Supergravity Localization & AdS Black Holes

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based on

[1803.05920] with K. Hristov and I. Lodato

July 11th, 2018

Workshop on Supersymmetric Localization and Holography – ICTP, Trieste

Black hole entropy

- Our starting point and motivation: black holes.
- Black holes are a theorist's laboratory to understand gravity.
  Key property:

$$\mathcal{S} = \frac{k_B\, c^3}{\mathcal{G}_N\, \hbar}\, \frac{A_H}{4} + \alpha \log A_H + \ldots + e^{-\beta\, A_H} + \ldots$$

- Semi-classical physics gives the leading term in a large area expansion.

[Hawking'71],[Bekenstein'73]

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- Semi-classical physics gives the leading term in a large area expansion.
  [Hawking'71],[Bekenstein'73]
- Corrections to Bekenstein-Hawking probe the quantum gravity regime.
- A natural question: can we give a Boltzmann interpretation of the exact entropy $\mathcal{S}$ in terms of microscopic degeneracies?

$$\mathcal{S} \stackrel{?}{=} k_B \log d_{\text{micro}}$$

Thermodynamic *vs.* Boltzmann entropy

- Progress has been made for supersymmetric models.
- Asymptotically flat BHs: string theory successfully accounts for Bekenstein-Hawking entropy by realizing the black hole as a brane system.

[Strominger,Vafa'96]

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- Asymptotically flat BHs: string theory successfully accounts for Bekenstein-Hawking entropy by realizing the black hole as a brane system. [Strominger,Vafa'96]
- In fact, brane picture is very powerful: also allows for the computation of sub-leading corrections to the entropy. [Maldacena,Strominger,Witten'97] [Dijkgraaf,Verlinde,Verlinde'97],[Maldacena,Moore,Strominger'99]
- For certain supersymmetric black holes, microscopic degeneracies are fully known as functions of the charges carried by the brane system.
- Generating functions obtained from topological invariants (elliptic genus and generalizations).

Thermodynamic *vs.* Boltzmann entropy (cont.)

- Asymptotically AdS BHs: recent progress for AdS_4 spherically symmetric BPS black holes via microstate counting in the dual field theory.

  [Benini,Hristov,Zaffaroni'16]

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- Resulting matrix model is valid for all N, but difficult to evaluate exactly. At large N, it reproduces the Bekenstein-Hawking entropy of the BH.
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- Resulting matrix model is valid for all N, but difficult to evaluate exactly. At large N, it reproduces the Bekenstein-Hawking entropy of the BH.
- The sub-leading contributions are encoded in the matrix model.
- Followed (a lot of) generalizations to other models, other dimensions, etc...

  [Azzurli,Bobev,Cabo-Bizet,Crichigno,Hosseini,Liu,Min,Nedelin,Giraldo-Rivera, Pando Zayas,Passias,Pilch,Rathee,Zhao...]

Aim of this talk

- Given the degeneracies computed in the microscopic picture, can we define (and compute!) the corrections to $\mathcal{S}$ directly in the macroscopic picture?
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- We will examine these questions using
  - The quantum entropy of asymptotically AdS black holes
  - Localization in gauged supergravity (gSUGRA)

    [Dabholkar,Drukker,Gomes'14],[Nian,Zhang'17]
- See also talks by B. de Wit and I. Jeon.

Outline

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- In general, want to examine solutions of 4d $\mathcal{N} = 2$ gauged SUGRA, with electric and magnetic charges and AdS asymptotics.

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- Full BH solution interpolates between AdS_4 vacuum at infinity and the near-horizon $\text{AdS}_2 \times S^2$ geometry.
- The solution preserves 2 supercharges (1/4-BPS), and in the near-horizon region there is an enhancement to 4 supercharges (1/2-BPS).
- General feature of the attractor mechanism in gauged SUGRA.

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- Focus on the entropy contribution from the near-horizon.
- Allows to consider a large class of asymptotically AdS BHs at once, namely all those with near-horizon attractor geometry $\mathrm{AdS}_2 \times S^2$.
  See also talk by K. Hristov.

