Lecture 4

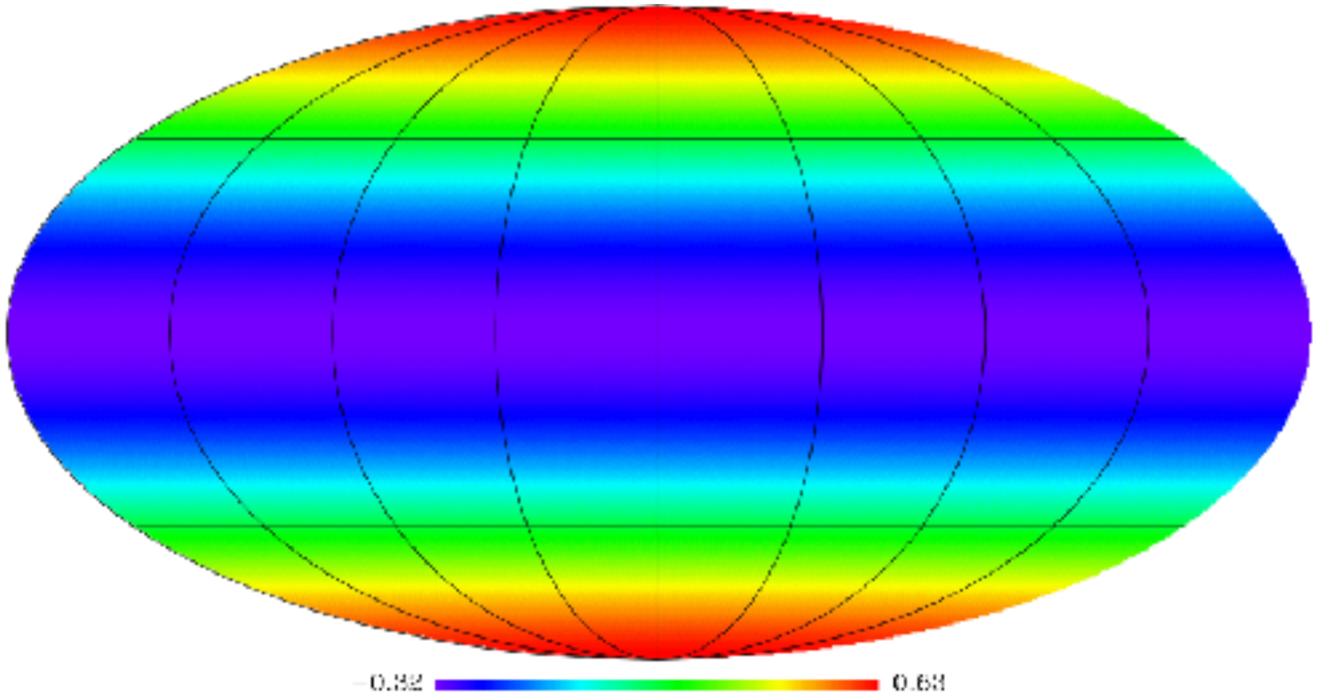
- Polarisation of the CMB (continued)

- Gravitational waves and their imprints on the CMB

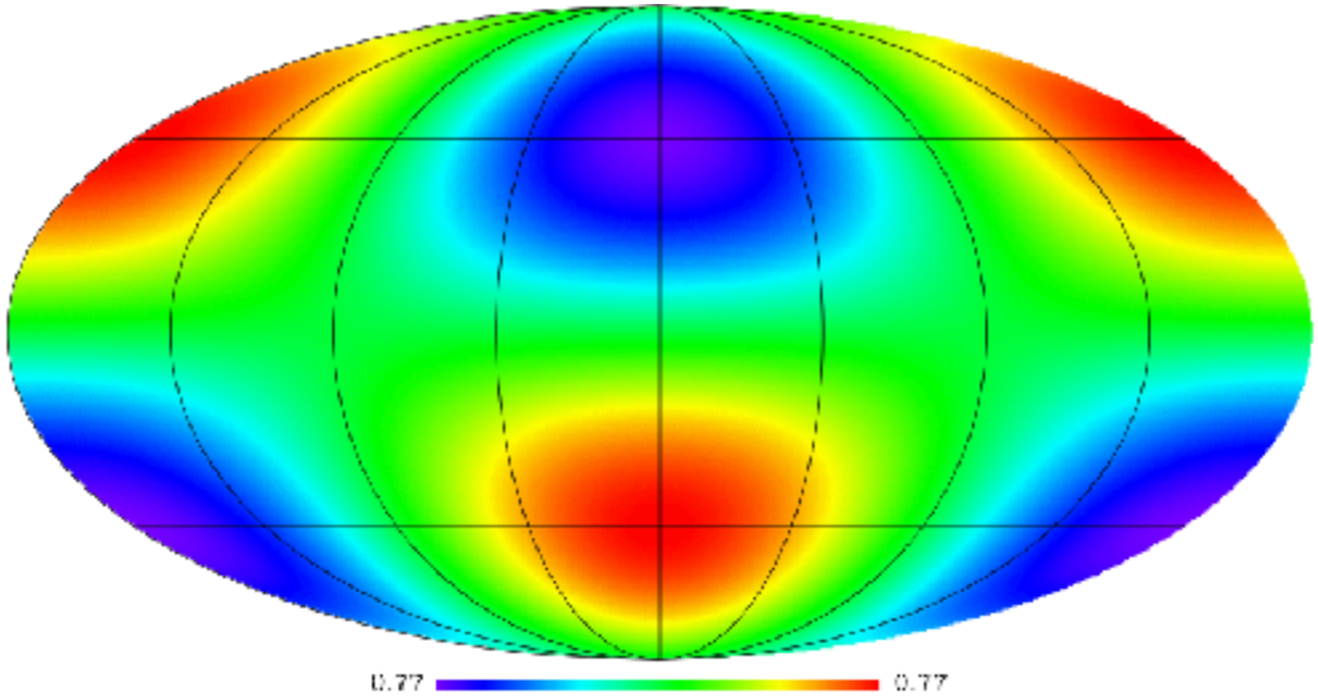
The Single Most Important Thing You Need to Remember

- Polarisation is generated by the local quadrupole temperature anisotropy, which is proportional to viscosity

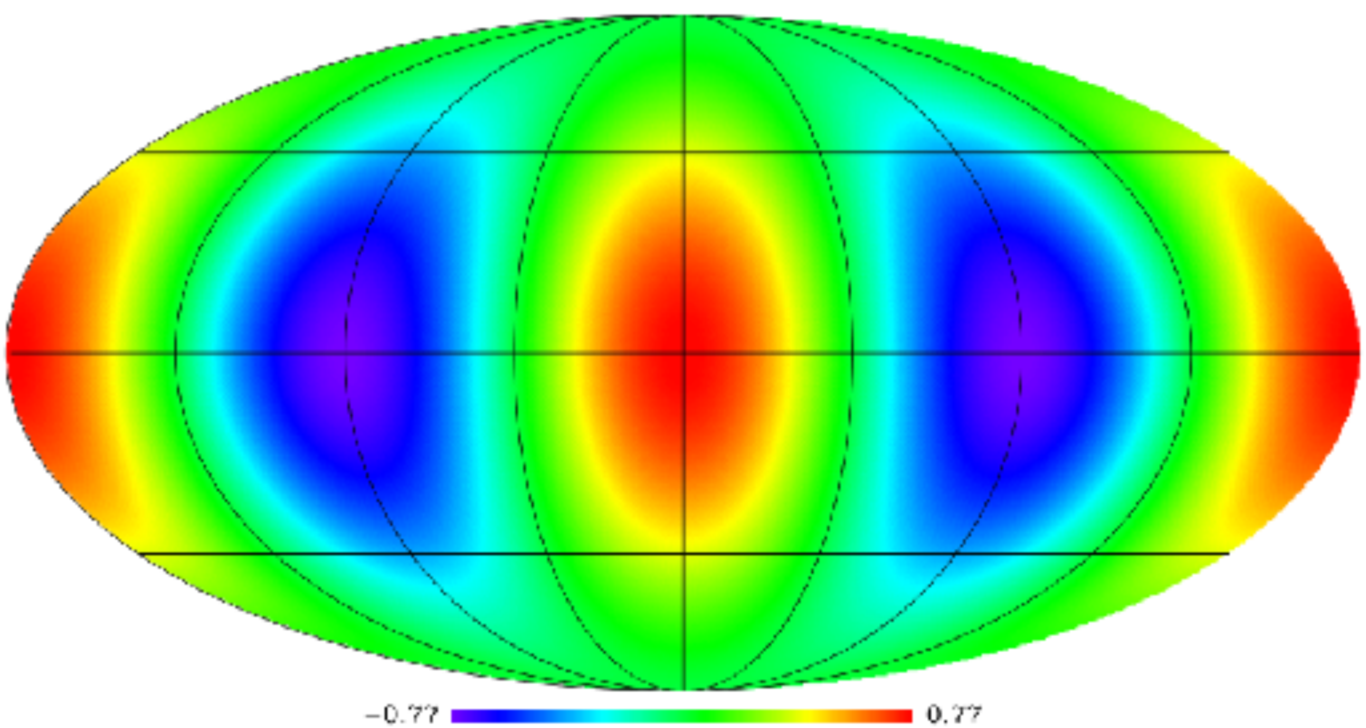
(l,m)=(2,0)



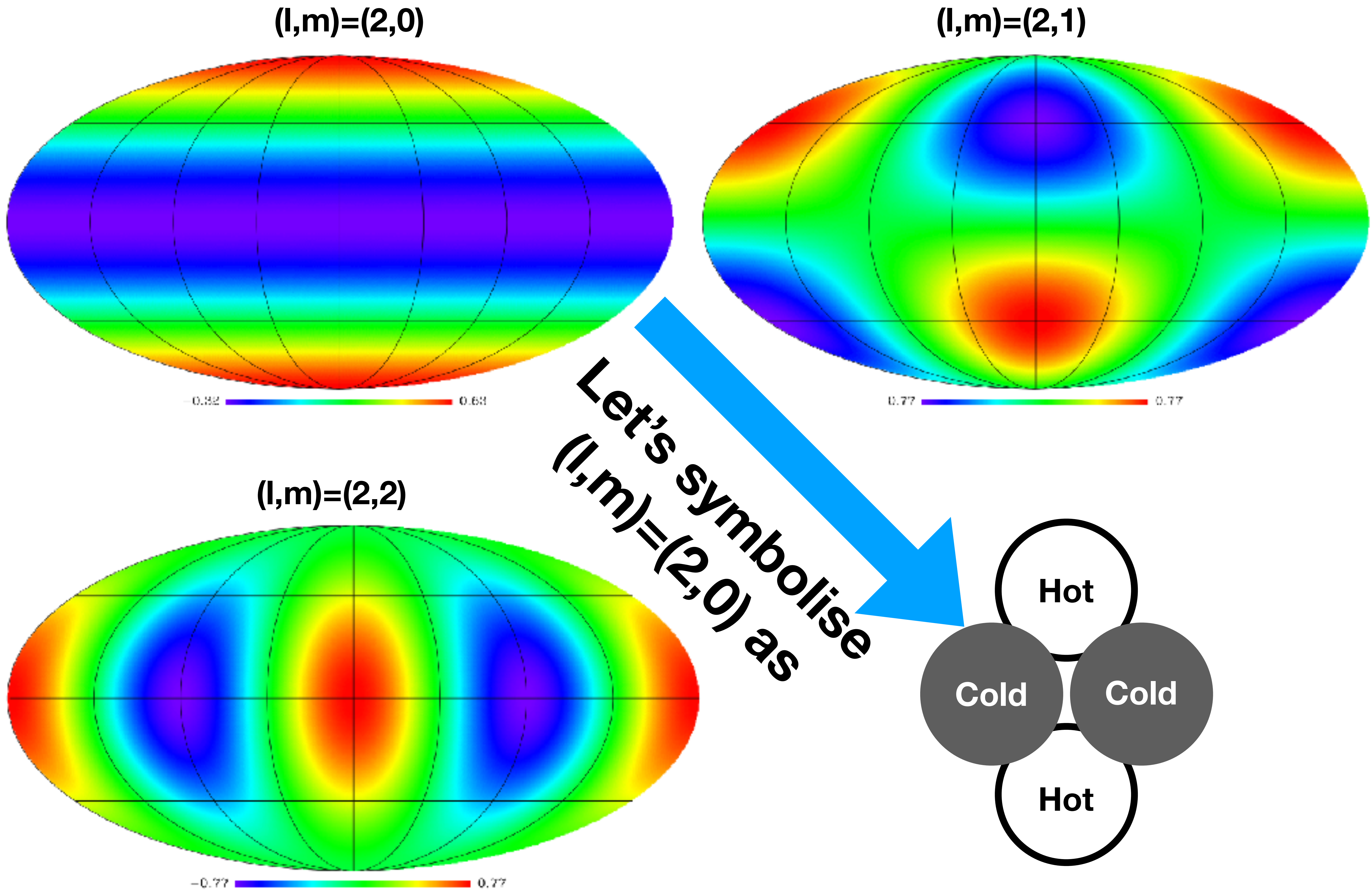
(l,m)=(2,1)



(l,m)=(2,2)

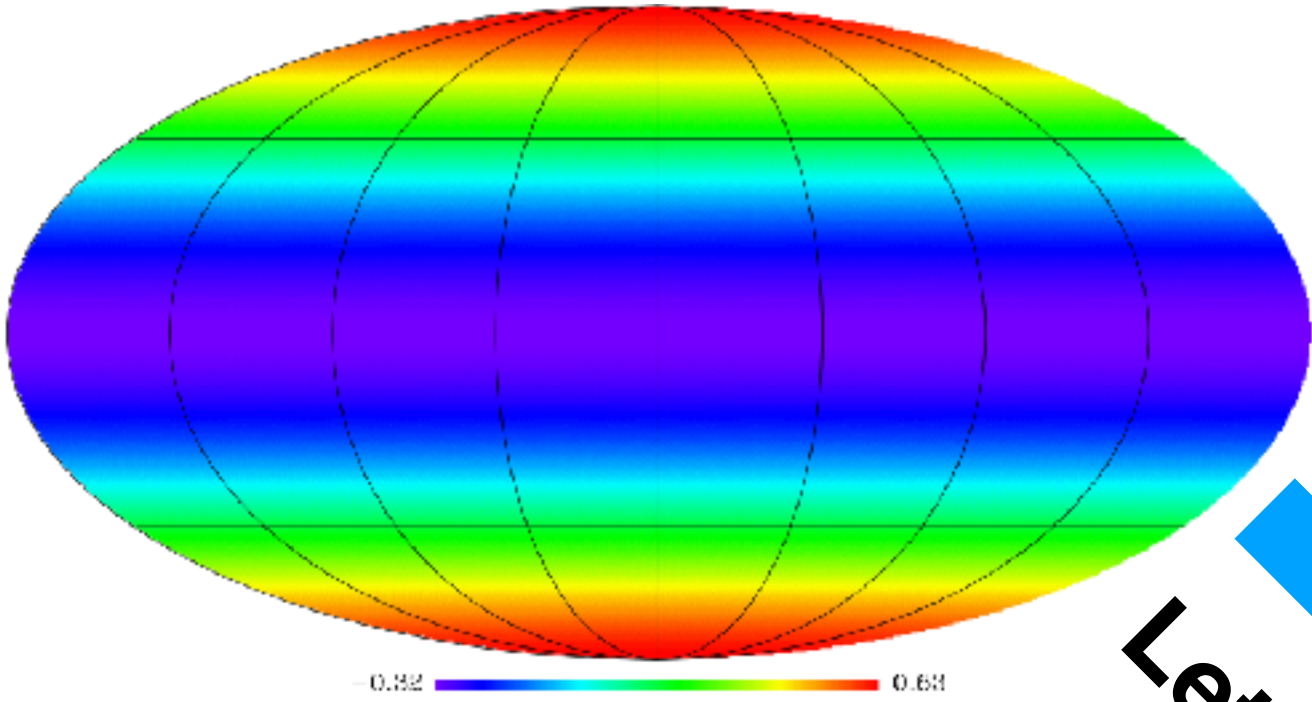


Local quadrupole temperature anisotropy seen from an electron

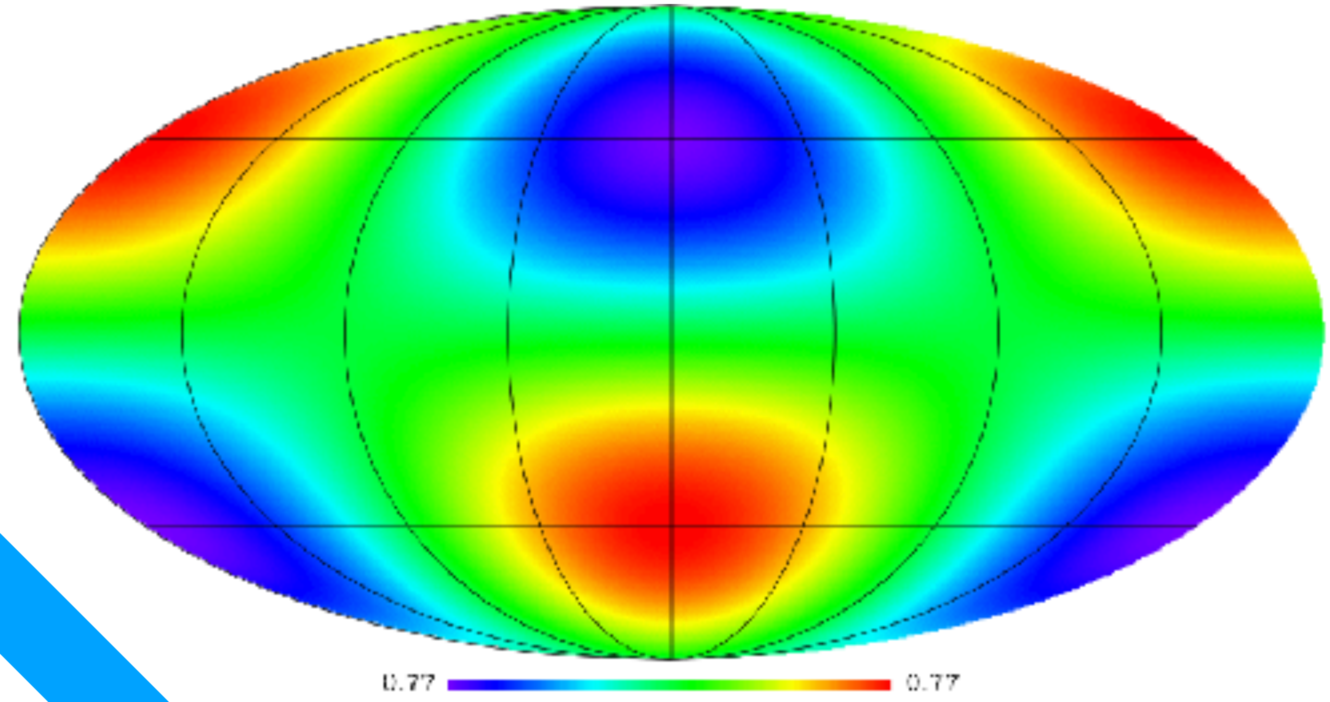


(l,m)=(2,0)
(l,m)=(2,1)
0.77
0.77
(l,m)=(2,2)
−0.77
0.77
Let's symbolise (l,m)=(2,0) as
Hot
Cold
Cold
Hot

(l,m)=(2,0)



(l,m)=(2,1)



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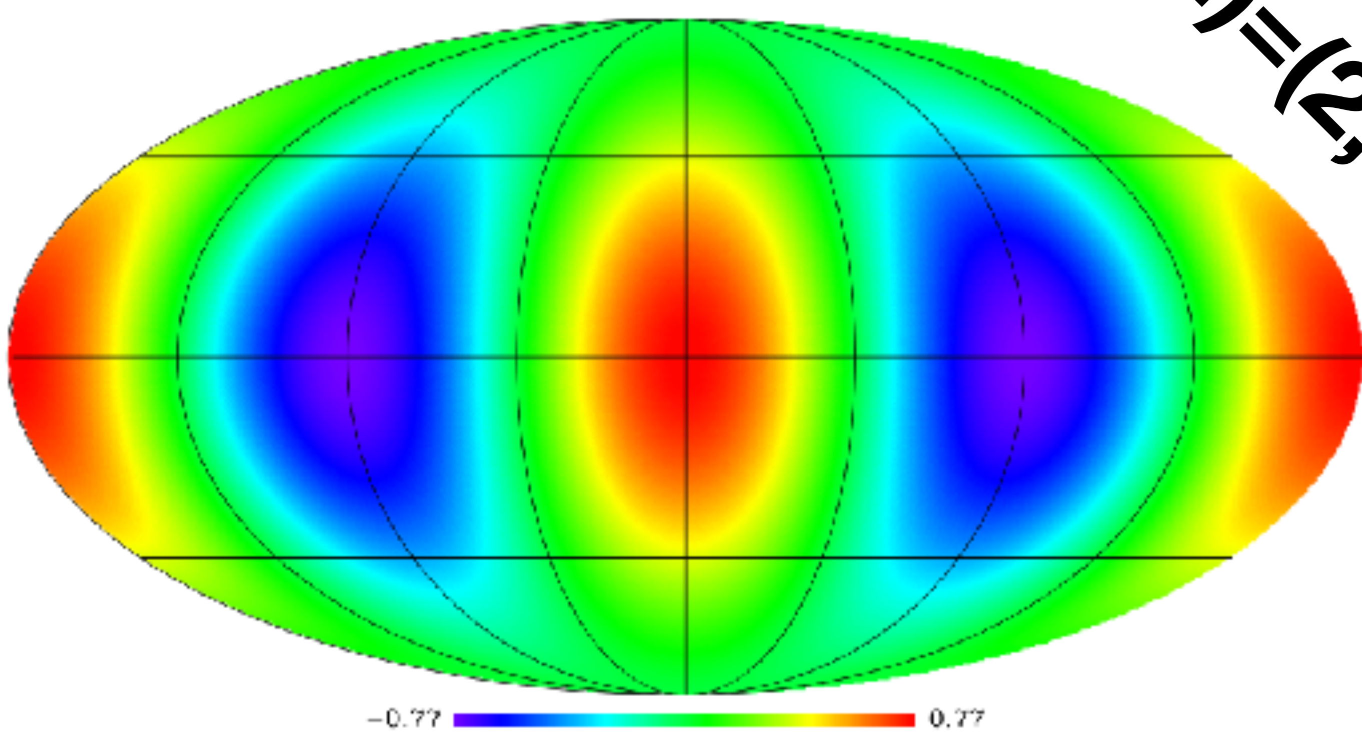

