Lectures on
the Effective Field Theory of Large-Scale structure

Leonardo Senatore

Stanford Institute for Theoretical Physics
Department of Physics, Stanford University, Stanford, CA 94306

Kavli Institute for Particle Astrophysics and Cosmology,
Stanford University and SLAC, Menlo Park, CA 94025

Abstract

Planning to explore the Large-Scale Structure of the Universe and to do fundamental physics with those? A lightweight *guide du routard* to the Effective Field Theory for you.

Introduction

Disclaimer: these notes should be considered as my hand-written notes. They are not a publication, they do not satisfy many of the required standards. They are a collections of extracts from papers that I used to prepare these lectures. References are mainly only given to expose where the material has been taken from, look at the cited papers for a complete list of references.

1. number of modes is almost everything. Example is tilt, non-Gaussianity, etc.

2. CMB has been great, but the primordial CMB has almost exhausted its information. Still order one improvement.

3. We want large improvements, for example to cross some interesting theory threshold, or to have a real chance at measuring non-Gaussianities.

4. Large-scale structure has in principle lots of modes

$$N_{\rm modes} \sim \sum_i \sim V \int^{k_{\rm max}} \frac{d^3k}{(2\pi)^3} \sim \frac{V k_{\rm max}^3}{2\pi^2} \tag{1}$$

We have $V \sim (10^4 {\rm Mpc}/h)^3$, if $k_{\rm max} \sim 0.3\, h\, {\rm Mpc}^{-1}$, we have

$$N_{\rm modes} \sim 10^{10} \gg 10^7 \sim N_{\rm modes\ CMB} \tag{2}$$

5. Given that Volume is fixed, we can gain by going to high $k_{\rm max}$, which means that we need to understand the short distance dynamics. But this is made complicated by the formation of very non-linear structures such as galaxies where physics is very complicated

6. My purpose is to do fundamental physics, therefore we need precision and accuracy. Can we do this?.

7. Let us observe the dark matter power spectrum. This is the change of matter contained in a box of size $1/k$ as we change its location in the universe. We see that at long distances the fluctuations are very small, and instead at short distances, as expected, they are very large. We also see that there is an intermediate regime where perturbations are still smaller than one, though not yet order one. There is the temptation that there we could perform a rigorous, accurate description of the dynamics in that regime, by expanding in the smallness of the fluctuations.

8. Notice that large scale structure are very complicated: there is matter, there are gravitationally bound objects, such as halos, galaxies, and clusters of galaxies. Again, our hope is to develop a perturbative and rigorous approach valid at long wavelengths.

9. **The EFTofLSS:** Why we think this can be possible? Let us remember the case of dielectric materials. For the propagation of weak-field, long-wavelength photons in a material, we have developed a set of equations, called the Maxwell dielectric equations, that describe such a propagation in *any* medium. The properties associated to the various different media are encoded in a few coefficients. We do not need to know the complicated structure of the material at atomic level due derive those equations. The only thing we need to know is that the fundamental constituent of the material satisfy some normal principal of physics such as locality, causality, etc.; once this is assumed, the equations can be written and they ar eright.

10. In a sense, dielectric=composite objects+EM, EFTofLSS=composite objects+GR

    Here we are going to do the same for LSS. We start from dark matter

1 From Dark Matter Particles to Cosmic ∼Fluid

See [1]. We take dark matter to be fundamentally described by a set of identical collisionless classical non-relativistic particles interacting only gravitationally. This is a very good approximation for all dark matter candidates apart from very light axions. We discuss baryons later on. As we discuss later, we also neglect general relativistic effects and radiation effects. In this approximation, numerical N-body simulations exactly solve our UV theory. The coefficients of the effective fluid that we will define can therefore be extracted directly from the N-body simulations, following directly the procedure described in [1].

As we will see, we will see that the long-range dynamics is described by fluid-like equations with some coefficients. The coefficients are determine by the UV physics. Here, the UV theory is described by a Boltzmann equation. Therefore, in order to be able to extract the fluid parameters from N-body simulations, we need to derive the fluid equations from the Boltzmann equations and subsequently express the parameters of the effective fluid directly in terms of quantities measurable in an N-body simulation. This is one of the task of this section.

1.1 Boltzmann Equation

Let us start from a one-particle phase space density $f_n(\vec{x},\vec{p})$ such that $f_n(\vec{x},\vec{p})d^3x d^3p$ represents the probability for the particle n to occupy the infinitesimal phase space volume $d^3x d^3p$. For a point particle, we have

$$f_n(\vec{x},\vec{p}) = \delta^{(3)}(\vec{x}-\vec{x}_n)\delta^{(3)}(\vec{p}-m\,a\,\vec{v}_n) \ . \tag{3}$$

The total phase space density f is defined such that $f(\vec{x},\vec{p})d^3x d^3p$ is the probability that there is a particle in the infinitesimal phase space volume $d^3x d^3p$:

$$f(\vec{x},\vec{p}) = \sum_n \delta^{(3)}(\vec{x}-\vec{x}_n)\delta^{(3)}(\vec{p}-m\,a\,\vec{v}_n) \ . \tag{4}$$

We define the mass density ρ, the momentum density π^i and the kinetic tensor σ^{ij} as

$$\rho(\vec{x},t) = \frac{m}{a^3}\int d^3p\; f(\vec{x},\vec{p}) = \frac{m}{a^3}\sum_n \delta^{(3)}(\vec{x}-\vec{x}_n) \ , \tag{5}$$

$$\pi^i(\vec{x},t) = \frac{1}{a^4}\int d^3p\; p^i f(\vec{x},\vec{p}) = \frac{m}{a^3}\sum_n v_n^i \delta^{(3)}(\vec{x}-\vec{x}_n) \ , \tag{6}$$

$$\sigma^{ij}(\vec{x},t) = \frac{1}{ma^5}\int d^3p\; p^i p^j f(\vec{x},\vec{p}) = \sum_n \frac{m}{a^3} v_n^i v_n^j \delta^{(3)}(\vec{x}-\vec{x}_n) \ .$$

The particle distribution f_n evolves accordingly to the Boltzmann equation

$$\frac{Df_n}{Dt} = \frac{\partial f_n}{\partial t} + \frac{\vec{p}}{ma^2}\cdot\frac{\partial f_n}{\partial \vec{x}} - m\sum_{\bar{n}\neq n}\frac{\partial \phi_{\bar{n}}}{\partial \vec{x}}\cdot\frac{\partial f_n}{\partial \vec{p}} = 0 \ , \tag{7}$$

where ϕ_n is the single-particle Newtonian potential. There are two important points to highlight about the former equation. First, we have taken the Newtonian limit of the full general relativistic Boltzmann equation. This is an approximation we make for simplicity. All our results can be trivially extended to include general relativistic effects. However, it is easy to realize that the Newtonian approximation is particularly well justified. Non-linear corrections to the evolution of the dark matter evolution are concentrated at short scales, with corrections that scale proportional to k/k_{NL}. General relativistic corrections are expected to scale as $(aH)^2/k^2$. This means that we should be able to wavelength shorter than order 300 Mpc before worrying about per mille General-Relativity corrections.

Furthermore, one of the goals of this construction is to recover the parameters of the effective fluid of the universe from very short scale simulations valid on distances of order of the non-linear scale. The parameters we will extract in the Newtonian approximation are automatically valid also for the description of an effective fluid coupled to gravity in the full general relativistic setting.

A second important point to highlight in the former Boltzmann equation is about the single-particle Newtonian potential ϕ_n. Following [1], the Newtonian potential ϕ is defined through the Poisson equation

$$\partial^2\phi = 4\pi G a^2 \left(\rho - \rho_b\right) \ , \tag{8}$$

with ρ_b being the background density and $\partial^2 = \delta^{ij}\partial_i\partial_j$. We raise and lower spatial indexes with δ_{ij}. The solution reads

$$\phi = \sum_n \phi_n + \frac{4\pi G a^2 \rho_b}{\mu^2} \ , \tag{9}$$

$$\phi_n(\vec{x}) = -\frac{G\,m}{|\vec{x}-\vec{x}_n|} e^{-\mu|\vec{x}-\vec{x}_n|} \ . \tag{10}$$

Notice that the overall $\phi(\vec{x})$ is IR divergent in an infinite universe. This is due to a breaking of the Newtonian approximation. We have regulated it with an IR cutoff μ that we will take to zero at the end of the calculation. Our results do not depend on μ, as indeed we are interested in very short distance physics.

By summing over n, we obtain the Boltzmann equation for f

$$\frac{Df}{Dt} = \frac{\partial f}{\partial t} + \frac{\vec{p}}{ma^2}\cdot\frac{\partial f}{\partial \vec{x}} - m \sum_{n,\bar{n};\bar{n}\neq n} \frac{\partial \phi_{\bar{n}}}{\partial \vec{x}}\cdot\frac{\partial f_n}{\partial \vec{p}} = 0 \ . \tag{11}$$

1.2 Smoothing

Following [2], we construct the equations of motion for the effective fluid by smoothing the Boltzmann equations and by taking moments of the resulting long-distance Boltzmann equation. The smoothing guarantees that the Boltzmann hierarchy can be truncated, leaving us with an effective fluid. Indeed, notice that it is not trivial at all that we should end up with an effective fluid. Fluid equations are usually valid over distances longer than the mean free path of the particles. But here for dark matter particles the mean free path is virtually infinite. What saves us is that the dark matter particles have had a finite amount of proper time, of order H^{-1}, to travel since reheating, and they traveled at a very non-relativistic speed. This defines a length scale $vH^{-1} \sim 1/k_{NL} sim 1/10\,h\,\mathrm{Mpc}^{-1}$ which is indeed of order of the non-linear scale. This length scale plays the role of a mean free path, as verified in [2]. The truncation of the Boltzmann hierarchy is regulated by powers $k/k_{NL} \ll 1$ [1].

Mote: in general, EFT's can be just written down without need to smooth directly the UV equations. They can be written just based in terms of low-energy degrees of freedom

[1] This comes from dimensional analysis and inspection of the terms: each successive term in the Boltzmann equation contributes as $v/H\cdot\partial_i \sim k/k_{\rm NL}$

and symmetries. Maybe I will sketch a derivation of this later on. However, once on has the UV-completion, it is particularly enlightening to derive the EFT from te UV theory.

We define the Gaussian smoothing

$$W_\Lambda(\vec{x}) = \left(\frac{\Lambda}{\sqrt{2\pi}}\right)^3 e^{-\frac{1}{2}\Lambda^2 x^2} \ , \qquad W_\Lambda(k) = e^{-\frac{1}{2}\frac{k^2}{\Lambda^2}} \ , \tag{12}$$

with Λ^2 representing a k-space, comoving cutoff scale. This will smooth out quantities with wavenumber $k \gtrsim \Lambda$, or equivalently with waveleghts smaller than $\lambda \lesssim 1/\Lambda$. The idea is to take $\Lambda \sim k_{\rm NL} \sim 1/10\, h\,{\rm Mpc}^{-1}$. We regularize our observable quantities $\mathcal{O}(\vec{x},t)$, $\rho, \pi, \phi, \ldots$, by taking convolutions in real space with the filter, defining long-wavelength quantities as

$$\mathcal{O}_l(\vec{x},t) = [\mathcal{O}]_\Lambda(\vec{x},t) = \int d^3x'\, W_\Lambda(\vec{x}-\vec{x}')\mathcal{O}(\vec{x}') \ . \tag{13}$$

Notice that in Fourier space $W(k) \to 1$ as $k \to 0$: our fields are asymptotically untouched at long distances.

