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Hydrodynamic transport in electron systems

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Collaborators

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Fluid flow through an empty 2D channel

What happens when we drive a classical incompressible fluid through a '2D pipe' by applying a pressure gradient along *x*?

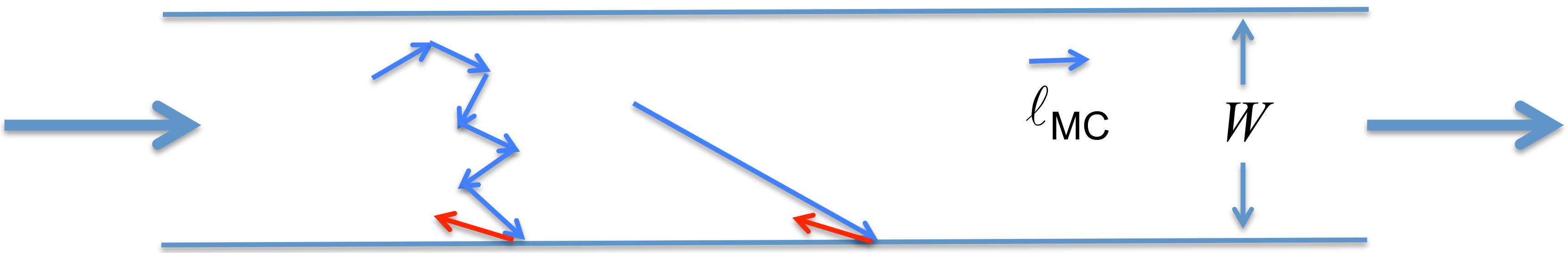
Fluid velocity is greater in the middle of the channel than at the edges

The only transfer of momentum to the outside world is at the edges

We parameterise the strength of the coupling of the fluid to itself along *y* with the *shear viscosity*, η.

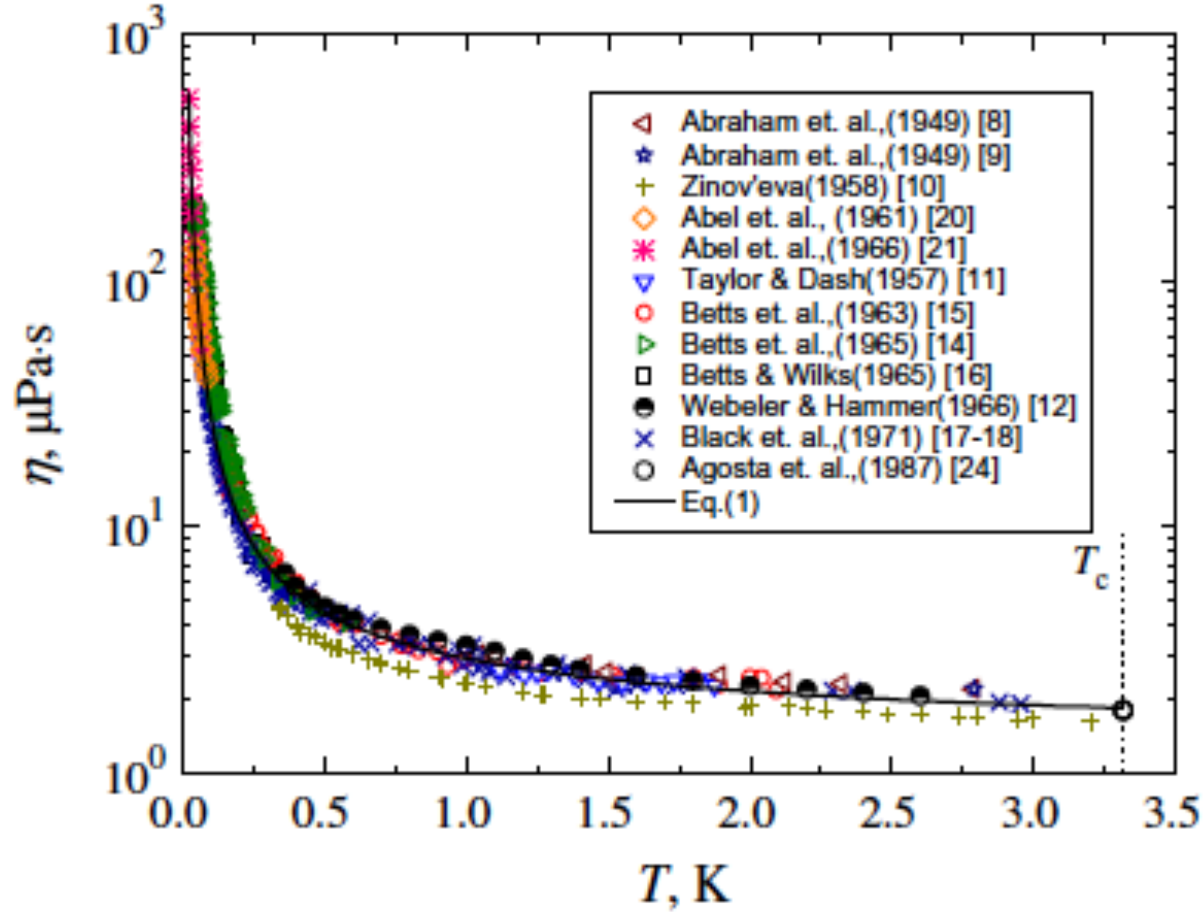
Often this is quoted as a *kinematic viscosity* $\eta_K = \eta/\rho$ where ρ is the density.

Fluid flow through an empty 2D channel



- Mean free path ℓ_{MC} is for scattering of the fluid particles from each other. These events conserve the overall fluid momentum.
- Only momentum-relaxing collisions are with the outside world, i.e. at the walls of the channel.
- For longer ℓ_{MC} the particles find the walls more efficiently so the rate of momentum relaxation goes up.
- The same applies to all transverse coupling so ***η is proportional to ℓ_{MC}***. A 'pure' particle fluid with a low internal scattering rate is a viscous one!
- Most appropriate theory is based on hydrodynamics, e.g. on the Navier-Stokes equations

What about a quantum fluid? Consider ^3He



This is non-intuitive at first sight – the 'better' the fluid (lower scattering) the more viscous it becomes!

It is a very real effect though – it dictates the low temperature limit of dilution fridge operation.

High scattering rate implies low viscosity: is there a lower bound on viscosity?

A natural uncertainty-principle-based definition of a characteristic time is:

$$\tau \sim \hbar / k_B T$$

Is this more than a piece of dimensional analysis – does it have physical significance as a minimum time?

S. Sachdev, Quantum Phase Transitions, Cambridge University Press, 1999

J. Maldacena, S. Shenker & D. Stanford, arXiv:1503.01409

Same basic uncertainty principle idea applied to viscosity:

$$\frac{\eta}{s} \geq \frac{1}{4\pi} \frac{\hbar}{k_B}$$

P. Kovtun, D.T. Son and A.O. Starinets, Phys. Rev. Lett. ***94****, 111601 (2005)*

Hydrodynamics in contemporary condensed matter theory

The minimum time is of relevance to non-Fermi liquids, and leading to a large body of work discussing these strongly correlated systems using hydrodynamic theories, e.g.

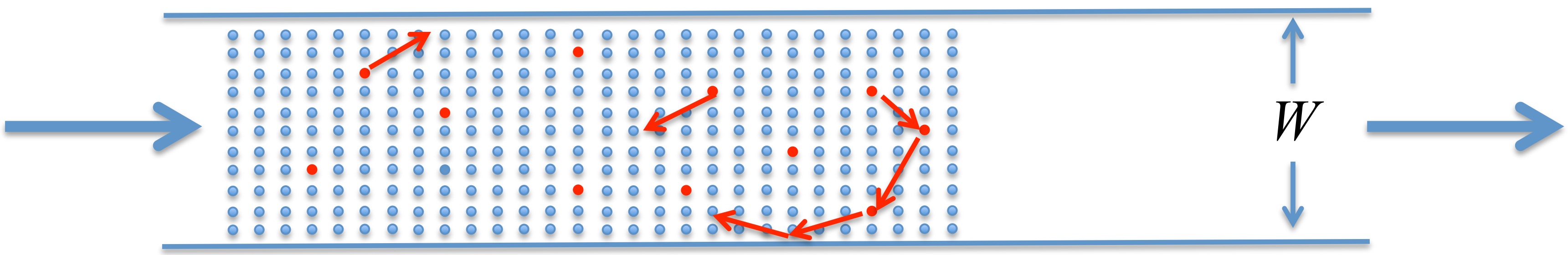
*S.A. Hartnoll, P.K. Kovtun, M. Mueller and S. Sachdev, Phys. Rev. B **76**, 144502 (2007)*
*M. Mueller & S. Sachdev, Phys. Rev. B. **78**, 115419 (2007)*
*M. Mueller, J. Schmalian & L. Fritz, Phys. Rev. Lett. **103**, 025301 (2009)*
*J. Sonner and A.G. Green, Phys. Rev. Lett. **109**, 091601 (2012)*
*R.A. Davison, K. Schalm and J. Zaanen, Phys. Rev. B **89**, 245116 (2014)*
*A. Lucas, J. Crossno, K.C. Fong, P. Kim and S. Sachdev, Phys. Rev. B **93**, 075426 (2016)*

Electronic hydrodynamics of standard metals also studied in recent years by e.g.

*B. Spivak, S. A. Kivelson, Ann. Phys. **321**, 2071–2115 (2006)*
*A.V. Andreev, S.A. Kivelson and B. Spivak, Phys. Rev. Lett. **106**, 256804 (2011)*
*I. Torre, A. Tomadin, A. K. Geim and M. Polini, Phys. Rev. B **92**, 165433 (2015)*
*L. Levitov and G. Falkovich, Nat. Phys. **12**, 672 (2016)*
A. Lucas and S.A. Hartnoll, arXiv:1706.04621

Question re-asked by a number of groups around 2014:
Are electronic hydrodynamics experimentally observable?

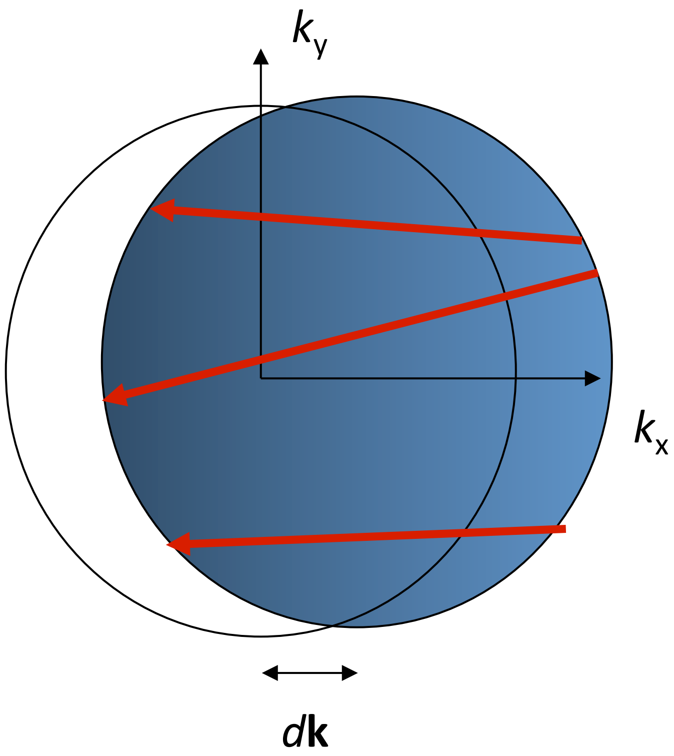
Why is electron hydrodynamics a challenge? Electrons flowing in a standard solid are *far* from the hydrodynamic regime



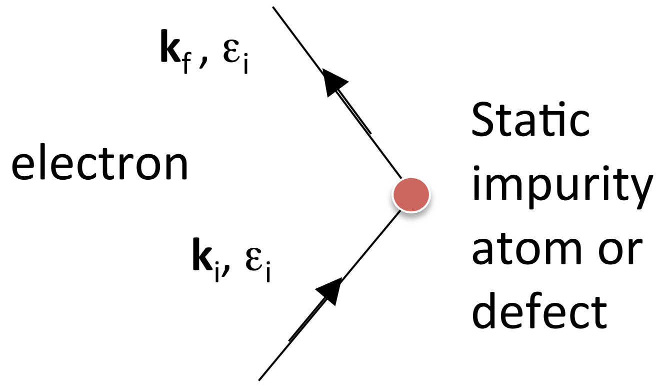
- Unlike the fluid in the empty tube, electrons in solids have many ways of making collisions in the bulk that relax the momentum to the solid.