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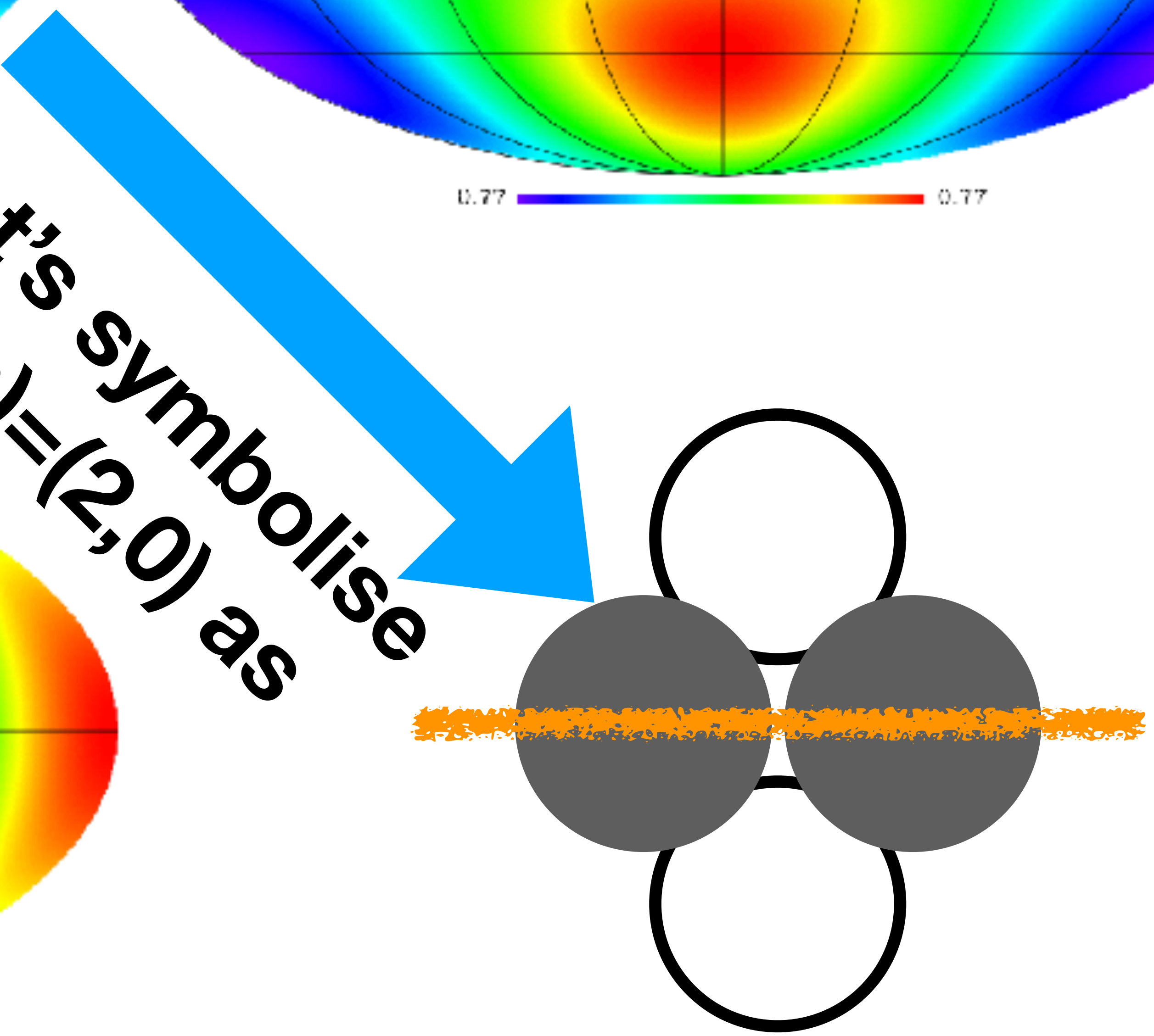
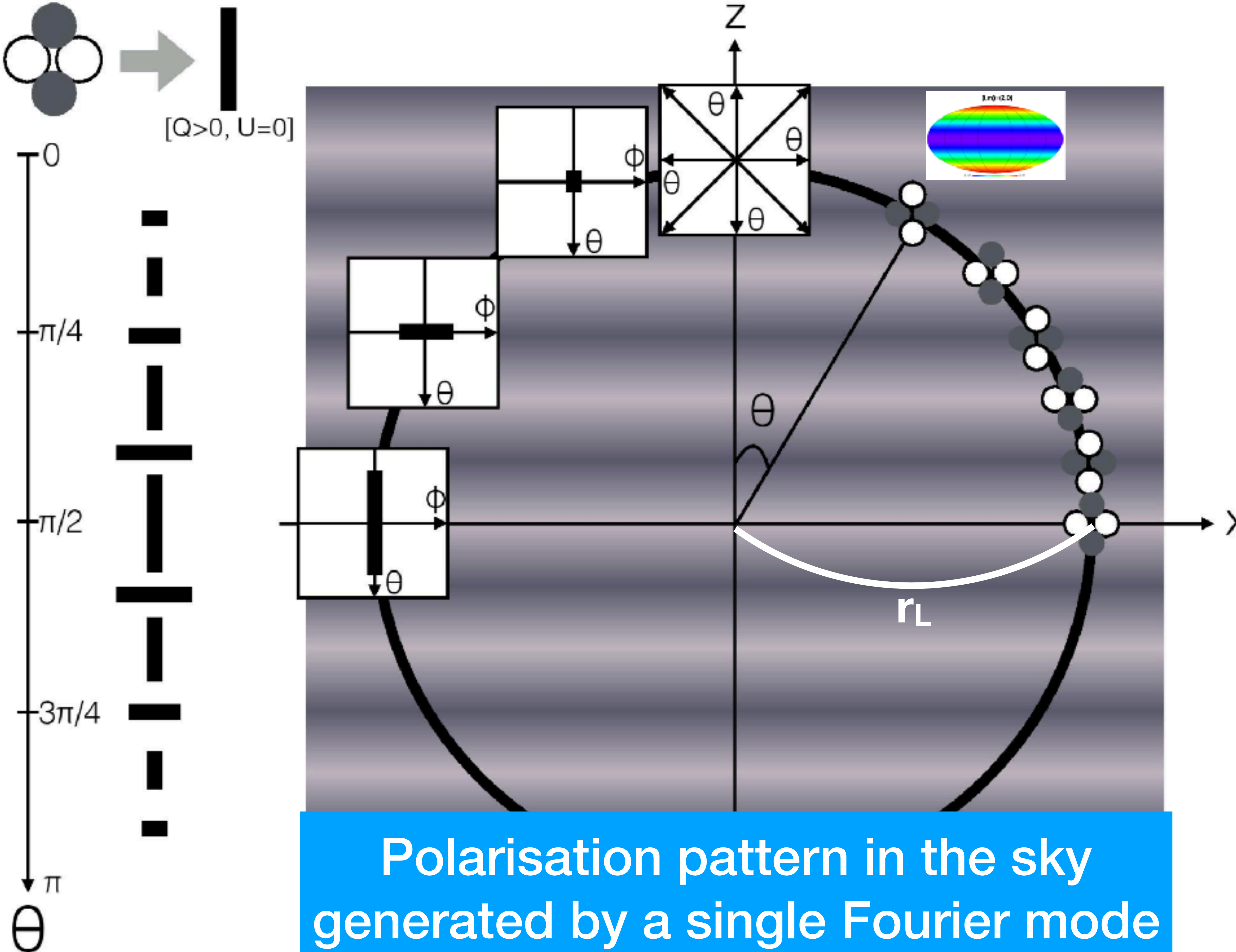
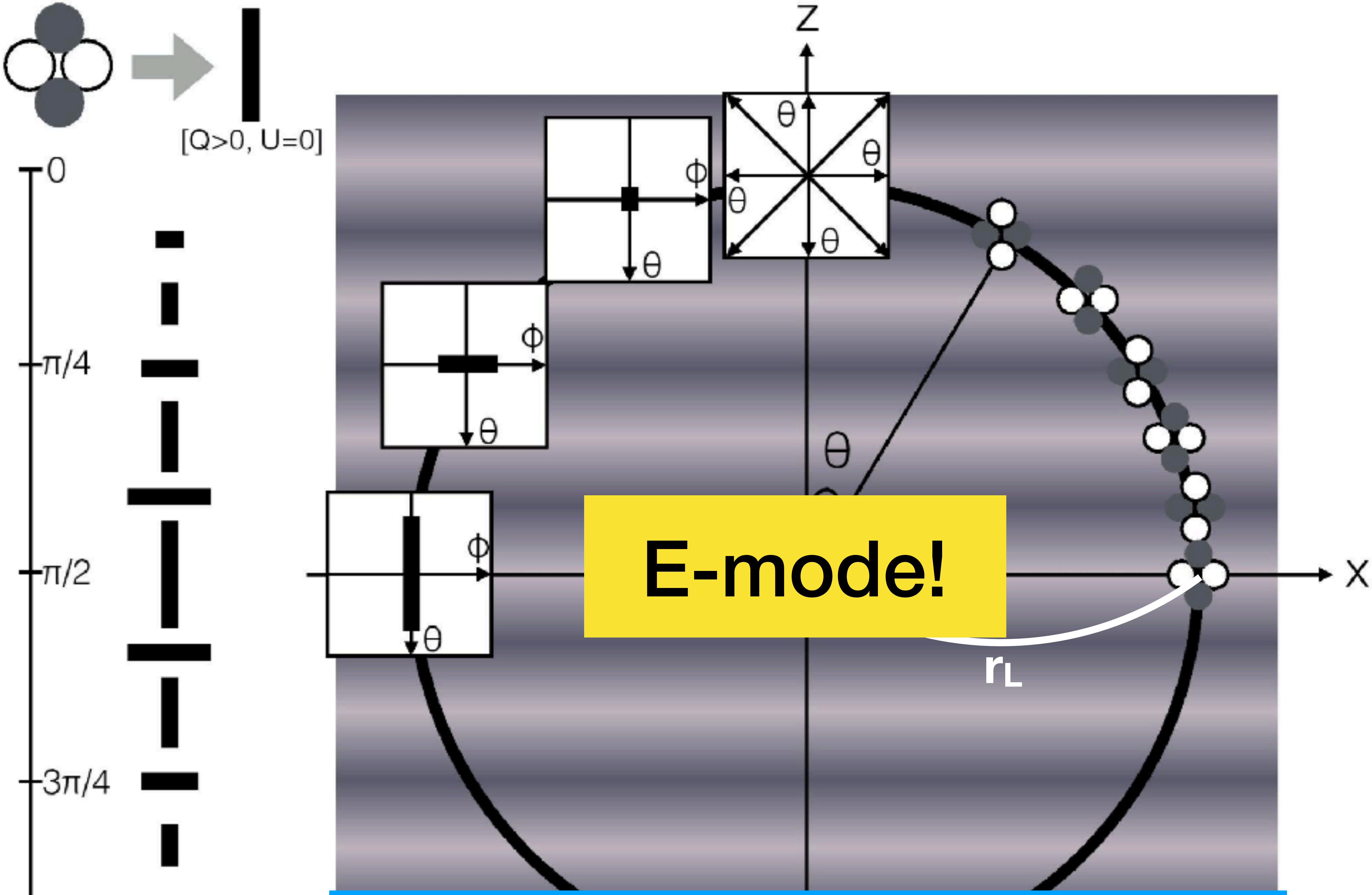


Polarisation pattern you will see



Polarisation pattern in the sky generated by a single Fourier mode



Polarisation pattern in the sky generated by a single Fourier mode

E-mode Power Spectrum

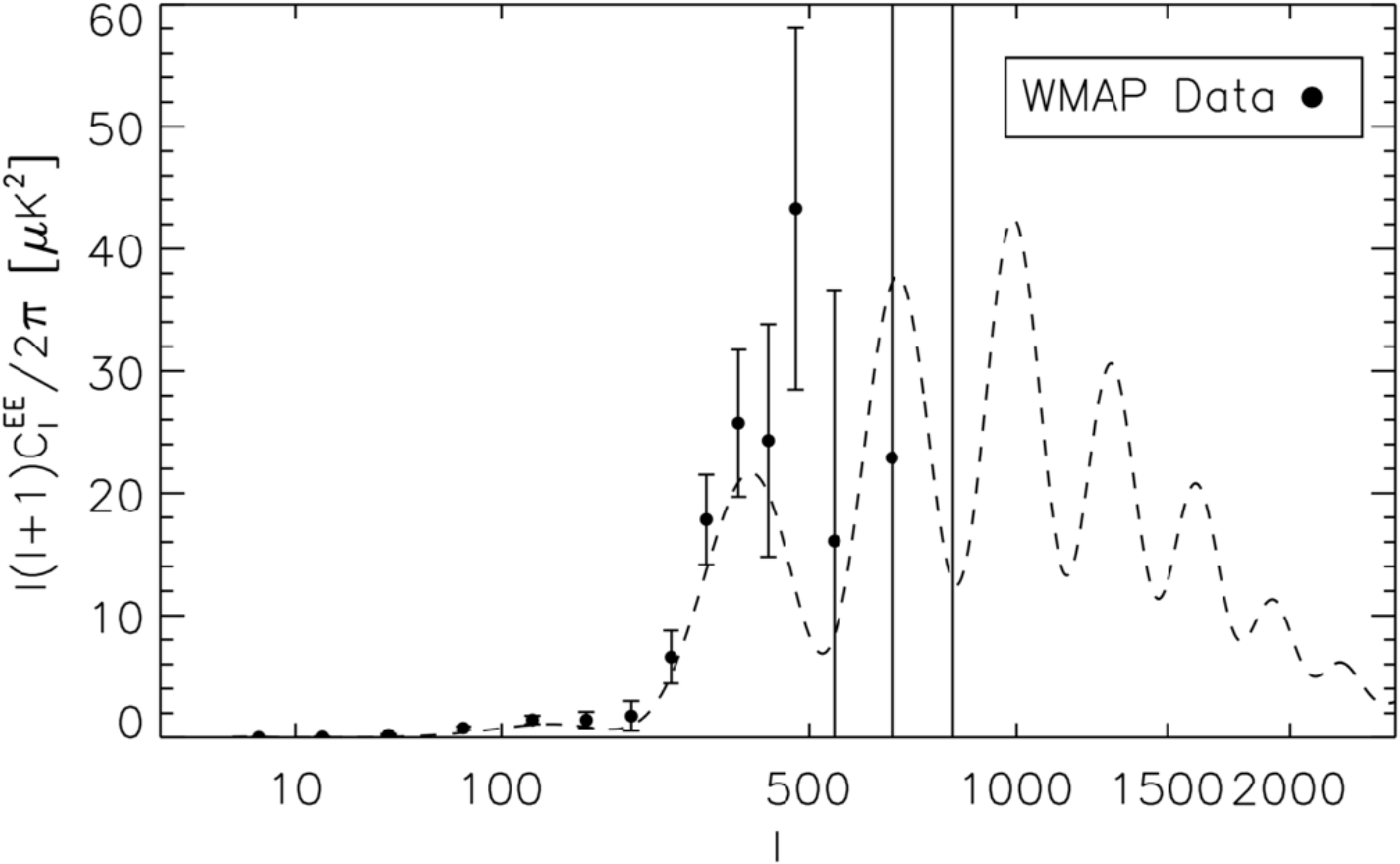
- Viscosity at the last-scattering surface is given by the velocity potential:

$$\pi_\gamma = -\frac{32}{45}\frac{\bar{\rho}_\gamma}{\sigma_T \bar{n}_e}\frac{\delta u_\gamma}{a^2}$$

- Velocity potential is $\mathrm{Sin}(qr_L)$, whereas the temperature power spectrum is predominantly $\mathrm{Cos}(qr_L)$

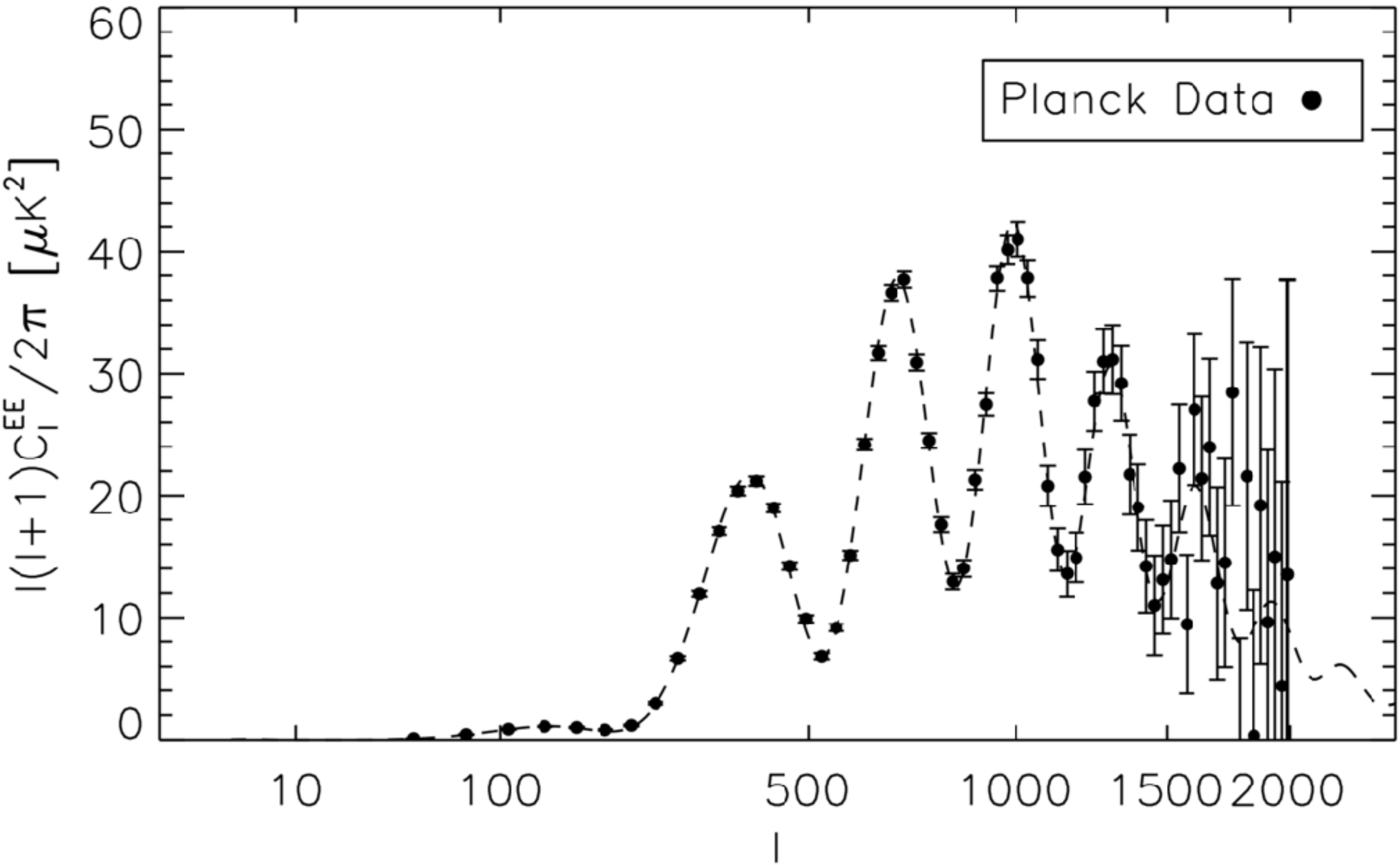
Bennett et al. (2013)

WMAP 9-year Power Spectrum

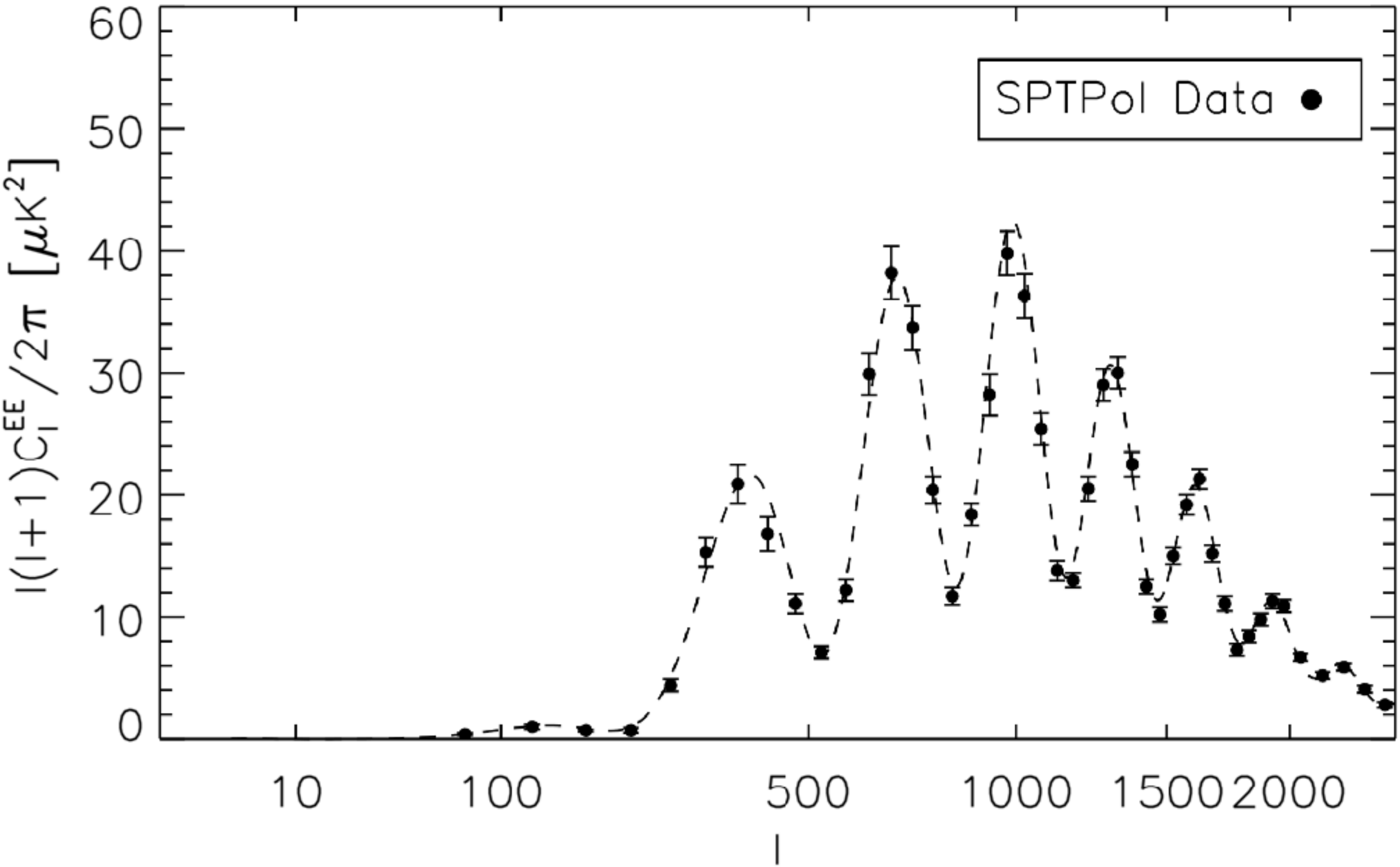


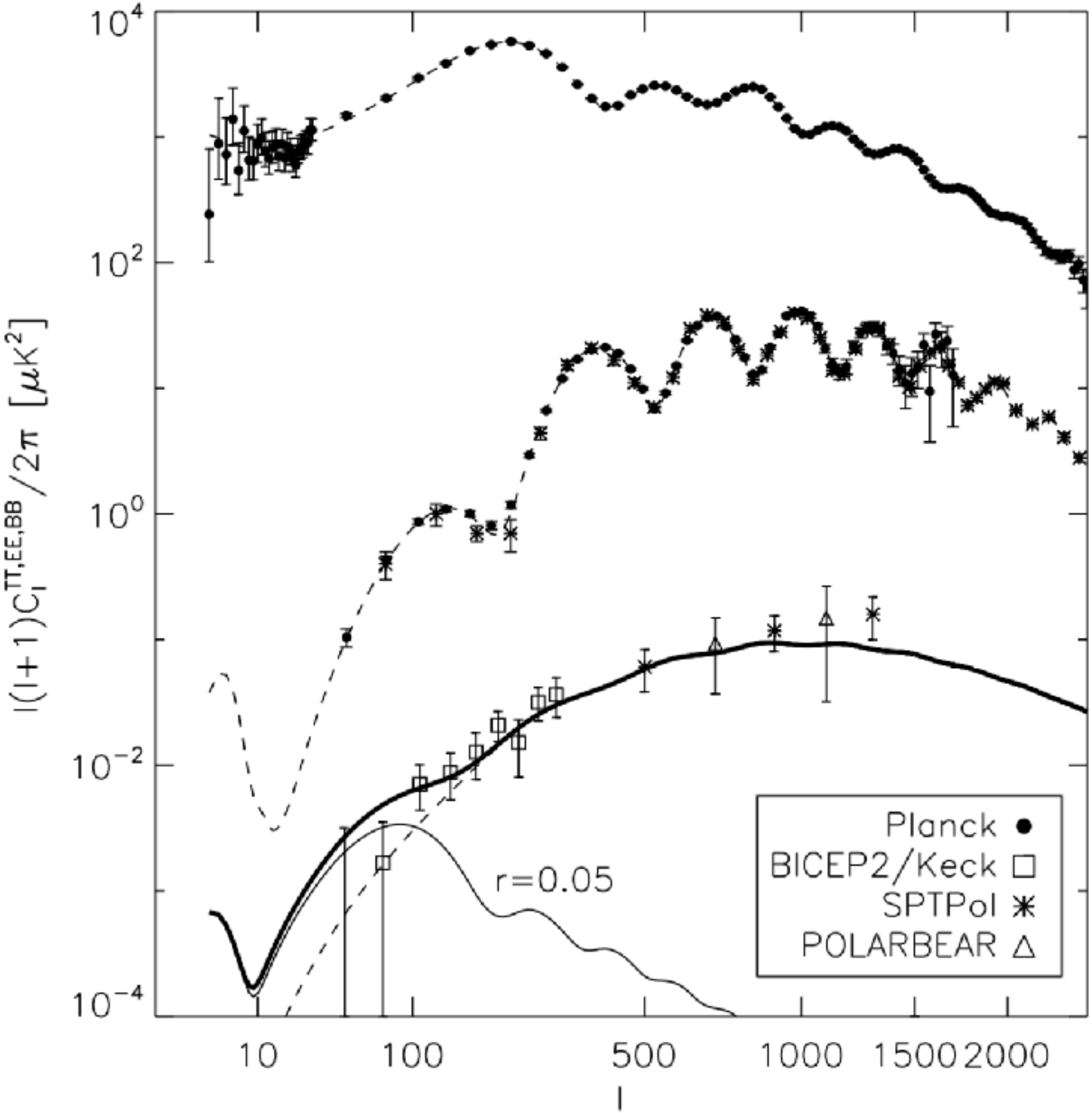
Planck Collaboration (2016)