The smoothed Boltzmann equation becomes

$$\left[\frac{Df}{Dt}\right]_\Lambda = \frac{\partial f_l}{\partial t} + \frac{\vec{p}}{ma^2}\cdot\frac{\partial f_l}{\partial \vec{x}} - m \sum_{n,\bar{n},n\neq\bar{n}} \int d^3x' W_\Lambda(\vec{x}-\vec{x}')\frac{\partial \phi_n}{\partial \vec{x}'}(\vec{x}')\cdot\frac{\partial f_{\bar{n}}}{\partial \vec{p}} \ . \tag{14}$$

Fluid equations are obtained by taking successive moments

$$\int d^3p\, p^{i_1}\ldots p^{i_n}\left[\frac{Df}{Dt}\right]_\Lambda(\vec{x},\vec{p}) = 0 \ , \tag{15}$$

creating in this way a set of coupled differential equations known as Boltzmann hierarchy. It is sufficient to stop at the first two moments: We obtain

$$\dot{\rho}_l + 3H\rho_l + \frac{1}{a}\partial_i(\rho_l v_l^i) = 0 \ , \tag{16}$$

$$\dot{v}_l^i + Hv_l^i + \frac{1}{a}v_l^j\partial_j v_l^i + \frac{1}{a}\partial_i\phi_l = -\frac{1}{a\rho_l}\partial_j\left[\tau^{ij}\right]_\Lambda \ . \tag{17}$$

Let us define the various quantities that enter in these equations. We define the long wavelength velocity field as the ratio of the momentum and the density

$$v_l^i = \frac{\pi_l^i}{\rho_l} \ . \tag{18}$$

The right hand side of the momentum equation (47) contains the divergence of an effective stress tensor which is induced by the short wavelength fluctuations. This is given by

$$\left[\tau^{ij}\right]_\Lambda = \kappa_l^{ij} + \Phi_l^{ij} \ , \tag{19}$$

where κ and Φ correspond to 'kinetically-induced' and 'gravitationally-induced' parts:

$$\kappa_l^{ij} = \sigma_l^{ij} - \rho_l v_l^i v_l^j \ , \tag{20}$$

$$\Phi_l^{ij} = -\frac{1}{8\pi G a^2}\left[w_l^{kk}\delta^{ij} - 2w_l^{ij} - \partial_k\phi_l\partial^k\phi_l\delta^{ij} + 2\partial^i\phi_l\partial^j\phi_l\right] \ ,$$

where

$$w_l^{ij}(\vec{x}) = \int d^3x' W_\Lambda(\vec{x}-\vec{x}') \left[\partial^i\phi(\vec{x}')\partial^j\phi(\vec{x}') - \sum_n \partial^i\phi_n(\vec{x}')\partial^j\phi_n(\vec{x}')\right] \ . \tag{21}$$

Note that we have subtracted out the self term from w_l^{ij}, as necessary when passing from the continuous to the discrete description in the Newtonian approximation, and used that $\partial^2\phi = 4\pi G a^2(\rho-\rho_b)$ and $\partial^2\phi_l = 4\pi G a^2(\rho_l - \rho_b)$ to express Φ_l in terms of ϕ and ϕ_l. In the limit in which there are no short wavelength fluctuations, and $\Lambda \to \infty$, κ_l and Φ_l vanish. In the literature [1,3] there are available the above expressions written just in terms of the short wavelength fluctuations.

This stress-tensor encodes how short-distance physics affects long distance one. Not only the kinetic jiggling, but also the gravitational one act as pressure.

1.3 Integrating out UV Physics

The effective stress tensor that we have identified is explicitly dependent on the short wavelength fluctuations. These are very large, strongly coupled, and therefore impossible to treat within the effective theory. The equations we derived so far are not very useful, as they depend explicitly on the short modes.

When we compute correlation functions of long wavelength fluctuations, we are taking expectation values. Since short wavelength fluctuations are not observed directly, we can take the expectation value over short-distances directly. This is the classical field theory analog of the operation of 'integrating out' the UV degrees of freedom in quantum field theory, now applied to classical field theory. The long wavelength perturbations will affect the result of the expectation value of the short modes, through, e.g., tidal like effects. This means that the expectation value will depend on the long modes. In practice, we take the expectation value on a long wavelength background. The resulting function depends only on long wavelength fluctuations as degrees of freedom. In this way, we have defined an effective theory that contains only long wavelength fluctuations. Since long wavelength fluctuations are perturbatively small, we can Taylor expand in the size of the long wavelength fluctuations. Schematically we have [1,2,4]

$$\begin{aligned}
&\tau^{ij}(\vec{x},t) = \langle \left[\tau^{ij}\right]_\Lambda \rangle_{\delta_l} + \Delta\tau^{ij} \qquad (22)\\
&= f_{\text{very complicated}}\left(H_0, \Omega_{\rm dm}, w, \ldots, m_{\rm dm}, \ldots, \rho_{\rm dm}(\vec{x}), \partial_i\partial_j\phi, \ldots\right)\big|_{\text{on past light cone}} + \Delta\tau^{ij}\\
&= \int^t dt' \left(\text{Ker}_0(t,t')\delta^{ij} + \delta^{ij}\text{Ker}_1(t,t')\partial^2\phi(\vec{x}_{\rm fl},t') + \delta^{ij}\text{Ker}_2(t,t')_{lm}\partial^l\partial^m\phi(t') + \mathcal{O}\left(\delta^2, \frac{\partial}{k_{\rm NL}}\right)\right)\\
&\quad +\Delta\tau^{ij}
\end{aligned}$$

- We write down all the terms that are allowed by general relativity (no dependence on velocity, no dependence on Φ or $\partial\Phi$: only locally observable quantities are included).
- there is a stochastic term: it accounts for the renormalization of the short-wavelength wavefunction of the modes.

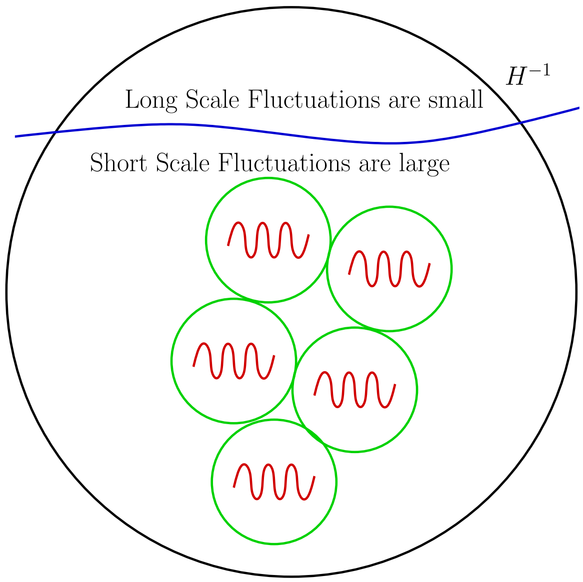

Figure 1:

- coefficients depend explicitly on time: time-translation are spontaneously broken in our universe.

- Evaluation on the past light cone: the theory is non-local in time: this is very unusual. In order to obtain a local field theory, we need an hierarchy of scales. $k \ll k_{\rm NL}$, so in space we have locality. In time, the short modes evolve with H time scales, which is the same as the long modes. Notice that when we evaluate on the past like cone, position of evaluation is given by the fluid location.

$$\vec{x}_{\rm fl}(\vec{x},t,t') = \vec{x} - \int_{\tau(t')}^{\tau(t)} d\tau''\; v_{\rm dm}(\tau'', \vec{x}_{\rm fl}(\vec{x},\tau,\tau''))\ , \tag{23}$$

- The stochastic term is characterized by short distance correlation, on length of order $1/k_{\rm NL}$. It therefore has the following Poisson-like correlation functions:

$$\langle \Delta\tau(\vec{x}_1)\dots\Delta\tau(\vec{x}_n)\rangle = \frac{\delta^{(3)}(\vec{x}_1-\vec{x}_2)}{k_{\rm NL}^3}\dots\frac{\delta^{(3)}(\vec{x}_{n-1}-\vec{x}_n)}{k_{\rm NL}^3} \tag{24}$$

- Notice that the stress tensor enters the equations with a derivative acting on it. The effective stress tensor encodes how short distance physics affects long distance one.

For the precision we pursue in the rest of the paper, we will stop at linear level in the long wavelength fluctuations, though nothing stops us from going to higher order.

Finally, the ellipses $(\dots)$ represent terms that are either higher order in δ_l, or higher order on derivatives of δ_l. Indeed, higher derivative terms will be in general suppressed by $k/k_{NL} \ll 1$, and, as typical in effective field theories, we take a derivative expansion in those.

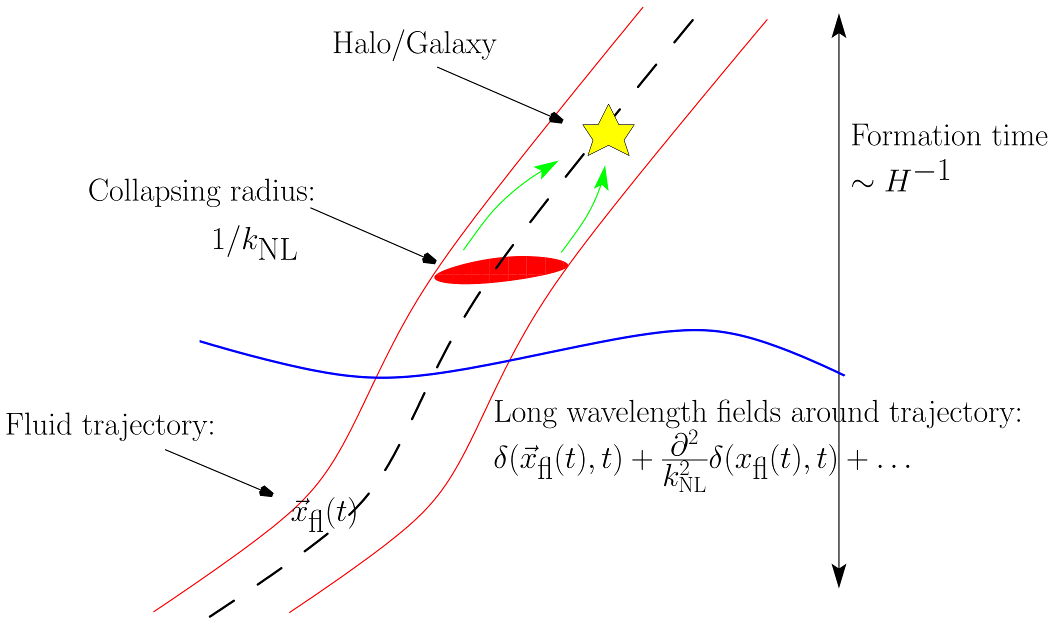

Figure 2:

Astrophysically, these terms would corresponds to the effects induced by a sort of higher-derivative tidal tensor. Once we expand in derivatives of the long wavelength fluctuations, we take the parameters in (22) to be spatially independent, but time dependent.

