Constraining cosmic curvature by using age of galaxies and gravitational lenses

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Outline

1. Introduction

2. Test of FLRW metric : Using cosmic chronometers

3. Test of curvature : Using the mean image separation of strong gravitational lenses

4. Discussion

Introduction

- The spatial curvature is one of the most fundamental issue of modern cosmology

- Estimation of curvature of the Universe (Ω_{k0}) is directly linked with

  - The validity of FLRW metric,
  - Degeneracy with dark energy equation of state parameter
  - Cosmic inflation and
  - The ultimate fate of the Universe.

- The recent constraint on curvature ($|\Omega_{k0}| < 0.005$) was obtained by the newest Planck 2015 observations.

Test of FLRW metric

- FLRW metric represents the homogeneous and isotropic Universe at sufficiently large scales

$$ds^2 = -c^2dt^2 + a^2(t)\left[\frac{dr^2}{(1-kr^2)} + r^2d\theta^2 + r^2sin^2\theta d\varphi^2\right]$$

where k = 1, 0,−1 for closed, flat and open geometry of the space.

- In FLRW Universe, the transverse comoving distance can be written as

$$D_c(z) = \frac{c}{H_0\sqrt{|\Omega_{k0}|}} \; S\left[\sqrt{|\Omega_{k0}|}\int_0^z \frac{dz}{E(z)}\right] \qquad where\ \Omega_{k0} = \frac{-kc^2}{{H_0}^2{a_0}^2}\ \&\ H(z) = H_0\,E(z)$$

- By taking the first derivative of comoving distance, we can redefine

$$\Omega_{k0} = \frac{H(z)^2{D_c}'^2 - c^2}{{H_0}^2{D_c}^2}$$

Clarkson et.al. (2008)

- For FLRW metric to hold, estimate of present curvature density derived using observables at different redshift must remain constant.

Test of FLRW metric

- To check the consistency of this relation, we need an Independent datasets of
  - comoving distance D_c
  - its first derivative D_c'
  - And Hubble parameter $H(z)$
- Calculation of transverse comoving distance

$$H(z) = \frac{-1}{(1+z)} \left(\frac{dt}{dz} \right)^{-1}$$

we obtain,

$$D_c = \begin{cases} \frac{c}{H_0 \sqrt{|\Omega_{k0}|}} \; sinh \left[H_0 \sqrt{|\Omega_{k0}|} \int_z^0 (1+z') \frac{dt}{dz'} \, dz' \right] & for \;\; \Omega_{k0} > 0 \\ c \int_z^0 (1+z') \frac{dt}{dz'} \, dz' & for \;\; \Omega_{k0} = 0 \\ \frac{c}{H_0 \sqrt{|\Omega_{k0}|}} \; sin \left[H_0 \sqrt{|\Omega_{k0}|} \int_z^0 (1+z') \frac{dt}{dz'} \, dz' \right] & for \;\; \Omega_{k0} < 0 \end{cases}$$

