Integrating out matter loops, 1-loop effective action
a la Coleman Weinberg

$$e^{-S_{eff}(\bar{g}_{\mu\nu})} = \int D\phi \; e^{-S(\bar{g},\phi)} \qquad \phi = \bar{\Phi} + \chi$$

$$= \int D\chi \; e^{-\chi \frac{\delta^2 S(\bar{g},\bar{\phi})}{\delta\chi^2}\chi} \qquad \bar{\Box} = \bar{g}^{\mu\nu}\bar{\nabla}_\mu\bar{\nabla}_\nu$$

$$= \int D\chi \; e^{-\int d^4x\, d^4y\, \chi(x)\left[\bar{\Box} - m^2\right]\chi(y)}$$

Coleman - Weinberg

$$S_{eff} = \frac{1}{2}\log\det\left[\bar{\Box} - m^2\right]$$

$$= \frac{1}{2}\,\mathrm{Tr}\log\left[\bar{\Box} - m^2\right]$$

$$\sim \int \frac{d^4k}{(2\pi)^4}\log\left[\bar{g}^{ab}k_a k_b + m^2\right]$$

$$k_a \to \hat{k}_a \qquad \bar{g}^{ab}k_a k_b = \delta^{ab}\hat{k}_a\hat{k}_b$$

$$d^4k \to \sqrt{\bar{g}}\, d^4\hat{k}$$

$$S_{eff} \sim \sqrt{\bar{g}}\,\underbrace{\int \frac{d^4\hat{k}}{(2\pi)^4}\log\left[\delta^{ab}\hat{k}_a\hat{k}_b + m^2\right]}_{\text{Indep of metric}}$$

if we don't include a reg scale,
so if we perform dim-reg, the only
scale this can depend on is m^2

$$\Rightarrow \sim m^4\log\left(\frac{\mu}{m}\right) + \begin{matrix}\text{derivative}\\ \text{corrections}\end{matrix}$$

So for every field coupled to gravity,
we expect it to contribute to the vacuum
energy with an amount $\sim m^4$
(actually it turns out that bosons contribute positively
and fermions negatively...)

Integrating out matter loops, 1-loop effective action a la Coleman Weinberg

So we expect

$$\Lambda_{vacuum} \sim -m_{e^-}^4 - m_{\mu}^4 - m_t^4 + m_{Higgs}^4 + m_Z^4 + m_{W^+}^4 + m_{W^-}^4 + \dots$$

$$\sim (200\ \mathrm{GeV})^4$$

This would lead to

$$H_0 \sim \frac{(200\ \mathrm{GeV})^2}{(10^{18}\ \mathrm{GeV})} \sim \frac{10^4\, 10^{18}}{10^{18}\, 10^9}\,\mathrm{eV} \sim 10^{-5}\,\mathrm{eV} \sim (2\,\mathrm{cm})^{-1}$$

If this was correct, the size of the "observable Universe" would roughly the same as a lychee...

Old Cosmological Constant problem

formulated in the 80's, how come $\Lambda = 0$ when matter loops contribute with such a large amount...

how can we set $\Lambda = 0$?

Since then, we know that the Universe is accelerating

New Cosmological Constant Problem

why is the observed C.C. so small ?

Most models of Dark Energy ignore the old C.C. problem (ie assume that $\Lambda = 0$ for a reason or another) and try to explain the late-time acceleration of the Universe using instead a dynamical fluid.