- The AdS_2 factor is a key ingredient to explore the quantum entropy of BHs.
- Proposal: compute the following expectation value [Sen'08]

The quantum entropy

$$e^{\mathcal{S}(p,q)} := W(p,q) = \left\langle \exp\left[-i\, q_I \oint A^I_\tau \, d\tau\right] \right\rangle^{\text{finite}}_{\text{AdS}_2}$$

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- 'finite' denotes a regularization procedure, due to non-compactness of AdS_2.
- In a (suitable) large charge limit, this is expected to reproduce the Bekenstein-Hawking(-Wald) entropy of the BH.
- Holographically, corresponds to a Witten index in the dual CFT_1 counting the number of ground states (microstates).

- The quantum entropy defines the non-perturbative entropy of extremal BPS BHs as a Euclidean path integral (expectation value of a Wilson line).
- Localization gives an exact one-loop evaluation of path integrals.

[Witten‘88],[Pestun‘07],...

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- Apply localization in 4d $\mathcal{N} = 2$ gSUGRA. Will use superconformal formulation of the theory, which ensures off-shell closure of the gauge algebra.

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- Field content: Weyl multiplet, $n_v + 1$ vector multiplets and 1 hypermultiplet, including the gauge compensators.
- Specify the near-horizon 1/2-BPS field configuration, and look for off-shell BPS fluctuations around this background satisfying the attractor b.c.'s.

- The Weyl multiplet:

$$\mathbb{W} = (e_\mu{}^a,\, \psi_\mu{}^i,\, b_\mu,\, A_\mu,\, \mathcal{V}_\mu{}^i{}_j \mid T_{ab},\, \chi^i,\, D)$$

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- The (on-shell) compensating hypermultiplet:

$$\mathbb{H} = (A_i{}^\alpha,\, \zeta^\alpha)$$

- The gauging is specified by $(\xi_I,\, \xi^I = 0)$ and generators $t^\alpha{}_\beta$,

$$\mathcal{D}_\mu A_i{}^\alpha = (\partial_\mu - b_\mu) A_i{}^\alpha + \tfrac{1}{2}\, \mathcal{V}_\mu{}_i{}^j\, A_j{}^\alpha - \xi_I\, W_\mu^I\, t^\alpha{}_\beta\, A_i{}^\beta$$

- In the gauge-fixed Poincaré theory, the gravitini are electrically charged.

- Weyl multiplet (bosonic) attractor configuration:

$$\mathring{g}_{\mu\nu}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu = v_1\Big[(r^2-1)\,\mathrm{d}\tau^2 + \frac{\mathrm{d}r^2}{r^2-1}\Big] + v_2\,\mathrm{d}\Omega_2{}^2$$

$$\mathring{T}^{\mp}_{12} = \pm\frac{2}{\sqrt{v_1}} \qquad \mathring{D} = -\frac{1}{6}\Big(\frac{1}{v_1}+\frac{2}{v_2}\Big)$$

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- The hypermultiplet compensator fixes the $\mathrm{SU}(2)_\mathrm{R}$ gauge,

$$\chi_\mathrm{H}^{-1/2}\,\mathring{A}_i{}^\alpha = \delta_i{}^\alpha \;\text{ where }\; \chi_\mathrm{H} = \tfrac{1}{2}\,\varepsilon^{ij}\Omega_{\alpha\beta}\,\mathring{A}_i{}^\alpha\,\mathring{A}_j{}^\beta$$

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- Vector multiplets (bosonic) attractor configuration:

$$\xi_I\, \mathring{F}^{\mp}_{34}{}^I = \frac{1}{4 v_2} \qquad \xi_I\, \mathring{X}^I_{\mp} = \frac{1}{4\sqrt{v_1}}$$

$$\mathring{Y}^I_{ij} = 2\, \chi_{\mathrm{H}}\, \mathring{N}^{IJ}(X_+, X_-)\, \xi_J\, \varepsilon_{ik}\, t^k{}_j$$

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$$\mathring{Y}^I_{ij} = 2\,\chi_\mathrm{H}\,\mathring{N}^{IJ}(X_+,X_-)\,\xi_J\,\varepsilon_{ik}\,t^k{}_j$$

- $\mathrm{SU}(2)_\mathrm{R}$ gauge field fixed in terms of the vectors, $\mathring{\mathcal{V}}_\mu{}^i{}_j = -2\,\xi_I\,\mathring{W}^I_\mu\,t^i{}_j$.