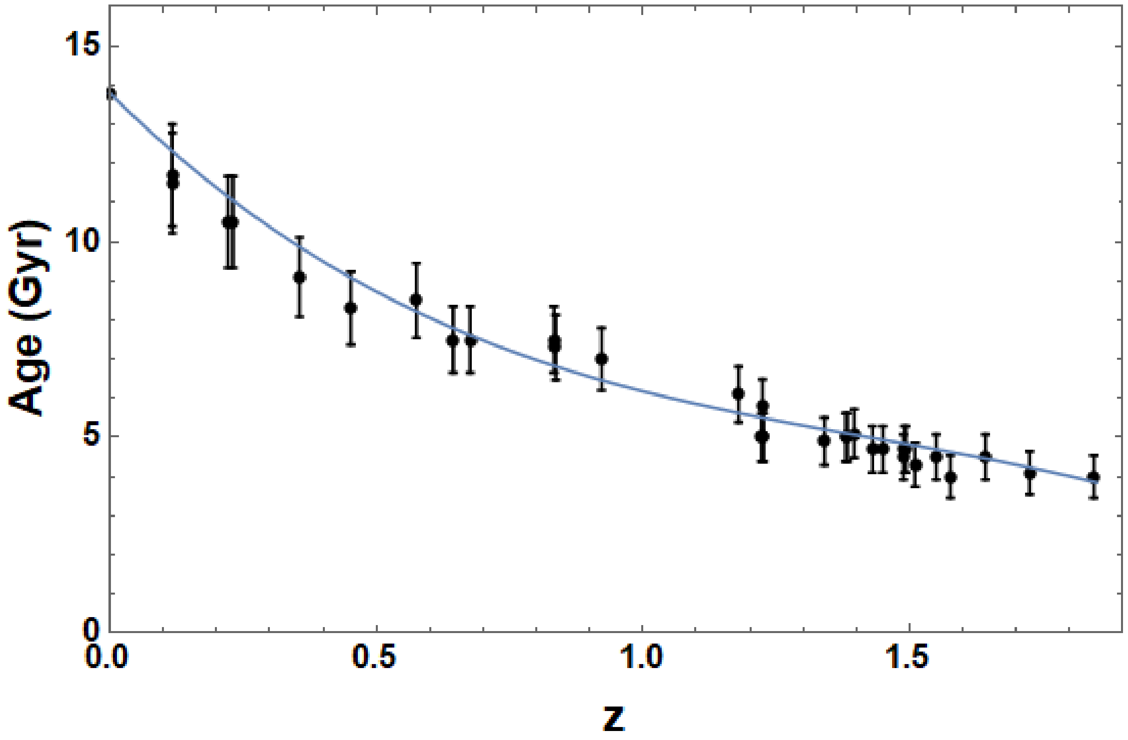
Test of Homogeneity: Datasets

- **Age of galaxies dataset**

  32 old and passive galaxies ***(0.11 < z < 1.84).***

  Incubation time $t_{inc} = 1.50 \pm 0.45\ Gyr$ **(Wei et al.2015)**

  Present age of universe, $t_0 = 13.799 \pm 0.021\ Gyr$ **(Planck 2015)**



$\chi^2{}_{reduced} = 0.61$

- **Polynomial fit**

$$t = A + Bz + Cz^2$$

$$\frac{dt}{dz} = B + 2Cz$$

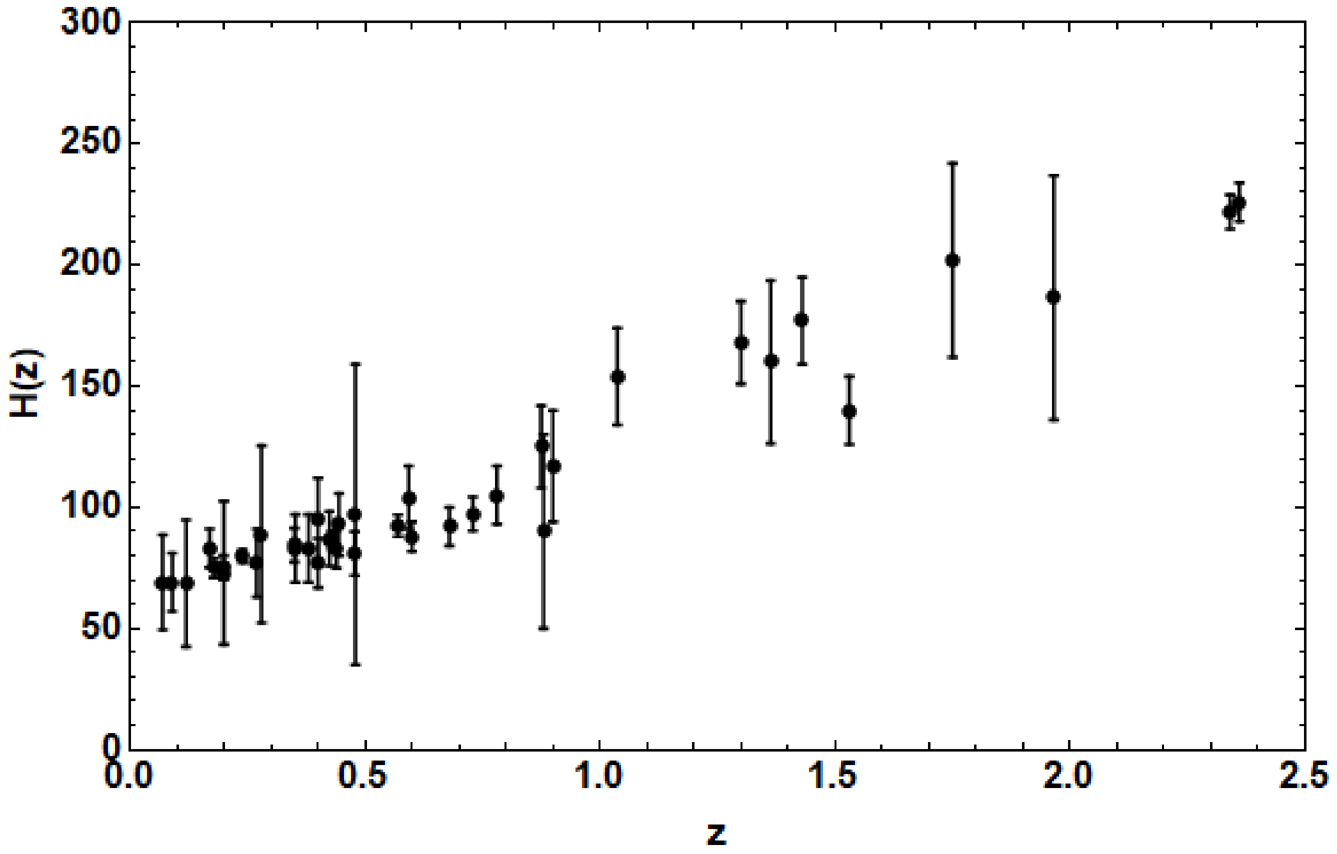
- **For a flat universe**

$$D_c = c\left[-B\left(z + \frac{z^2}{2}\right) - C(z^2 + \frac{2z^3}{3})\right]$$

$$D_c{}' = c[-B(1+z) - 2C(z + z^2)]$$

Test of Homogeneity: Dataset

- **Hubble dataset**
  - *H(z)* data consisting of 38 data points in the redshift range *(0.07 < z < 2.36).*



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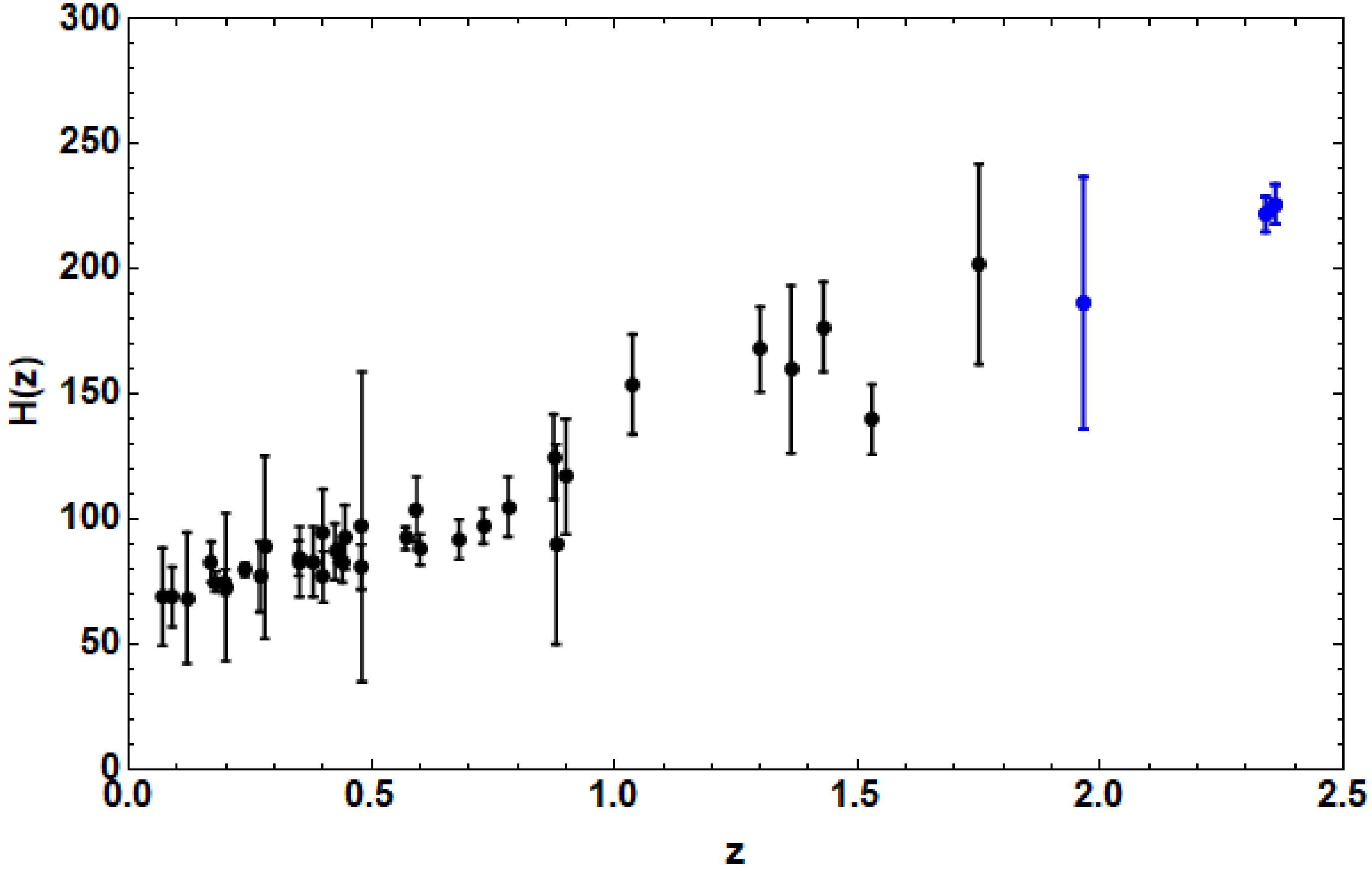