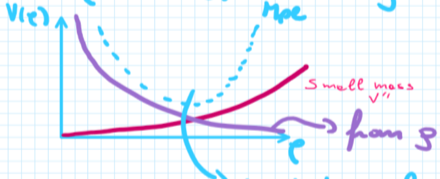
Some models of modified gravity try to tackle the old C.C. problem...

c) Chameleon Mechanism

$$\mathcal{L} = \int d^4x \left[-\frac{1}{2}(\partial\phi)^2 - V(\phi) + e^{-\frac{\beta\phi}{M_{pl}}} \rho \right]$$

environment, eg. a star, or density of galaxy

$$\Box\phi = V''(\phi) + \frac{\beta}{M_{pl}} e^{-\frac{\beta\phi}{M_{pl}}} \rho$$

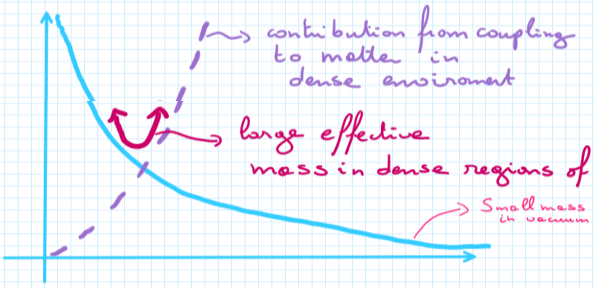


In practise we take rather

$$V(\phi) = \frac{\Lambda^{n+4}}{\phi^n}$$

and coupling to matter $A(\phi)\rho$

with $A(\phi) = e^{\phi/M_c}$



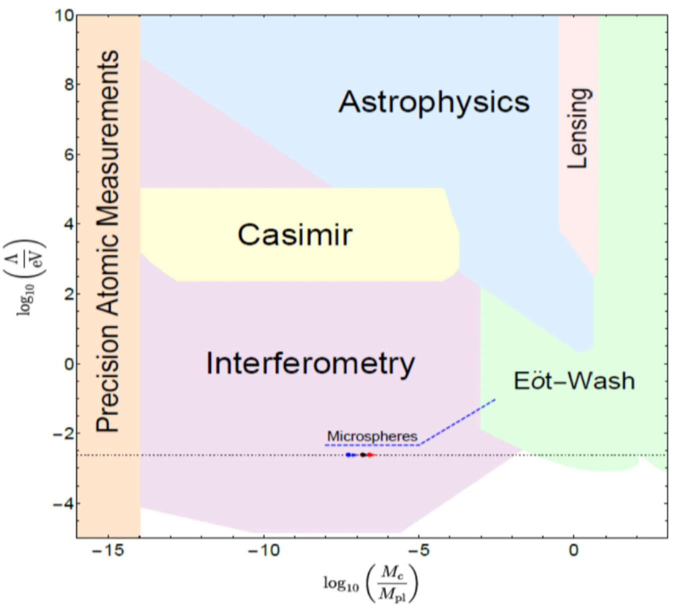


Figure 4: Current bounds on the parameters M_c and Λ for $n = 1$ chameleon models. The regions excluded by each specific test are indicated in the figure; the region labelled astrophysics contains the bounds from both Cepheid and rotation curve tests. The dashed line indicates the dark energy scale $\Lambda = 2.4$ meV. The black, red, and blue arrows show the lower bound on M_c coming from neutron bouncing and interferometry. The blue corresponds to the bounds of [133] and the red to the bounds of [134].

From 1709.09071

Only way an $f(R)$ model of gravity (or dark energy) is viable is if it has a chameleon mechanism (some of them do - but it depends on function f)

II Dark Energy

Tuesday, June 26, 2018 2:48 PM

3) Screening Mechanisms
eg. of the symmetron

In many models of dark Energy, we need to "hide" or "screen" the dark energy field in Solar System, Galaxy, maybe for LSS,...

