Neutrino Mass Models: A sampler

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Prime message from atmospheric neutrinos

$$\theta_{23} \cong \pi/4$$

$$|\Delta m^2|_{atm} \gg |\Delta m^2|_{sol}$$

$\theta_{23} = \pi/4$?

and/or

$|\Delta m^2|_{sol} \cong 0$?

Simple mixing patterns

- **Maximal ν_μ-ν_τ mixing $\Rightarrow \theta_{23} = \pi/4$ Also $\Rightarrow \theta_{13} = 0$**
- **Only θ_{12} varies from model to model**

$$U_0 = \begin{pmatrix} \cos\theta^0_{12} & \sin\theta^0_{12} & 0 \\ -\frac{\sin\theta^0_{12}}{\sqrt{2}} & \frac{\cos\theta^0_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{\sin\theta^0_{12}}{\sqrt{2}} & -\frac{\cos\theta^0_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} ,$$

- **Tribimaximal (TBM), Bimaximal (BM), Golden Ratio (GR), etc.**

Model	TBM	BM	GR	NSM
θ^0_{12}	$35.3°$	$45.0°$	$31.7°$	$0.0°$

Mixing angles either 0 or maximal ($\pi/4$)

- **Flavour symmetries can be used to realize such mixings**

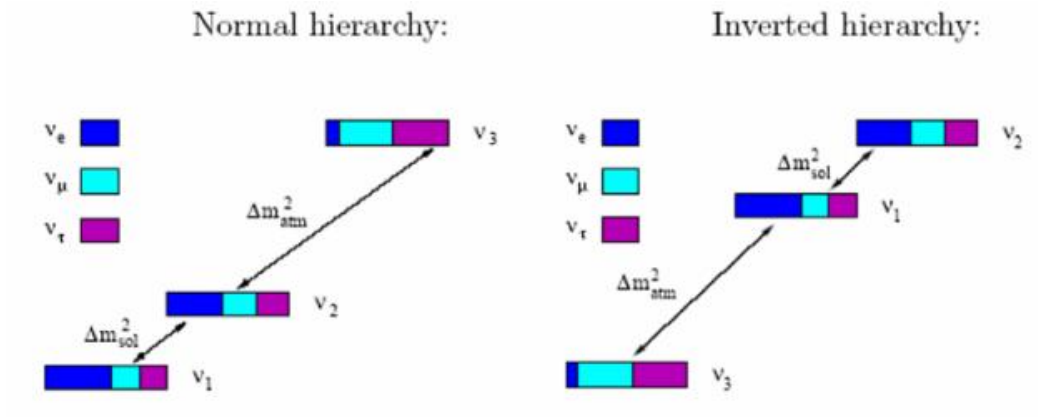
Dark Matter?

$\theta_{13}\neq 0$, these can only be starting points

Major issues

$$U_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} + s_{23}s_{13}c_{12}e^{i\delta} & c_{23}c_{12} + s_{23}s_{13}s_{12}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} + c_{23}s_{13}c_{12}e^{i\delta} & -s_{23}c_{12} + c_{23}s_{13}s_{12}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

- **Mass ordering unknown**
- **Which octant for θ_{23}**
- **What is the value of the CP-phase δ?**
- **Origin of small neutrino mass?**



Notation/Conventions

Dirac mass
For charged leptons

Fermion mass term: $m_D\bar{\Psi}\Psi \equiv m_D(\Psi^c)^T C^{-1}\Psi \equiv m_D\bar{\Psi}_L\Psi_R + h.c.$

Lepton No. conserved

Majorana mass $\Psi^c = \Psi \Rightarrow m_M(\Psi)^T C^{-1}\Psi$

Useful for ν

ΔL = 2 (For neutral fermions only!)

$$\text{In SM } L_{iL} \equiv \begin{pmatrix} \nu_i \\ l_i \end{pmatrix}_L \quad l_{iR}, \quad (i = e,\ \mu,\ \tau) \text{ No } \nu_{iR}$$

Charged lepton mass

$$\mathcal{L}_{charged\ lepton} = -Y_{ij}\bar{L}_{iL}Hl_{jR}$$

Neutrino Masses

In SM no RH $\nu \Rightarrow$ No Dirac mass.
Also, Lepton No. is conserved $\Rightarrow$ No Majorana mass.