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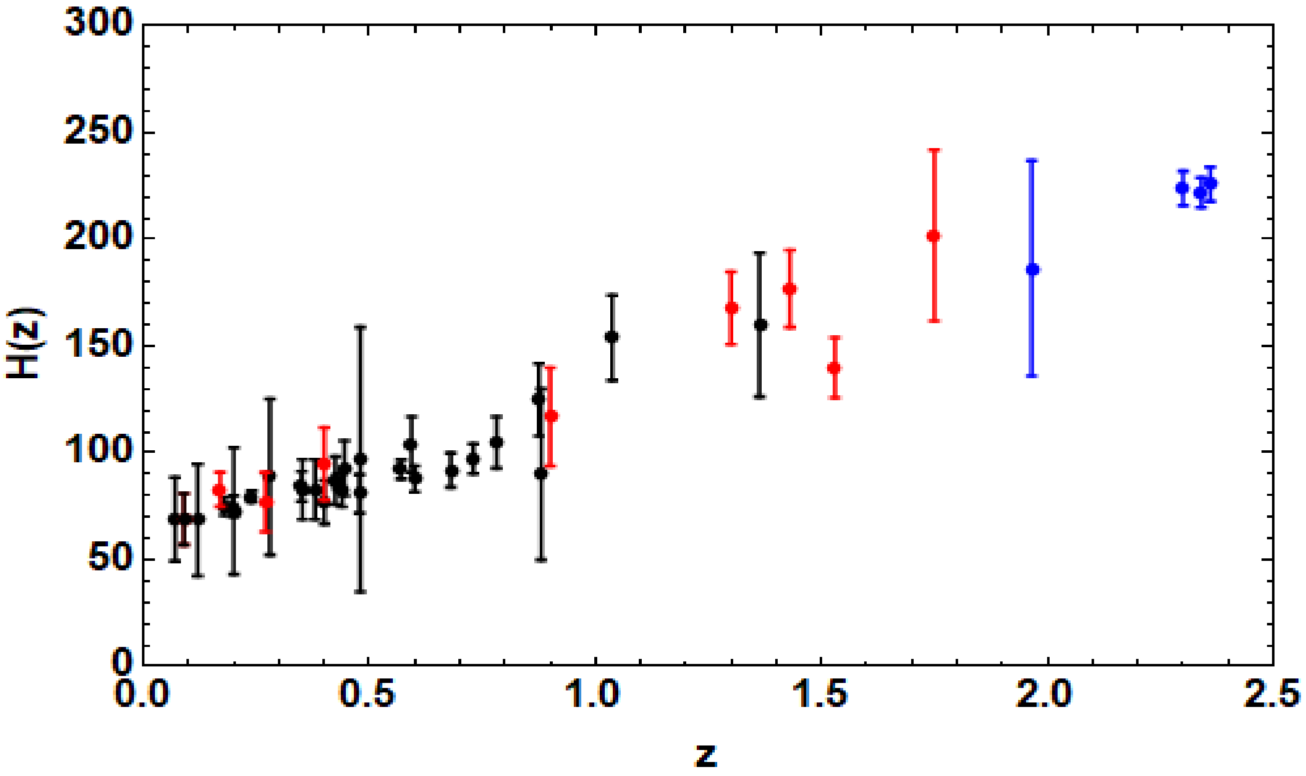

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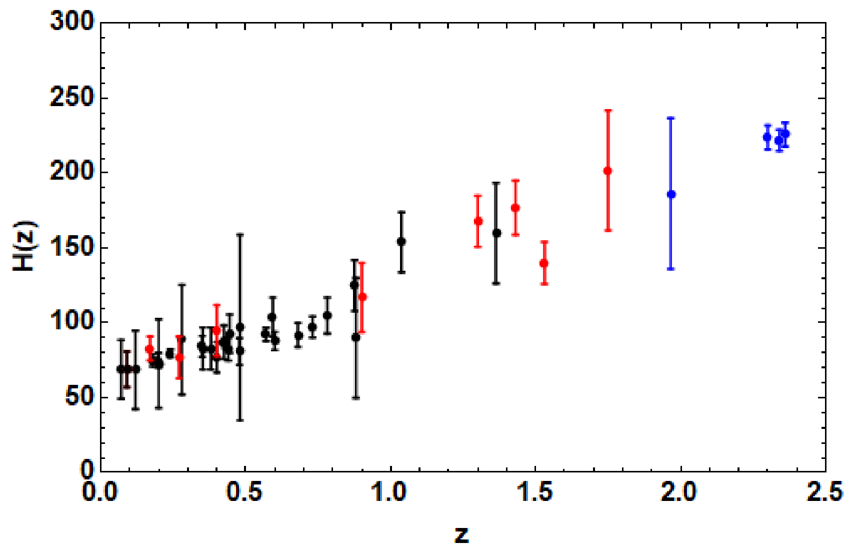
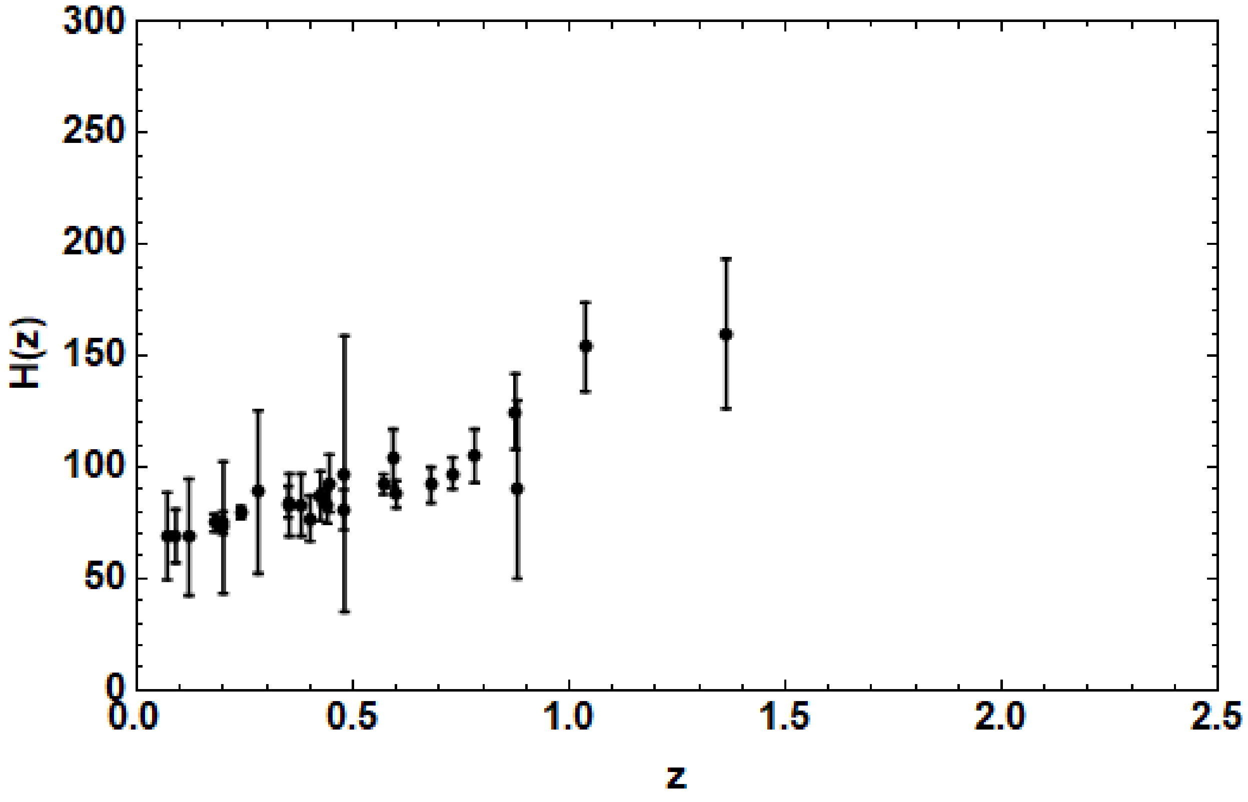


