Stanislav (Stas) Babak.

AstroParticule et Cosmologie, CNRS (Paris)

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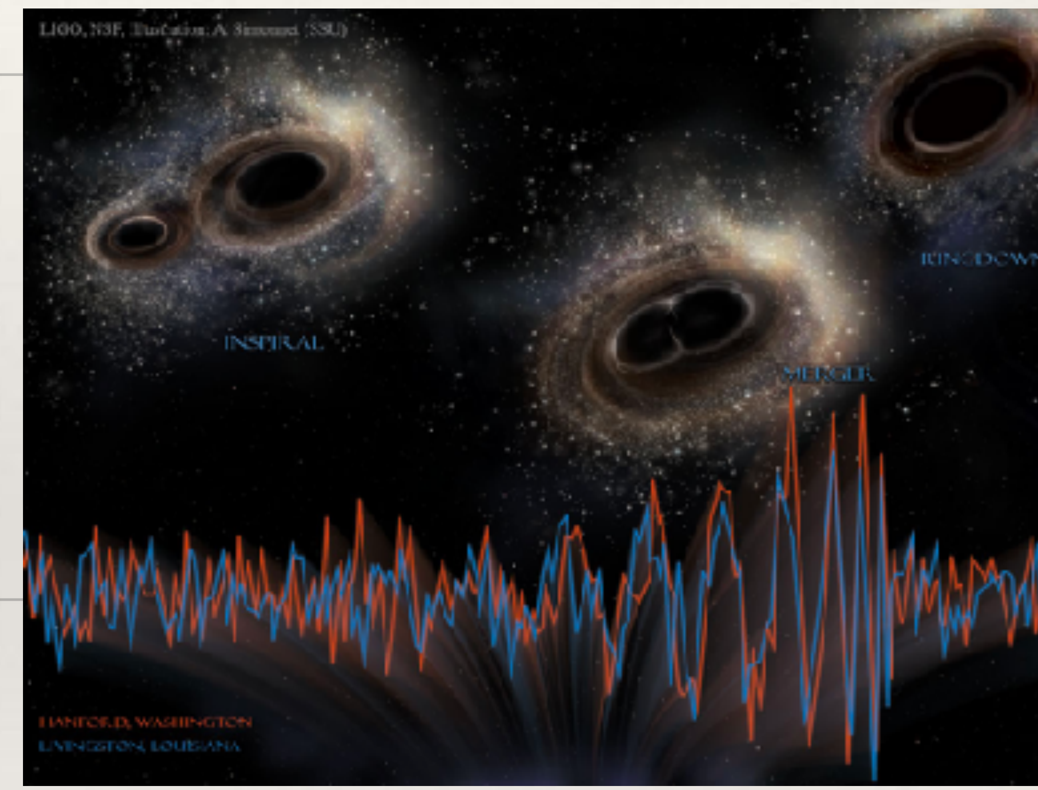
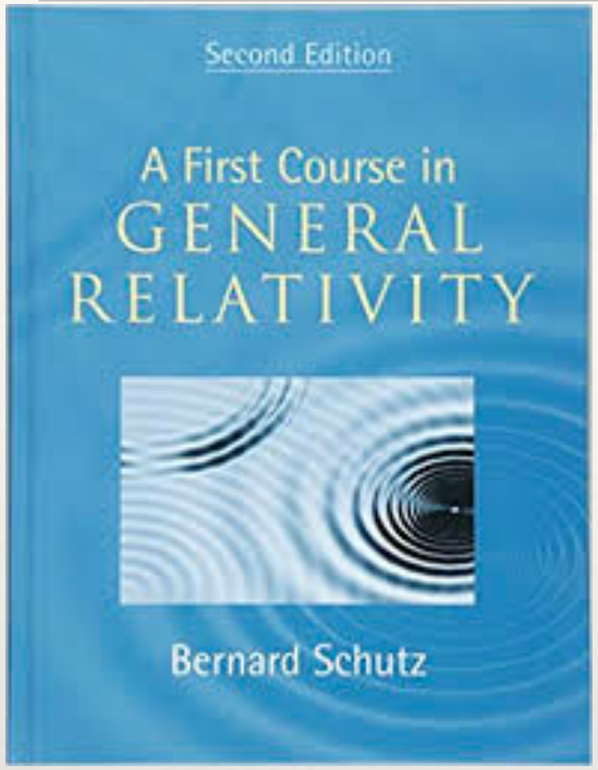
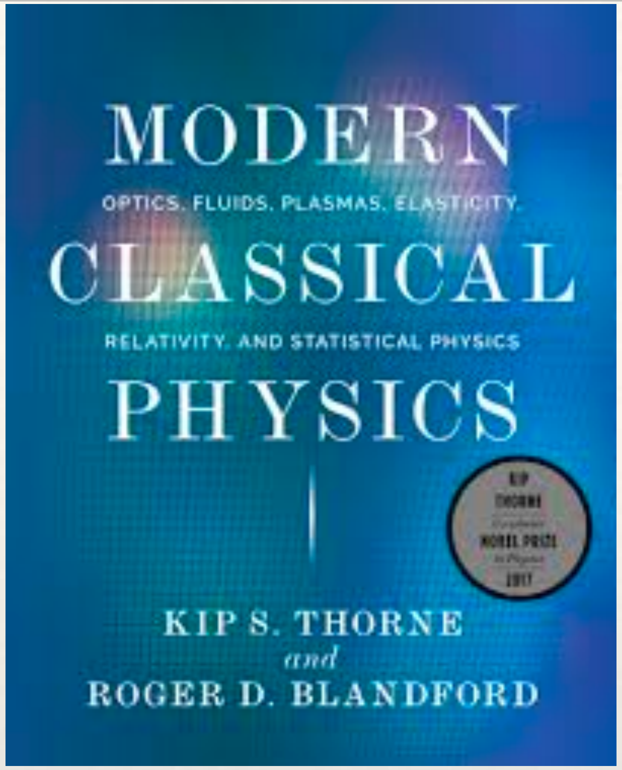
Gravitational waves
Part I



ICTP, 18-22 June 2018

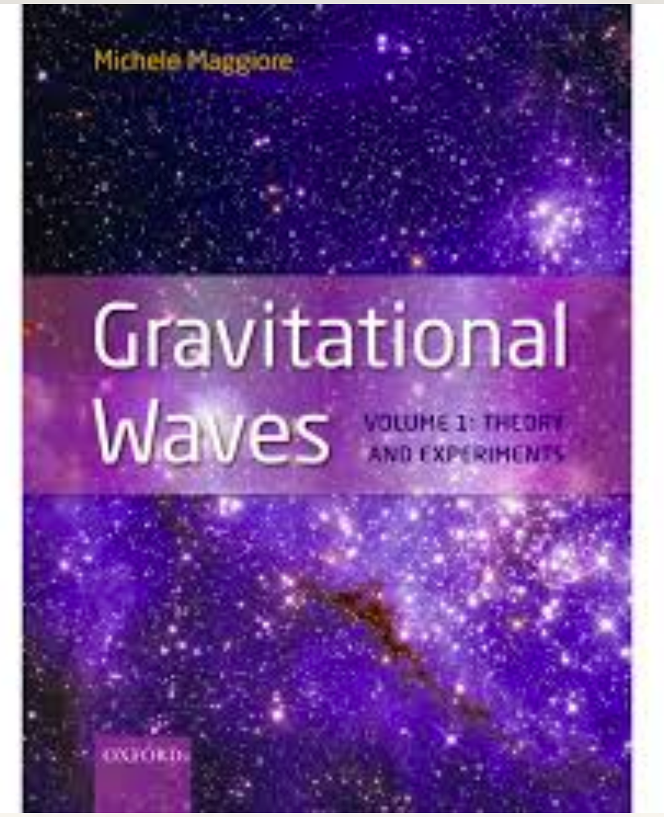
Literature

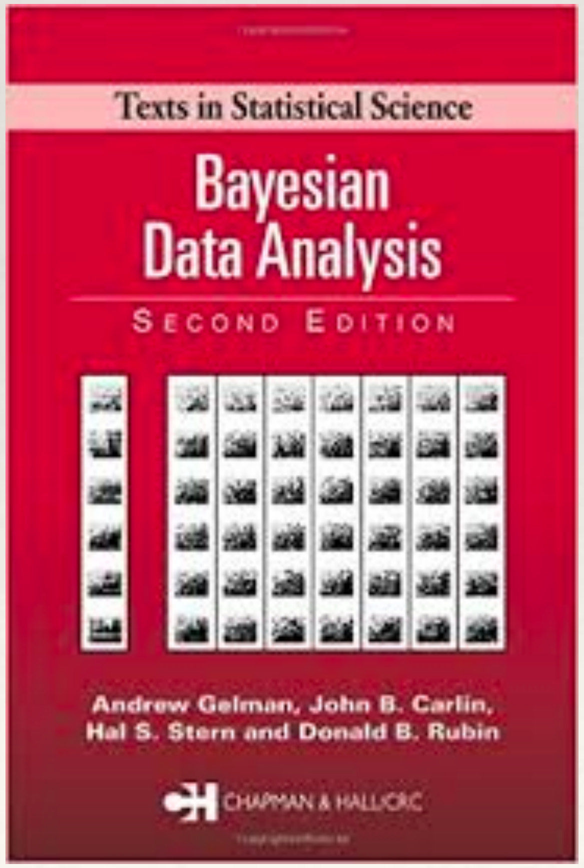


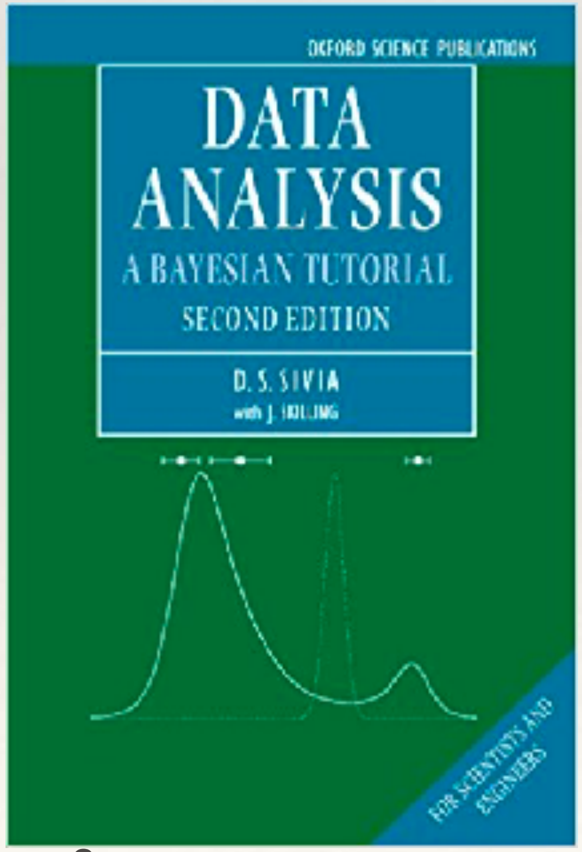


Some figures in these lectures are borrowed from theses books

LSC+Virgo, Phys. Rev. Lett. 116, 221101 (2016), LSC+Virgo, Phys. Rev. X 6, 041015 (2016), LSC+Virgo, Phys.Rev.Lett. 116 241102 (2016), LSC+Virgo, Phys.Rev. X6 041014 (2016), LSC+Virgo, Phys. Rev. Lett. 118, 221101 (2017), Berti et al., Class.Quantum Grav. 32, 243001 (2015), LSC+VIRGO, ArXiV: 1805.11579, S. Khan+, Phys.Rev. D93 (2016) 044007, LSC+Virgo arXiv:1805.11579 , LIGO_Virgo, Astrophys.J. 848 (2017) L12, LIGO+Virgo Phys.Rev.Lett. 119 (2017) 161101, Babak+ Phys.Rev. D95 (2017) 103012, LISA consortium arXiv:1702.00786, Klein+ Phys.Rev. D93 (2016) 024003, LISA consortium, arXiv:1305.5720, Amaro-Seoane+ Class.Quant.Grav. 29 (2012) 124016 .





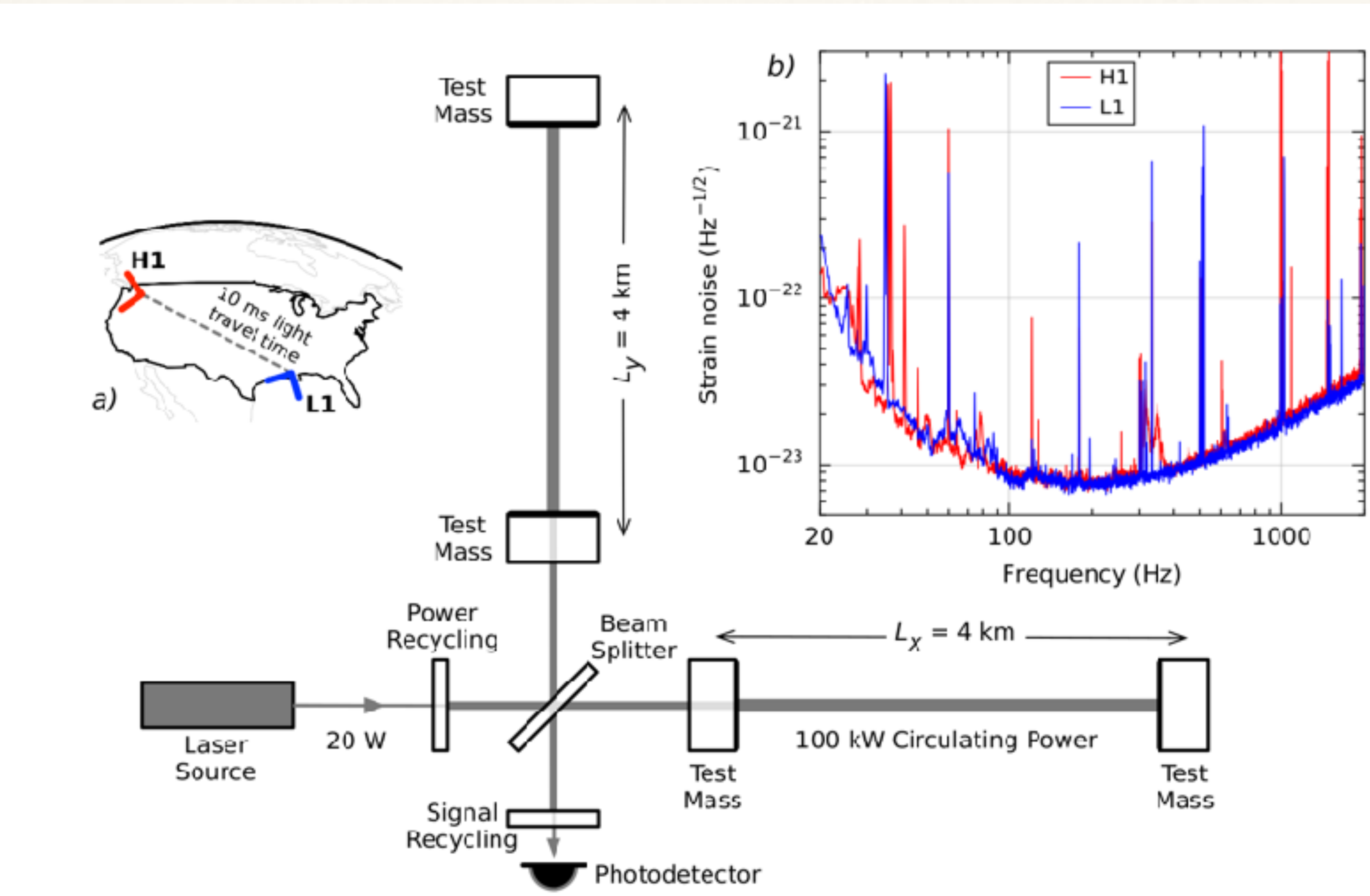


Lecture 3

- Laser interferometer in space: LISA
- Sources and event rate
- Data analysis: Part 1