Chameleon is a way to make mass dependent on environment but there is other ways..

$$\mathcal{L} = \underbrace{-\frac{1}{2}(\partial\phi)^2}_{\text{Vainshtein}} - \frac{1}{2}\underbrace{m^2}_{\text{chameleon}}\phi^2 + \underbrace{A(\phi)}_{\text{weak coupling}}T$$

many models rely on a weak coupling to matter,

effectively

$$A(\phi)T \sim \frac{\phi}{M_c}T$$

weak coupling if $M_c \ggg M_{pl}$

but naively we would expect $M_c \simeq M_{pl}$

→ leads to naturalness issues

Chameleon, Symmetron, Vainsthein are Screening mechanism to have an order $M_c \sim M_{pl}$ coupling and yet avoid 5^{th} force constraints

The Symmetron mechanism works by making the coupling itself being suppressed in dense regions

This time take a symmetry-breaking potential

$$V(\phi) = -\frac{1}{2}\mu^2\phi^2 + \frac{\lambda}{4}\phi^4$$

and coupling to matter

$$A(\phi) = 1 + \frac{\phi^2}{2M_S^2}$$

So $V_{eff}(\phi) = -\frac{1}{2}\mu^2\left(1 - \frac{\rho}{\mu^2 M_S^2}\right)\phi^2 + \frac{\lambda}{4}\phi^4$

for $\boxed{\rho < \rho_* = \mu^2 M_S^2}$

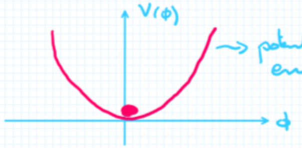
we see the symmetry-breaking effective potential and the field background is close to minimum at $\pm\bar{\phi} \neq 0$

perturbations on top of this backgrd see a small effective mass and large coupling to matter

$$A(\bar{\phi} + \delta\phi)\,T = \underbrace{\frac{\bar{\phi}}{M_S^2}}_{\substack{\text{not a small} \\ \text{coupling if } \rho < \rho_*}} \delta\phi\, T$$

however if $\boxed{\rho > \rho_*}$ situation is dramatically different...

potential's minimum is then at $\phi = 0$



→ potential in dense environments where $\rho > \rho_*$

Vacuum at $\bar{\phi} = 0$ and about this solution, the field fluctuation does not couple to matter: $\phi = \underbrace{\bar{\phi}}_{=0} + \delta\phi$

$$\rightarrow \quad A(\bar{\phi} + \delta T) = \frac{\bar{\phi}}{M_S^2}\,\delta\phi\, T$$

$= 0$ if $\rho > \rho_*$ and $\bar{\phi} = 0$

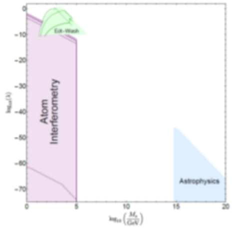


Figure 10: The current bounds on the symmetron parameters M_s and λ. The region of parameter space excluded by each specific test is indicated in the figure. The Eöt-Wash region corresponds to $\mu = 2.4$ meV; the outlines for values $\mu = \{10^{-4}, 10^{-3}, 10^{-2}\}$ eV are shown by the solid, dashed, and dotted green lines respectively. The atom interferometry lines correspond to the regions excluded for $\mu = \{10^{-4}, 10^{-4.5}, 10^{-5}, 10^{-5}, 2.4 \times 10^{-3}\}$ eV from top to bottom respectively, the latter value corresponding to the dark energy scale. The astrophysical bounds are insensitive to the value of μ for the values considered here.

From 1709.09071

4) Kinetic screening or Vainshtein

Another alternative we can explore is to have a Screening mechanism based on the kinetic term, imagine,

$$\mathcal{L} = -\frac{1}{2} Z (\partial \chi)^2 - \frac{1}{2} m^2 \chi^2 + \frac{1}{M_{pl}} \chi T$$

then we should normalize the field

$$\chi = Z^{-1/2} \phi$$

$$\mathcal{L} = -\frac{1}{2} (\partial \phi)^2 - \frac{1}{2} \frac{m^2}{Z} \phi^2 + \frac{1}{M_{pl} Z^{1/2}} \phi T$$

if in dense regions of Universe we had $Z \gg 1$, then the coupling of ϕ to matter is Suppressed...

how can we make $Z \gg 1$?
answer: Through kinetic interactions...