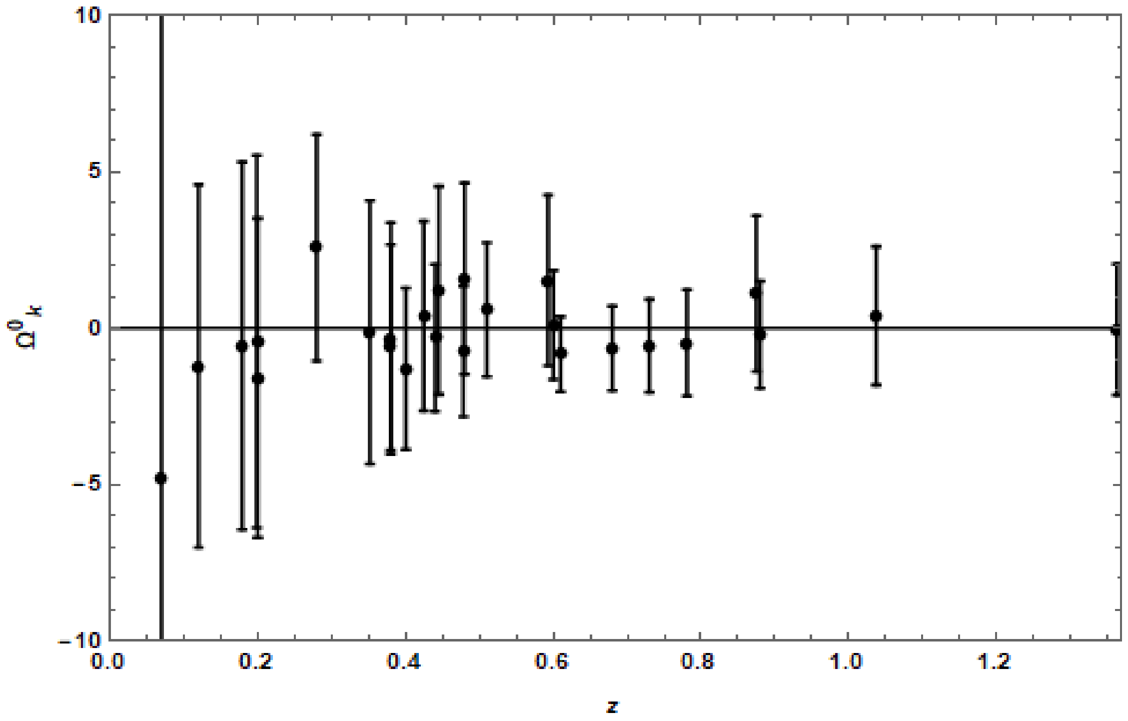


Test of Homogeneity: Result

- Error propagation in Ω_{k0}

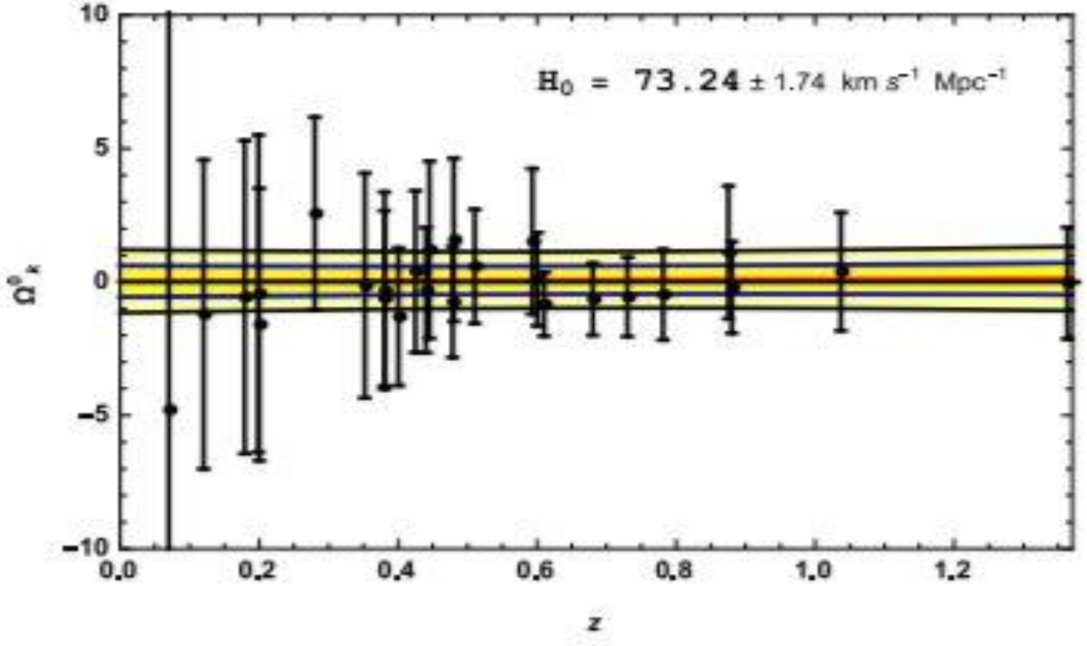
$$\sigma^2_{\Omega_{k0}} = 4\,(\Omega_{k0})^2 \left[\left(\frac{\sigma_{D_c}}{D_c}\right)^2 + \left(\frac{\sigma_{H_0}}{H_0}\right)^2\right] + 4\left[\Omega_{k0} + \left(\frac{c}{H_0 D_c}\right)^2\right]^2 \left[\left(\frac{\sigma_{D_c'}}{D_c'}\right)^2 + \left(\frac{\sigma_H}{H}\right)^2\right]$$