The coefficient in the stress tensor are determined by the UV physics and by our smoothing cutoff Λ, and are not predictable within the effective theory. They must be measured from either N-body simulations, or fit directly to observations. This is akin to what happens in the Chiral Lagrangian for parameters that can be measured in experiments or in lattice simulations, such as F_π. We first define the correlation functions that will allow us to extract these parameters from small N-body simulations.

Once we plug (22) into (47), we find a set of equations that depend only on the long modes: all the dependence on the short modes has been encoded in the few coefficients appearing in (22).

1.4 Smoothing out a fluid

In order to gain some intuition, it is worth to see the same formalism applied to the toy example where we imagine that the UV system is a perfect pressureless fluid, and we integrate out short distance fluctuations. We follow [2].

It is instructive to present the derivation of our effective stress-tensor $\tau_{\mu\nu}$ in the Newtonian context in yet another way. As we saw earlier, we will later define the effective theory for long-wavelength fluctuations by smoothing the stress-energy tensor $\tau_{\mu\nu}$ on a scale Λ and declaring that long-wavelength gravitational fields are coupled to it. It is particularly illuminating to see how $\tau_{\mu\nu}$ arises in we perform the smoothing immediately at the level of the Euler and

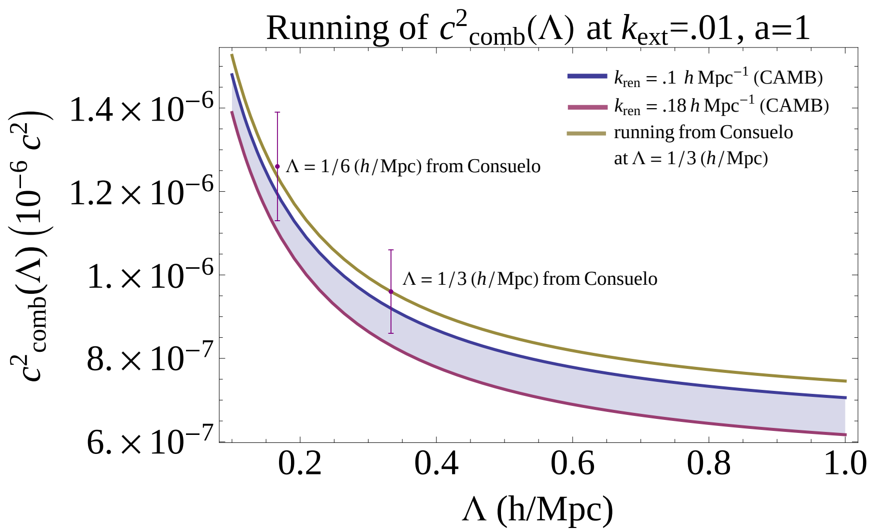

Figure 3:

Poisson equations. We take the Euler equation (in flat space, for simplicity):

$$\left\{\ \rho_m\left[\dot{v}^i+v^j\nabla_j v^i\right]+\rho\nabla_i\Phi\ \right\}=0 \tag{25}$$

We apply a filter on scales of order Λ^{-1} to the Euler equation

$$\int d^3x'\ W_\Lambda(|\vec{x}-\vec{x}'|)\cdot\left\{\ \rho_m\left[\dot{v}^i+v^j\nabla_j v^i\right]+\rho\nabla_i\Phi\ \right\}=0\,. \tag{26}$$

We define smoothed quantities of all fields $X\equiv\{\rho_m,\Phi,\rho_m\vec{v}\}$ as

$$X_\ell\equiv[X]_\Lambda(\vec{x})=\int d^3x'\,W_\Lambda(|\vec{x}-\vec{x}'|)X(\vec{x}')\,, \tag{27}$$

and split the fields into short-wavelength and long-wavelength fluctuations $X\equiv X_\ell+X_s$. Straightforward algebra then shows (see Appendix of [2]) that the Euler equation can be recast in the following way

$$\rho_\ell\left[\dot{v}^i_\ell+v^j_\ell\nabla_j v^i_\ell\right]+\rho_\ell\nabla_i\Phi_\ell=-\nabla_j\left[\tau^j{}_i\right]^s, \tag{28}$$

where

$$\left[\tau_{ij}\right]^s\equiv\left[\rho_m v^s_i v^s_j\right]_\Lambda+\frac{1}{8\pi G}\left[2\partial_i\Phi_s\partial_j\Phi_s-\delta_{ij}(\nabla\Phi_s)^2\right]_\Lambda\,. \tag{29}$$

We see that the long-wavelength fluctuations obey an Euler equation in which the stress tensor τ_{ij} receives contributions from the short-wavelength fluctuations. Eqn. (29)shows explicitly how the effective long-wavelength fluid is different from the pressureless fluid we started with in the continuity and Euler equations (25).

We can also formulate an ansatz for τ^{00}:

$$\tau^{00}=\rho_m+\frac{1}{2}\rho_m v^2-\frac{1}{8\pi G}\left(\nabla\Phi\right)^2\,. \tag{30}$$

There are a few ambiguities in this choice, which correspond to the usual ambiguities of the definition of the local stress tensor, to which we can add

$$\partial_\alpha \partial_\beta \Sigma^{[\alpha\mu][\beta\nu]} \, . \tag{31}$$

Here, the tensor Σ is symmetric under the exchange of the two index pairs, and antisymmetric within each pair. We now impose that this obeys the 0-component of stress-energy conservation

$$0 = \partial_\mu \tau^{\mu 0} = \partial_0 \tau^{00} + \partial_i \tau^{i0} \, . \tag{32}$$

We do *not* assume that τ^{i0} is the same as τ^{0i} defined in (**??**). As we will see in a moment this is an interesting point. Taking the time-derivative of (30) and using repeatedly the continuity, Euler, and Poisson equations, we get

$$\partial_0 \tau^{00} = -\partial_i \Big[\rho_m v^i \Big(1 + \frac{1}{2} v^2 + \Phi \Big) + \frac{1}{4\pi G} \Phi \partial_i \dot{\Phi} \Big] \, . \tag{33}$$

This is consistent with the local conservation law for

$$\tau^{i0} = \rho_m v^i \Big(1 + \frac{1}{2} v^2 + \Phi \Big) + \frac{1}{4\pi G} \Phi \partial_i \dot{\Phi} \simeq \rho v^i \tag{34}$$

up to relativistic corrections that we neglected.

1.5 Renormalization of the Background

From the above analysis it is straightforward to see that integrating out short-wavelength fluctuations leads to a renormalization of the background. We define the new background as the $k \ll \Lambda$ limit of the effective fluid,

$$\bar{\rho}_{\rm eff} \equiv - \lim_{k \ll \Lambda} \langle \tau^0{}_0 \rangle \, , \qquad 3\bar{p}_{\rm eff} \equiv \lim_{k \ll \Lambda} \langle \tau^i{}_i \rangle \, , \qquad (\bar{\Sigma}^i{}_j)_{\rm eff} \equiv \lim_{k \ll \Lambda} \langle \hat{\tau}^i{}_j \rangle \, . \tag{35}$$

Eqn. (35) describes the fluid on very large scales, where spatial fluctuations are suppressed by $k^2/q_\star^2$, with $q_\star$ the typical scale of non-linearities. In particular, on superhorizon scales these fluctuations are highly suppressed.

Let us define:

$$\kappa_{ij} \equiv \frac{1}{2} \langle (1+\delta) v_i v_j \rangle \tag{36}$$

$$\omega_{ij} \equiv -\frac{\langle \phi_{,i} \phi_{,j} \rangle}{8\pi G a^2 \bar{\rho}} \approx \frac{\langle \phi_{,ij} \, \phi \rangle}{8\pi G a^2 \bar{\rho}} \, . \tag{37}$$

$$\kappa = \kappa_{ii} = \frac{1}{2} \langle (1+\delta) v^2 \rangle \qquad \text{and} \qquad \omega = \omega_{ii} = \frac{1}{2} \langle \delta \, \phi \rangle \, < \, 0 \, . \tag{38}$$

Density. We find that the *effective energy density* receives contributions from the kinetic and potential energies associated with small-scale fluctuations

$$\bar{\rho}_{\rm eff} = \bar{\rho}_m (1 + \kappa + \omega) \, . \tag{39}$$

This shows that the *background energy density is corrected precisely by the total kinetic and potential energies associated with non-linear small-scale structures.*

Pressure. The *effective pressure* of the fluid is

$$3\bar{p}_{\rm eff} = \bar{\rho}_m(2\kappa + \omega)\,, \tag{40}$$

and its *equation of state* is

$$\bar{w}_{\rm eff} \equiv \frac{\bar{p}_{\rm eff}}{\bar{\rho}_{\rm eff}} = \frac{1}{3}(2\kappa+\omega)\,. \tag{41}$$

We see that for virialized scales the effective pressure vanishes. As intuitively expected, *a universe filled with virialized objects acts like pressureless dust.* (This agrees with the conclusion reached by Peebles in [5].) Non-virialized structures, however, do have a small effect on the long-wavelength universe, giving corrections to the background of order the velocity dispersion, $\mathcal{O}(v^2)$. In Ref. [2] (and references therein), it is shown in perturbation theory that $2\kappa+\omega>0$ (*e.g.* in linear theory $2\kappa_{\rm L}+\omega_{\rm L}=\frac{1}{2}\kappa_{\rm L}>0$ in Einstein-de Sitter), and that *the induced effective pressure is always positive*, $\bar{p}_{\rm eff}>0$.

Anisotropic stress. On very large scales the anisotropic stress $(\bar{\Sigma}^i{}_j)_{\rm eff}$ averages to zero, *i.e.* it has no long-wavelength contribution:

$$\lim_{k\ll\Lambda}(\bar{\Sigma}^i{}_j)_{\rm eff}\approx 0\,. \tag{42}$$

This straightforwardly follows from the isotropy of the fluctuation power spectrum. On very large scales, the gravitationally-induced fluid therefore acts like an isotropic fluid; its only effects are small $\mathcal{O}(v^2)$ corrections to the background density and pressure. Anisotropic stress, however, does become important when studying the evolution of perturbations on subhorizon scales.

This predictions and analytic explanations have recently numerically verified by numerical codes that solve the GR equations expanded linearly in the metric fluctuations ($\delta g^{\mu\nu}\ll 1$) [6].

Comment later on non-renormalization theorem.

