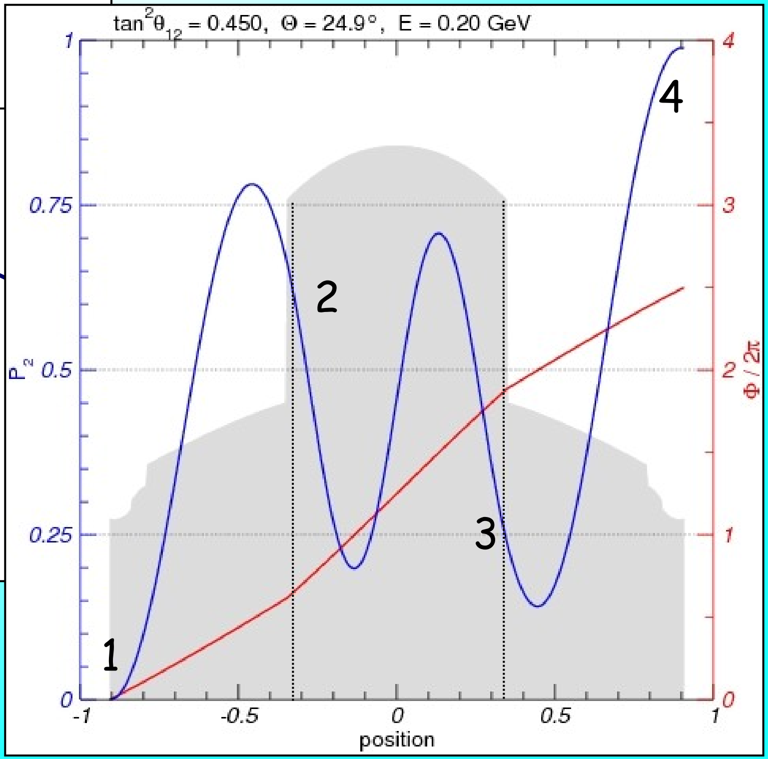






Parametric enhancement of 1-2 mode

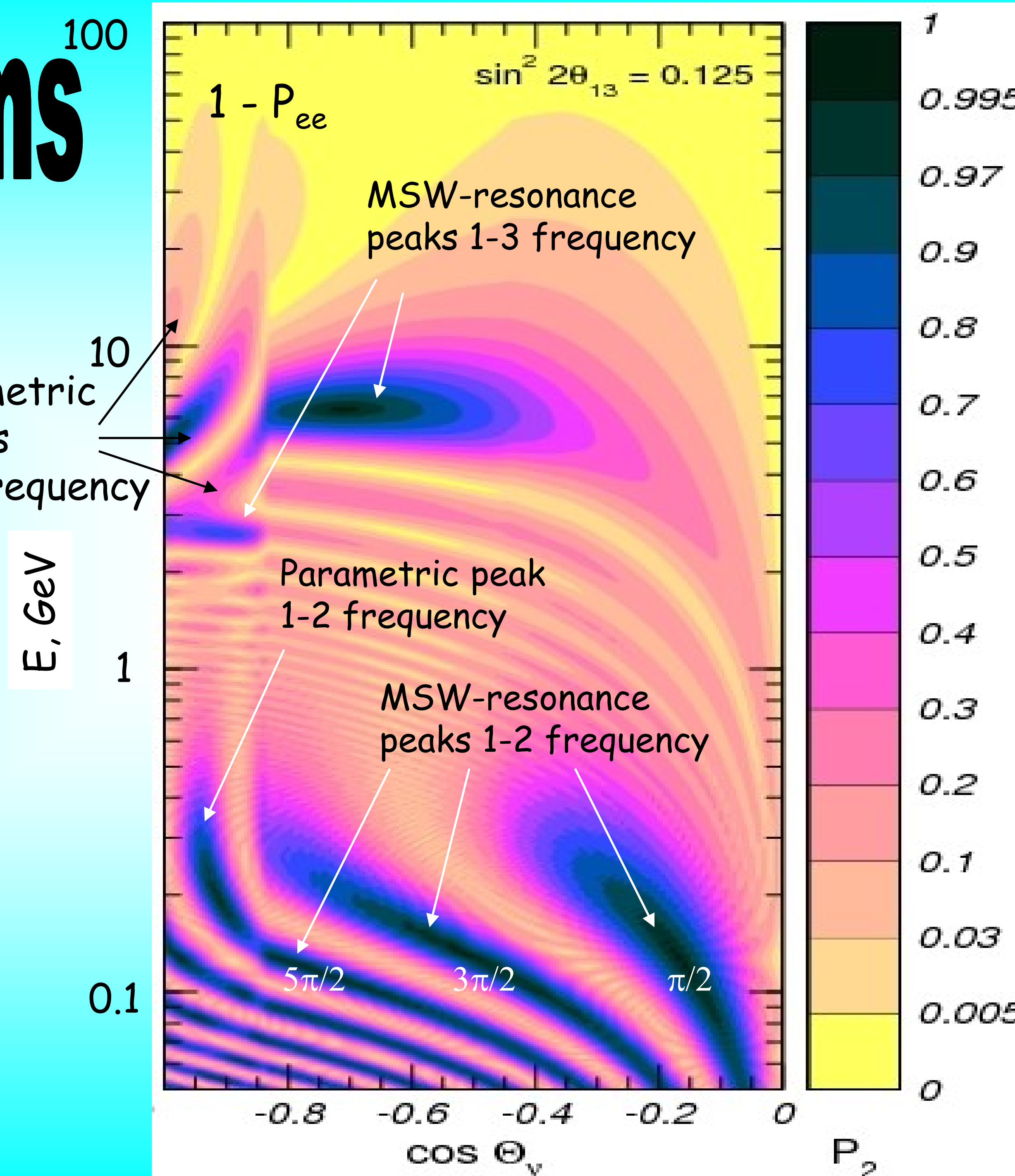




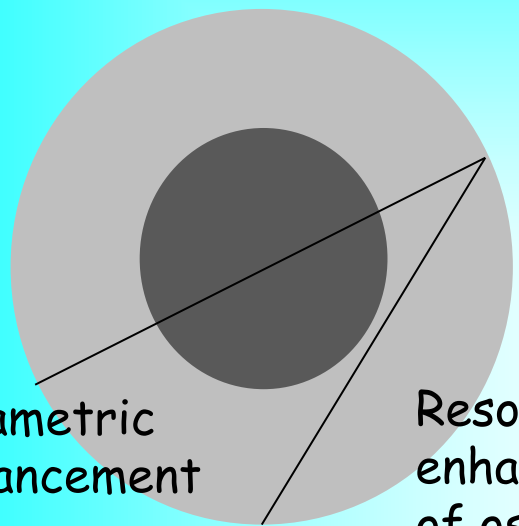
Oscillograms of the Earth

Lines of equal probability in the $E - \theta_z$ plane

$\nu_e \rightarrow \nu_\mu + \nu_\tau$

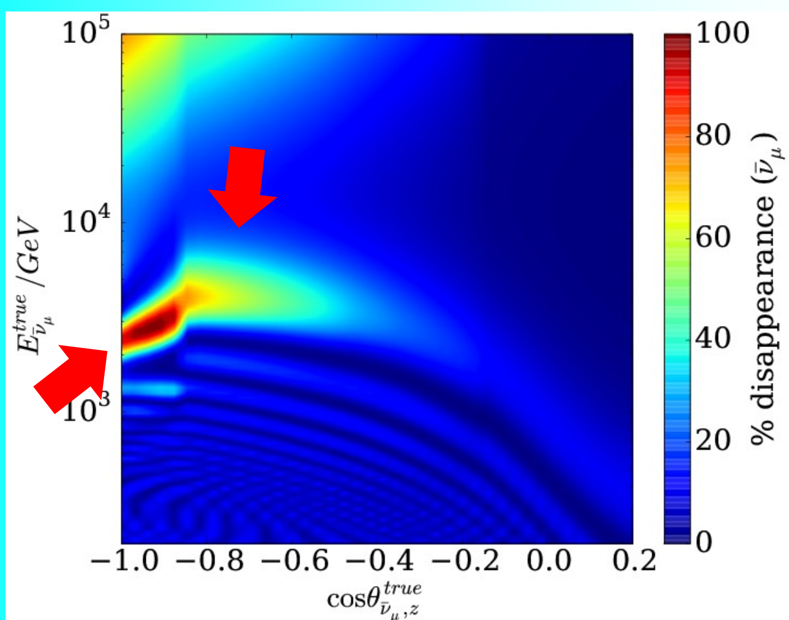


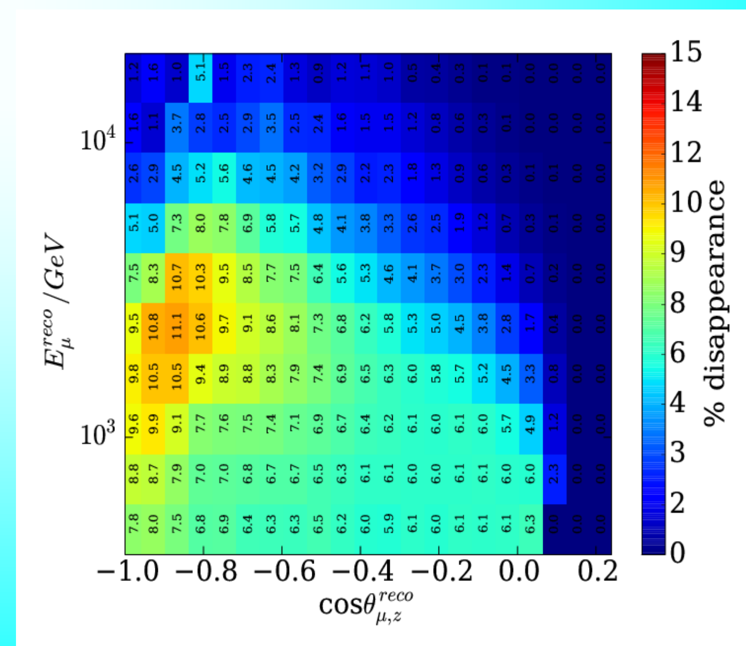
IceCube searches for sterile neutrinos



M.G. Aartsen et al,
(IceCube Collaboration)
1605.01990 (hep-ex)