LIGO/VIRGO

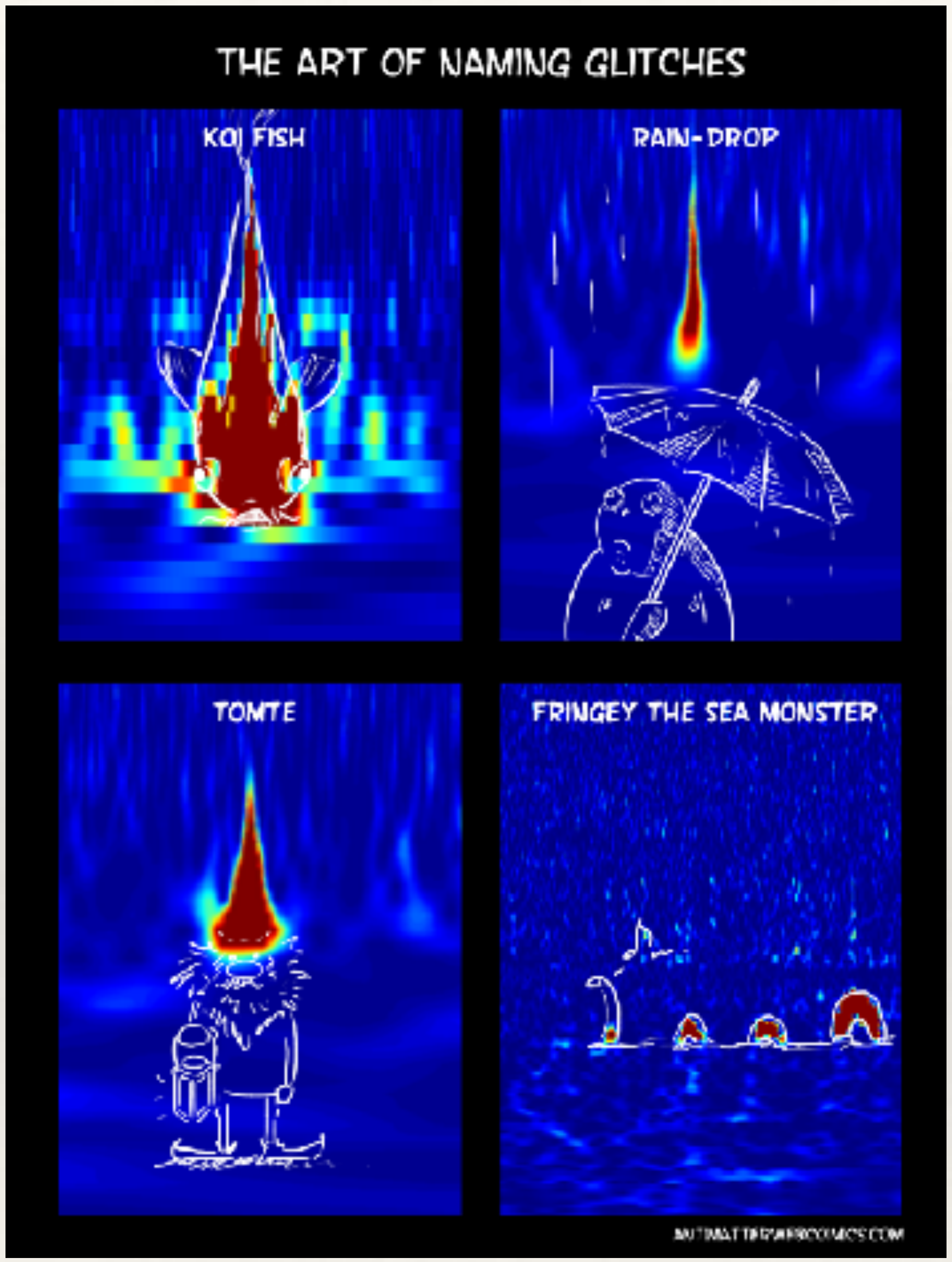
$$\Delta L = \frac{1}{2} h^{GW} L \qquad h \approx 10^{-22}, \rightarrow \; L = 4km, \; \rightarrow \; \Delta L \approx 10^{-16} cm$$



LIGO/VIRGO noise

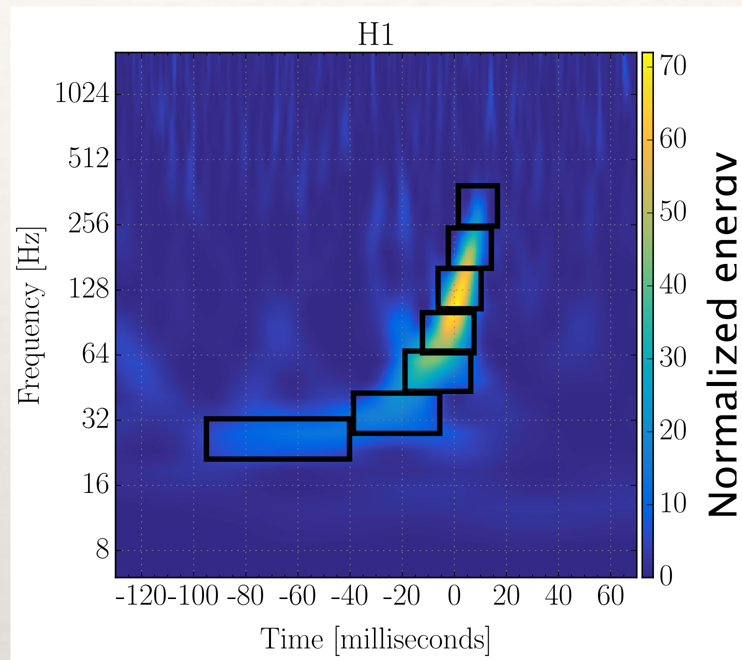
The noise in LIGO/VIRGO is stationary only on relatively short time scale

frequency

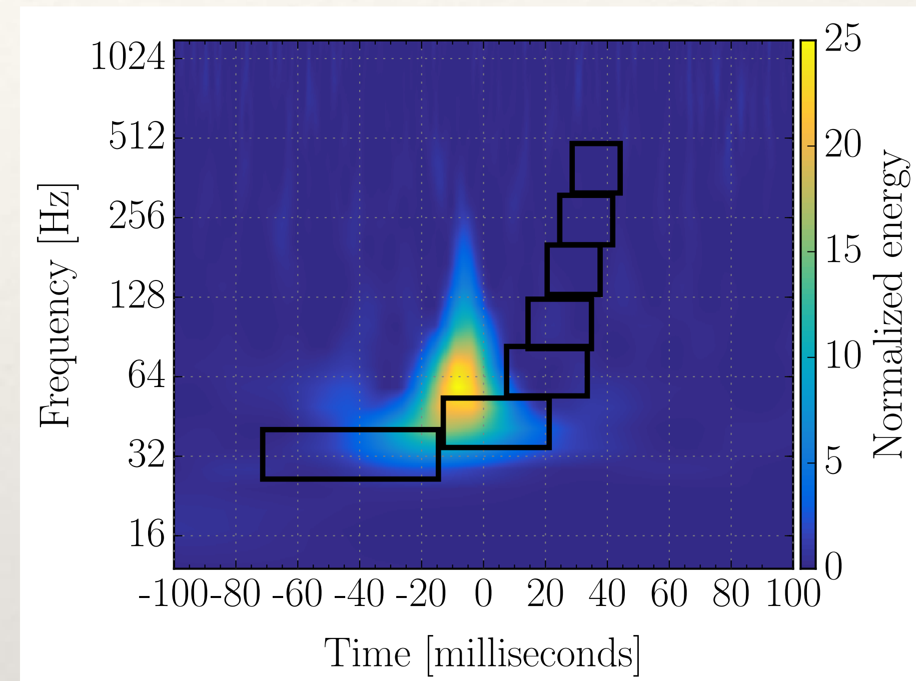


time

Real

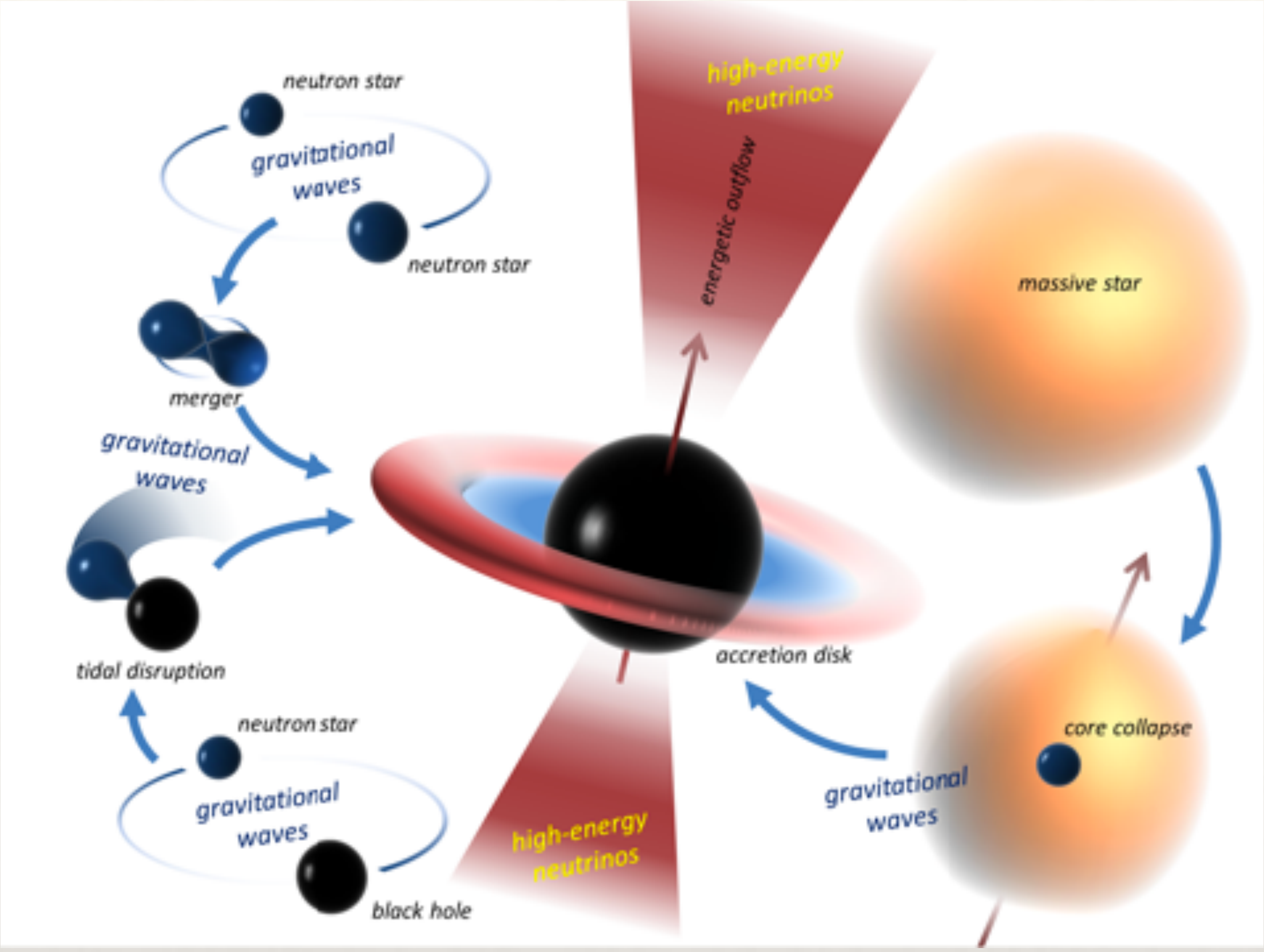


Instrumental



The noise is not Gaussian: need to introduce additional consistency checks into the detection statistic

Other GW sources in LIGO/VIRGO frequency band



Credit: I. Bartos/Based on arXiv:1212.2289

- Core collapse
- Merging NS binary systems
- Binary: NS-BH
- Monochromatic signal from deformed single NSs
- Stochastic GW signal from the energetic processes in early Universe