2 Perturbation Theory (including Renormalization)

We are now ready to use our long wavelength effective equations to compute perturbatively correlation function. It is immediate to expand the effective equations in the smallness of $\delta\rho/\rho$, and solve perturbatively.

Let us write the equation for the vorticity $w_l^i=\epsilon^{ijk}\partial_j v_k$. Neglecting the stochastic terms that we argued are small, we have

$$\left(\frac{\partial}{\partial t}+H-\frac{3c_{sv}^2}{4Ha^2}\partial^2\right)w_l^i=\epsilon^{ijk}\partial_j\left(\frac{1}{a}\epsilon_{kmn}v_l^m w_l^n\right)\,. \tag{43}$$

In linear perturbation theory the vorticity is driven to zero, and this occurs even the more so at this order in perturbation theory, as the source is proportional to w_l. While at higher vorticity is generated [4], at the lowest order that we keep in this lectures, and therefore for the purposes of this paper, we can take it to be zero. This means that we can work directly with the divergence of the velocity

$$\theta_l = \partial_i v_l^i \tag{44}$$

Let us first neglect the contribution of the stress tensor, which will be included later perturbatively. Using a as our time variable, the equations

$$\frac{\partial^2}{a^2}\phi_l = H^2\delta_l \tag{45}$$

$$\dot\rho_l + 3H\rho_l + \frac{1}{a}\partial_i(\rho_l v_l^i) = 0\ , \tag{46}$$

$$\dot v_l^i + H v_l^i + \frac{1}{a}v_l^j\partial_j v_l^i + \frac{1}{a}\partial_i\phi_l = -\frac{1}{a\rho_l}\partial_j\left[\tau^{ij}\right]_\Lambda\ . \tag{47}$$

reduce to

$$\begin{aligned} a\mathcal{H}\delta_l' + \theta_l &= -\int \frac{d^3q}{(2\pi)^3}\alpha(\vec q,\vec k-\vec q)\delta_l(\vec k-\vec q)\theta_l(\vec q)\ , \\ a\mathcal{H}\theta_l' + \mathcal{H}\theta_l + \frac{3}{2}\frac{\mathcal{H}_0^2\Omega_m}{a}\delta_l &= -\int \frac{d^3q}{(2\pi)^3}\beta(\vec q,\vec k-\vec q)\theta_l(\vec k-\vec q)\theta_l(\vec q)\ , \end{aligned} \tag{48}$$

where $\mathcal{H} = a^{-1}\partial a/\partial\tau$, subscript $_0$ for a quantity means that the quantity is evaluated at present time, we have set $a_0 = 1$, $'$ represents $\partial/\partial a$ and

$$\alpha(\vec k,\vec q) = \frac{\left(\vec k+\vec q\right)\cdot\vec k}{k^2}\ , \qquad \beta(\vec k,\vec q) = \frac{\left(\vec k+\vec q\right)^2\vec k\cdot\vec q}{2q^2\vec k^2}\ . \tag{49}$$

Since the correlation function of matter overdensities is small at large distances, we can solve the above set of equations (48) perturbatively in the amplitude of the fluctuations. For the computation of the power spectrum at one loop, it is enough to solve these equations iteratively up to cubic order. Order by order, the solution is given by convolving the retarded Green's function associated to the linear differential operator with the non-linear source term evaluated on lower order solutions.

Schematically, if $\mathcal{D}_{\{x,t\}}$ is a differential operator, we have

$$\begin{aligned} &\mathcal{D}_{\{x,t\}}\delta_l = J \quad\Rightarrow\quad \delta_l(\vec x,t) = \int d^4x' G_R(x,t;x',t')\ J(x',t') \\ &\mathcal{D}_{\{x,t\}}G_R(x,t;x',t') = \delta^{(4)}(x^\mu - x'^\mu) \end{aligned} \tag{50}$$

At linear level, we have the following. If $D(a)$ represents the growth factor at scale-factor-time a, we write the linear solution as

$$\delta_l^{(1)}(k,a) = \frac{D(a)}{D(a_0)}\delta s_1(\vec k)\ , \tag{51}$$

with a_0 being the present time, and δs_1 representing a classical stochastic variable with variance equal to the present power spectrum

$$\langle \delta s_1(\vec{k}) \delta s_1(\vec{q}) \rangle = (2\pi)^3 \delta^{(3)}(\vec{k}+\vec{q}) P_{11,l}(k) \ , \tag{52}$$

with $P_{11,l}(k)$.

At second order we obtain

$$\begin{aligned} \delta_l^{(2)}(\vec{k},a) &= \frac{1}{16\pi^3 D(a_0)^2} \\ &\left[\left(\int_0^a d\tilde{a}\, G(a,\tilde{a}) \tilde{a}^2 \mathcal{H}^2(\tilde{a}) D'(\tilde{a})^2 \right) \left(2 \int d^3 q \beta(\vec{q},\vec{k}-\vec{q}) \delta s_1(\vec{k}-\vec{q}) \delta s_1(\vec{q}) \right) \right. \\ &+ \left(\int_0^a d\tilde{a}\, G(a,\tilde{a}) \left(2\tilde{a}^2 \mathcal{H}^2(\tilde{a}) D'(\tilde{a})^2 + 3\mathcal{H}_0^2 \Omega_m \frac{D(\tilde{a})^2}{\tilde{a}} \right) \right) \\ &\left. \times \left(\int d^3 q \alpha(\vec{q},\vec{k}-\vec{q}) \delta s_1(\vec{k}-\vec{q}) \delta s_1(\vec{q}) \right) \right] \ . \end{aligned} \tag{53}$$

Let us explain some of the relevant expressions that appear here. $G(a,\tilde{a})$ is the retarded Green's function for the second order linear differential operator associated with δ that is obtained after substituting θ in the second equation of (48) with the value obtained from the first, and linearizing. In doing this, it is important to neglect all the terms of order c_s^2 because, in our power counting, they count as non-linear terms. The Green's function is given by

$$\begin{aligned} &-a^2 \mathcal{H}^2(a) \partial_a^2 G(a,\tilde{a}) - a \left(2\mathcal{H}^2(a) + a\mathcal{H}(a)\mathcal{H}'(a) \right) \partial_a G(a,\tilde{a}) + 3\frac{\Omega_m \mathcal{H}_0^2}{2a} G(a,\tilde{a}) = \delta(a-\tilde{a}) \ , \\ &G(a,\tilde{a}) = 0 \quad \text{for} \quad a < \tilde{a} \ . \end{aligned} \tag{54}$$

For a ΛCDM cosmology the result can be expressed[2] as a hypergeometric function, although its form is not particularly illuminating. For all calculations presented here it is sufficient to numerically solve the above differential equation. This can be easily accomplished by replacing the $\delta(a-\tilde{a})$ on the RHS of the first equation with zero, but starting with the boundary conditions being $G(a,\tilde{a})|_{a=\tilde{a}} = 0$, and $\frac{\partial}{\partial a} G(a,\tilde{a})|_{a=\tilde{a}} = 1/(\tilde{a}\mathcal{H}(\tilde{a}))^2$. In principle, it is possible to include in the linear equations that determine the Green's function and the growth functions also the higher-order linear terms proportional to c_s^2. Doing this amounts to resumming the effect of these pressure or viscous terms. The resulting linear equation can be easely solved numerically, finding for example that the growth factor becomes k-dependent, being the more suppressed the higher is the wavenumber [7]. However, it is not fully consistent to resum these terms without including the relevant loop corrections.

Iterating, we obtain the solution for δ at cubic order $\delta^{(3)}$.

A very useful simplification is due to the fact the growth factor and the Green's function are k-independent. This is due to the fact that at linear level we can neglect the pressure and viscosity terms that would otherwise induce a k-dependence. Because of this, the convolution integrals that would couple time integration and momentum integration nicely split into

[2] Using, e.g., Mathematica's "DSolve" function.

separate time integrals and momentum integrals that can be simply performed separately. We have tried to underline this in (53) by adding suitable parenthesis. In fact, to a very good numerical approximation, we have that

$$\delta^{(2)}(\vec{k},a) \simeq \frac{D(a)^2}{D(a_0)^2}\int \frac{d^3q_1}{(2\pi)^3}\int \frac{d^3q_2}{(2\pi)^3}(2\pi)^3\delta_D^{(3)}(\vec{k}-\vec{q}_1-\vec{q}_2)F_2(\vec{q}_1,\vec{q}_2)\delta^{(1)}(\vec{q}_1,a_0)\delta^{(1)}(\vec{q}_2,a_0)$$
$$\delta^{(3)}(\vec{k},a) \simeq \frac{D(a)^3}{D(a_0)^3}\int \frac{d^3q_1}{(2\pi)^3}\int \frac{d^3q_2}{(2\pi)^3}\int \frac{d^3q_3}{(2\pi)^3}(2\pi)^3\delta_D^{(3)}(\vec{k}-\vec{q}_1-\vec{q}_2-\vec{q}_3)$$
$$F_3(\vec{q}_1,\vec{q}_2,\vec{q}_3)\delta^{(1)}(\vec{q}_1,a_0)\delta^{(1)}(\vec{q}_2,a_0)\delta^{(1)}(\vec{q}_3,a_0) \qquad (55)$$

where $F_{2,3}$ are simple expressions of the q's (such as $1+\frac{\vec{q}_2\cdot\vec{q}_3}{q_2^2}$). Show picture.

We can now form Feynamn diagrams by contracting then linear fluctuations. At fourth order in the fluctuations, we have two diagrams, that we denote by P_{22} and P_{13}.

$$P_{22}(\vec{k},a) = \langle\delta^{(2)}(\vec{k})\delta^{(2)}(\vec{k}')\rangle = (2\pi)^3\delta_D^{(3)}(\vec{k}+\vec{k}')D(a)^4\int\frac{d^3q}{(2\pi)^3}F_2(\vec{k}-\vec{q},\vec{q})^2P_{11}(q)P_{11}(|\vec{k}-\vec{q}|)$$
$$P_{13}(\vec{k},a) = \langle\delta^{(1)}(\vec{k})\delta^{(3)}(\vec{k}')\rangle = (2\pi)^3\delta_D^{(3)}(\vec{k}+\vec{k}')D(a)^4\int\frac{d^3q}{(2\pi)^3}F_3(\vec{k},-\vec{q},\vec{q})^2P_{11}(q)P_{11}(k)$$
$$P_{1-\mathrm{loop}} = P_{22}+P_{13}+P_{31} \qquad (56)$$

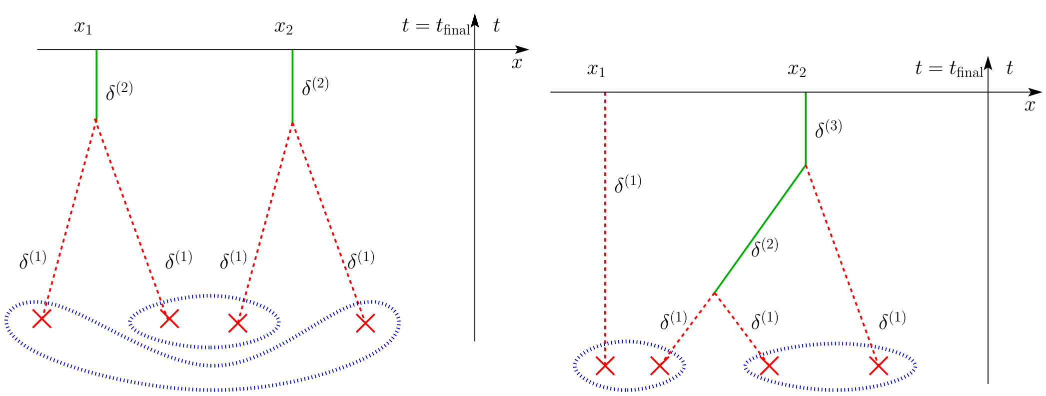

Figure 4:

Now, for simplicity, let us imagine that the initial power spectrum was a simple power law. This is not the case in the true universe, but it helps to make the physics clear.