Planck 29-mo Power Spectrum



SPTPol Power Spectrum





[1] Trough in T -> Peak in E

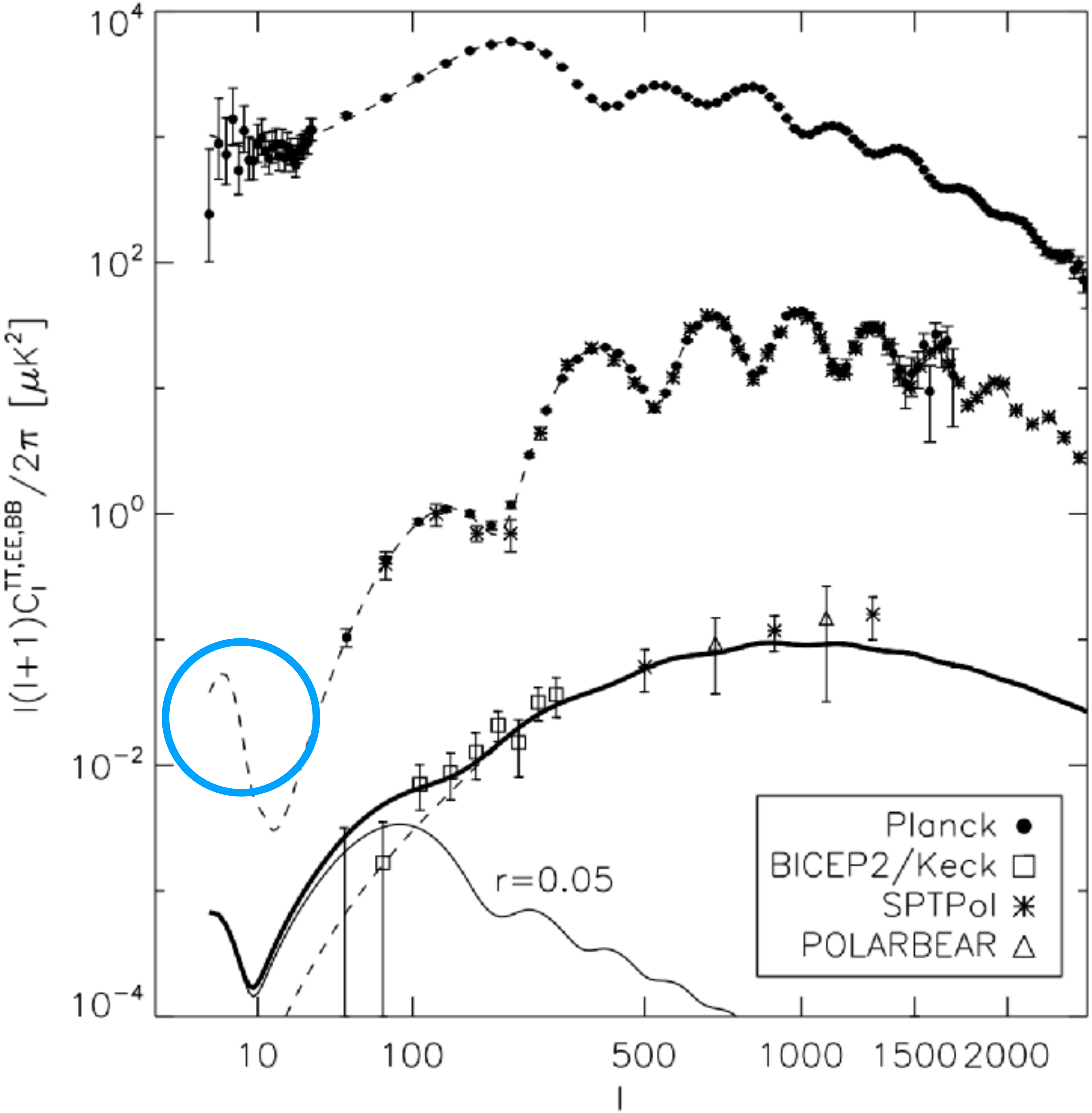
because $C_l^{TT} \sim \cos^2(qr_s)$ whereas $C_l^{EE} \sim \sin^2(qr_s)$

[2] T damps -> E rises

because T damps by viscosity, whereas E is created by viscosity

[3] E Peaks are sharper

because C_l^{TT} is the sum of $\cos^2(qr_L)$ and Doppler shift's $\sin^2(qr_L)$, whereas C_l^{EE} is just $\sin^2(qr_L)$



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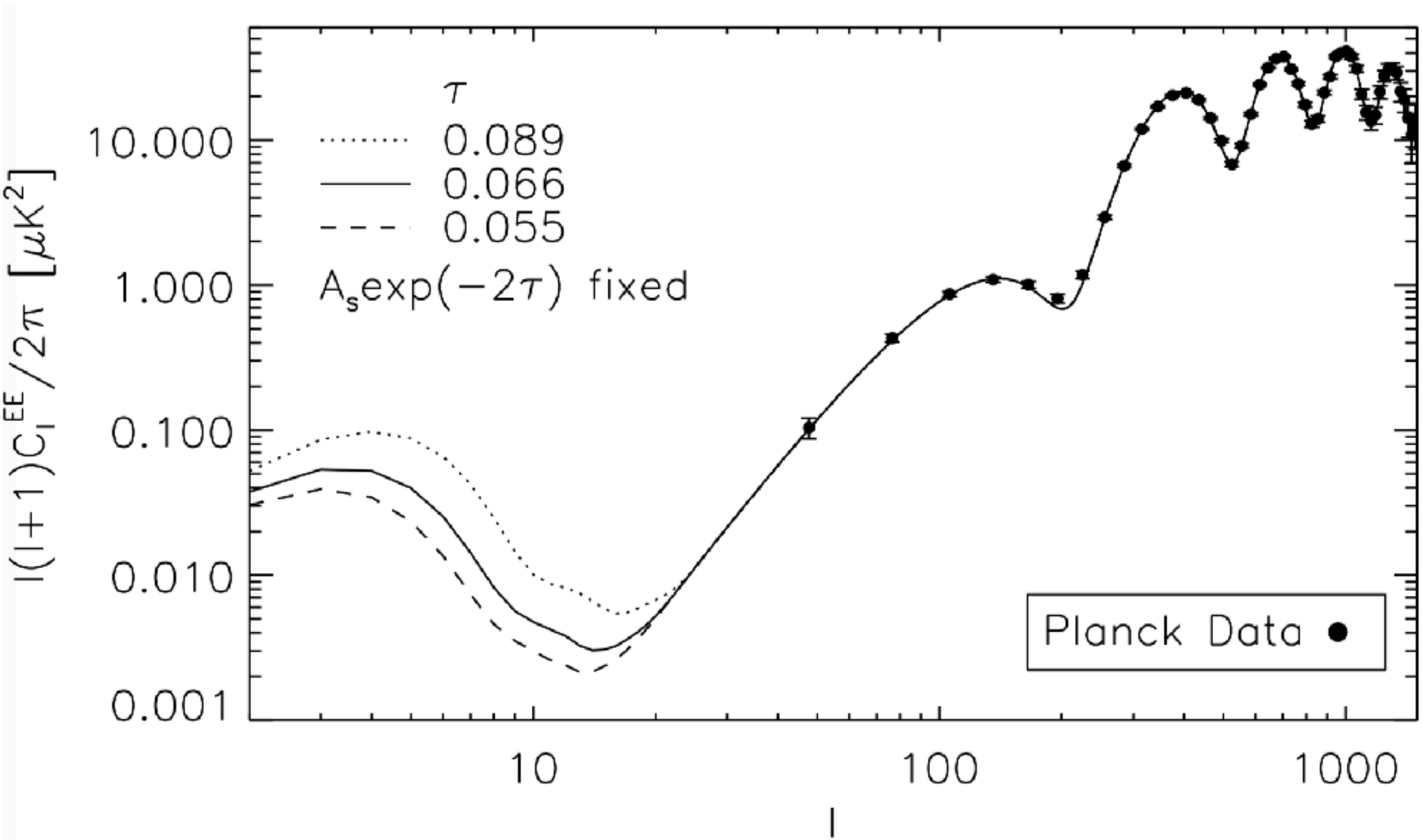
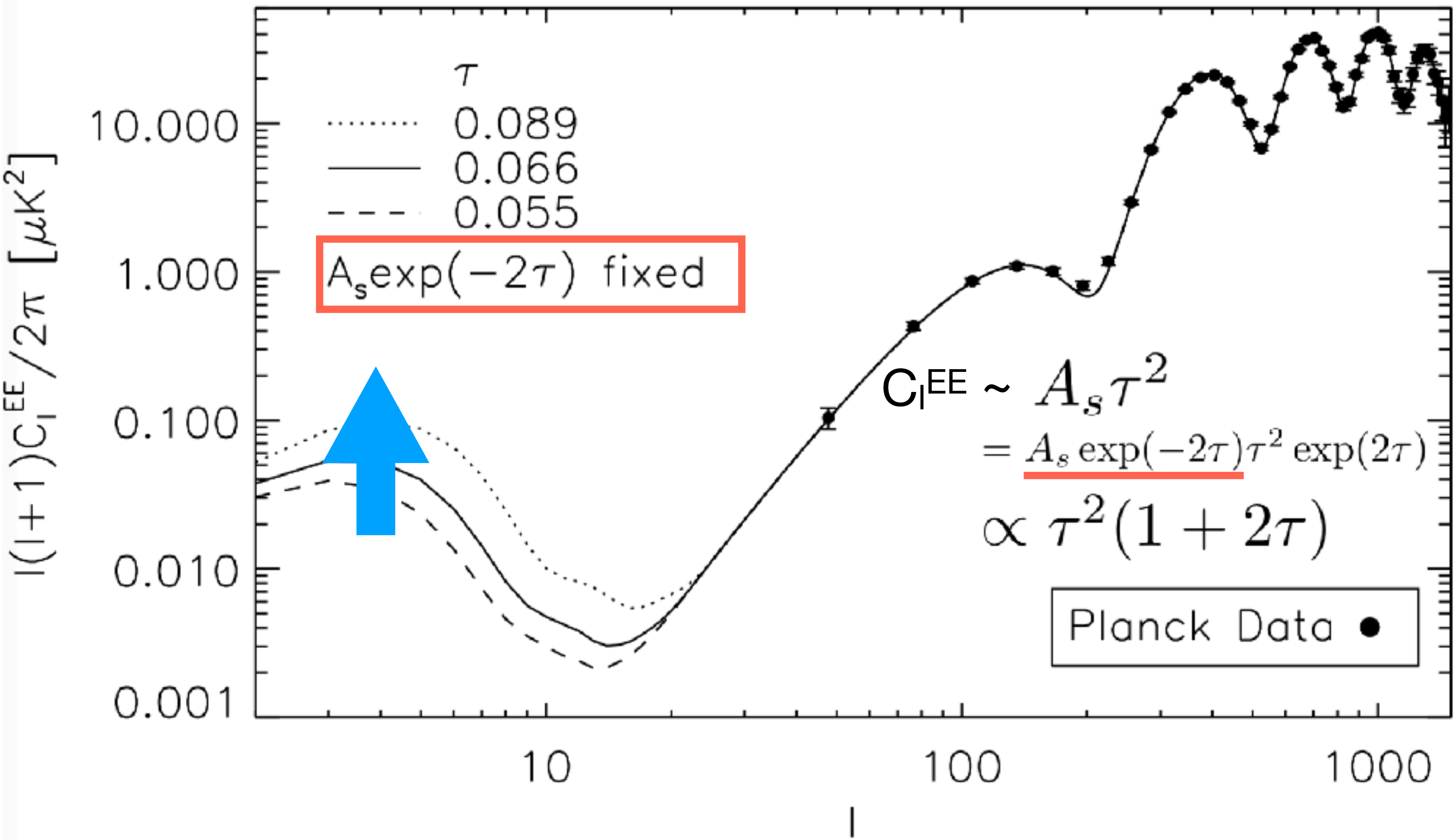
Polarisation from Re-ionisation



Polarisation from Re-ionisation

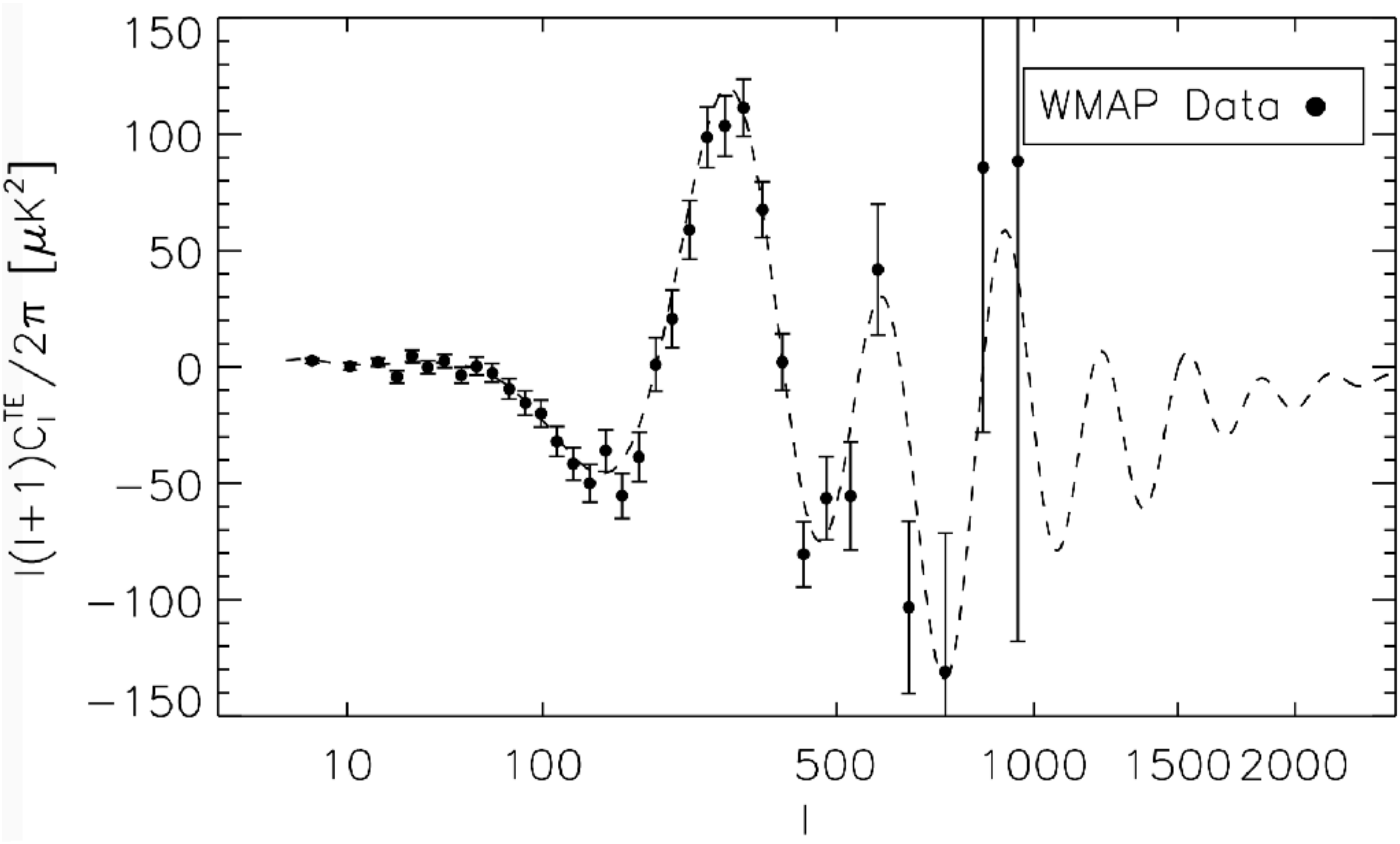


Cross-correlation between T and E

- Velocity potential is $Sin(qr_L)$, whereas the temperature power spectrum is predominantly $Cos(qr_L)$
- Thus, the TE correlation is $Sin(qr_L)Cos(qr_L)$ which can change sign

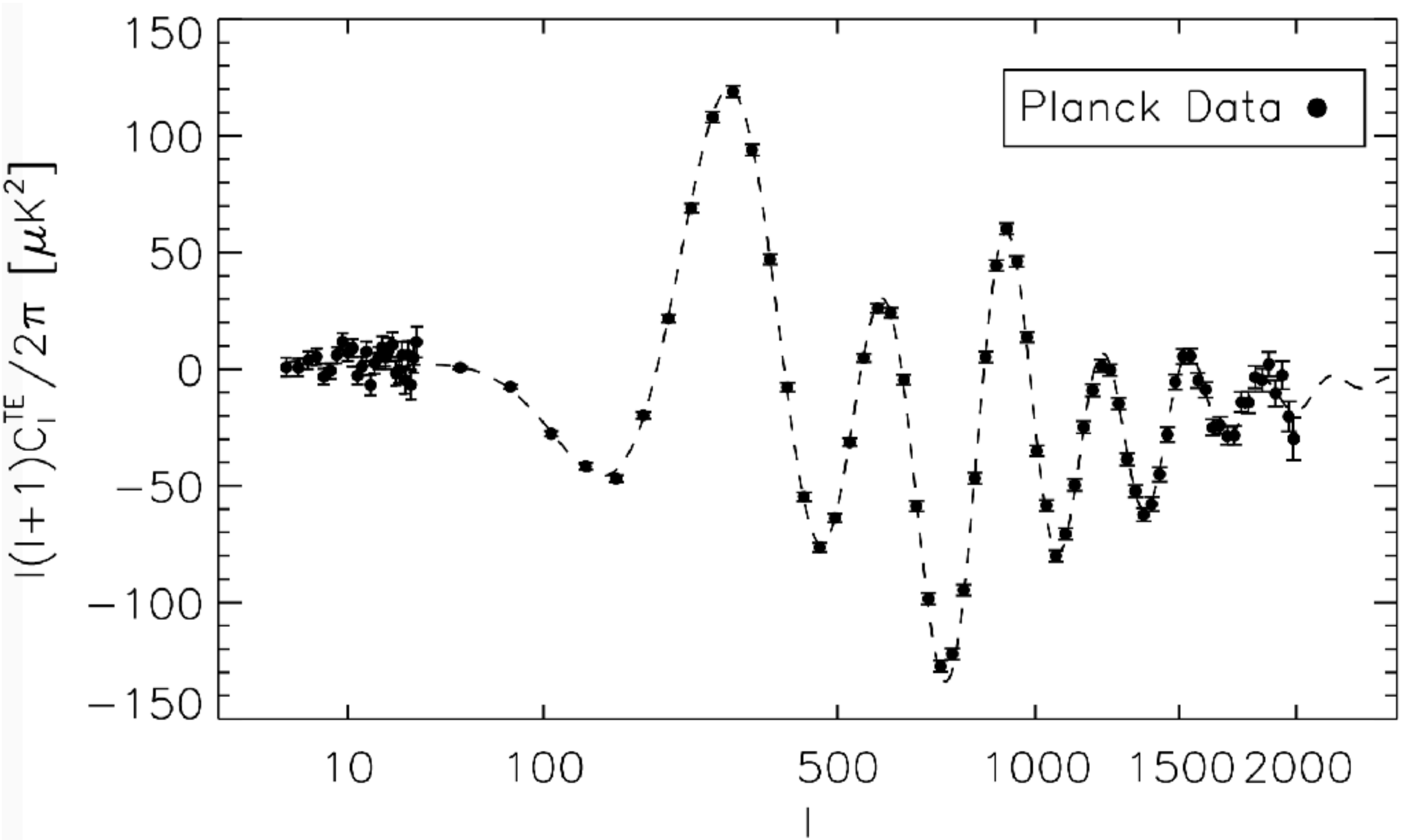
Bennett et al. (2013)

WMAP 9-year Power Spectrum



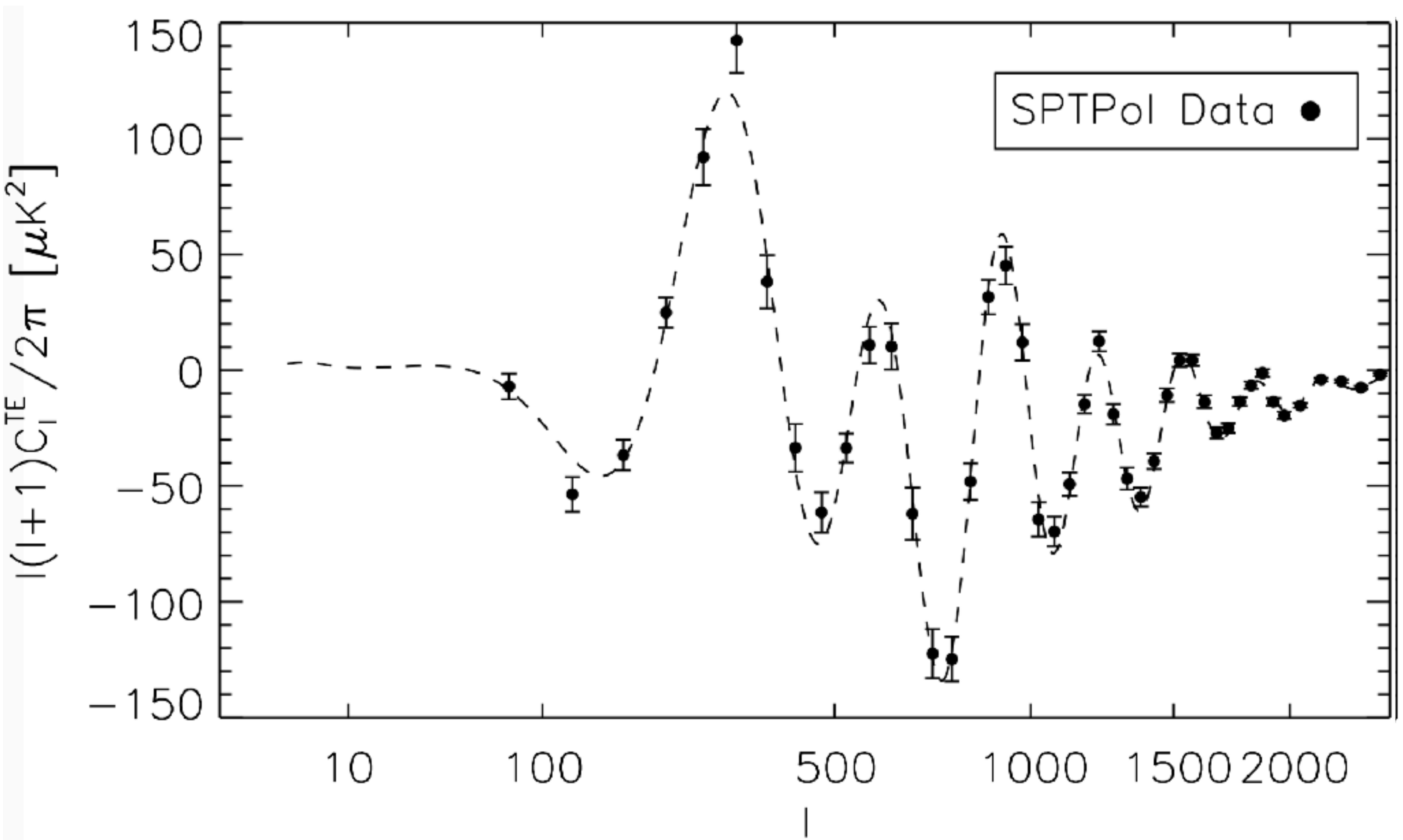
Planck Collaboration (2016)

Planck 29-mo Power Spectrum



South Pole Telescope Collaboration (2018)

SPTPol Power Spectrum



TE correlation is useful for understanding physics

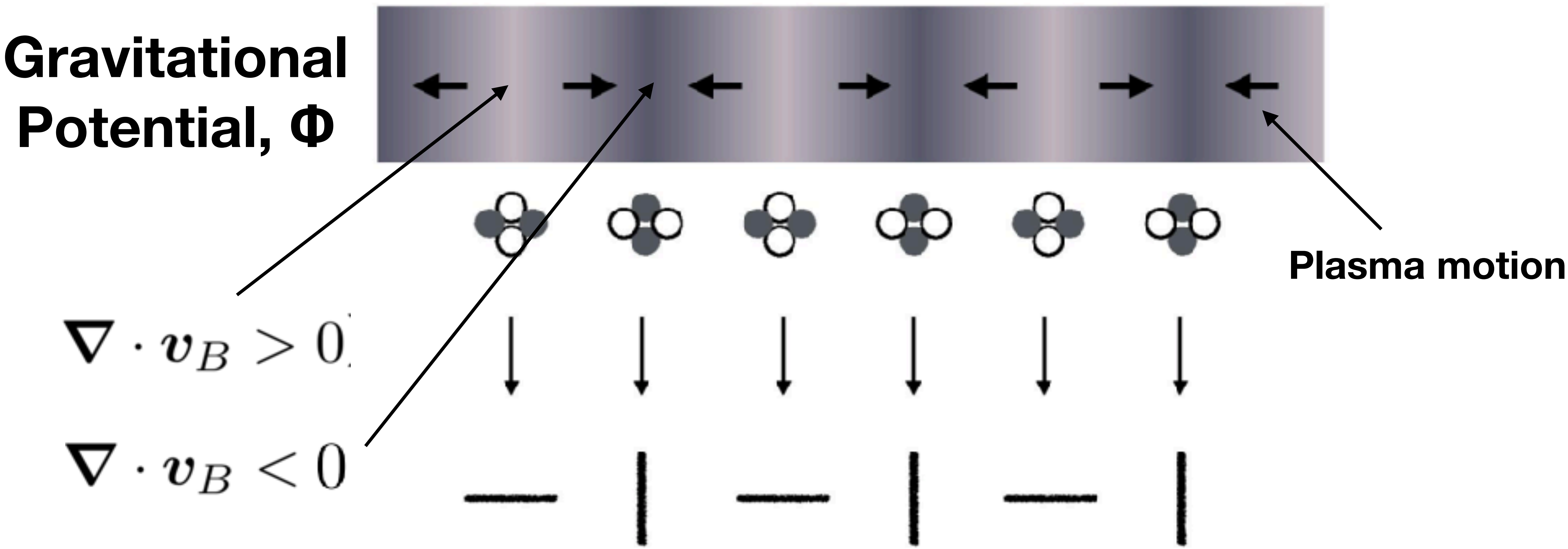
- T roughly traces gravitational potential, while E traces velocity

$$q^2\pi_\gamma \propto -q^2\delta u_\gamma \propto \boldsymbol{\nabla}\cdot\boldsymbol{v}_B$$

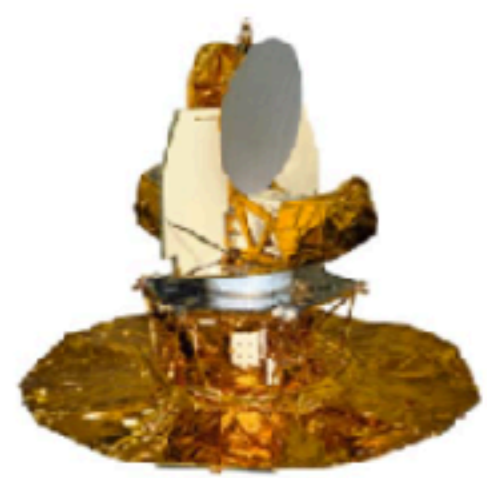
- With TE, we witness how plasma falls into gravitational potential wells!

