# Probing non-unitarity with atmospheric neutrinos

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arXiv:1609.08623, 1712.02798

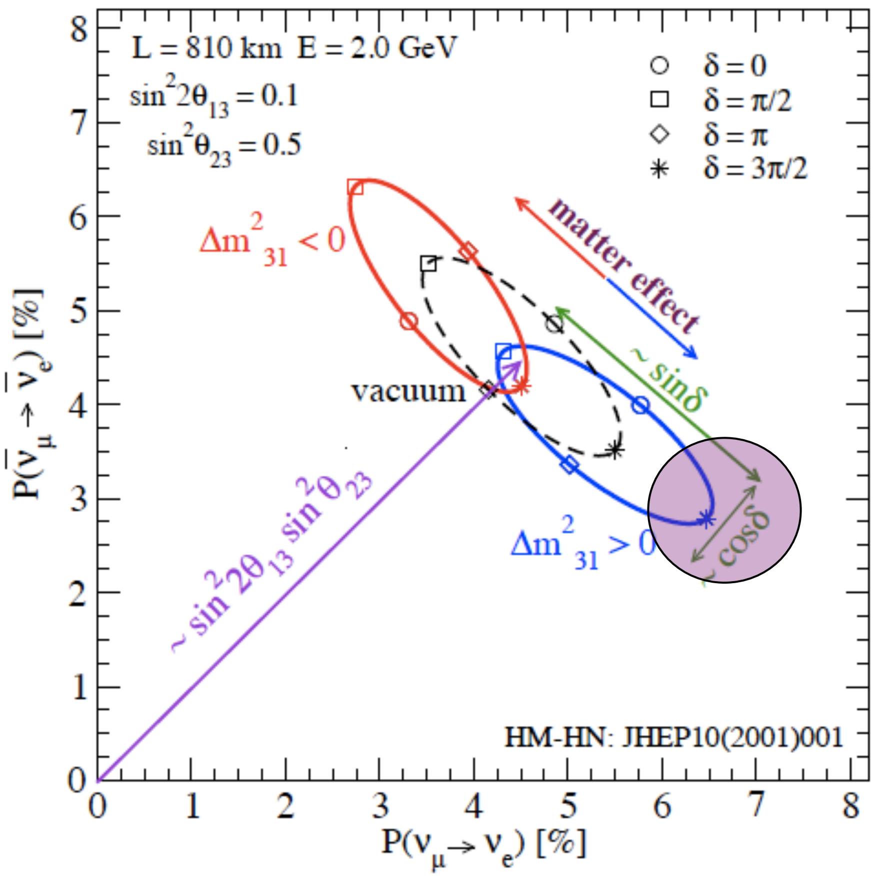
# We see things converge

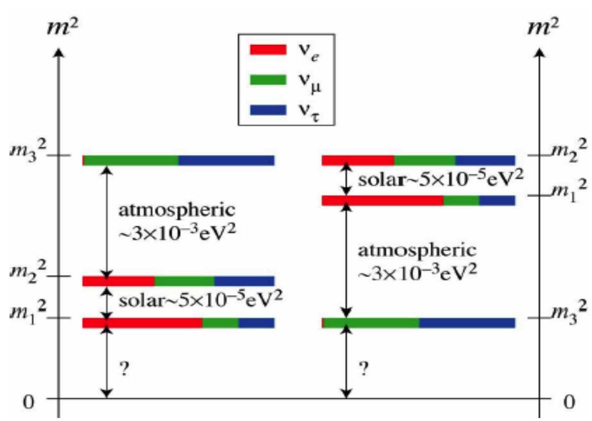
- T2K see more and more preference of $\delta \sim 3\pi/2$
- T2K II (proposed, run until 2026) they could see CPV at ~3 $\sigma$ (expected for ~$15 \times 10^{21}$ POT)
- $\delta \sim 3\pi/2$ is the best case for NOvA for mass ordering (see bi-P plot), will see mass hierarchy at ~3 $\sigma$
- Already Bari global analysis says "normal at 3 $\sigma$"
- Everybody vs NOvA discrepancy about $\theta_{23}$ seems resolved → best fit near maximal
- We must think about what is next?

My answer is "paradigm test"

# $\delta=3\pi/2$ (or $-\pi/2$) implies that we are at the tip of the ellipse → the best case for NOvA

P-\bar{P} bi-probability diagram, proposed by HM-H.Nunokawa, JHEP 2001



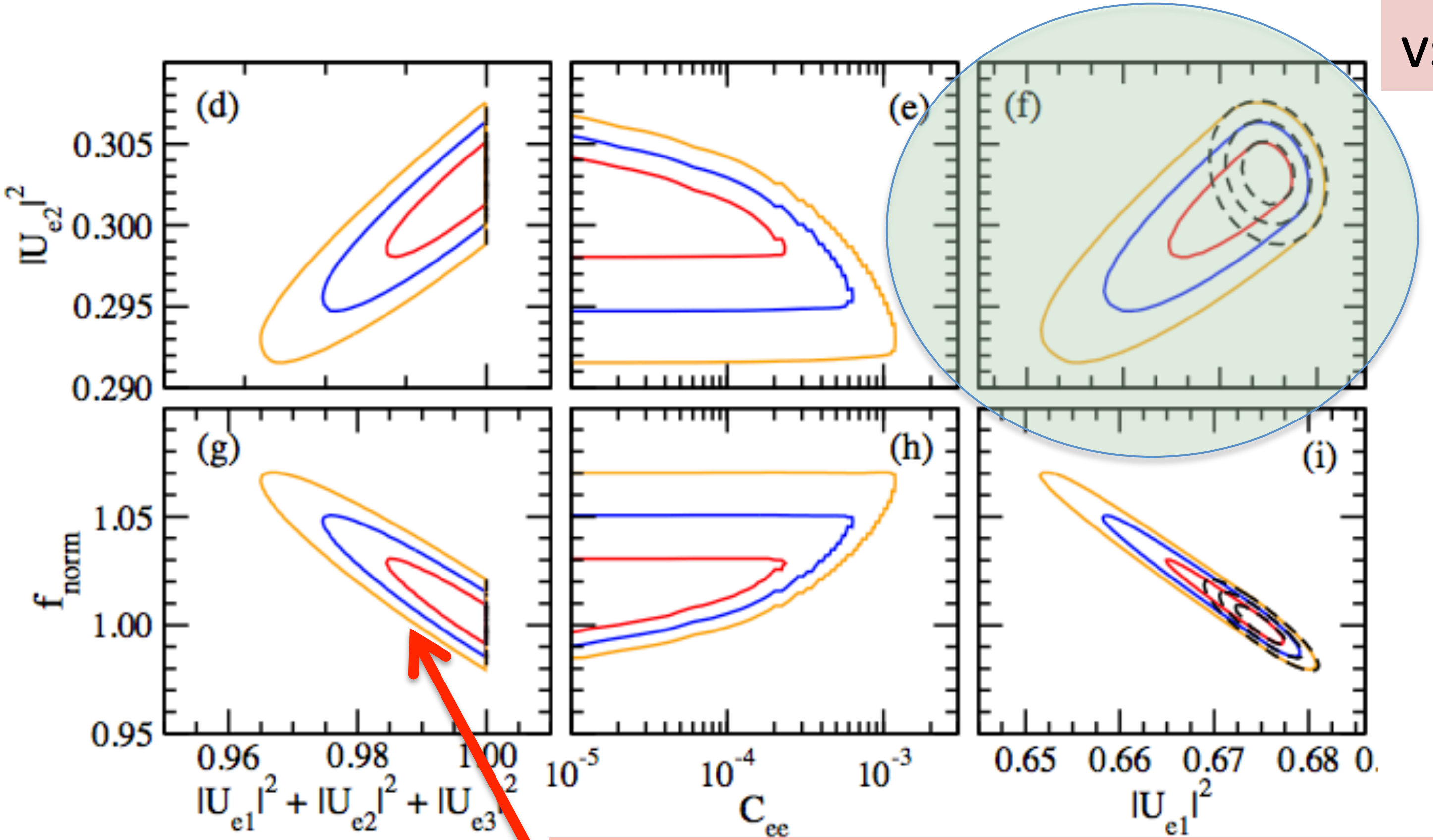

Sign of $\Delta m^2_{31}$ distinguishes normal vs inverted mass ordering

$\delta$ and sign $\Delta m^2_{31}$ couple because ($\Delta m^2_{31} \rightarrow -\Delta m^2_{31}$, $\delta \rightarrow \pi-\delta$) symmetry in vacuum (JHEP 2001)