- Now look for off-shell BPS fluctuations. Must solve the gravitini SUSY variation for arbitrary metric and Killing spinor respecting b.c.'s.
- Hard problem. In ungauged SUGRA, solved for fluctuations around the full-BPS attractor configuration where, in particular, the $SU(2)_R$ vanishes.

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- In gSUGRA, we assume that this is also the case: $\mathbb{W} = \mathring{\mathbb{W}}$ and $\epsilon^i = \mathring{\epsilon}^i$.
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- As a result, we use the attractor geometry and Killing spinors and look for off-shell BPS fluctuations of gauge and matter fields only.
- Among the 4 supercharges of the attractor background, we pick

Algebra of the localizing supercharge

$$\left(Q_{\text{loc}}\right)^2 = \mathcal{L}_\tau + \delta_{\text{SU(2)}}\left(\tfrac{1}{\sqrt{v_1}}\, t^i{}_j\right) + \delta_{\text{gauge}}(X_\pm)$$

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- In the vector multiplet sector, impose the vanishing of gaugini variation

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- Supercharge parametrized by a particular attractor Killing spinor. Due to the non-trivial $\mathrm{SU}(2)_{\mathrm{R}}$ gauge field, the gauging effects a twist on the S^2.
- The localizing KS is constant on the 2-sphere. The localizing equations are non-trivial in the radial direction, less constraining in the angular directions.

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- The localizing KS is constant on the 2-sphere. The localizing equations are non-trivial in the radial direction, less constraining in the angular directions.
- Parametrization for the off-shell fluctuations

$$X^I_\pm = \mathring{X}^I_\pm + x^I_\pm \qquad F^I_{ab} = \mathring{F}^I_{ab} + f^I_{ab} \qquad Y^I_{ij} = \mathring{Y}^I_{ij} + y^I_{ij}$$

- The localizing equations constrain the fluctuations $(x^I_\pm,\, f^I_{ab},\, y^I_{ij})$.

- Imposing b.c.'s, we find for each vector multiplet

$$X_\pm = \mathring{X}_\pm + \sum_{k=1}^{\infty} \frac{C_k^\pm(\theta, \varphi)}{r^k}$$

  and f_{ab}, y_{ij} are given in terms of $C_k^\pm$.

- Two real functional parameters for each vector multiplet.

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- There is however an additional constraint: a specific linear combination of the fluctuation parameters is independent of the angular coordinates

$$\begin{aligned} \mathcal{M}_{\text{loc}}^{\text{vec}} &= \left\{ C_k^{+}(\theta,\varphi)^I, C_k^{-}(\theta,\varphi)^I \right\} \quad \forall\, k \geq 1, \;\; \forall\, I = 0 \ldots n_v \\ &= \left\{ \phi_+^I, \phi_\perp^I(\theta,\varphi) \right\} \end{aligned}$$

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- Here we assumed smoothness of the BPS solutions. Additional singular configurations may contribute, similar to the orbifolded geometries $AdS_2/\mathbb{Z}_c$ in the asymptotically flat case. Set aside for now.