Coulson et al. (1994)

Example: Gravitational Effects



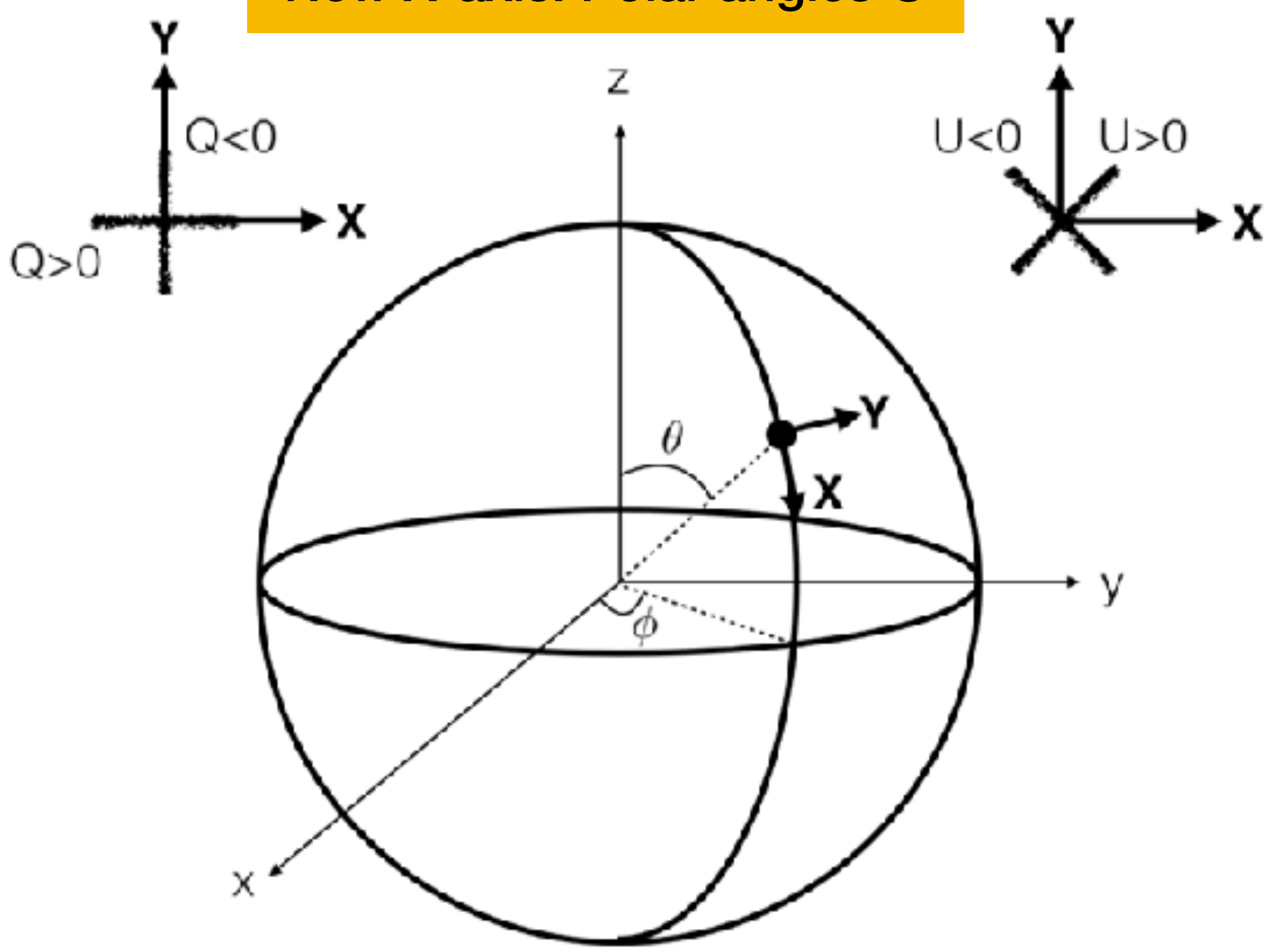
$$q^2 \pi_\gamma \propto -q^2 \delta u_\gamma \propto \nabla \cdot \boldsymbol{v}_B$$

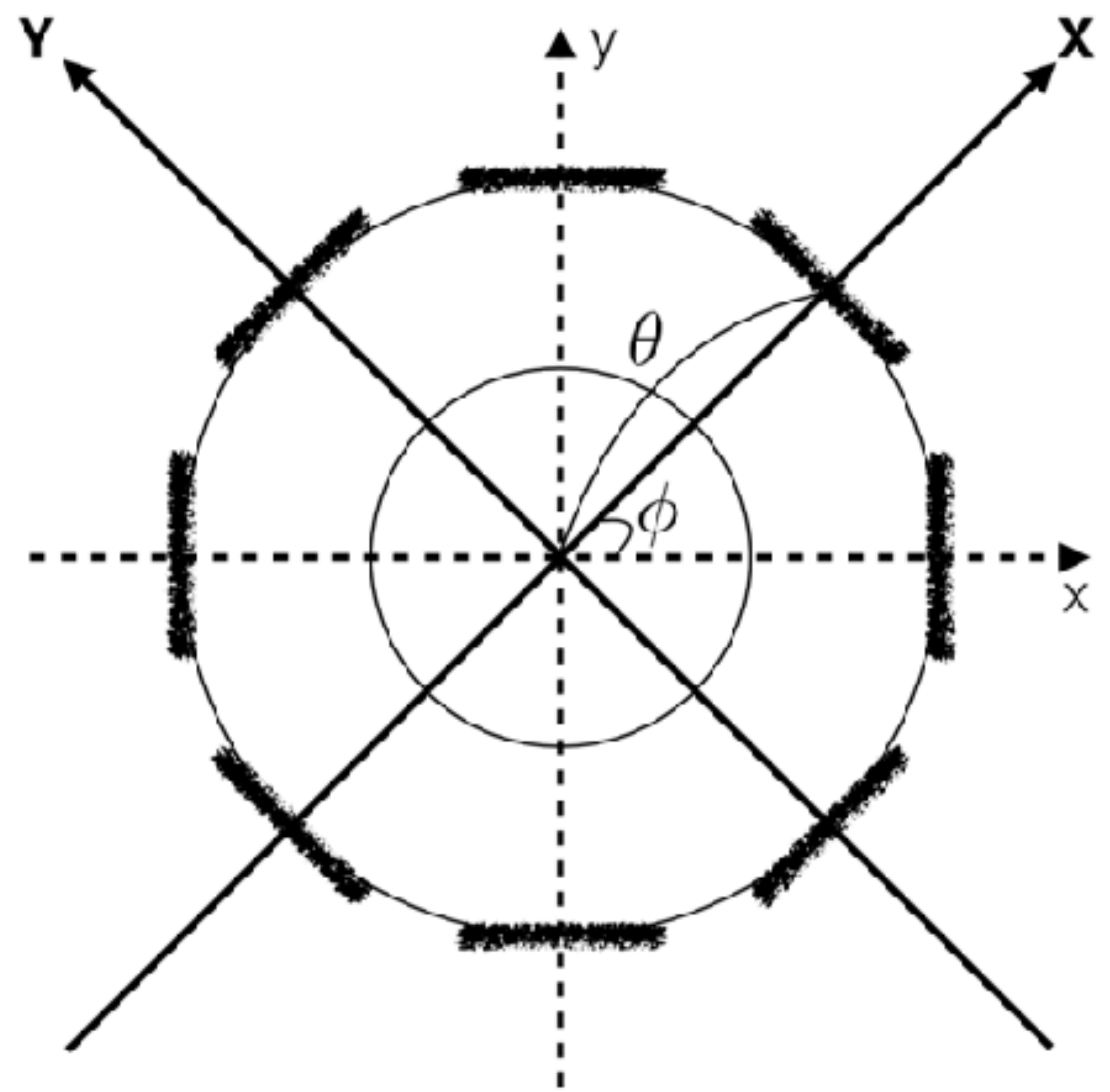


TE correlation in angular space

First, let's define Stokes parameters in sphere

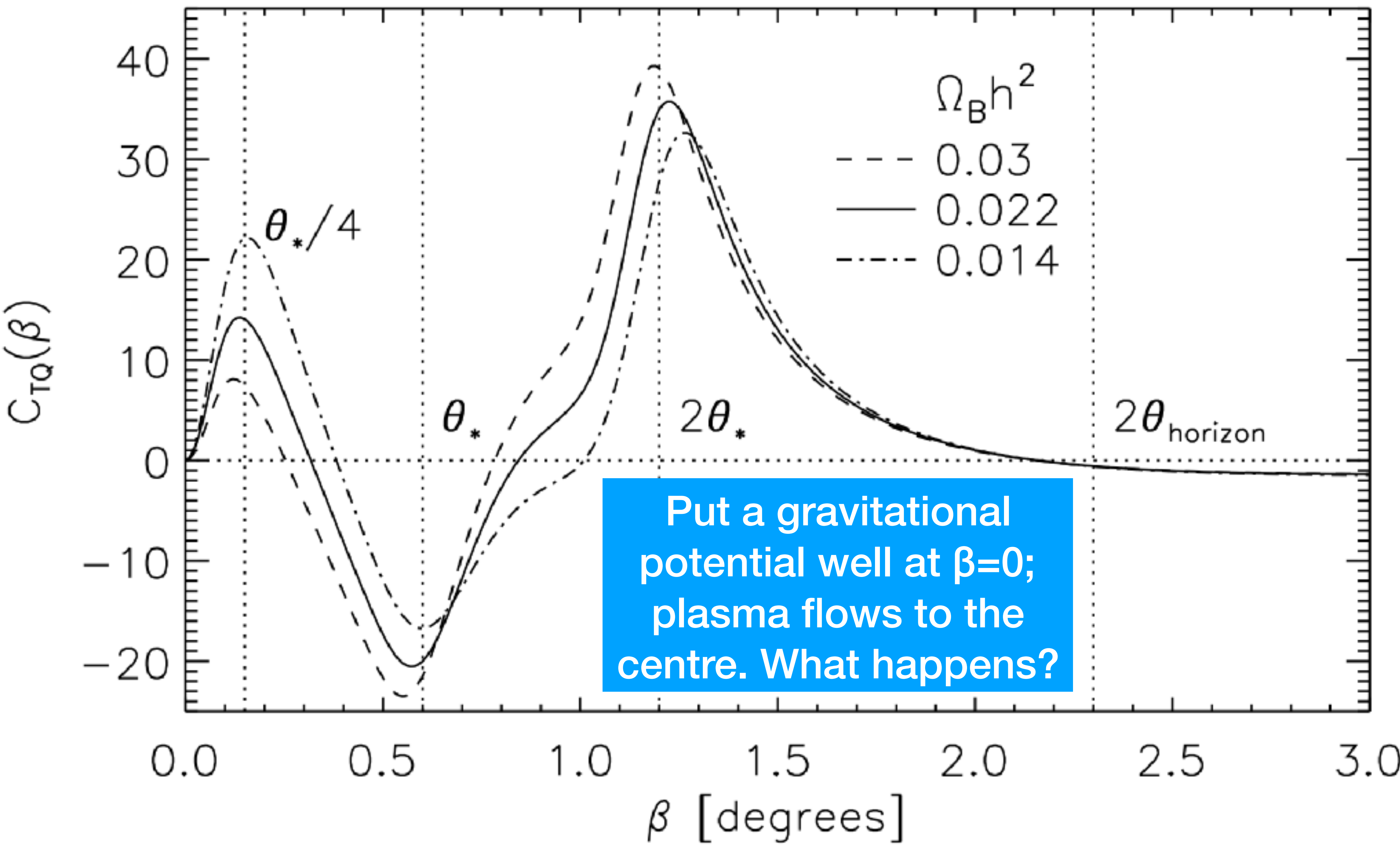
New X-axis: Polar angles θ





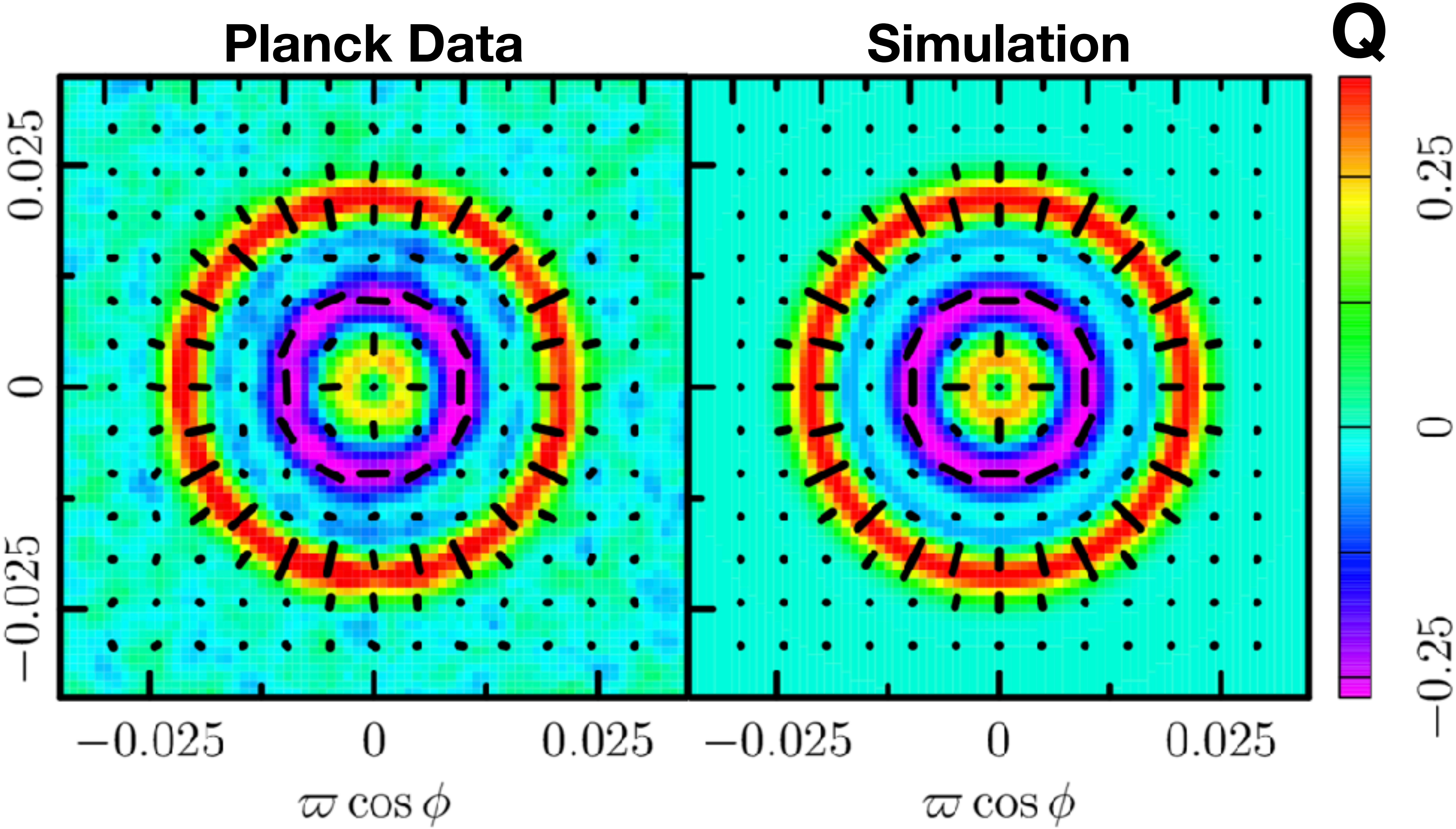
In this example, they are all Q<0

TE correlation in angular space



Komatsu et al. (2011); Planck Collaboration (2016)

Average Q polarisation around temperature hot spots

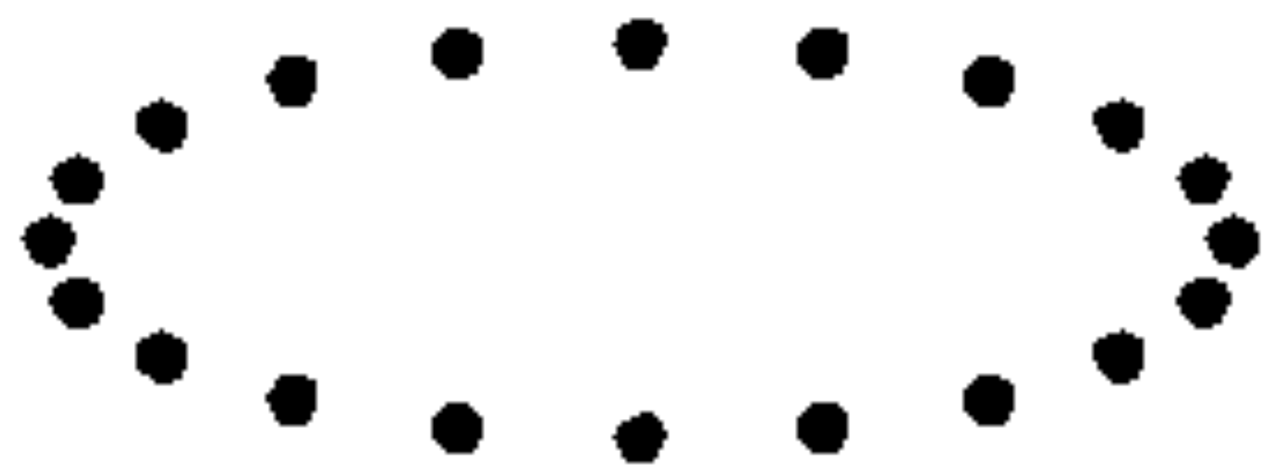


Gravitational Waves

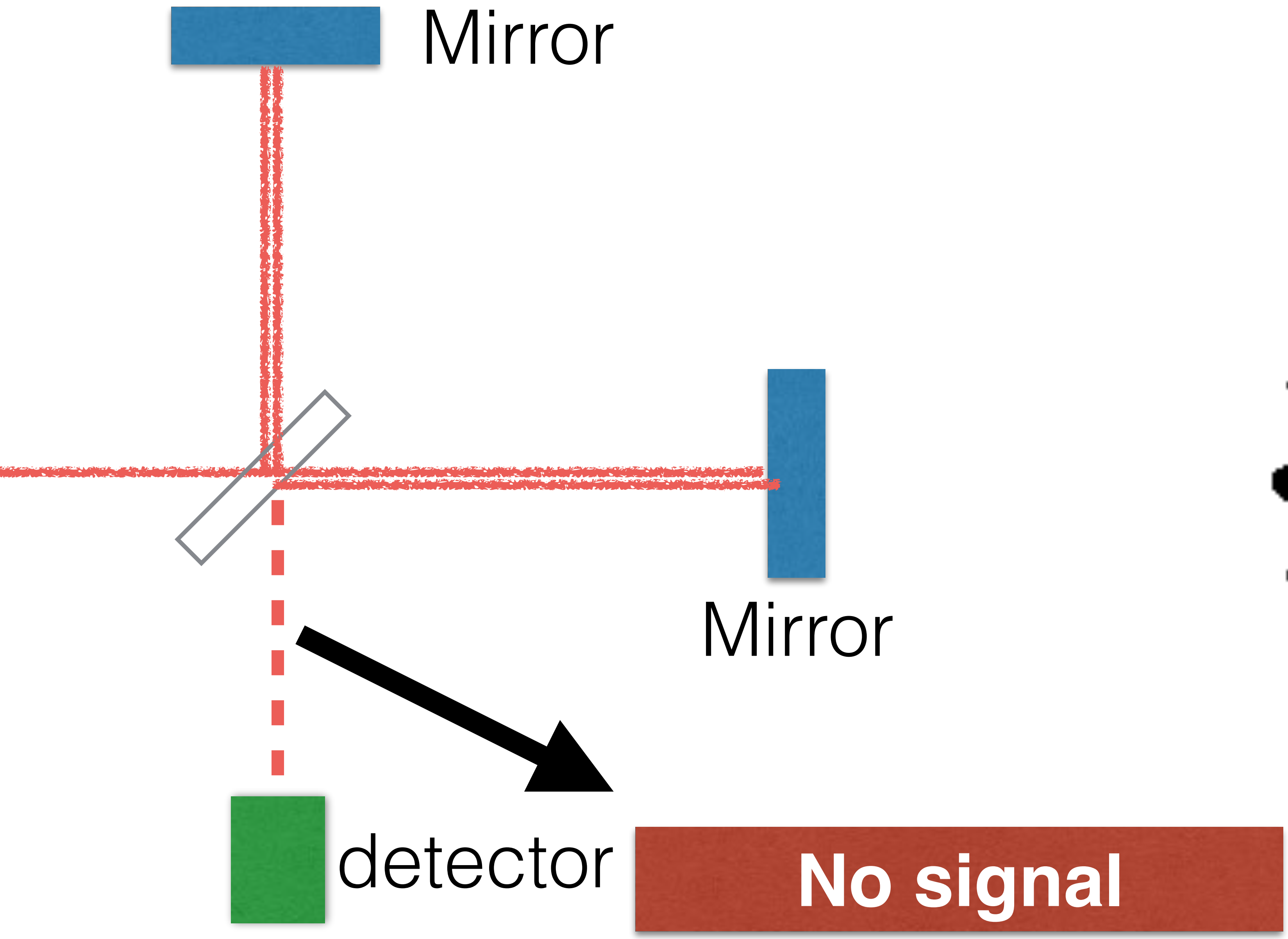
- GW changes the distances between two points