JUNO can measure $|U_{e1}|^2 + |U_{e2}|^2 + |U_{e3}|^2$

# JUNO can do Separate determination $|U_{e1}|^2$ and $|U_{e2}|^2$ : 5 years

Resolve $\Delta m^2_{31}$ vs. $\Delta m^2_{32}$ waves




3% flux error assumed

$1 - (|U_{e1}|^2 + |U_{e2}|^2 + |U_{e3}|^2)$ can be constrained to ~1% level by JUNO !

Chee Sheng Fong, HM, Hiroshi Nunokawa, JHEP 2017

# High vs low scale unitarity violation

# New Physics at high energies:

- This is Orthodox way, well studied ..
- high scale UV → pioneering work by Antusch, Biggio, Fernandez-Martinez, Gavela, and Lopez-Pavon, JHEP2006
- But, neutrino experiments will not be the best player for unitarity test
- the reason is: high-energy → SU(2) x U(1) prevails → charge lepton gives stronger constraints

→ My suggestion today is low-E UV

# New Physics at low energies: relatively new option

- "low scale": heavy leptons/neutrinos do communicate with light $\nu$ system, i.e., participate to nu oscillation
- Various scenarios are proposed which involve "new physics" at low energies: motivated by LSND-MiniBoone, DAMA, etc.


[21] A. E. Nelson and J. Walsh, "Short Baseline Neutrino Oscillations and a New Light Gauge Boson," Phys. Rev. D **77** (2008) 033001 doi:10.1103/PhysRevD.77.033001 [arXiv:0711.1363 [hep-ph]].

[22] M. Pospelov and J. Pradler, "Elastic scattering signals of solar neutrinos with enhanced baryonic currents," Phys. Rev. D **85** (2012) 113016 Erratum: [Phys. Rev. D **88** (2013) no.3, 039904] doi:10.1103/PhysRevD.85.113016, 10.1103/PhysRevD.88.039904 [arXiv:1203.0545 [hep-ph]].

[23] R. Harnik, J. Kopp and P. A. N. Machado, "Exploring nu Signals in Dark Matter Detectors," JCAP **1207** (2012) 026 doi:10.1088/1475-7516/2012/07/026 [arXiv:1202.6073 [hep-ph]].

[24] K. S. Babu, A. Friedland, P. A. N. Machado and I. Mocioiu, "Flavor Gauge Models Below the Fermi Scale," JHEP **1712** (2017) 096 doi:10.1007/JHEP12(2017)096 [arXiv:1705.01822 [hep-ph]].


Plus many more !!

- Orthodoxy seems challenged, e.g., WIMP dark matter, low-E SUSY, day one NP, ..

# High- vs low-energy unitarity violation

**Low-energy UV**

- lepton flavor universality: YES
- zero distance neutrino flavor transition: NO
- Kinematical effect of sterile nu emission: YES

**High-energy UV**

- lepton flavor universality: NO
- zero distance neutrino flavor transition: YES
- Kinematical effect of sterile nu emission: YES (if kinematically allowed)

# 3 active+N-sterile $\nu$ model for Low-E unitarity violation

Other models of Low-E UV?

# 3 active +N sterile unitary model in vacuum

3x3

$$\nu_\alpha = \mathbf{U}_{\alpha i}\tilde{\nu}_i,$$

$$\mathbf{U} = \begin{bmatrix} U & W \\ Z & V \end{bmatrix}$$

NxN

$$i\frac{d}{dx}\nu = H\nu.$$

$$H = \mathbf{U}\begin{bmatrix} \Delta_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \Delta_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \Delta_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & \Delta_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & 0 & \Delta_{3+N} \end{bmatrix}\mathbf{U}^\dagger$$

$$\Delta_i \equiv \frac{m_i^2}{2E} \quad (i = 1, 2, 3), \qquad \Delta_J \equiv \frac{m_J^2}{2E} \quad (J = 4, \cdots, 3+N).$$

# 3+N model for low-E UV and modest requests on it

- By 3+N model I mean (3+N) space is unitary, but not in 3 active nu space

Unique? Probably not. General Low-E UV model hard to construct. My strategy is …

- Requirement: The prediction of the 3+N model must be independent of details of N sterile sector
- After fulfilling this criterion we will show what is the *difference* between High-E vs Low-E UV

# Probability in vacuum

$$P(\nu_\beta \to \nu_\alpha) = \left|\sum_{k=1}^{3} U_{\alpha k} U_{\beta k}^*\right|^2 \tag{4.5}$$

$$-2\sum_{j\neq k} \mathrm{Re}\,(U_{\alpha j}^* U_{\beta j} U_{\alpha k} U_{\beta k}^*) \sin^2\frac{(\Delta_k - \Delta_j)x}{2} + \sum_{j\neq k} \mathrm{Im}\,(U_{\alpha j}^* U_{\beta j} U_{\alpha k} U_{\beta k}^*) \sin(\Delta_k - \Delta_j)x$$

$$+\sum_J |W_{\alpha J}|^2 |W_{\beta J}|^2$$

$$+\sum_{J\neq K} [\mathrm{Re}\,(W_{\alpha J}^* W_{\beta J} W_{\alpha K} W_{\beta K}^*) \cos(\Delta_K - \Delta_J)x + \mathrm{Im}\,(W_{\alpha J}^* W_{\beta J} W_{\alpha K} W_{\beta K}^*) \sin(\Delta_K - \Delta_J)x]$$

$$+2\sum_{j=1}^{3}\sum_{K=4}^{3+N} [\mathrm{Re}\,(U_{\alpha j}^* U_{\beta j} W_{\alpha K} W_{\beta K}^*) \cos(\Delta_K - \Delta_j)x + \mathrm{Im}\,(U_{\alpha j}^* U_{\beta j} W_{\alpha K} W_{\beta K}^*) \sin(\Delta_K - \Delta_j)x]\,.$$

What is this?