Let us therefore image that $P_{11} = \frac{1}{k_{\rm NL}^3}\left(\frac{k}{k_{\rm NL}}\right)^n$, with $-1 < n < 1$. In this case, we have that P_{13} is divergent. If we cut it off at $q = \Lambda$.

$$P_{13}(k) = D(a)^4\left\{\left(\frac{\Lambda}{k_{\rm NL}}\right)^{n+1}\frac{k^2}{k_{\rm NL}^2}+c_n\left(\frac{k}{k_{\rm NL}}\right)^{n+3}\right\}P_{11}(k)\ . \qquad (57)$$

We notice that the result would be infinitely large. We have cut it off, but at the cost at an unphysical cutoff dependence.

This is cannot be the right result, as cutoff dependence is unphysical. The reason of the mistake is that our contribution is UV sensitive. But in that regime perturbation theory is not supposed to apply, and not even our equations are correct. What do we do?

The effect of short distance physics was encoded in the stress tensor. With its free coefficient, it should be able to cancel the error that we make in perturbation theory and give the correct result. Let us therefore use the stress tensor perturbatively. At leading order, we can take the stress tensor at linear level. We obtain

$$\delta^{(3)}_{c_s^2}(\vec{k},a) = -k^2 \left\{ \int_0^a da'\ G(a,a') \left[\int^{a'} da'' \mathrm{Ker}_1(a',a'') \frac{D(a'')}{D(a)} \right] \right\} \delta^{(1)}(\vec{k},a) \equiv c_s^2(a) D(a)^3 \frac{k^2}{k_{\rm NL}^2} \delta^{(1)}(\vec{k},a) \ . \tag{58}$$

show diagram

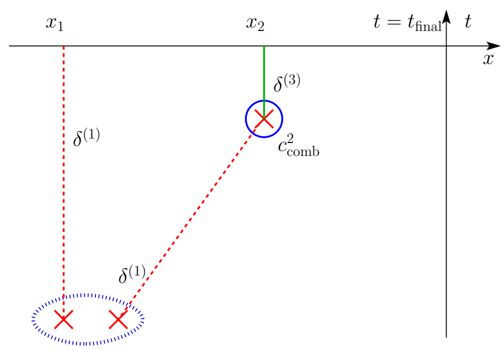

Figure 5:

Notice that we have defined a time-dependent speed of sound. There are two things to discuss:

1. since time-translations are spontaneously broken, the coefficients are time-dependent

2. since the theory was non-local in time, parameters are time-dependent kernels, and there are additional time integrals in the solutions. However, thanks to the fact that the solution has the factorized structure

$$\delta(k,a) \sim \sum_n D^{(n)}(a) \int dq_1 \ldots dq_n\ \delta_D^{(3)}(\vec{k} - \vec{q}_1 - \ldots \vec{q}_n) F_n(q_1, \ldots, q_n)\ \delta(\vec{q}_1) \ldots \delta(\vec{q}_n) \tag{59}$$

then we can always symbolically do the time integrals over the kernels

$$\left(\int da' \mathrm{Ker}(a,a') \sum_n D^{(n)}(a') \right) \int dq_1 \ldots dq_n \delta_D^{(3)}(\vec{k} - \vec{q}_1 - \ldots \vec{q}_n) F_n(q_1, \ldots, q_n)\ \delta(\vec{q}_1) \ldots \delta(\vec{q}_n) = = \sum_i c_{n,i}(a) \int dq_1 \ldots dq_n\ \delta_D^{(3)}(\vec{k} - \vec{q}_1 - \ldots \vec{q}_n)\ F_n(q_1, \ldots, q_n)\ \delta(\vec{q}_1) \ldots \delta(\vec{q}_n) \tag{60}$$

So, we just get a different value of the counterterm for each order in the perturbative expansion we use a counterterm.

Instead, if the theory were to be local in time, we would get the same coefficient associated to the counterterm as for each different order in perturbation theory at which we evaluate the counterterm. In local in time field theories, the perturbative time-non-locality in encoded in the small higher derivative terms $\partial_t/\omega_{\rm UV}$.

So we have

$$P_{13,c_s} = c_s^2 D^4 \frac{k^2}{k_{\rm NL}^2} P_{11}(k) \tag{61}$$

Notice that the counterterm has the same k-functional form as the UV divergent part of the one loop diagram in (57). This means that we can define

$$c_s^2 = -\left(\frac{\Lambda}{k_{\rm NL}}\right)^{n+1} + c^2_{s,\,\rm finite} \tag{62}$$

to obtain

$$P_{1-\rm loop} = 2P_{13} + P_{22} + 2P_{13,c_s} = D^4 \left\{ c^2_{s,\,\rm finite} \frac{k^2}{k_{\rm NL}^2} + c_n \left(\frac{k}{k_{\rm NL}}\right)^{n+3} \right\} P_{11}(k)\ . \tag{63}$$

The result is finite and cutoff independent. And furthermore, we have that

$$P_{1-\rm loop} \ll P_{11} \quad \text{for} \quad k \ll k_{\rm NL} \tag{64}$$

so, perturbation theory is well defined. To achieve this, it was essential to introduce the stress tensor, which allowed us to reabsorb the UV difergencies.

Was this good? We have found a well defined perturbative expansion at the cost of introducing some counterterms. The prefactor of the non-analytic part, $\left(\frac{k}{k_{\rm NL}}\right)^{n+3} P_{11}(k)$, called c_n, is known and cannot be changed by the counterterms. It is predictive. Instead, the prefactor of the analytic part, $k^2 P(k)$, instead can be changed by the counterterm. The factor of c_s is a new coupling constant that can be either measured in the data (as we have done for the standard model of particle physics), or measured in simulations (one can also use some approximate treatments such as the mass functions, to have a prior for these parameters). Overall, the theory is still predictive (just bit less than a theory will a smaller number of parameters).

Let us check better the expansion parameters. The integrand of P_{13} has the following limits:

$$\frac{P_{13}(k)}{P_{11}(k)} \supset \epsilon_{\delta<} = \int_{q\sim k} d^3q P_{11}(q) \tag{65}$$

$$\begin{aligned} \frac{P_{13}(k)}{P_{11}(k)} &\supset \epsilon_{s>} = \int_{q\gg k} d^3q\ \frac{k^2}{q^2} P_{11}(q) \equiv (k\delta s_>)^2 \\ \frac{P_{13}(k)}{P_{11}(k)} &\supset \epsilon_{s<} = \int_{q\ll k} d^3q\ \frac{k^2}{q^2} P_{11}(q) \equiv (k\delta s_<)^2 \end{aligned} \tag{66}$$

The first contribution is the effect of $\partial^2\phi$, the force of gravity. The second is the ratio of the wavelength of interest with respect to the displacement associated to the shorter wavelength modes. In fact, the displacement is, at linear level,

$$s \sim v\, H^{-1} \sim \frac{\partial^i}{\partial^2}\delta\ , \qquad P_s \sim P_\delta/k^2 \tag{67}$$

The third is the ratio of the wavelength of interest with respect to the displacement associated to the longer wavelength modes [3] Notice that δ modes of wavelength shorter than the one of interest do not contribute. At the level of the UV, the contribution is suppressed by k^2/q^2: this was indicated by the stress tensor, indeed.

Show pictures

Infrared modes: There is also a contribution from infrared modes. In reality, this contribution is order one for modes larger than $k \sim 0.1\, h\,\mathrm{Mpc}^{-1}$. It is possible to resum the contribution of these modes because they simply correspond to long modes *displacing, translating* short modes. This can be done. Intuition was there for a long time, but the first correct formula (consistent with the principles of physics) was developed in the context of the EFT in [8] (see [9–11] for some simplifications of different power and of different level of accuracy). I do not have time to talk about it... maybe.

What about the stochastic counterterm? The UV limit of P_{22} is

$$P_{22}(k) \supset \int_{q\gg k} d^3q \frac{k^4}{q^4} P(q)^2 \sim k^4 \tag{68}$$

But indeed

$$\delta^{(2)}_{\rm stoch}(\vec k, a) = k_i k_j \int^a da'\ G(a,a')\ \Delta\tau^{ij}(a')\ , \tag{69}$$

$$\Rightarrow \quad \langle \delta^{(2)}_{\rm stoch}(\vec k, a)\delta^{(2)}_{\rm stoch}(\vec k', a)\rangle = k_i k_j k'_l k'_m \int^a da' \int^a da''\ G(a,a')G(a,a'')\langle \Delta\tau_{ij}(k,a')\Delta\tau_{lm}(k,a'')\rangle$$

Using that

$$\Delta\tau_{ij}(k,a')\Delta\tau_{lm}(k,a'') = \frac{\delta^{(3)}_D(\vec k+\vec k')}{k^3_{\rm NL}}\left(\epsilon_{\rm stoch,1}(a',a'')\ \delta^{il}\delta^{jm} + \epsilon_{\rm stoch,2}(a',a'')\ \delta^{ij}\delta^{lm}\right)\ , \tag{70}$$

we have

$$\begin{aligned} &\langle \delta^{(2)}_{\rm stoch}(\vec k, a)\delta^{(2)}_{\rm stoch}(\vec k', a)\rangle \\ &\quad = k^4 \int^a da' \int^a da''\ G(a,a')G(a,a'')\left(\epsilon_{\rm stoch,1}(a',a'') + \epsilon_{\rm stoch,2}(a',a'')\right) = \frac{k^4}{k^4_{\rm NL}}\widetilde\epsilon_{\rm stoch}(a) \end{aligned} \tag{71}$$

So, this has the exact k-dependence to correct the UV contribution from P_{22}.

[3] For IR-safe quantities, $\epsilon_{s<}$ should be modified to $\epsilon_{s<} = \int_{k_{\rm BAO}\simeq q \ll k} d^3q\ \frac{k^2}{q^2} P_{11}(q) \equiv (k\delta s_<)^2$, as only modes longer than the BAO scalem $k_{\rm BAO}$, contribute. Quantitatively, for the power spectrum of our universe, this does not change the answer.

