Bounds on Graviton mass by using weak lensing and SZ effect in Galaxy Clusters

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Outline

1. Introduction
2. Implications of a Massive Graviton
3. Methodology
4. Galaxy Cluster datasets: Weak lensing and SZ effect
5. Results and discussion

Probing Graviton mass using weak lensing and SZ effect in Galaxy Clusters

Akshay Rana, Deepak Jain, Shobhit Mahajan, Amitabha Mukherjee

Physics Letters B, Volume 781, (2018) p. 220-226. arXiv:1801.03309

Introduction

Sir I. Newton @ IUCAA, Pune, India

- All fundamental interactions are governed through the mediating particles.
- Gravity mediates through Gravitons.
- Described by Einstein's General theory of Relativity (GTR)
- In GTR, gravitational attraction is a consequence of space-time curvature
- Mediating particle: Massless spin 2 graviton
- In the weak field limits, Einstein's GTR reproduce Newtonian gravity.

$g_{\mu\upsilon} \approx \eta_{\mu\upsilon} + h_{\mu\upsilon}$ where $h_{\mu\upsilon} << 1$ (static weak field metric)

GR: Success story : From millimeter to solar length scales

- Perihelion advance of Mercury
- Deflection of light by the Sun
- Tests of Equivalence principle
- Frame-dragging effect
- Hulse-Taylor binary pulsar
- Direct observation of gravitational radiation

A. Einstein @ IUCAA, Pune, India

GR: Challenges

Sub-millimeter length scales

- Difficult to get enough matter in close enough proximity at length scales smaller than 1 mm
- Strength of gravity: Hierarchy problem
- Alternative of GR: Extra dimensions theories
- Observation tests : through Torsion Balance or Collider experiments

Cosmological length scales

- Needed Dark component in the energy budget of the Universe
- Cosmic acceleration: Dark Energy
- Rotation curves of galaxy: Dark matter
- Cosmological Constant Problem

GR: Challenges

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A possible alternative of GR: Massive gravity theories

- Fierz and Pauli (1939) proposed a theory of **massive spin 2 gravitons** by adding a mass term in Einstein-Hilbert action.
- It suffered from several discontinuities and ghosts.
- de Rham, Gabadadze, Tolley (dRGT 2011) provided a nonlinear completion to Fierz-Pauli's massive gravity theory.
- **DGP model , Bigravity models** appear as alternative of GR.

If graviton can be massive then motivation to look for the mass of the graviton.

Implications of a Massive Graviton

❑ Modified Dispersion Relation

- If gravitation got propagated by a massive field (a massive graviton). Then the modified dispersion relation would modify,

$$\frac{v_g^2}{c^2} = 1 - \frac{m_g^2\, c^2}{E^2}$$

where m_g and E are the graviton rest mass and energy, respectively.

❑ Yukawa potential

- The gravitational potential of a static point-like source M changes from the standard Newtonian form to Yukawa form,

$$V = -\frac{GM}{r}\, exp[-r/\lambda_g]$$

Where $\lambda_g = {}^{h}/_{m_g c}$; Compton wavelength

❑ Fifth force like behavior

- Additional degrees of freedom
- Vainshtein mechanism to take care of the non-linear terms,
- Decoupling limit generates a fifth force like scale in theory. These results are theory dependent hence comparatively less reliable

Various Bounds on Graviton mass

Hypothesis	Method	m_g in eV
Yukawa potential	1σ bound from weak lensing power spectrum of cluster at z= 1.2 (Choudhury et.al. 2002) Using Holmberg cluster by assuming scale size around 580 kpc (Goldhaber et.al 1974) 1.64σ (90%) bound from galaxy cluster Abell 1689 (Desai 2017) 2σ bound from the precession of Mercury (Finn et.al. 2002) 1.64σ bound using trajectories of S2 stars near the galactic center(Zakharov et.al. 2017)	6.0×10^{-32} 1.10×10^{-29} 1.37×10^{-29} 7.20×10^{-23} 2.91×10^{-21}
Dispersion Relation	90% upper limit from GW150914 (Abbott et. al. 2016: LIGO Scientific Collaboration) 90% upper bound from binary pulsar observations (Manchester et. al. 2010) 90% upper limit from GW170104 (Abbott et. al. 2017: LIGO Scientific Collaboration) Impacts of graviton mass on the B-mode polarization of CMB (Lin et.al. 2016)	1.20×10^{-22} 7.60×10^{-20} 7.70×10^{-23} $\sim 9.7 \times 10^{-30}$
Fifth force	From earth-moon precession for cubic Galilean theories (Dvali et. al 2002) Observations of altered structure formation from fifth force (Park et.al. 2015)	$\sim 10^{-32}$ $\sim 10^{-32}$

Present work

Motivation: **Study of the implication of graviton mass in static gravitational field of Galaxy Clusters**

Probing Graviton mass using weak lensing and SZ effect in Galaxy Clusters
Akshay Rana, Deepak Jain, Shobhit Mahajan, Amitabha Mukherjee

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Methodology

- Given the mass of a galaxy cluster M_Δ at any particular radial distance R_Δ, the gravitational acceleration a_n in **Newtonian gravity** is

$$a_n = \frac{G\ M_\Delta}{R_\Delta^2}$$

- If we assume a modified theory with massive gravitons, the corresponding gravitational acceleration at any particular radial distance would take the **Yukawa form**

$$a_y = \frac{G\ M_\Delta}{R_\Delta} \exp\left[\frac{-R_\Delta}{\lambda_g}\right]\left(\frac{1}{R_\Delta} + \frac{1}{\lambda_g}\right)$$

where λ_g is a length scale that represents the range of interaction due to the exchange of gravitons of mass $m_g = {}^{h}/_{\lambda_g c}$

- For galaxy clusters, R_Δ = Distance from the core of cluster at which the density of galaxy cluster becomes **Δ times the critical density ρ_c of the Universe** at that epoch.