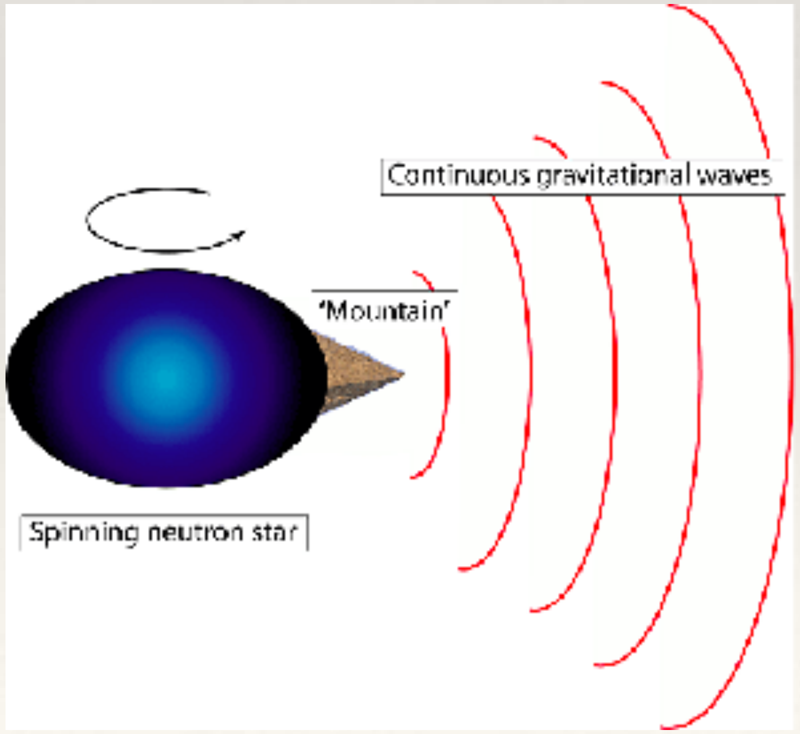
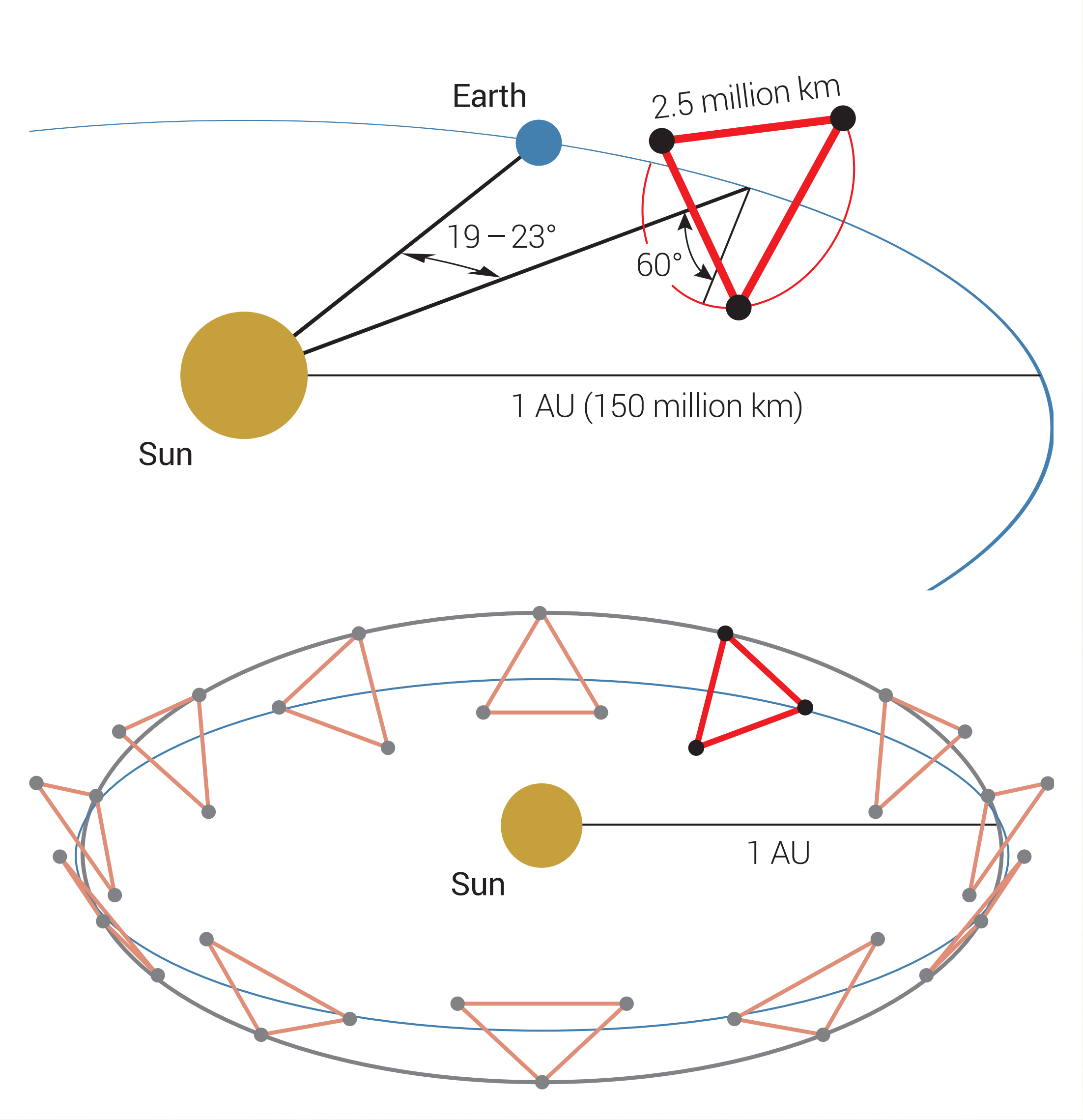


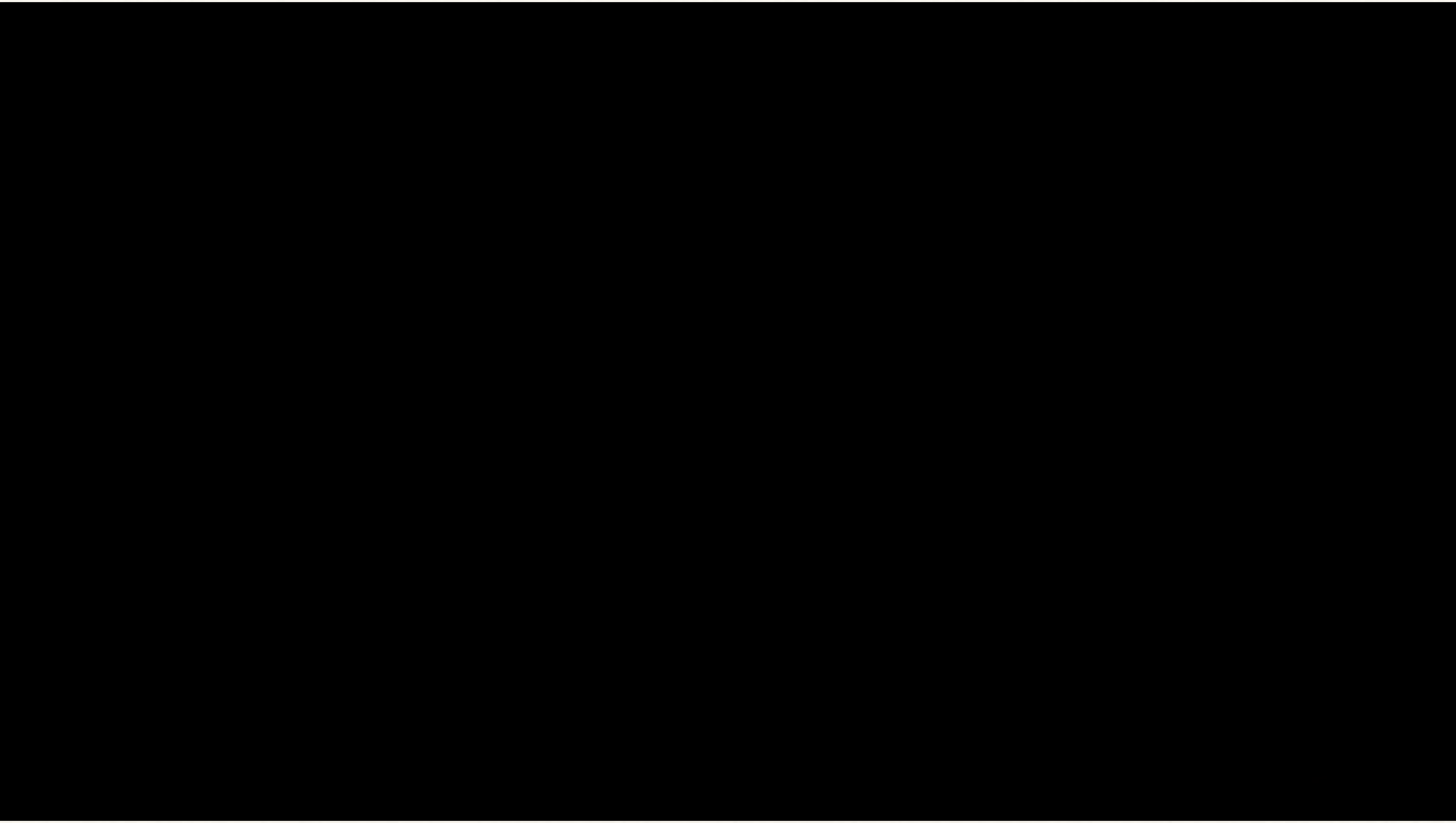
Image from https://physics.anu.edu.au/quantum/cgp/research/datatheory/neutronstars.php

LISA: Laser Interferometric Space Antenna

- LISA: GW observatory in space: The launch date 2032 - 2034. Leading by European Space Agency.
- LISAPathfinder - Technological mission to prove the technical readiness of LISA - fantastic results, order of magnitude better than minimum requirement

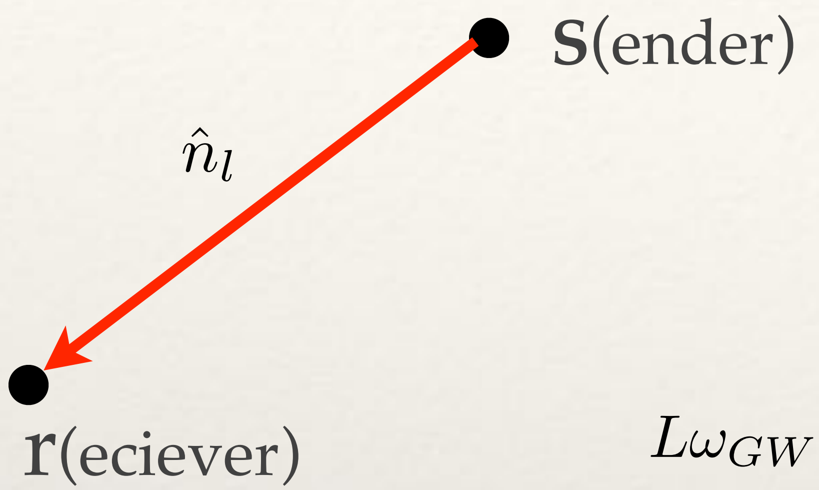


LISA (cartoon)



LISA

Principle of measurement



LISA: three satelites in free falling orbits around the sun, constellation forms equelateral triangle

L1~L2~L3= 2.5 mln. km.

Operates in freq. range 0.1 mHz - 0.1 Hz.

Exchange laser light - measurement of the proper distance between satellites.

$L\omega_{GW} = 1 \quad \rightarrow \; f_{GW} \approx 20 mHz$ Long wavelength is not applicable

We cannot cover the detector by LIF, use "TT" frame: in this frame GW can be seen as affecting the phase (or frequency) of the laser light.

$$\frac{\Delta\nu}{\nu_0} = \frac{n_l^i n_l^j \Delta h_{ij}}{2(1-\hat{k}.\hat{n}_l)}$$

Change in laser freq. due to GWs → $\Delta\nu$

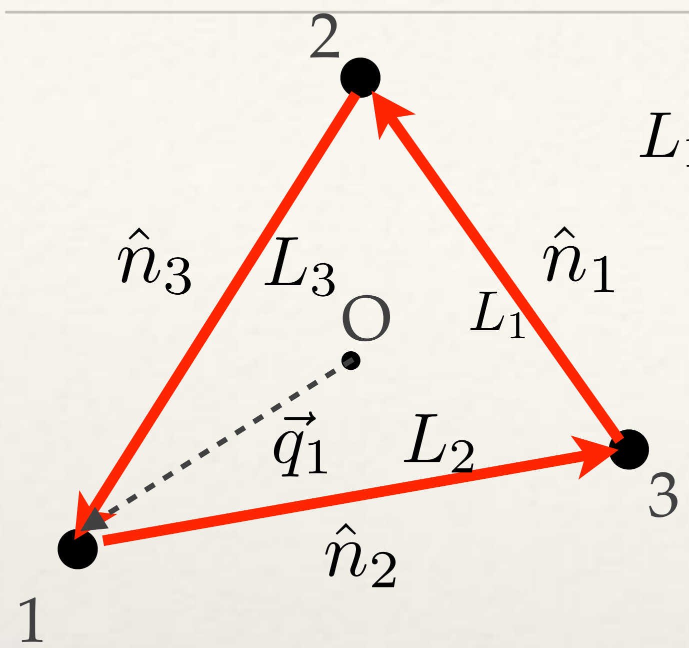
unperturbed freq. of a laser → ν_0

direction of GW propagation → $\hat{k}$

$\Delta h^{ij} = h^{ij}(t_s) - h^{ij}(t_r)$

$$t = t_r, \;\; t_s = t - |\vec{R}_r(t) - \vec{R}_s(t_s)| \approx t - |\vec{R}_r(t) - \vec{R}_s(t)| \approx L_l$$

LISA



$L_1 \approx L_2 \approx L_3 \approx L$ — but not exactly!