IC86, 2011 – 2012, 343,7 days,
20,145 muon events
(reconstructed tracks) with
E = 320 GeV – 20 TeV

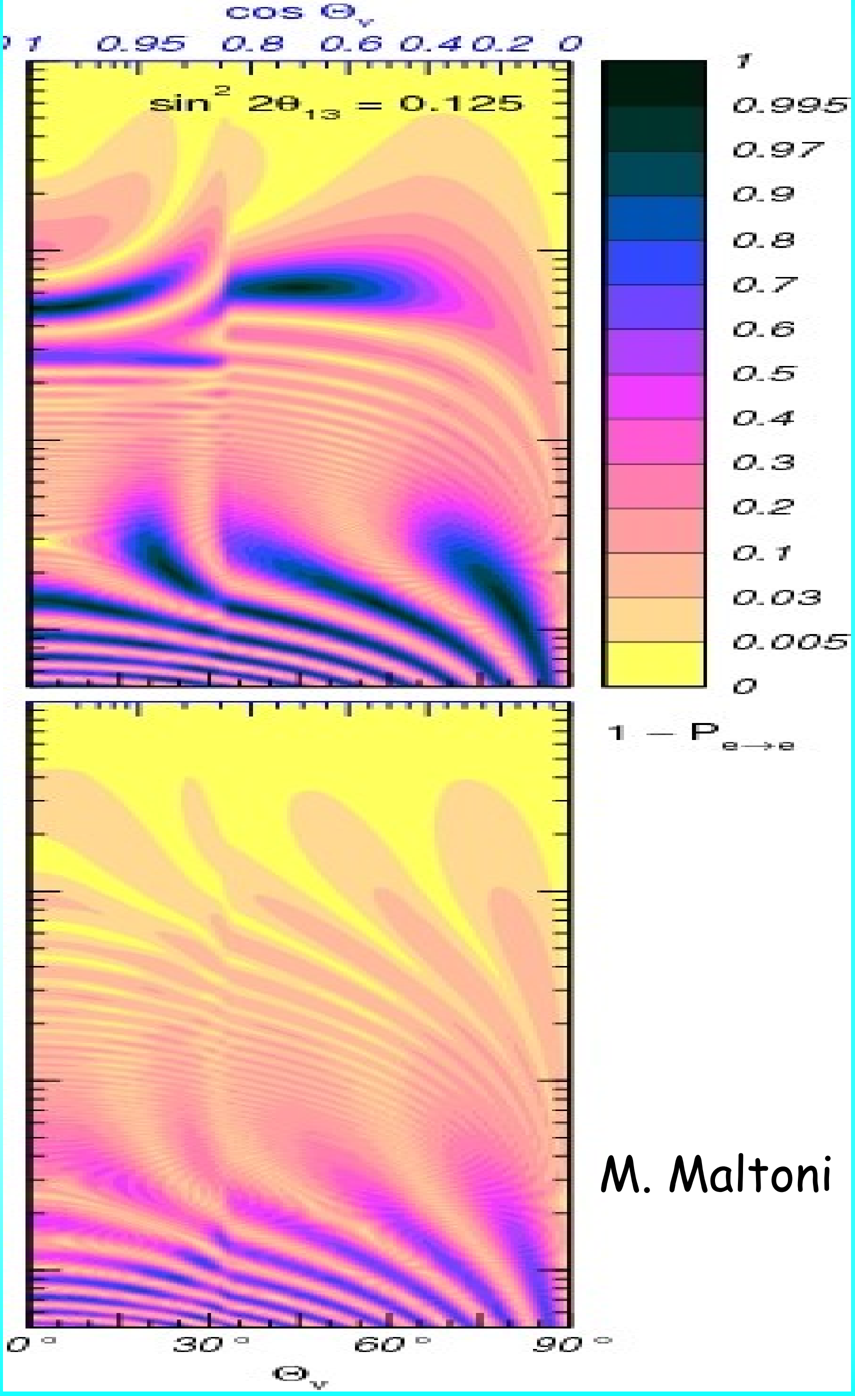




Neutrinos and antineutrinos

Normal mass hierarchy

No resonances



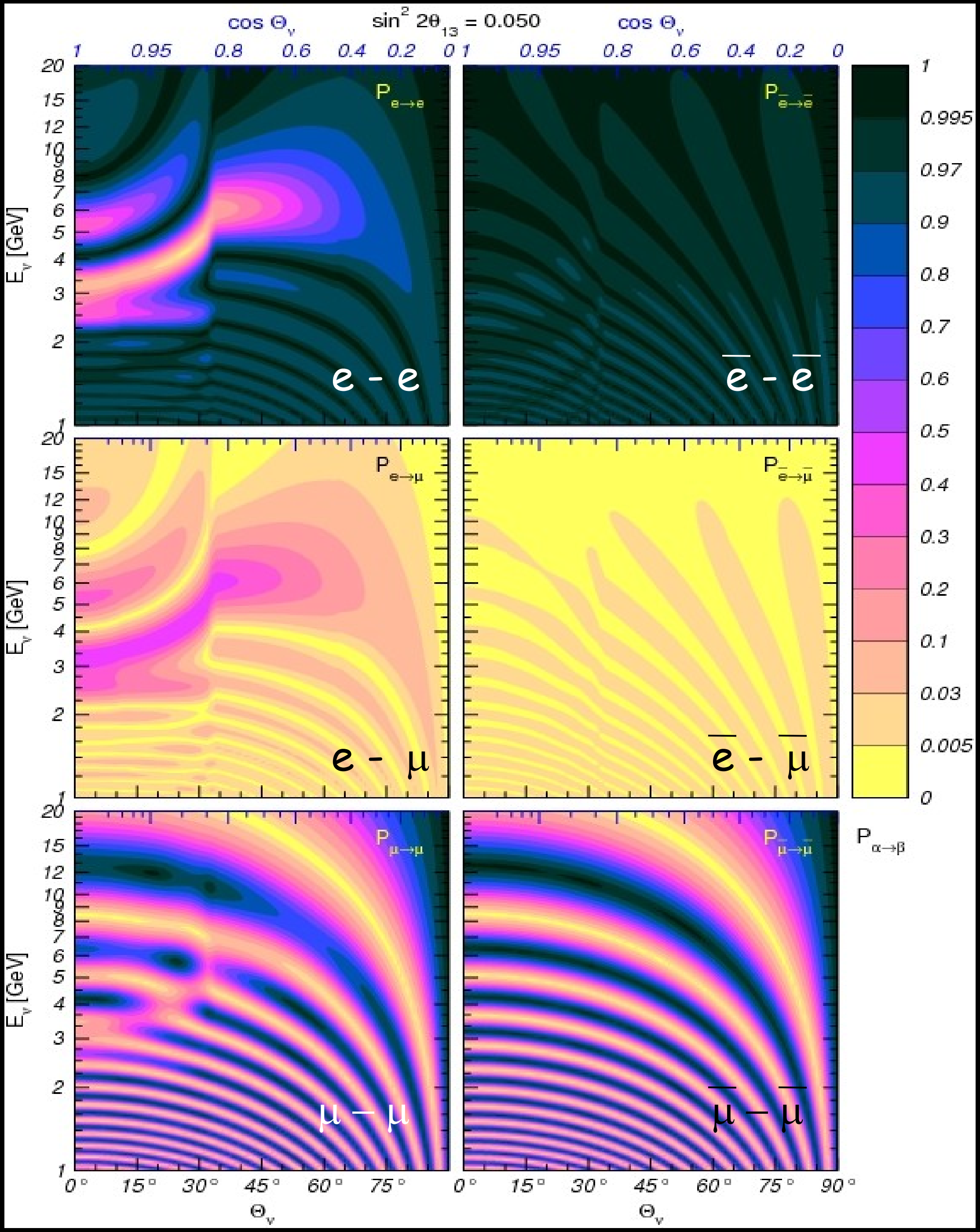
M. Maltoni

Other channels

For 2ν system

normal → inverted

neutrino → antineutrino



Interference CP-violation

Dependence on CP-phase

E.K. Akhmedov, M Maltoni, A Y S, arXiv: 0804.1466

δ -dependent parts of probabilities - interference of amplitudes driven by solar and atmospheric frequencies

$$P_{\mu e}{}^{\delta} = \sin 2\theta_{23}|A_{e2}||A_{e3}|\cos(\phi - \delta)$$

$$\phi = \arg (A_{e2} A_{e3}{}^*)$$

$$P_{\mu\mu}{}^{\delta} = - \sin 2\theta_{23}|A_{e2}||A_{e3}|\cos \phi \cos\delta - D_{23}$$

$$P_{\mu\tau}{}^{\delta} = - \sin 2\theta_{23}|A_{e2}||A_{e3}|\sin \phi \sin \delta + D_{23}$$

Dependences on ϕ and δ factorize

$$D_{23} = \tfrac{1}{2} \sin 4\theta_{23} \cos\delta\, [\mathrm{Re}\, A_{23}{}^* (A_{33} - A_{22})]$$