- Active-active, active-sterile, sterile-sterile oscillations
- If $\Delta m^2_{as}$ ($\Delta m^2_{ss}$) > 0.1 eV$^2$, "fast oscillation" due to active-sterile and sterile-sterile $\Delta m^2$ are averaged out

$$\left\langle \sin\left(\frac{\Delta m^2_{Ji} x}{2E}\right)\right\rangle \approx \left\langle \sin\left(\frac{\Delta m^2_{JK} x}{2E}\right)\right\rangle \approx 0,$$

# P looks almost standard one, but there is a new term

Appearance

Probability leakage !

$$P(\nu_\beta \to \nu_\alpha) = \mathcal{C}_{\alpha\beta} + \left| \sum_{j=1}^{3} U_{\alpha j} U^*_{\beta j} \right|^2 - 2 \sum_{j \neq k} \mathrm{Re}\left( U_{\alpha j} U^*_{\beta j} U^*_{\alpha k} U_{\beta k} \right) \sin^2 \frac{(\Delta_k - \Delta_j)x}{2}$$

$$- \sum_{j \neq k} \mathrm{Im}\left( U_{\alpha j} U^*_{\beta j} U^*_{\alpha k} U_{\beta k} \right) \sin(\Delta_k - \Delta_j)x,$$

U = non-unitary "MNS"

Disappearance

$$P(\nu_\alpha \to \nu_\alpha) = \mathcal{C}_{\alpha\alpha} + \left( \sum_{j}^{3} |U_{\alpha j}|^2 \right)^2 - 4 \sum_{k>j}^{3} |U_{\alpha j}|^2 |U_{\alpha k}|^2 \sin^2 \frac{(\Delta_k - \Delta_j)x}{2},$$

$$\mathcal{C}_{\alpha\beta} \equiv \sum_{J=1}^{N} |W_{\alpha J}|^2 |W_{\beta J}|^2, \qquad \mathcal{C}_{\alpha\alpha} \equiv \sum_{J=1}^{N} |W_{\alpha J}|^4$$

Order ~ $W^4$, small!!

# P-leaking term: It must be obvious to exist, right?

- There is a N sterile sector which can communicate with active nu sector
- So the probability leaks to sterile sector
- Yet, not emphasized before…
- ~ $W^{4,}$ Too small?

$$\delta_{\alpha\beta} = \sum_{j=1}^{3} U_{\alpha j} U^*_{\beta j} + \sum_{J=4}^{N+3} W_{\alpha J} W^*_{\beta J}.$$

Then, $\left|\sum_{j=1}^{3} U_{\alpha j} U^*_{\beta j}\right|^2 = \left|\sum_{J=4}^{N+3} W_{\alpha J} W^*_{\beta J}\right|^2$ in the appearance channel ($\alpha \neq \beta$),

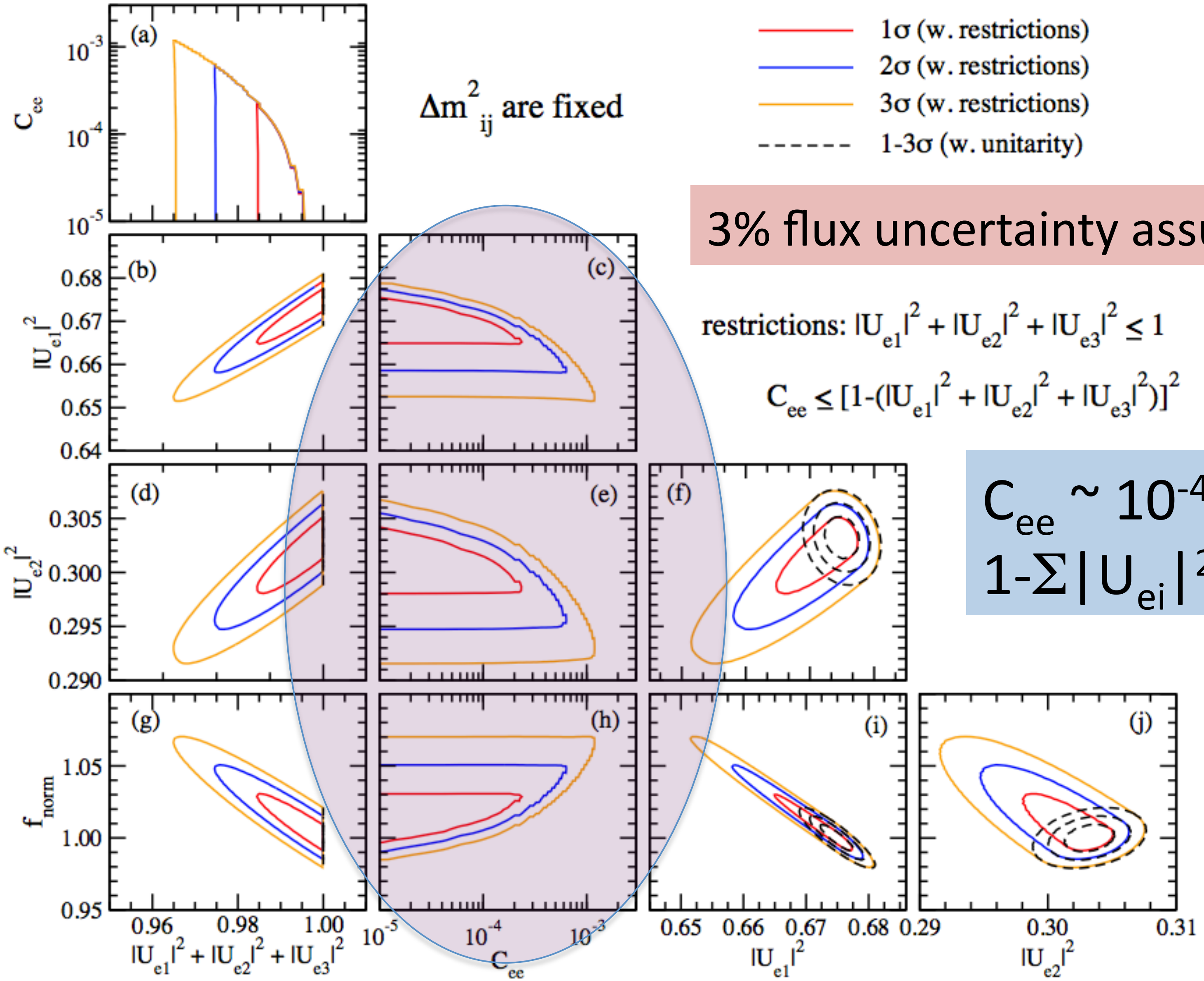
$\left(\sum_{j=1}^{3} |U_{\alpha j}|^2\right)^2 = \left(1 - \sum_{J=4}^{N+3} |W_{\alpha J}|^2\right)^2 = 1 - \mathcal{O}(W^2)$ in the disappearance channel

Term kept by S. Parke and M. Ross Lonergan, PRD 2017 is also 4th order in W

Can one detect $C_{ee}$?

Invitation to non-unitary world..

# Sensitivity to $C_{ee}$ and $1-\Sigma|U_{ei}|^2$ from JUNO 5 years




3% flux uncertainty assumed

$C_{ee} \sim 10^{-4}$ (1 σ)
$1-\Sigma|U_{ei}|^2 \sim 0.01$

# Parke-Ross-Lonergan PRD2016

Without P leaking term!