$\frac{\Delta\nu}{\nu_0} \to y_{slr}$ — for a single link

Consider GW signal from a binary system

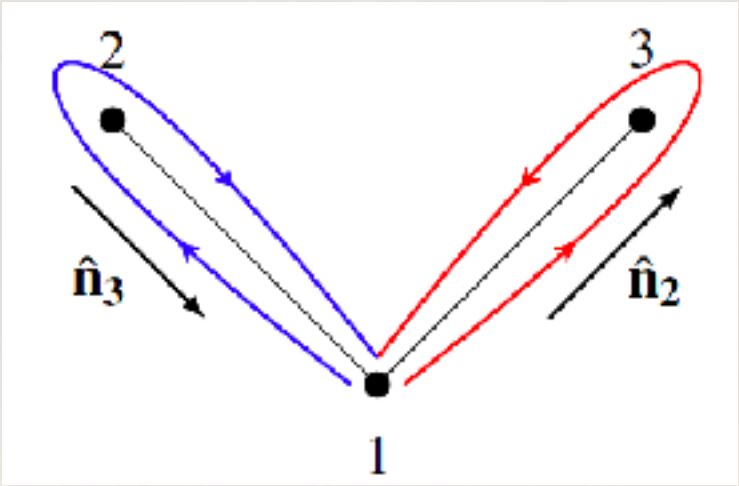
$$y_{slr}^{GW} = -i\frac{\omega L}{2} A_l(\iota, \psi, \theta_s, \phi_s) e^{i\Phi(t-\hat{k}.\vec{R}_0)} e^{-i\omega\hat{k}.\vec{q}_r} Sinc\left[\frac{\omega L}{2}(1-\hat{k}.\hat{n}_l)\right] e^{-i\frac{\omega L}{2}(1-\hat{k}.\hat{n}_l)}$$

- A_l Amplitude of GW times antenna beam function
- Sinc — zero of sinc function gives freq. of GW signal which cannot be measured (f-n of sky position) - wiggles in the sensitivity at high frequencies
- Phase: $t - \hat{k}.\vec{R}_0(t)$ Doppler modulation (dominant) due to relative motion of the detector and the source

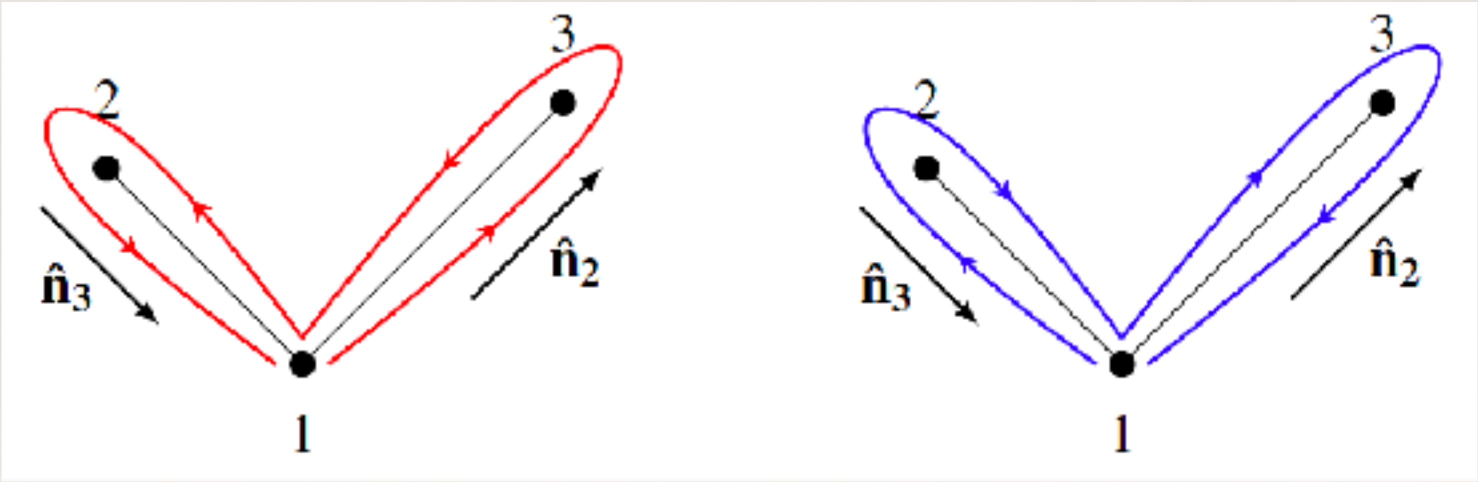
LISA

- The localization of the sources in the sky comes from the Doppler modulation of the phase and from the amplitude modulation (time dependent antenna beam f-n)
- The term(s) dependent on the position of each spacecraft explicitely (q-vectors): important at very high frequency: constellation "feels" GW propagation

TDI — Time Delay Interferometry: technique which we apply to cancel the laser noise (in post-processing the data)



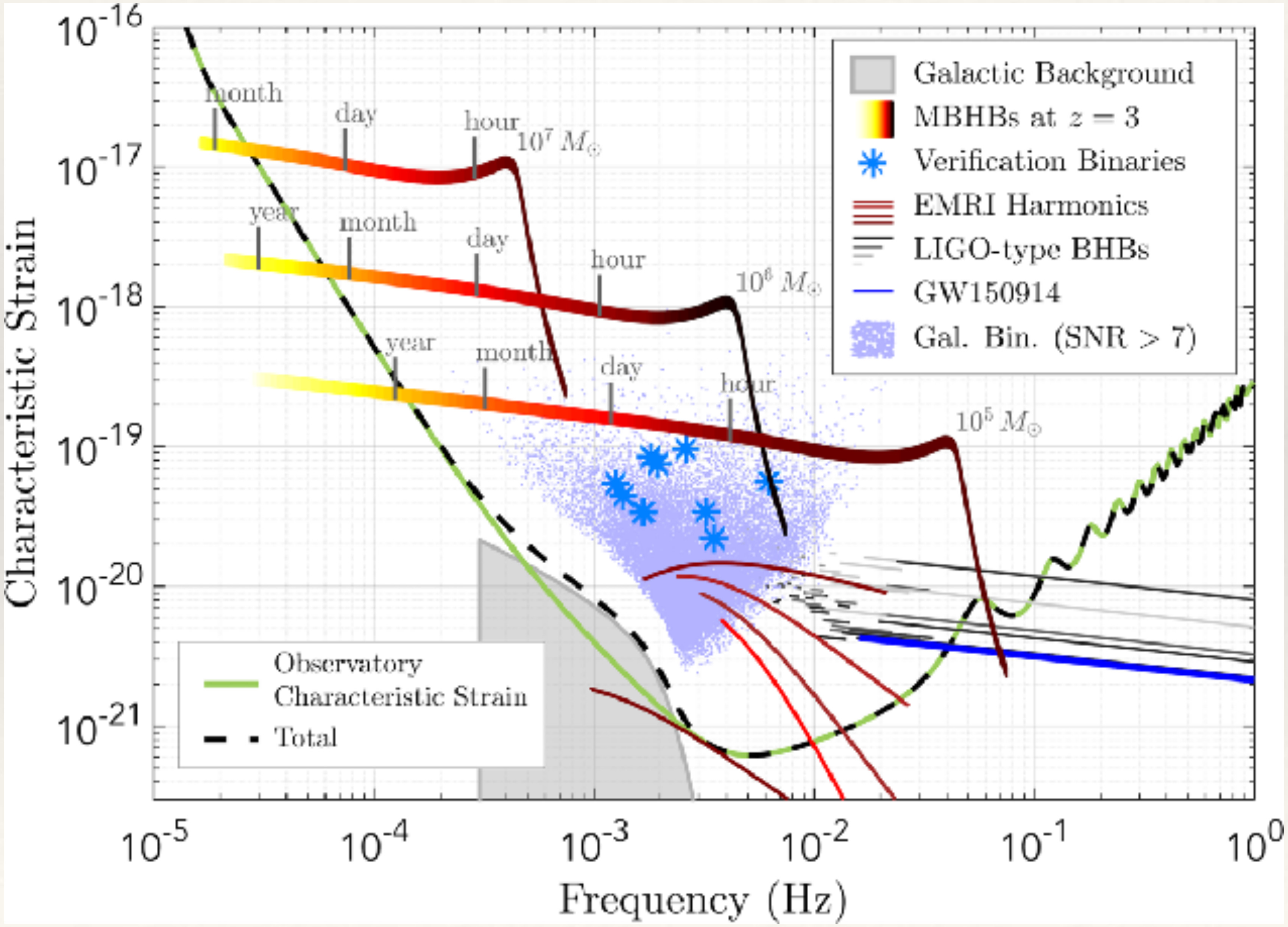
Equal arm Michelson



Unequal arm Michelson

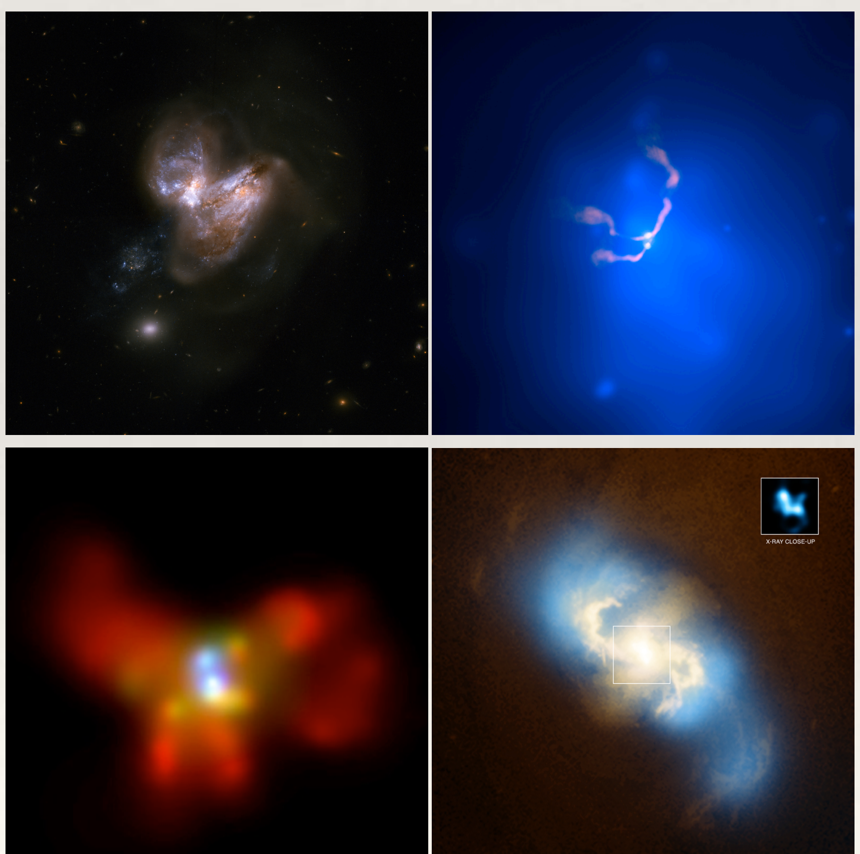
GW sources in LISA band

- GW signals in LISA are strong and long-lived.
- LISA data will contain thousands of GW signals simultaneously: need to separate and characterize them
- Non-stationary noise

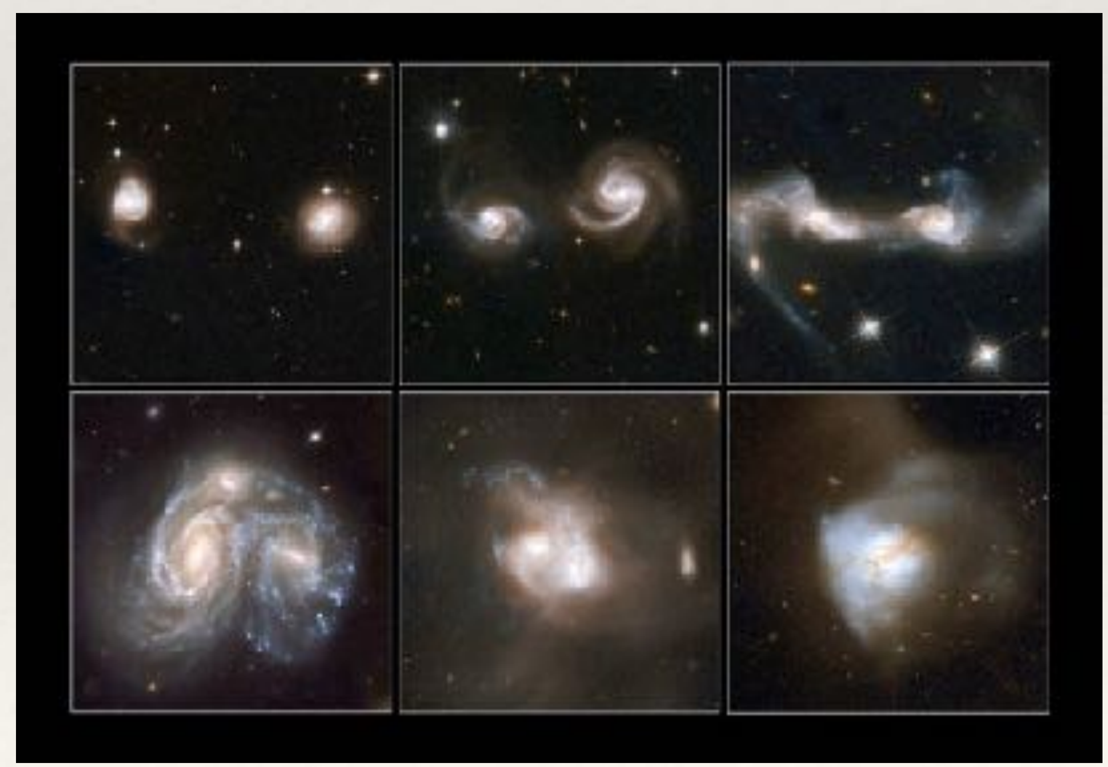


LISA sources

- We believe that all galactic nuclei host Massive Black Holes: Milky Way has 4 mln. solar mass BH
- Galaxies merge: we can form Massive Black Hole Binary (MBHB) system
- We need stars and gas to bring MBHs close together for GW to be efficient (binary is merging within Hubble time)



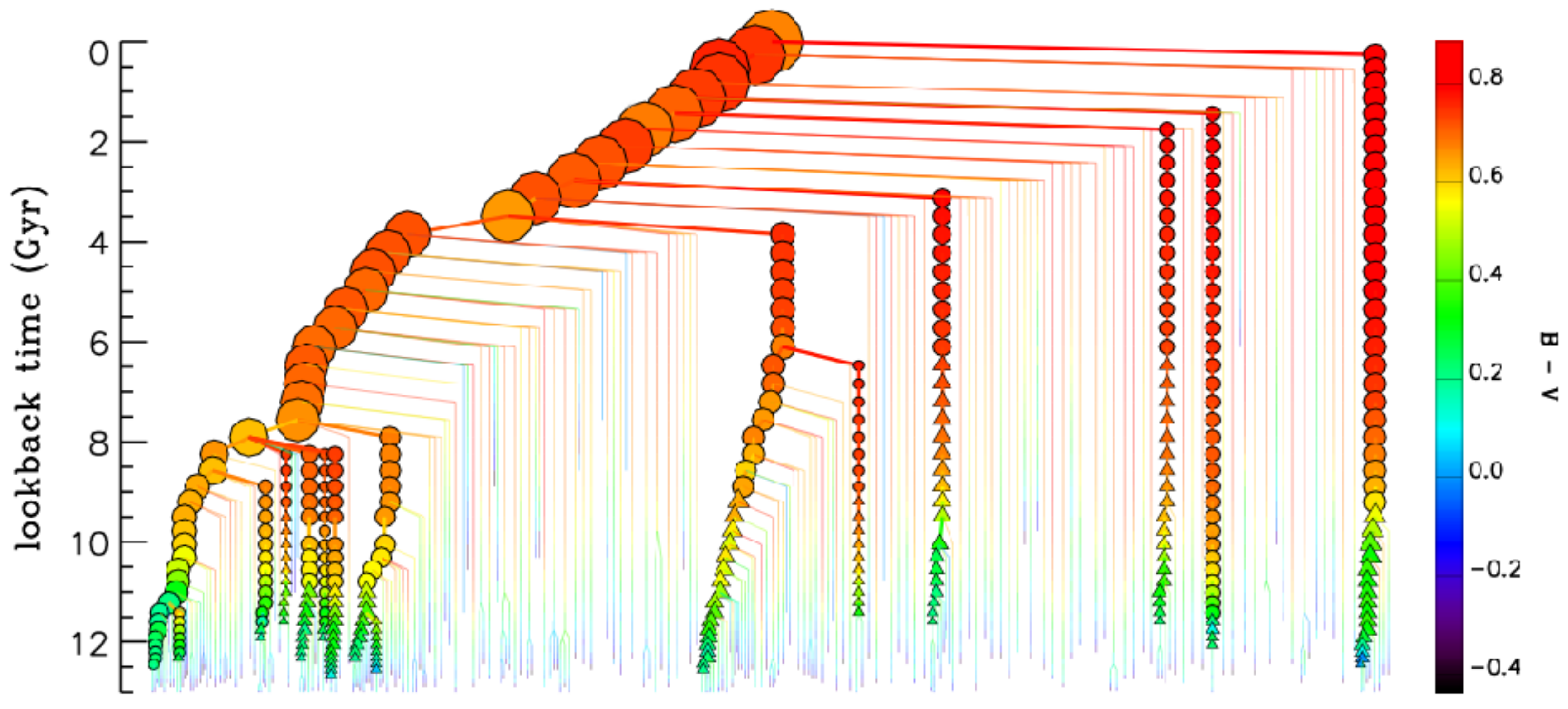
[Credits: Hassinger+, VLA, Chandra , NASA]