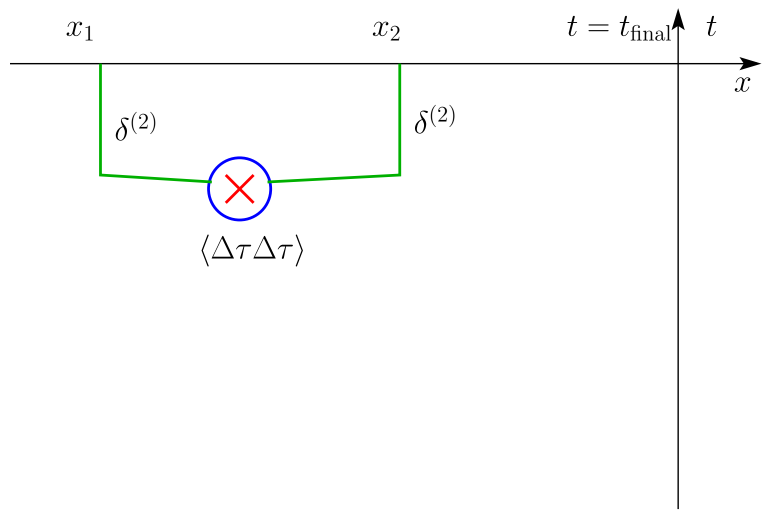

Figure 6:

For a particularly pedagogical discussion of renormalization in the EFTofLSS in the context of scaling universes, see [12] (see also [1, 13])

It is time to show some results on dark matter.

Before doing so, it might be useful to notice that there is an equivalent description. As for fluids there are the Eulerian and the Lagrangian coordinates, we can do the same also for our system. We can think of each non-linear scale as a particle endowed of a finite size, given by the non-linear scale. Particle with finite size evolve not as point-like particles. Since they have an extension, they feel the tidal tensor, and also, they can overlap. This leads to the following equation:

$$\frac{d^2\vec{z}_L(\vec{q},\eta)}{d\eta^2} + \mathcal{H}\frac{d\vec{z}_L(\vec{q},\eta)}{d\eta} \;=\; -\vec{\partial}_x \left[\Phi_L[\vec{z}_L(\vec{q},\eta)] + \frac{1}{2}Q^{ij}(\vec{q},\eta)\partial_i\partial_j\Phi_L[\vec{z}_L(\vec{q},\eta)] + \cdots\right] + \vec{a}_S(\vec{q},\eta)\;, \tag{72}$$

The quadrupole and the higher moments, as well as the stochastic force, are the counterterms that can be expressed in terms of long wavelength field using vevs, responses, and stochastic terms. This approach to the EFTofLSS based in Lagrangian space was developed in [14].

Uniqueness: One of the reasons why we know that EFT's are the correct description of the system is their universality, which means also their *uniqueness*. Indeed, only one set of equations can describe a given system. Therefore, different descriptions, if correct, can at best be equivalent. The difference between the Lagrangian-space EFTofLSS and the Eulerian-space EFTofLSS is the different number of parameters in which they Taylor-expand. The Lagrangian-space formulation does not expand in $\epsilon_{s<}$, while the Eulerian-space EFT does. However, the IR-resummed Eulerian-space EFTofLSS does not expand in $\epsilon_{s<}$ as well. So, all EFTofLSS's are the same and should give the same result, up to higher order terms that were not computed and that constitute the theoretical error.

3 Baryons

See [15]. So far we have talked about dark matter. But we know that there are baryons, which contribute and are affected by star formation physics, which moves them around. Can we develop an accurate description of baryons, notwithstanding the huge complications associated to star formation physiscs? In fact, star formation physics is so complicated that it cannot be even simulated. One can find simulations around, but they are models, nobody is claiming to describe the ab-initio physics, which means that they are creating some ad hoc recipes. This is the reason why there are many star formation models (AGN, feedback, no feedback, Supernovae, wind, no wind).

But let us observe nature. For how complicated star formation events are, baryons are still inside a cluster: they are not moved much around. This means that their overall displacement is of order $1/k_{\rm NL}$. The construction of the EFTofLSS for dark matter was just based on the fact that dark matter particles could move only $1/k_{\rm NL}$, and on longer distances we had a fluid-like system.

Here with baryons we have the same situation. So, baryons are just another fluid-like system! A universe of dark matter plus baryons is just a universe with two fluid-like systems.

The only difference with respect to the case of only dark matter is that there is number conservation for dark matter and for baryons separately. So, both fluids satisfy an exact continuity equation, so that

$$\frac{\partial N_{\rm dm}}{\partial t}=\int d^3x\dot\delta=-\int d^3x\partial_i(\pi^i)=0 \tag{73}$$

but the two system can exchange momentum.

In the case of dark matter only, we had on the right hand side

$$\dot\pi^i+\ldots=\partial_j\tau^{ij}, \tag{74}$$

so that

$$\frac{\partial\Pi^i}{\partial t}=\int d^3x\frac{\partial\pi^i}{\partial t}\supset\int d^3x\partial_j\tau^{ij}=0\ , \tag{75}$$

that is short distance physics could not change the overall momentum: momentum was conserved.

Instead, baryons and dark matter can exchange momentum, however, the overall momentum of the system is conserved. So we can write:

$$\begin{aligned}
&\nabla^2\phi=\frac{3}{2}H_0^2\frac{a_0^3}{a}(\Omega_c\delta_c+\Omega_b\delta_b) \qquad (76)\\
&\dot\delta_c=-\frac{1}{a}\partial_i((1+\delta_c)v_c^i)\\
&\dot\delta_b=-\frac{1}{a}\partial_i((1+\delta_b)v_b^i)\\
&\partial_i\dot v_c^i+H\partial_iv_c^i+\frac{1}{a}\partial_i(v_c^j\partial_jv_c^i)+\frac{1}{a}\partial^2\phi=-\frac{1}{a}\partial_i\left(\partial\tau_\rho\right)_c^i+\frac{1}{a}\partial_i(\gamma)_c^i\ ,\\
&\partial_i\dot v_b^i+H\partial_iv_b^i+\frac{1}{a}\partial_i(v_b^j\partial_jv_b^i)+\frac{1}{a}\partial^2\phi=-\frac{1}{a}\partial_i\left(\partial\tau_\rho\right)_b^i+\frac{1}{a}\partial_i(\gamma)_b^i\ ,
\end{aligned}$$

where

$$(\partial\tau_\rho)^i_\sigma = \frac{1}{\rho_\sigma}\partial_j\tau^{ij}_\sigma \ , \qquad (\gamma)^i_c = \frac{1}{\rho_c}V^i \ , \qquad (\gamma)^i_b = -\frac{1}{\rho_b}V^i \ . \tag{77}$$

There is, on top of the effective stress tensor, an effective force.

Again, as before, we can write

$$\begin{aligned} &-(\partial\tau_\rho)^i_\sigma\,(a,\vec{x}) + (\gamma)^i_\sigma(a,\vec{x}) = \\ &\qquad \int da' \left[\kappa^{(1)}_\sigma(a,a')\,\partial^i\partial^2\phi(a',\vec{x}_{\rm fl}(\vec{x};a,a')) + \kappa^{(2)}_\sigma(a,a')\,\frac{1}{H}\partial^i\partial_j v^j_\sigma(a',\vec{x}_{\rm fl}(\vec{x};a,a'))\ldots\right] \ , \end{aligned} \tag{78}$$

This theory is supposed to be able to describe the baryons and dark matter analytically, at long distances, with arbitrary precision.

Let us study a bit of the dynamics. In our universe, baryons and dark matter start at the CMB time with different velocities. However, the relative velocity rapidly decays. So, we can consider that they have the same velocity in the dark ages. At some point then, star formation begins, and baryons move differently due to the radiation pressure. This short distance physics effect is encoded in the effect of the counterterms.

The leading effect is again a c_s-like effect. We have

$$\Delta P_{b-c} \sim c_\star^2 k^2 P(k) \tag{79}$$

This means that the analytic form of the baryonic effect is known: all different star formation physics effects are encoded in a different $c_\star$ (similar to the fact that different dielectric coefficients fit all dielectric material). From the figure, we see that this seems to work.

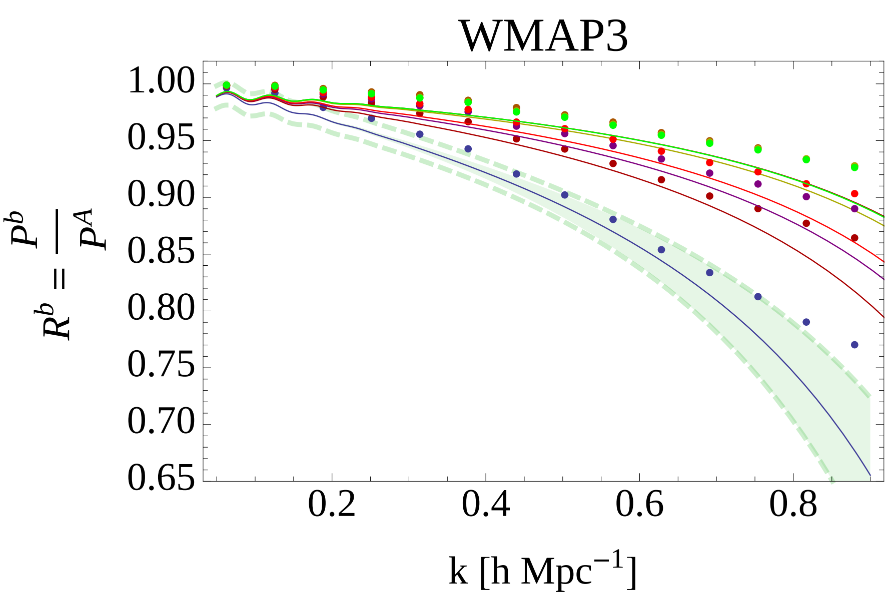

Figure 7: Fit of the EFTofLSS to the total-matter power spectrum with different starformation models. By adjusting $c_{s,\star}$ we seem to fit all star formation models.

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4 Galaxies, Halos, biased tracers

See [16, 17]. We wish to write how the distribution of galaxies depends on the distribution of the dark matter. Galaxies form because of gravitational collapse, therefore they will depend on the underlying values of the gravitational field and dark matter field. Since the overdensities of galaxies is a scalar quantity, it can only depend on similarly scalar quantities built out of these fields. Let us consider each of these terms one at a time (this discussion can be interpreted also as a more detailed discussion of the terms that can enter in the dark matter stress tensor).

Tidal tensor: Concerning the gravitational field, because of the equivalence principle, the number of galaxies at a given location can only depend on the gravitational potential ϕ with at least two derivatives acting on it, as it is for the curvature. ϕ without derivatives does appear in curvature terms only at non-linear level in terms such as $\phi\partial^2\phi$ or $(\partial\phi)^2$. These are general relativistic corrections, which are important only at long distances of order Hubble, where perturbations can be treated as linear to a very good approximation. We will therefore neglect these terms.