Reminder of scattering processes in solids

For illustration, use two dimensions:



Electron-impurity scattering



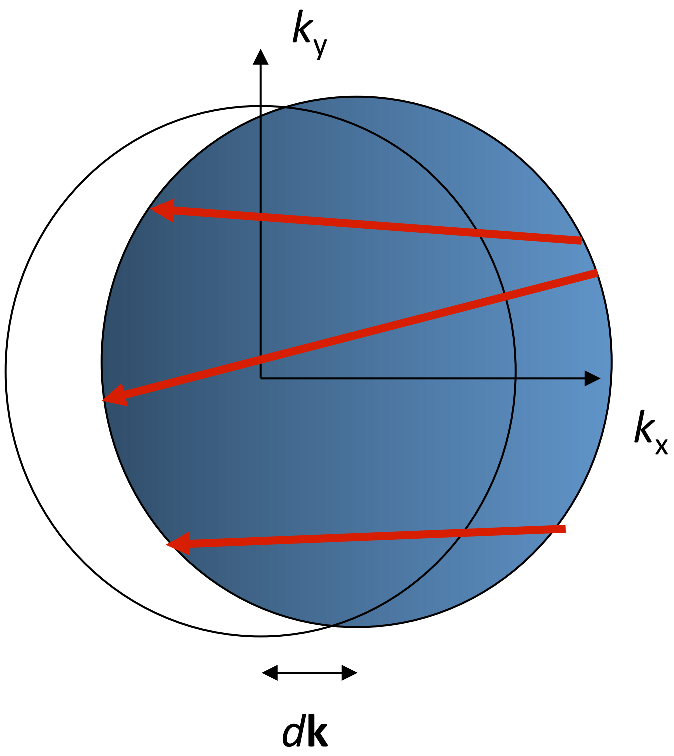
Always momentum relaxing

To give resistivity, you must relax the total momentum of the conduction electrons.

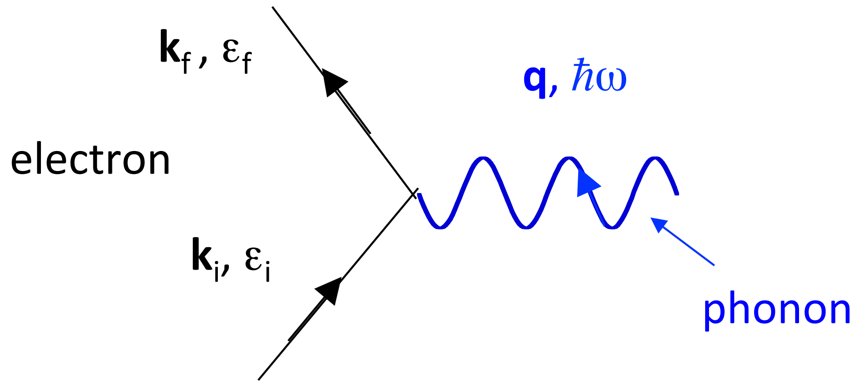
Which microscopic scattering processes do this?

Reminder of scattering processes in solids

For illustration, use two dimensions:



'Normal' electron-phonon scattering



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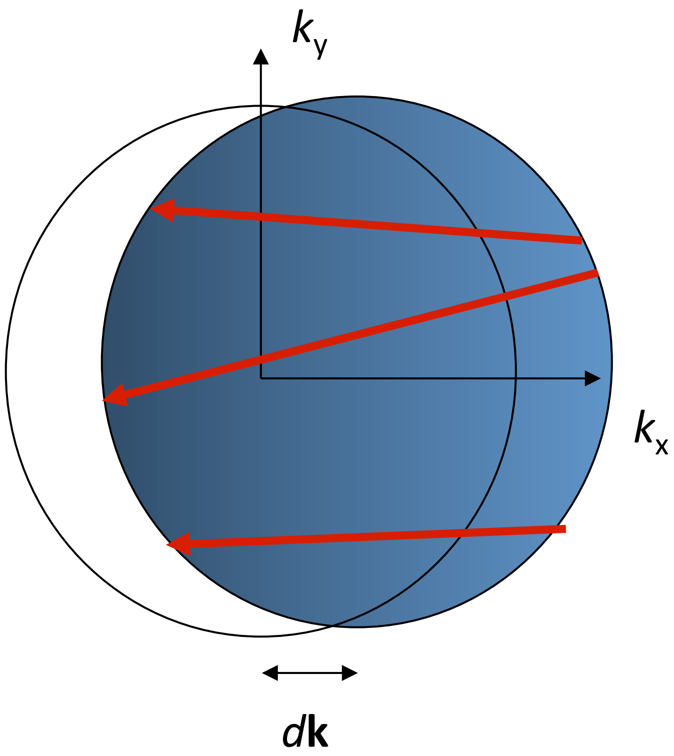
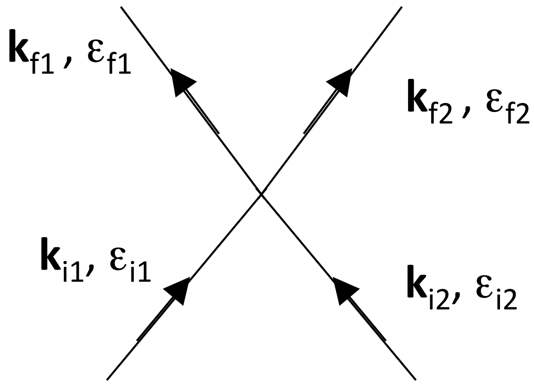
Reminder of scattering processes in solids

For illustration, use two dimensions:



'Normal' electron-electron scattering

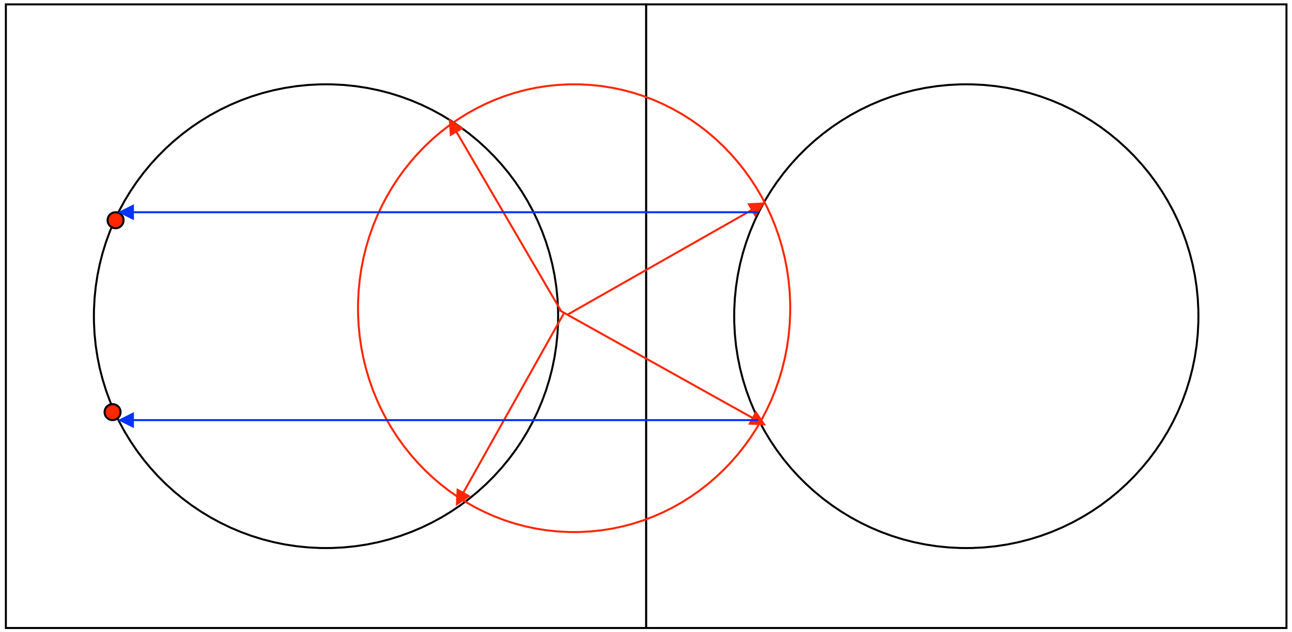


Momentum-conserving: individual electrons change momentum but the overall assembly of electrons conserves that momentum.

To give resistivity, you must relax the total momentum of the conduction electrons.

Which microscopic scattering processes do this?

A striking example of the difference between crystal momentum and 'real' momentum: *Umklapp* processes.



Scattering circle with radius *q* from point **k**. In free electron case, only two allowed **k'**.

But in a solid, can go to repeated zone scheme. Two extra allowed **k'**.

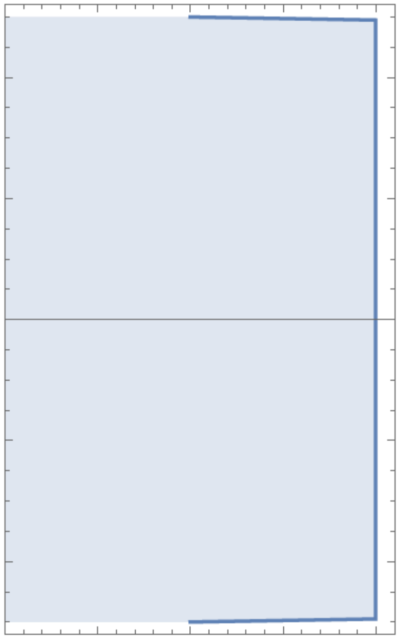
Electron-phonon and electron-electron Umklapp processes, if allowed, always relax momentum

The approach to electronic flow in 99.9999% of metals

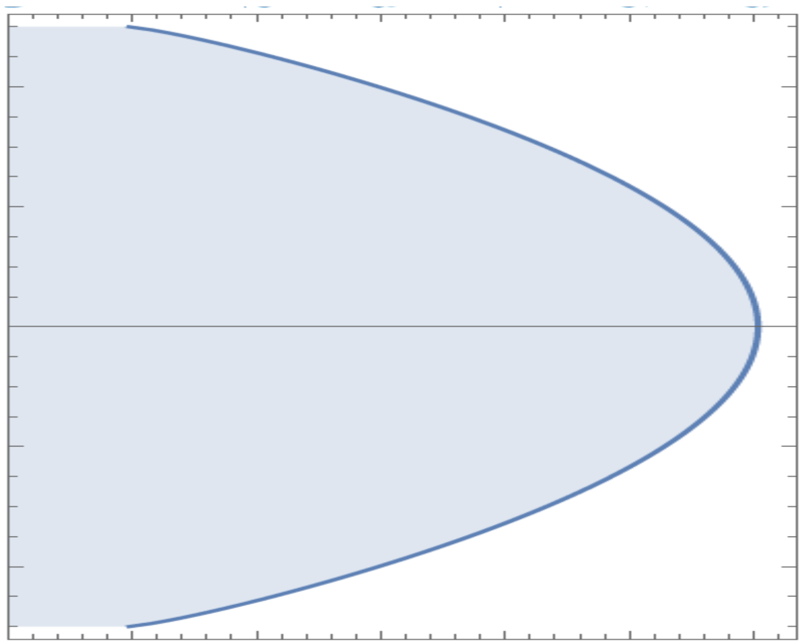
The electron fluid can usually relax its momentum very efficiently in the bulk of the material.

The boundaries are therefore more or less irrelevant, so viscous contributions are more or less irrelevant as well.

Velocity profile: standard metal with internal momentum relaxing scattering dominant



Velocity profile: hydrodynamic metal with internal momentum conserving scattering dominant



Normal strategy: simply ignore processes that would be relevant to electronic viscosity.

The 0.00001%

MINIMUM OF RESISTANCE IN IMPURITY-FREE CONDUCTORS

R. N. GURZHI

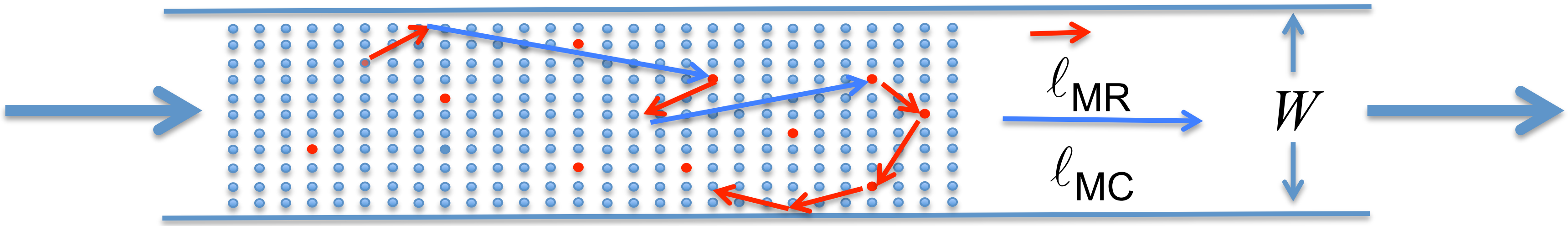
J. Exptl. Theoret. Phys. (U.S.S.R.) **44**, 771-772 (February, 1963)

HYDRODYNAMIC EFFECTS IN SOLIDS AT LOW TEMPERATURE

R. N. GURZHI

Usp. Fiz. Nauk **94**, 689–718 (April, 1968)

Key point introduced by Gurzhi: In solids, hydrodynamic effects can be parameterised in terms of the relationship between the three length scales ℓ_{MR}, ℓ_{MC} and sample dimension (here *W).*



$\ell_{MR} << \ell_{MC} << W$

Standard theory applies;
R is determined entirely by solid resistivity ρ and usual geometrical factors

$\ell_{MC} << W << \ell_{MR}$

Hydrodynamic theory applies; *R* is determined entirely by fluid viscosity η, boundary scattering and 'Navier-Stokes' geometrical factors

The early 1990s – availability of ultra-high mobility 2DEGs

Achieving the hydrodynamic condition $\ell_{MC} << W << \ell_{MR}$ is not easy. In fact it took 30 years.