e.g.: Screening in $P(X)$
(called k-essence if leading to dark energy and if it also has a kinetic Screening, then it is k-mouflage)

$$X = -\frac{1}{2} \frac{(\partial \chi)^2}{\Lambda^4}$$

$$\mathcal{L} = \Lambda^4 P(X) + \frac{\chi}{M_{pl}} T .$$

Consider the case where

$$T_0 = M\, \delta^{(3)}(r)$$

mass localized at the origin.

$$\pi = \pi(r) \qquad X = -\frac{1}{2} \frac{\pi'(r)^2}{\Lambda^4}$$

$$\mathcal{L} = r^2 \left[\Lambda^4\, P\left(-\frac{\pi'^2}{2\Lambda^4}\right) - \frac{M}{M_{pl}} \pi \frac{\delta(r)}{r^2} \right]$$

$$\frac{\delta}{\delta \pi} : \quad \partial_r \left[r^2 \Lambda^4 P' \left(-\frac{\pi'}{\Lambda^4}\right) \right] = -\frac{M}{M_{pl}} \delta(r)$$

$$\Big\downarrow \int dr$$

$$\pi' P' = -\frac{M}{M_{pl}} \frac{1}{r^2}$$

if $P = X \quad P' = 1$

$$\Rightarrow \pi' = -\frac{M}{M_{pl}} \frac{1}{r^2}$$

if for instance $\Lambda^4 P = X + \frac{X^2}{2\Lambda^4}$

then eom is

$$\pi' \left(1 + \frac{\pi'^2}{2\Lambda^4}\right) = -\frac{M}{M_{pl}} \frac{1}{r^2}$$

as r large, π' is small, linear term dominates

and $\pi' = -\frac{M}{M_{pl}} \frac{1}{r^2}$

if r is small, π' is large, cubic term dominates

$$\rightarrow \pi' \sim -(2\Lambda^4)^{1/3} \left(\frac{M}{M_{pl}}\right)^{1/3} \frac{1}{r^{2/3}}$$

Transition occurs when $\pi'^2 \sim 2\Lambda^4$

ie when $\left(\frac{M}{M_{pl}}\right)^2 \frac{1}{r^4} \sim 2\,\Lambda^4$

$$r \sim \left(\frac{M}{M_{pl}}\right)^{1/2} \Lambda^{-1}$$

Now look at fluctuations on that background

$$T = T_0 + \delta T, \quad \chi = \chi(r) + \delta\chi$$

$$\mathcal{L} \sim \underbrace{P'(\bar{x})}\,(\partial\delta\chi)^2 + \text{interactions} + \frac{\delta\chi}{M_{pl}}\delta T$$

$$\hookrightarrow \sim \frac{M}{M_{pl}} \frac{1}{r^2} \frac{1}{\chi'} \gg 1 \quad \text{as } r \ll r_*$$

In this scenario, the redressing Z of the kinetic term is given by $P' \gg 1$ close to the source.

As $r \gg r_*$, Scalar field is can mediate DE
as $r \ll r_*$, Scalar field gets frozen and decouples from the rest of matter.

II Dark Energy

Tuesday, June 26, 2018 8:11 PM

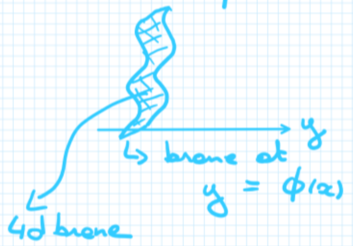
5) Galileons

Galileon Scalar fields are 4d Scalar fields $\phi(x^\mu)$ that enjoy a shift & galileon Symmetry.