- The mass of the galaxy cluster can be defined as

$$M_\Delta = \Delta \times \rho_c \times \frac{4\pi}{3} R_\Delta^3$$

Methodology

- The critical density of the Universe is given by,

$$\rho_c = \frac{3H(z)^2}{8\pi G}$$

- By using the definition of R_Δ and ρ_c , one can rewrite the acceleration expressions for a_n and a_y

$$a_n(z\,, H(z), M_\Delta) = (GM_\Delta)^{1/3}\left(\frac{\Delta \times H(z)^2}{2}\right)^{2/3}$$

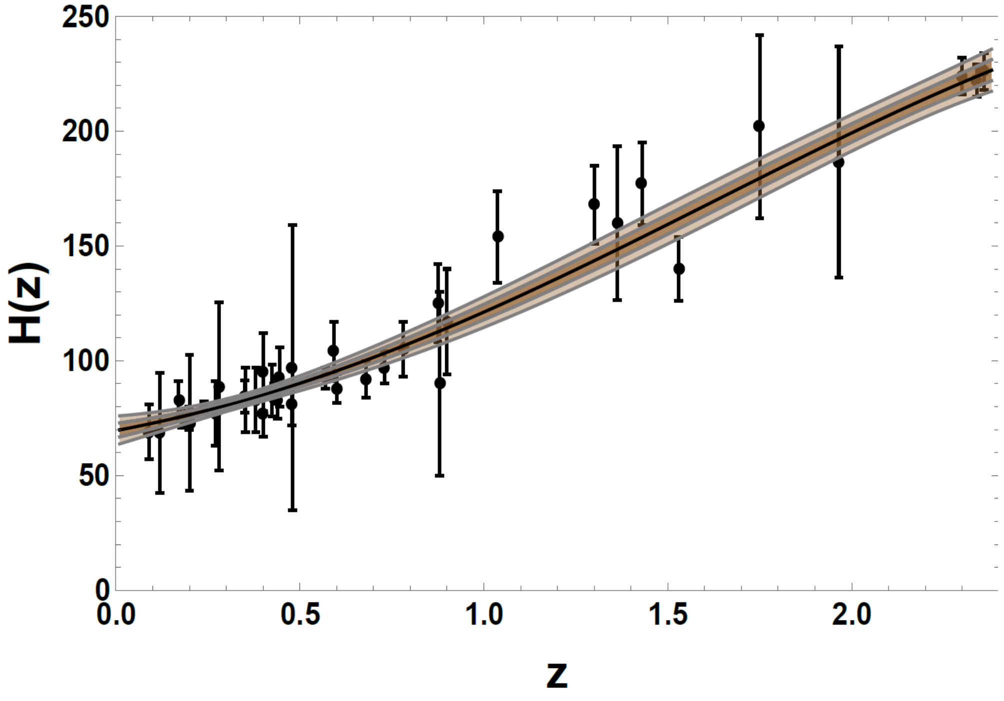
$$a_y(z, H(z), M_\Delta, \lambda_g) = (GM_\Delta)^{\frac{2}{3}}\left(\frac{\Delta \times H(z)^2}{2}\right)^{\frac{1}{3}} \exp\left[\frac{-1}{\lambda_g}\left(\frac{2M_\Delta G}{\Delta \times H(z)^2}\right)^{\frac{1}{3}}\right] \left(\frac{1}{\lambda_g} + \frac{\Delta \times H(z)^2}{2M_\Delta G}\right)$$

- In the expressions of a_n and a_y, the quantities of interest
  - Model independent measurement of $\boldsymbol{H(z)}$
  - Measurements of $\boldsymbol{M_\Delta}$ for galaxy clusters

Methodology

- For Hubble parameter calculation, we use the **38 observed Hubble parameter values** of H(z) in the redshift range $0.07 < z < 2.34$ calculated by using the
  - Differential ages of galaxies
  - Baryonic Acoustic Oscillation (BAO)

- We apply a nonparametric technique **(Gaussian process)** to smoothen it which enables us to get **model independent value of H(z)** at all desired redshifts of the galaxy clusters.

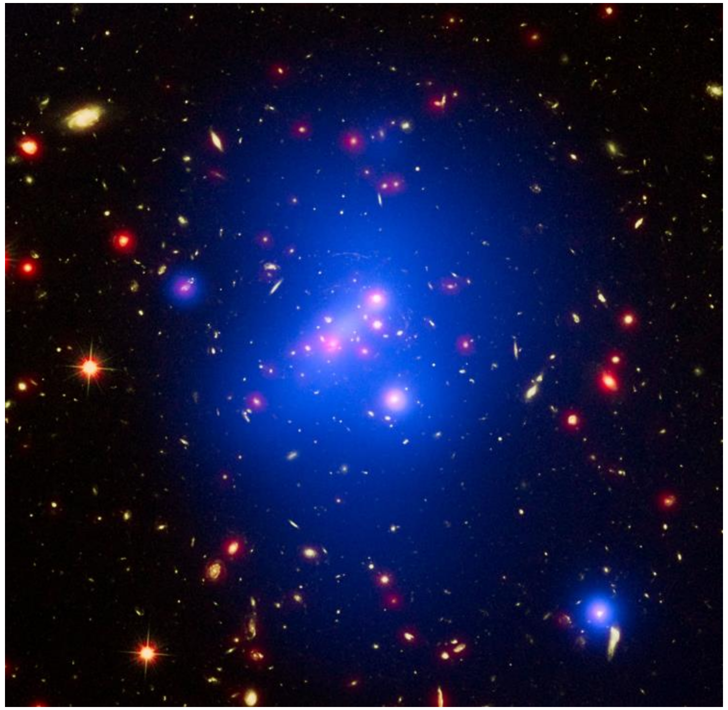


Gaussian Process

- Widely used non parametric smoothing technique in cosmology.
- Parametric relationship is replaced by parametrizing a probability model over the data.

Galaxy cluster

- Galaxy clusters largest known gravitationally bound structures in the universe.
- The Inter cluster medium of galaxy clusters consists of heated gas between the galaxies and has a peak temperature between 2–15 keV
- Methods to calculate the mass of the galaxy clusters.
  - Stellar light
  - Velocity Dispersion
  - X-Ray emission from bremsstrahlung mechanism
  - Sunyeav- Zel’dovich effect
  - Weak gravitational lensing (Cleanest method)



Galaxy cluster IDCS J1426
Multi-wavelength image
Source: http://www.spacetelescope.org/images/opo1602a/

Galaxy cluster : Mass estimate using Weak Lensing

From weak lensing

- **Local Cluster Substructure Survey (LoCuSS). [Okaba et. al (2014)]**
  - mass measurement of 50 most massive galaxy clusters in the local universe (redshift range 0.15 < z < 0.3)

- **Mass estimates:**
  - We use mass estimates of galaxy clusters calculated by using the same approach at radius R_{200}, R_{500}, R_{1000}, R_{2500} and defined as M_{200}^{WL}, M_{500}^{WL}, M_{1000}^{WL} and M_{2500}^{WL}.