Semiconductor 2DEGs are ideal:

- Hetero-doping allows very low impurity scattering.
- Small k_F so no e-e Umklapp.
- Possibility of suppressing e-ph scattering.

→ large ℓ_{MR}

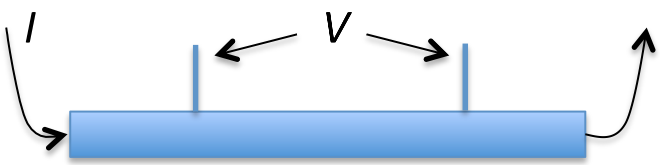
- Possibility of working with non-degenerate electron gases so quite small ℓ_{MC}
- Microfabrication and gating – control of W.

Successful hydrodynamic prediction of now widely observed THz plasma oscillations and other effects.

M. Dyakonov and M. Shur, Phys. Rev. Lett. ***71****, 2465 (1993)*

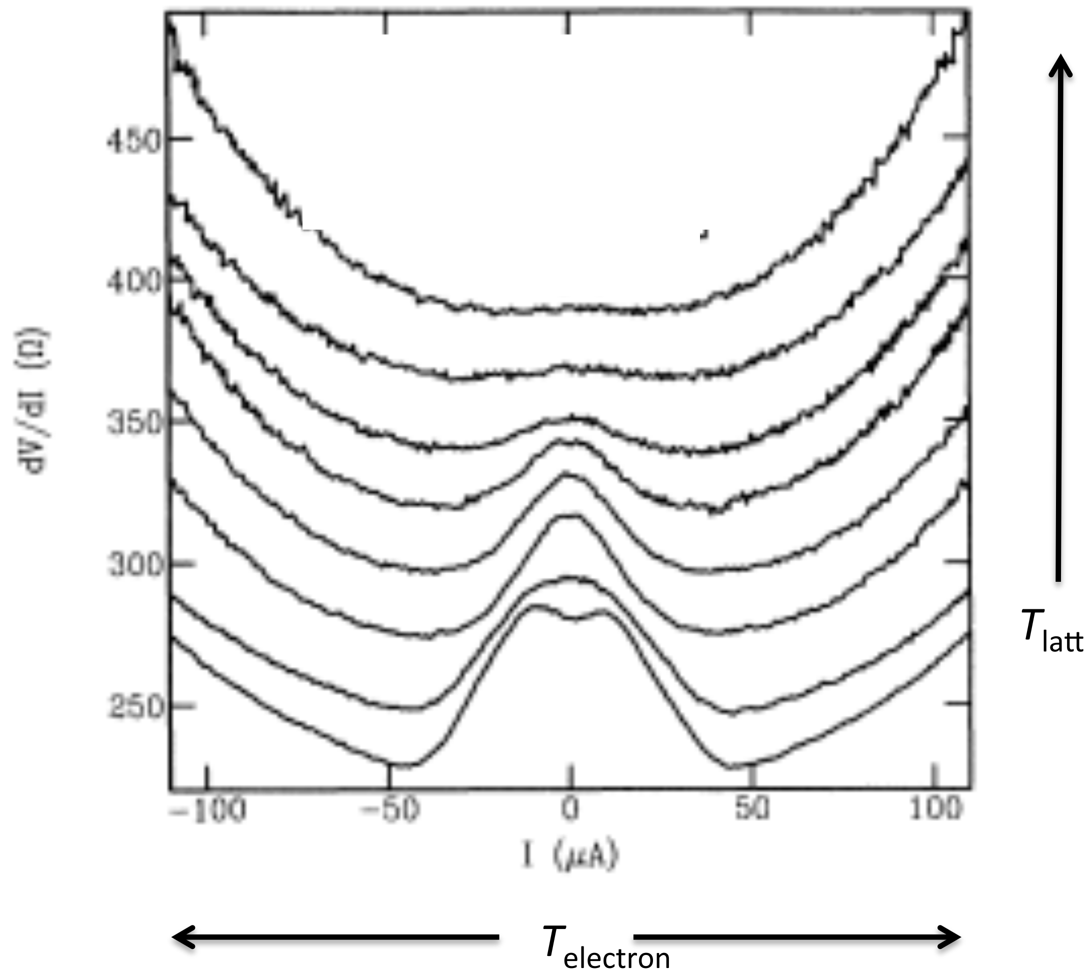
E. Chou, H.P. Wei, S.M. Girvin and M. Shayegan, Phys. Rev. Lett. ***77****, 1143 (1996)*

The Molenkamp – de Jong experiment



Molenkamp & de Jong:
use gating to define wires
with $W \sim 4$ μm.

Key idea: use high currents to differentially heat the electrons while leaving the lattice at an externally fixed temperature.



L.W. Molenkamp & M.J.M de Jong , Phys. Rev. B ***49****, 5038 (1994)*

The de Jong – Molenkamp theory

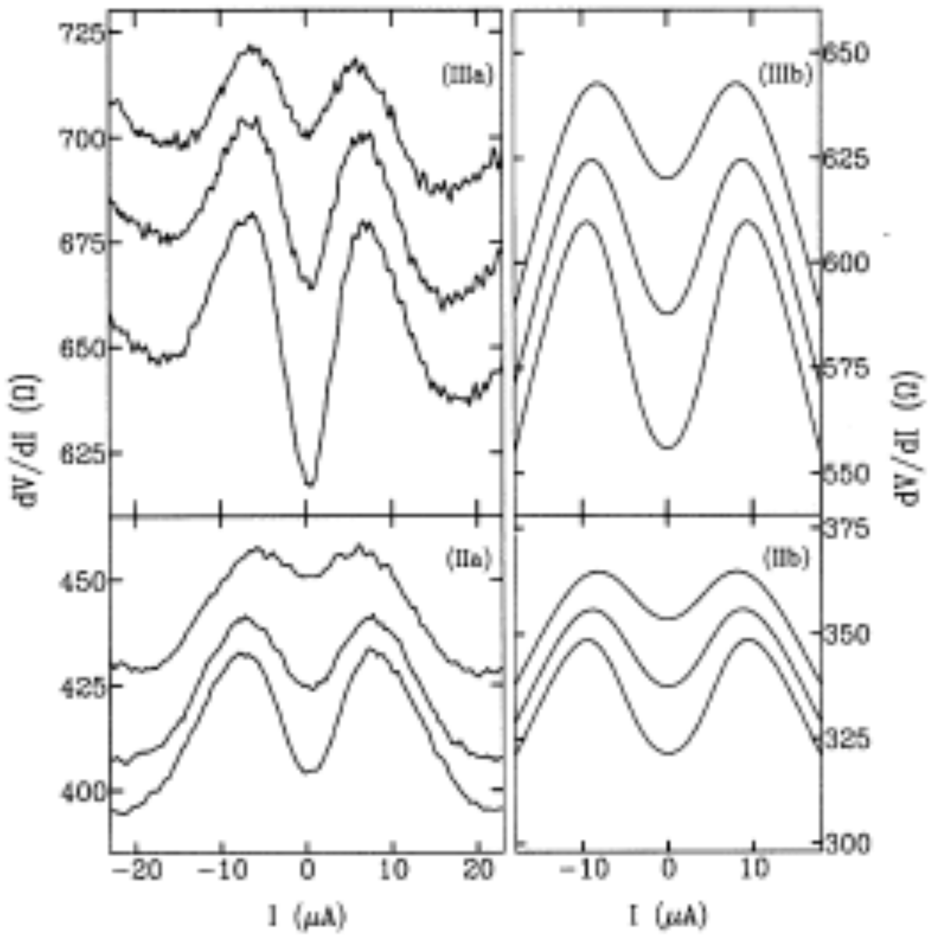
Rewrite standard Boltzmann theory explicitly including momentum-conserving scattering.

Convenient and (eventually!) intuitive parameterisation in terms of the three length scales introduced by Gurzhi.

Predictive capability in principle for any combination of ℓ_{MR}, ℓ_{MC} and W

Elegant, **useful**, but for some reason widely ignored.

*M.J.M de Jong & L.W. Molenkamp, Phys. Rev. B **51**, 13389 (1995)*



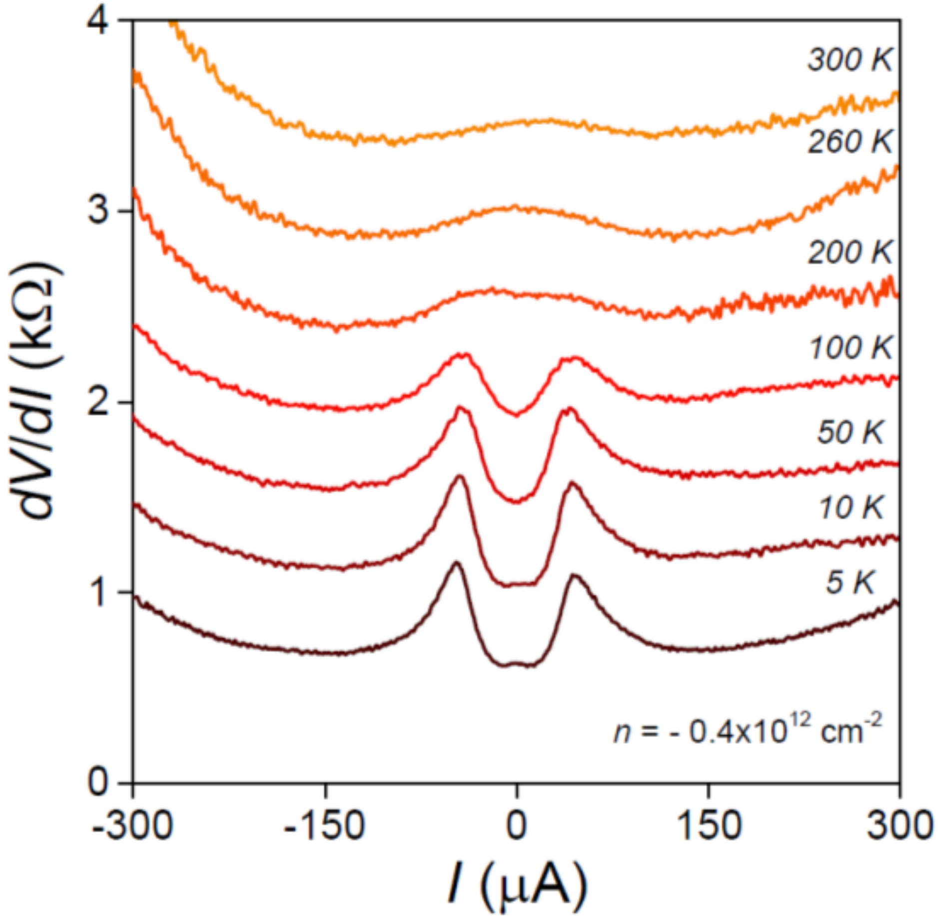
Note: *any* hydrodynamic theory incorporates assumptions about boundary conditions. For a good discussion in electron systems see *E.I. Kiselev and J. Schmalian, ArXiv:1806.03933 (2018)*

Fast-forward 20 years: Graphene hydrodynamics

Graphene shares many of the advantages of semiconductor 2DEGs with additional topicality and new physics due to the Dirac dispersion etc.