In the Eulerian EFT, the dark matter field is identified by the density field δ and the momentum field π^i [4]. This is a useful quantity because its divergence is related to the time derivative of the matter overdensity by the continuity equation. Due to Newton's equation, the density field is constrained to be proportional to $\partial^2\phi$, so it can be discarded as an independent field. Concerning the momentum field, clearly a spatially constant momentum field cannot affect the formation of galaxies. Indeed, the momentum is not a scalar quantity. Under a spatial diffeomorphism

$$x^i \quad \to \quad x^i + \int^\tau d\tau'\, V^i \tag{80}$$

the momentum shifts as

$$\pi^i \quad \to \quad \pi^i + V^i \rho \ . \tag{81}$$

where ρ is the dark matter density $\rho = \rho_b(1+\delta)$, with ρ_b being the background density.

gradient of velocity: Working with the field π^i has the advantage, as discussed in [4], that no new counterterm is needed to define correlation functions of $\partial_i\pi^i$ once the correlation functions of δ have been renomalized. Alternatively, one can work with the velocity field v^i, defined as

$$v(\vec{x},t)^i = \frac{\pi(\vec{x},t)^i}{\rho(\vec{x},t)} \ . \tag{82}$$

The velocity field has the advantage that $\partial_i v^j$ is a scalar quantity. However, v^i is defined as the ratio of two operators at the same location. It is therefore a composite operator that requires its own counterterm and a new renormalization even after the matter correlation functions have been renormalized [4] (see also [18]). As we will see, when dealing with biased tracer, one has to define contact operators in any event, and v^i has simpler transformation properties than π^i. Therefore, instead of working with π^i, we work with v^i. In analogy to what we have just discussed, the galaxy field can depend on v^i only through $\partial_j v^i$ and its derivatives.

$k_{\rm M}$**:** The field of collapsed objects at a given location will not depend just on the gravitational field or the derivatives of the velocity field at the same location. There will be a length scale enclosing the points of influence. This length scale will be of order the spatial range covered by the matter that ended up collapsing in a given collapsed object. We call the wavenumber associated to this scale k_M, as it depends on the nature of the object, most probably prominently through its mass. We expect $k_{\rm M} \sim 2\pi(\frac{4\pi}{3}\frac{\rho_{b,0}}{M})^{1/3}$, where M is the mass of the object and $\rho_{b,0}$ is the present day matter density. In particular, $k_{\rm M}$ can be different from $k_{\rm NL}$, the scale as which the dark matter field becomes non-linear [4]. If we are interested on correlations on collapsed objects of wavenumbers $k \ll k_{\rm M}$, we can clearly Taylor expand this spatially non-local dependence in spatial derivatives.

Stochastic: In addition, in general there is a difference between the average dependence of the galactic field on a given realization of the long wavelength dark matter fields, and its actual response in a specific realization. To account for this, we add a stochastic term ϵ to the general dependence of the galaxy field. ϵ is a stochastic variable with zero mean but with other non-trivial, Poisson-in-space-distributed, correlation functions.

Time derivative and their more physical description:

This suggest that we should add in the bias terms that go as $\frac{1}{\omega_{\rm short}}\frac{\partial}{\partial t}$, such as $\frac{1}{\omega_{\rm short}}\frac{\partial\, \partial^2\phi}{\partial t}$. It is pretty clear that these term are not diff. invariant. Under a time-dependent spatial diff., $\partial/\partial t$ shifts as [5]

$$\frac{\partial}{\partial t} \quad\to\quad \frac{\partial}{\partial t} - V^i\frac{\partial}{\partial x^i} \ . \tag{83}$$

A diff. invariant combination can be formed by allowing the presence of the dark matter velocity field v^i without derivatives acting on it, and defining a *flow time-derivative*, familiar from fluid dynamics, as

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + v^i\frac{\partial}{\partial x^i} \ . \tag{84}$$

We are therefore led to naively lead to include terms of the form

$$\delta_M(\vec x,t) \supset c_{D_t\partial^2\phi}(t)\ \frac{1}{H^2}\frac{1}{\omega_{\rm short}}\frac{D\,\partial^2\phi}{Dt} + \dots \ . \tag{85}$$

In reality, the situation is even more peculiar, at least at first. In fact, let us ask ourselves what is the scale $\omega_{\rm short}$ that suppresses the higher derivative operators. Naively, $\omega_{\rm short}$ is of order H, as this is the timescale of the short modes collapsing into halos. This is the same time-scale as the long modes we are keeping in in our effective theory! This means that the parameters controlling the Taylor expansion in $\frac{1}{\omega_{\rm short}}\frac{D}{Dt} \sim \frac{H}{\omega_{\rm short}}$ is actually of order one. Therefore, what we have to do is to generalize these formulas: since the formation time of a collapsed object is of order Hubble, we have to allow for the density of the collapsed objects to depend on the underlying long-wavelength fields evaluated at all times up to an

[4] $k_{\rm NL}$ can be unambiguously defined as the scale at which dark matter correlation functions computed with the EFT stop converging.

[5] People familiar with the Effective Field Theory of Inflation [19,20] might remember that $g^{0\mu}\partial_\mu$ is invariant, not $\partial/\partial t$.

order one Hubble time earlier. This means that the formula relating compact objects and long-wavelength fields will actually be non-local in time. Therefore we have

$$\begin{aligned}\delta_M(\vec{x},t) \simeq \int^t dt'\ H(t')\ \Bigg[&\bar{c}_{\partial^2\phi}(t,t')\ \frac{\partial^2\phi(\vec{x}_{\rm fl},t')}{H(t')^2} \\ &+\bar{c}_{\partial_i v^i}(t,t')\ \frac{\partial_i v^i(\vec{x}_{\rm fl},t')}{H(t')} + \bar{c}_{\partial_i\partial_j\phi\partial^i\partial^j\phi}(t,t')\ \frac{\partial_i\partial_j\phi(\vec{x}_{\rm fl},t')}{H(t')^2}\frac{\partial^i\partial^j\phi(\vec{x}_{\rm fl},t')}{H(t')^2} + \dots \\ &+\bar{c}_\epsilon(t,t')\ \epsilon(\vec{x}_{\rm fl},t') + \bar{c}_{\epsilon\partial^2\phi}(t,t')\ \epsilon(\vec{x}_{\rm fl},t')\frac{\partial^2\phi(\vec{x}_{\rm fl},t')}{H(t')^2} + \dots \\ &+\bar{c}_{\partial^4\phi}(t,t')\ \frac{\partial^2_{x_{\rm fl}}}{{k_{\rm M}}^2}\frac{\partial^2\phi(\vec{x}_{\rm fl},t')}{H(t')^2} + \dots\ \Bigg] .\end{aligned} \tag{86}$$

Here $\bar{c}_{\dots}(t,t')$ are dimensionless kernels with support of order one Hubble time and with size of order one, and $\vec{x}_{\rm fl}$ is defined iteratively as

$$\vec{x}_{\rm fl}(\vec{x},\tau,\tau') = \vec{x} - \int_{\tau'}^{\tau} d\tau''\ \vec{v}(\tau'',\vec{x}_{\rm fl}(\vec{x},\tau,\tau''))\ . \tag{87}$$

where τ is conformal time.

Another way to derive the above formula (86) is to notice that the local number density of galaxies, $n_{\rm gal}(\vec{x},t)$, is given by a very-complicated formula. This complicated formula depends on a huge amount of variables: all the cosmological parameters, all the local density of dark matter and baryons, the local gradients of the velocities, the local curvature, but also the electron and proton mass and the electroweak charges (as they affect the molecular levels that affect the cooling mechanism and consequently the star formation mechanism), and many more variables like this one. And everything must be evaluated on the past light cone of the point under consideration [6]. We can write:

$$\begin{aligned}&n_{\rm gal}(\vec{x},t) = \\ &f_{\text{very complicated}}(\{H,\Omega_m,\Omega_b,w,\rho_{\rm dm}(\vec{x}',t'),\rho_{\rm b}(\vec{x}',t),\partial_i\partial_j\phi(\vec{x}',t'),\dots,m_p,m_e,g_{ew},\dots\}_{\text{on past ligh cone}})\end{aligned} \tag{88}$$

However, if we are interested only in long-wavelength correlations of this quantity, we notice that the only variables that carry spatial dependence are a few and that these quantities, at long wavelengths, have small fluctuations. We can therefore Taylor expand (88) in those quantities, to obtain (86).

In this way, correlation functions of galaxies can be computed in terms of correlation functions of dark-matter density and velocity fields, that we compute before. In particular, the non-locality in time is treated exactly as before: each perturbative solution has a factorized form in terms of time and spatial dependence, and we can ultimately perform the integration easely.

Again, this theory is supposed to match distribution of galaxies with arbitrary precision.

[6] In other words, Galaxies are very UV-sensitive objects. This is one way to say why it is so complicated to simulate their formation from first principles.

In summary, we have the following schematic structure of the perturbative expansion for dark-matter, δ, and galaxies, δ_M, correlation functions:

$$\langle\delta(\vec{k})\delta(\vec{k})\rangle' \sim \tag{89}$$
$$\langle\delta(\vec{k})\delta(\vec{k}))\rangle'_{\text{tree}} \times \underbrace{\left[1+\left(\frac{k}{k_{\text{NL}}}\right)^2+\ldots+\left(\frac{k}{k_{\text{NL}}}\right)^D\right]}_{\text{Derivative Expansion}} \underbrace{\left[1+\left(\frac{k}{k_{\text{NL}}}\right)^{(3+n)}+\ldots+\left(\frac{k}{k_{\text{NL}}}\right)^{(3+n)L}\right]}_{\text{Loop Expansion}}$$
$$+\underbrace{\left[\left(\frac{k}{k_{\text{NL}}}\right)^4+\left(\frac{k}{k_{\text{NL}}}\right)^6+\ldots\right]}_{\text{Stochastic Terms}},$$