Dirac mass (Add right-handed neutrinos N_j)

N_j are singlets under SM (a sterile fermion)

$$\mathcal{L}_{Dirac\ mass} = -y^{\nu}_{ij} \bar{L}_i \tilde{H} N_j \ , \ \tilde{H} = i\sigma_2 H$$

Why is y^{ν}_{ij} so very small compared to Yukawa couplings for quarks and charged leptons?

Separate Higgs doublet with tiny vev for neutrinos?

Neutrino Masses

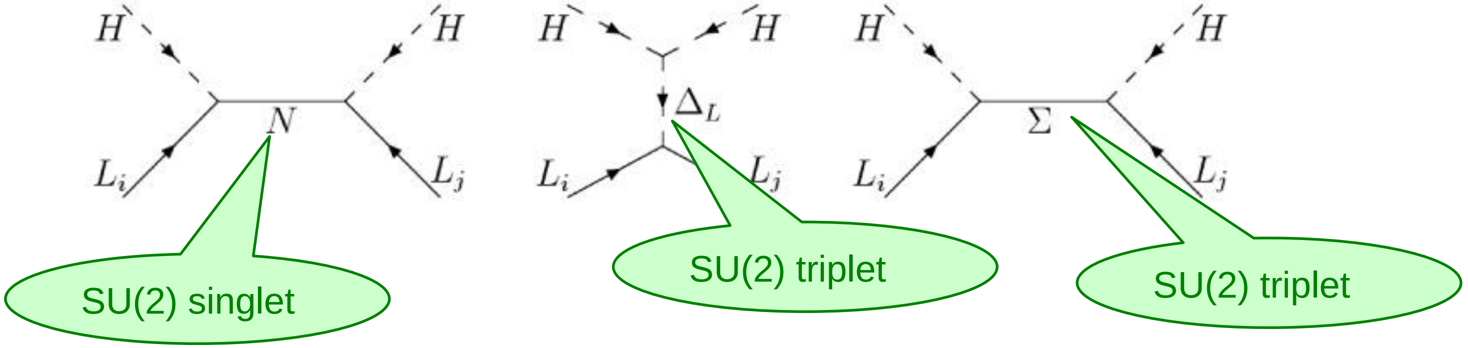
⇒Leptogenesis?

Majorana mass (Violate Lepton number)

e.g., Lepton No. violating interactions in BSM, R-parity violation in SUSY, etc.

$$\mathcal{L}_{Weinberg} = \frac{1}{2}\frac{1}{M_\nu} L_i L_j H H$$

See-saw (Add new fermions/scalars)



Neutrino mass (Loops)

$1/(16\ \pi^2)^n$ suppression for n-loop diagram $\Leftarrow$ smallness of m_ν

See e.g., Farzan, Pascoli, Schmidt, JHEP 1303, 107 (2013)
Cai, Herrero-Garcia, Schmidt, Vicente, Volkas, Front. Phys. 5, 63 (2017)

One loop examples
Zee model: Singly-charged scalar
Scotogenic model (Ma): Dark matter
SUSY: Superparticles, R-parity violation

Two loop example
Zee-Babu: Doubly-charged scalars

Three loop example
Krauss-Nasri-Trodden: Charged scalars, neutral fermions, Z2 symmetry

Mass and mixing information

NuFIT 3.0 (2016)

	Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 0.83$)		Any Ordering
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range	3σ range
$\sin^2\theta_{12}$	$0.306^{+0.012}_{-0.012}$	$0.271 \to 0.345$	$0.306^{+0.012}_{-0.012}$	$0.271 \to 0.345$	$0.271 \to 0.345$
$\theta_{12}/^\circ$	$33.56^{+0.77}_{-0.75}$	$31.38 \to 35.99$	$33.56^{+0.77}_{-0.75}$	$31.38 \to 35.99$	$31.38 \to 35.99$
$\sin^2\theta_{23}$	$0.441^{+0.027}_{-0.021}$	$0.385 \to 0.635$	$0.587^{+0.020}_{-0.024}$	$0.393 \to 0.640$	$0.385 \to 0.638$
$\theta_{23}/^\circ$	$41.6^{+1.5}_{-1.2}$	$38.4 \to 52.8$	$50.0^{+1.1}_{-1.4}$	$38.8 \to 53.1$	$38.4 \to 53.0$
$\sin^2\theta_{13}$	$0.02166^{+0.00075}_{-0.00075}$	$0.01934 \to 0.02392$	$0.02179^{+0.00076}_{-0.00076}$	$0.01953 \to 0.02408$	$0.01934 \to 0.02397$
$\theta_{13}/^\circ$	$8.46^{+0.15}_{-0.15}$	$7.99 \to 8.90$	$8.49^{+0.15}_{-0.15}$	$8.03 \to 8.93$	$7.99 \to 8.91$
$\delta_{CP}/^\circ$	261^{+51}_{-59}	$0 \to 360$	277^{+40}_{-46}	$145 \to 391$	$0 \to 360$
$\frac{\Delta m^2_{21}}{10^{-5}\ \text{eV}^2}$	$7.50^{+0.19}_{-0.17}$	$7.03 \to 8.09$	$7.50^{+0.19}_{-0.17}$	$7.03 \to 8.09$	$7.03 \to 8.09$
$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \text{eV}^2}$	$+2.524^{+0.039}_{-0.040}$	$+2.407 \to +2.643$	$-2.514^{+0.038}_{-0.041}$	$-2.635 \to -2.399$	$\begin{bmatrix} +2.407 \to +2.643 \\ -2.629 \to -2.405 \end{bmatrix}$

Concrete examples

Massless neutrinos

J. Kersten and A. Yu. Smirnov, PRD 76, 073005 (2007)
R. Adhikari and A.R., PRD 84, 033002 (2011)

Type-I see-saw models which automatically lead to 3 massless neutrinos due to symmetries.

$$M = \begin{pmatrix} 0 & M_D \\ M_D^T & M_R \end{pmatrix} \; . \quad \text{condition for zero eigenvalues of M}$$

$$M_D = \begin{pmatrix} x_1 & x_2 & x_3 \\ \alpha_1 x_1 & \alpha_2 x_2 & \alpha_3 x_3 \\ \beta_1 x_1 & \beta_2 x_2 & \beta_3 x_3 \end{pmatrix} \quad \text{and} \quad M_R = \begin{pmatrix} M_1 & M_4 & M_5 \\ M_4 & M_2 & M_6 \\ M_5 & M_6 & M_3 \end{pmatrix}$$

1 zero eigenvalue $\Rightarrow$

$$\alpha_1 = \alpha_2 = \alpha_3 \text{ or } \; \beta_1 = \beta_2 = \beta_3$$
$$\text{or} \;\; \alpha_i = \beta_i, \alpha_j = \beta_j, (i \neq j) \; , \; \text{etc.}$$

3 zero eigenvalues $\Rightarrow$

$$\alpha_1 = \alpha_2 = \alpha_3 \text{ and } B = 0 \; , \; \text{etc.}$$

$$B = \sum_{i,j,k} \left[(\beta_i - \beta_j)^2 x_i^2 x_j^2 M_k + 2 x_1 x_2 x_3 (\beta_i - \beta_j)(\beta_j - \beta_k) x_j M_{(7-j)} \right]$$
$$(i, j, k \text{ cyclic})$$

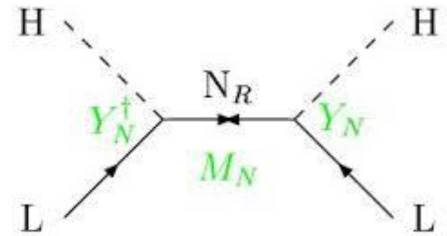
Scaling ansatz realistic models:
e.g., R.Samanta, P. Roy, and A.Ghosal
EPJC 76 , 662 (2016)