The galileon sym is a **space dependent global Symetry**

$$\delta\phi = c + v_\mu x^\mu$$

These Symmetries can be seen to be the reminiscent of Poincaré in 1 extra dim,



translation invariance in 5d

$\Rightarrow \quad y \to y + c$

↳ Sym for ϕ

$\phi \to \phi + c$

boosts in 5d $\quad y \to \Lambda^5{}_\mu x^\mu \quad \to$ lead to galileon invariance $\phi \to \phi + v_\mu x^\mu$ in the non-relativistic limit

To construct a Lagrangian which is invariant under the Shift & Galileon Symetry, we could simply construt it out of $\Pi_{\mu\nu} = \partial_\mu \partial_\nu \pi$

eg. $\mathcal{L} = \Box\pi + (\Box\pi)^2 + \Box\pi[\pi^2] + \dots$

however, $\Box\pi$ total derivative

$(\Box\pi)^2 + \dots$ higher derivative in eq. of motion

and ... no kinetic term

however we can see that just kinetic term

$$\mathcal{L}_2 = -\frac{1}{2}(\partial\pi)^2 = \frac{1}{2}\pi\Box\pi$$

already satisfies the Symmetry

$$\delta\mathcal{L}_2 = \frac{1}{2}\delta\pi\,\Box\pi + \frac{1}{2}\pi\underbrace{\Box\delta\pi}_{=0}$$

$$= \frac{1}{2}\underbrace{(\Box\delta\pi)}_{=0}\pi$$

$$= 0$$

$$\mathcal{L}_2 \sim \pi\,\varepsilon^{\mu\nu\alpha\beta}\varepsilon^{\mu'}{}_{\nu\alpha\beta}\,\Pi_{\mu\mu'}$$

ε: antisymmetric Levi-Civita Symbol

This shows us how to generalize this:

$$\mathcal{L}_1 \sim \pi\,\underbrace{\varepsilon^{\mu\nu\alpha\beta}\varepsilon_{\mu\nu\alpha\beta}}_{=4!}$$

$$\mathcal{L}_2 \sim \pi\,\varepsilon^{\mu\nu\alpha\beta}\varepsilon_{\mu'\nu\alpha\beta}\,\Pi^{\mu'}{}_{\mu}$$

$$\sim (\partial\pi)^2$$

$$\mathcal{L}_3 \sim \pi\,\varepsilon^{\mu\nu\alpha\beta}\varepsilon^{\mu'\nu'}{}_{\alpha\beta}\,\Pi_{\mu\mu'}\Pi_{\nu\nu'}$$

$$\sim (\partial\pi)^2\Box\pi \sim \pi\left((\Box\pi)^2 - (\partial_\mu\partial_\nu\pi)^2\right)$$

$$\mathcal{L}_4 \sim \pi\,\varepsilon\,\varepsilon\,\Pi\,\Pi\,\Pi$$

$$\sim (\partial\pi)^2\left((\Box\pi)^2 - (\partial_\mu\partial_\nu\pi)^2\right)$$

$$\mathcal{L}_5 \sim \pi\,\varepsilon\,\varepsilon\,\Pi\,\Pi\,\Pi\,\Pi$$

$$\sim (\partial\pi)^2\left((\Box\pi)^3 - 3\Box\pi(\partial_\mu\partial_\nu\pi)^2 + 2(\partial_\mu\partial_\nu\pi)^3\right)$$

And in d-dimensions there would be $(d+1)$ galileon invariant of that form.

They are all invariant under galileon symmetry:

$$\mathcal{L}_{n+1} \sim \pi\,\varepsilon\varepsilon\,\overbrace{\Pi\,\Pi\cdots\Pi}^{n\text{-times}}$$

$$\delta\mathcal{L}_{n+1} \sim \delta\pi\,\varepsilon\varepsilon\,\Pi\,\Pi\cdots\Pi$$

$$+\cancel{n\,\pi\,\varepsilon\varepsilon\,\underbrace{\delta\Pi}_{=0}\,\Pi\cdots\Pi}$$

$$\sim \delta\pi\,\varepsilon^{\mu_1\cdots\mu_n}\varepsilon^{\nu_1\cdots\nu_n}\,\partial_{\mu_1}\partial_{\nu_1}\pi\cdots\partial_{\mu_n}\partial_{\nu_n}\pi$$