From SZ effect

- **Atacama Cosmology Telescope (ACT) survey [Hilton et. al. (2017)]**
  - 182 optically confirmed galaxy clusters detected via the SZ effect in redshift range 0.1 < z < 1.4 at radius R_{500} and defined as M_{500}^{SZ}.
  - Universal pressure profile (UPP) modeled by using a generalized Navarro, Frank & White (NFW) density profile for dark matter halo. [Arnaud et. al (2009)]

Galaxy cluster : Mass estimate using Weak Lensing

From weak le

- ❑ **Local Cluster**
  - ➢ mass mea … redshift range 0.15 < z < 0.3)

- ❑ **Mass estimate**
  - ➢ We use ma … ach at radius R_{200}, R_{500}, R_{1000}

From SZ e

- ❑ **Atacama Cosm**
  - ➢ 182 optica … ange 0.1 < z < 1.4 at radius R
  - ➢ Universal … nk & White (NFW) density pr

Name	M_{vir} $10^{14}\ h^{-1} M_{\odot}$	M_{200} $10^{14}\ h^{-1} M_{\odot}$	M_{500} $10^{14}\ h^{-1} M_{\odot}$	M_{1000} $10^{14}\ h^{-1} M_{\odot}$	M_{2500} $10^{14}\ h^{-1} M_{\odot}$
ABELL2697	$10.82^{+3.32}_{-2.29}$	$8.39^{+1.92}_{-1.52}$	$4.86^{+0.85}_{-0.85}$	$2.87^{+0.73}_{-0.80}$	$1.18^{+0.62}_{-0.54}$
ABELL0068	$8.09^{+1.78}_{-1.49}$	$6.93^{+1.37}_{-1.18}$	$4.98^{+0.79}_{-0.72}$	$3.67^{+0.51}_{-0.48}$	$2.22^{+0.35}_{-0.36}$
ABELL2813	$9.84^{+2.56}_{-2.05}$	$8.50^{+1.94}_{-1.63}$	$6.15^{+1.07}_{-0.98}$	$4.56^{+0.72}_{-0.70}$	$2.78^{+0.58}_{-0.65}$
ABELL0115	$9.92^{+4.64}_{-3.01}$	$7.38^{+2.72}_{-2.02}$	$3.98^{+1.16}_{-1.06}$	$2.19^{+0.78}_{-0.78}$	$0.81^{+0.51}_{-0.41}$
ABELL0141	$5.90^{+1.47}_{-1.24}$	$4.91^{+1.10}_{-0.97}$	$3.34^{+0.65}_{-0.61}$	$2.32^{+0.48}_{-0.48}$	$1.27^{+0.38}_{-0.38}$
ZwCl0104.4+0048	$2.82^{+1.68}_{-1.11}$	$2.33^{+1.03}_{-0.82}$	$1.54^{+0.57}_{-0.71}$	$1.05^{+0.48}_{-0.70}$	$0.54^{+0.46}_{-0.53}$
ABELL0209	$16.06^{+3.18}_{-2.59}$	$13.28^{+2.30}_{-1.95}$	$9.02^{+1.17}_{-1.07}$	$6.27^{+0.69}_{-0.66}$	$3.42^{+0.46}_{-0.49}$
ABELL0267	$7.58^{+1.60}_{-1.36}$	$6.22^{+1.18}_{-1.03}$	$4.10^{+0.65}_{-0.60}$	$2.76^{+0.44}_{-0.42}$	$1.42^{+0.31}_{-0.31}$
ABELL0291	$7.81^{+2.73}_{-1.96}$	$5.89^{+1.65}_{-1.31}$	$3.28^{+0.72}_{-0.67}$	$1.87^{+0.51}_{-0.52}$	$0.72^{+0.36}_{-0.31}$
ABELL0383	$6.52^{+1.85}_{-1.45}$	$5.44^{+1.32}_{-1.09}$	$3.80^{+0.67}_{-0.61}$	$2.71^{+0.40}_{-0.38}$	$1.55^{+0.29}_{-0.33}$
ABELL0521	$7.03^{+1.60}_{-1.38}$	$5.85^{+1.20}_{-1.07}$	$3.94^{+0.70}_{-0.66}$	$2.72^{+0.51}_{-0.51}$	$1.46^{+0.40}_{-0.40}$
ABELL0586	$7.97^{+3.00}_{-2.08}$	$6.91^{+2.17}_{-1.64}$	$5.24^{+1.14}_{-1.00}$	$4.08^{+0.70}_{-0.65}$	$2.72^{+0.53}_{-0.67}$
ABELL0611	$11.34^{+2.50}_{-2.16}$	$9.61^{+1.95}_{-1.74}$	$6.65^{+1.18}_{-1.10}$	$4.71^{+0.83}_{-0.81}$	$2.65^{+0.58}_{-0.60}$
ABELL0697	$13.18^{+4.95}_{-3.22}$	$10.20^{+2.97}_{-2.19}$	$5.67^{+1.10}_{-1.02}$	$3.23^{+0.76}_{-0.81}$	$1.25^{+0.57}_{-0.52}$
ZwCl0857.9+2107	$3.26^{+1.60}_{-1.22}$	$2.56^{+1.10}_{-0.93}$	$1.52^{+0.70}_{-0.76}$	$0.92^{+0.57}_{-0.60}$	$0.40^{+0.45}_{-0.43}$
ABELL0750	$7.95^{+3.99}_{-2.37}$	$6.55^{+2.73}_{-1.78}$	$4.50^{+1.22}_{-0.97}$	$3.16^{+0.62}_{-0.55}$	$1.76^{+0.39}_{-0.46}$
ABELL0773	$11.55^{+1.69}_{-1.47}$	$9.94^{+1.30}_{-1.16}$	$7.34^{+0.76}_{-0.71}$	$5.56^{+0.50}_{-0.49}$	$3.53^{+0.37}_{-0.39}$
ABELL0781	$8.52^{+3.06}_{-2.30}$	$6.86^{+2.01}_{-1.69}$	$4.17^{+1.12}_{-1.18}$	$2.59^{+0.93}_{-1.10}$	$1.16^{+0.77}_{-0.71}$
ZwCl0949.6+5207	$5.21^{+1.74}_{-1.36}$	$4.61^{+1.34}_{-1.11}$	$3.61^{+0.81}_{-0.74}$	$2.90^{+0.56}_{-0.53}$	$2.05^{+0.44}_{-0.53}$
ABELL0901	$3.23^{+1.27}_{-0.98}$	$2.75^{+0.96}_{-0.78}$	$2.03^{+0.58}_{-0.51}$	$1.54^{+0.40}_{-0.37}$	$0.98^{+0.29}_{-0.31}$
ABELL0907	$19.79^{+8.14}_{-4.77}$	$14.93^{+4.69}_{-3.07}$	$8.47^{+1.41}_{-1.21}$	$4.92^{+0.75}_{-0.78}$	$1.96^{+0.63}_{-0.62}$
ABELL0963	$8.94^{+1.90}_{-1.59}$	$7.42^{+1.40}_{-1.22}$	$5.08^{+0.79}_{-0.73}$	$3.57^{+0.54}_{-0.53}$	$1.97^{+0.41}_{-0.42}$
ZwCl1021.0+0426	$6.35^{+1.45}_{-1.24}$	$5.45^{+1.10}_{-0.98}$	$3.88^{+0.62}_{-0.58}$	$2.84^{+0.43}_{-0.42}$	$1.69^{+0.34}_{-0.37}$
ABELL1423	$5.24^{+1.68}_{-1.28}$	$4.47^{+1.21}_{-0.99}$	$3.24^{+0.63}_{-0.58}$	$2.41^{+0.43}_{-0.43}$	$1.47^{+0.37}_{-0.45}$
ABELL1451	$10.69^{+1.66}_{-1.44}$	$8.89^{+1.23}_{-1.10}$	$6.14^{+0.68}_{-0.64}$	$4.34^{+0.44}_{-0.42}$	$2.44^{+0.30}_{-0.31}$
RXCJ1212.3-1816	$2.54^{+1.50}_{-1.10}$	$2.20^{+1.19}_{-0.92}$	$1.62^{+0.76}_{-0.64}$	$1.22^{+0.54}_{-0.49}$	$0.76^{+0.38}_{-0.40}$
ZwCl1231.4+1007	$7.51^{+2.97}_{-2.13}$	$5.83^{+1.86}_{-1.52}$	$3.39^{+1.02}_{-1.03}$	$2.01^{+0.84}_{-0.87}$	$0.83^{+0.66}_{-0.51}$
ABELL1682	$10.78^{+1.87}_{-1.60}$	$9.02^{+1.40}_{-1.23}$	$6.23^{+0.76}_{-0.71}$	$4.41^{+0.47}_{-0.45}$	$2.48^{+0.31}_{-0.33}$
ABELL1689	$12.80^{+2.09}_{-1.80}$	$11.39^{+1.68}_{-1.48}$	$9.13^{+1.09}_{-1.00}$	$7.50^{+0.73}_{-0.70}$	$5.49^{+0.47}_{-0.47}$
ABELL1758N	$7.02^{+2.10}_{-1.67}$	$6.11^{+1.55}_{-1.33}$	$4.50^{+0.99}_{-0.98}$	$3.41^{+0.88}_{-1.02}$	$2.15^{+0.91}_{-1.02}$
ABELL1763	$21.55^{+5.03}_{-3.78}$	$17.63^{+3.48}_{-2.76}$	$11.56^{+1.55}_{-1.39}$	$7.76^{+0.87}_{-0.84}$	$3.97^{+0.68}_{-0.73}$
ABELL1835	$11.94^{+2.46}_{-2.08}$	$10.48^{+1.90}_{-1.66}$	$7.98^{+1.11}_{-1.02}$	$6.24^{+0.75}_{-0.73}$	$4.18^{+0.64}_{-0.68}$
ABELL1914	$11.51^{+2.91}_{-2.27}$	$9.12^{+1.96}_{-1.63}$	$5.73^{+0.96}_{-0.89}$	$3.69^{+0.64}_{-0.64}$	$1.76^{+0.47}_{-0.48}$
ZwCl1454.8+2233	$5.85^{+4.59}_{-2.34}$	$4.62^{+2.53}_{-1.66}$	$2.75^{+1.04}_{-1.01}$	$1.67^{+0.79}_{-0.92}$	$0.72^{+0.68}_{-0.67}$
ABELL2009	$10.73^{+5.81}$	$8.13^{+3.26}$	$4.70^{+1.04}$	$2.78^{+0.65}$	$1.14^{+0.57}$