Standard see-saws

- **Type – I**

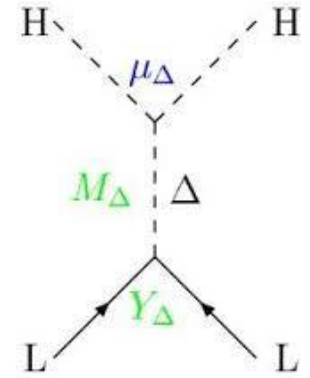
$$(m_\nu)_{ij} = v^2 \Sigma_k \left[\frac{(Y_N)^\dagger_{ik} (Y_N)_{kj}}{M_{N_k}} \right]$$

Majorana mass of N_k. Lepton No. violated.



- **Type – II** An SU(2) triplet scalar Δ. No RH ν
  Either Δ gets a vev violating lepton number
  or it has a trilinear coupling to a pair of H which get vev.

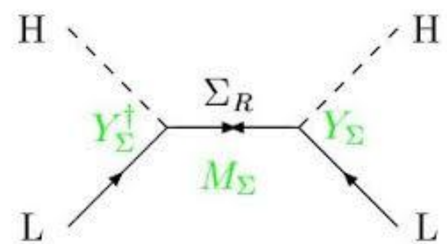
$$(m_\nu)_{ij} = (Y_\Delta)_{ij} \frac{\mu_\Delta v^2}{M_\Delta^2}$$



- **Type – III**
  Similar to Type – I but Σ_R are SU(2) triplets

(Three RH SU(2)-singlet neutrinos N_R /Σ_R to give masses to all neutrinos)

$$(m_\nu)_{ij} = v^2 \Sigma_k \left[\frac{(Y_\Sigma)^\dagger_{ik} (Y_\Sigma)_{kj}}{M_{\Sigma_k}} \right]$$



Majorana mass of Σ_k. Lepton No. violated.

Generalised see-saw

- **Inverse see-saw (in addition to the RH neutrinos add an SM singlet Majorana neutrino, S)**
  The Majorana mass, μ, breaks lepton no.

$$M = \begin{pmatrix} 0 & M_D & 0 \\ M_D^T & 0 & M_R \\ 0 & M_R^T & \mu \end{pmatrix} .$$

For $\mu < M_D << M_R$ one gets $m_\nu = M_D (M_R^T)^{-1} \mu (M_R)^{-1} M_D^T$

Smallness of m_ν can be partially attributed to the smallness of μ. M_R ~ 1 TeV OK

Other see-saw variants, texture zeros of the matrices, etc.

Common feature: N_R(sterile). Where is it? Many studies: sterile mixing, colliders, etc.

See e.g., Rasmussen, Winter, PRD 94 073004 (2016)
Bambhaniya, Chakrabortty, Goswami, Konar, PRD 88 075006 (2013)

Combine see-saws?

Small quantities in the neutrino sector: θ_{13}, $R = \Delta m^2_{atm}/\Delta m^2_{sol}$, $(\theta_{23} - \pi/4)$
Perhaps they are vanishing to start with (due to symmetry) and have a different origin from the rest?

Example:

Type II see-saw $\Rightarrow$ θ_{23}, θ_{12}, (e.g., TBM) and Δm^2_{atm}
Type I see-saw (sub-dominant) $\Rightarrow$ θ_{13}, δ, and Δm^2_{sol}
and nudges θ_{23}, θ_{12} into correct range

Consequence:
Interrelationships

Implementation:

- Choose discrete symmetry G (e.g., S3, A4)
- Include RH neutrinos
- Assign Left- and Right-handed leptons to chosen multiplets of G
- Choose scalars in appropriate G representations
- Obtain mass matrices and find consequences

Soumita Pramanick and AR PRD 93, 033007 (2016)
Soumita Pramanick, arXiv: 1711.03510
Similar with dominant Type 1: D. Borah and M.K. Das, PRD 90, 015006 (2014)

Combine see-saws?