$\downarrow$ integrate by parts

$$\sim \cancel{\underbrace{\partial_{\mu_1}\partial_{\nu_1}\delta\pi}_{=0}\,\varepsilon\varepsilon\,\pi\,\Pi\cdots\Pi}$$

$$+\,\partial_{\nu_1}\delta\pi\,\underbrace{\varepsilon^{\mu_1\cdots\mu_n}}_{\text{antisymmetric in }\mu_1,\mu_2}\varepsilon^{\nu_1\cdots\nu_n}\,\pi\,\underbrace{\partial_{\mu_1}\partial_{\mu_2}\partial_{\nu_2}}_{\text{symmetric in }\mu_1,\mu_2}\pi\cdots$$

$\longleftrightarrow$

$\rightarrow\ =0$

$$+\,\delta\pi\,\varepsilon\,\varepsilon\,\pi\,\partial_{\mu_1}\partial_{\nu_1}\partial_{\mu_2}\partial_{\nu_2}\pi\cdots$$

$=0$ as well

So $\delta\left[\int d^4x\,\mathcal{L}_n\right]=0$

The galileon action are galileon invariant (even though the Lagrangian is not....)

and the e.o.m. are 2^{nd} derivative:

$$\frac{\delta \mathcal{L}_n}{\delta \pi} \sim n \, \mathcal{E}\,\mathcal{E} \, \overbrace{\Pi \Pi \cdots \Pi}^{n \text{ times}}$$

$\rightarrow$ at most 2^{nd} order en derivative.

These galileon Lagrangian satisfy a non-renormalization theorem which means that within this framework, higher order operators can be generated by quantum corrections (like for all the EFT, this is entirely normal), but these Galileon operators themselves do not get renormalized

can consider

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 + \frac{1}{\Lambda^3}(\partial\phi)^2 \Box\phi + \frac{\phi}{M_{pl}} T$$

with $\Lambda \ll M_{pl}$

and not get detuned by quantum corrections

Can have Dark-Energy Solution

$$\phi \sim \frac{\Lambda^3}{H_0} t \rightarrow H_0^2 \sim \frac{\Lambda^3}{M_{pl}}$$

To have dark-energy behaviour,

$$\Lambda^3 \sim H_0^2 M_{pl} \sim (1000\,km)^{-3}$$
$$\sim (10^{-13}\,eV)^3$$

$\Lambda <<< M_{pl}$

of course $\frac{\Lambda^3}{M_{pl}^3} \sim \frac{H_0^2}{M_{pl}^2}$ Same tuning as original cosmological constant, but it is a tuning which remains stable against quantum corrections.

This Galileon models also exhibit a Vainshtein Screening Mechanism

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 + \frac{1}{\Lambda^3}(\partial\phi)^2\Box\phi + \frac{\phi T}{M_{pl}}$$

$$\phi = \bar{\phi} + \delta\phi$$
$$T = \bar{T} + \delta T$$

$$\mathcal{L} = -\frac{1}{2} Z^{\mu\nu}\partial_\mu\delta\phi\,\partial_\nu\delta\phi + \frac{1}{\Lambda^3}(\delta\phi)^2\Box\delta\phi + \frac{\delta\phi\,\delta T}{M_{pl}}$$

$$Z^{\mu\nu} = \eta^{\mu\nu} + \underbrace{\frac{1}{\Lambda^3}\left[\partial^\mu\partial^\nu\bar{\phi} - \eta^{\mu\nu}\Box\bar{\phi}\right]}_{\gg 1 \text{ in vicinity of matter}}$$