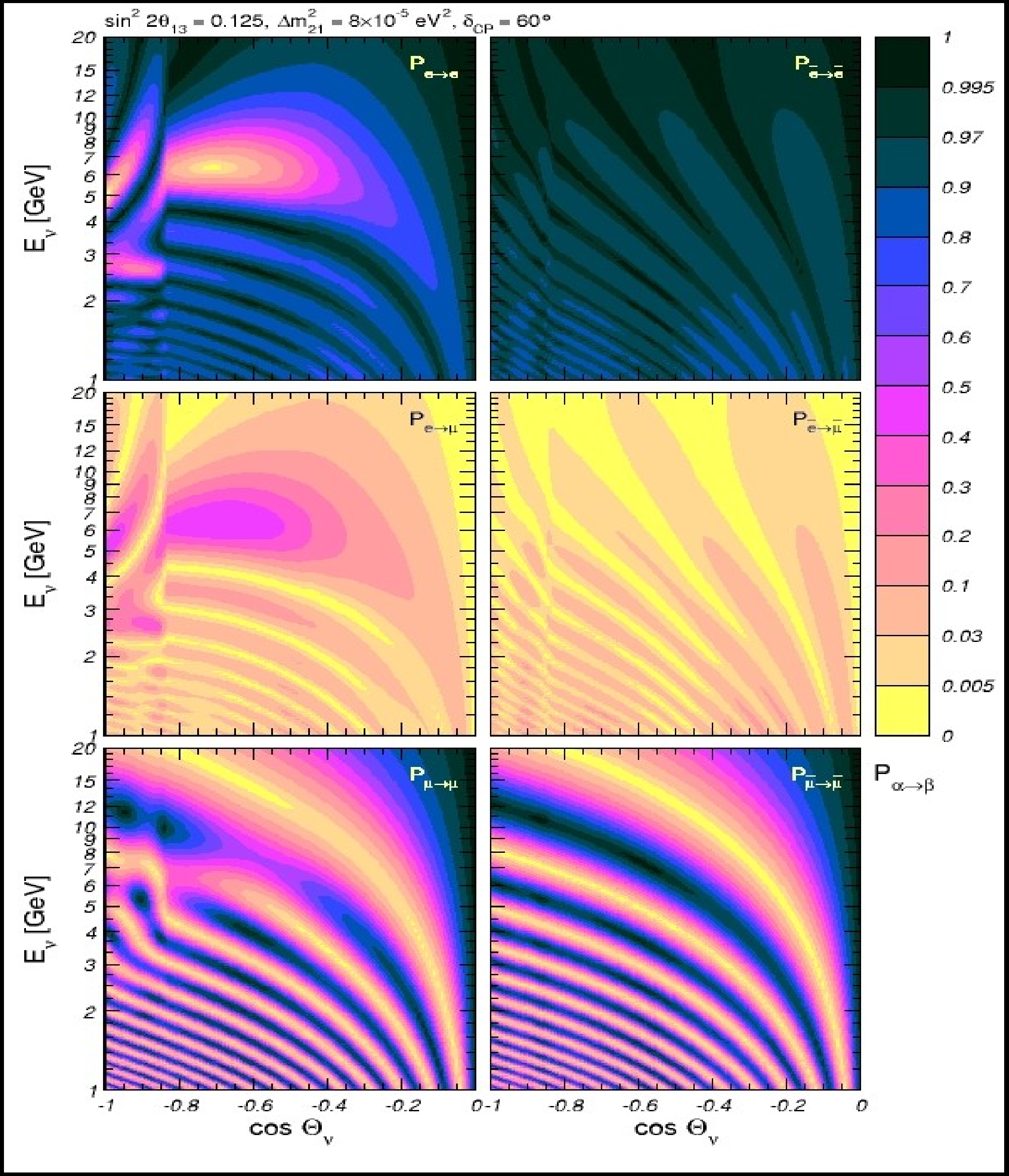
small amplitude

Sum: $\Sigma_{\alpha}\ P_{\mu\alpha}{}^{\delta} = 0$

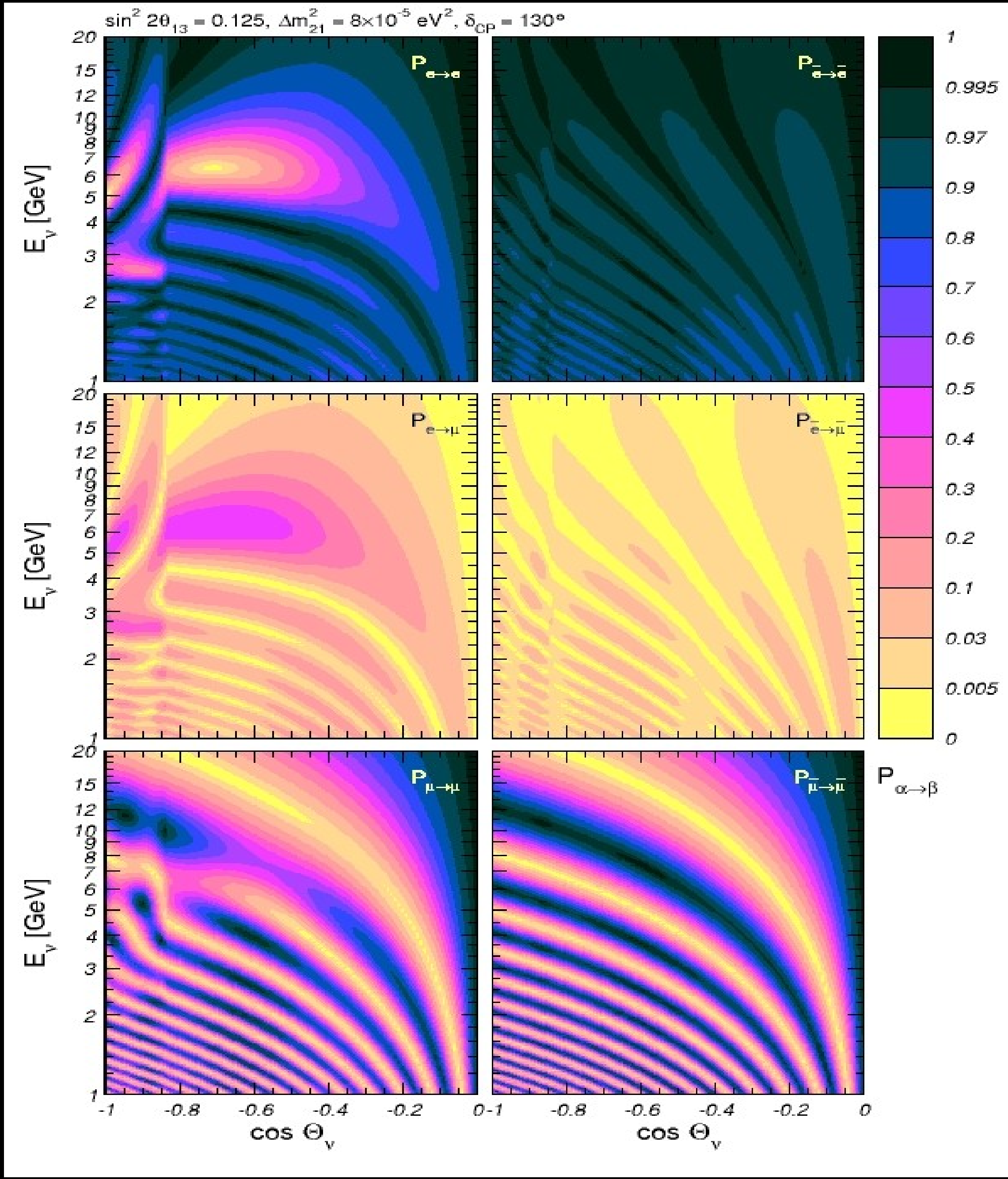
CP-violation

$\delta = 60^o$

Standard parameterization

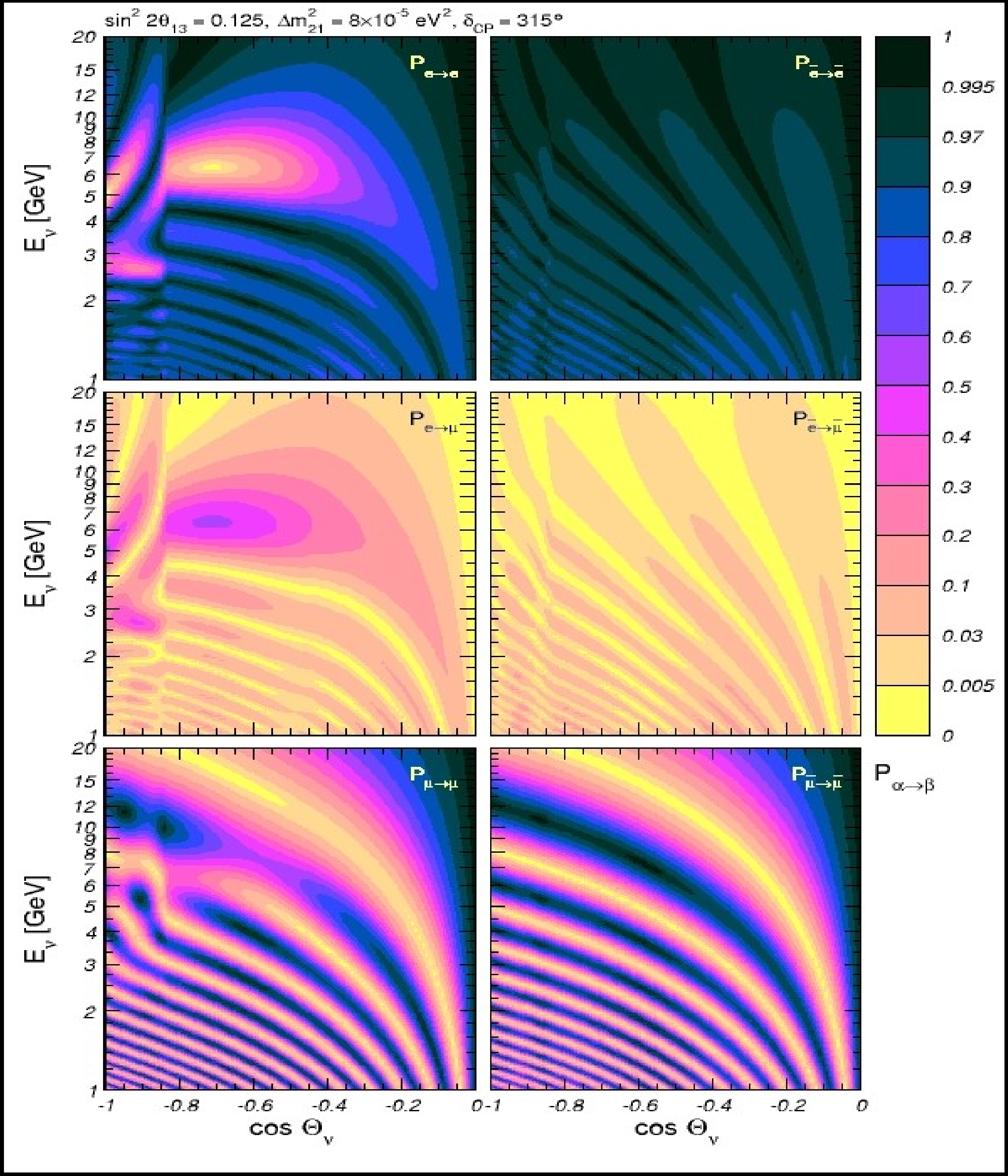


$\delta = 130^o$



sin² 2θ13 = 0.125, Δm²21 = 8×10⁻⁵ eV², δCP = 130°
Pe→e
Pē→ē
Pe→μ
Pē→μ̄
Pμ→μ
Pμ̄→μ̄
Eν [GeV]
cos Θν
Pα→β
1
0.995
0.97
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.03
0.005
0