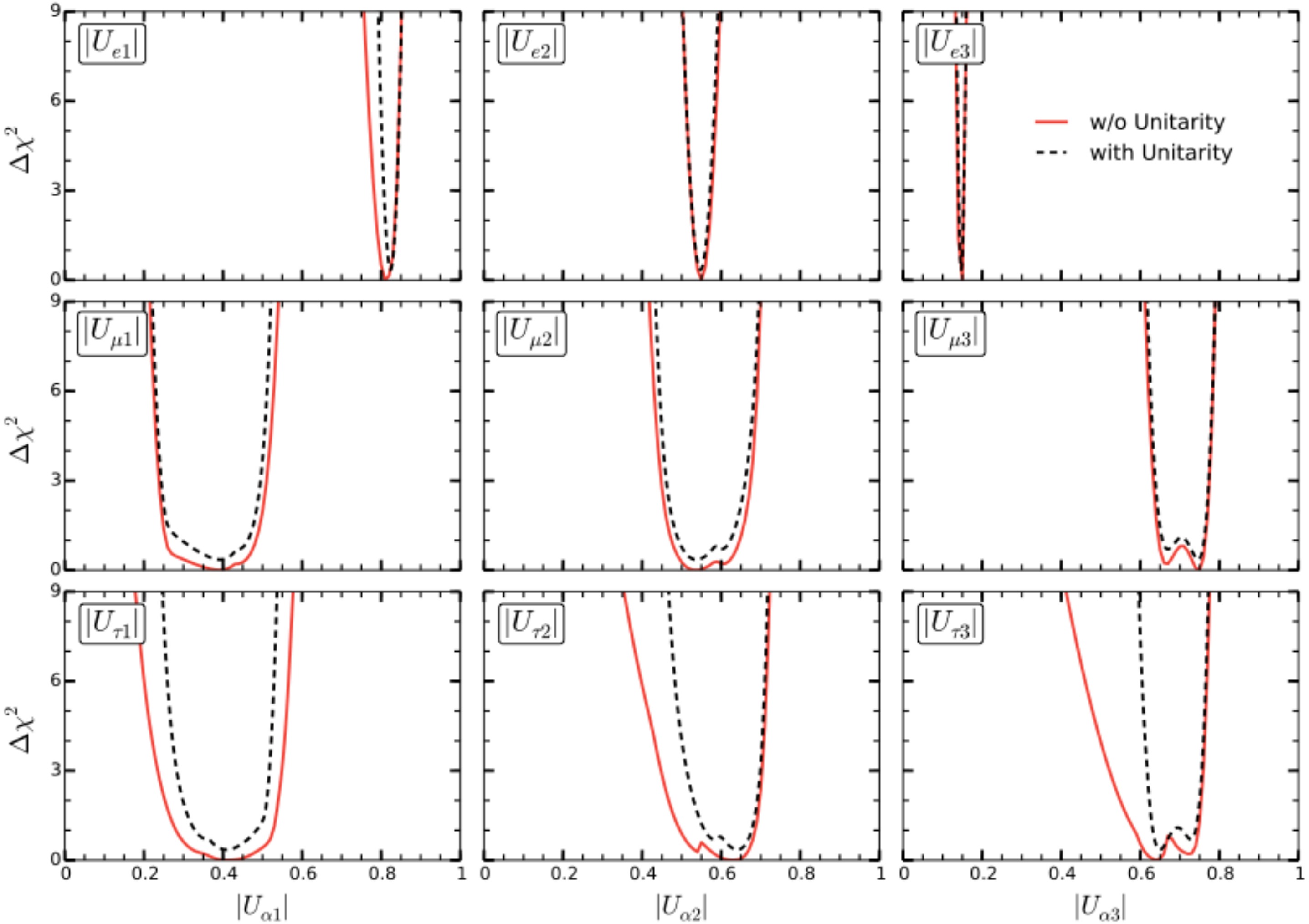



FIG. 1. Marginalized 1-D $\Delta\chi^2$ for each of the magnitudes of the $3 \times 3$ neutrino mixing matrix elements, without (red solid) and with (black dashed) the assumption of unitarity. The x-axis is the magnitude of each individual matrix element, and the y-axis is the associated $\Delta\chi^2$ after marginalization over all parameters other than the one in question. This analysis was performed for the normal hierarchy, the inverse hierarchy providing the same qualitative result.

# Constraints on unitarity violation (Parke-Ross-Lonergan)

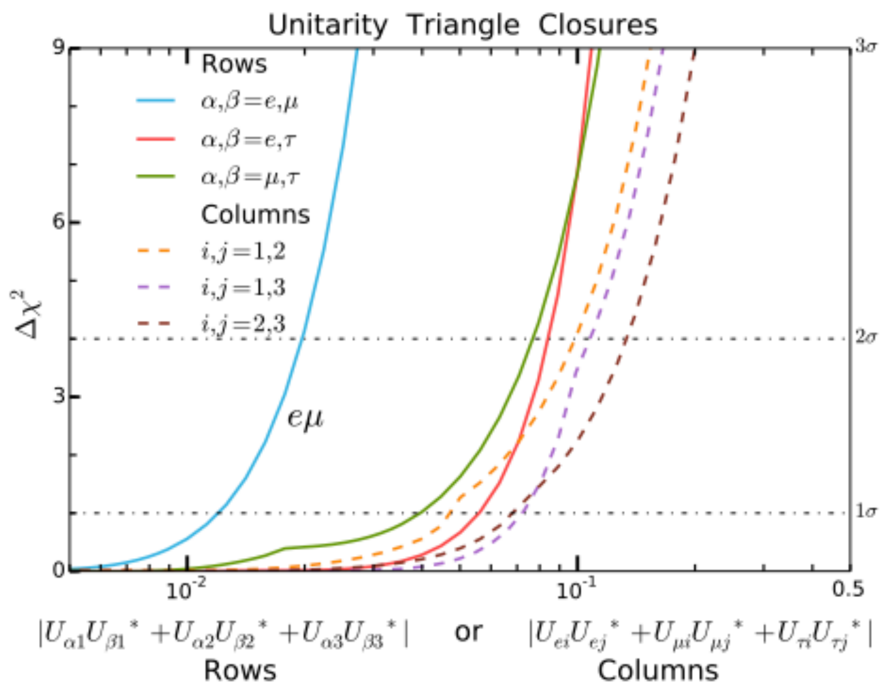



FIG. 2. 1-D $\Delta\chi^2$ for the absolute value of the closure of the three row (solid) and three column (dashed) unitarity triangles when considering new physics that enters above $|\Delta m^2| \geq 10^{-2}$ eV$^2$. There is one unique unitarity triangle, the $\nu_e \nu_\mu$ row unitarity triangle, in that it does not contain any $\nu_\tau$ elements and hence is constrained to be unitary at a level half an order of magnitude better than the others. By comparison to Fig. 3 one can clearly see that the Cauchy-Schwartz constraints are satisfied.

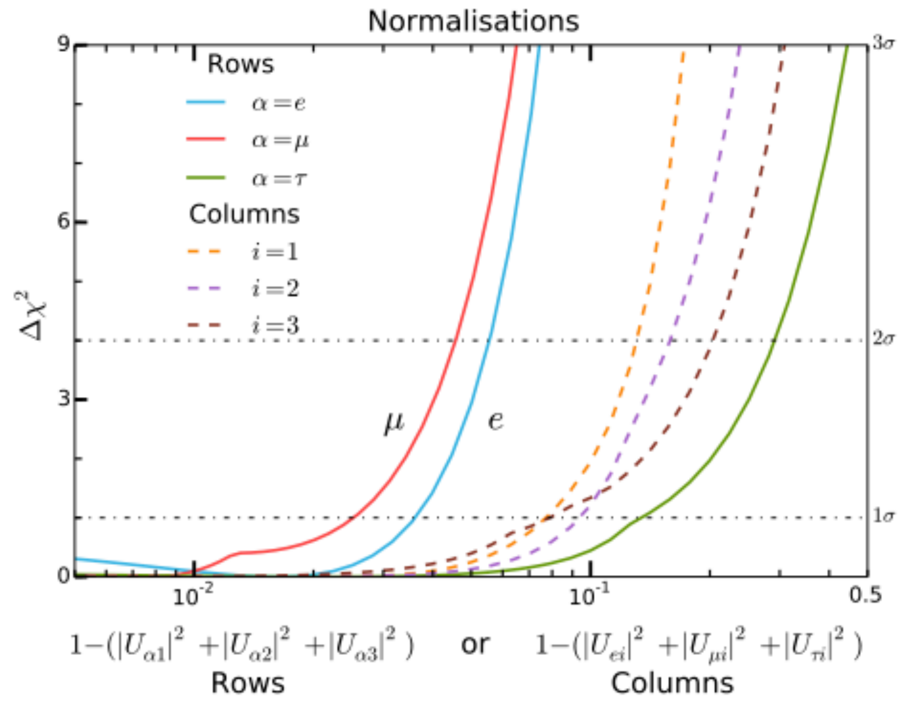



FIG. 3. 1-D $\Delta\chi^2$ for deviation of both $U_{\mathrm{PMNS}}$ row (solid) and column (dashed) normalizations, when considering new physics that enters above $|\Delta m^2| \geq 10^{-2}$ eV$^2$.