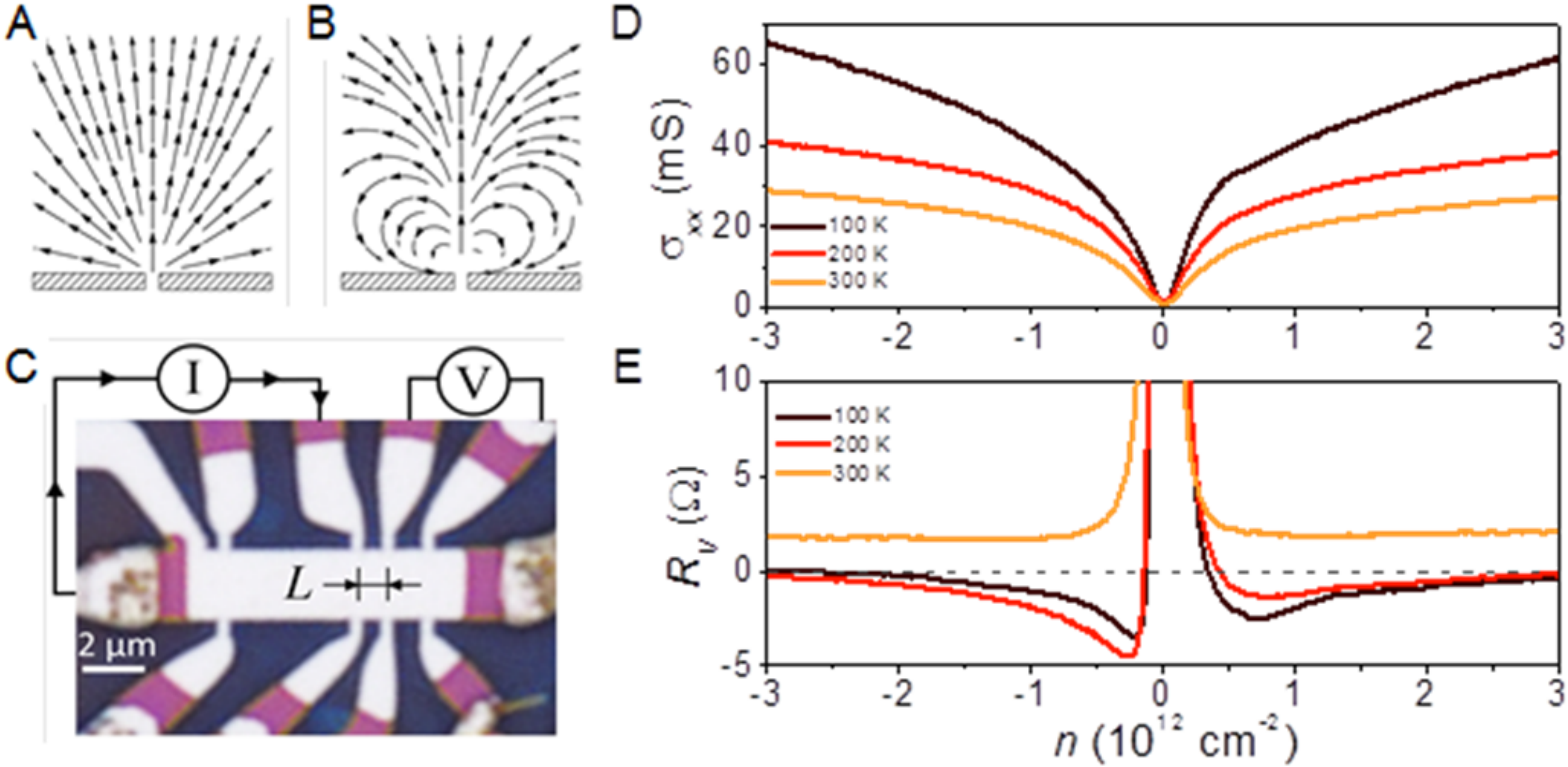
M. Mueller, J. Schmalian & L. Fritz, Phys. Rev. Lett. ***103****, 025301 (2009)*

Very recently – Molenkamp-de Jong result reproduced by Manchester group for semi-metallic dopings away from the Dirac point.



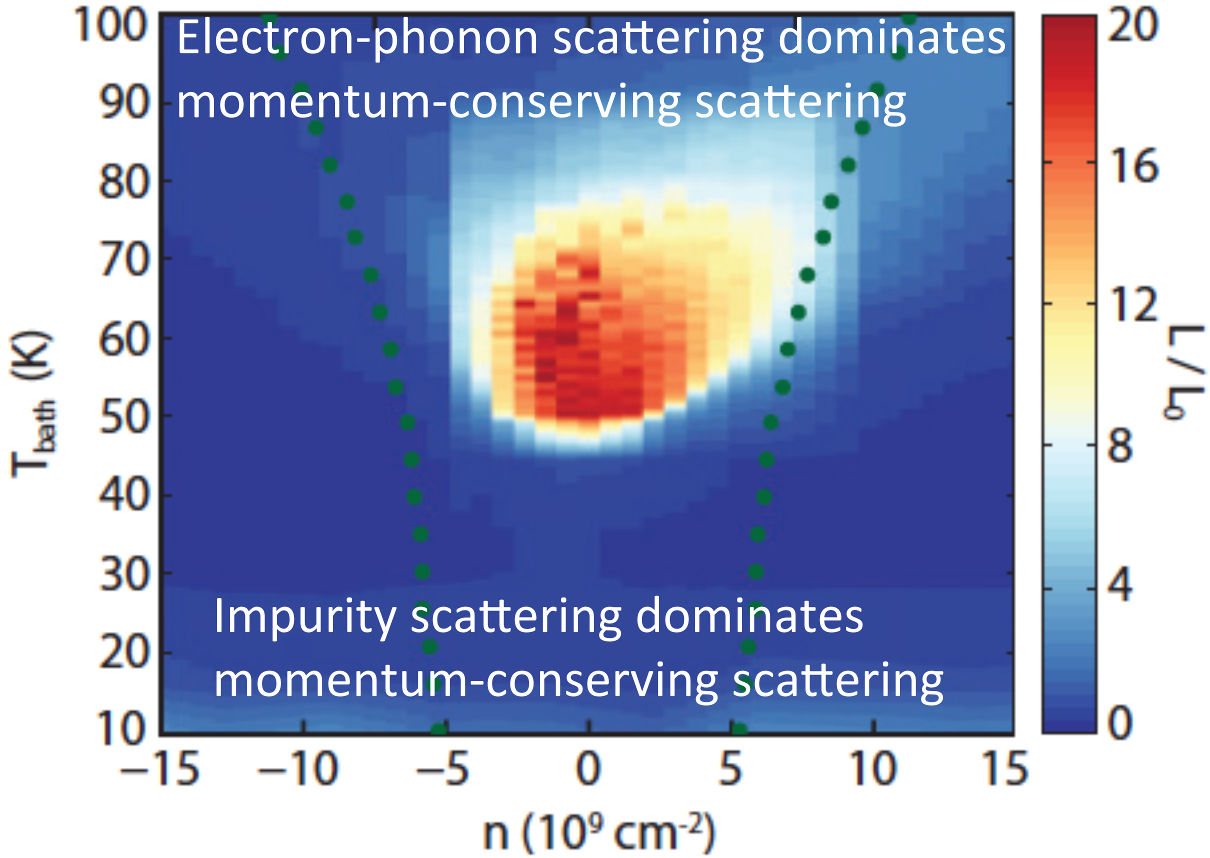
D. A. Bandurin, I. Torre, R. Krishna Kumar, M. Ben Shalom, A. Tomadin, A. Principi, G. H. Auton, E. Khestanova, K. S. Novoselov, I. V. Grigorieva, L. A. Ponomarenko, A. K. Geim, M. Polini, Science ***351****, 1055 (2016)*

Signatures of viscous current vortices observed



See also independent theory: *L. Levitov & G. Falkovich, arXiv:1508.00836; Nature Physics* ***12****, 672 (2016)*

Also thermal conductivity hydrodynamic signatures in graphene



Experiment observing hydrodynamic Wiedemann-Franz law violation in graphene*:*
J. Crossno, J. K. Shi, K. Wang, X. Liu, A. Harzheim, A. Lucas, S. Sachdev, P. Kim, T. Taniguchi, K. Watanabe, T. A. Ohki & K. C. Fong, Science ***351****, 1058 (2016)*

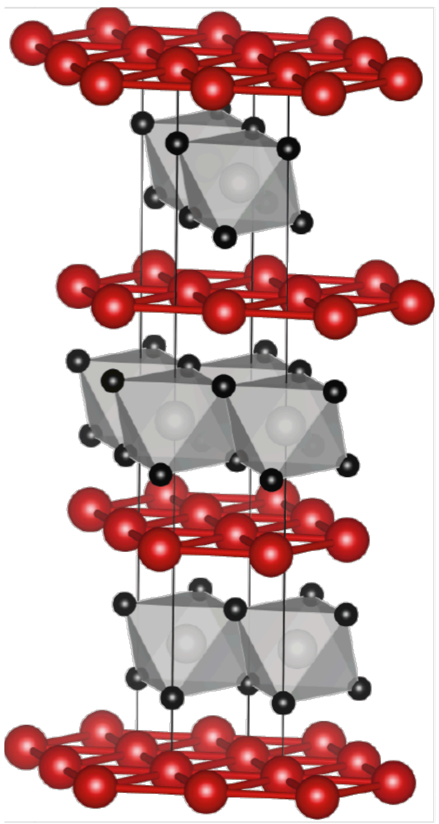
Can hydrodynamic effects be observed in a true metal?

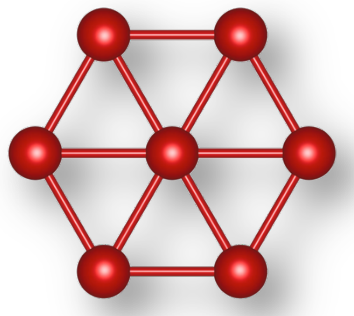
Recall hydrodynamic condition: $\ell_{MC} << W << \ell_{MR}$

Looks extremely difficult: Must often work far below T_F so ℓ_{ee} is very long, *and* e-e Umklapp is in principle efficient. Expectation is that $\ell_{MC} >> \ell_{MR}$.

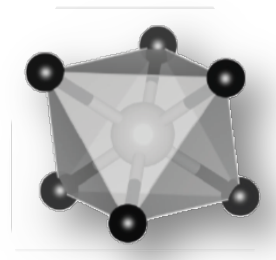
However, *delafossites* seem not to be standard metals.

ABO_2





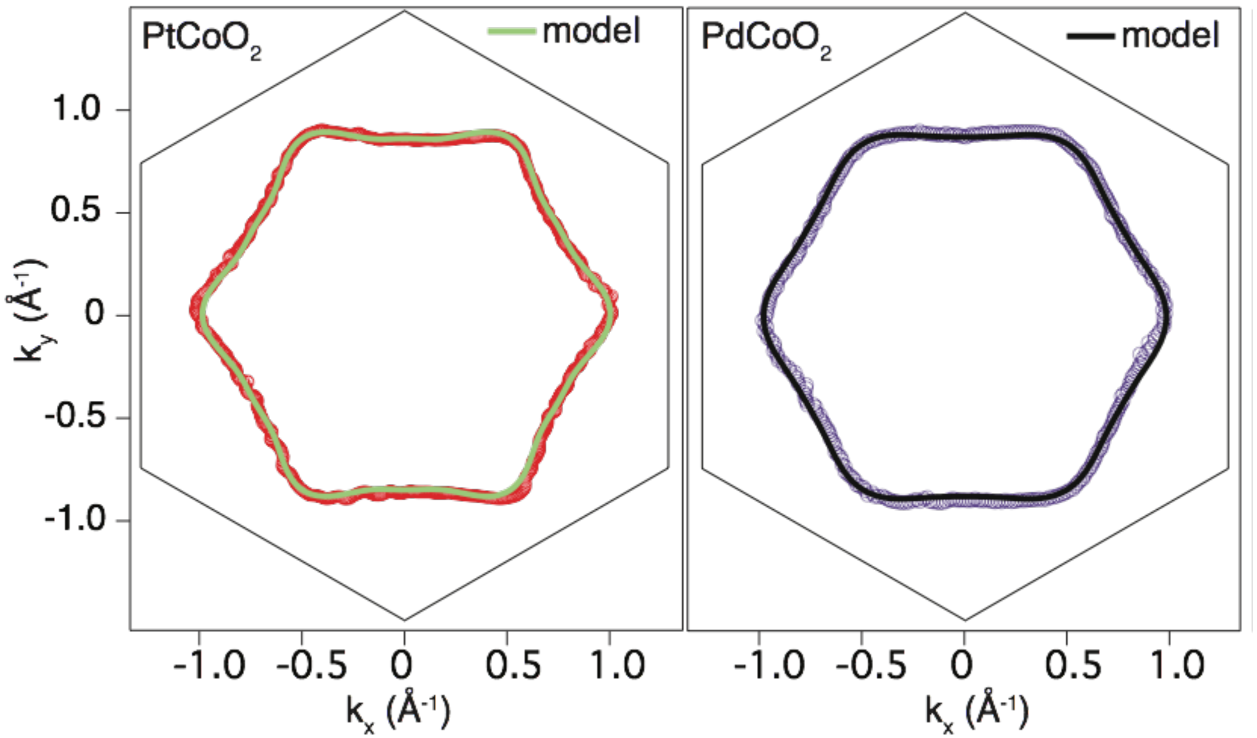
A site:
Pd or Pt



CoO_2 octahedra

Non-magnetic low-spin Co^{3+}

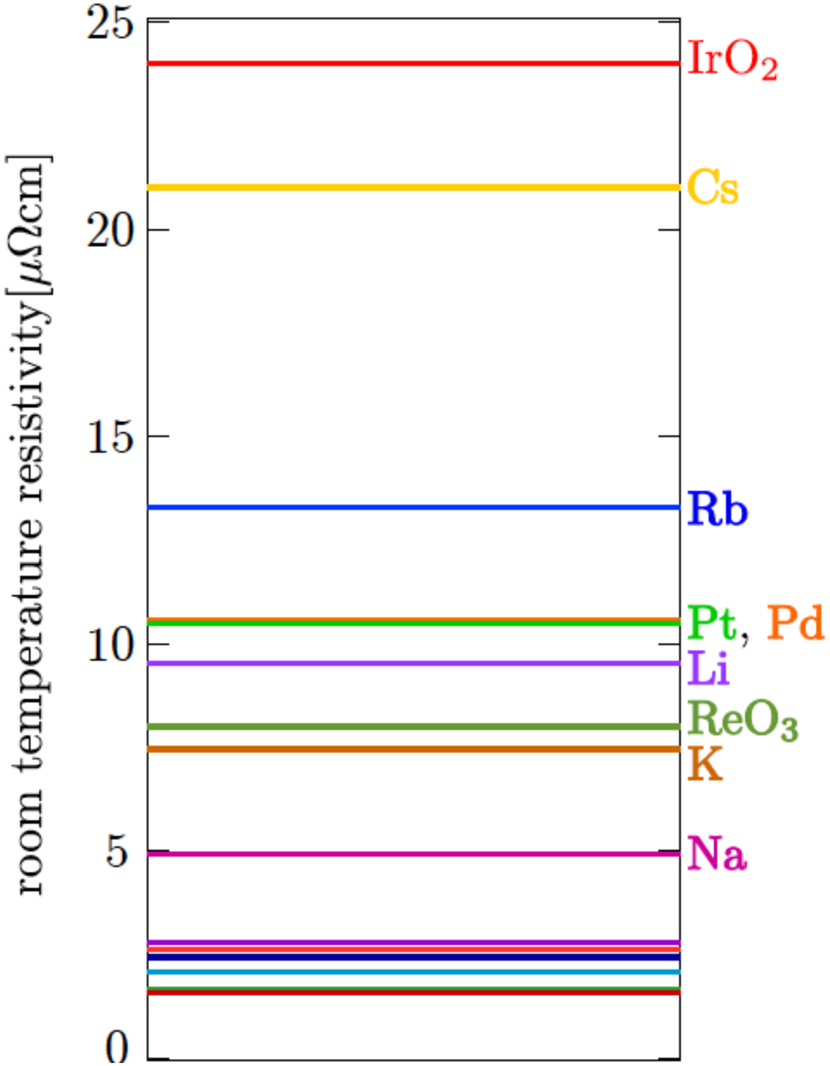
$PdCoO_2$ and $PtCoO_2$: simplicity of electronic structure