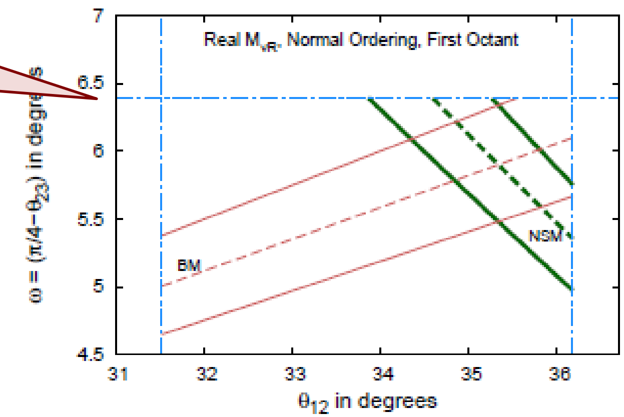
Features:

For S3/A4 possible to arrange appropriate fermion and scalar assignments

Rich scalar sector

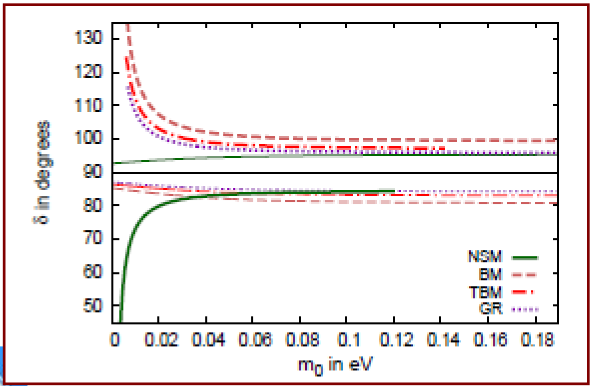
CP-violation arises from complexity of right-handed ν mass matrix M_R

TBM and GR give θ_{23} beyond range



For Real M_R only normal ordering is allowed.

Complex M_R permits inverted ordering, CP-violation.



Mixing Pattern	Normal Ordering		Inverted Ordering	
	δ quadrant	θ_{23} octant	δ quadrant	θ_{23} octant
NSM	First/Fourth	First	Second/Third	Second
BM, TBM, GR	Second/Third	First	First/Fourth	Second

Soumita Pramanick and AR PRD 93, 033007 (2016)

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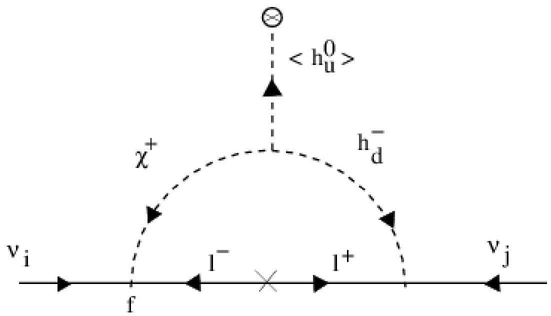


The Zee Model

$$\mathcal{L}_{scalars} = \mu \epsilon_{ij} h_i h_j (\chi^+)* \ , \ \epsilon_{12} = -\epsilon_{21} = 1 \ , \epsilon_{ii} = 0 \ ,$$

where $h_1 \equiv h_u \ , \ h_2 \equiv h_d$ and

$$\mathcal{L}_{Yukawa} = Y^a_{ij} \bar{L}_{iL} h_a e_{jR} + f_{ij} (L^c_{iL})^T L_{jL} \chi^+ \ , \ \ f_{ij} = -f_{ji}$$

A. Zee, PLB 93, 389 (1980)

Lepton No. violation

Note no N_R

With only h_u giving masses to the leptons, i.e., $Y^2_{ij} = 0$, one has the neutrino mass matrix: $m_{ij} \propto f_{ij}(m^2_{ej} - m^2_{ei})$ Imposing the solar/atmospheric mass splitting ratio yields a too large θ_{12}. So $Y^2_{ij} \neq 0$ must be taken.