- For the compensating hypermultiplet, need to put it off-shell with respect to the localizing supercharge $Q_{\rm loc}$.
- Do so by introducing auxiliary scalar fields and constrained parameters

[Berkovits'93],[Hama,Hosomichi'12]

$$Q_{\rm loc}\zeta^i_\pm = \not{D} A_j{}^i\,\xi^j_\mp - 2\,\xi_I\,X^I_\mp\,t^i{}_j\,A_k{}^j\,\xi^k_\pm + H_j{}^i\,\check{\xi}^j$$

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- The gauging relates the off-shell BPS fluctuations in the hyper sector to those in the vector multiplet sector.
- Fluctuations of $A_j{}^i$ and $H_j{}^i$ controlled by the parameters C^+_k and C^-_k.
- In addition, constraint on the constant linear combination

$$\xi_I\,\phi^I_+ = 2\pi$$

- We have characterized the localizing manifold in the gauge/matter sector

The localizing manifold in gSUGRA

$$\mathcal{M}_{\text{loc}} = \left\{ \phi^I_+, \phi^I_\perp(\theta, \varphi) \quad \text{s.t.} \quad \xi_I\, \phi^I_+ = 2\pi \right\}$$

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- Now evaluate the gSUGRA action on $\mathcal{M}_{\text{loc}}$. In the Euclidean formalism, this action is specified by two prepotentials (not related by complex conjugation)

$$\mathcal{F}^+(X_+) \quad \text{and} \quad \mathcal{F}^-(X_-)$$

- Important observation: S_{bulk} only depends on the constant parameter ϕ_+^I.

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- Important observation: S_{bulk} only depends on the constant parameter ϕ^I_+.
- Add the Wilson line contribution and appropriate boundary terms to renormalize (according to the 'finite' prescription).
- Final form:

$$S_{\text{ren}}(\phi_+) = p^I\,\mathcal{F}^+_I(\phi_+) - q_I\,\phi^I_+$$

- Because $\phi^I_\perp(\theta,\varphi)$ are zero-modes of the renormalized action, they can only enter via the one-loop determinants and not via the classical action.
- We split the one-loop det. into a contribution from the ϕ_+-modes and one from the $\phi_\perp$-modes. The latter still contains a functional integration since $\phi^I_\perp(\theta,\varphi)$ are functions on the 2-sphere of the near-horizon geometry.
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- Putting everything together,

Quantum entropy for 1/4-BPS BHs in $\mathcal{N} = 2$ gSUGRA

$$\widehat{W}(p, q) = \int_{-\infty}^{+\infty} \prod_{I} \mathrm{d}\phi_{+}^{I}\, \delta(\xi_I \phi_{+}^{I} - 2\pi)\, \exp\left[q_I \phi_{+}^{I} - p^I \mathcal{F}_I^{+}\right] Z_{\text{1-loop}}(\phi_{+}^{I})\, Z_{\perp}(\phi_{\perp}^{I})$$

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- One-loop determinants currently being investigated.

[Hristov,Lodato,VR - WIP]

- Setting aside the unknown factors, we can analyze the saddle-point.
- Define the familiar combinations

$$\mathcal{Z}(\phi) = q_I \phi_+^I - p^I \mathcal{F}_I^+ \,, \quad \mathcal{L}(\phi) = \xi_I \phi_+^I$$

The delta function imposes $\mathcal{L} = 2\pi$ and the saddle-point equations coincide with the standard attractor equations in gSUGRA,

[Cacciatori,Klemm'09],[Dall'Agata,Gnecchi'10],[Hristov,Vandoren'10]

$$\frac{\partial}{\partial \phi_+^I} \frac{\mathcal{Z}}{\mathcal{L}} = 0$$

- The value of $\mathcal{Z}/\mathcal{L}$ at the extremum is the Bekenstein-Hawking entropy.

- Setting aside the unknown factors, we can analyze the saddle-point.
- Define the familiar combinations

$$\mathcal{Z}(\phi) = q_I \phi_+^I - p^I \mathcal{F}_I^+ \,, \quad \mathcal{L}(\phi) = \xi_I \phi_+^I$$

The delta function imposes $\mathcal{L} = 2\pi$ and the saddle-point equations coincide with the standard attractor equations in gSUGRA,

[Cacciatori,Klemm‘09],[Dall’Agata,Gnecchi‘10],[Hristov,Vandoren‘10]

$$\frac{\partial}{\partial \phi_+^I} \frac{\mathcal{Z}}{\mathcal{L}} = 0$$

- The value of $\mathcal{Z}/\mathcal{L}$ at the extremum is the Bekenstein-Hawking entropy.
- Strong hint that the product of one-loop and perp. factors does not contribute at the level of the saddle-point.
  Otherwise, would spoil the leading order agreement.
- At least for one-loop, reminiscent of the flat BH case where $Z_{\text{1-loop}}$ contributes at order $\log A_H$ and beyond.