Canonically normalizing,

$$\hat{\phi} = \sqrt{Z}\,\delta\phi \quad \text{(Symbolically)}$$

$$\mathcal{L} = -\frac{1}{2}(\partial\hat{\phi})^2 + \frac{1}{\Lambda^3 Z^{3/2}}(\partial\hat{\phi})^2\Box\hat{\phi} + \frac{1}{M_{pl}\,Z^{1/2}}\hat{\phi}\,\delta T$$

then the new Scale of interactions is suppressed:

$$\Lambda_*^3 = \Lambda^3 Z^{3/2} \gg \Lambda^3$$

Typically if we take $T \sim M_{Earth}$

$$\Lambda \sim (1000\,km)^{-1} \Rightarrow \Lambda_* \sim (1\,cm)^{-1}$$

$$Z \sim 10^{16}$$

III Modified Gravity

Tuesday, June 26, 2018 9:47 PM

Models of modified gravity often come with additional degrees of freedom

eg. Spin-2

massless GR: $+2$, -2

massive (massive gravity, DGP, ...): $+2$, $+1$, 0, -1, -2

If you like to give a mass to graviton break diff invariance -

$$S = \int d^4x \, \frac{M_{Pl}^2}{2} \sqrt{-g} \left[R - \frac{m^2}{2} \left(h_{\mu\nu}^2 - h^2 \right) + \cdots \right]$$

Fierz-Pauli mass term

$$h_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}$$

gauge invariant

not gauge invariant

instead promote $\eta_{\mu\nu} \longrightarrow \tilde{\eta}_{\mu\nu} = \eta_{ab} \partial_\mu \phi^a \partial_\nu \phi^b$

ϕ^a : 4 Scalar fields

can write ϕ^a as $\phi^a = x^a + A^a + \partial^a \pi$

let's ignore these

$$\tilde{\eta}_{\mu\nu} = \eta_{\mu\nu} + 2 \partial_\mu \partial_\nu \pi + (\partial_\mu \partial_\nu \pi)^2$$

π always enters with Galileon Symmetry

If we want the result to be free of any Ostrogradsky instability (ghost) then the result will need to look a Galileon (at least in some limit)

Motivation behind IR modifications of gravity is to directly tackle C.C. problem (degravitation)

No known explicit way to make it work yet...

GR $\Rightarrow H^2 = \frac{8\pi G}{3}\Lambda$ large $\Lambda \to$ large H

modification so that $H^2 = \frac{8\pi}{3}\int dk\, G_{eff}(k)\, \rho(k)$

with $G_{eff}(0) \to 0$ then Λ = const

so $\rho_\Lambda(k) = \Lambda\,\delta(k)$

$\to$ effect on acceleration is small

way to do this effectively is through a mass term

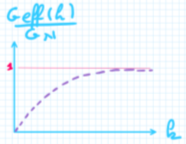
in GR, $\Box h_{\mu\nu} = G_N T_{\mu\nu}$

in mG, $(\Box - m^2) h_{\mu\nu} = G_N T_{\mu\nu}$

$\Box h_{\mu\nu} = G_{eff}(\Box) T_{\mu\nu}$

with $G_{eff}(\Box) = \frac{G_N \Box}{\Box - m^2}$

$\to G_{eff}(0) \to 0$



In this case C.C. no longer needs to be small

the problem is replaced by a smallness of m

but m is much more stable against quantum corrections

$\delta m^2 \sim m^2$

So m could be small without needing to worry as much

Modified Gravity - Galileons - Horndeski

Wednesday, June 27, 2018 11:16 AM

Dark-energy in GR

$m_\nu \sim 10^{-3}\,eV$

$m_{de} \sim 10^{-33}\,eV$

$m = 0$
Spin-2

Multi gravity (GR + exotic dark energy)

$m_\nu \sim 10^{-3}\,eV$

$m_{new} \sim 10^{-33}\,eV$
Spin-2

$m = 0$
Spin-2

If only the massless m=0 mode and one massive mode, corresponds to bi-gravity, if multiple new massive state then multi-gravity.
In either case, we can think of that as gravity + exotic type of dark energy or dark matter

Modified gravity

$m_\nu \sim 10^{-3}\,eV$

$m_g \sim 10^{-33}\,eV$

~~$m = 0$ Spin-2~~