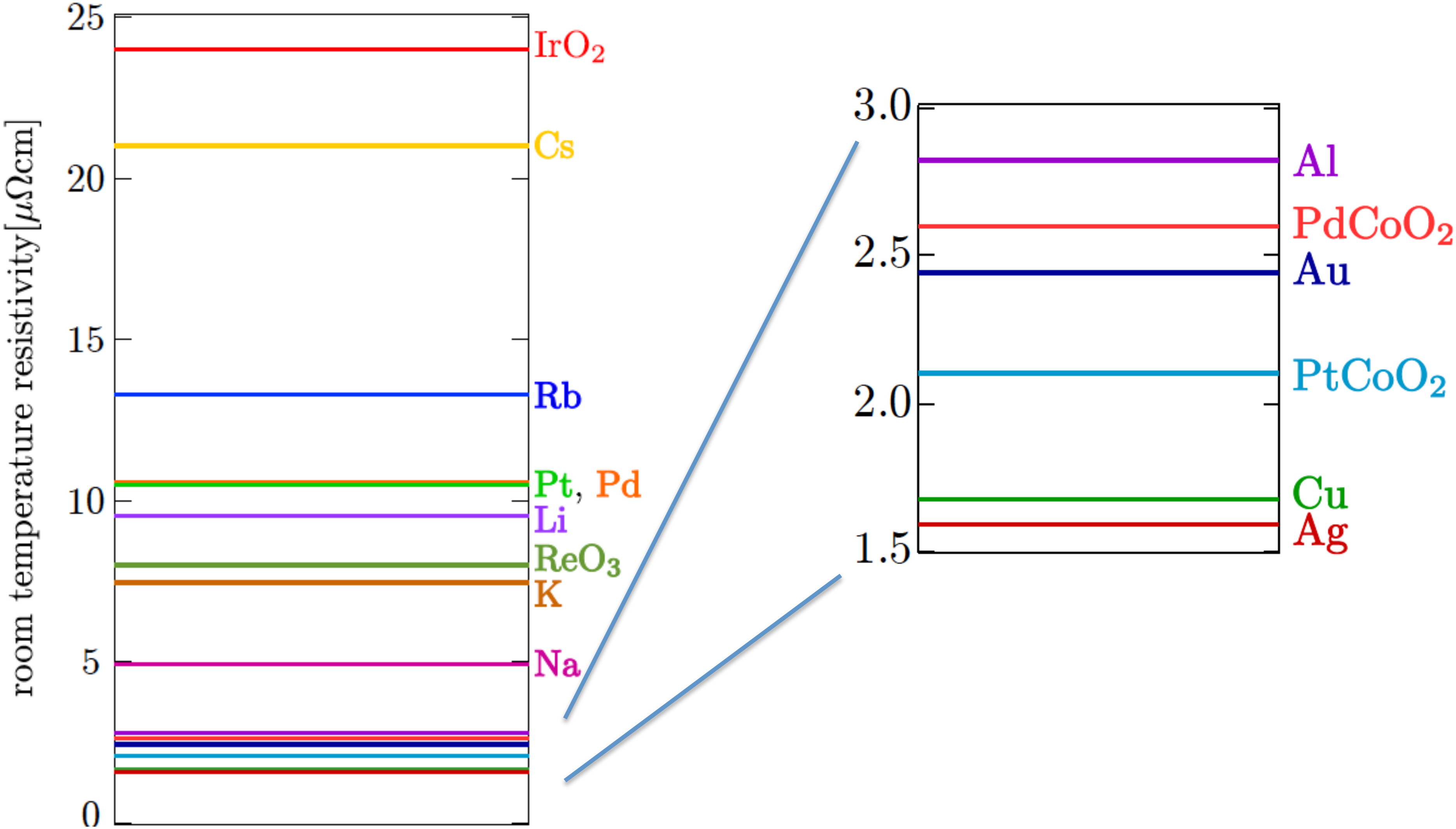
Single Fermi surface sheets, experimentally established from angle-resolved photoemssion and de Haas-van Alphen effect measurements.

*H.J. Noh et al., Phys. Rev. Lett. **102**, 256404 (2009)*
*C.W. Hicks et al., Phys. Rev. Lett. **109**, 116401 (2012)*
*P. Kushwaha, V. Sunko et al., Science Advances **1**, 1500692 (2015)*

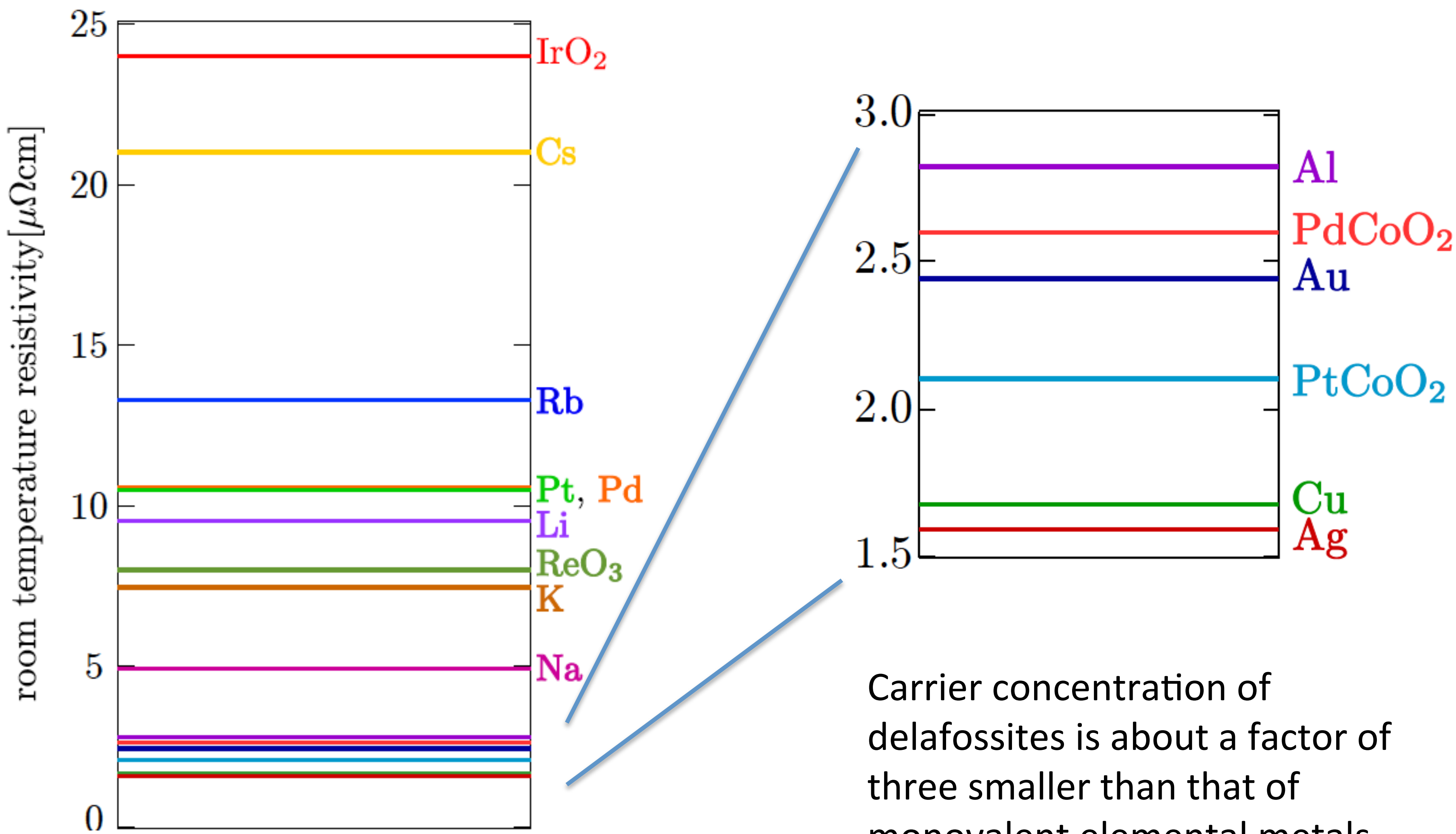
$PdCoO_2$ and $PtCoO_2$: record-breaking conductivity