δ = 315°



sin² 2θ₁₃ = 0.125, Δm²₂₁ = 8×10⁻⁵ eV², δ_CP = 315°
P e→e
P ē→ē
P e→μ
P ē→μ̄
P μ→μ
P μ̄→μ̄
E_ν [GeV]
cos Θ_ν
1
0.995
0.97
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.03
0.005
0
P α→β

"Magic lines"

V. Barger, D. Marfatia, K Whisnant
P. Huber, W. Winter, A.S.

$$P(\nu_e \to \nu_\mu) = |\cos\theta_{23} A_{e2} + e^{i\delta}\sin\theta_{23} A_{e3}|^2$$

$$P_{int} = 2 s_{23} c_{23} |A_{e2}||A_{e3}| \cos(\phi - \delta)$$

$$\phi = \arg(A_{e2} A_{e3}^*)$$

Dependence on δ disappears, interference term is zero if

$P_{int} = 0$

$A_{e2} = 0$ - solar magic lines

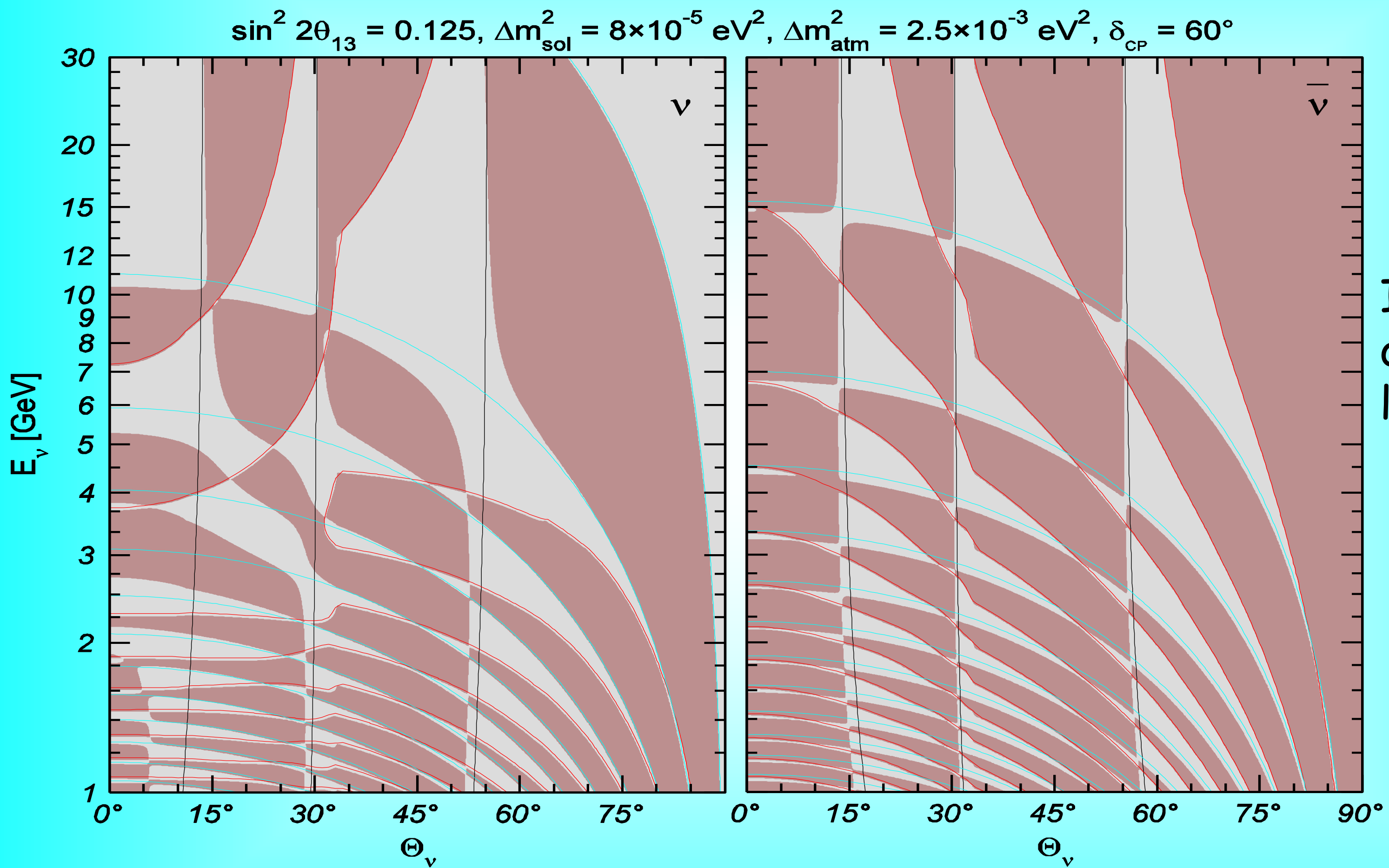
$A_{e3} = 0$ - atmospheric magic lines

$(\phi - \delta) = \pi/2 + 2\pi k$ - interference phase condition

$\phi(E, L) = \delta + \pi/2 + \pi k$ depends on δ

$\phi = \pi/2 + 2\pi k$ for $\nu_\mu \to \nu_\mu$

CP violation domains



Interconnection of lines due to level crossing

factorization is not valid

Regions of different sign of ΔP

solar magic lines
atmospheric magic lines
relative phase lines

$\Delta P = P(\delta) - P(\delta_f) = \text{const}$

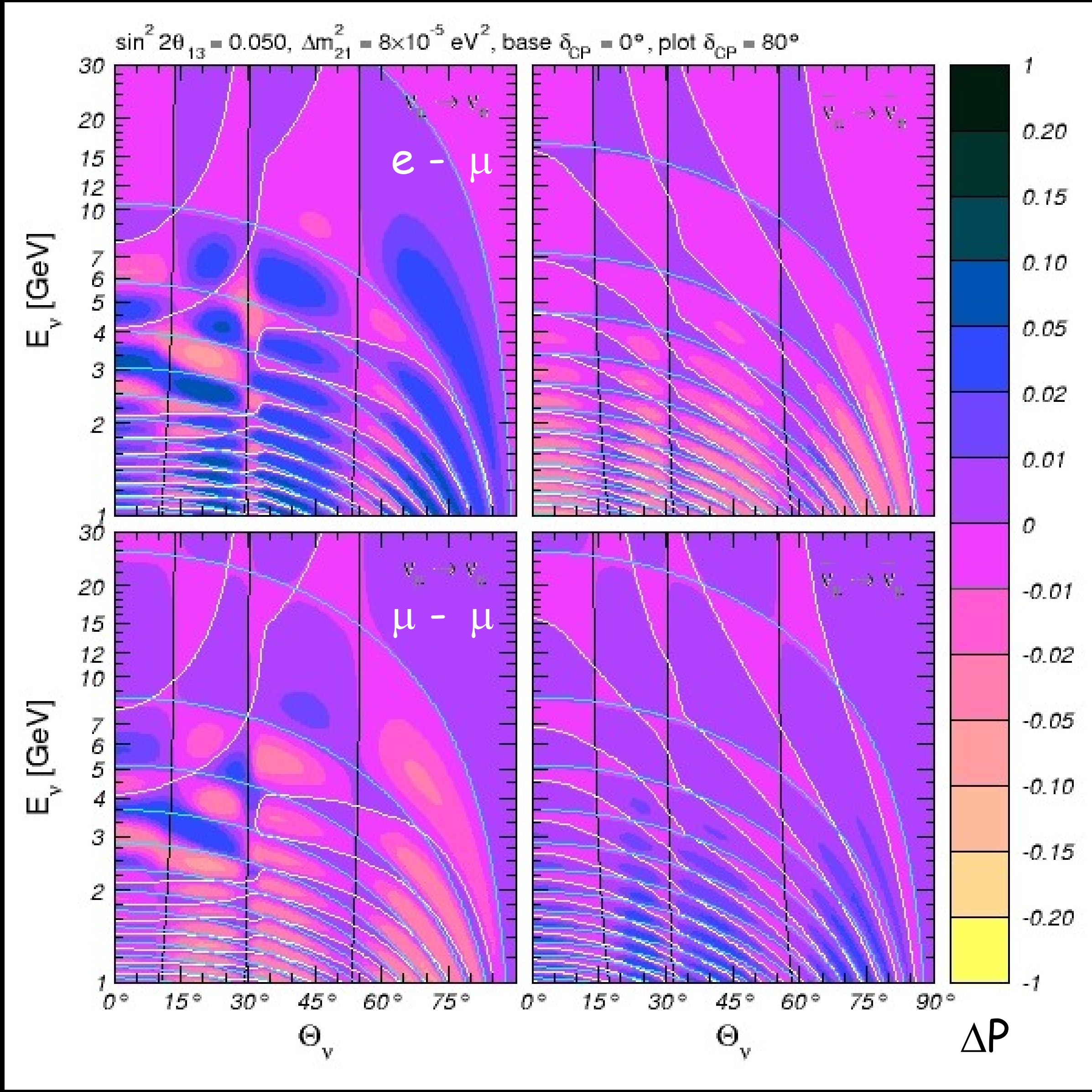
Magic grid

Int. phase line (blue) moves with δ-change