[Image: Hubble telescope]

LISA sources: MBHB

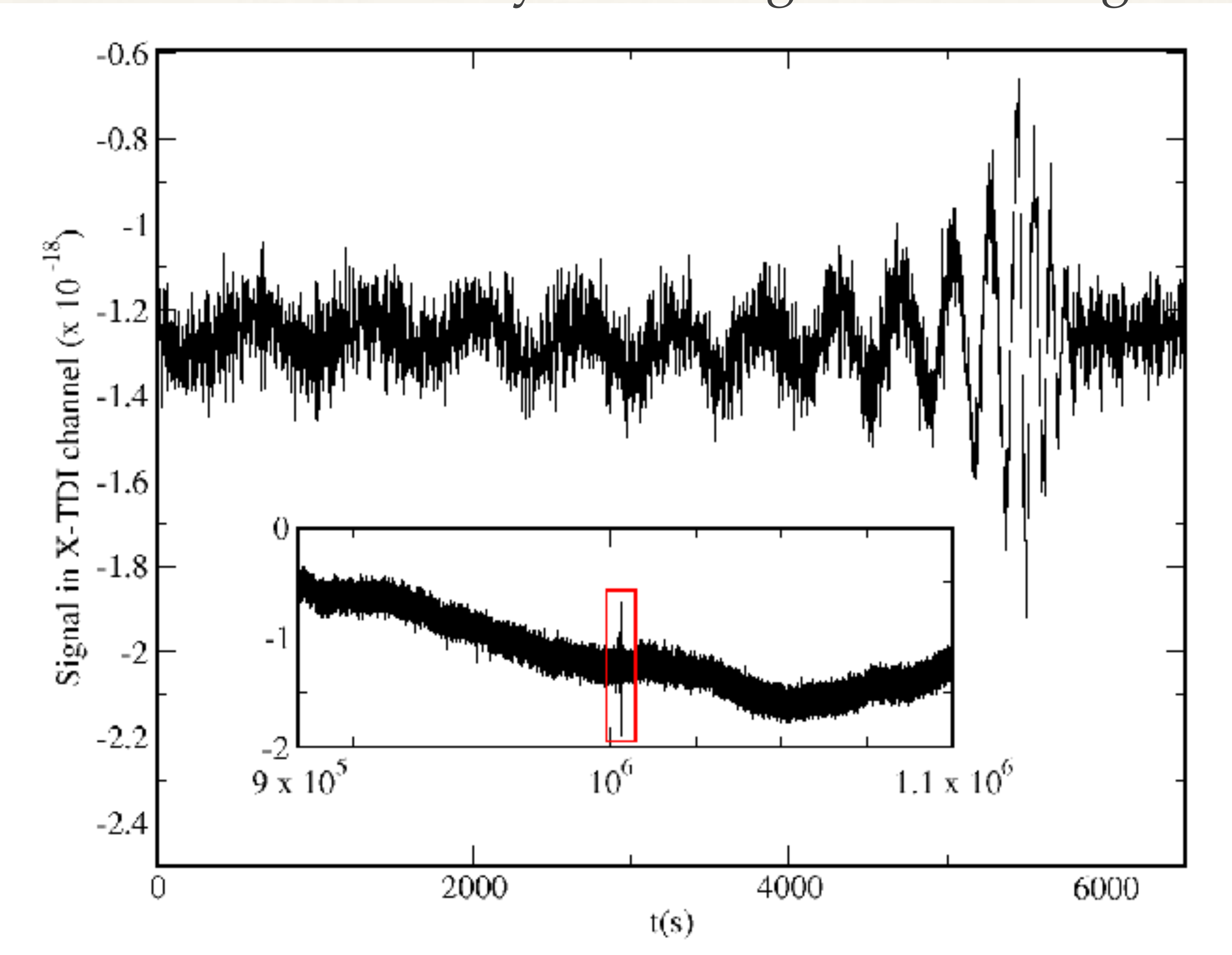
MBHs are formed from the initial BH seed. Those seeds could be "light" remnant of the first generation of stars or "heavy" from the direct collapse of a giant gas cloud. BHs accumulated the mas through gas accretion and merging.



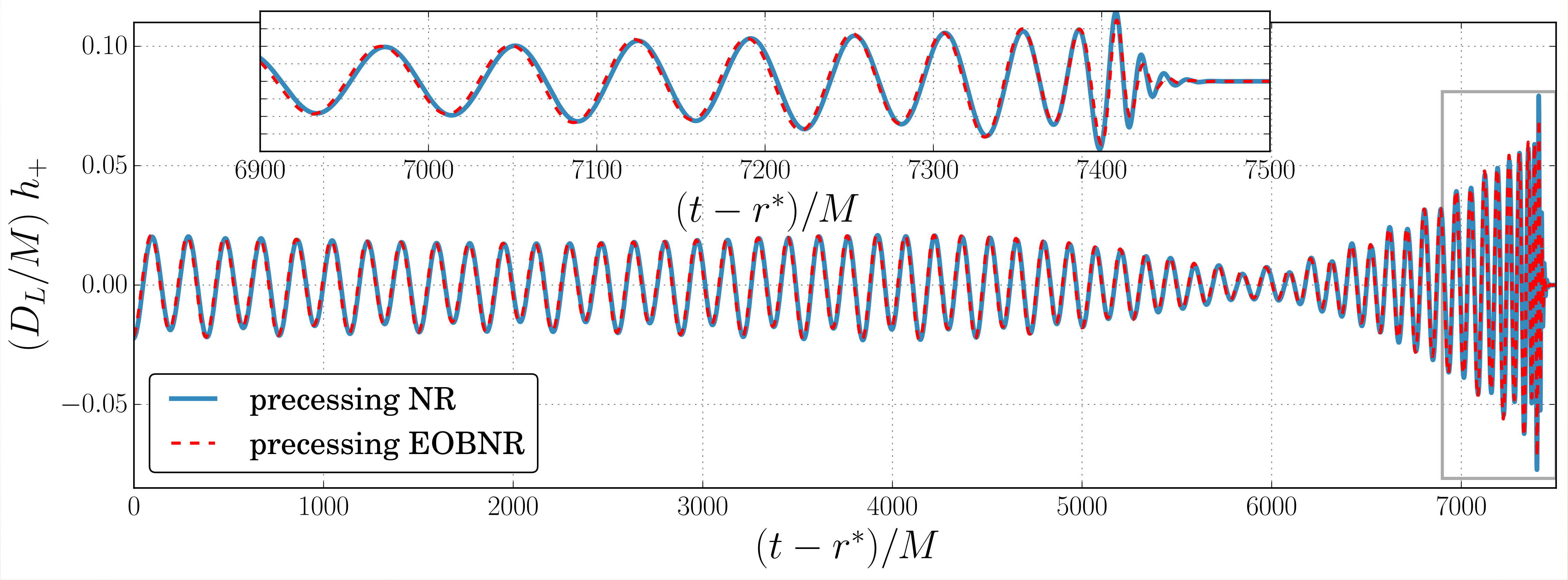
[Credits: Gabriella De Lucia]

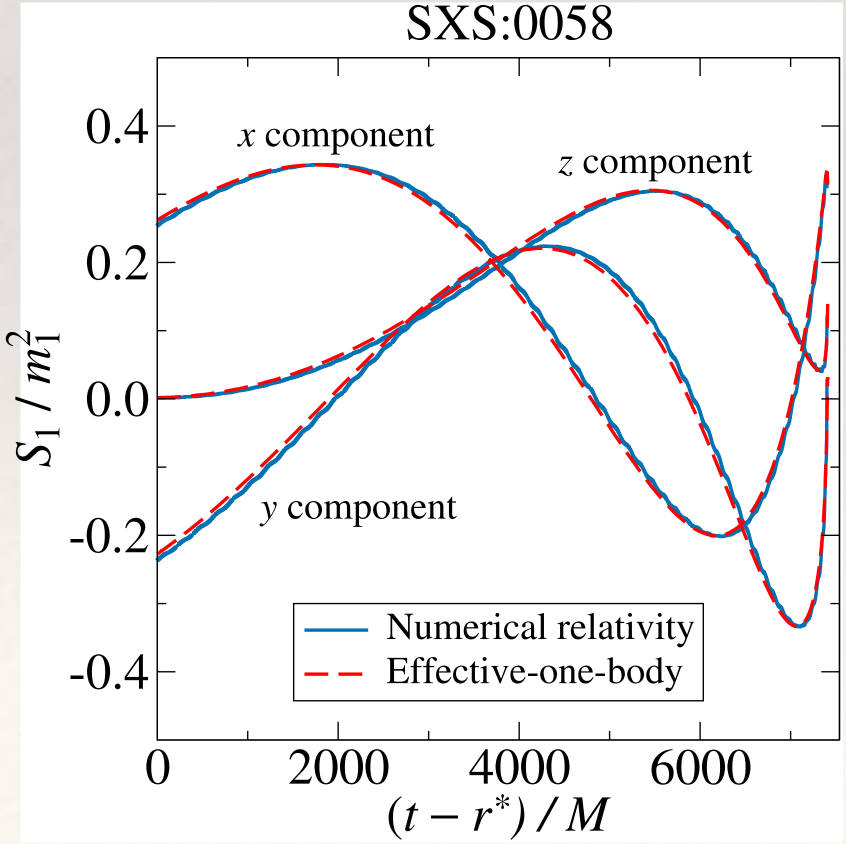
LISA: GW signal from MBHB

The signal from MBHB is similar to what we have observed in LIGO (scaled up in the amplitude and streched in time). GW signal from MBHB is expected to be the strongest signal (seen by eye in the simulated data). Imposes stringent demands on the accuracy of GW signal modelling



Precessing binary

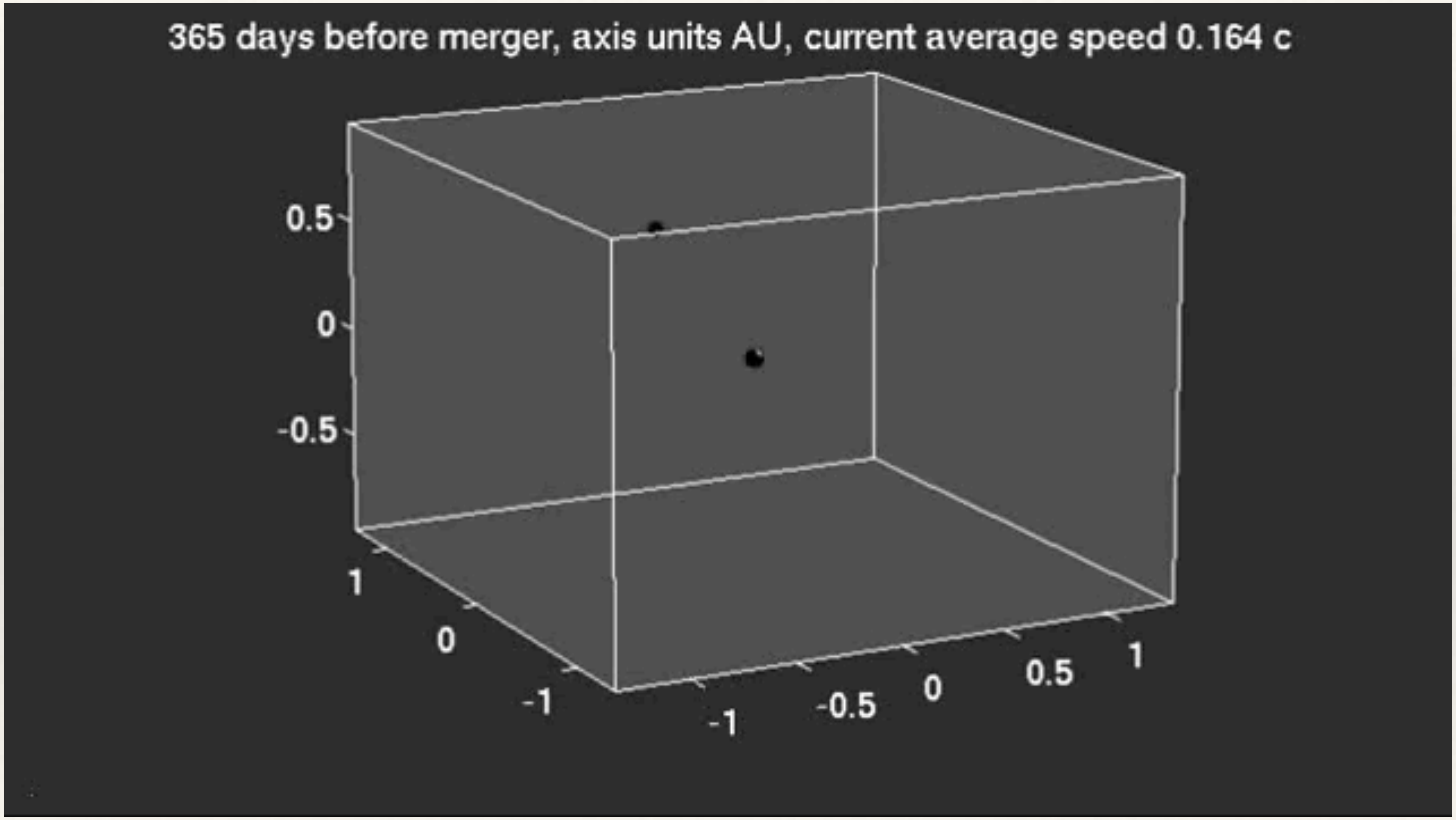




EMRIs (extreme mass ratio inspirals)

- Massive BHs in galactic nuclei surrounded by stars and gas with quite high density
- MBH could capture a compact object (BH, NS, WD) which is thrown on a very eccentric orbit (due to N-body interaction). The orbit shrinks and circularizes due to grav. radiation.
- EMRI: binary system with extreme mass ratio of component 10^{-7} - 10^{-5}
- Compact object revolves 10^{6} orbits in the proximity of MBH before the plunge.