Methodology

- Once the acceleration corresponding to the Newtonian potential and Yukawa potential are known, we defined chi-square χ^2

$$\chi^2 = \sum_i \left[\frac{a_{n,i}(z,H(z),M_\Delta) - a_{y,i}(z,H(z),M_\Delta,\lambda_g)}{\sigma_{n,i}}\right]^2$$

where σ_a gives the error in acceleration obtained by adding the errors of mass estimate, σ_M and Hubble parameter $\sigma_{H(z)}$ in quadrature, given by,

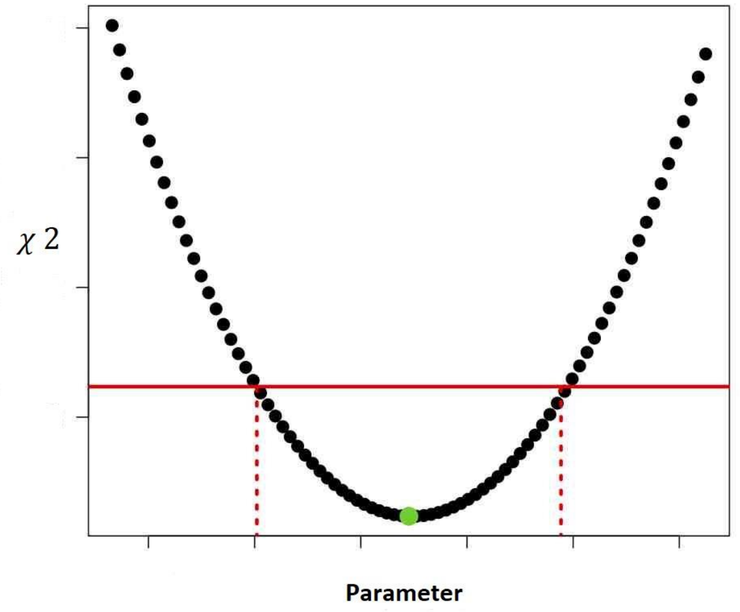
$$\sigma_n = \frac{a_n}{3}\sqrt{\left(\frac{\sigma_{M_\Delta}}{M_\Delta}\right)^2 + 16\left(\frac{\sigma_H}{H(z)}\right)^2}$$

Methodology

- Once the acceleration corresponding to the Newtonian potential and Yukawa potential are known, we defined chi-square χ^2

$$\chi^2 = \sum_i \left[\frac{a_{n,i}(z,H(z),M_\Delta) - a_{y,i}(z,H(z),M_\Delta,\lambda_g)}{\sigma_{n,i}}\right]^2$$