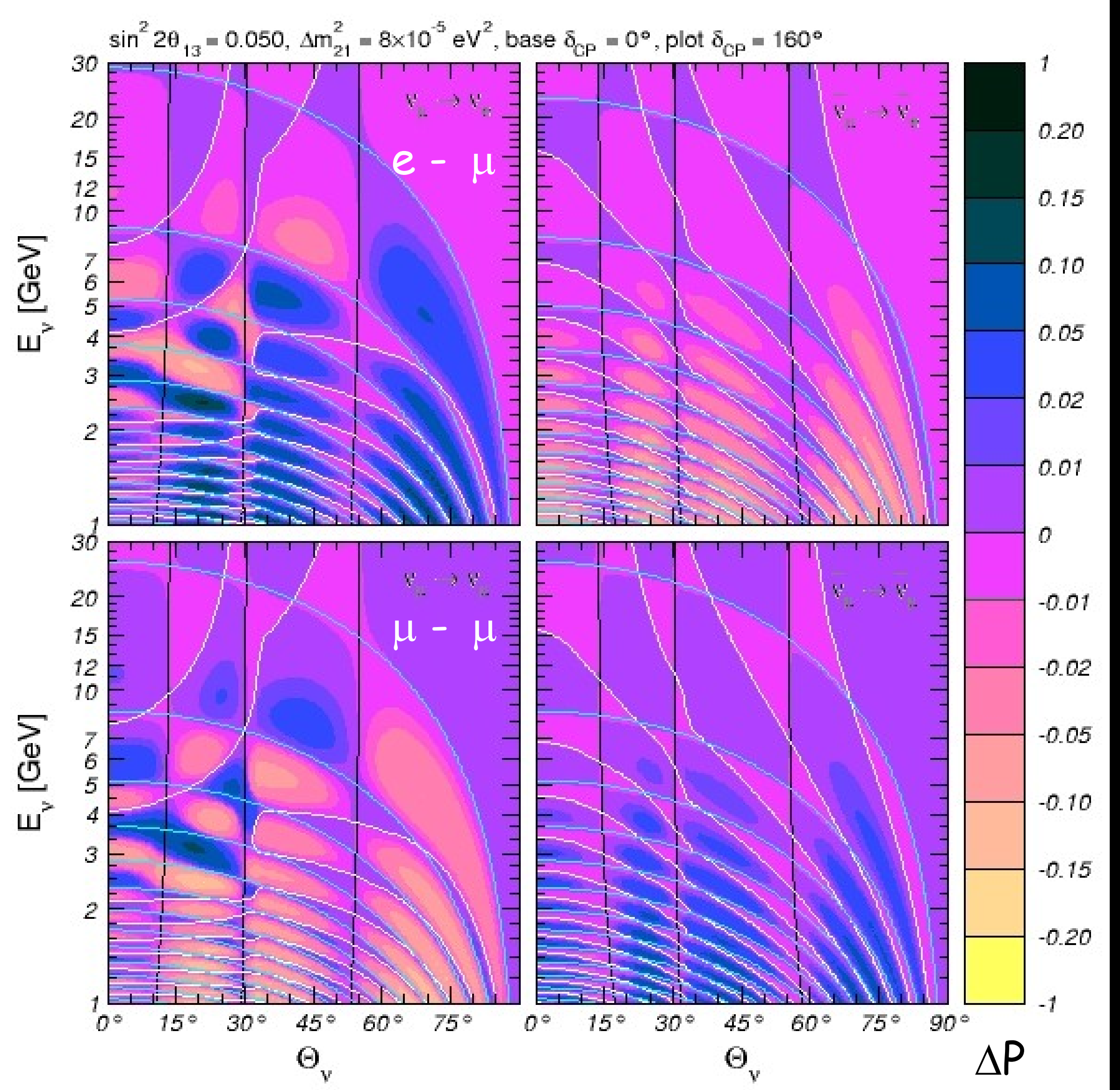
Grids do not change with δ

Black: solar

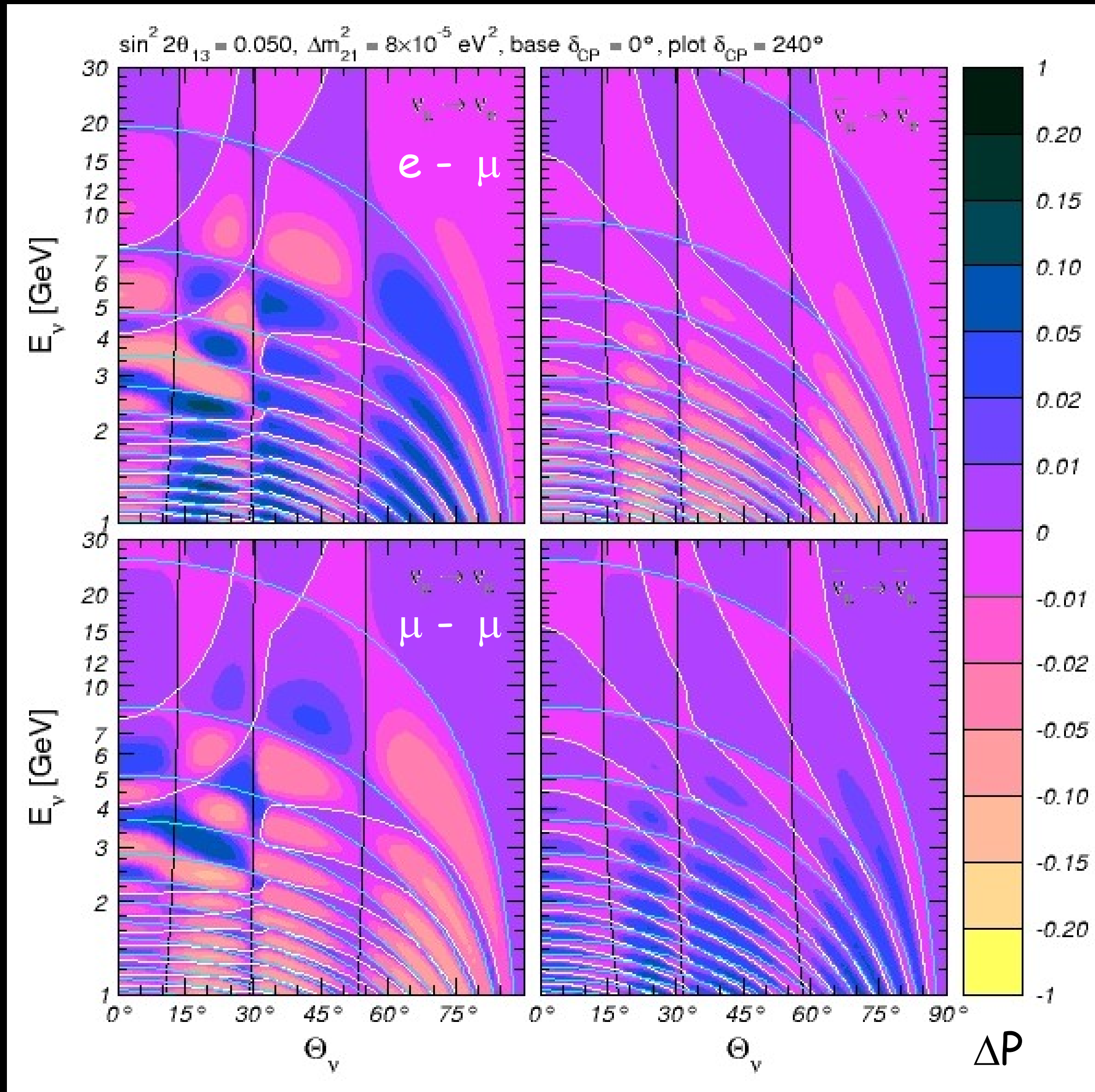
White: atmospheric



Neutrinos Antineutrinos



$\sin^2 2\theta_{13} = 0.050$, $\Delta m^2_{21} = 8\times10^{-5}$ eV^2, base $\delta_{CP} = 0°$, plot $\delta_{CP} = 160°$
e - μ
μ - μ
E_ν [GeV]
Θ_ν
ΔP



sin² 2θ13 = 0.050, Δm²21 = 8×10⁻⁵ eV², base δCP = 0°, plot δCP = 240°
e - μ
μ - μ
Eν [GeV]
Θν
0° 15° 30° 45° 60° 75° 90°
1 2 3 4 5 6 7 10 12 15 20 30
1 0.20 0.15 0.10 0.05 0.02 0.01 0 -0.01 -0.02 -0.05 -0.10 -0.15 -0.20 -1
ΔP

Fluxes at a detector

Screening

Charge suppression

ν_e - flux at the detector

$$\Phi_\alpha = \Phi_e^0 P_{e\alpha} + \Phi_\mu^0 P_{\mu\alpha} = \Phi_e^0 (P_{e\alpha} + r P_{\mu\alpha})$$

$r = \Phi_\mu^0 / \Phi_e^0$

$$\frac{\Phi_e}{\Phi_e^0} = 1 + (r \sin^2\theta_{23} - 1) P_{e3} + (r \cos^2\theta_{23} - 1) P_{e2} + r P_{\mu e}^{\delta}$$

screening

$$P_{\mu e}^{\delta} = \sin 2\theta_{23} \; [P_{e2} P_{e3}]^{1/2} \cos(\phi - \delta)$$

$\phi = \arg (A_{e2}^* A_{e3})$,

P_{e3} (P_{e2}) appears in all the probabilities with the screening factor $(r \sin^2\theta_{23} - 1)$, ($(r \cos^2\theta_{23} - 1)$)