Koide, PRD 64, 77301 (2001)
Frampton, Oh, and Yoshikawa, PRD 65, 073014 (2002)
See also X-G He, EPJC 34, 371 (2004),

Phenomenology of the model has been extensively studied

e.g., LRS Model implementation Collider search: Khan, Mitra, Patra, arXiv:1805.09844

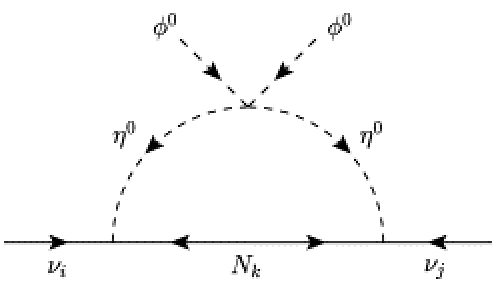
Scotogenic Model

E. Ma, PRD 73, 077301 (2006)

- **Add discrete Z_2 symmetry to the standard model**

  - All SM particles even under Z_2
  - Include RH neutrinos N_k and scalar doublet η, both odd under Z_2
  - Z_2 is unbroken, $< \eta^0 > = 0 \Rightarrow$ Neutrino Dirac mass = 0.
  - N_k Majorana mass is allowed
  - Also allowed is a quartic scalar term: $\lambda_5 (\phi^\dagger \eta)^2$

$$\mathcal{L}_{Yukawa} = f_{ij}\bar{L}_{iL}\phi e_{jR} + h_{ij}\bar{L}_{iL}\tilde{\eta}N_{jR} + h.c.$$



- **One-loop diagram for neutrino mass**

$$(M_\nu)_{ij} = \frac{\lambda_5 v^2}{16\pi^2} \sum_k \frac{h_{ik} hjk}{M_k} \text{ , if } m_\eta \simeq M_k$$

- **Lightest of N_k or *Re* (η^0)/*Im* (η^0) is a dark matter candidate protected by Z_2 symmetry**

A SUSY example

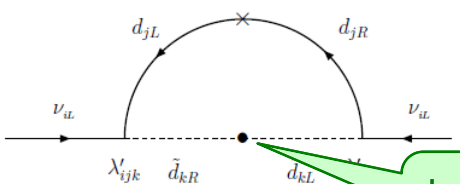
S. Rakshit, G. Bhattacharyya, and AR PRD 59, 091701 (R) (1999)
Review: S. Rakshit, MPLA 19, 2239 (2004)
Bilinear R-parity: S. Roy, B. Mukhopadhyaya, and F. Vissani, PLB 443, 191 (1998)

R-parity violating superpotential

$$W_{\not{R}} = \frac{1}{2}\lambda_{ijk} L_i L_j E_k^c + \lambda'_{ijk} L_i Q_j D_k^c + \frac{1}{2}\lambda''_{ijk} U_i^c D_j^c D_k^c + \mu_i L_i H_u ,$$

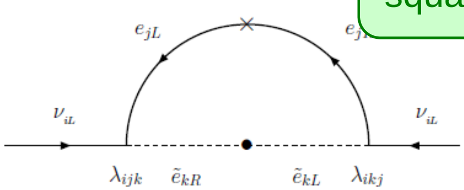
$$\lambda_{ijk} = -\lambda_{jik} \ , \ \lambda''_{ijk} = -\lambda''_{ikj} \ , \ (i, j, k = 1, 2, 3)$$

$\lambda_{ijk}, \lambda'_{ijk}$ violate L, λ''_{ijk} violates B



Left-right squark mixing

$$\Rightarrow \quad (m_\nu)_{mn} \sim \frac{3}{8\pi^2} \lambda'_{mjk} \lambda'_{nkj} \frac{m_{d_j} \Delta m_k^2(d)}{m_{\tilde{q}}^2}$$