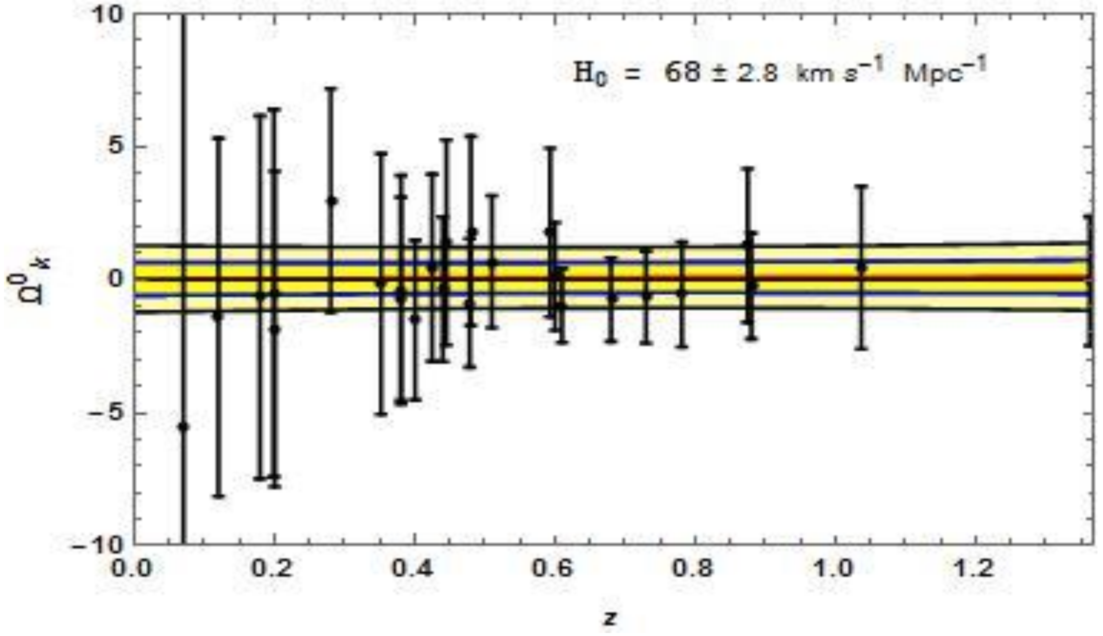
Where σ_{H_0}, σ_H, σ_{D_c} and $\sigma_{D_c'}$ are the error in H_0, ***H(z)***, D_c and D_c' respectively



- On applying model independent non-parametric smoothing technique Gaussian Process

Test of Homogeneity: Result





H_0 ($km\ sec^{-1} Mpc^{-1}$)	Ω_{k0}
73.24 ± 1.74	0.025 ± 0.57
68 ± 2.8	0.036 ± 0.62

- The reconstructed plot completely encloses Ω_{k0}=0 within 1σ confidence level and remains constant along *z*
- This shows consistency with the assumption of homogeneity of the universe and also concordance with the *FLRW* metric

Test of Curvature: Using Strong Gravitational Lenses

- Strong gravitational lensing (SGL) is a phenomenon in which light coming from a distant source get distorted to form multiple images of source in the presence of a foreground galaxy.

- **Statistical properties of SGL**

- Let us assume that lensing galaxies are non evolving with comoving density n_0

- Let effective Einstein radius of the lens is given by a_{cr} .

- The differential probability $\mathrm{d}\tau$ of a beam of light to interact with uniformly distributed lenses at redshift z would be

$$d\tau = n_0(1+z_L)^3 \sigma \frac{cdt}{dz_L}\, dz_L \qquad \text{where } \sigma = \pi {a_{cr}}^2$$

and
$$\frac{cdt}{dz_L} = \frac{c}{H_0(1+z_L)} \frac{1}{\sqrt{\Omega_m(1+z_L)^3+\Omega_\Lambda+(1-\Omega_m-\Omega_\Lambda)(1+z_L)^2}}$$