Reason why oscillations of ν_e have not been observed from the beginning

ν_μ - flux at the detector

$r = \Phi_\mu{}^0 / \Phi_e{}^0$

$$\frac{\Phi_\mu}{\Phi_\mu{}^0} = 1 - \tfrac{1}{2} \sin^2 2\theta_{23} \,[1 - \mathrm{Re}\,(A_{33} A_{22}{}^*)] -$$

$$- \frac{s_{23}{}^2}{r} (r\, s_{23}{}^2 - 1)\, P_{e3} - \frac{c_{23}{}^2}{r} (r\, c_{23}{}^2 - 1)\, P_{e2} +$$

$$+ P_{\mu\mu}{}^\delta + \frac{1}{r} P_{e\mu}{}^\delta$$

Screened

CP-violation

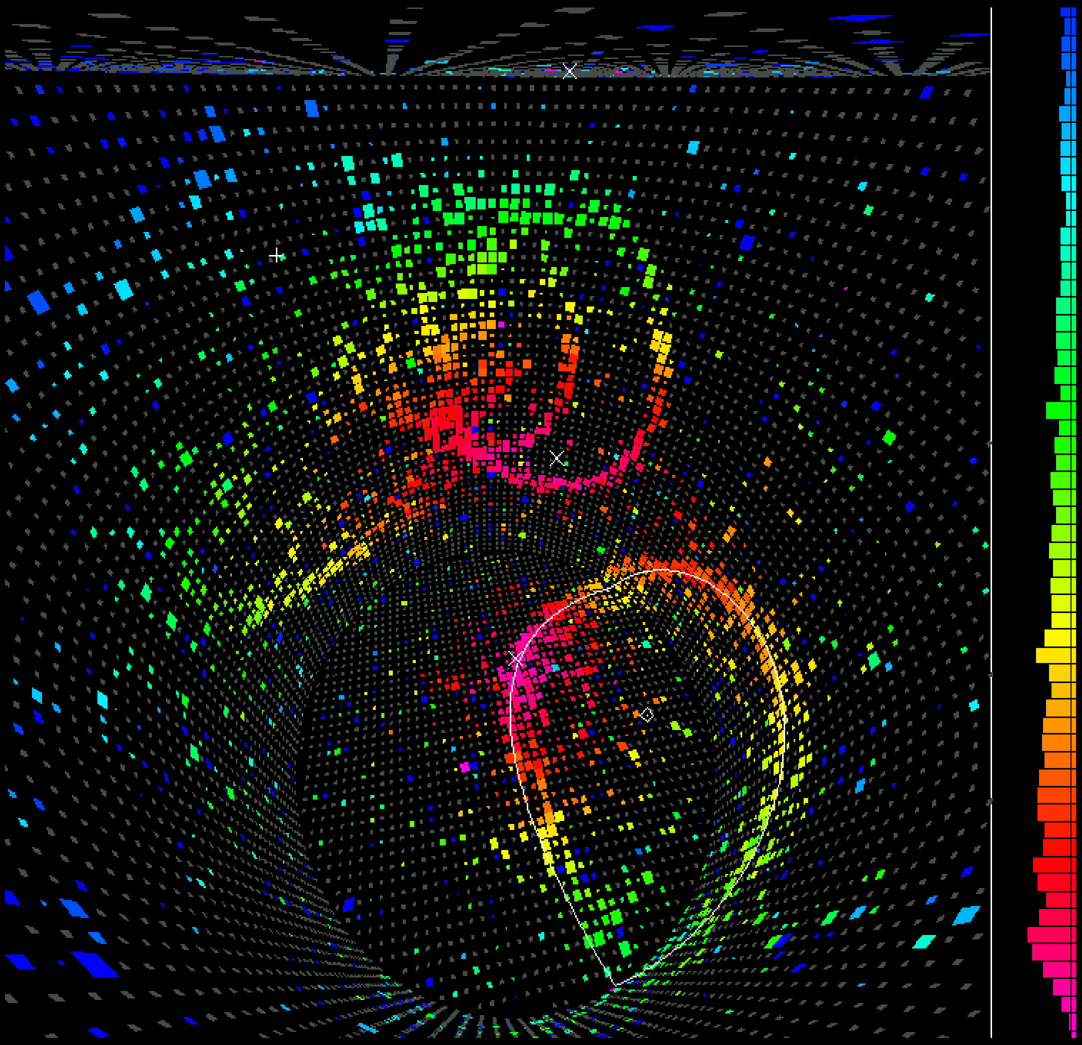
$$\frac{\Phi_\tau}{\Phi_\mu{}^0} = \tfrac{1}{2} \sin^2 2\theta_{23} \,[1 - \mathrm{Re}\,(A_{33} A_{22}{}^*)] -$$

$$- \frac{c_{23}{}^2}{r} (r\, s_{23}{}^2 - 1)\, P_{e3} - \frac{s_{23}{}^2}{r} (r\, c_{23}{}^2 - 1)\, P_{e2} +$$

$$+ P_{\mu\tau}{}^\delta + \frac{1}{r} P_{e\tau}{}^\delta$$

Screened

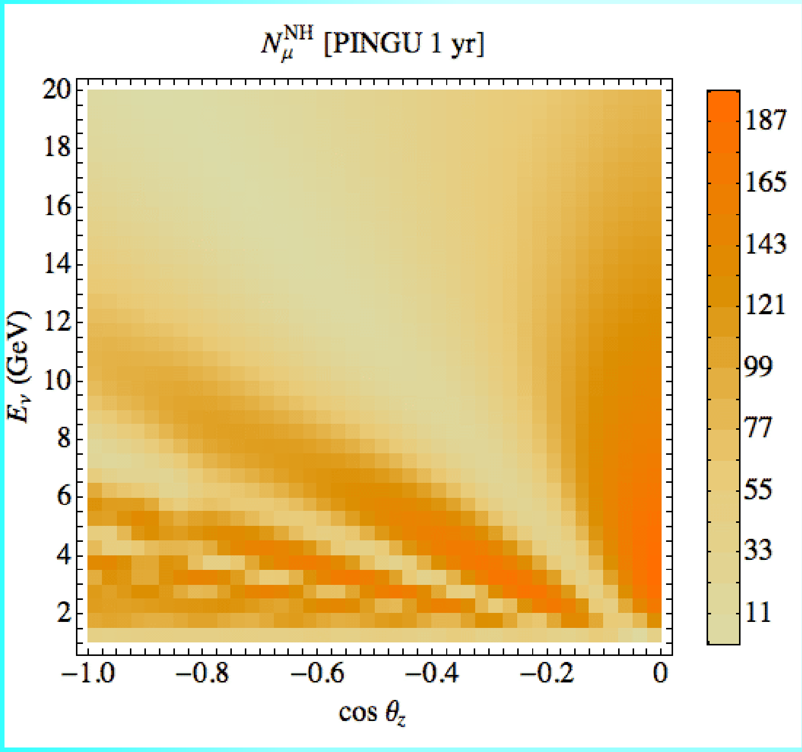
Ordering
Octant
CP-violation.

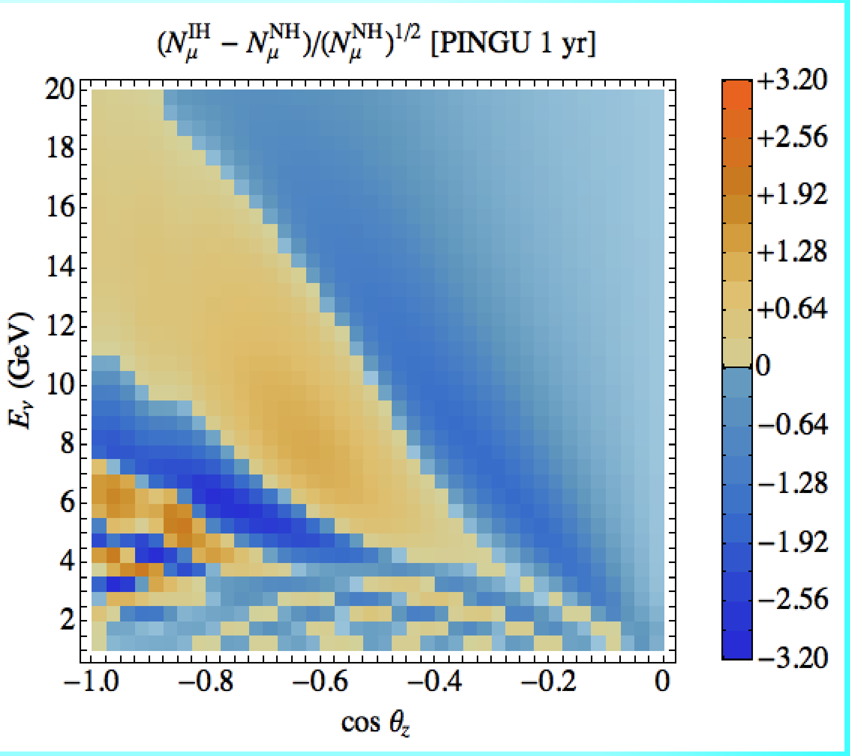


Track events

~ 10^5 events/year

``Distinguishability"