$$\langle\delta_M(\vec{k})\delta(\vec{k})\rangle' \sim P_{11}(k_1) \tag{90}$$
$$\times\Bigg\{\underbrace{\left[c_{\partial^2\phi}+c_{\partial^4\phi}\left(\frac{k}{k_{\text{M}}}\right)^2+\ldots+c_{\partial^{2D}\phi}\left(\frac{k}{k_{\text{M}}}\right)^{2D-2}\right]}_{\text{Linear Bias Derivative Expansion}} \underbrace{\left[1+\left(\frac{k}{k_{\text{NL}}}\right)^{(3+n)}+\ldots+\left(\frac{k}{k_{\text{NL}}}\right)^{(3+n)L}\right]}_{\text{Matter Loop Expansion}}$$
$$+\underbrace{\left[c_{(\partial^2\phi)^2}+c_{\partial^2(\partial^2\phi)^2}\left(\frac{k}{k_{\text{M}}}\right)^2+\ldots+c_{\partial^{2D-2}(\partial^2\phi)^2}\left(\frac{k}{k_{\text{M}}}\right)^{2D-2}\right]}_{\text{Quadratic Bias Derivative Expansion}}$$
$$\times\underbrace{\left(\frac{k}{k_{\text{NL}}}\right)^{3+n}}_{\text{Quadratic Bias}} \underbrace{\left[1+\left(\frac{k}{k_{\text{NL}}}\right)^{(3+n)}+\ldots+\left(\frac{k}{k_{\text{NL}}}\right)^{(3+n)L}\right]}_{\text{Matter Loop Expansion}}$$
$$+\underbrace{\left[c_{(\partial^2\phi)^3}+c_{\partial^2(\partial^2\phi)^3}\left(\frac{k}{k_{\text{M}}}\right)^2+\ldots+c_{\partial^{2D-2}(\partial^2\phi)^3}\left(\frac{k}{k_{\text{M}}}\right)^{2D-2}\right]}_{\text{Cubic Bias Derivative Expansion}}$$
$$\times\underbrace{\left(\frac{k}{k_{\text{NL}}}\right)^{2(3+n)}}_{\text{Cubic Bias}} \underbrace{\left[1+\left(\frac{k}{k_{\text{NL}}}\right)^{(3+n)}+\ldots+\left(\frac{k}{k_{\text{NL}}}\right)^{(3+n)L}\right]}_{\text{Matter Loop Expansion}}\Bigg\}$$
$$+\underbrace{\left[c_{\epsilon_0}\,c_{m,\text{stoch},1}+c_{a,\,\epsilon_2}\left(\frac{k}{k_{\text{M}}}\right)^2+c_{m,\text{stoch},2}\left(\frac{k}{k_{\text{NL}}}\right)^2+\ldots\right]}_{\text{Stochastic Bias Derivative Expansion}} \underbrace{\frac{1}{({k_{\text{M}}}^3 k_{\text{NL}}^3)^{1/2}}\left(\frac{k}{k_{\text{NL}}}\right)^2}_{\text{Stochastic Bias}}+\ldots\ .$$

For the additional fields that galaxies and dark matter can depend on in the presence of baryons, see [17], and in the presence of primordial non-gaussianities [17, 21, 22]. These expressions need to be IR-resummed. By the equivalence principle, the formula is exactly the same as for dark matter (see [17]).

An equivalent but different basis to the one developed in [16], (that is a change of basis), that some people might find more easy to handle than the one presented here has been then proposed in [23].

5 Redshift space distortions

See [24]. When we look at objects in redshift space, we look at them in redshift space, not in real-space coordinates. The relation between the position in real space $\vec{x}$ and in redshift space $\vec{x}_r$ is given by (see for example [25]):

$$\vec{x}_r = \vec{x} + \frac{\hat{z}\cdot\vec{v}}{aH}\hat{z}\ . \tag{91}$$

Mass conservation relates the density in real space $\rho(\vec{x})$ and in redshift space $\rho_r(\vec{x}_r)$:

$$\rho_r(\vec{x}_r)\ d^3x_r = \rho(\vec{x})\ d^3x\ , \tag{92}$$

which implies

$$\delta_r(\vec{x}_r) = \left[1+\delta\left(\vec{x}(\vec{x}_r)\right)\right]\left|\frac{\partial\vec{x}_r}{\partial\vec{x}}\right|^{-1}_{\vec{x}(\vec{x}_r)} - 1\ . \tag{93}$$

In Fourier space, this relationship becomes

$$\delta_r(\vec{k}) = \delta(\vec{k}) + \int d^3x\, e^{-i\vec{k}\cdot\vec{x}}\left(\exp\left[-i\frac{k_z}{aH}v_z(\vec{x})\right]-1\right)\left(1+\delta(\vec{x})\right)\ . \tag{94}$$

We now assume we can Taylor expand the exponential of the velocity field to obtain an expression that is more amenable to perturbation theory (this is where the Eulerian approach that we describe here, and the Lagrangian approach that we mentioned earlier differ, but once the Eulerian-space has been IR-resummed, they are equivalent). For the purpose of this paper, we will show formulas that are valid only up to one loop. We therefore can Taylor expand up to cubic order, to obtain

$$\begin{aligned}
\delta_r(\vec{k}) \simeq\ & \delta(\vec{k}) + \\
& \int d^3x\, e^{-i\vec{k}\cdot\vec{x}}\left[\left(-i\frac{k_z}{aH}v_z(\vec{x}) + \frac{i^2}{2}\left(\frac{k_z}{aH}\right)^2 v_z(\vec{x})^2 - \frac{i^3}{3!}\left(\frac{k_z}{aH}\right)^3 v_z(\vec{x})^3\right)\right. \\
& \qquad \left. + \left(-i\frac{k_z}{aH}v_z(\vec{x}) + \frac{i^2}{2}\left(\frac{k_z}{aH}\right)^2 v_z(\vec{x})^2\right)\delta(\vec{x})\right] \\
=\ & \delta(\vec{k}) - i\frac{k_z}{aH}v_z(\vec{k}) + \frac{i^2}{2}\left(\frac{k_z}{aH}\right)^2[v_z^2]_{\vec{k}} - \frac{i^3}{3!}\left(\frac{k_z}{aH}\right)^3[v_z^3]_{\vec{k}} - i\frac{k_z}{aH}[v_z\delta]_{\vec{k}} + \frac{i^2}{2}\left(\frac{k_z}{aH}\right)^2[v_z^2\delta]_{\vec{k}}\ ,
\end{aligned} \tag{95}$$

where in the last line we have introduced the notation $[f]_{\vec{k}} = \int d^3x\, e^{-i\vec{k}\cdot\vec{x}}\, f(\vec{x})$.

The product of fields at the same location is highly UV sensitive. As usual, we need to correct for every dependence we get from the non-linear scale. Therefore, we need to replace:

$$
\begin{aligned}
[v_z^2]_{R,\vec{k}} &= \hat{z}^i\hat{z}^j\left\{[v_iv_j]_{\vec{k}}+\left(\frac{aH}{k_{\rm NL}}\right)^2\left[c_1\delta^{ij}+\left(c_2\delta^{ij}+c_3\frac{k^ik^j}{k^2}\right)\delta(\vec{k})\right]+\ldots\right\} \qquad (96)\\
&= [v_z^2]_{\vec{k}}+\left(\frac{aH}{k_{\rm NL}}\right)^2\left[c_1+c_2\,\delta(\vec{k})\right]+\left(\frac{aH}{k_{\rm NL}}\right)^2 c_3\,\frac{k_z^2}{k^2}\,\delta(\vec{k})+\ldots\ ,\\
[v_z^3]_{R,\vec{k}} &= \hat{z}^i\hat{z}^j\hat{z}^l\left\{[v_iv_jv_l]_k+\left(\frac{aH}{k_{\rm NL}}\right)^2 c_1\left(\delta_{ij}\,v_l(\vec{k})+2\text{ permutations}\right)+\ldots\right\}\\
&= [v_z^3]_{\vec{k}}+\left(\frac{aH}{k_{\rm NL}}\right)^2 3\,c_1\,v_z(\vec{k})+\ldots\ ,\\
[v_z^2\delta]_{R,\vec{k}} &= \hat{z}^i\hat{z}^j\left\{[v_iv_j\delta]_k+\left(\frac{aH}{k_{\rm NL}}\right)^2 c_1\ \delta_{ij}\,\delta(\vec{k})+\ldots\right\}\\
&= [v_z^2\delta]_{\vec{k}}+\left(\frac{aH}{k_{\rm NL}}\right)^2 c_1\,\delta(\vec{k})+\ldots\ .
\end{aligned}
$$

So, computing correlation functions in redshift space have reduced to compute correlation functions in physical space of dark matter density and velocities.

In the presence of primordial non-Gaussianities and baryonic fields, additional counterterms are needed. See [26].

IR-resummation in redshift space was developed in [24, 26].

6 Calculations and comparison with numerical simulations

Here is an incomplete list of calculations and comparisons with simulations that have been performed in the context of the EFTofLSS.

1. **dark matter**: power spectrum at one-loop [1], at two-loops [4, 27–30]. Bispectrum at one-loop [9, 31].

2. **biased tracers:** power spectrum at one-loop [17]. Tree-level Bispectrum [17], leading-in-mass one-loop bispectrum [32]. Leading-in-high-mass higher-derivative terms in tree-level bispectrum [33].

3. **dark matter in redshift space:** one-loop power spectrum [26]

4. **tracers in redshift space:** one-loop power spectrum [34].

5. **Baryonic effects:** one-loop power spectrum: [15].

6. **dark-energy**: one-loop power spectrum: [37, 38].

7. **neutrinos:** one-loop power spectrum [35] and tree-level bispectrum [36].

7 More stuff

Interesting things that I have not time to discuss about.

1. **IR-Resummation** [8] (see [9–11] for some simplifications of different power and of different level of accuracy). For IR-resummation for biased tracers (which is the same as for dark-matter, see [17]). In redshift space, see [26]. For an application of the IR-resummation-in-redshift-space to biased tracers, see [34].

2. **primordial non-gaussianities:** the presence of primordial non-Gaussianities implies that the UV-sensitive terms could depend on terms that are different than the ones allowed by diff. invariance. For real space, see [17, 21, 22]. For the new counterterms that arise in redshift space, see [26].

3. **neutrinos:** the EFTofLSS has been upgraded to describe also the effect due to neutrinos [35, 36].

4. **dark energy**: the EFTofLSS has been upgraded to describe also the effect due to dark energy [37, 38].

5. **analytic calculation:** a formalism to compute correlation functions in a practically analytical way has been developed in [39].

8 Looking Ahead

1. say that it is the result of 30 years of reaserch: we could not do this before, now that we can, we have thousands of things to compute. For example, to start, take every n-point function of any observable, find out what is the maximum order at which it was computed, see if you can compute the next order

2. I do not think we should expect many older people to work on this. This means that, if the EFTofLSS is right (as I think it is), they are leaving an open door for the younger people to contribute.

3. Most importantly, there are LSS observations that are already limited by lack of theoretical predictions (evidently, all the methods that had been developed before the EFTofLSS had not been sufficient to do this). So, we should compute all correlation functions at the highest order possible, as much as we can, and compare to data. We should also use the available models and/or simulations to obtain priors for the parameters of the EFTofLSS, so that we limit the price we pay due to the free parameters.

4. To me, we are in the following situation. It is as if QCD had been discovered, and LHC was going to turn on in a couple of years (actually, it has already turned on, as my understanding is such that we are not analyzing the data because of lack of accurate-enough theory predictions). At that time, people started to do computations and those results now stay in the history of physics, as QCD happened to be the right theory. It seems to me (though I could be wrong), that we are in the same situation now with LSS and the EFTofLSS, as I think that the EFTofLSS is right (or equivalently, if it is not right, I think it being wrong would represent a revolution of physics).

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