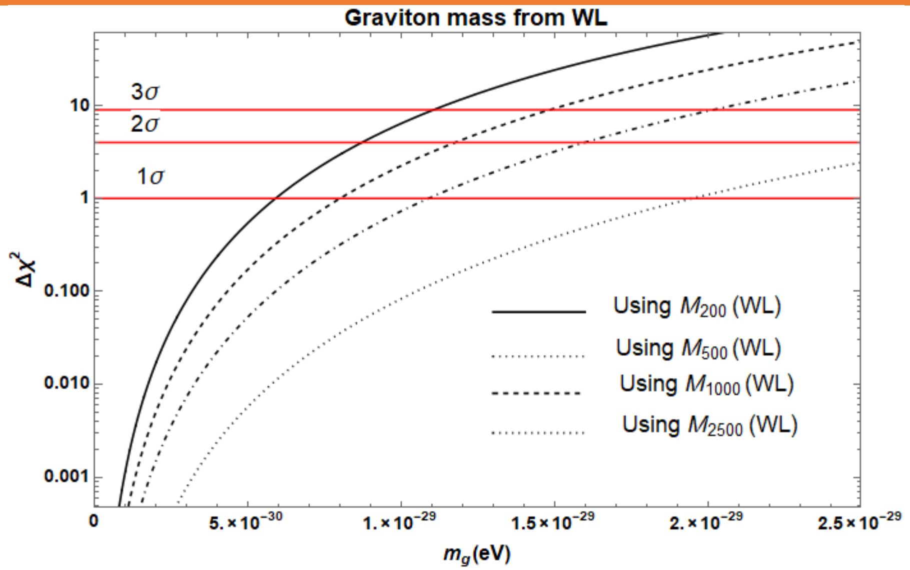
where $m_g = {}^{h}/_{\lambda_g c}$

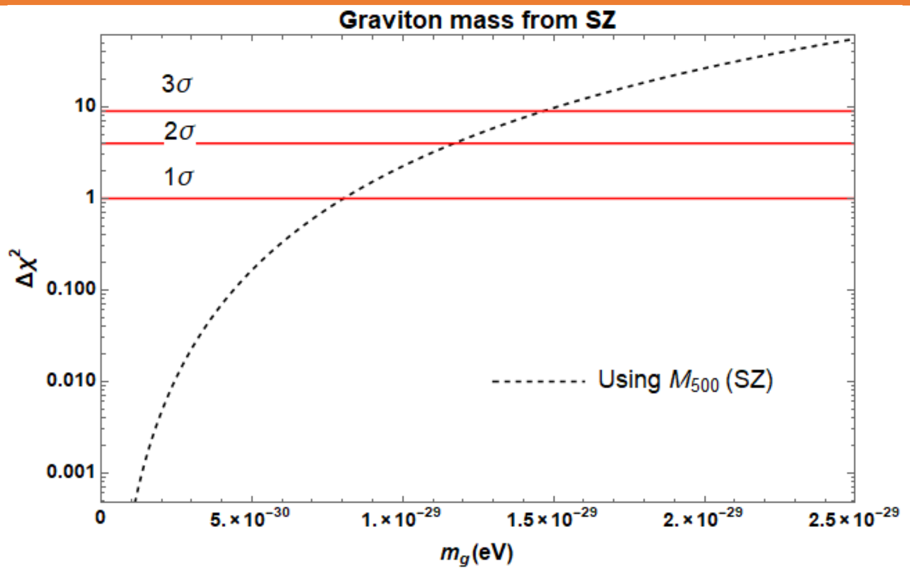


- As $\lambda_g \longrightarrow \infty$ or $m_g \longrightarrow 0$, $a_{y,i}(z, H(z), M_\Delta, \lambda_g)$ will reduce to $a_{n,i}(z, H(z), M_\Delta)$. Hence the minimum value of χ^2_{min} would be zero.

- Hence it is obvious that the best value of m_g for which χ^2 would minimize is zero. To get a bound on graviton mass with different confidence levels are defined as $\Delta\chi^2 = \chi^2 - \chi^2_{min}$.