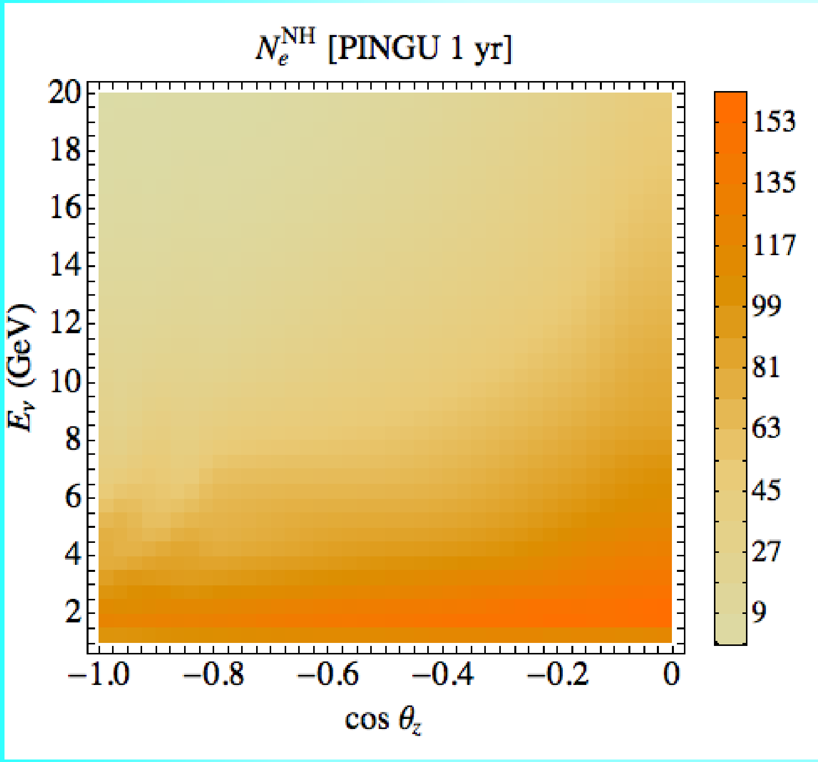


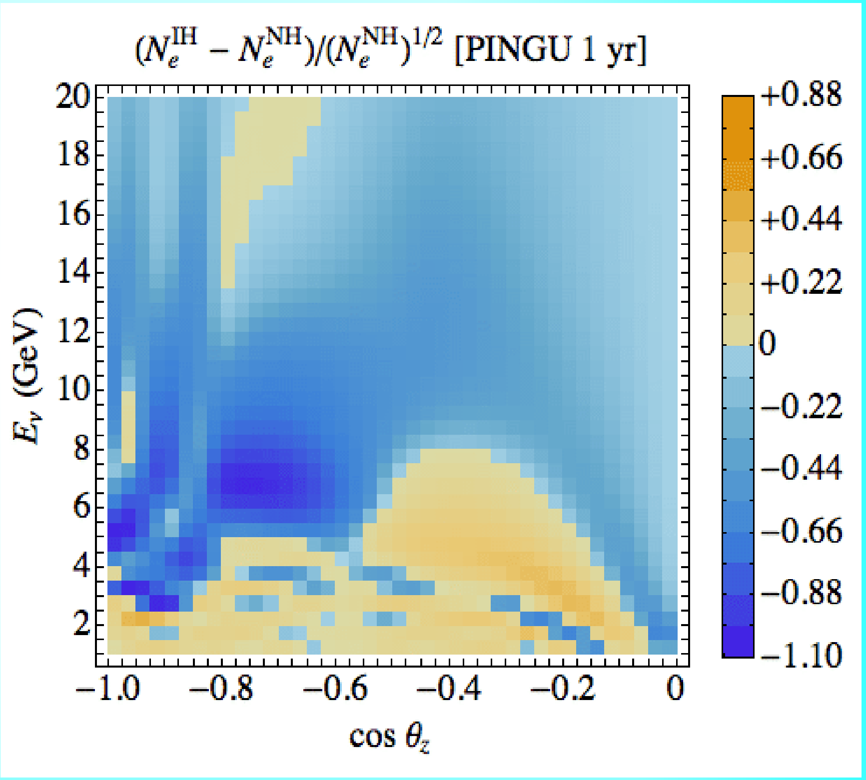


Estimator of sensitivity
S – asymmetry
|S| - significance

Cascade events

``Distinguishability"



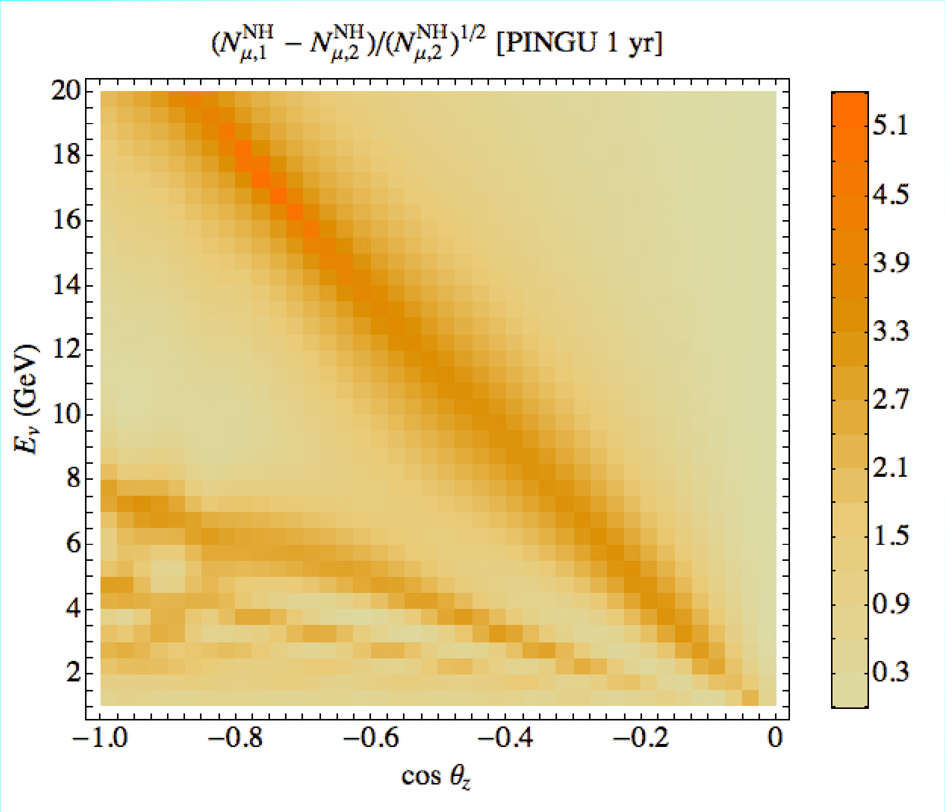


Statistical significance

Theta_23

Deviation of 2-3 mixing from maximal

quadrant



$\sin^2 \theta_{32,\,fit} = 0.50$

$\sin^2 \theta_{32,\,true} = 0.42$

Future measurements – improve accuracy of determination of the angle

Effect has the same sign

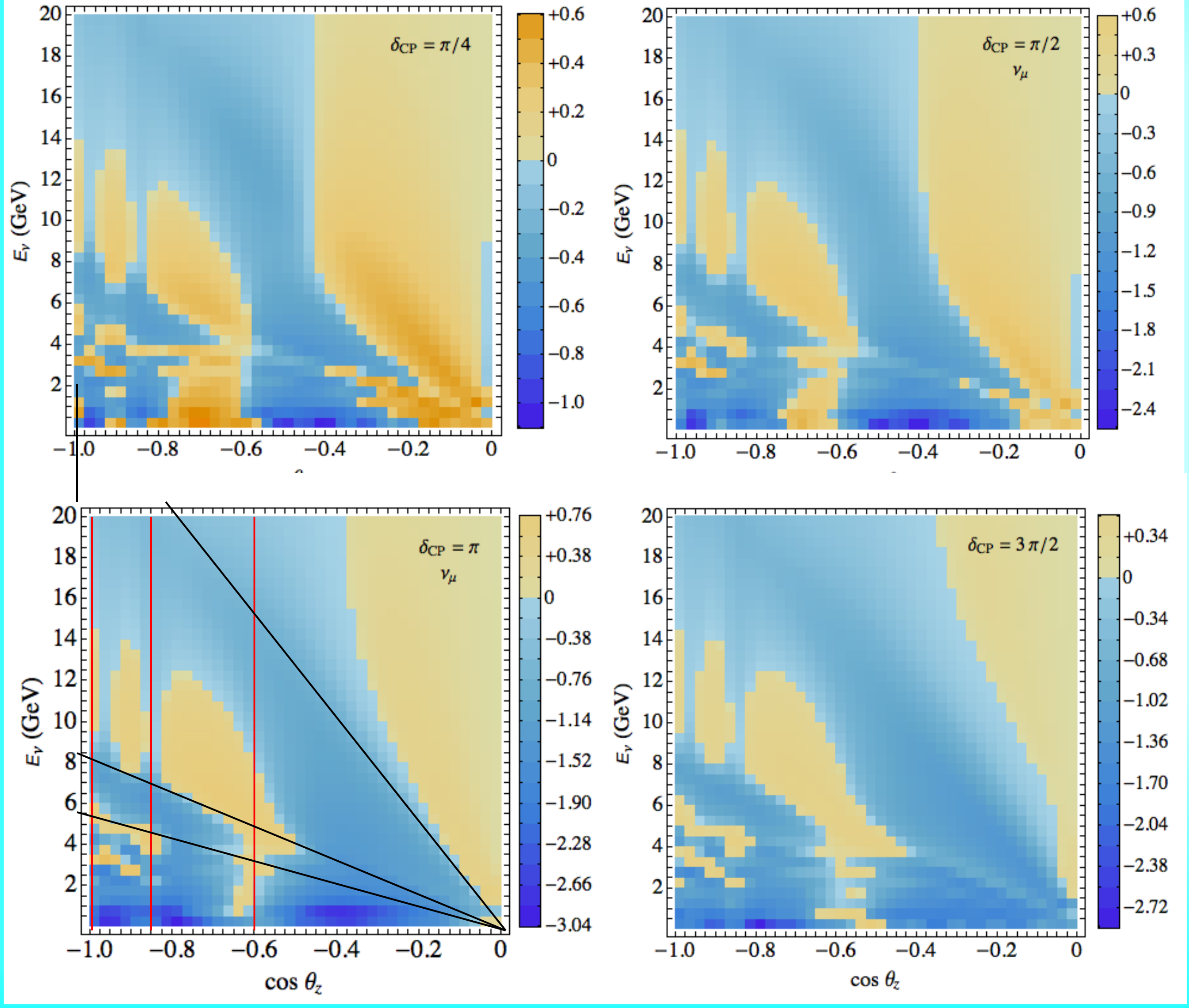
Large effect is in he region of small number of events

Regions of strong effects of the angle and hierarchy do not overlap significantly