- The total probability of interaction

$$\tau = \int_0^{z_S} \frac{d\tau}{dz_L}\, dz_L$$

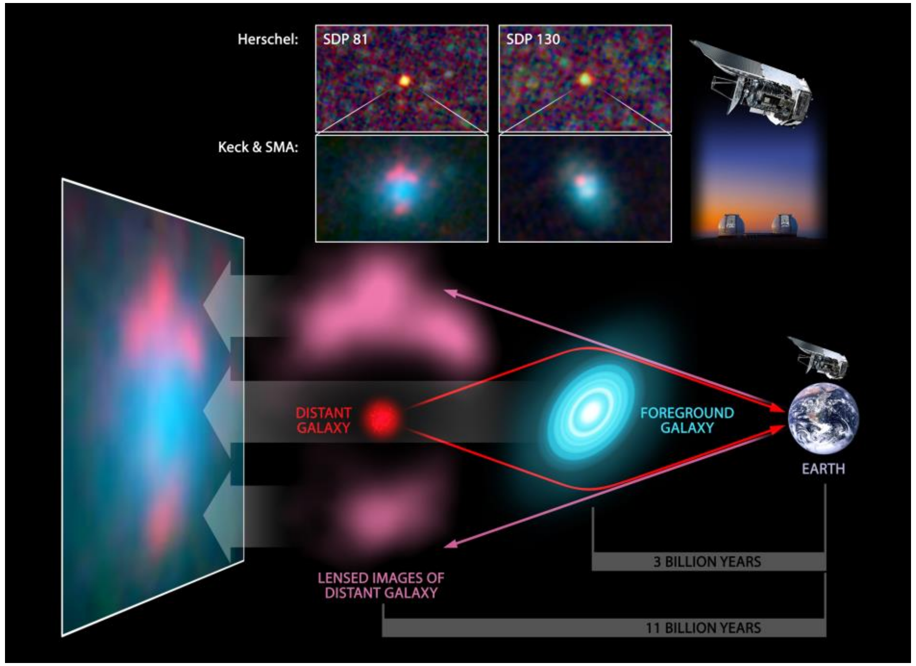
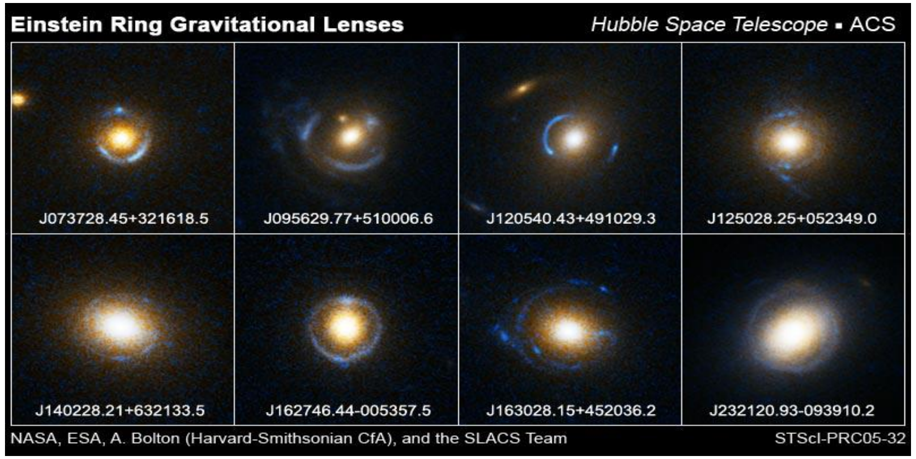


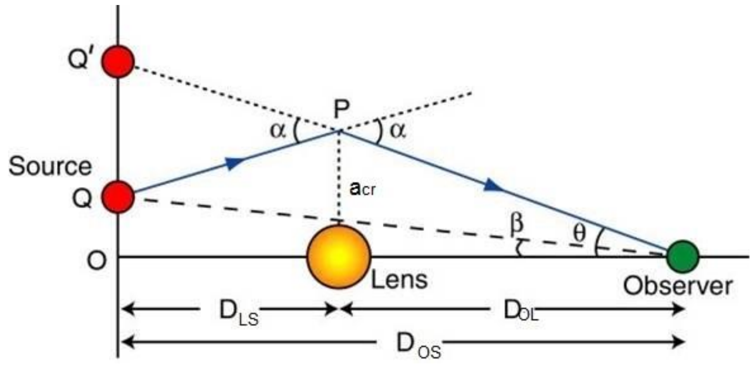
Image credits: (Credit: ESA / NASA /JPL-Caltech / Keck / SMA)



Test of Curvature: Using Strong Gravitational Lenses

- On integrating over the full range of z_L we get the overall mean image separation

$$< \Delta\theta > = \frac{1}{\tau}\left[\int_0^{z_s} \overline{\Delta\theta}\, \frac{d\tau}{dz_L}\, dz_L\right] \qquad \text{where } \overline{\Delta\theta}(z_L) = \frac{2a_{cr}}{D_{OL}}$$



Credits: http://www.jb.man.ac.uk/distance/frontiers/glens/section2.html

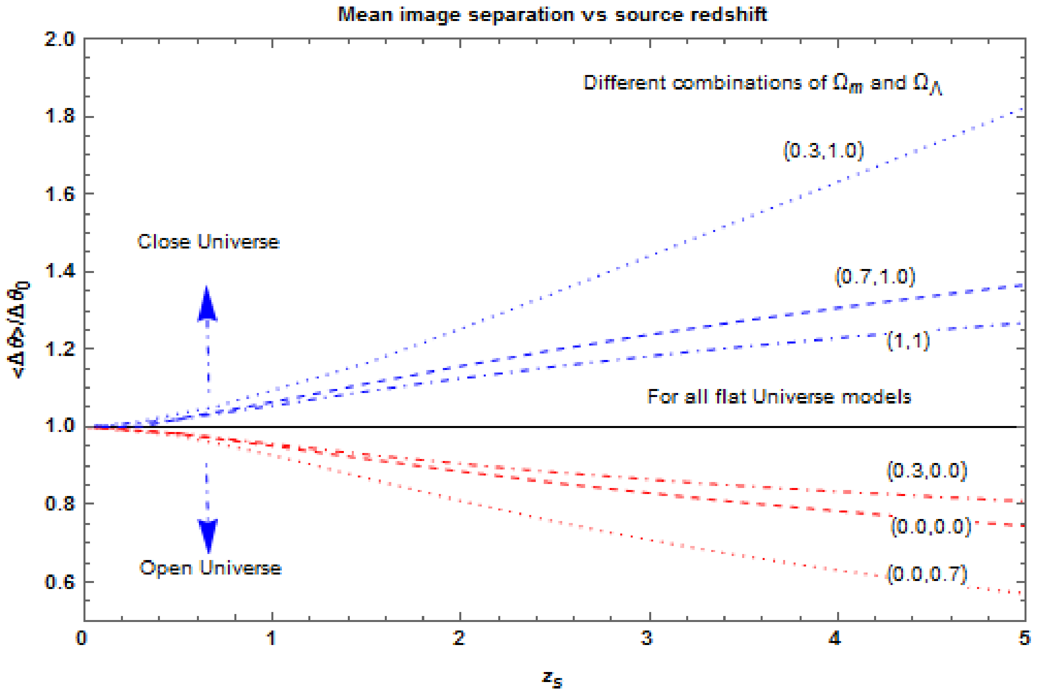
- For SIS mass profile of lens galaxy with velocity dispersion (v)

$$a_{cr} = 4\pi \left(\frac{v}{c}\right)^2 \frac{D_{OL} D_{LS}}{D_{OS}}$$

- On solving

$$\frac{< \Delta\theta(z_s) >}{\Delta\theta_0} = \frac{\int_0^{z_s} \frac{D_{LS}^3 D_{OL}^2 (1+z_L)^2}{D_{OS}^3 E(z)}\, dz_L}{\int_0^{z_s} \frac{D_{LS}^2 D_{OL}^2 (1+z_L)^2}{D_{OS}^2 E(z)}\, dz_L}$$

Where, $\Delta\theta_0 = 8\pi \left(\frac{v}{c}\right)^2$



- For the singular isothermal sphere (SIS) lensing galaxies, the mean image separation is completely independent of the source redshift for the all FLRW based cosmological models having flat Universe i.e. Ω_{k0} = 0