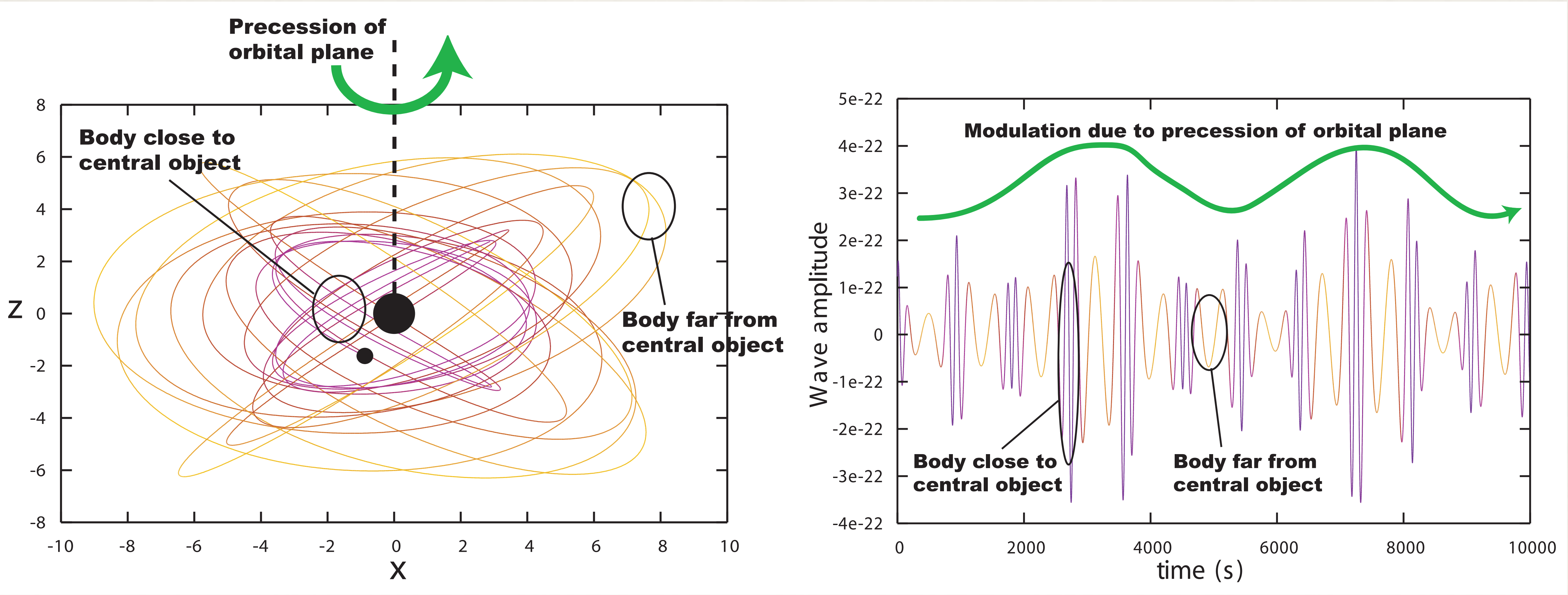
EMRI



[Credits: S Draco, CalTech]

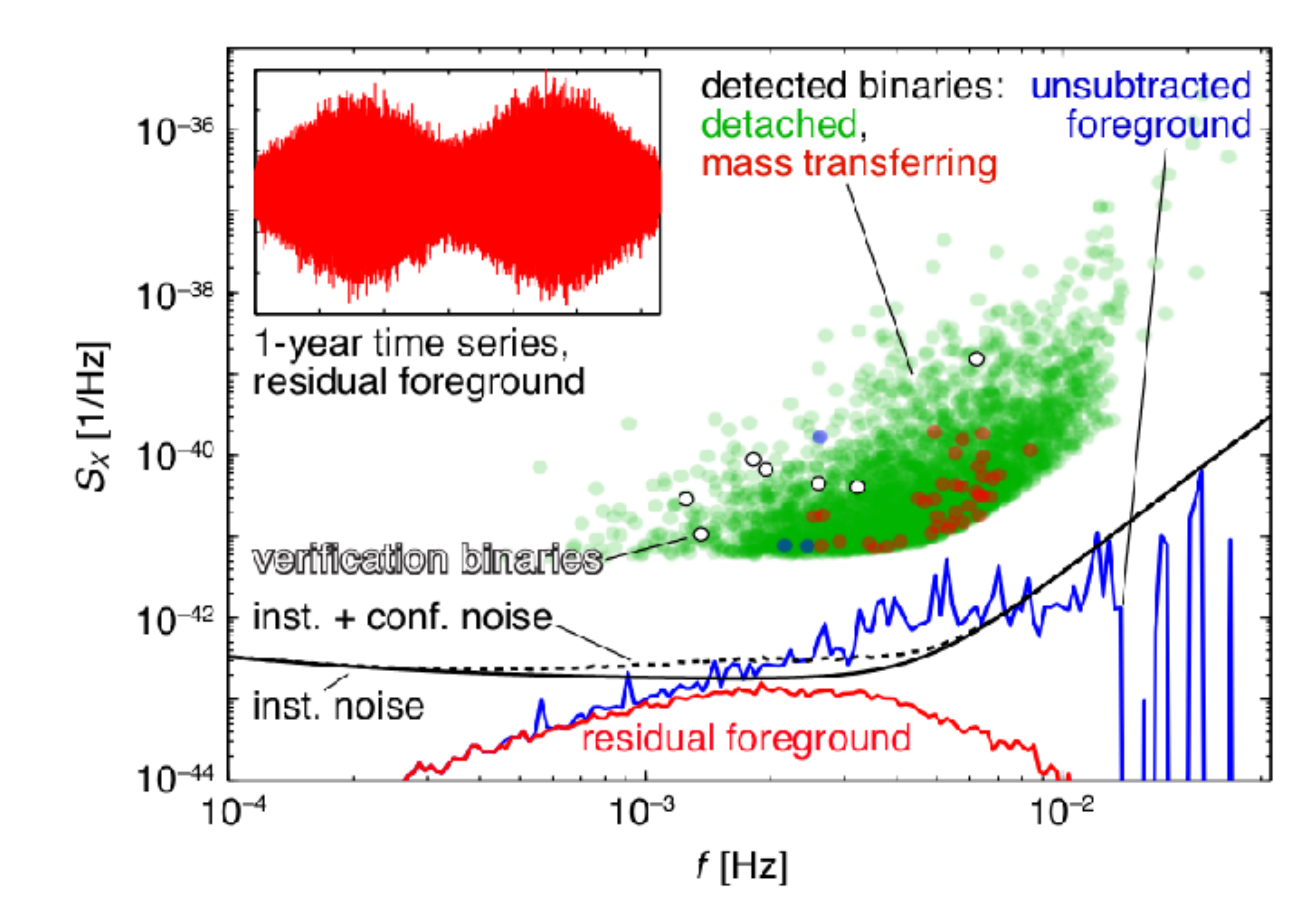
EMRI

- Orbital motion: (almost) elliptical with a strong relativistic precession + orbital precession due to spin-orbital coupling
- Signal is very rich in structure (hard to detect but gives a lot of information)
- Ultra-precise parameter determination (if detected). Can map spacetime of a heavy object: holiodesy



Galactic white dwarf binaries

- We expect to have 10^7 WD binaries all emitting GWs in the LISA band, only 10^4 can be resolved individually, other form stochastic GW signal (foreground)
- GW signal is almost monochromatic
- Verification binaries: known from current e/m observations (+GAIA,+ LSST)

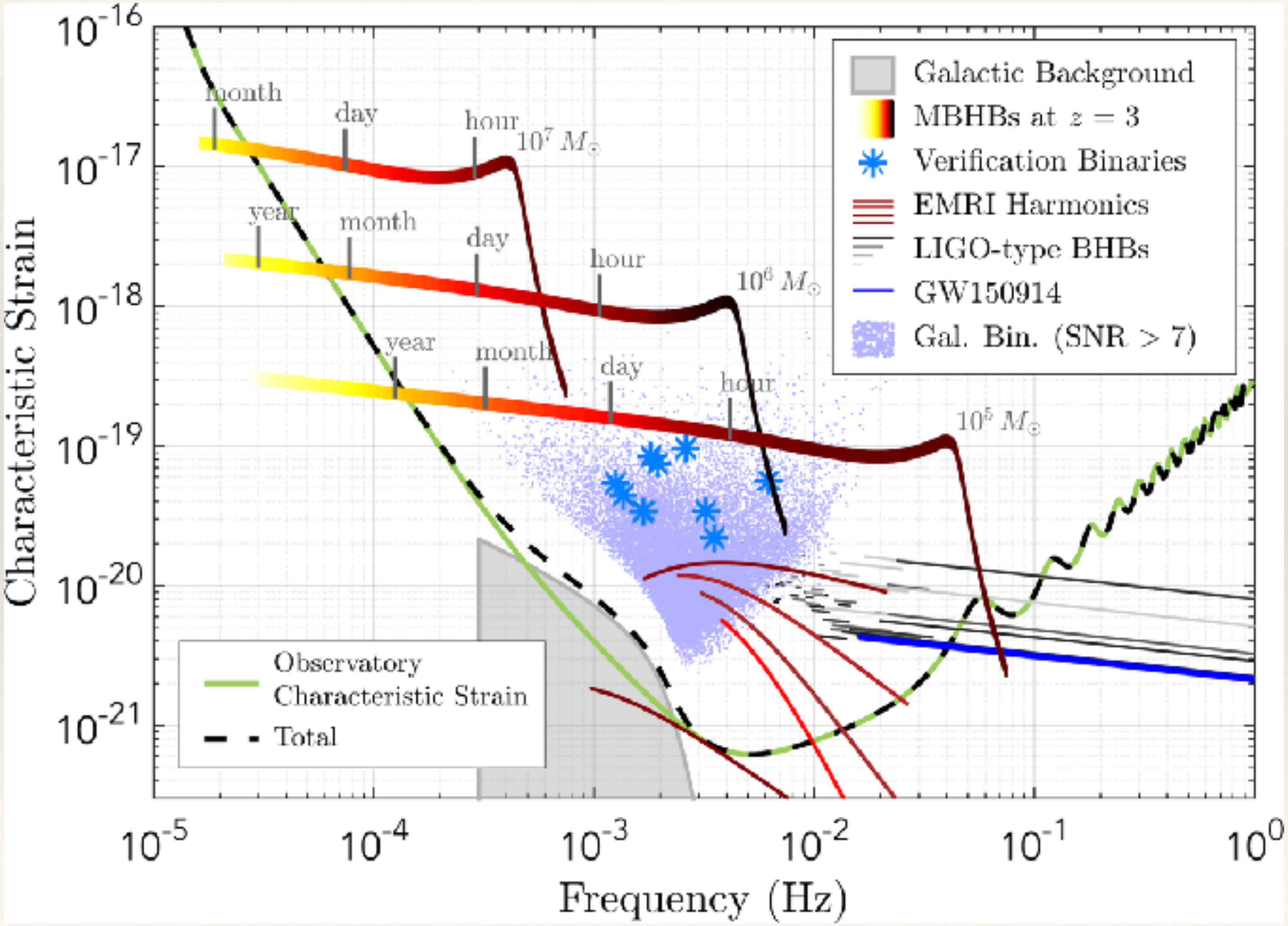


Expected event rate in LISA

- MBHB : high uncertainties in the event rate - from few to few hundreds per year
- EMRIs: even more uncertain - from few to few thousands of detectable GW signals per year.
- GW signal from solar mass BBH (LIGO/VIRGO sources). We expect to observe about 10 sources: GW signal first observed in LISA and then 5-10 years later with the ground based detectors.
- Possible detection of the stochastic GW signal from energetic processes in the early Universe.

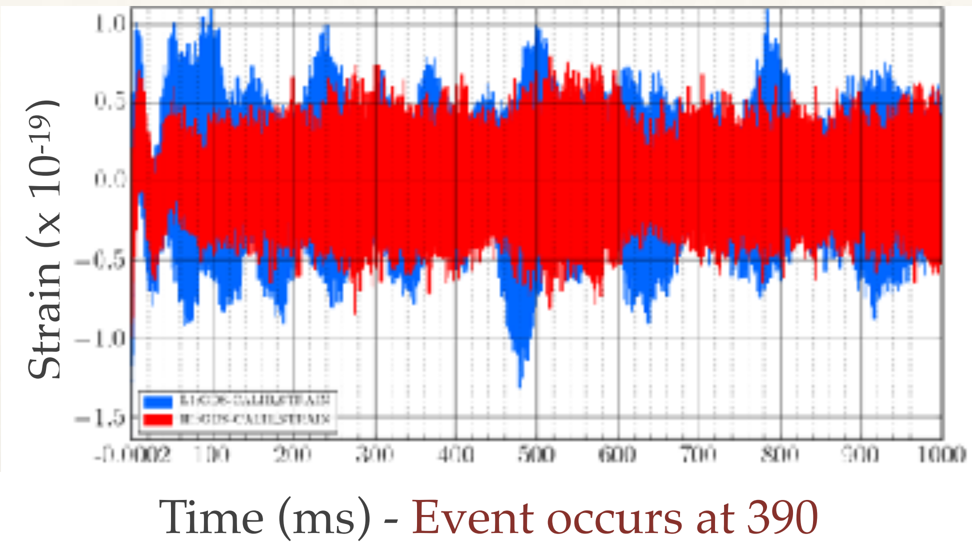
LISA data

LISA data analysis is quite a complex task. We organize the LISA data challenge: https://lisa-ldc.lal.in2p3.fr/home . We simulate LISA data (noise and GW signals) and anyone can download the data and analyse it.