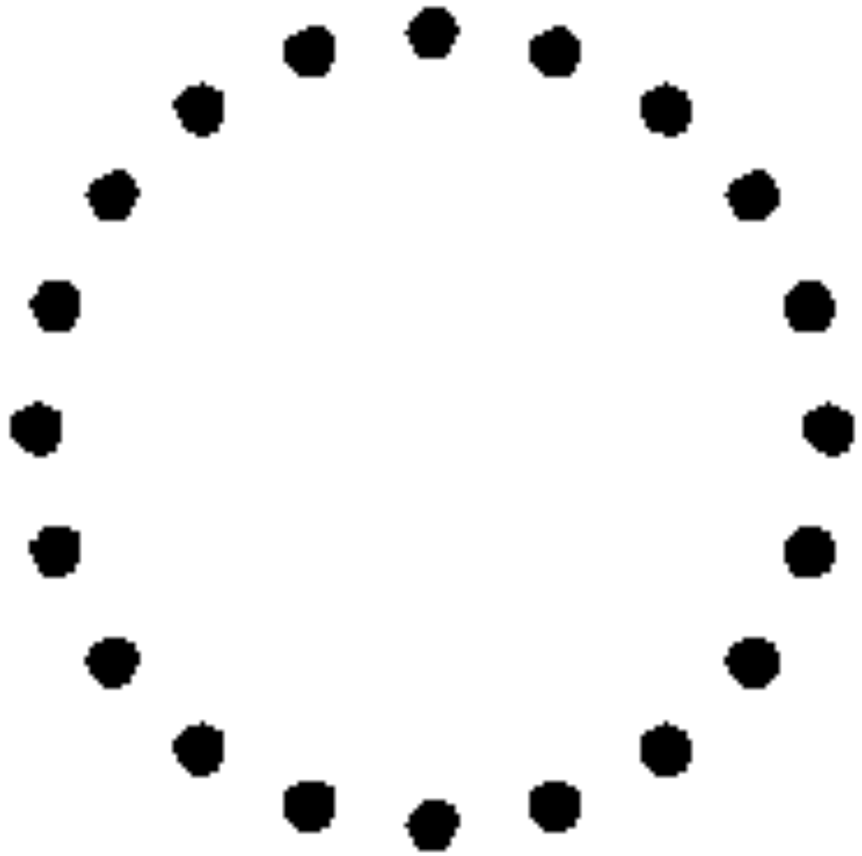
$$d\ell^2 = d\mathbf{x}^2 = \sum_{ij} \delta_{ij} dx^i dx^j$$

$$d\ell^2 = \sum_{ij} (\delta_{ij} + D_{ij}) dx^i dx^j$$

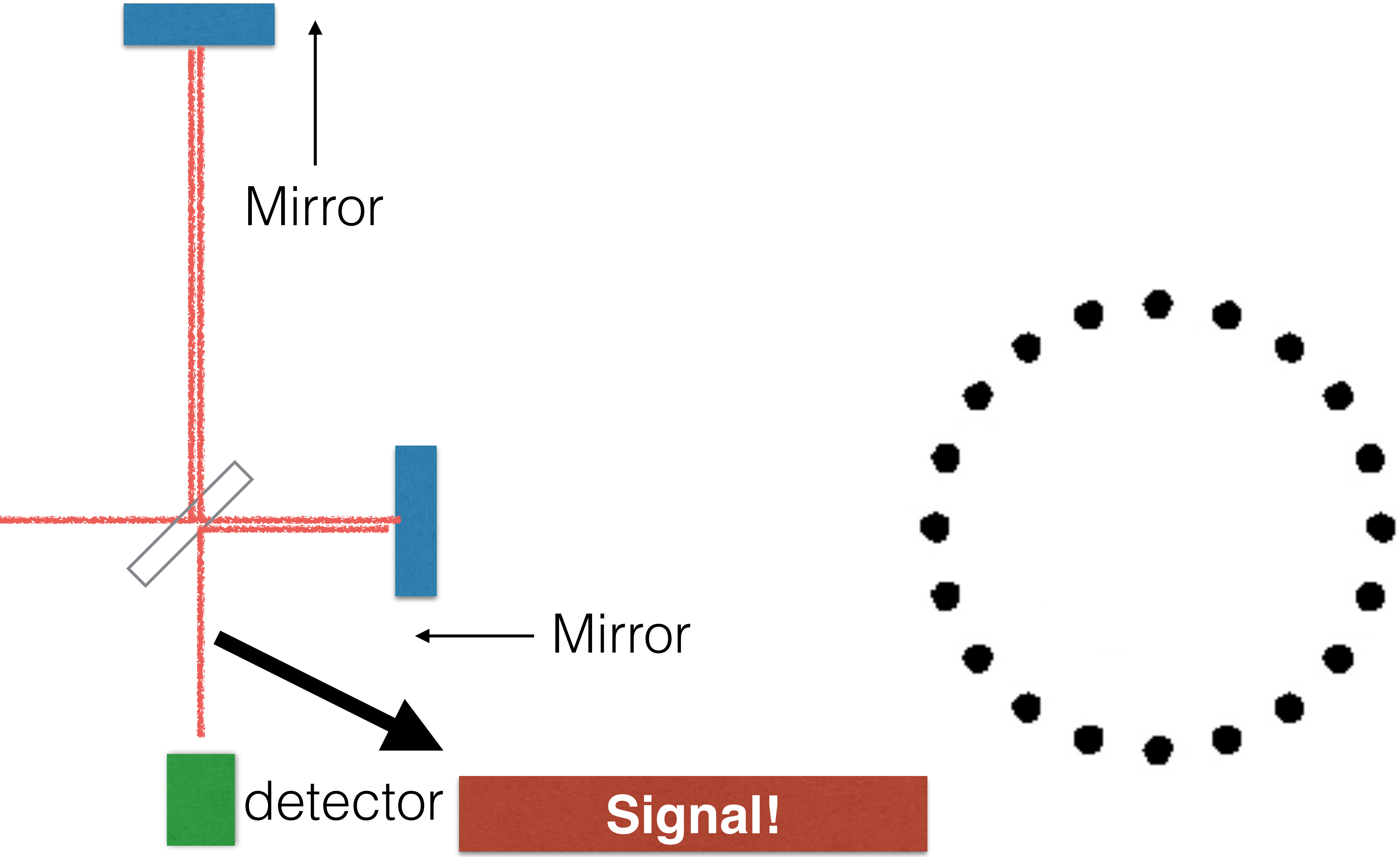


Laser Interferometer

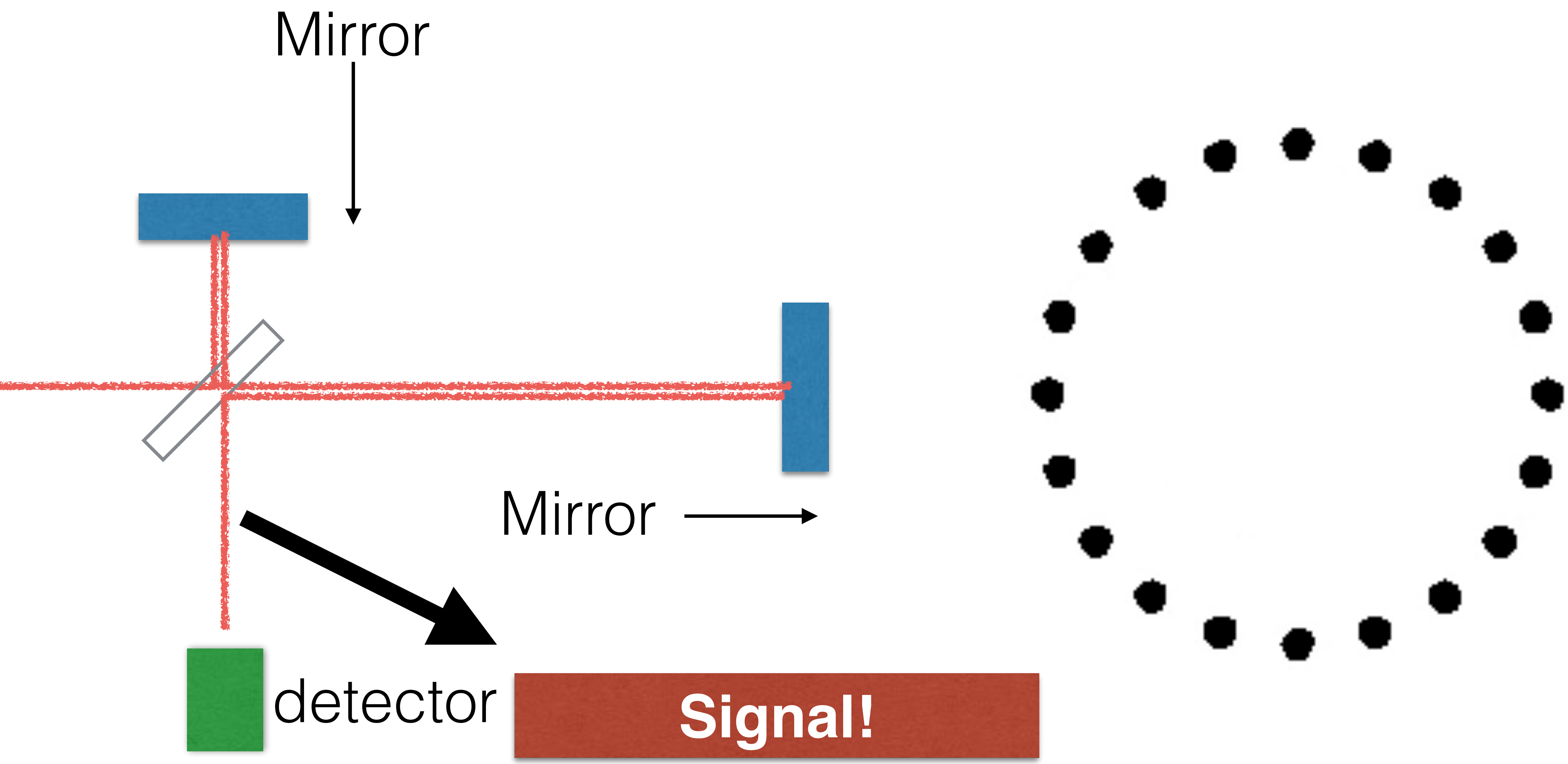




Laser Interferometer



Laser Interferometer



LIGO detected GW from binary blackholes, with the wavelength of thousands of kilometres

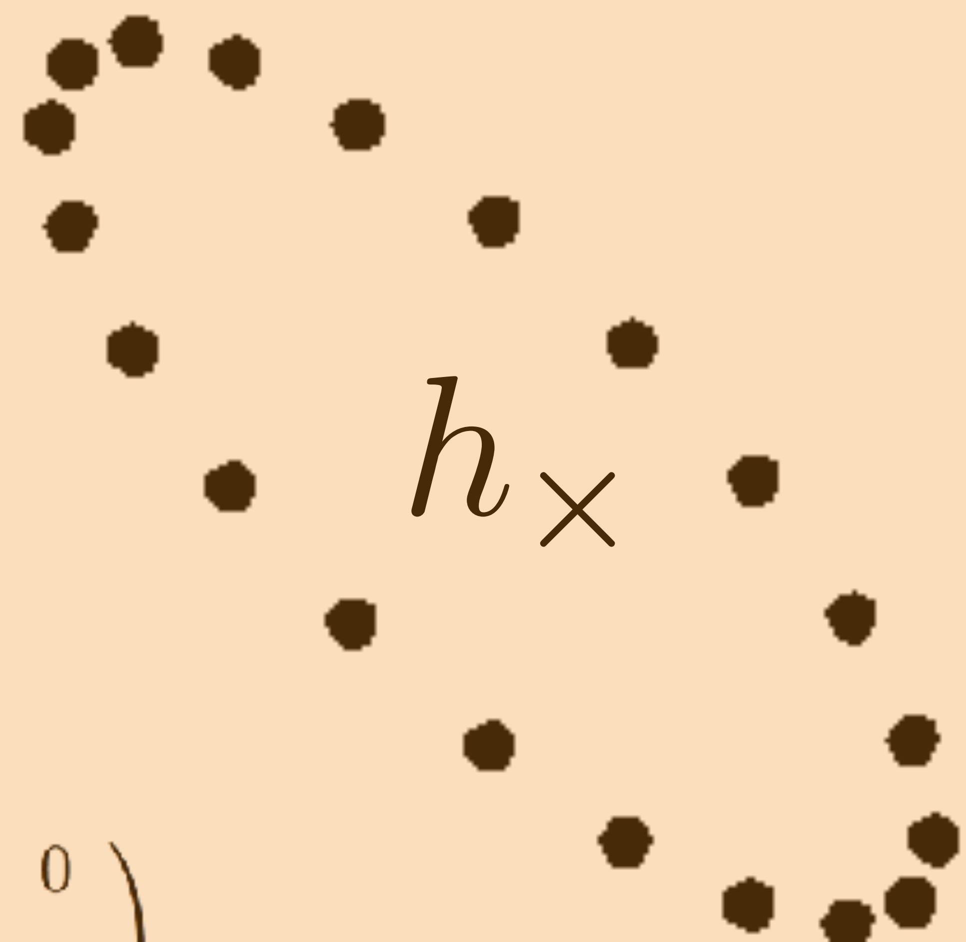
But, the primordial GW affecting the CMB has a wavelength of **billions of light-years**!! How do we find it?

Detecting GW by CMB

Isotropic electro-magnetic fields

Detecting GW by CMB

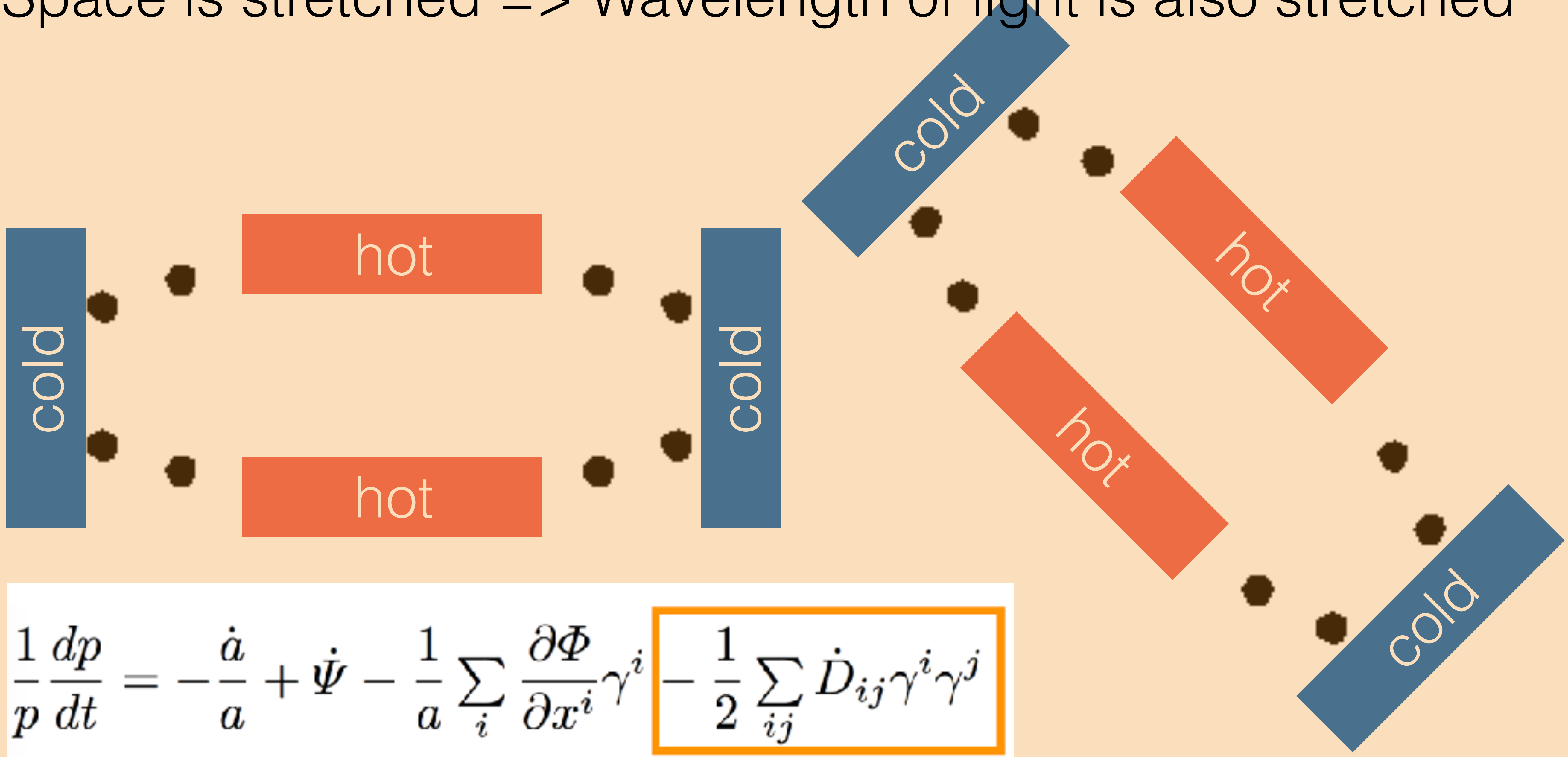
GW propagating in isotropic electro-magnetic fields



$$D_{ij} = \begin{pmatrix} h_+ & h_\times & 0 \\ h_\times & -h_+ & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Detecting GW by CMB

Space is stretched => Wavelength of light is also stretched



$$\frac{1}{p}\frac{dp}{dt} = -\frac{\dot{a}}{a} + \dot{\Psi} - \frac{1}{a}\sum_i \frac{\partial \Phi}{\partial x^i}\gamma^i - \frac{1}{2}\sum_{ij} \dot{D}_{ij}\gamma^i\gamma^j$$

Generation and erasure of tensor quadrupole (viscosity)

- Gravitational waves create quadrupole temperature anisotropy [i.e., tensor viscosity of a photon-baryon fluid] gravitationally, **without velocity potential**
- Still, tight-coupling between photons and baryons erases the tensor viscosity exponentially before the last scattering

$$\left[\frac{\Delta T(\hat{n})}{T_0}\right]_{\rm ISW} = -\frac{1}{2}\sum_{ij}\int_{t_L}^{t_0} dt\ \dot{D}_{ij}(t,\hat{n}r)\hat{n}^i\hat{n}^j$$

negligible contribution before the last scattering

Propagation of cosmological gravitational waves

$$\ddot{D}_{ij} + \frac{3\dot{a}}{a}\dot{D}_{ij} - \frac{1}{a^2}\nabla^2 D_{ij} = 16\pi G \pi_{ij}^{\text{tensor}}$$

- Tensor anisotropic stress can do two things:
  - It can ***generate*** gravitational waves
  - It can ***damp*** gravitational waves (neutrino anisotropic stress)

But we shall ignore the tensor anisotropic stress for this lecture

Super-horizon Solution

$$\ddot{D}_{ij} + \frac{3\dot{a}}{a}\dot{D}_{ij} = 0$$

$$D_{ij} = \text{constant} + \text{decaying term}$$

- Super-horizon tensor perturbation is conserved! [Remember ζ for the scalar perturbation]
  - Thus, **no ISW temperature anisotropy on super-horizon scales**
- It does not look like “gravitational waves”, but it will start oscillating and behaving like waves once it enters the horizon

Matter-dominated Solution

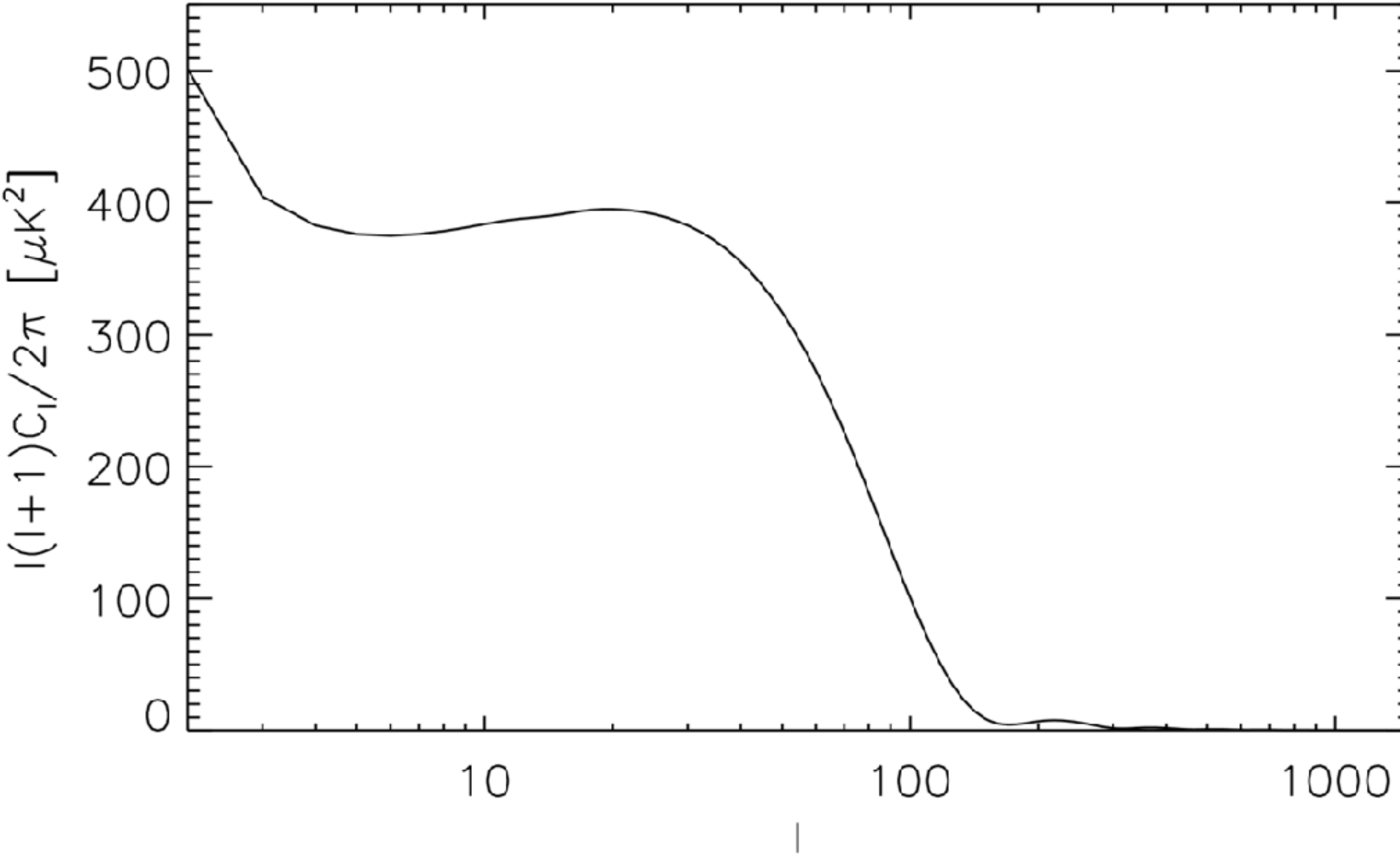
$$D_{ij,\boldsymbol{q}}(t) = C_{ij,\boldsymbol{q}} \frac{3j_1(q\eta)}{q\eta} \propto \frac{1}{a(t)}$$

$$\dot{D}_{ij,\boldsymbol{q}}(t) = -C_{ij,\boldsymbol{q}} \frac{q}{a(t)} \frac{3j_2(q\eta)}{q\eta} \propto \frac{1}{a^2(t)}$$

- $\partial D_{ij}/\partial t$ gives the ISW. It peaks at the horizon crossing, $q\eta \sim 2$
- The energy density is given by $(\partial D_{ij}/\partial t)^2$, which indeed decays like radiation, a^{-4}

Scale-invariant

Temperature C_l from GW



Scale-invariant

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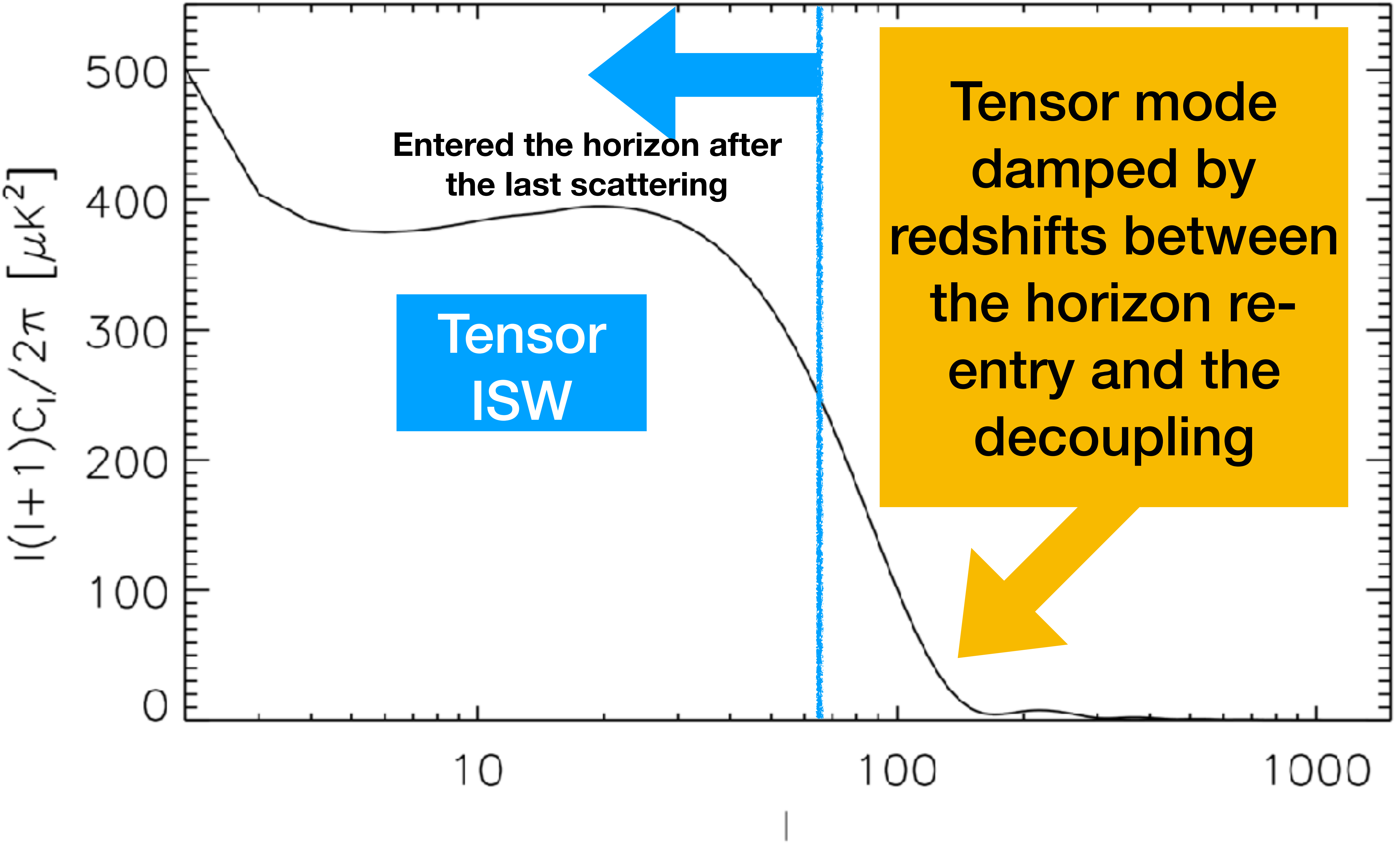