Results

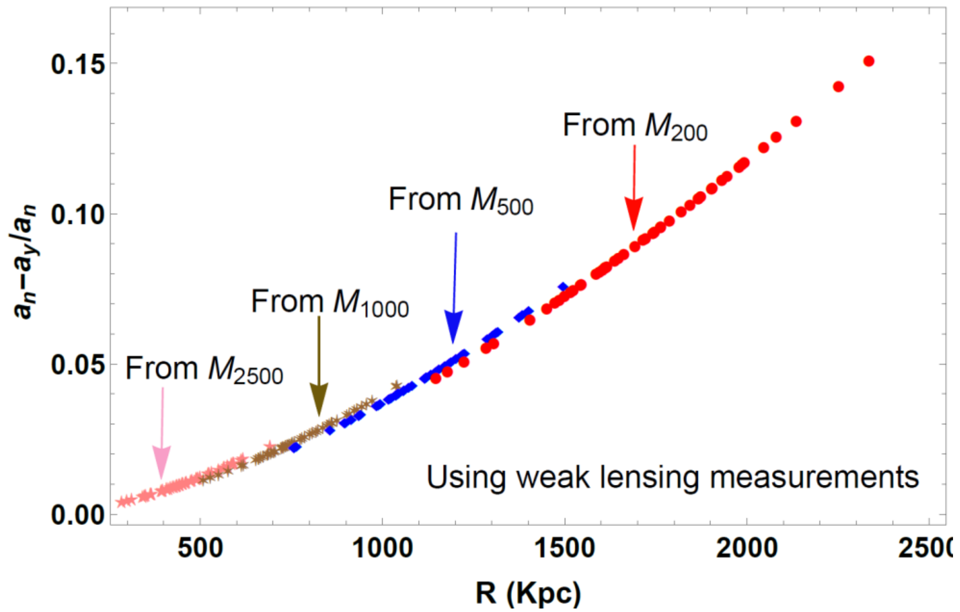


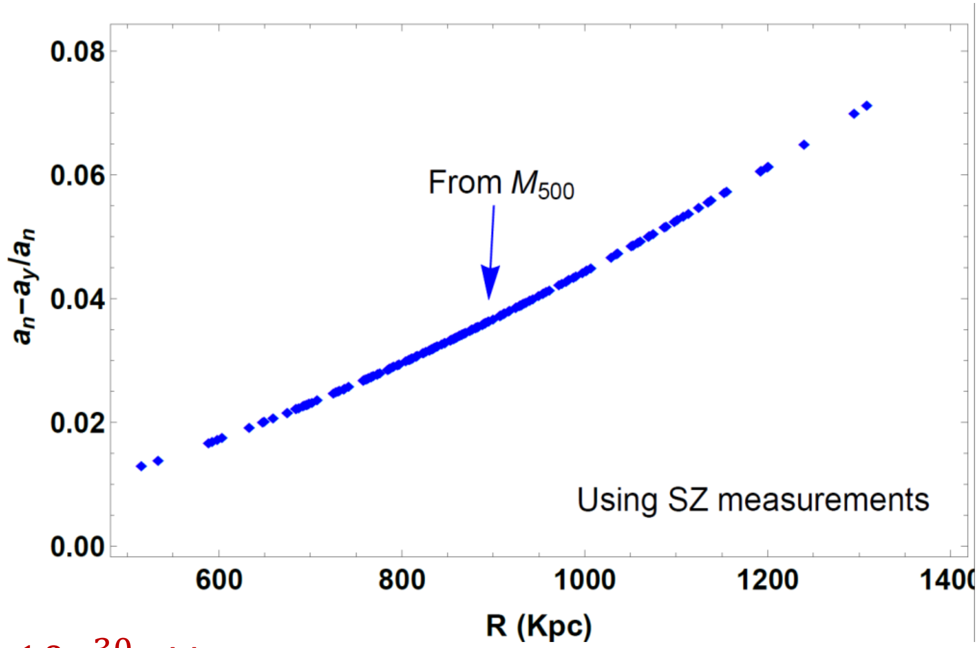


Upper Bound on Graviton mass m_g (in eV) and lower bound on λ_g (in Mpc)					
Data	Parameter	1 σ (68.3%)	1.64 σ (90%)	2 σ (95.5%)	3 σ (99.7 %)
M_{200}^{WL}	$m_g <$ (in eV)	5.902×10^{-30}	7.849×10^{-30}	8.715×10^{-30}	1.105×10^{-29}
	$\lambda_g >$(Mpc)	6.822	5.132	4.622	3.643
M_{500}^{WL}	$m_g <$ (in eV)	8.003×10^{-30}	1.053×10^{-29}	1.175×10^{-29}	1.48×10^{-29}
	$\lambda_g >$(in Mpc)	5.033	3.824	3.427	2.713
M_{1000}^{WL}	$m_g <$ (in eV)	1.008×10^{-29}	1.42×10^{-29}	1.59×10^{-29}	2.017×10^{-29}
	$\lambda_g >$(in Mpc)	3.700	2.821	2.520	1.997
M_{2500}^{WL}	$m_g <$ (in eV)	1.952×10^{-29}	2.583×10^{-29}	2.894×10^{-29}	3.641×10^{-29}
	$\lambda_g >$(in Mpc)	2.060	1.560	1.390	1.100
M_{500}^{SZ}	$m_g <$ (in eV)	8.307×10^{-30}	1.051×10^{-29}	1.169×10^{-29}	1.461×10^{-29}
	$\lambda_g >$(Mpc)	5.012	3.831	3.443	2.747

Table 1: Bounds on the graviton mass m_g and corresponding Compton length scale λ_g within 1σ, 1.64σ, 2σ and 3σ confidence limits estimated by using M_{200}^{WL}, M_{500}^{WL}, M_{1000}^{WL}, M_{2500}^{WL} and M_{500}^{SZ}

Results





For $m_g = 5.9 \times 10^{-30}$ eV

- In left panel of above figure, the fractional change is **approx. 15% at 2.3 Mpc** (From Weak lensing)

- In right panel of figure, this difference is **approx. 7%** at a radial distance **1.3 Mpc** (From SZ effect)

- It confirms that the difference between Newtonian potential and Yukawa potential become significant at large lengths. Hence the motivation for such a test using large scale structures.

Various Bounds on Graviton mass

Hypothesis	Method	m_g in eV
Yukawa potential	1σ bound from weak lensing power spectrum of cluster at z= 1.2 (Choudhury et.al. 2002)	6.0×10^{-32}
	Using Holmberg cluster by assuming scale size around 580 kpc (Goldhaber et.al 1974)	1.10×10^{-29}
	1.64σ (90%) bound from galaxy cluster Abell 1689 (Desai 2017)	1.37×10^{-29}
	2σ bound from the precession of Mercury (Finn et.al. 2002)	7.20×10^{-23}
	1.64σ bound using trajectories of S2 stars near the galactic center(Zakharov et.al. 2017)	2.91×10^{-21}
	1σ bound from M_{WL}^{200} mass estimate of 50 galaxy cluster (This work)	$\mathbf{5.90 \times 10^{-30}}$
	1σ bound from M_{SZ}^{500} mass estimate of 182 galaxy cluster (This work)	$\mathbf{8.31 \times 10^{-30}}$
Dispersion Relation	90% upper limit from GW150914 (Abbott et. al. 2016)	1.20×10^{-22}
	90% upper bound from binary pulsar observations (Manchester et. al. 2010)	7.60×10^{-20}
	90% upper limit from GW170104 (Abbott et. al. 2017)	7.70×10^{-23}
	Impacts of graviton mass on the B-mode polarization of CMB (Lin et.al. 2016)	$\sim 9.7 \times 10^{-30}$
Fifth force	From earth-moon precession for cubic Galilean theories (Dvali et. al 2002)	$\sim 10^{-32}$
	Observations of altered structure formation from fifth force (Park et.al. 2015)	$\sim 10^{-32}$

Take home message

❑ What's New

- ❖ Novel approach to probe the graviton mass by using the presently available observational catalogs of mass measurements of galaxy clusters instead of a single galaxy cluster.

- ❖ Significant improvement in the upper limit of graviton mass

❑ Limitations

- ❖ The mass estimates of galaxy clusters indirectly depend upon the form of the potential. It requires input about the mass profiles for dark matter halos. Here NFW density profile have been used which is an empirical mass profile identified in N-body simulations of structure formation performed under the preview of GR and widely accepted in the literature.

❑ Result

- ❖ $m_g \leq 5.9 \times 10^{-30}\, eV$ **corresponding to** $\lambda_g \geq 6.822\, Mpc$ from weak lensing measurements of clusters
- ❖ $m_g \leq 8.307 \times 10^{-30}\, \mathrm{eV}$ **corresponding to** $\lambda_g \geq 5.012\, Mpc$ from SZ effect measurements of clusters

- ❖ With the ongoing and future surveys, our understanding of mass distribution in large scale structures like galaxies, clusters, super-clusters and filaments will improve and more reliable and precise bounds can be obtained with this analysis.

Thank You.

Research Interest

1) Completed and ongoing projects

a) Dark energy and alternative models of cosmology
b) Test of homogeneity and Isotropy of space-time (Rana et. al, 2017a)
c) Model independent test to check the cosmic curvature (Rana et. al, 2017a)
d) Testing fundamental cosmological relations like; Cosmic distance duality (Rana et. al, 2016, 17b)
e) Model independent estimate of Angular diameter distance (Rana et. al, 2017b)
f) Constraints on graviton mass using galaxy clusters (Rana et. al, 2018)
g) Distances in the Inhomogeneous Universe

2) Observational Probes

SNe IA, BAO, Galaxy clusters, Gravitational lensing, Cosmic Chronometers, GWs, H21 etc.

3) Astro-statistics

a) Bayesian analysis, MCMC

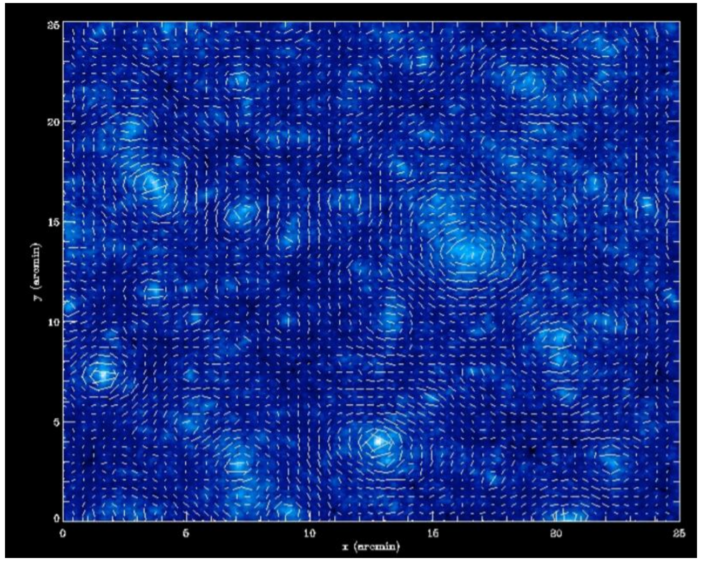
b) Non-parametric: Gaussian process, LOESS+SIMEX, Median statistics)

Appendix.