- Another check: M-theory on $\mathrm{AdS}_4 \times S^7$ has dual holographic description as ABJM theory (with $k = 1$).
- For 3d $\mathcal{N} = 2$ gauge theory on $S^1 \times S^2$ with a topological twist on S^2, the partition function is given by the topologically twisted index.

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- This index can be computed using (rigid) localization. [Benini,Zaffaroni'15]
- The degeneracies of supersymmetric ground states as a function of the charges are obtained after a Fourier transform of the index:

$$d_{\mathrm{CFT}}(p,q) = \int_0^{2\pi} \mathrm{d}\Delta_a \, \delta\Big(\sum_a \Delta_a - 2\pi\Big) \, Z(p,\Delta) \, \exp[-q_a \, \Delta_a]$$

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- At large N, the index $Z(p,\Delta) = \mathrm{Tr}\,(-1)^F \, e^{-\beta H + \Delta_a J_a}$ can be evaluated in a variety of examples, including ABJM.
- Again setting aside the one-loop and perp. factors, integrand of $\widehat{W}$ matches the one of d_{CFT} at large N upon identifying ϕ_+^I with the chemical potentials Δ_a and using the holographic dictionary.

- An observation: we obtained, in gauged SUGRA,

$$\widehat{W}_{\text{AdS}}(p,q) \sim \int_{-\infty}^{+\infty} \prod_I \mathrm{d}\phi^I\, \delta(\xi_I\phi^I - 1)\, \exp\left[q_I\phi^I - p^I\mathcal{F}_I\right] Z_{\text{1-loop}}(\phi^I)\, Z_\perp$$

- Compare with the ungauged SUGRA result [Dabholkar,Gomes,Murthy'10]

$$\widehat{W}_{\text{flat}}(p,q) \sim \int_{-\infty}^{+\infty} \prod_I \mathrm{d}\phi^I\, \exp\left[q_I\phi^I - \mathrm{Im}\mathcal{F}(\phi^I + ip^I)\right] Z_{\text{1-loop}}(\phi^I)$$

with (see talk by I. Jeon) [Murthy,VR'15],[Gupta,Ito,Jeon'15],[Jeon,Murthy'18]

$$Z_{\text{1-loop}}(\phi^I) = \exp\left[-\left(2 - \frac{n_v + 1 - n_h}{12}\right)\mathcal{K}(\phi^I)\right]$$

- For specific models (IIB@T^6), $\widehat{W}_{\text{flat}}$ can be computed exactly.
Precise non-perturbative match with exact string theory results.
Beautiful connection to number theory (Rademacher expansion).

Outline

Conclusions and future directions

- We presented the first step towards the computation of the quantum entropy for BPS BHs in gSUGRA with near-horizon $\mathrm{AdS}_2 \times S^2$.
- Unsurprisingly, the saddle-point of the quantum entropy reproduces the (semi-)classical Bekenstein-Hawking entropy via the attractor mechanism.
- The answer takes a very similar form to the computation of the topologically twisted index in the dual field theory. Encouraging...

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- Unsurprisingly, the saddle-point of the quantum entropy reproduces the (semi-)classical Bekenstein-Hawking entropy via the attractor mechanism.
- The answer takes a very similar form to the computation of the topologically twisted index in the dual field theory. Encouraging...
- Main priority: completing the one-loop determinant computation in gSUGRA. Should lead to a better understanding of the role of the zero-modes $\phi^I_\perp$. Concrete goal that could teach us about the dual matrix model at finite N.
- Thorough investigation of our assumption that $\mathbb{W} = \mathring{\mathbb{W}}$.
- Generalization to $\mathrm{AdS}_2 \times \Sigma_g$ geometries and more general gaugings ($\xi^I \neq 0$).

Thank you for your attention