Scale-invariant

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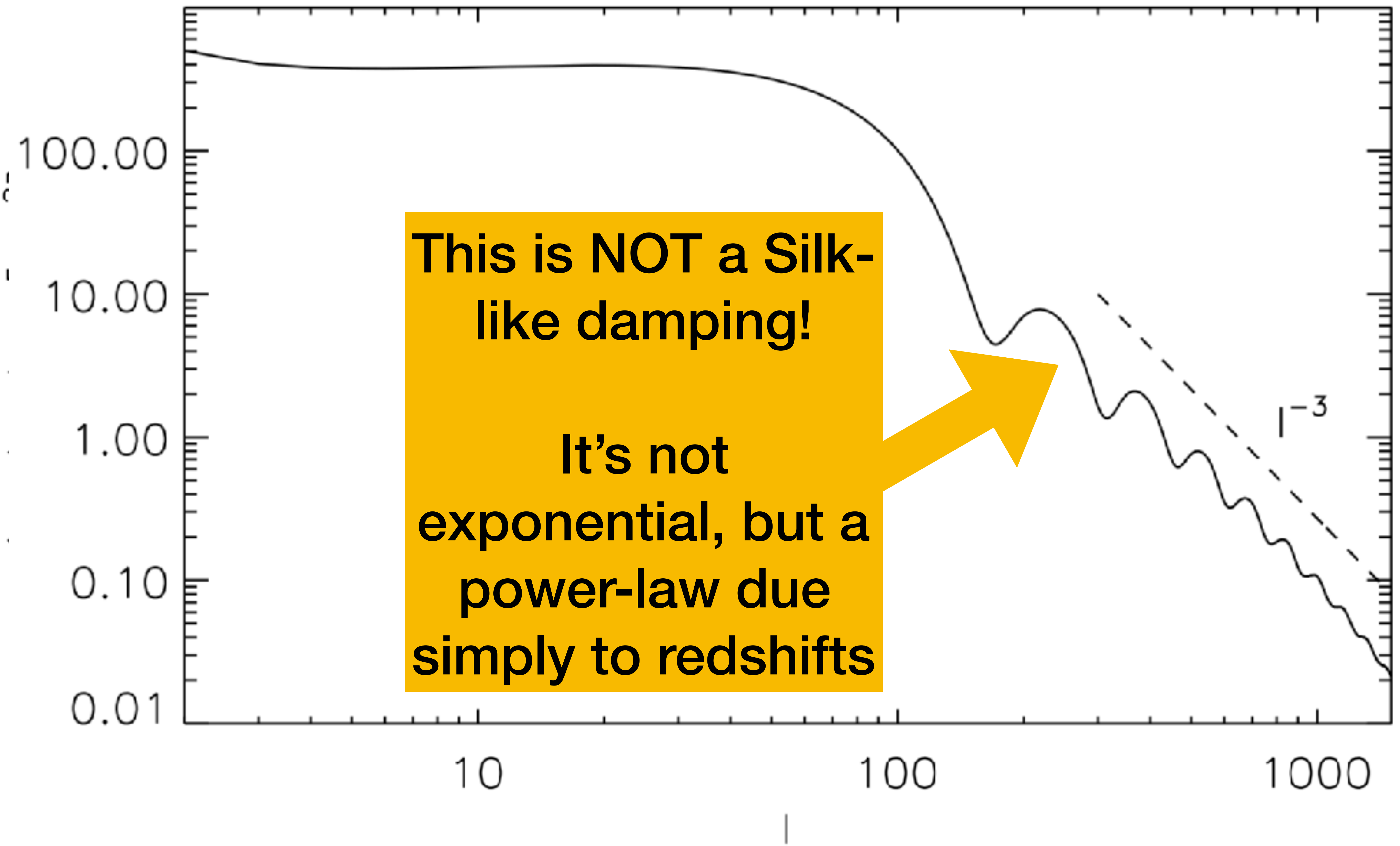
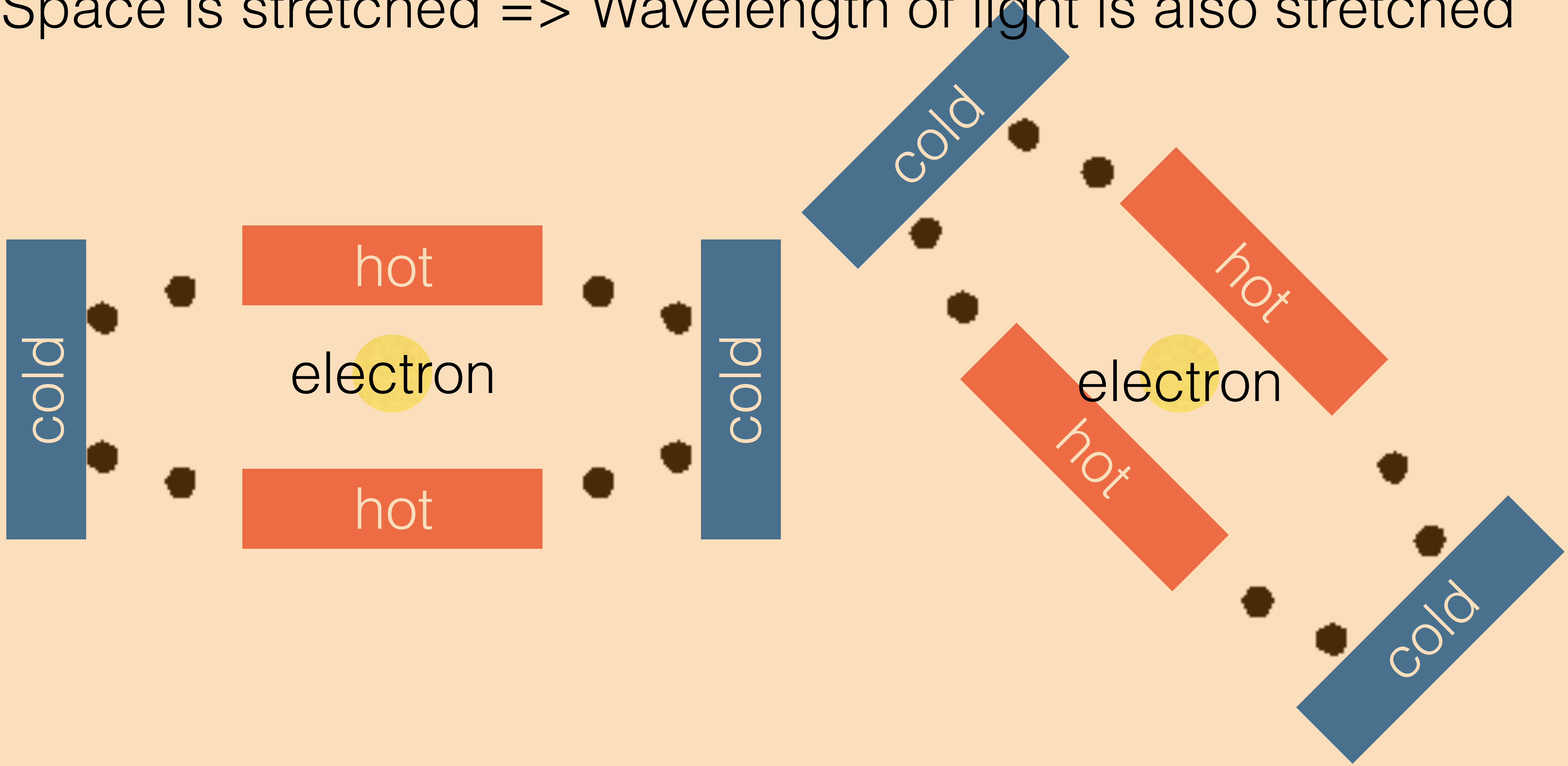
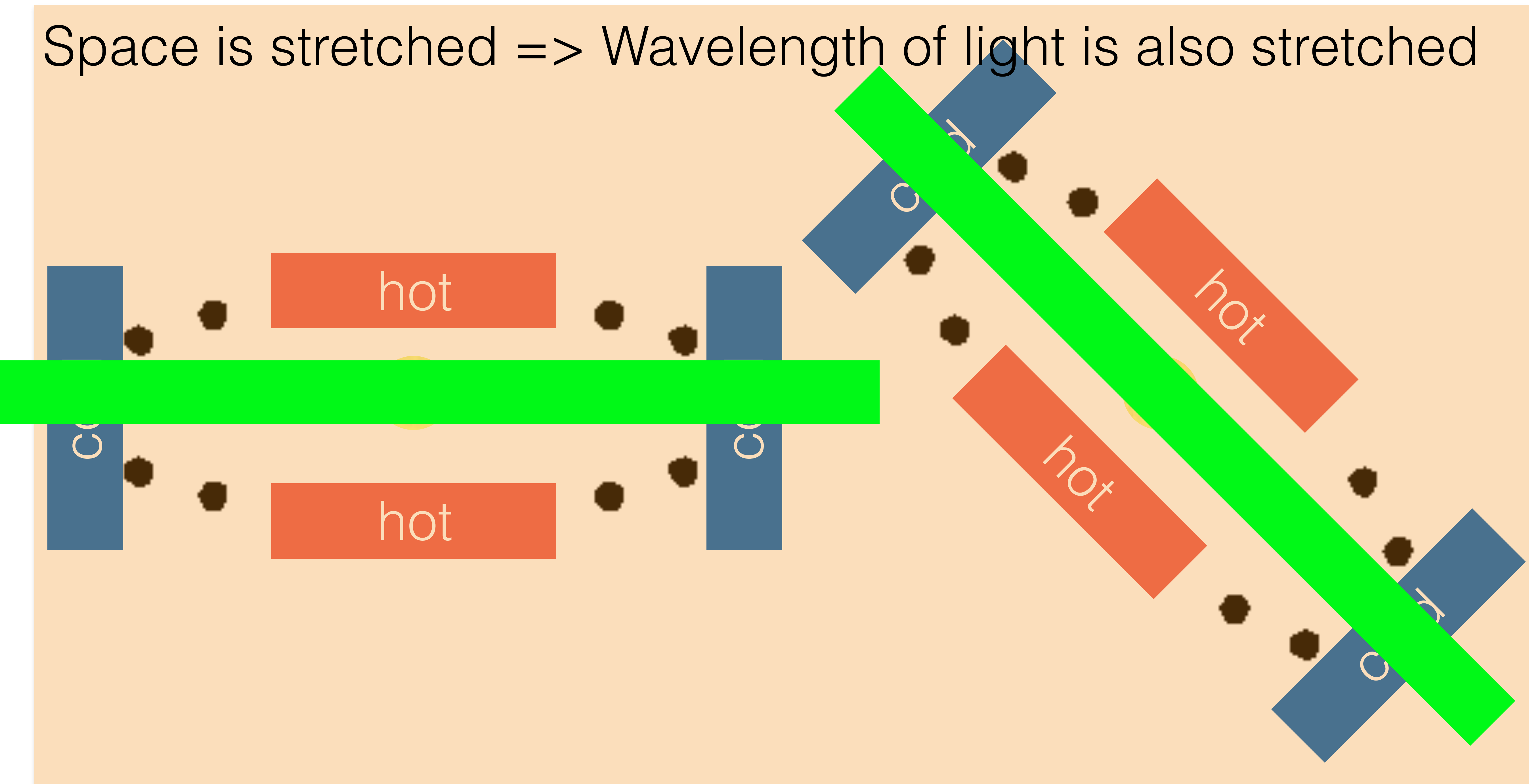


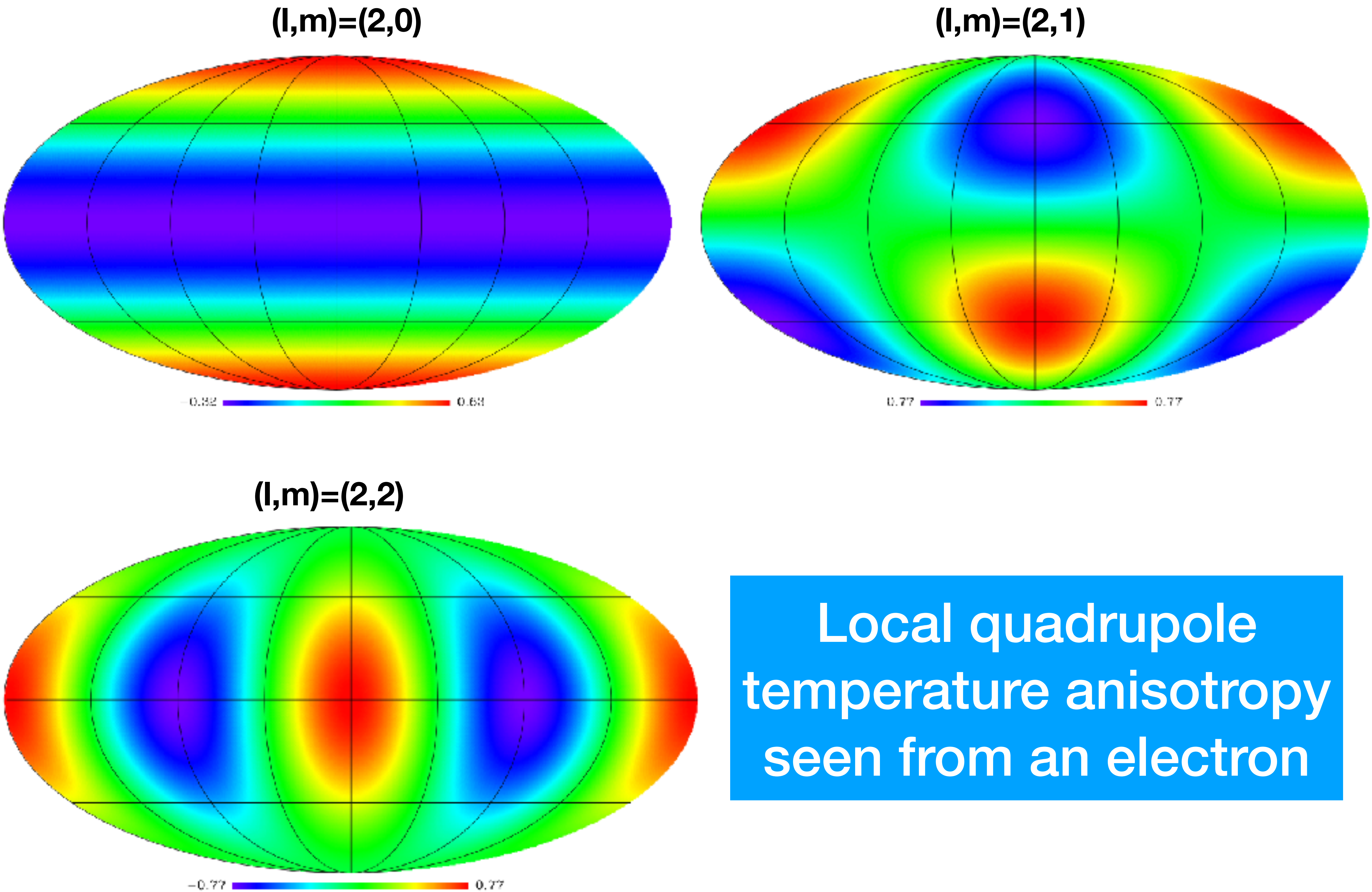
Detecting GW by CMB
Polarisation

Space is stretched => Wavelength of light is also stretched



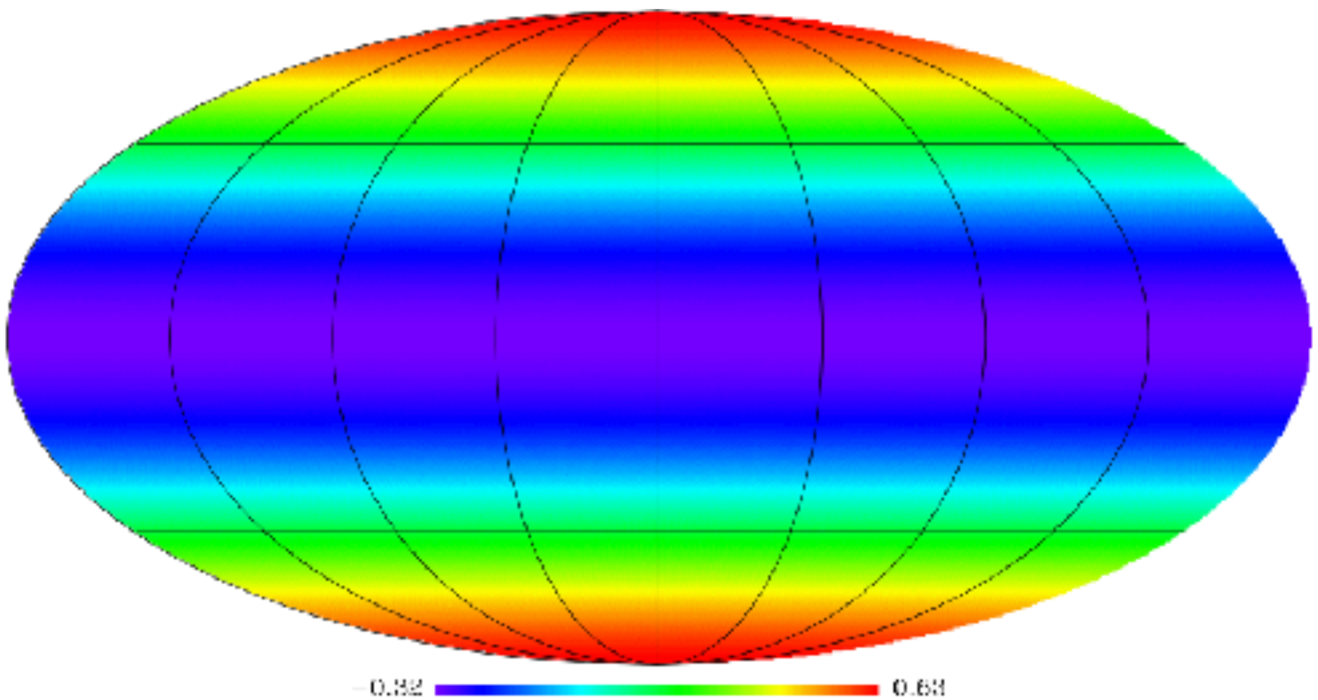
Detecting GW by CMB
Polarisation



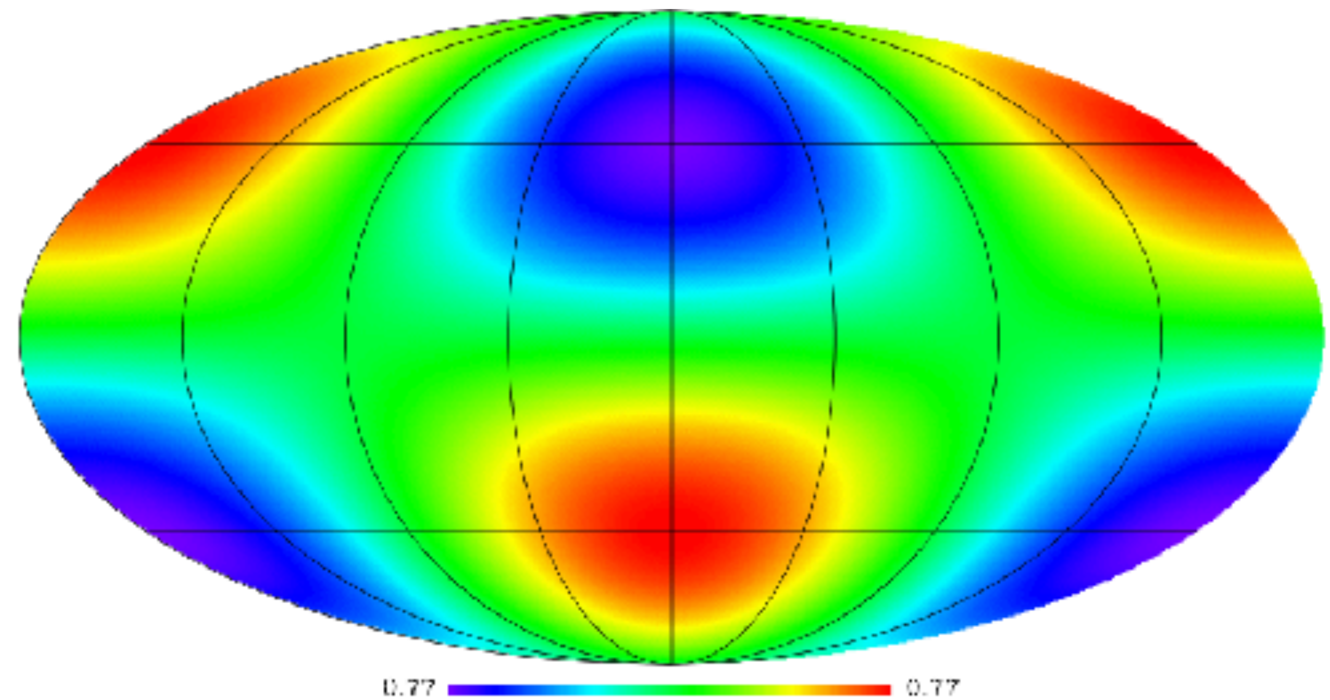


Local quadrupole temperature anisotropy seen from an electron

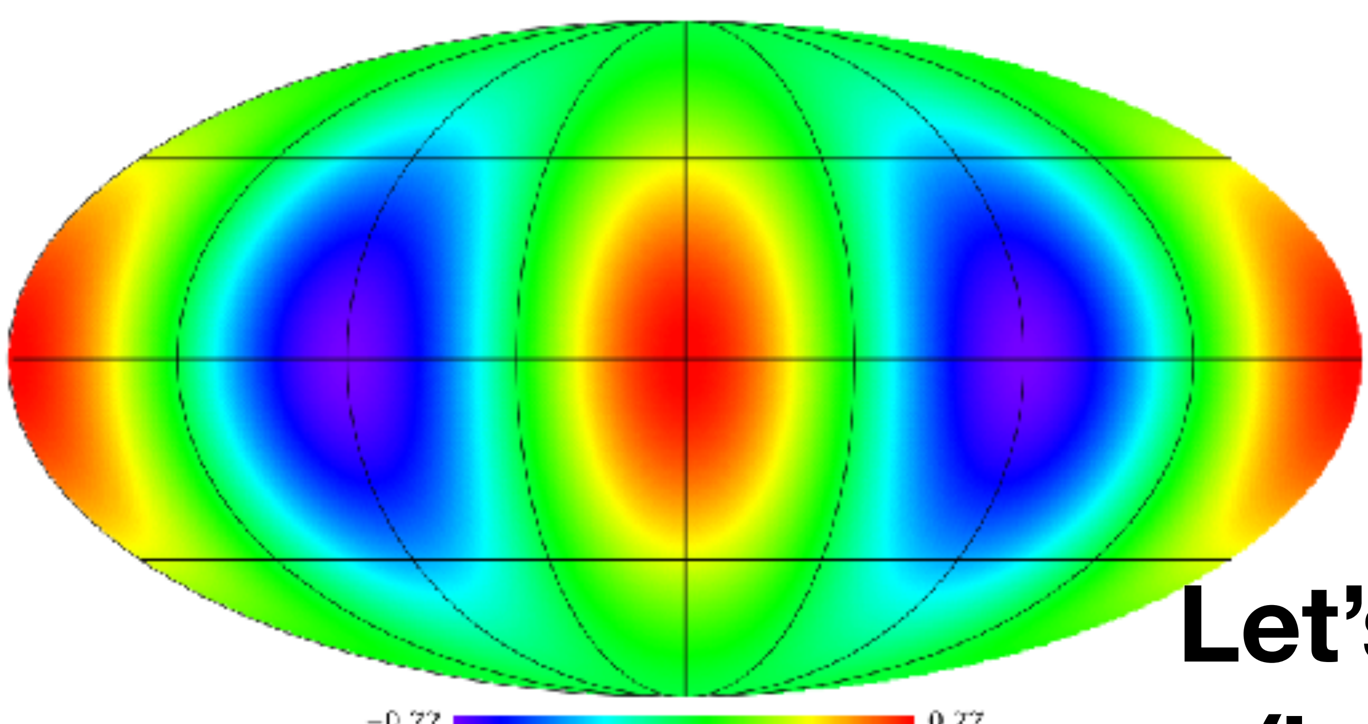
(l,m)=(2,0)