# Model-independent P: Prevail to "in matter" ?

# Small-UV perturbation theory

Chee Sheng Fong, HM, Hiroshi Nunokawa, arXiv:1712.02798

$$H = \mathbf{U}\begin{bmatrix} \mathbf{\Delta_a} & 0 \\ 0 & \mathbf{\Delta_s} \end{bmatrix}\mathbf{U}^\dagger + \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix} \equiv H_{\rm vac} + H_{\rm matt}$$

where $\mathbf{\Delta_a} = {\rm diag}(\Delta_1, \Delta_2, \Delta_3)$ and $\mathbf{\Delta_s} = {\rm diag}(\Delta_4, \Delta_5, \cdots, \Delta_{N+3})$.

$$A = \begin{bmatrix} \Delta_A - \Delta_B & 0 & 0 \\ 0 & -\Delta_B & 0 \\ 0 & 0 & -\Delta_B \end{bmatrix}$$

$$\mathbf{U} = \begin{bmatrix} U & W \\ Z & V \end{bmatrix}$$

$$\Delta_A \equiv \frac{a}{2E}, \qquad \Delta_B \equiv \frac{b}{2E},$$

$$a = 2\sqrt{2} G_F N_e E \approx 1.52 \times 10^{-4} \left(\frac{Y_e \rho}{\rm g\, cm^{-3}}\right)\left(\frac{E}{\rm GeV}\right) {\rm eV}^2,$$

$$b = \sqrt{2} G_F N_n E = \frac{1}{2}\left(\frac{N_n}{N_e}\right) a.$$

# Small-UV perturbation theory: change to vacuum mass basis

$$\tilde{H} = \tilde{H}_{\rm vac} + \tilde{H}_{\rm matt} = \begin{bmatrix} \mathbf{\Delta_a} & 0 \\ 0 & \mathbf{\Delta_s} \end{bmatrix} + \mathbf{U}^\dagger \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix} \mathbf{U}.$$

$$\tilde{H}_0 = \begin{bmatrix} \mathbf{\Delta_a} + U^\dagger A U & 0 \\ 0 & \mathbf{\Delta_s} \end{bmatrix}, \qquad \tilde{H}_1 = \begin{bmatrix} 0 & U^\dagger A W \\ W^\dagger A U & W^\dagger A W \end{bmatrix}.$$

X=unitary matrix, diagonalize $H_0$(3x3)

$$\mathbf{X}^\dagger \tilde{H}_0 \mathbf{X} = \begin{bmatrix} X^\dagger \left(\mathbf{\Delta_a} + U^\dagger A U\right) X & 0 \\ 0 & \mathbf{\Delta_s} \end{bmatrix} = \begin{bmatrix} \mathbf{h} & 0 \\ 0 & \mathbf{\Delta_s} \end{bmatrix} \equiv \hat{H}_0,$$

$$\hat{H}_1 = \mathbf{X}^\dagger \tilde{H}_1 \mathbf{X} = \begin{bmatrix} 0 & (UX)^\dagger A W \\ W^\dagger A (UX) & W^\dagger A W \end{bmatrix}.$$

Do perturbation theory in hat basis

$$S_{aa} = (UX)\hat{S}_{aa}(UX)^\dagger + (UX)\hat{S}_{aS}W^\dagger + W\hat{S}_{Sa}(UX)^\dagger + W\hat{S}_{SS}W^\dagger,$$

# Do W perturbation to 4th order to keep P leaking term

- Did we find ~$W^4$ P leaking term? → Yes!
- How about what is the role of the rest?

Typical W$^4$ term

$$
\begin{aligned}
&P(\nu_\beta \to \nu_\alpha)^{(4)}_{2nd} \equiv 2\mathrm{Re}\left[\left(S^{(0)}_{\alpha\beta}\right)^* S^{(4)}_{\alpha\beta}[4]_{\text{ diag}}\right] \\
&= 2\mathrm{Re}\Bigg\{\sum_n\sum_k\sum_K\Bigg[-\frac{x^2}{2}\frac{1}{(\Delta_K-h_k)^2}e^{-i(h_k-h_n)x} - \frac{2(ix)}{(\Delta_K-h_k)^3}e^{-i(h_k-h_n)x} \\
&- \frac{(ix)}{(\Delta_K-h_k)^3}e^{-i(\Delta_K-h_n)x} - \frac{3}{(\Delta_K-h_k)^4}\left(e^{-i(\Delta_K-h_n)x} - e^{-i(h_k-h_n)x}\right)\Bigg] \\
&\times (UX)_{\alpha k}(UX)^*_{\beta k}(UX)^*_{\alpha n}(UX)_{\beta n} \\
&\times \left\{(UX)^\dagger AW\right\}_{kK}\left\{W^\dagger A(UX)\right\}_{Kk}\left\{(UX)^\dagger AW\right\}_{kK}\left\{W^\dagger A(UX)\right\}_{Kk} \\
&+ \sum_n\sum_k\sum_K\sum_{m\neq k}\Bigg[\frac{(ix)}{(\Delta_K-h_k)^2(h_m-h_k)}e^{-i(h_k-h_n)x} \\
&- \frac{(ix)}{(\Delta_K-h_k)^2(\Delta_K-h_m)}e^{-i(\Delta_K-h_n)x} + \frac{(h_k+2h_m-3\Delta_K)}{(\Delta_K-h_k)^3(\Delta_K-h_m)^2}e^{-i(\Delta_K-h_n)x} \\
&+ \frac{1}{(\Delta_K-h_m)^2(h_m-h_k)^2}e^{-i(h_m-h_n)x} - \frac{(\Delta_K+2h_m-3h_k)}{(\Delta_K-h_k)^3(h_m-h_k)^2}e^{-i(h_k-h_n)x}\Bigg] \\
&\times (UX)_{\alpha k}(UX)^*_{\beta k}(UX)^*_{\alpha n}(UX)_{\beta n} \\
&\times \left\{(UX)^\dagger AW\right\}_{kK}\left\{W^\dagger A(UX)\right\}_{Km}\left\{(UX)^\dagger AW\right\}_{mK}\left\{W^\dagger A(UX)\right\}_{Kk} \\
&+ \sum_n\sum_k\sum_{K\neq L}\Bigg[-\frac{x^2}{2}\frac{1}{(\Delta_K-h_k)(\Delta_L-h_k)}e^{-i(h_k-h_n)x} - (ix)\frac{(\Delta_K+\Delta_L-2h_k)}{(\Delta_K-h_k)^2(\Delta_L-h_k)^2}e^{-i(h_k-h_n)x} \\
&- \frac{1}{(\Delta_K-h_k)^3(\Delta_L-\Delta_K)}e^{-i(\Delta_K-h_n)x} + \frac{1}{(\Delta_L-h_k)^3(\Delta_L-\Delta_K)}e^{-i(\Delta_L-h_n)x} \\
&+ \frac{1}{(\Delta_K-h_k)^3(\Delta_L-h_k)^3}\left\{\Delta_L^2+\Delta_L\Delta_K+\Delta_K^2-3h_k(\Delta_L+\Delta_K)+3h_k^2\right\}e^{-i(h_k-h_n)x}\Bigg] \\
&\times (UX)_{\alpha k}(UX)^*_{\beta k}(UX)^*_{\alpha n}(UX)_{\beta n} \\
&\times \left\{(UX)^\dagger AW\right\}_{kK}\left\{W^\dagger A(UX)\right\}_{Kk}\left\{(UX)^\dagger AW\right\}_{kL}\left\{W^\dagger A(UX)\right\}_{Lk} \\
&+ \sum_n\sum_k\sum_{K\neq L}\sum_{m\neq k}\Bigg[\frac{(ix)}{(\Delta_K-h_k)(\Delta_L-h_k)(h_m-h_k)}e^{-i(h_k-h_n)x} \\
&- \frac{1}{(\Delta_K-h_k)^2(\Delta_L-h_k)^2(h_m-h_k)^2}\left\{\Delta_K\Delta_L+(h_m-2h_k)(\Delta_K+\Delta_L)+3h_k^2-2h_mh_k\right\}e^{-i(h_k-h_n)x} \\
&+ \frac{1}{(\Delta_K-h_m)(\Delta_L-h_m)(h_m-h_k)^2}e^{-i(h_m-h_n)x} \\
&+ \frac{1}{(\Delta_K-\Delta_L)(\Delta_K-h_k)^2(\Delta_K-h_m)}e^{-i(\Delta_K-h_n)x} - \frac{1}{(\Delta_K-\Delta_L)(\Delta_L-h_k)^2(\Delta_L-h_m)}e^{-i(\Delta_L-h_n)x}\Bigg] \\
&\times (UX)_{\alpha k}(UX)^*_{\beta k}(UX)^*_{\alpha n}(UX)_{\beta n} \\
&\times \left\{(UX)^\dagger AW\right\}_{kK}\left\{W^\dagger A(UX)\right\}_{Km}\left\{(UX)^\dagger AW\right\}_{mL}\left\{W^\dagger A(UX)\right\}_{Lk}\Bigg\}. \qquad \text{(C.5)}
\end{aligned}
$$