Galaxy cluster : Mass estimate using Weak Lensing

- **Why, Weak lensing (WL)**
  - Cleanest method for mass estimation of galaxy cluster
  - Sensitive to the total matter distribution, Not affected by the physical and dynamical state.

- **Observable quantity: Cosmic Shear**
  - Small change in the ellipticity of background objects or the tidal distortion of a galaxy's image
  - Shear directly related to the projected foreground mass of lensing objects.



Simulated Shear Map
Jain, Seljak & White 1997

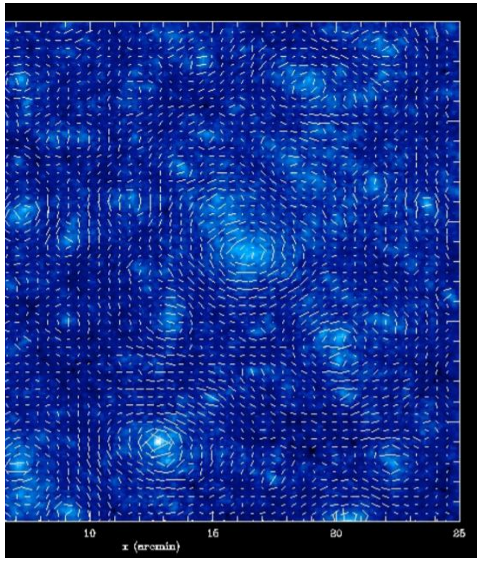
Data set

- **Local Cluster Substructure Survey (LoCuSS). [Okaba et. al (2014)]**
  - mass measurement of 50 most massive galaxy clusters in the local universe (redshift range 0.15 < z < 0.3)

- **Mass estimates:**
  - We use mass estimates of galaxy clusters calculated by using the same approach at radius R_{200}, R_{500}, R_{1000}, R_{2500} and defined as M_{200}^{WL}, M_{500}^{WL}, M_{1000}^{WL} and M_{2500}^{WL}.

Galaxy cluster : Mass estimate using Weak Lensing

- **Why, Weak le**
  - ➢ Cleanest
  - ➢ Sensitive … namical state.



- **Observable qu**
  - ➢ Small cha distortion
  - ➢ Shear di objects.

Data set

- **Local Cluster**
  - ➢ mass mea … redshift range 0.15 < z < 0.3)