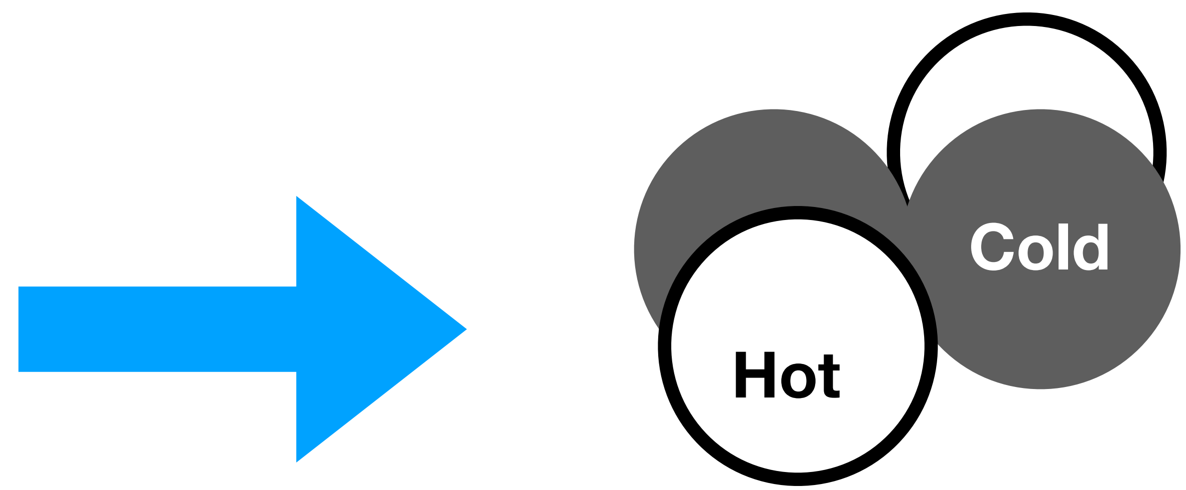


(l,m)=(2,1)

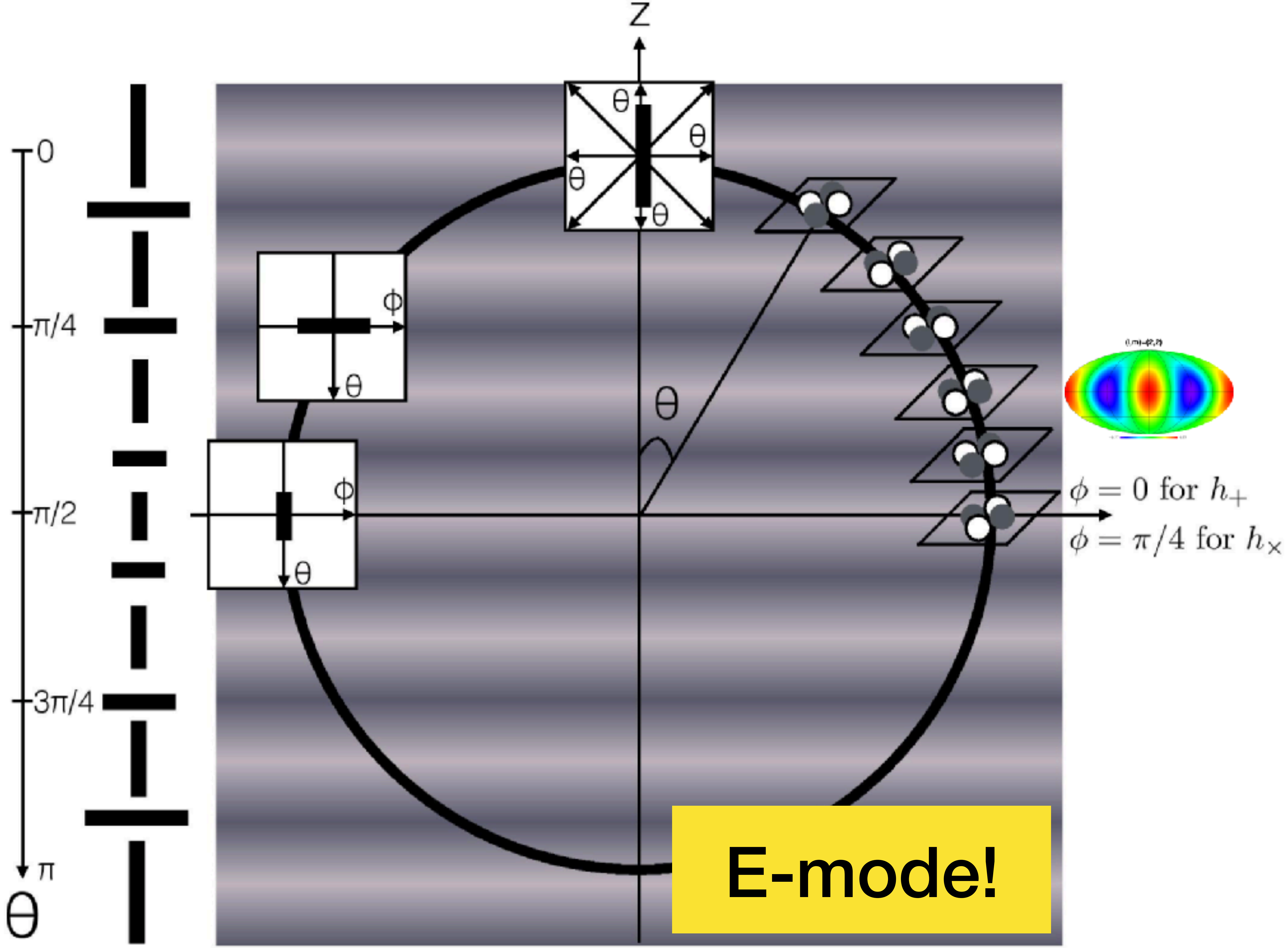


(l,m)=(2,2)

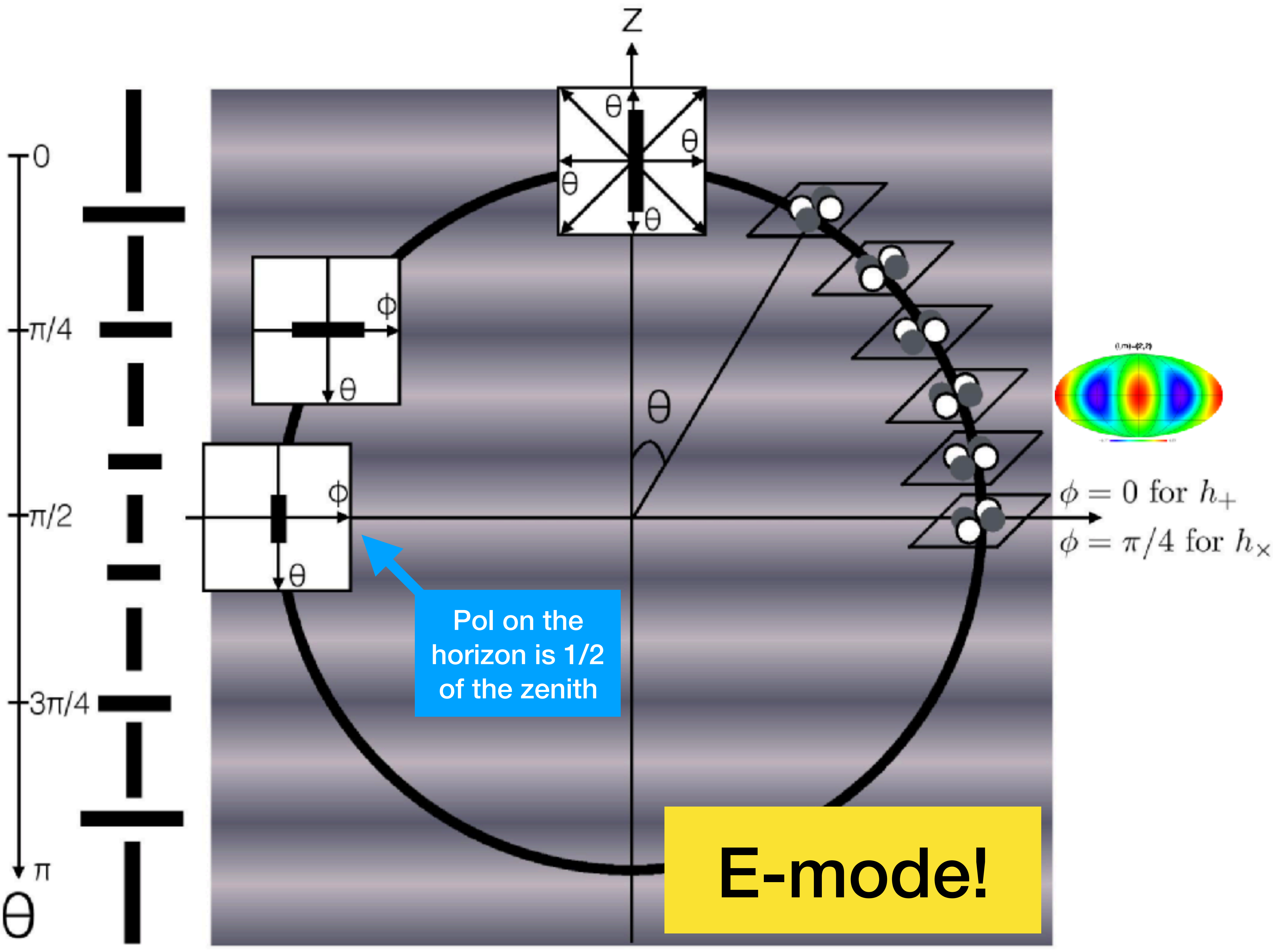


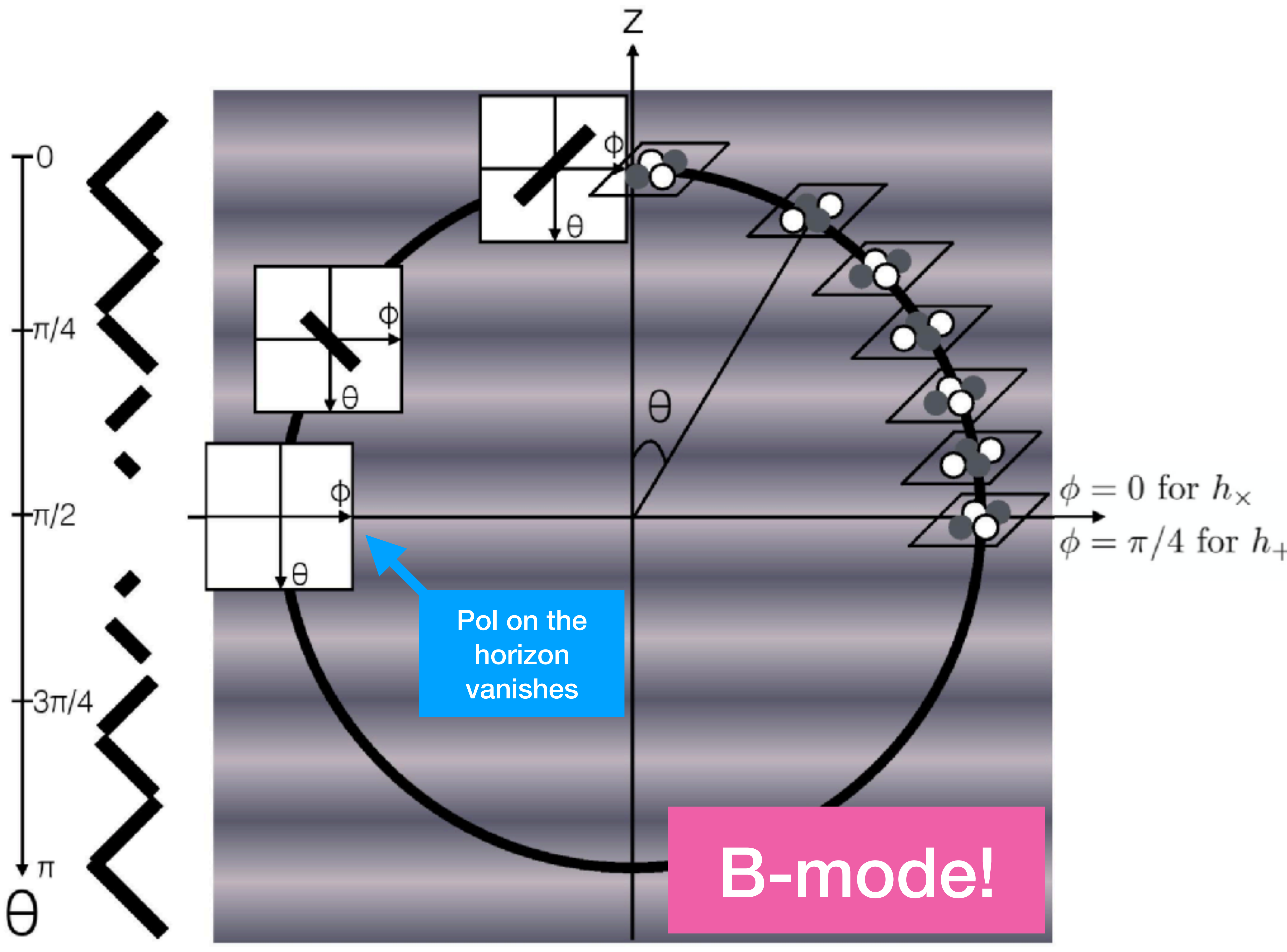


Let's symbolise (l,m)=(2,2) as

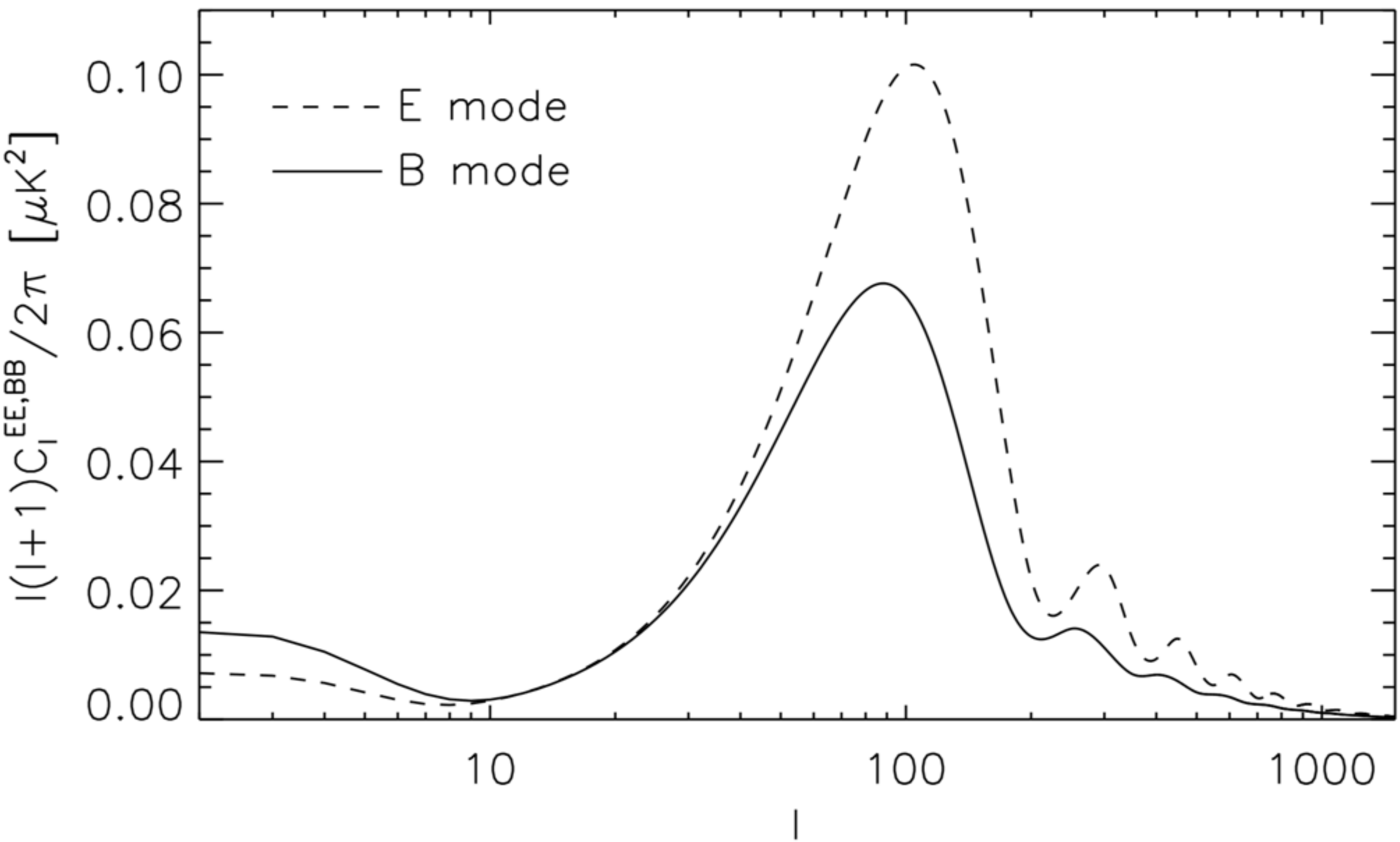


Z
θ
ϕ
0
π/4
π/2
3π/4
π
θ
ϕ = 0 for h₊
ϕ = π/4 for h×
E-mode!

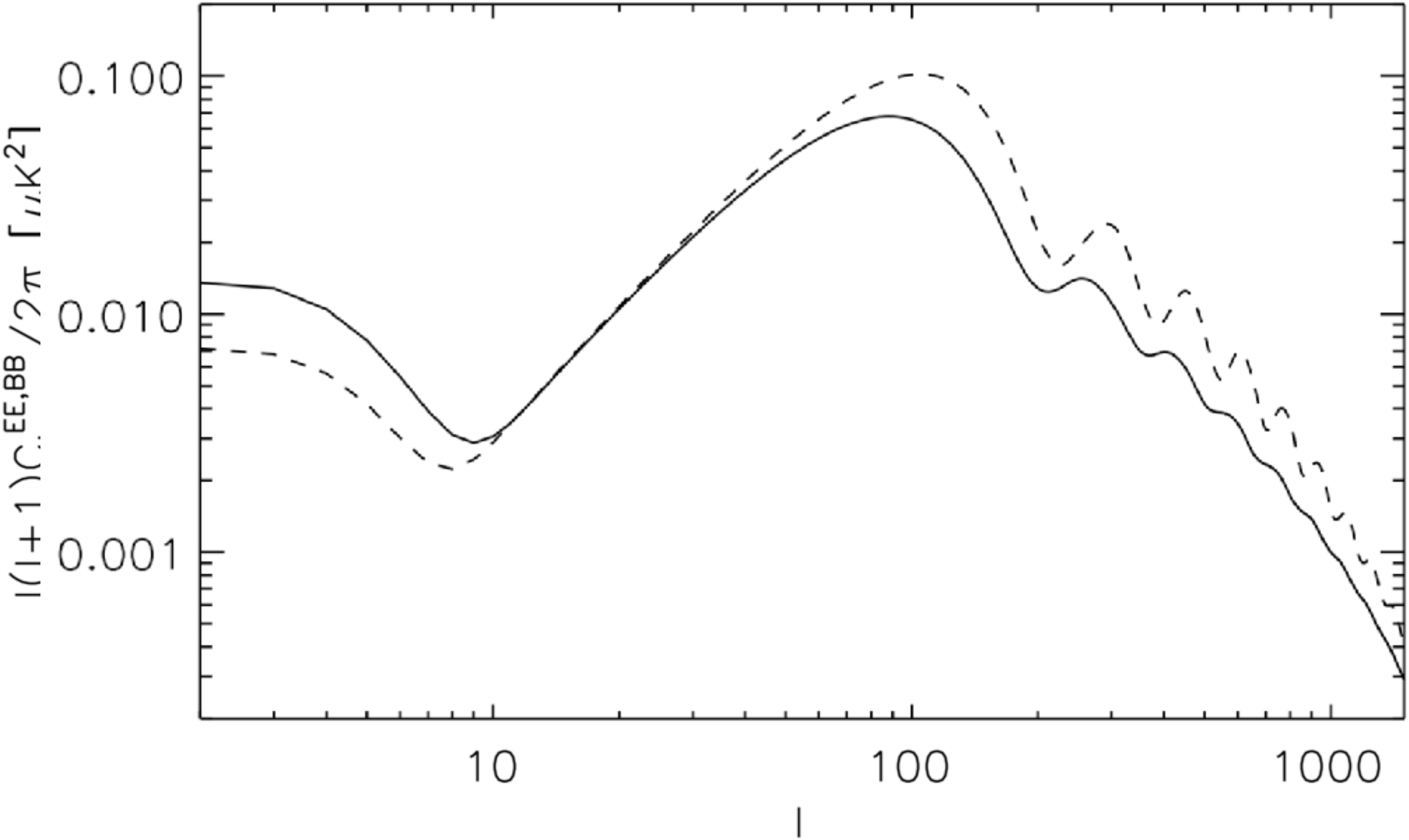

Z
θ
θ
θ
θ
φ
θ
φ
θ
θ
0
π/4
π/2
3π/4
π
θ
$\phi = 0$ for h_+
$\phi = \pi/4$ for $h_\times$
Pol on the horizon is 1/2 of the zenith
E-mode!



z
0
π/4
π/2
3π/4
π
θ
ϕ
θ
$\phi = 0$ for $h_\times$
$\phi = \pi/4$ for h_+
Pol on the horizon vanishes
B-mode!