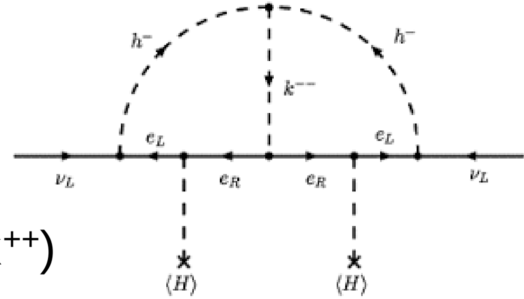
$$\Rightarrow \quad (m_\nu)_{mn} \sim \frac{1}{8\pi^2} \lambda_{mjk} \lambda_{nkj} \frac{m_{e_j} \Delta m_k^2(l)}{m_{\tilde{l}}^2}$$

$m_{\tilde{q}}$ $(m_{\tilde{l}})$ average squark (slepton) mass

$\Delta m^2(f) \approx m_f \tilde{m}$ left-right sfermion (squark or slepton) mixing

Neutrino mass and mixing can be reproduced

The Zee-Babu Model



Add to SM, a singly-charged (h^+) and a doubly charged scalar (k^{++})

Both SU(2) singlets

$$\mathcal{L}_{scalars} = \mu h^- h^- k^{++} h.c. \, , \quad \text{and}$$

$$\mathcal{L}_{Yukawa} = Y_{ij} \bar{L}_{iL} H e_{jR} + f_{ij} (L^c_{iL})^T L_{jL} (h^-)^* + g_{ij} (e^c_{iR})^T e_{jR} (k^{--})^* + h.c. \, ,$$

with $f_{ij} = -f_{ji}$, $g_{ij} = g_{ji}$

Lepton No. violated

No N_R

Together, the couplings result in a 2-loop contribution to the Majorana neutrino mass matrix

$$m_{ij} \propto f_{im} h_{mn} f^*_{nj} m_m m_n I_{mn}$$

where $I_{mn} = I_{nm}$ is a loop integral from the diagram.

A. Zee, NPB 264,99 (1986)
K.S. Babu, PLB 203, 132 (1988).

$f_{ij} = - f_{ji} \Rightarrow$ one massless neutrino.
Mixing and mass splittings can be reproduced

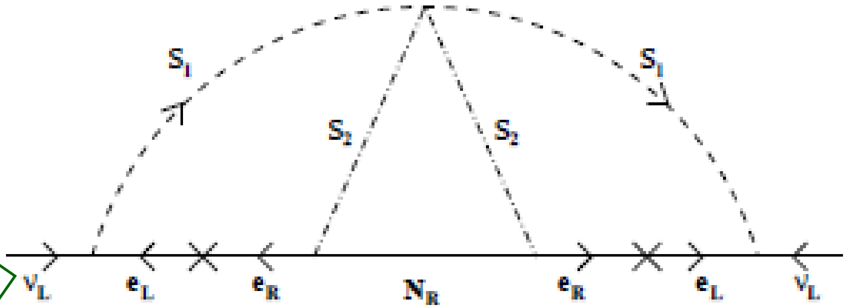
Nebot, Oliver, Palao, Santamaria, PRD 77, 093013 (2008)
K.S. Babu, PLB 203, 132 (1988).

A three-loop model

Krauss-Nasri-Trodden model

Krauss, Nasri, Trodden, PRD 67, 085002 (2003)
Ahrice, Chen, McDonald, Nasri PRD 90, 015024 (2014)

- Right-handed neutrino N_R
- Two charged SU(2) singlet scalars S_1,
- Additional Z_2 symmetry under which
  - SM particles and S_1 are even
  - S_2 and N_R are odd



$$\mathcal{L} = \mathcal{L}_{SM} + f_{\alpha\beta} L_\alpha^T C L_\beta S_1^\dagger + g_\alpha N_R S_2^\dagger l_{\alpha R} + M_R N_R^T C N_R + V(S_1, S_2) + h.c.$$

$$f_{\alpha\beta} = -f_{\beta\alpha} \text{ , and } V(S_1, S_2) \supset \lambda_s (S_1 S_2^*)^2$$

$$M_R < M_{S_1} < M_{S_2} \sim \text{ TeV}$$

This results in

$$m_\nu \sim v^2/\Lambda \text{ , with } \Lambda \propto \frac{(4\pi^2)^3}{f^2 \lambda_s Y_l^2 g_\alpha^2} M_{S_2} \text{ , } m_\nu \sim 0.1\text{eV}$$