- **Mass estimate**
  - ➢ We use ma … ach at radius R_{200}, R_{500}, R_{1000}

Name	$M_{\rm vir}$ $10^{14}\ h^{-1} M_\odot$	M_{200} $10^{14}\ h^{-1} M_\odot$	M_{500} $10^{14}\ h^{-1} M_\odot$	M_{1000} $10^{14}\ h^{-1} M_\odot$	M_{2500} $10^{14}\ h^{-1} M_\odot$
ABELL2697	$10.82^{+3.32}_{-2.29}$	$8.39^{+1.92}_{-1.52}$	$4.86^{+0.85}_{-0.85}$	$2.87^{+0.73}_{-0.80}$	$1.18^{+0.62}_{-0.54}$
ABELL0068	$8.09^{+1.78}_{-1.49}$	$6.93^{+1.37}_{-1.18}$	$4.98^{+0.79}_{-0.72}$	$3.67^{+0.51}_{-0.48}$	$2.22^{+0.35}_{-0.36}$
ABELL2813	$9.84^{+2.56}_{-2.05}$	$8.50^{+1.94}_{-1.63}$	$6.15^{+1.07}_{-0.98}$	$4.56^{+0.72}_{-0.70}$	$2.78^{+0.58}_{-0.65}$
ABELL0115	$9.92^{+4.64}_{-3.01}$	$7.38^{+2.72}_{-2.02}$	$3.98^{+1.16}_{-1.06}$	$2.19^{+0.78}_{-0.78}$	$0.81^{+0.51}_{-0.41}$
ABELL0141	$5.90^{+1.47}_{-1.24}$	$4.91^{+1.10}_{-0.97}$	$3.34^{+0.65}_{-0.61}$	$2.32^{+0.48}_{-0.48}$	$1.27^{+0.38}_{-0.38}$
ZwCl0104.4+0048	$2.82^{+1.68}_{-1.11}$	$2.33^{+1.03}_{-0.82}$	$1.54^{+0.57}_{-0.71}$	$1.05^{+0.48}_{-0.70}$	$0.54^{+0.46}_{-0.53}$
ABELL0209	$16.06^{+3.18}_{-2.59}$	$13.28^{+2.30}_{-1.95}$	$9.02^{+1.17}_{-1.07}$	$6.27^{+0.69}_{-0.66}$	$3.42^{+0.46}_{-0.49}$
ABELL0267	$7.58^{+1.60}_{-1.36}$	$6.22^{+1.18}_{-1.03}$	$4.10^{+0.65}_{-0.60}$	$2.76^{+0.44}_{-0.42}$	$1.42^{+0.31}_{-0.31}$
ABELL0291	$7.81^{+2.73}_{-1.96}$	$5.89^{+1.65}_{-1.31}$	$3.28^{+0.72}_{-0.67}$	$1.87^{+0.51}_{-0.52}$	$0.72^{+0.36}_{-0.31}$
ABELL0383	$6.52^{+1.85}_{-1.45}$	$5.44^{+1.32}_{-1.09}$	$3.80^{+0.67}_{-0.61}$	$2.71^{+0.40}_{-0.38}$	$1.55^{+0.29}_{-0.33}$
ABELL0521	$7.03^{+1.60}_{-1.38}$	$5.85^{+1.20}_{-1.07}$	$3.94^{+0.70}_{-0.66}$	$2.72^{+0.51}_{-0.51}$	$1.46^{+0.40}_{-0.40}$
ABELL0586	$7.97^{+3.00}_{-2.08}$	$6.91^{+2.17}_{-1.64}$	$5.24^{+1.14}_{-1.00}$	$4.08^{+0.70}_{-0.65}$	$2.72^{+0.53}_{-0.67}$
ABELL0611	$11.34^{+2.50}_{-2.16}$	$9.61^{+1.95}_{-1.74}$	$6.65^{+1.18}_{-1.10}$	$4.71^{+0.83}_{-0.81}$	$2.65^{+0.58}_{-0.60}$
ABELL0697	$13.18^{+4.95}_{-3.22}$	$10.20^{+2.97}_{-2.19}$	$5.67^{+1.10}_{-1.02}$	$3.23^{+0.76}_{-0.81}$	$1.25^{+0.57}_{-0.52}$
ZwCl0857.9+2107	$3.26^{+1.60}_{-1.22}$	$2.56^{+1.10}_{-0.93}$	$1.52^{+0.70}_{-0.76}$	$0.92^{+0.57}_{-0.60}$	$0.40^{+0.45}_{-0.43}$
ABELL0750	$7.95^{+3.99}_{-2.37}$	$6.55^{+2.73}_{-1.78}$	$4.50^{+1.22}_{-0.97}$	$3.16^{+0.62}_{-0.55}$	$1.76^{+0.39}_{-0.46}$
ABELL0773	$11.55^{+1.69}_{-1.47}$	$9.94^{+1.30}_{-1.16}$	$7.34^{+0.76}_{-0.71}$	$5.56^{+0.50}_{-0.49}$	$3.53^{+0.37}_{-0.39}$
ABELL0781	$8.52^{+3.06}_{-2.30}$	$6.86^{+2.01}_{-1.69}$	$4.17^{+1.12}_{-1.18}$	$2.59^{+0.93}_{-1.10}$	$1.16^{+0.77}_{-0.71}$
ZwCl0949.6+5207	$5.21^{+1.74}_{-1.36}$	$4.61^{+1.34}_{-1.11}$	$3.61^{+0.81}_{-0.74}$	$2.90^{+0.56}_{-0.53}$	$2.05^{+0.44}_{-0.53}$
ABELL0901	$3.23^{+1.27}_{-0.98}$	$2.75^{+0.96}_{-0.78}$	$2.03^{+0.58}_{-0.51}$	$1.54^{+0.40}_{-0.37}$	$0.98^{+0.29}_{-0.31}$
ABELL0907	$19.79^{+8.14}_{-4.77}$	$14.93^{+4.69}_{-3.07}$	$8.47^{+1.41}_{-1.21}$	$4.92^{+0.75}_{-0.78}$	$1.96^{+0.63}_{-0.62}$
ABELL0963	$8.94^{+1.90}_{-1.59}$	$7.42^{+1.40}_{-1.22}$	$5.08^{+0.79}_{-0.73}$	$3.57^{+0.54}_{-0.53}$	$1.97^{+0.41}_{-0.42}$
ZwCl1021.0+0426	$6.35^{+1.45}_{-1.24}$	$5.45^{+1.10}_{-0.98}$	$3.88^{+0.62}_{-0.58}$	$2.84^{+0.43}_{-0.42}$	$1.69^{+0.34}_{-0.37}$
ABELL1423	$5.24^{+1.68}_{-1.28}$	$4.47^{+1.21}_{-0.99}$	$3.24^{+0.63}_{-0.58}$	$2.41^{+0.43}_{-0.43}$	$1.47^{+0.37}_{-0.45}$
ABELL1451	$10.69^{+1.66}_{-1.44}$	$8.89^{+1.23}_{-1.10}$	$6.14^{+0.68}_{-0.64}$	$4.34^{+0.44}_{-0.42}$	$2.44^{+0.30}_{-0.31}$
RXCJ1212.3-1816	$2.54^{+1.50}_{-1.10}$	$2.20^{+1.19}_{-0.92}$	$1.62^{+0.76}_{-0.64}$	$1.22^{+0.54}_{-0.49}$	$0.76^{+0.38}_{-0.40}$
ZwCl1231.4+1007	$7.51^{+2.97}_{-2.13}$	$5.83^{+1.86}_{-1.52}$	$3.39^{+1.02}_{-1.03}$	$2.01^{+0.84}_{-0.87}$	$0.83^{+0.66}_{-0.51}$
ABELL1682	$10.78^{+1.87}_{-1.60}$	$9.02^{+1.40}_{-1.23}$	$6.23^{+0.76}_{-0.71}$	$4.41^{+0.47}_{-0.45}$	$2.48^{+0.31}_{-0.33}$
ABELL1689	$12.80^{+2.09}_{-1.80}$	$11.39^{+1.68}_{-1.48}$	$9.13^{+1.09}_{-1.00}$	$7.50^{+0.73}_{-0.70}$	$5.49^{+0.47}_{-0.47}$
ABELL1758N	$7.02^{+2.10}_{-1.67}$	$6.11^{+1.55}_{-1.33}$	$4.50^{+0.99}_{-0.98}$	$3.41^{+0.88}_{-1.02}$	$2.15^{+0.91}_{-1.02}$
ABELL1763	$21.55^{+5.03}_{-3.78}$	$17.63^{+3.48}_{-2.76}$	$11.56^{+1.55}_{-1.39}$	$7.76^{+0.87}_{-0.84}$	$3.97^{+0.68}_{-0.73}$
ABELL1835	$11.94^{+2.46}_{-2.08}$	$10.48^{+1.90}_{-1.66}$	$7.98^{+1.11}_{-1.02}$	$6.24^{+0.75}_{-0.73}$	$4.18^{+0.64}_{-0.68}$
ABELL1914	$11.51^{+2.91}_{-2.27}$	$9.12^{+1.96}_{-1.63}$	$5.73^{+0.96}_{-0.89}$	$3.69^{+0.64}_{-0.64}$	$1.76^{+0.47}_{-0.48}$
ZwCl1454.8+2233	$5.85^{+4.59}_{-2.34}$	$4.62^{+2.53}_{-1.66}$	$2.75^{+1.04}_{-1.01}$	$1.67^{+0.79}_{-0.92}$	$0.72^{+0.68}_{-0.67}$
ABELL2009	$10.73^{+5.81}$	$8.13^{+3.26}$	$4.70^{+1.04}$	$2.78^{+0.65}$	$1.14^{+0.57}$

Galaxy cluster : Mass estimate using SZ effect

- ❑ **SZ effect (SZ)**
  - ➢ CMB photons below 218 GHz gain energy through inverse Compton scattering.

- ❑ **Observable quantity**
  - ➢ Compton parameter y, measure of gas pressure integrated along the line of sight.

$$y = \frac{\sigma_T}{m_e c^2} \int P\, dl$$

where c is the speed of light, m_e is the electron rest mass, σ_T is the Thomson cross section and $P = n_e T$ represents the product of electron density with temperature.

  - ➢ The gas pressure is directly related to the gravitational potential of clusters.

Data set

- ❑ Atacama Cosmology Telescope (ACT) survey [Hilton et. al. (2017)]
  - ➢ 182 optically confirmed galaxy clusters detected via the SZ effect in redshift range 0.1 < z < 1.4 at radius R_{500} and defined as M_{500}^{SZ}.
  - ➢ Universal pressure profile (UPP) modeled by using a generalized Navarro, Frank & White (NFW) density profile for dark matter halo.