$PdCoO_2$ and $PtCoO_2$: record-breaking conductivity

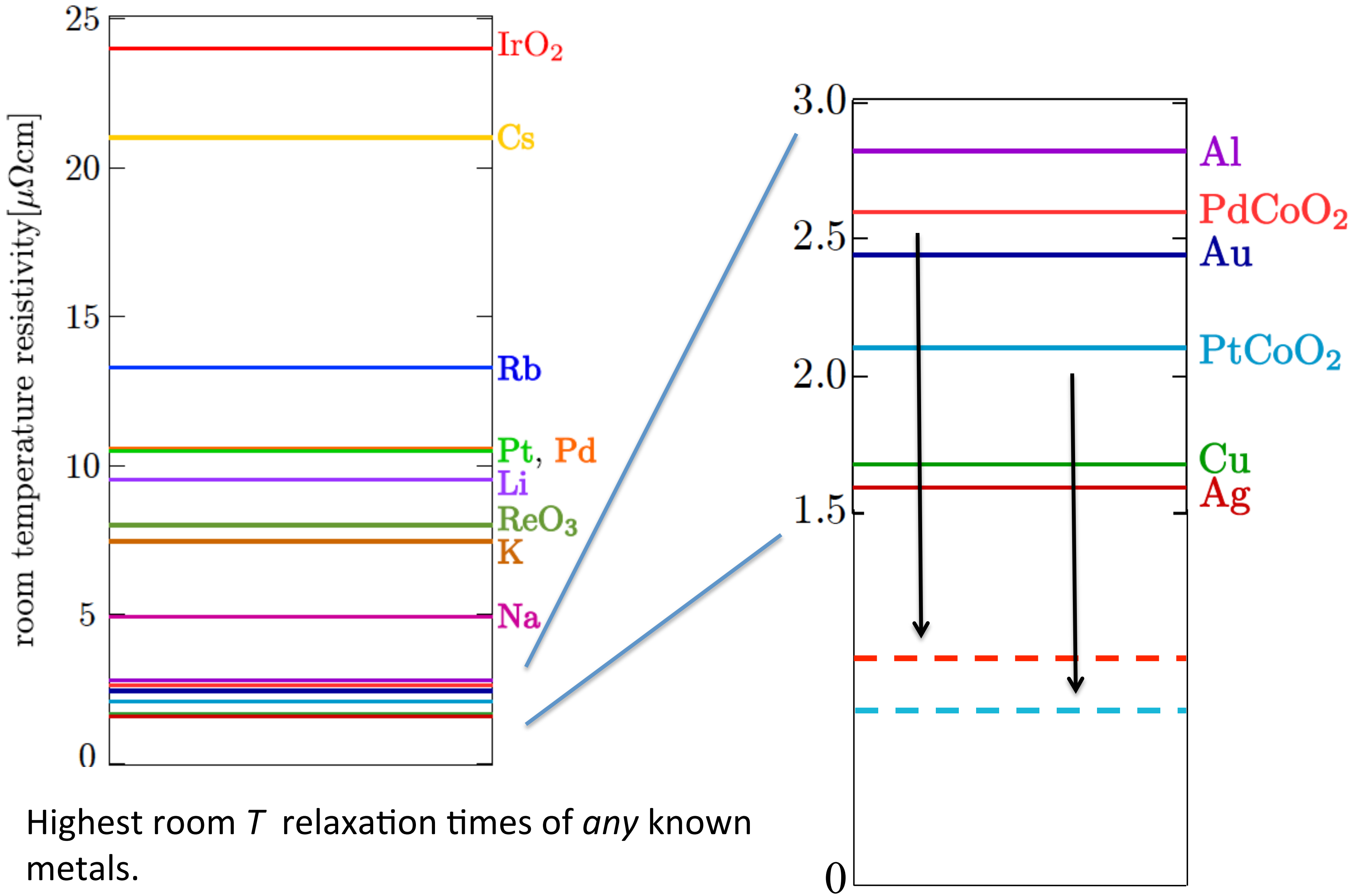


$PdCoO_2$ and $PtCoO_2$: record-breaking conductivity



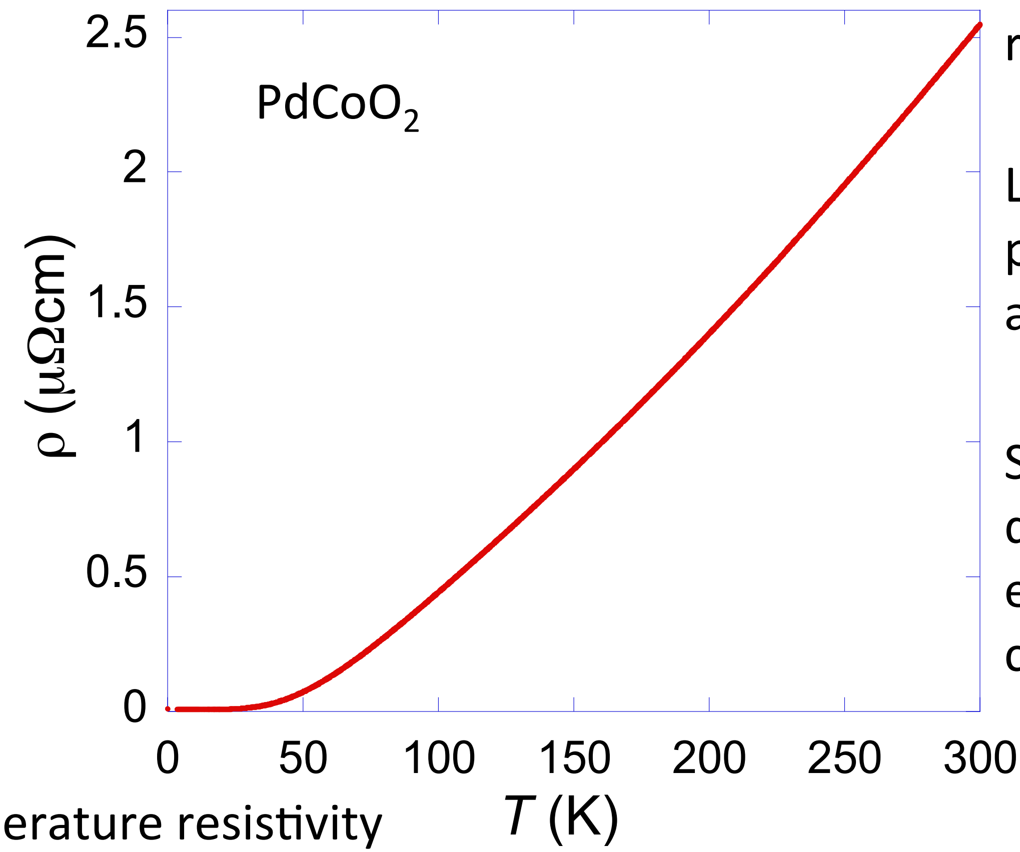
Carrier concentration of delafossites is about a factor of three smaller than that of monovalent elemental metals

PdCoO$_2$ and PtCoO$_2$: record-breaking conductivity



Highest room *T* relaxation times of *any* known metals.

Enormous low temperature conductivity and huge mean free paths



Room temperature resistivity about 2.5 μΩcm

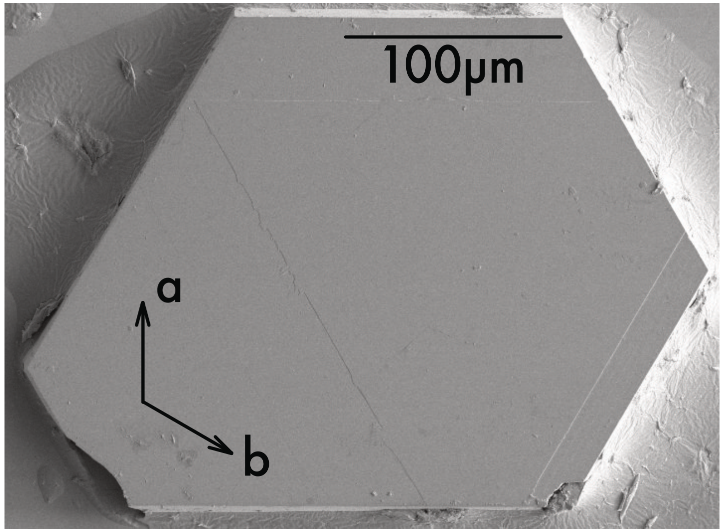
Large anisotropy: resistivity perpendicular to planes approximately 10^3 higher

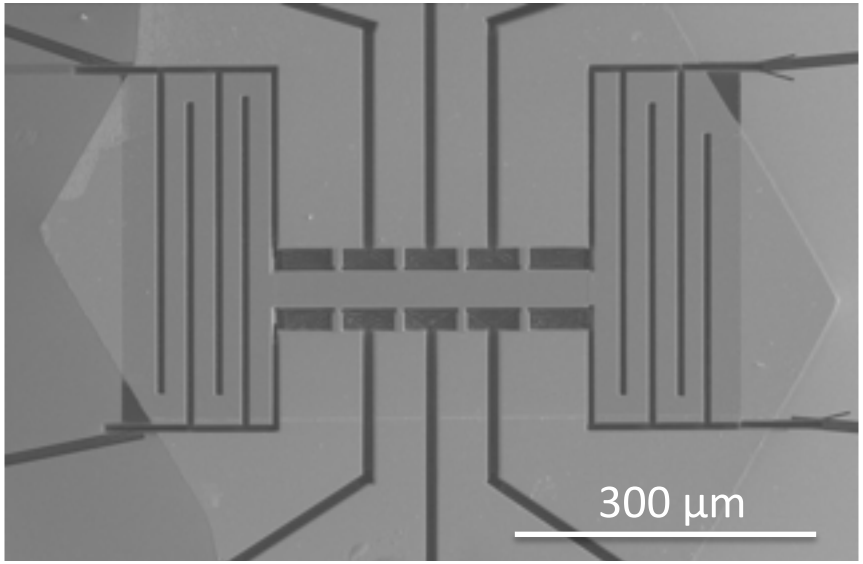
Some evidence for phonon drag which could lead to extra momentum-conserving processes

Low temperature resistivity a few nΩcm: in-plane mean free paths of tens of microns!

APM, Rep. Prog. Phys. ***80****, 032501 (2017)*

Clean fabrication - focused ion beam sculpting

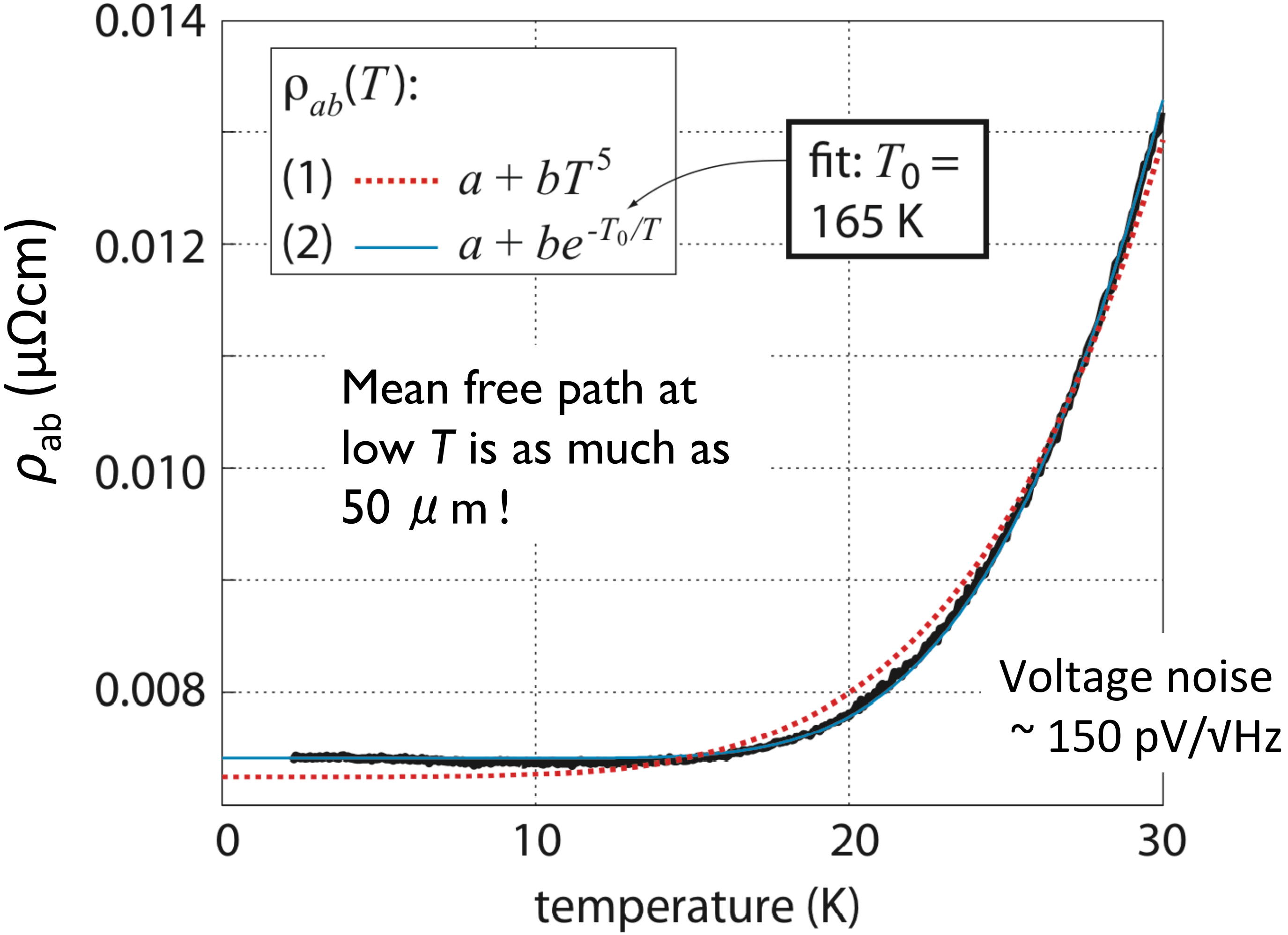




Crystals of $PdCoO_2$ grow 10-20 μm thick

Focused ion beams can be used to sculpt out arbitrary geometries; beyond a 10-20 nm damage layer this process is clean.

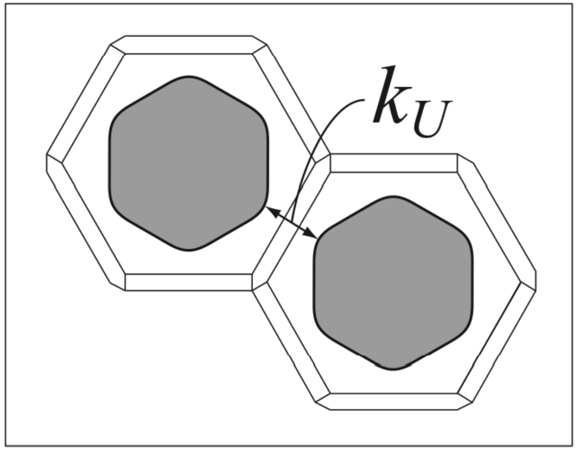
Exponential resistivity at low temperatures



*C.W. Hicks, A.S. Gibbs, A.P. Mackenzie, H. Takatsu, Y. Maeno & E.A. Yelland Phys. Rev. Lett. **109**, 116401 (2012)*

Could exponential resistivity be due to 'phonon drag'?

Idea (Peierls 1930s): phonons cannot equilibriate on the timescale of low temperature electron-phonon collisions and are dragged out of equilibrium by the electron distribution in an applied electric field at low temperatures.