$$
\begin{aligned}
\left|S^{(2)}_{\alpha\beta}\right|^2_{1\text{st}} &= \sum_{k,K}\sum_{l,L} \frac{1}{(\Delta_K - h_k)(\Delta_L - h_l)} \\
&\times \Bigg[ x^2 e^{-i(h_k-h_l)x} - (ix)\frac{e^{-i(\Delta_K-h_l)x} - e^{-i(h_k-h_l)x}}{(\Delta_K-h_k)} + (ix)\frac{e^{-i(h_k-\Delta_L)x}-e^{-i(h_k-h_l)x}}{(\Delta_L-h_l)} \\
&+ \frac{1}{(\Delta_K-h_k)(\Delta_L-h_l)}\left\{ e^{-i(\Delta_K-\Delta_L)x} + e^{-i(h_k-h_l)x} - e^{-i(\Delta_K-h_l)x} - e^{-i(h_k-\Delta_L)x}\right\}\Bigg] \\
&\times (UX)_{\alpha k}(UX)^*_{\beta k} \left\{(UX)^\dagger A W\right\}_{kK} \left\{W^\dagger A (UX)\right\}_{Kk} \\
&\times (UX)^*_{\alpha l}(UX)_{\beta l} \left\{(UX)^\dagger A W\right\}_{lL} \left\{W^\dagger A (UX)\right\}_{Ll} \\
&+ \sum_{k\neq m}\sum_{K}\sum_{l\neq n}\sum_{L} \frac{1}{(h_m-h_k)(\Delta_K-h_k)(\Delta_K-h_m)} \frac{1}{(h_n-h_l)(\Delta_L-h_l)(\Delta_L-h_n)} \\
&\times \left[(\Delta_K-h_k)\, e^{-ih_m x} - (\Delta_K-h_m)\, e^{-ih_k x} - (h_m-h_k) e^{-i\Delta_K x}\right] \\
&\times \left[(\Delta_L-h_l)\, e^{+ih_n x} - (\Delta_L-h_n)\, e^{+ih_l x} - (h_n-h_l) e^{+i\Delta_L x}\right] \\
&\times (UX)_{\alpha k}(UX)^*_{\beta m} \left\{(UX)^\dagger A W\right\}_{kK} \left\{W^\dagger A (UX)\right\}_{Km} \\
&\times (UX)^*_{\alpha l}(UX)_{\beta n} \left\{(UX)^\dagger A W\right\}_{nL} \left\{W^\dagger A (UX)\right\}_{Ll} \\
&+ \sum_{k,K}\sum_{l,L} \frac{1}{(\Delta_K-h_k)(\Delta_L-h_l)} \left(e^{-i\Delta_K x} - e^{-ih_k x}\right)\left(e^{+i\Delta_L x} - e^{+ih_l x}\right) \\
&\times \left[(UX)_{\alpha k} W^*_{\beta K} \left\{(UX)^\dagger A W\right\}_{kK} + W_{\alpha K}(UX)^*_{\beta k}\left\{W^\dagger A (UX)\right\}_{Kk}\right] \\
&\times \left[(UX)^*_{\alpha l} W_{\beta L} \left\{W^\dagger A (UX)\right\}_{Ll} + W^*_{\alpha L}(UX)_{\beta l}\left\{(UX)^\dagger A W\right\}_{lL}\right] \\
&+ \sum_{K} |W_{\alpha K}|^2 |W_{\beta K}|^2 + \sum_{K\neq L} e^{-i(\Delta_K-\Delta_L)x} W_{\alpha K} W^*_{\beta K} W^*_{\alpha L} W_{\beta L}. \qquad \text{(C.1)}
\end{aligned}
$$

P leaking term

# After averaging out fast oscillations..

$$P(\nu_\beta \to \nu_\alpha)^{(0+2)}$$

$$= \left|\sum_{j=1}^{3} U_{\alpha j} U^*_{\beta j}\right|^2 - 2\sum_{j\neq k} \mathrm{Re}\left[(UX)_{\alpha j}(UX)^*_{\beta j}(UX)^*_{\alpha k}(UX)_{\beta k}\right] \sin^2\frac{(h_k - h_j)x}{2}$$

$$- \sum_{j\neq k} \mathrm{Im}\left[(UX)_{\alpha j}(UX)^*_{\beta j}(UX)^*_{\alpha k}(UX)_{\beta k}\right] \sin(h_k - h_j)x$$