CP-domains

S-distributions for different values of δ

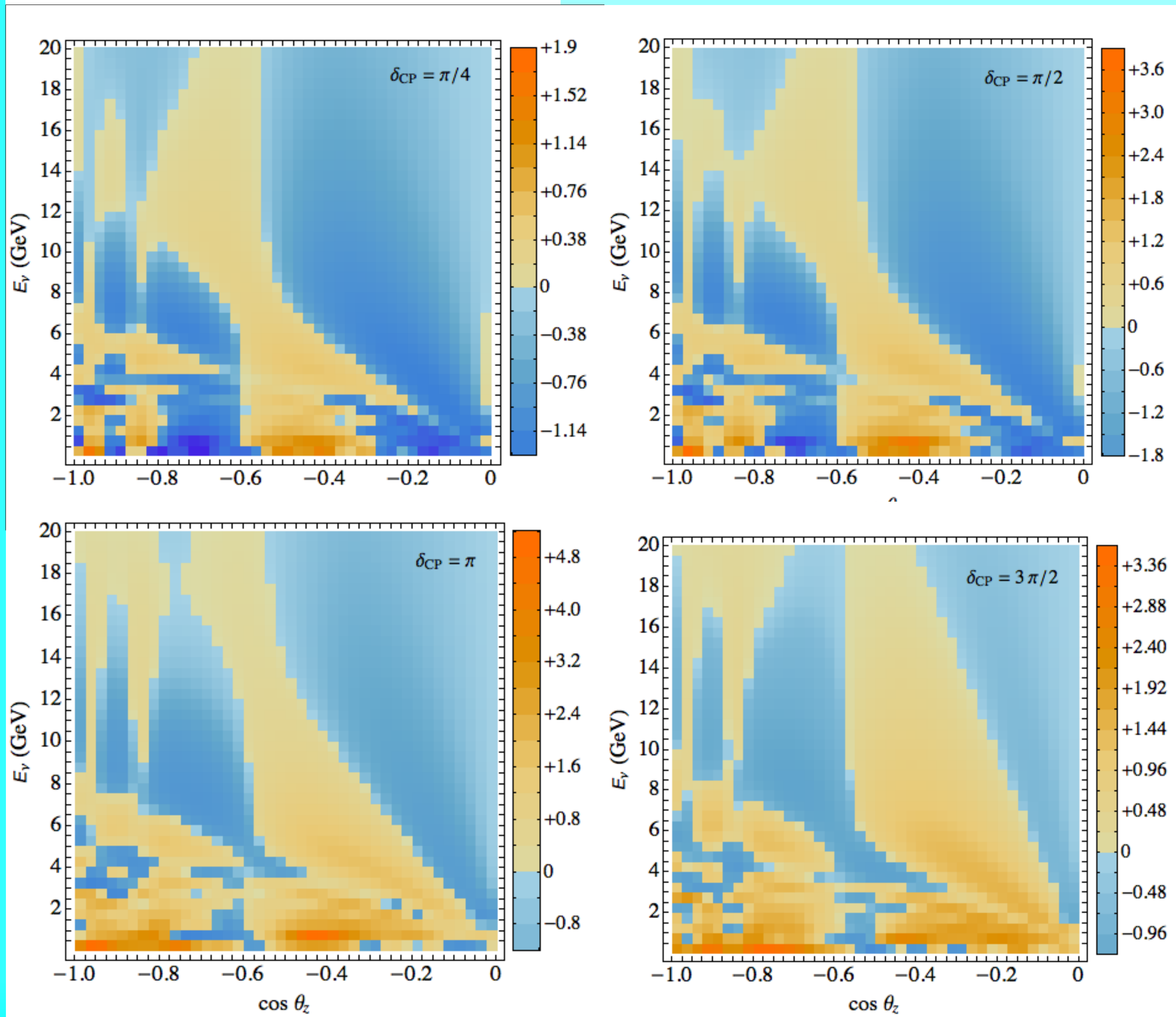
ν_μ-events (track + cascade)



CP-effect:
2 - 5 %
ΔN = 2 – 10 events in each small bin

CP-domains

Cascades (ν_e - events)



S-distributions for different values of δ

Strong asymmetry of CP differences

Have opposite sign at low energies with respect to ν_μ-events

Conclusions

Oscillations of the atmospheric neutrinos in the Earth in the standard 3ν framework are completely elaborated

Structure of the neutrino oscillograms is well understood

Physics includes
- Resonance enhancement of oscillations
- parametric effects
- interference of amplitudes with solar and atmospheric frequencies, CP violation, grid of magic lines, CP domains

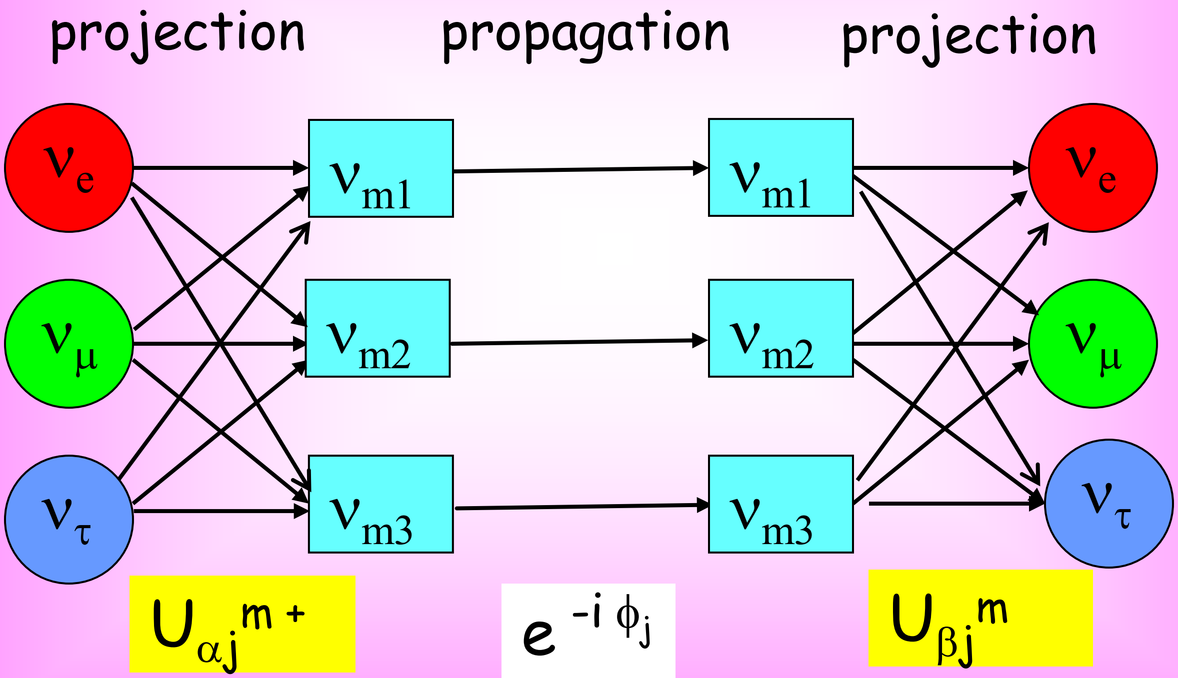
With this one can address open issues:
- Determination of the mass hierarchy
- Measurements of the CP-phase
- deviation of the 2-3 mixing from maximal
- searches new physics beyond the 3ν framework

Backup slides

Vacuum and constant density cases

Propagation basis – eigenstates in vacuum (mass states) or in matter

ν_{mi} - eigenstates in matter



$$P(\nu_\alpha \to \nu_\beta) = |\Sigma_j U_{\beta j}^{m} e^{-i\phi_j} U_{\alpha j}^{m+}|^2$$

A bit of history

Another way of getting strong transition

V. Ermilova ,V. Tsarev, V. Chechin, Krat. Soob. Fiz. # 5, 26, (1986)

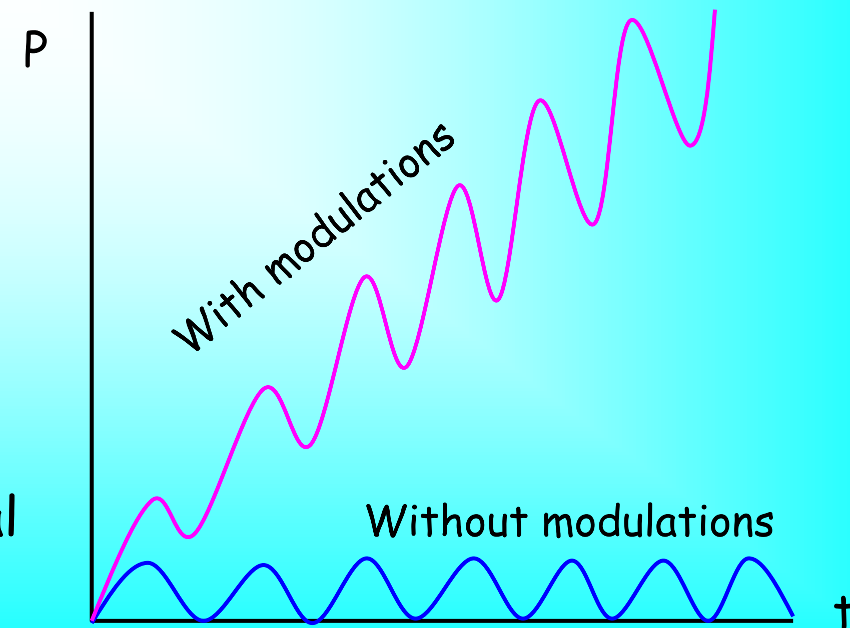
- no large vacuum mixing,
- no matter enhancement of mixing,
- no resonance conversion

Harmonic modulation of density $n(t) = \langle n \rangle + n_1 \cos \omega t$

Parametric resonance: $k\,\omega = 2\langle\omega\rangle, \quad k = 1, 2 \ldots$

$\langle\omega\rangle = \omega_m(\langle n \rangle)$ frequency of oscillations for the average density

$$\frac{\Delta m^2}{2E}$$



Solution from neutron-antineutron oscillations in magnetic field
G D Push, Nuovo Cim 74A, 2, 149 (1983)

Effect may play a role in astrophysical periodic structures