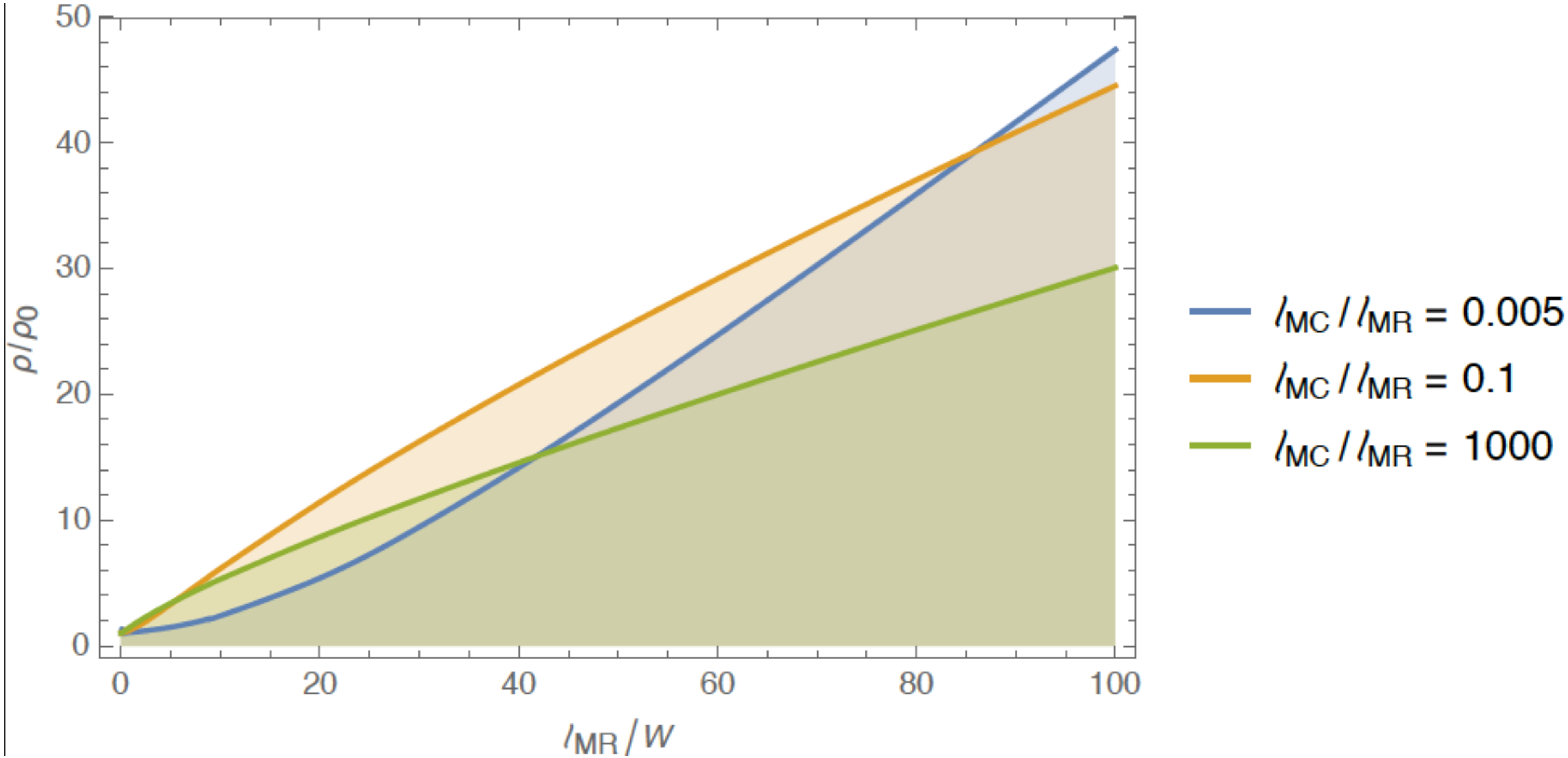
Standard el-ph scattering therefore does not relax the electron distribution's momentum at low temperatures.

Electron-phonon Umklapp processes then have an activation temperature $T_U = \hbar c k_U$ where c is the sound velocity.

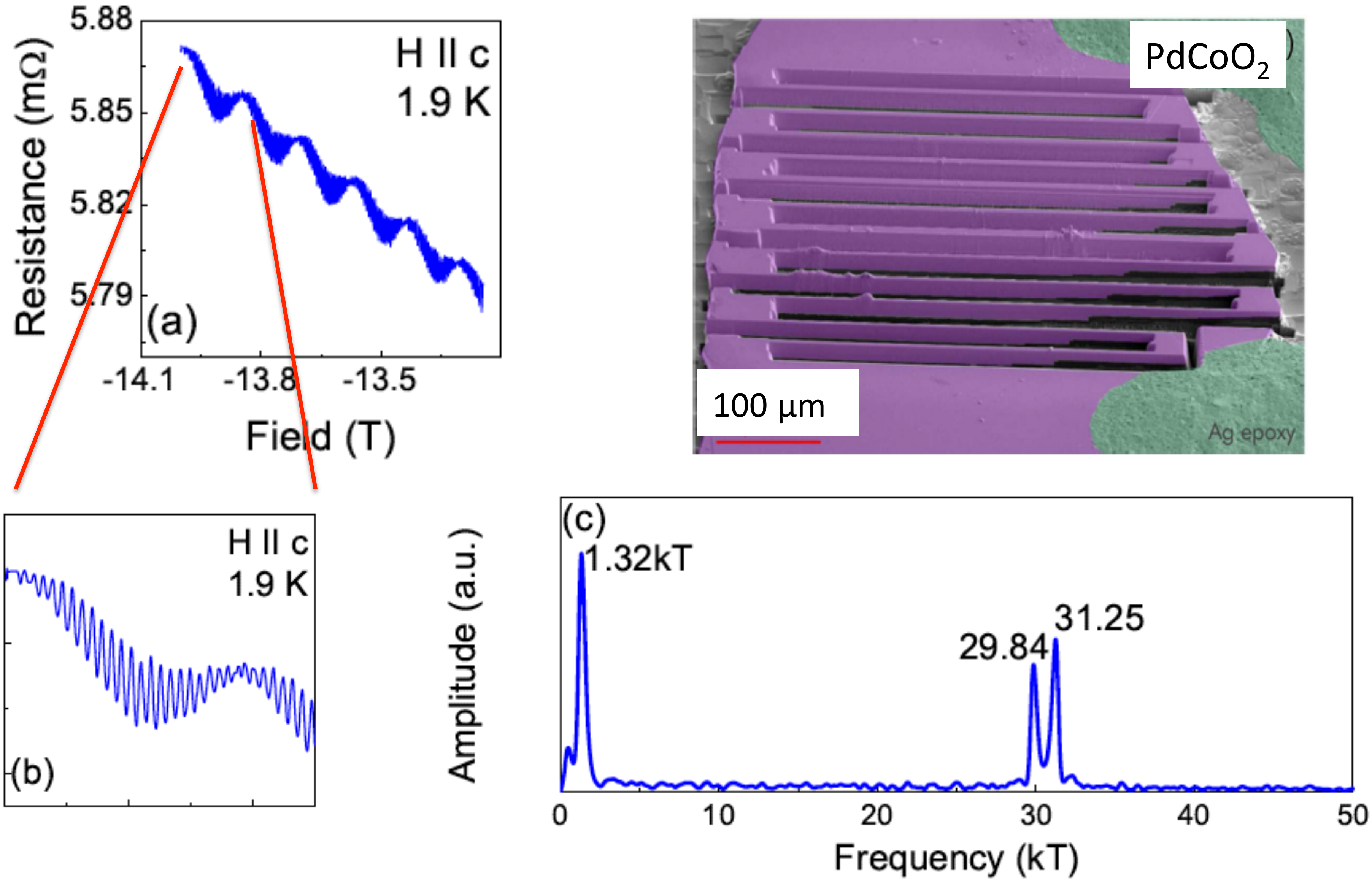
Estimating c from phonon specific heat and knowing k_U from the Fermi surface gives reasonable agreement between T_U and the measured T_o.

Dragged phonons would help rather than hinder reaching the hydrodynamic regime: our motivation to try an experiment.

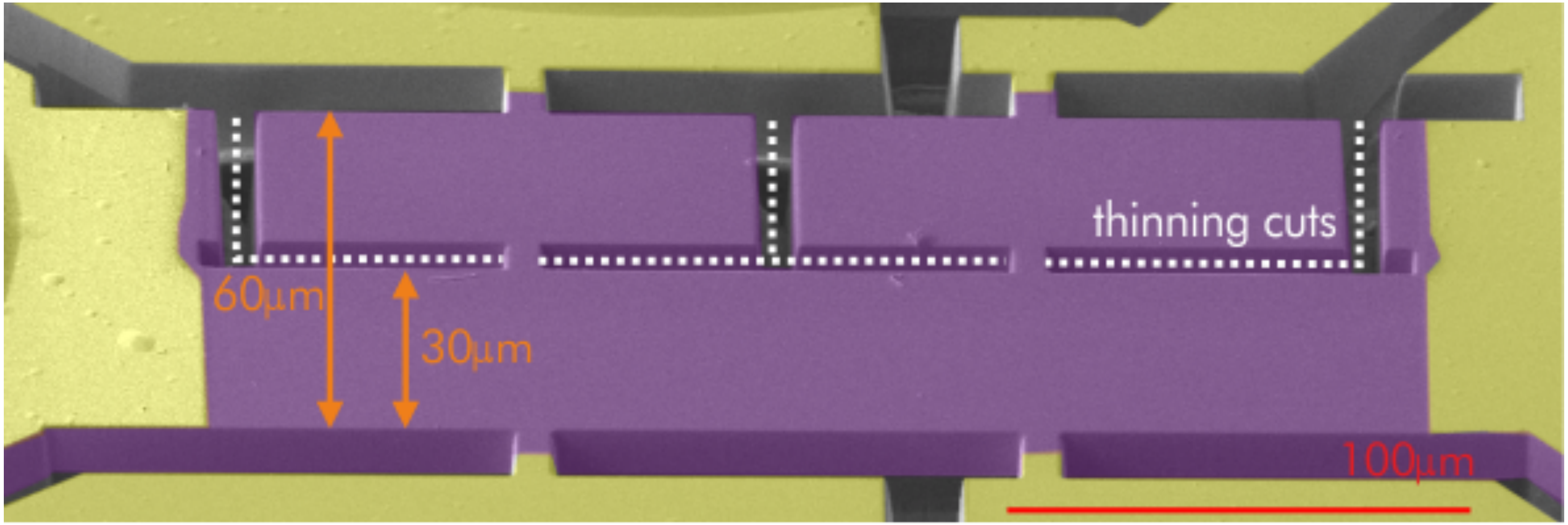
The ballistic – hydrodynamic crossover in the resistivity of mesoscopic wires



Focused ion beam processing does not damage beyond a very thin (20 nm) surface layer



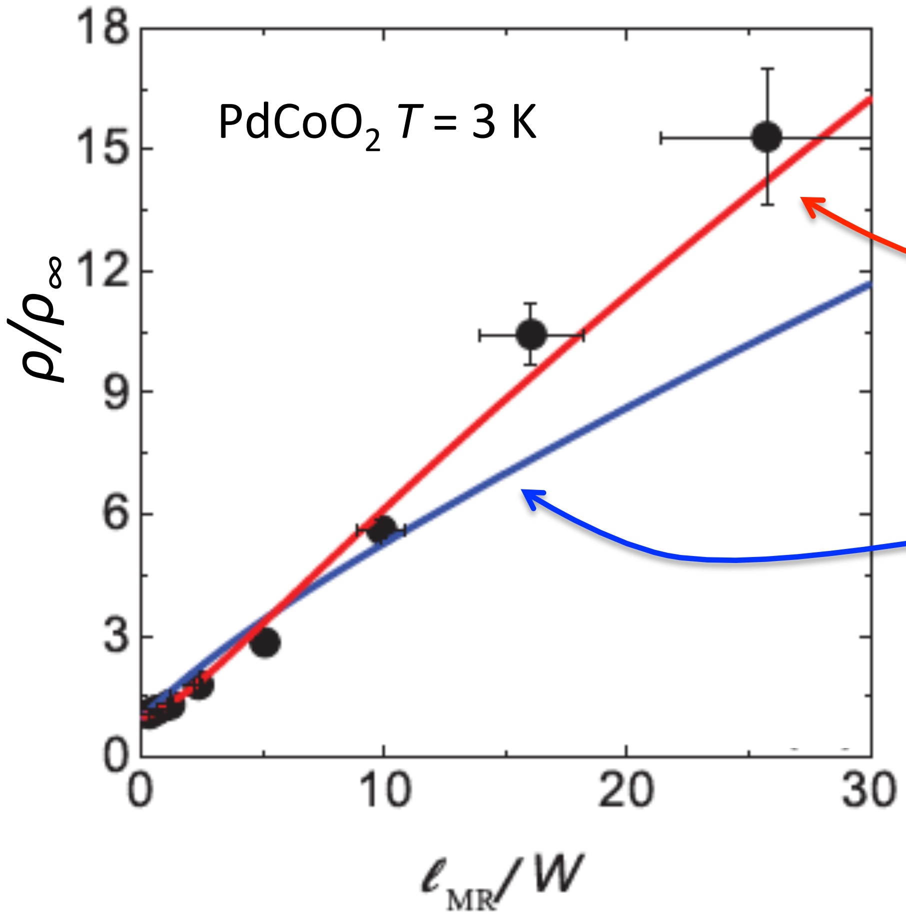
Search for signatures of Navier-Stokes hydrodynamic flow



Experiment: Successively narrow the channel in factors of 2, measuring the resistance after every step.