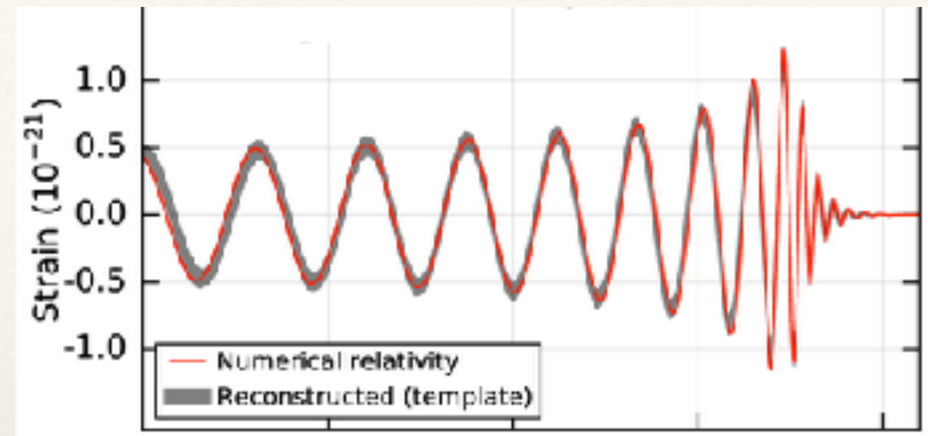


Data analysis: Matched filtering

GW150914 Raw data



GW signal from merging BHs (we search for)



Matched filtering: is used when we are searching for a signal of known form in the noisy data. The basis: we correlate the data with expected signal and search for a maximum of correlation.

$$\rho \sim 4\Re \int_0^\infty \frac{\tilde{d}(f)\tilde{h}^*(f)}{S(f)}df$$

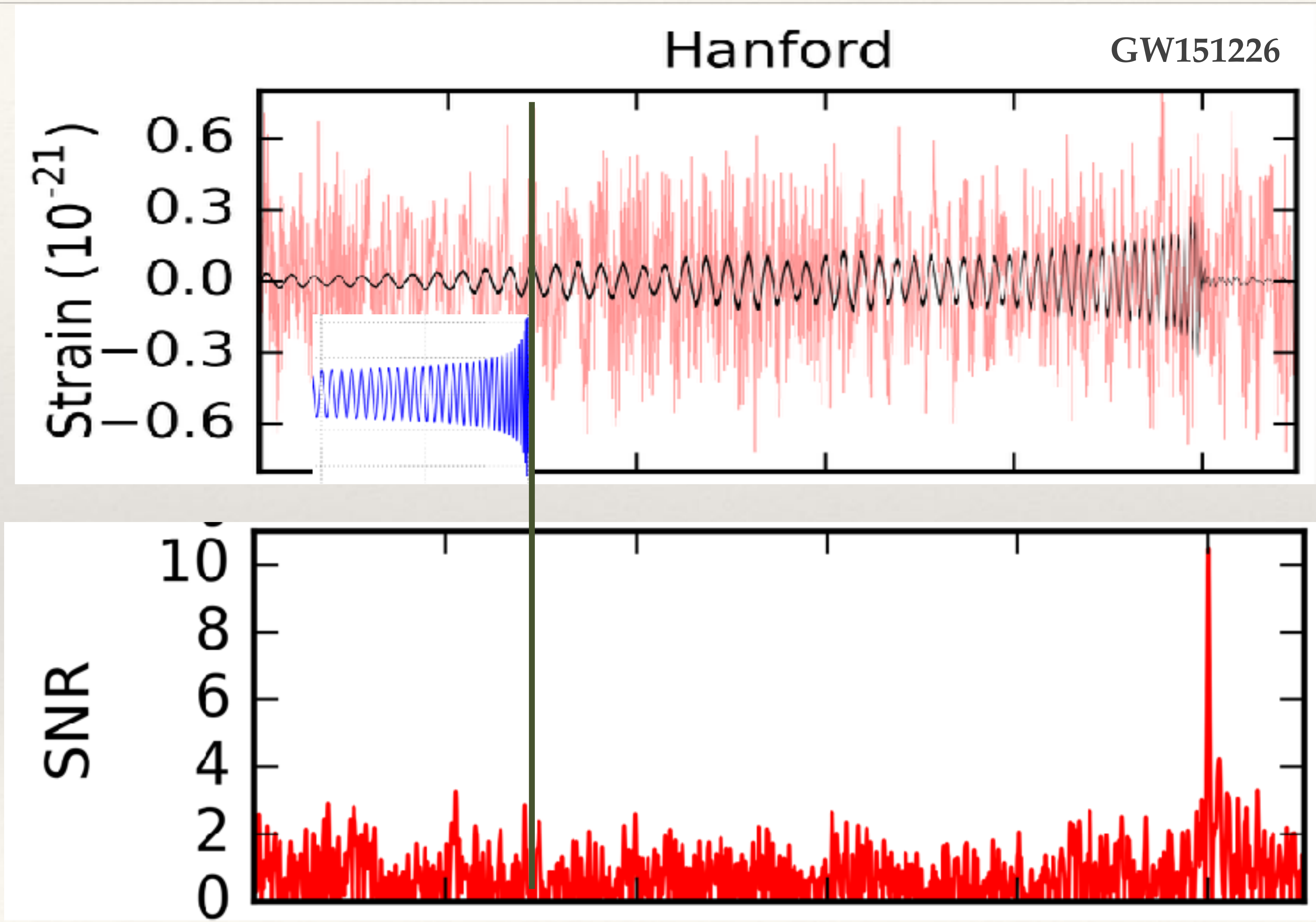
wavform/template we search for.