Zeroth order in W

$$+ 2\mathrm{Re}\Bigg\{\sum_m \sum_{k,K} \frac{1}{\Delta_K - h_k}\left[(ix)e^{-i(h_k-h_m)x} - \frac{e^{-i(h_k-h_m)x}}{(\Delta_K - h_k)}\right]$$

$$\times (UX)_{\alpha k}(UX)^*_{\beta k}(UX)^*_{\alpha m}(UX)_{\beta m}\left\{(UX)^\dagger A W\right\}_{kK}\left\{W^\dagger A (UX)\right\}_{Kk}$$

$$- \sum_m \sum_{k\neq l} \sum_K \frac{1}{(h_l - h_k)(\Delta_K - h_k)(\Delta_K - h_l)}$$

$$\times \left[(\Delta_K - h_k)\, e^{-i(h_l-h_m)x} - (\Delta_K - h_l)\, e^{-i(h_k-h_m)x}\right]$$

$$\times (UX)_{\alpha k}(UX)^*_{\beta l}(UX)^*_{\alpha m}(UX)_{\beta m}\left\{(UX)^\dagger A W\right\}_{kK}\left\{W^\dagger A (UX)\right\}_{Kl}$$

$$- \sum_m \sum_{k,K} \frac{e^{-i(h_k-h_m)x}}{(\Delta_K - h_k)}\Big[(UX)_{\alpha k} W^*_{\beta K}(UX)^*_{\alpha m}(UX)_{\beta m}\left\{(UX)^\dagger A W\right\}_{kK}$$

$$+ W_{\alpha K}(UX)^*_{\beta k}(UX)^*_{\alpha m}(UX)_{\beta m}\left\{W^\dagger A (UX)\right\}_{Kk}\Big]\Bigg\},$$

2[nd] order terms: Large denominator suppression → Always comes with matter potential

$$\frac{|A|}{\Delta m^2_{Jk}} = 2.13 \times 10^{-3}\left(\frac{\Delta m^2_{Jk}}{0.1\mathrm{eV}^2}\right)^{-1}\left(\frac{\rho}{2.8\,\mathrm{g/cm}^3}\right)\left(\frac{E}{1\ \mathrm{GeV}}\right),$$

# A simple formula for oscillation probability in matter w/o unitarity: leading order in W apart from $C_{\alpha\beta}$

$$P(\nu_\beta \to \nu_\alpha) = \mathcal{C}_{\alpha\beta} + \left| \sum_{j=1}^{3} U_{\alpha j} U_{\beta j}^{*} \right|^2$$

$$- 2 \sum_{j \neq k} \mathrm{Re} \left[ (UX)_{\alpha j} (UX)_{\beta j}^{*} (UX)_{\alpha k}^{*} (UX)_{\beta k} \right] \sin^2 \frac{(h_k - h_j)x}{2}$$

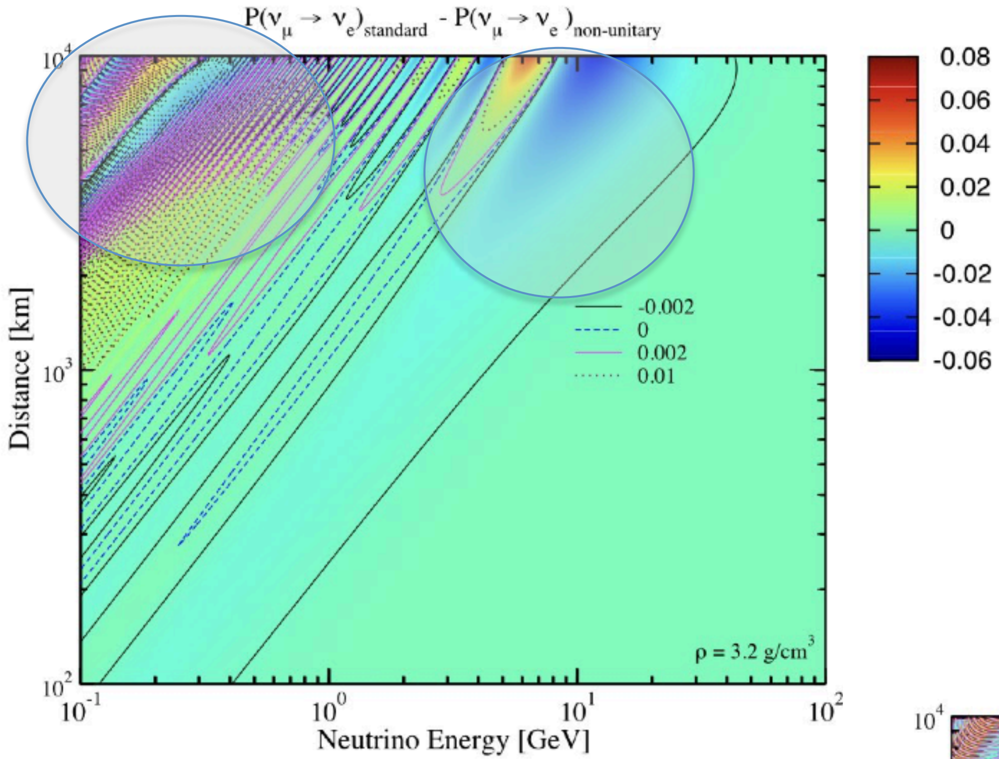
$$- \sum_{j \neq k} \mathrm{Im} \left[ (UX)_{\alpha j} (UX)_{\beta j}^{*} (UX)_{\alpha k}^{*} (UX)_{\beta k} \right] \sin(h_k - h_j)x,$$

X=...

$$\mathbf{U} = \begin{bmatrix} U & W \\ Z & V \end{bmatrix}$$

- All $W^2$ & $W^4$ terms avaraged out or suppressed if $\Delta m^2 > 0.1$ $eV^2$ except for P leaking term!!
- UV effect is in: (1) explicit W correction term, (2) non-unitary U matrix

# Where is the region of large UV?

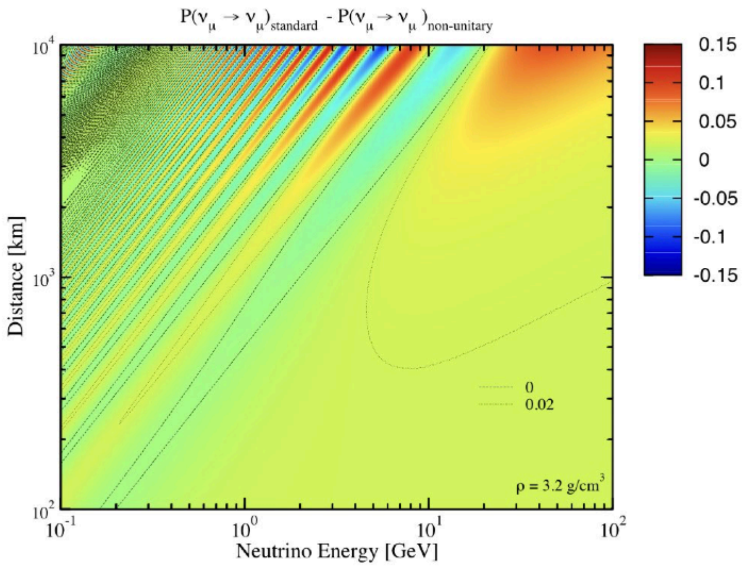



- Leading order terms = Zeroth order in W

$\Delta$P large at solar- and atm-MSW enhanced regions

$\alpha_{11} = 0.990,\ \alpha_{21} = -0.0141,\ \alpha_{22} = 0.995,\ \alpha_{31} = -0.0445,$