- E and B modes are produced nearly equally, but on small scales B is smaller than E because B vanishes on the horizon



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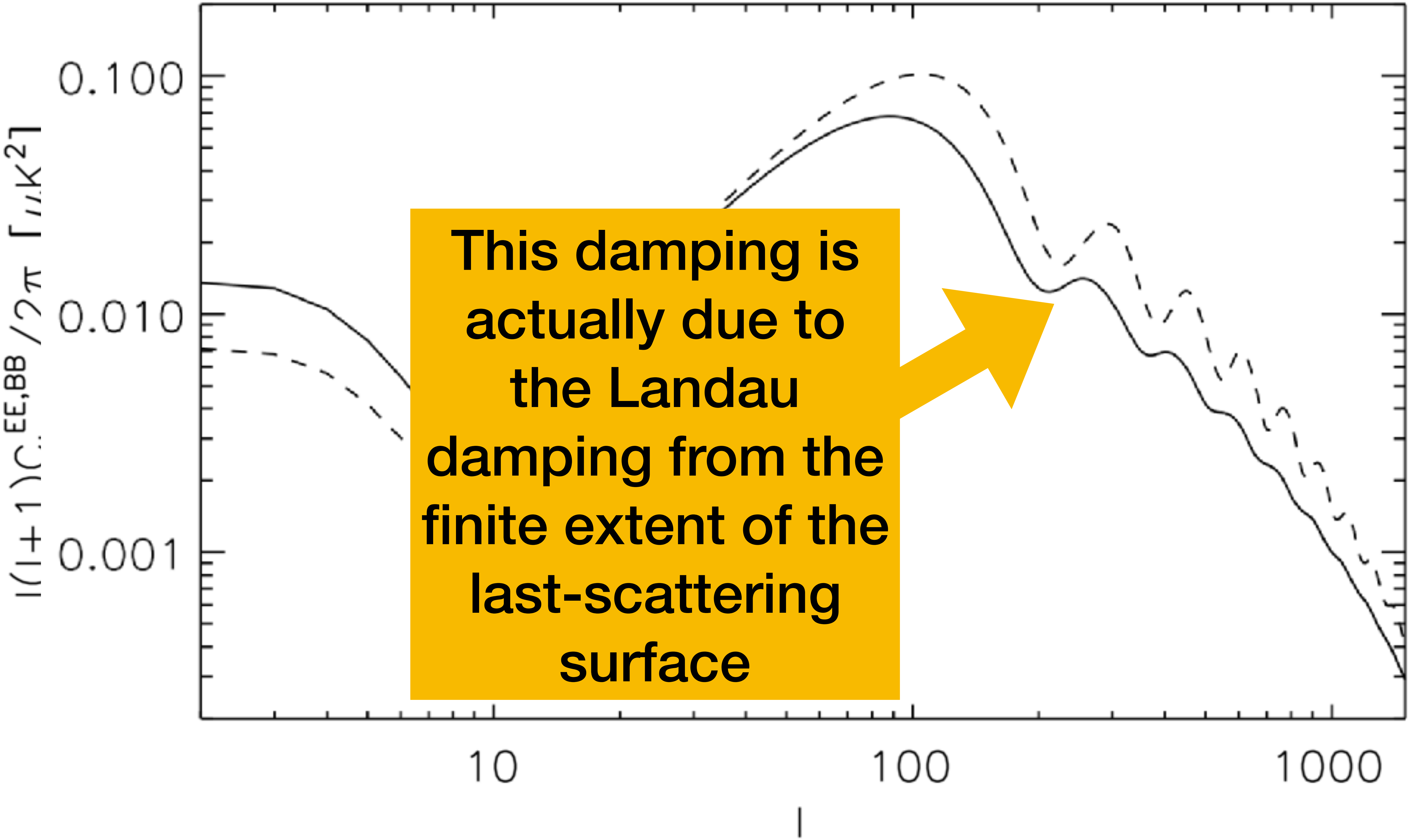

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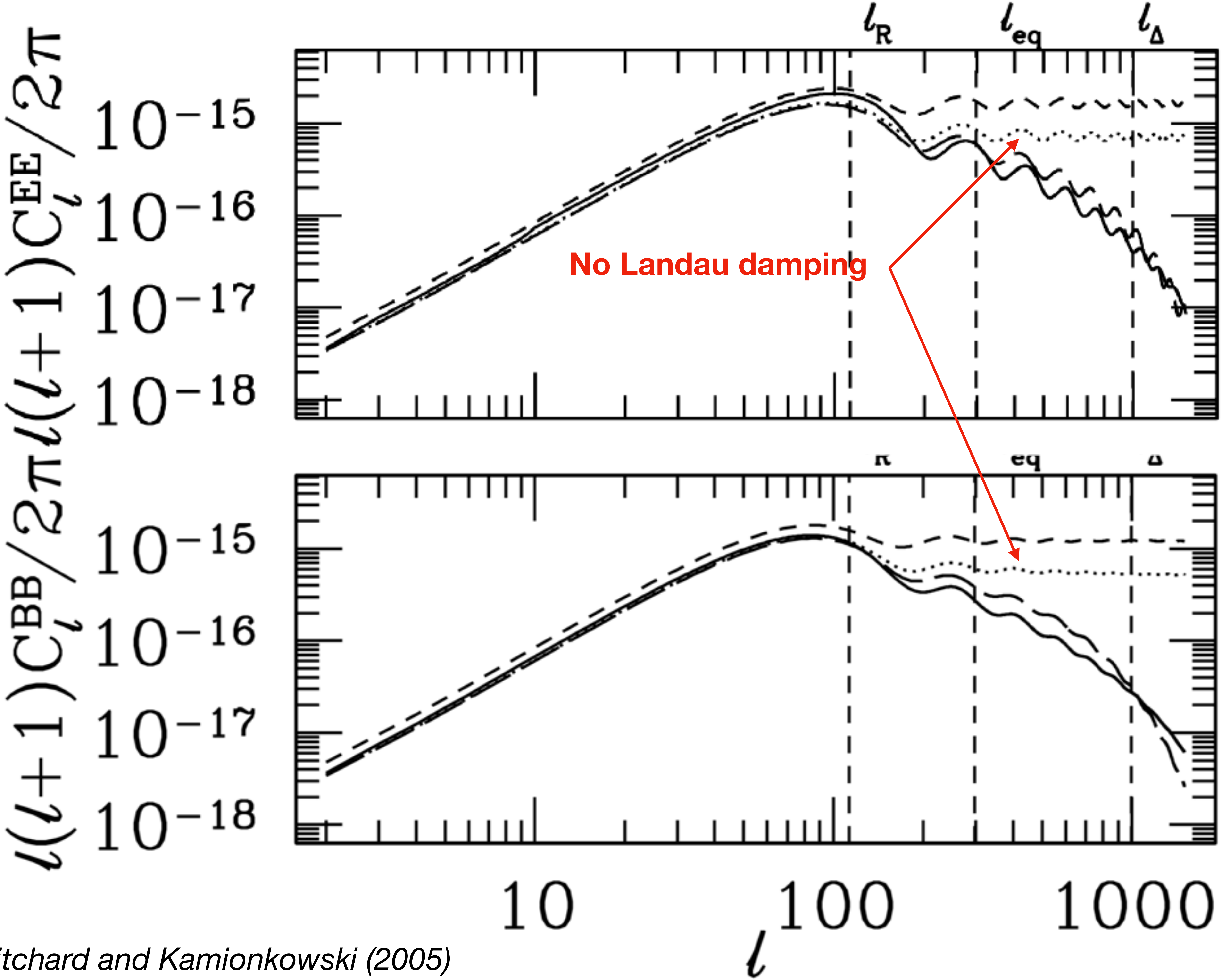
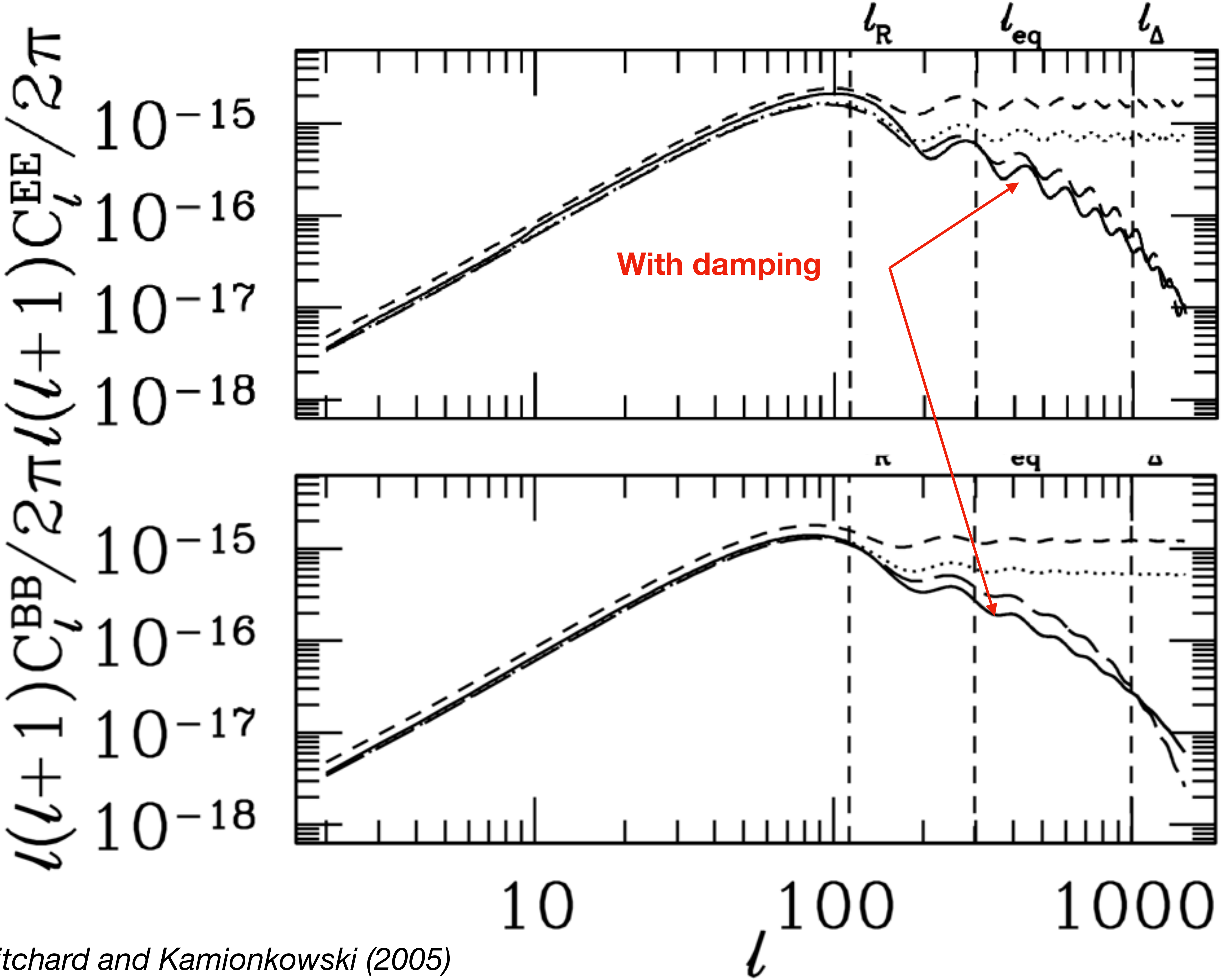
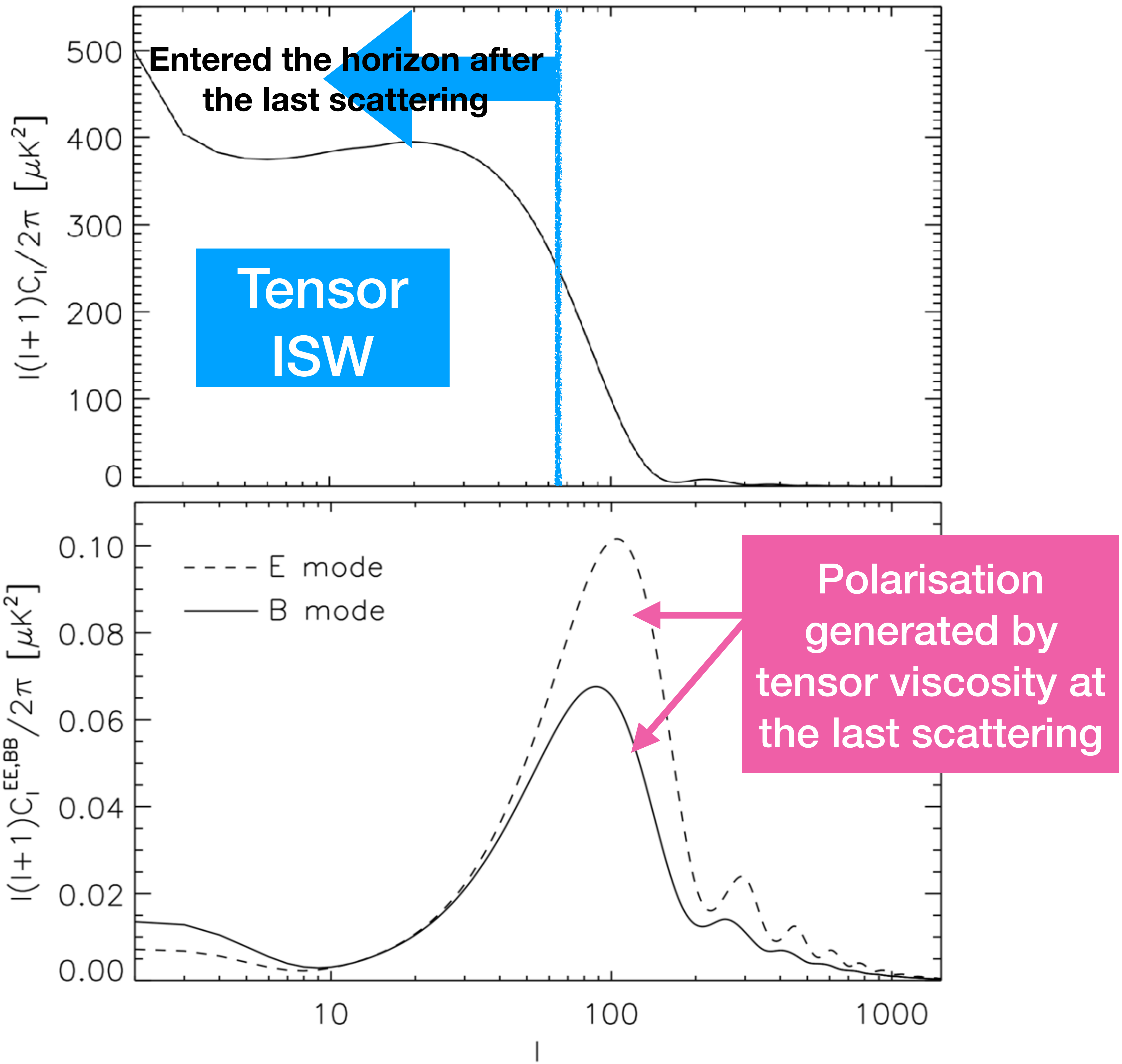


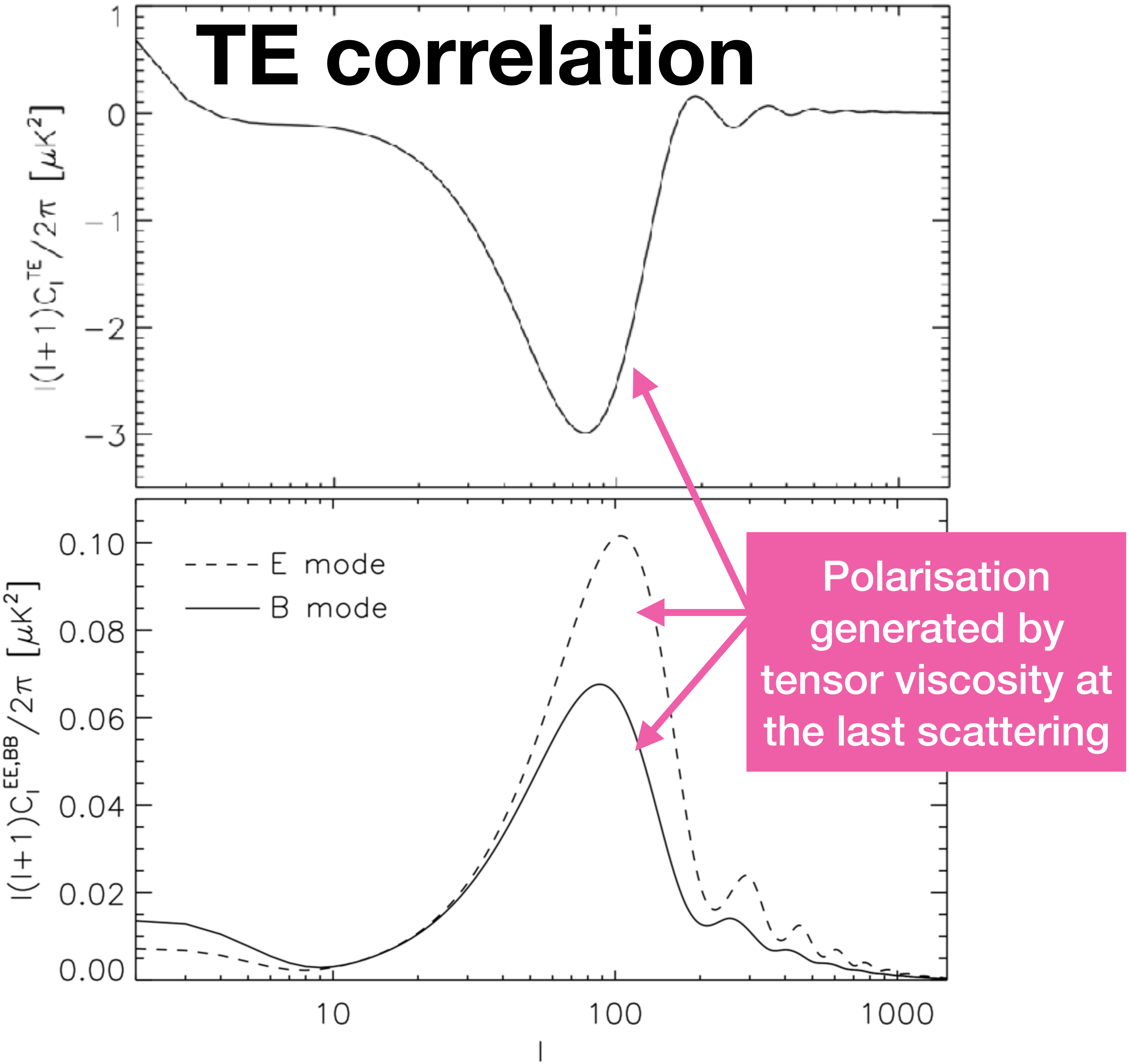
Pritchard and Kamionkowski (2005)



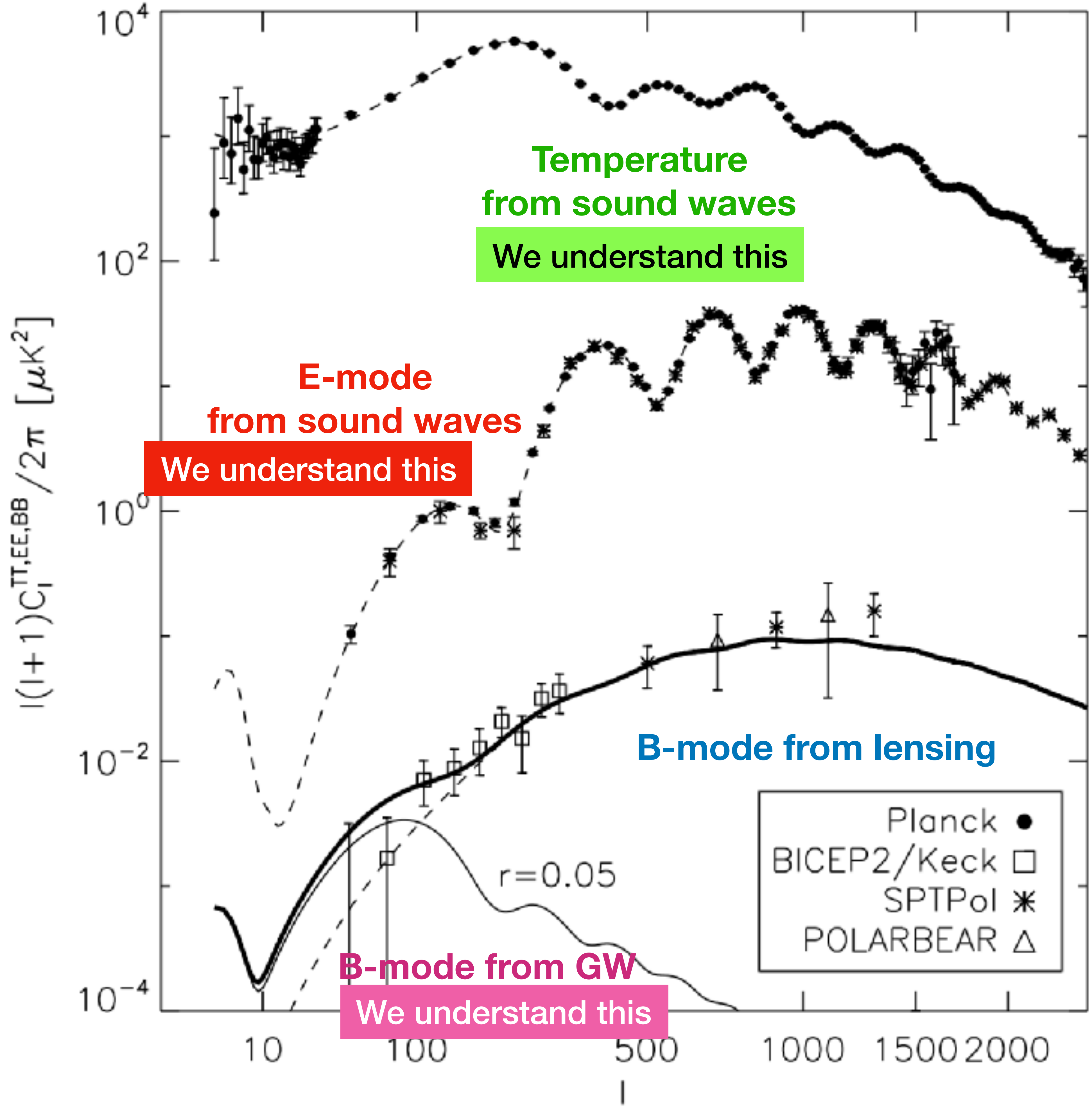
Pritchard and Kamionkowski (2005)



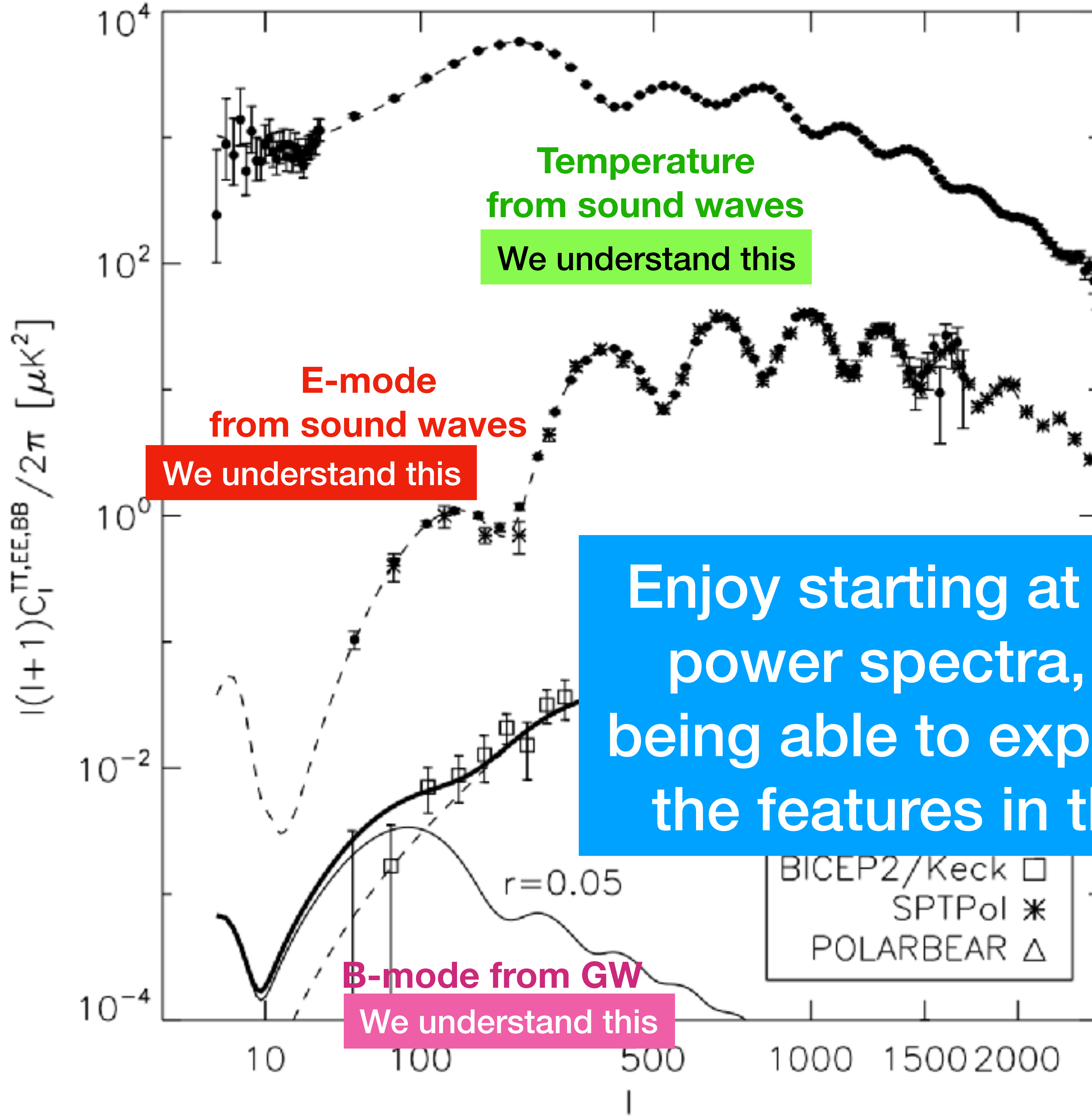
Entered the horizon after the last scattering
Tensor ISW
Polarisation generated by tensor viscosity at the last scattering
E mode
B mode
$l(l+1)C_l/2\pi$ [μK^2]
$l(l+1)C_l^{EE,BB}/2\pi$ [μK^2]
l
0
100
200
300
400
500
0.00
0.02
0.04
0.06
0.08
0.10
10
100
1000



TE correlation
$l(l+1)C_l^{TE}/2\pi$ [μK^2]
1
0
−1
−2
−3
$l(l+1)C_l^{EE,BB}/2\pi$ [μK^2]
0.10
0.08
0.06
0.04
0.02
0.00
E mode
B mode
10
100
1000
l
Polarisation generated by tensor viscosity at the last scattering



Temperature
from sound waves
We understand this
E-mode
from sound waves
We understand this
B-mode from lensing
B-mode from GW
We understand this
r=0.05
Planck
BICEP2/Keck
SPTPol
POLARBEAR
$l(l+1)C_l^{TT,EE,BB}/2\pi\ [\mu K^2]$
10^4
10^2
10^0
10^{-2}
10^{-4}
10
100
500
1000
1500
2000
l



Enjoy starting at these power spectra, and being able to explain all the features in them!