data

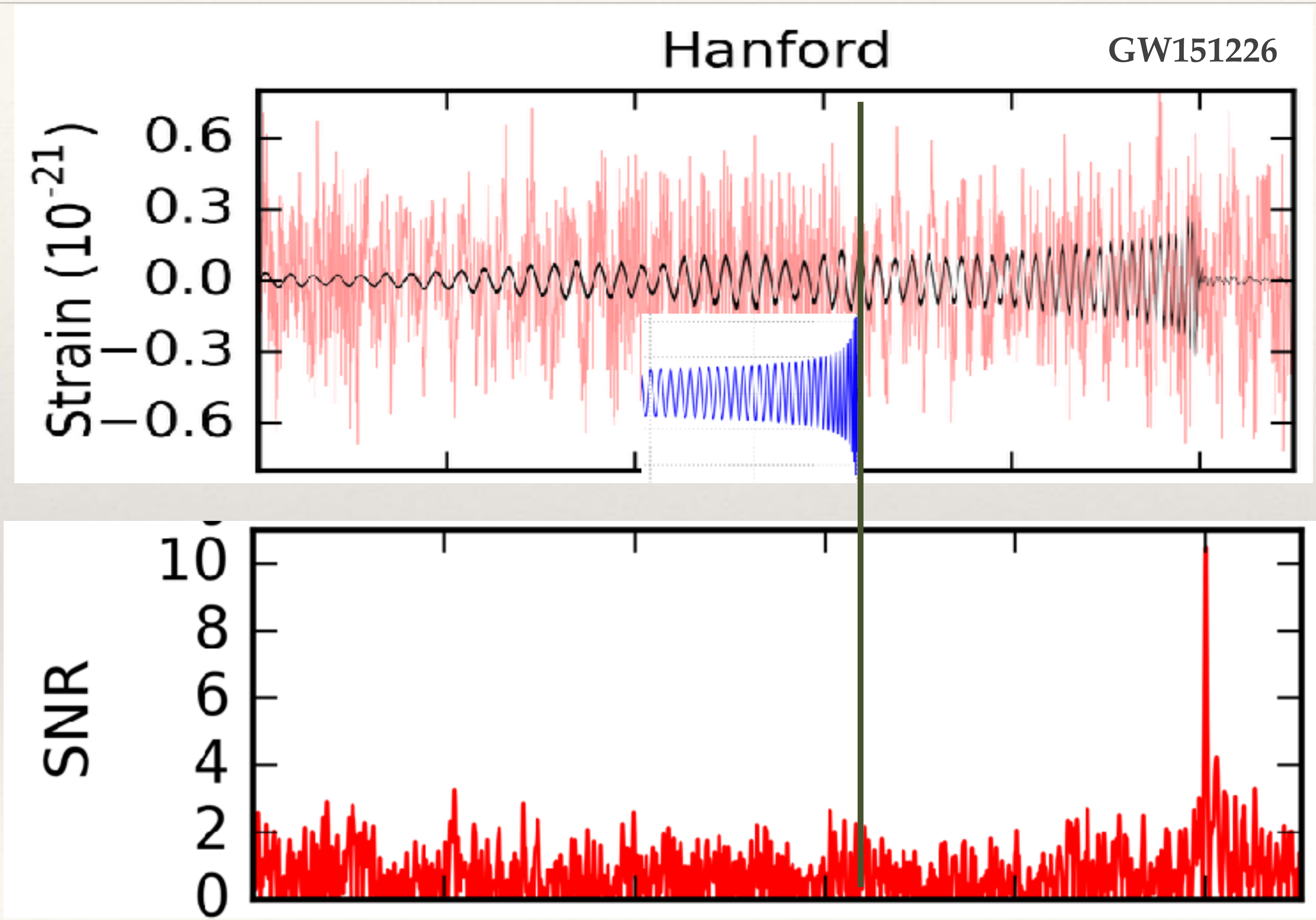
noise power spectral density

Correlation in frequency domain, weighted by detector sensitivity

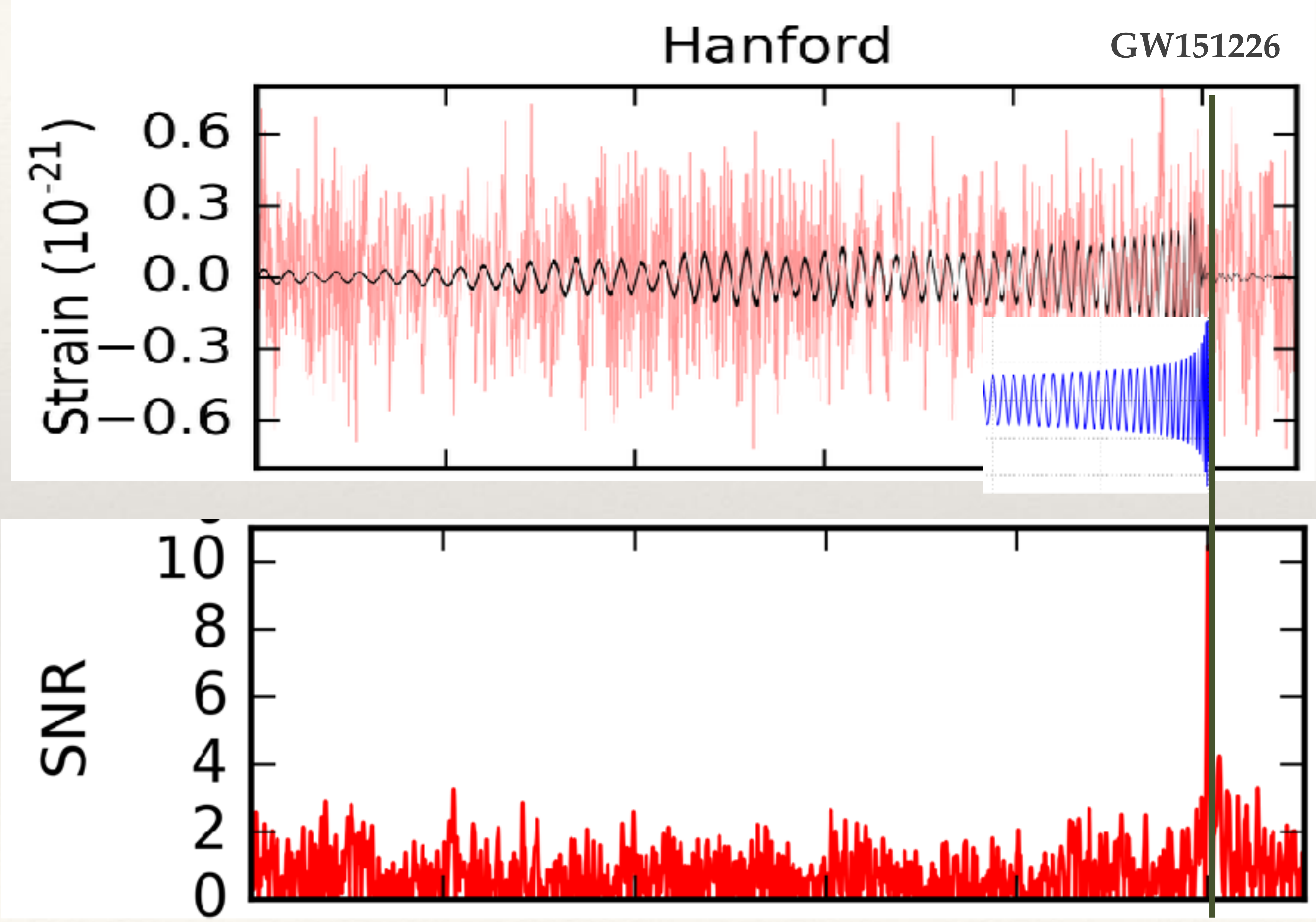
Matched filtering



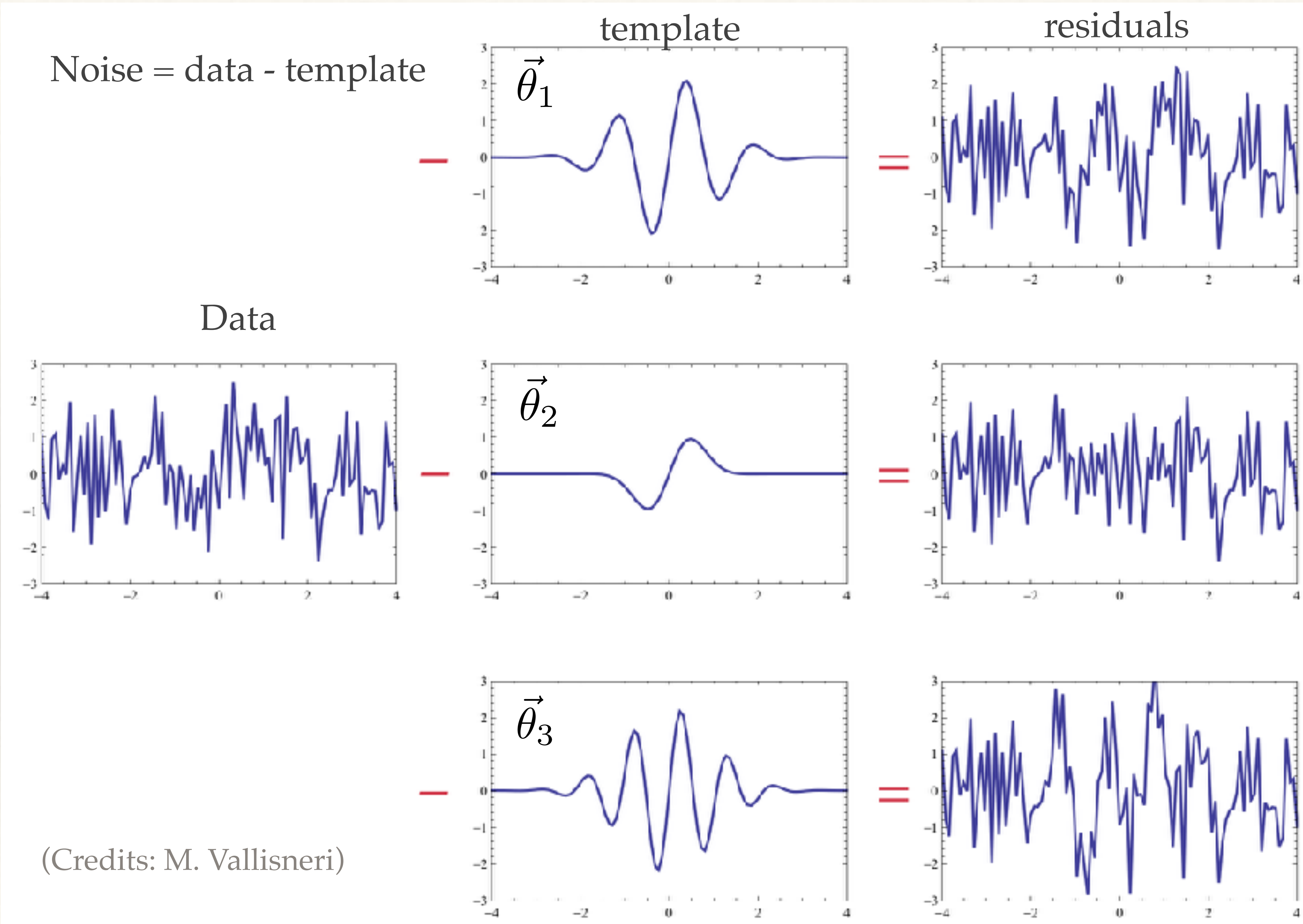
Matched filtering



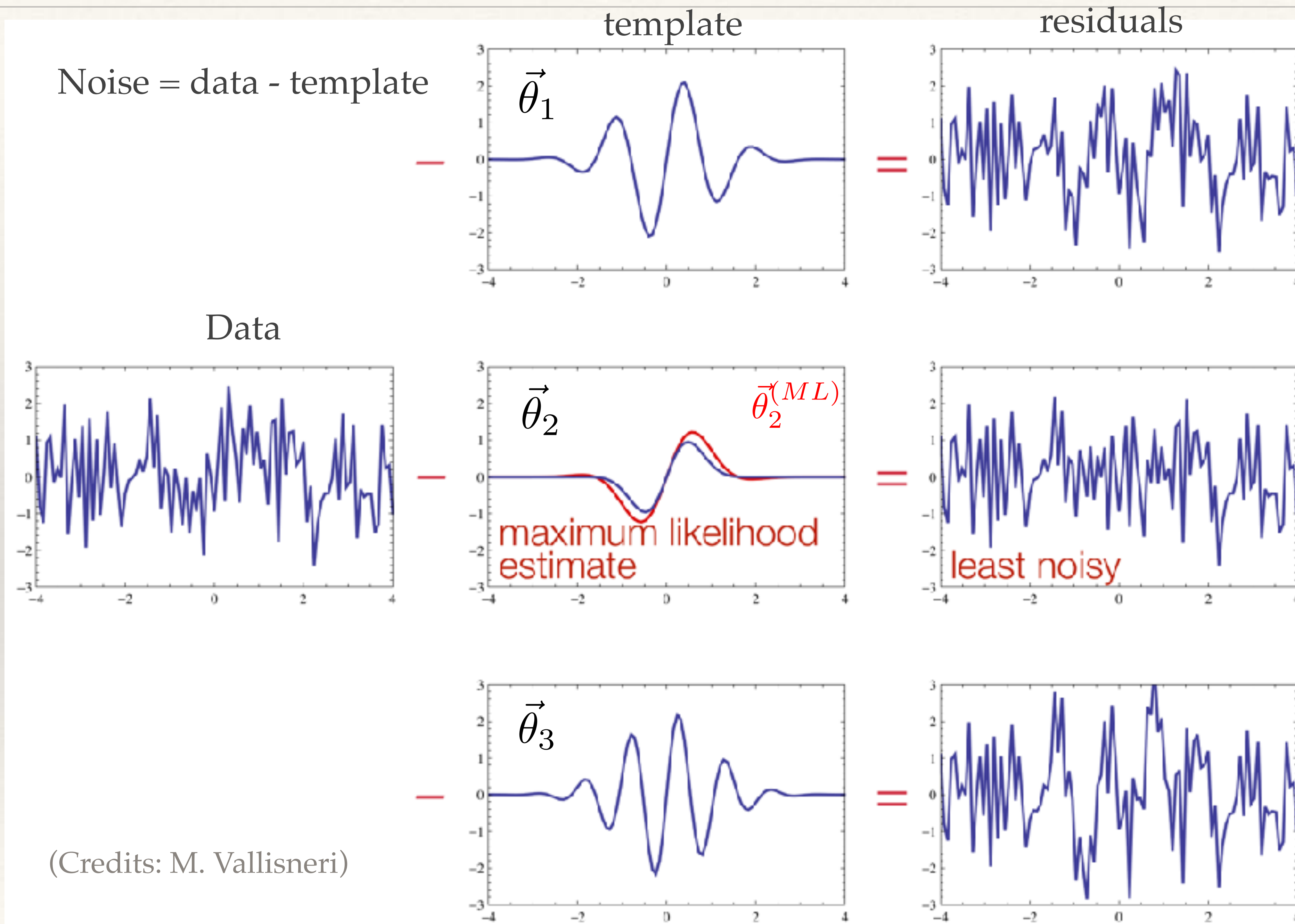
Matched filtering



Matched filtering and parameter estimation



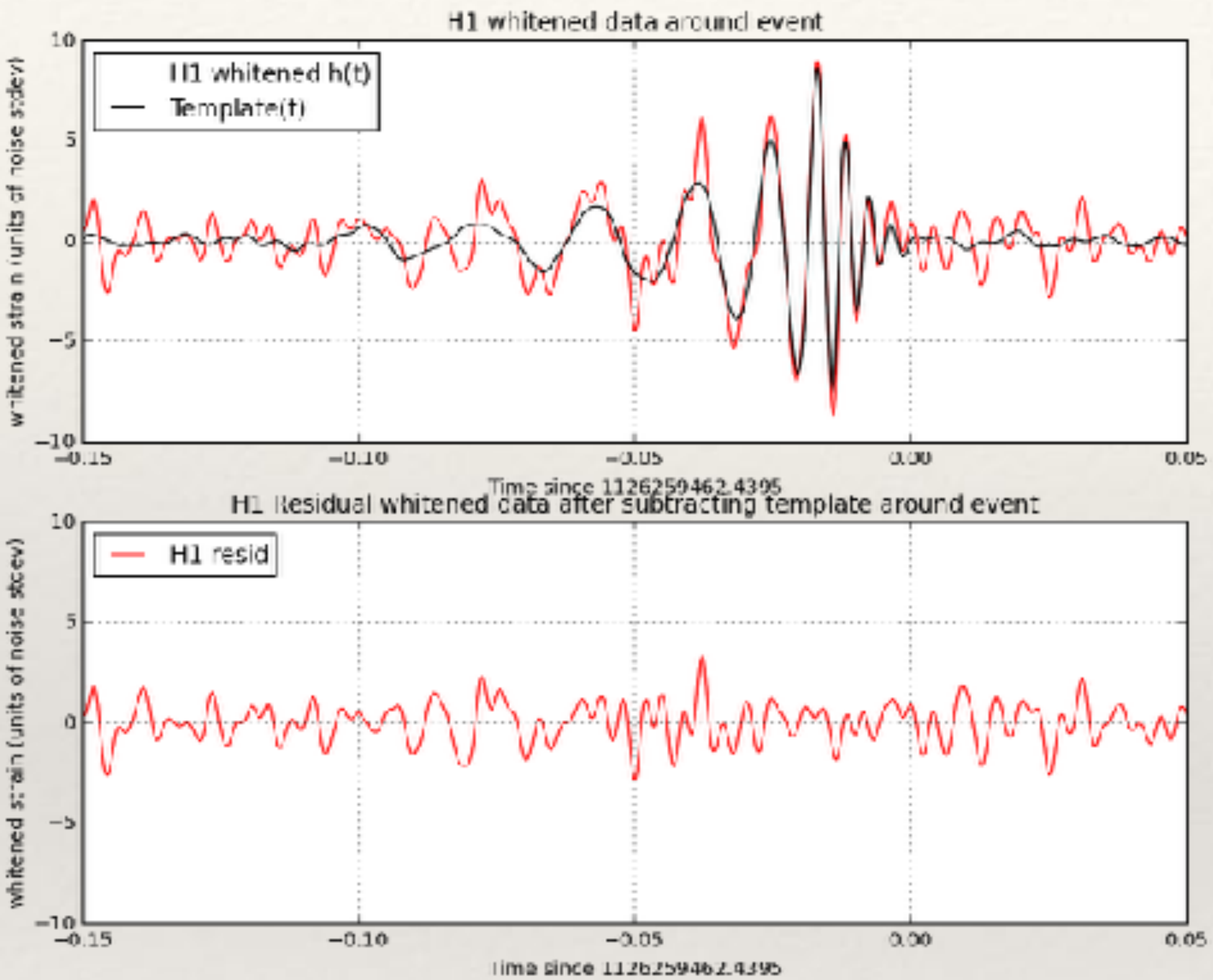
Matched filtering and parameter estimation



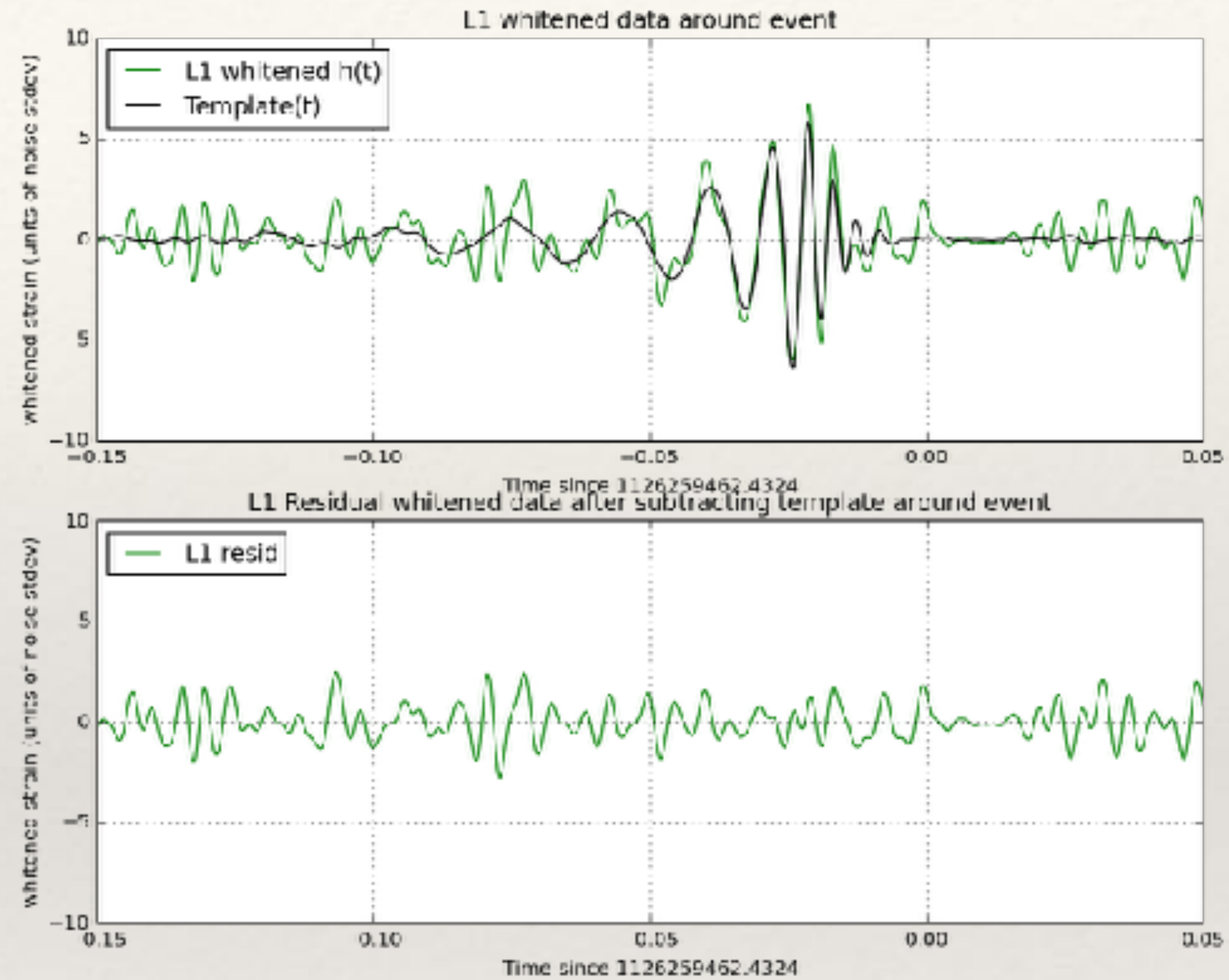
(Credits: M. Vallisneri)

Matched filtering for GW150914

H1



L1



[LOSC: https://losc.ligo.org/tutorials/]

Likelihood

Let us assume that the data contains the signal.: hypothesis (model) H_1

$d(t) = n(t) + s(t, \theta_i)$ signal "s" depends on parameters θ_i

data = noise + signal

If the template matches the GW signal exactly $h(t, \theta_i) = s(t, \theta_i) \longrightarrow d(t) - h(t, \theta_i) = n(t)$

$$p(d(t)|H_1, \bar{s}(t, \lambda)) = p(d(t) - s(t, \vec{\lambda})) = p_n$$

- Assume that the noise is Gaussian (but not necessarily white): non white noise has different variance at different frequencies. The the likelihood can be written as

Likelihood: $$p(d|H_1, \theta_i) \propto e^{-\frac{1}{2}(d-h(\theta_i)|d-h(\theta_i))}$$

The inner product: matched filtering $$(a|b) \equiv 4\Re \int_0^\infty \frac{\tilde{a}(f)\tilde{b}^*(f)}{S_n(f)}\, df$$

- We search for parameters which maximize the likelihood: making the residuals most noise-like — maximum likelihood estimators for parameters $\hat{\theta}_i$