Test of Curvature: Dataset

- We use the final statistical sample of lensed quasars from the SDSS Quasar Lens Search (SQLS). The SDSS DR7 quasar catalog consists of 26 lenses in a well-defined statistical sample and 36 additional lenses identified by various techniques

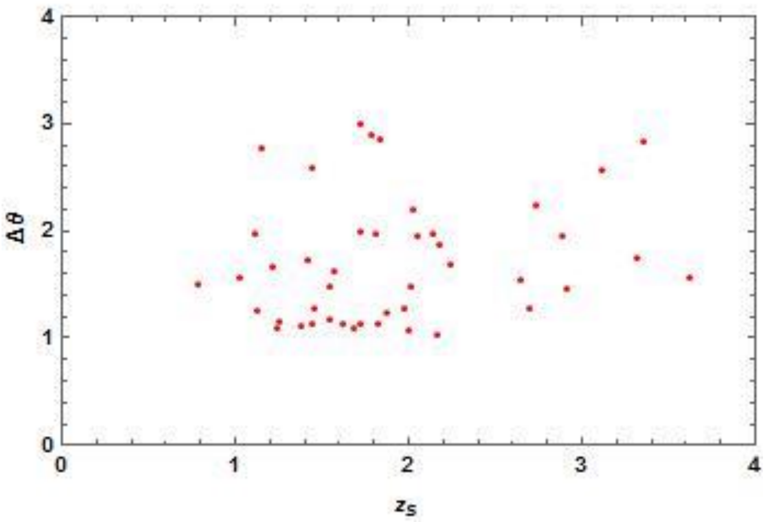
- **Selection Criterion and procedure**

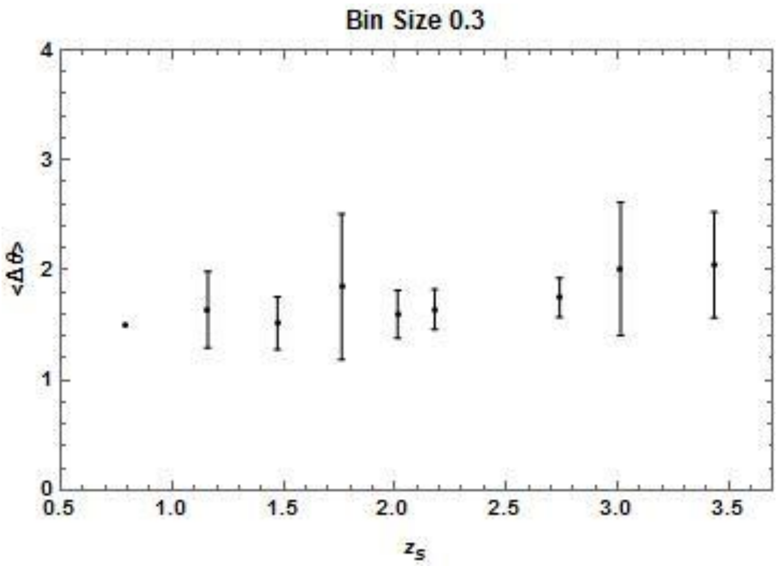
➢ Limits the number of source images to two.

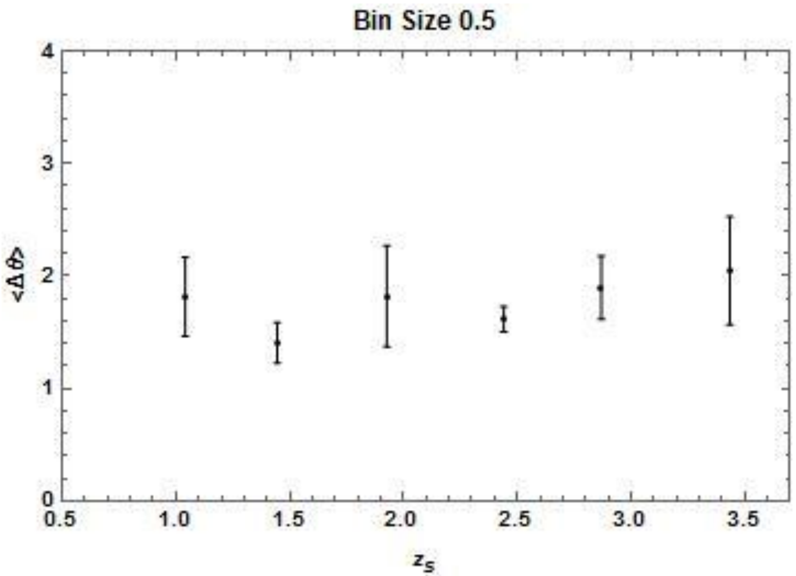
➢ Maximum image separation between two images should be less than 4″.

➢ After applying the selection criteria, out of 62 lensing systems, we are finally left with 44 galaxy lenses.

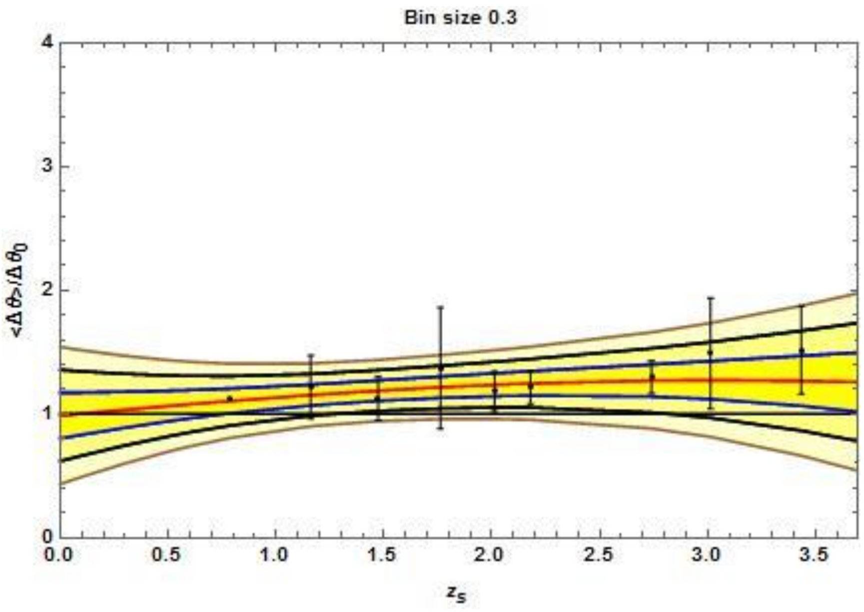
➢ For calculating the mean image separation first we divide this dataset in a redshift bin-size of 0.3 and 0.5 respectively and determine the mean value of $\Delta\theta$ in each interval.