Y → charged lepton Yukawa

Z_2 unbroken ⇒ N_R : A Dark Matter candidate

Left-Right symmetry, Grand Unification?

Left-right symmetry

- Provides a natural room for N_R.
- Under $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ lepton multiplets: (2,1,1) and (1,2,1)
  $Q = T_{3L} + T_{3R} + (B-L)/2$

$$L_L = \begin{pmatrix} e_L \\ \nu_L \end{pmatrix} \quad L_R = \begin{pmatrix} e_R \\ N_R \end{pmatrix}$$

Type-I

- Different types of see-saw can be readily incorporated
- Choice of scalars:

$$\Phi = \begin{pmatrix} \phi_1^0 & \phi_2^+ \\ \phi_1^- & \phi_2^0 \end{pmatrix} \equiv (2,2,0)\ ,\ \Delta_L = \begin{pmatrix} \delta_L^{++} \\ \delta_L^+ \\ \delta_L^0 \end{pmatrix} \equiv (3,1,2)\ ,\ \Delta_R = \begin{pmatrix} \delta_R^{++} \\ \delta_R^+ \\ \delta_R^0 \end{pmatrix} \equiv (1,3,2)$$

Type-II

- Type-III see-saw, loop masses etc. possible

See e.g., Hati, Patra, Pritimita, Sarkar, Found. Phys. 6, 19(2018)

GUT models

- SO(10) GUT provides a natural framework for inclusion of Left-Right symmetry through the Pati-Salam $SU(4)_c \times SU(2)_L \times SU(2)_R$ route.

$$L_L = \begin{pmatrix} e_L \\ \nu_L \end{pmatrix} \quad L_R = \begin{pmatrix} e_R \\ N_R \end{pmatrix} \subset \underline{16}$$

$$\text{Quark-lepton unification: } 16 \equiv (4,2,1) + (\bar{4},1,2)$$

$$\Phi = \begin{pmatrix} \phi_1^0 & \phi_2^+ \\ \phi_1^- & \phi_2^0 \end{pmatrix} \subset \underline{10}\ ,\ \Delta_L = \begin{pmatrix} \delta_L^{++} \\ \delta_L^+ \\ \delta_L^0 \end{pmatrix}\ ,\ \Delta_R = \begin{pmatrix} \delta_R^{++} \\ \delta_R^+ \\ \delta_R^0 \end{pmatrix} \subset \underline{126}$$

- Stepwise symmetry breaking provides the required high scales

$$SO(10) \xrightarrow[M_{GUT}]{} SU(4)_c \times SU(2)_L \times SU(2)_R \xrightarrow[M_R]{} SU(3)_c \times SU(2)_L \times U(1)_Y$$

- Proton decay!!

Summary

- Atmospheric neutrino oscillations tell us that θ_{23} is maximal (or nearly so).
- Also $(\Delta m^2)_{atm} \gg (\Delta m^2)_{sol}$.
- Solar, reactor, and accelerator oscillation results give additional information on neutrino mass and mixing.
- Is the neutrino Dirac or Majorana?
- Many models for neutrino mass and mixing.
- Smallness of mass is the primary target.
- See-saw is a popular direction.
- Loop models are also widely considered.
- Many other directions: Linear see-saw, Dirac see-saw, Extra-Dim, …
- Some models have falsifiable predictions.
- An early determination of θ_{23} , mass ordering, and CP-violation will considerably narrow down the options. Neutrino mass, not just splitting? Sterile neutrinos?

Thank you!!