$\alpha_{32} = -0.0316,\ \alpha_{33} = 0.949.$

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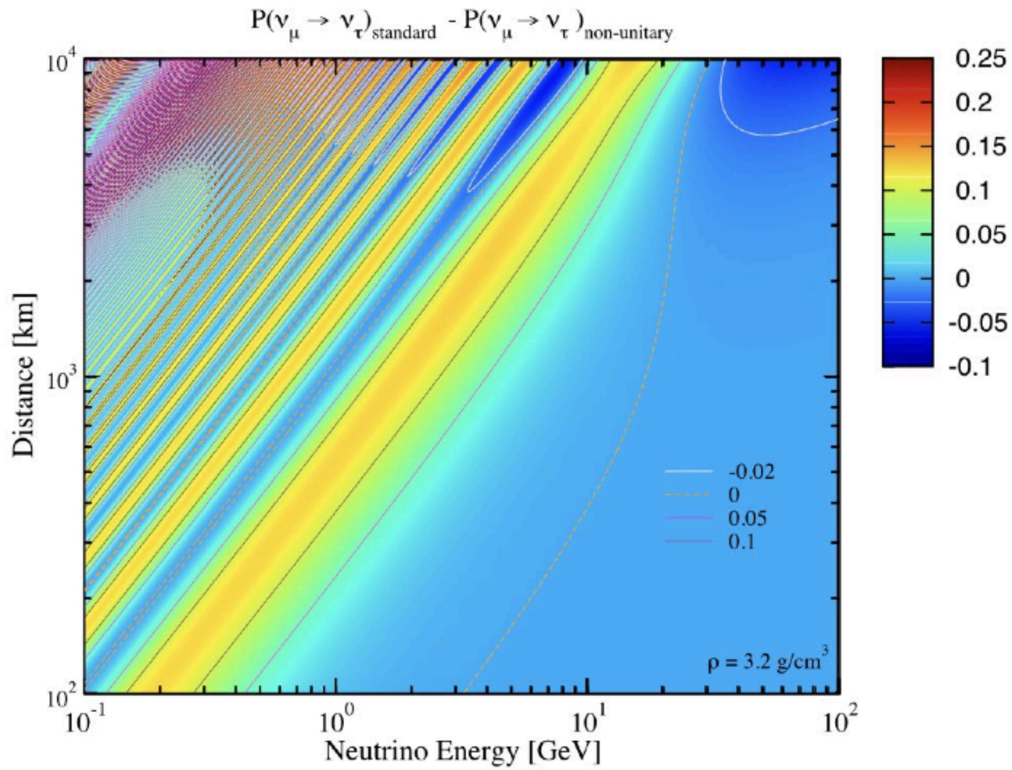
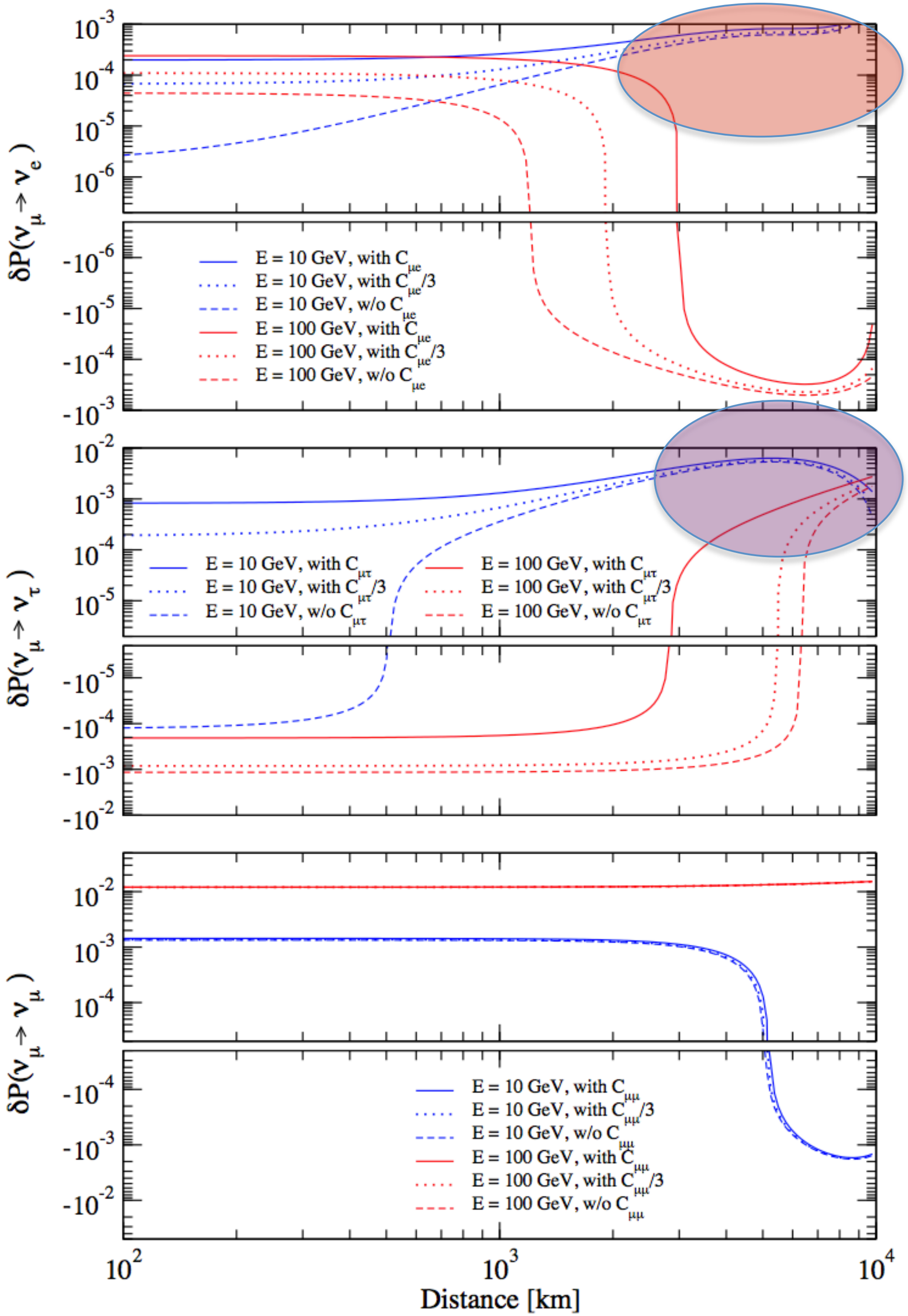


Large ~$W^2$ corrections?

- Order W2 correction terms

→ small in most of the regions of L-E, but sizeable in limited places

- High energy, long baseline → IceCube, PINGU, Hyper-K



# Conclusion (non-unitarity)

- General structure of nu oscillation in active nu sector of (3+N) unitary system is analyzed in vacuum and in matter in the context of low-E unitarity violation
- A new term, the "probability leaking term" found (leaking to sterile sector)
- → Distinguishes between Low-E vs High-E unitarity violation
- Conditions for sterile sector model-independent P in vacuum and in matter are elucidated → $m_J^2 > 0.1\ \mathrm{eV}^2$
- Likely to be insensitive to sterile interactions

# Conclusion (non-unitarity2)

- JUNO analysis shows one can constrain UV in $\nu_e$ row at a high level

→ $C_{ee} \sim 10^{-4}$, $1-\Sigma|U_{ei}|^2 \sim 0.01$ (both 1 $\sigma$)

- Non-unitarity effect in the leading order ($W^0$) seems sizeable in solar- and atm MSW regions (Probability level)
- generally requires $L \sim 3000\text{-}10^4$ km

Atmospheric nu important

- $W^2$ correction sizeable in limited L-E regions

→ distinguishes between low-E from high-E UV

→ $L \sim 3000\text{-}10^4$ km

Atmospheric nu important