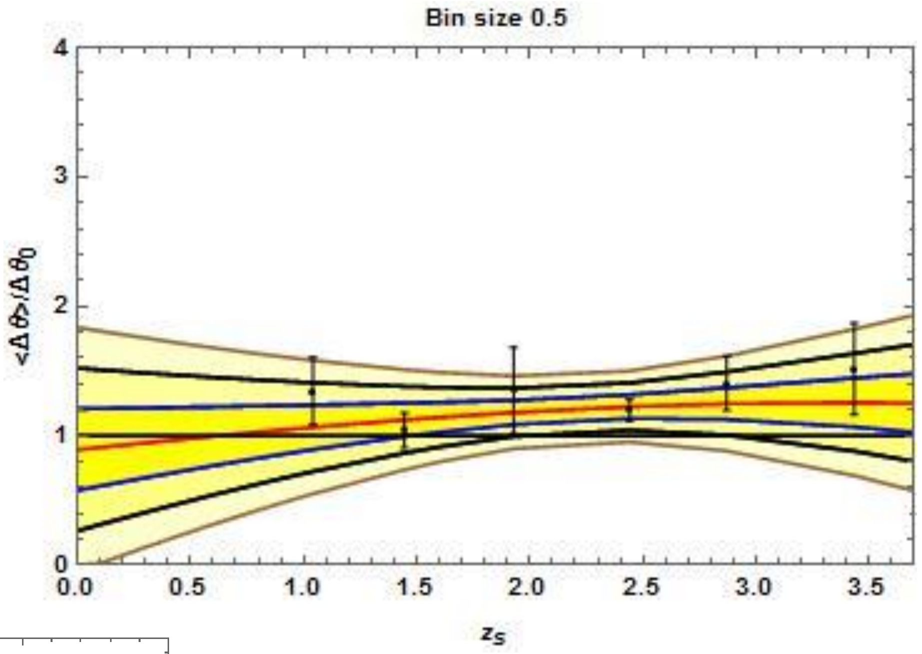


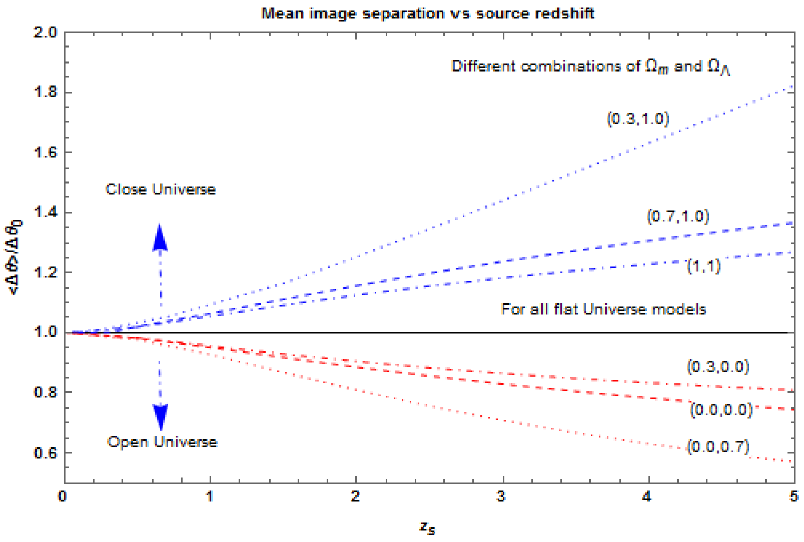




Test of Curvature: Results

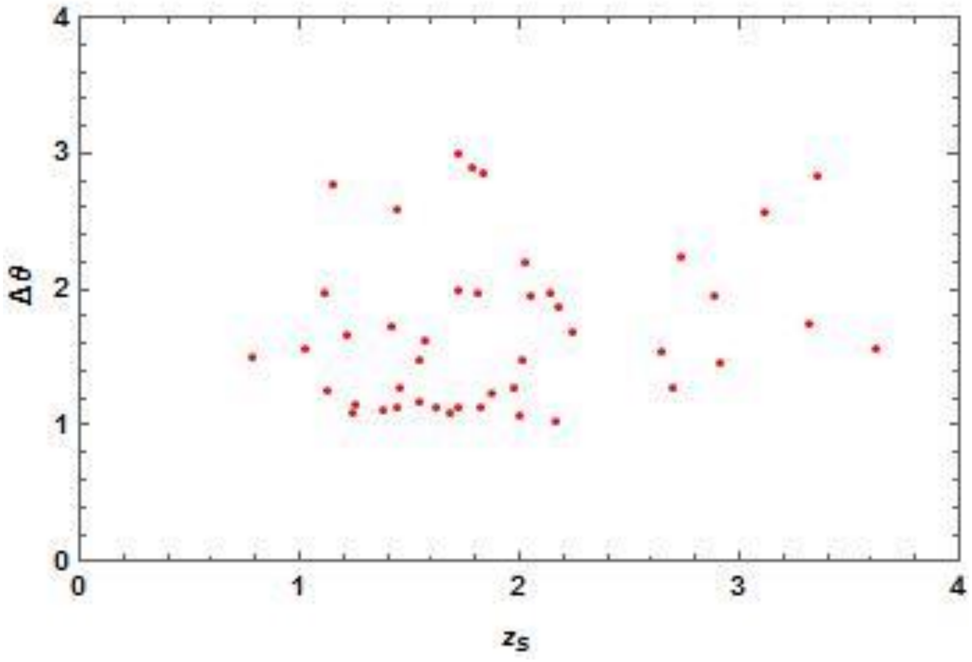






Test of Curvature: Results

- Statistical tests to check the correlation between image separation $(\Delta\theta)$ and source redshift, z_s.



Statistical tests	Values
Spearman's rank coefficient (ρ)	0.22 ± 0.09

Discussion

Level I : Test of FLRW metric

- Ω_{k0} estimates from different observations remain constant in redshift range 0 < z < 1.37. It shows the consistency with the assumption of homogeneity of the universe and also concordance with the FLRW metric.

Level II : Test of spatial Curvature

- The normalized mean image separation shows an inclination towards a close universe. However, within the 2σ region it also incorporates a flat universe.
- The value of correlation coefficients with the corresponding error bars indicates a weak positive correlation between image separation and source redshift z_s.
- Though the different bin sizes result in a difference in the number of data points, the overall trend remains the same . But with a small bin-size, we obtain comparatively tight bands.

Conclusion :

- This work jointly indicate a homogeneous but marginally closed universe.
- Though this inclination of best fit line towards a closed universe can't be directly taken as the strict deviation from a flat universe, it does motivate us to study some of the proposed non-flat dark energy models.
- In this era of precision cosmology, access to more dataset would improve the efficiency of such null test available in literature.

THANK YOU