P.J.W. Moll, P. Kushwaha, N. Nandi, B. Schmidt and A.P. Mackenzie, Science ***351****, 1061 (2016)*

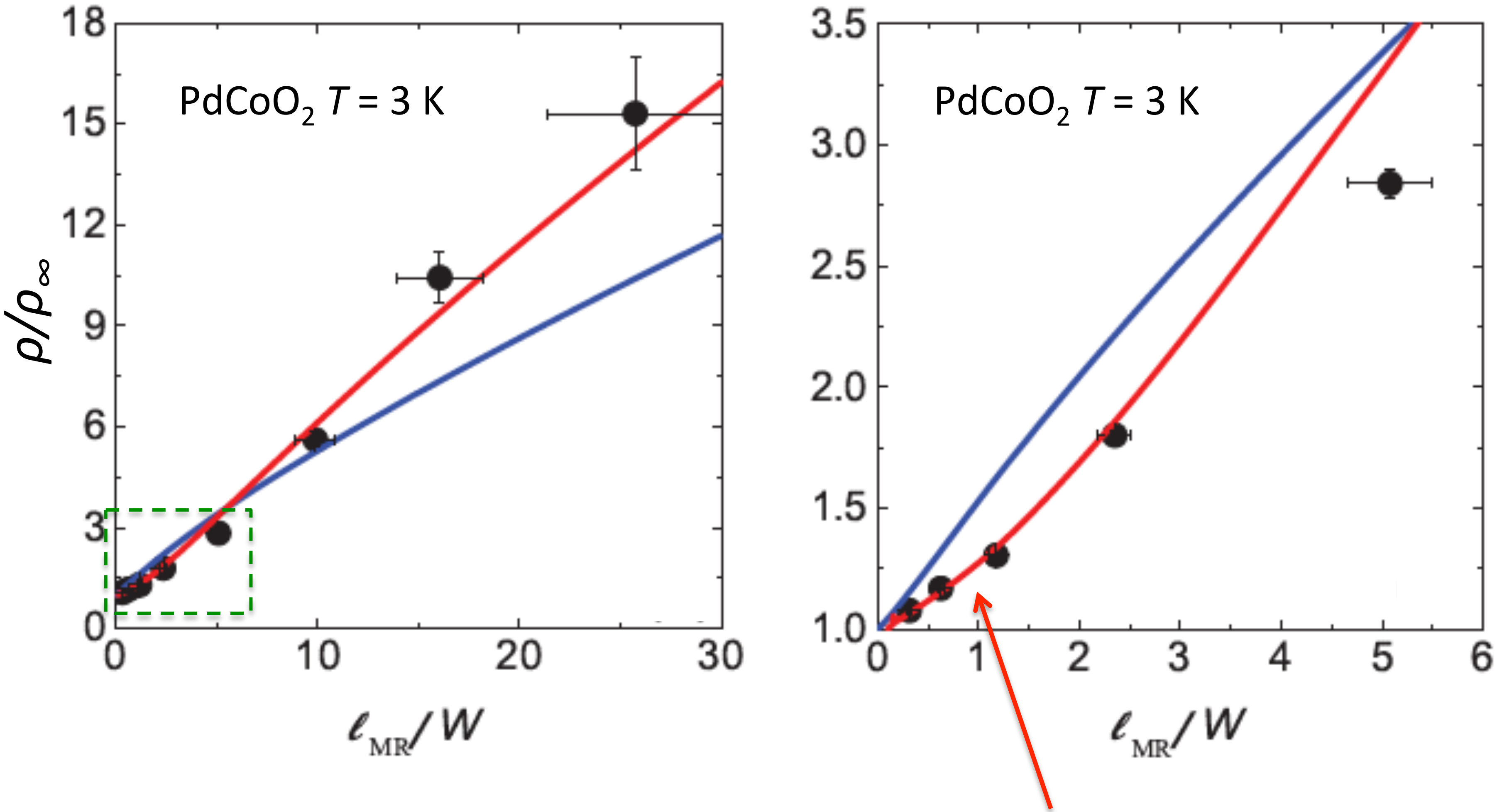
Width dependence of channel resistance analysed using the de Jong-Molenkamp theory



Effect of including momentum scattering such that $\ell_{MR} = 10\ell_{MC}$

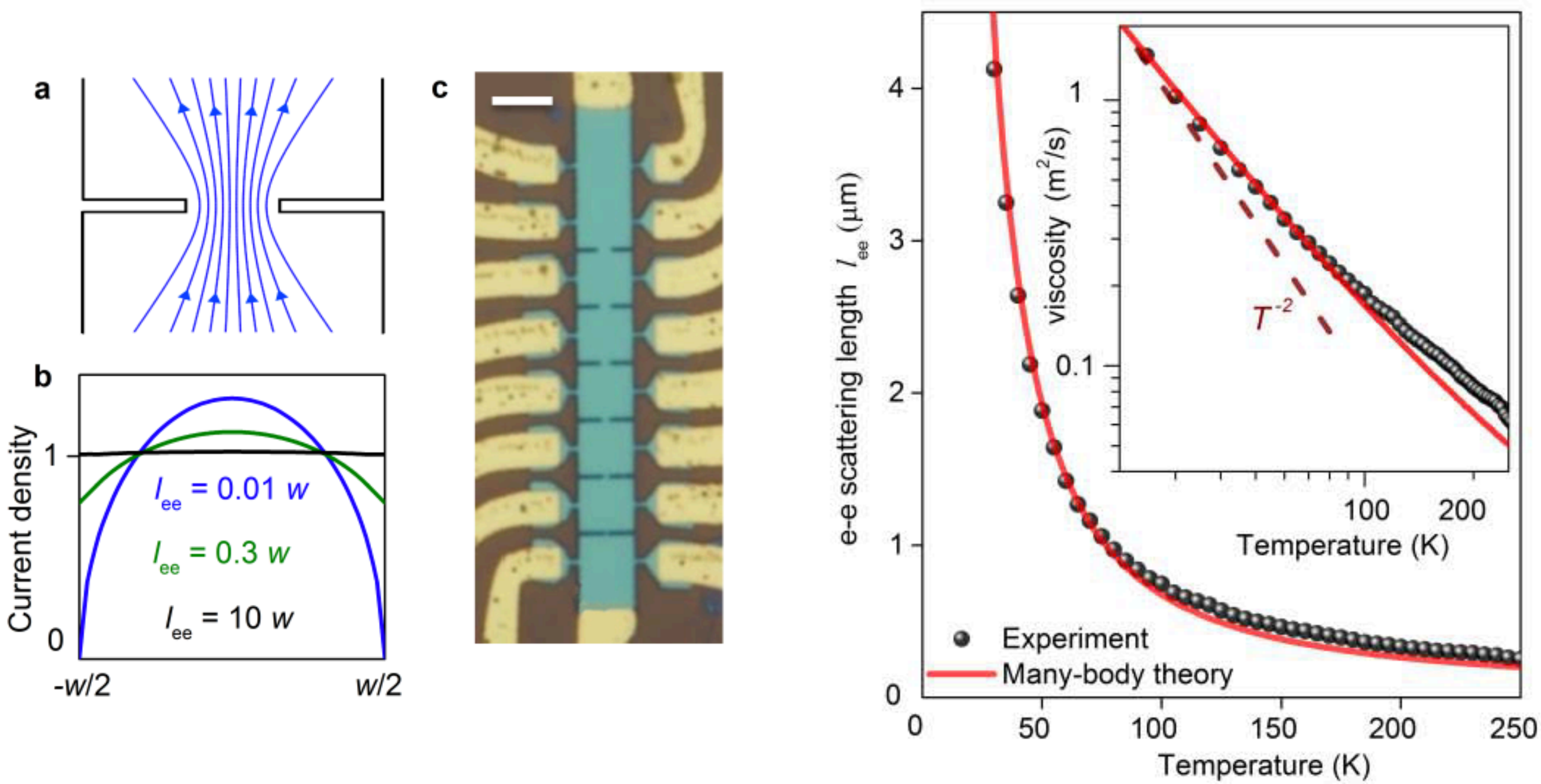
Prediction of standard transport theory neglecting momentum-conserving scattering

Width dependence of channel resistance analysed using the de Jong-Molenkamp theory



Curvature is the signature of a viscous contribution

Flow experiments on graphene in a similar spirit



*R. Krishna-Kumar et al., Nature Physics **13**, 1182 (2017)*

Magnetoresistance and Hall experiments on simple 'wires' at high temperatures (Nabhanila Nandi)

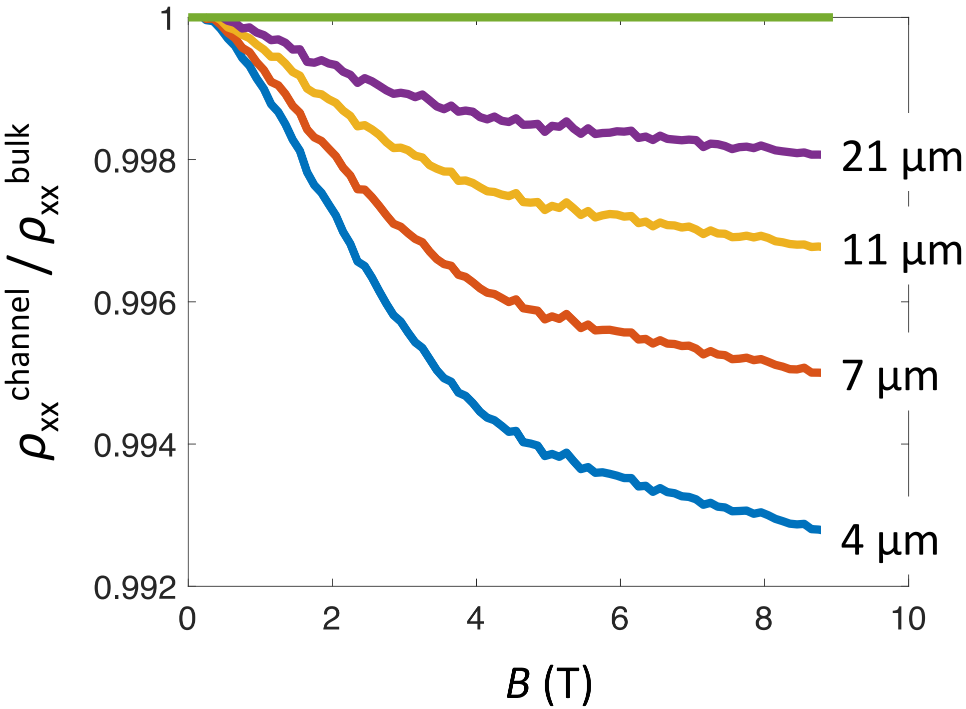
Recent extension of hydrodynamic transport theory to include magnetic fields:

Navier-Stokes expressions for MR: *P.S. Alekseev, Phys. Rev. Lett.* ***117****, 166601 (2016)*

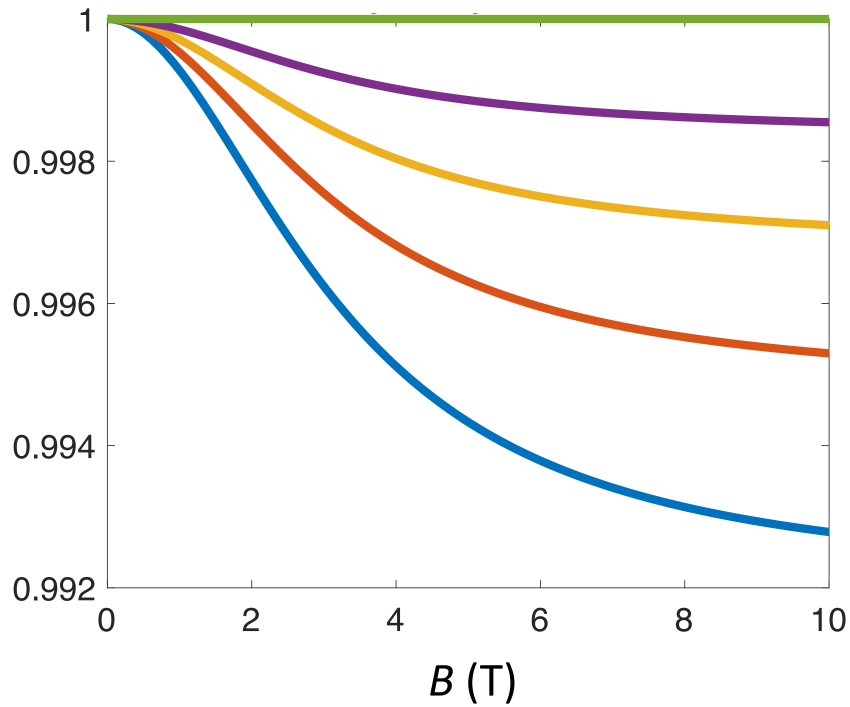
Navier-Stokes and kinetic calculations, longitudinal and transverse (MR and Hall):

T. Scaffidi, N. Nandi, B. Schmidt, APM and J.E. Moore, Phys. Rev. Lett. ***118****, 226601 (2017)*

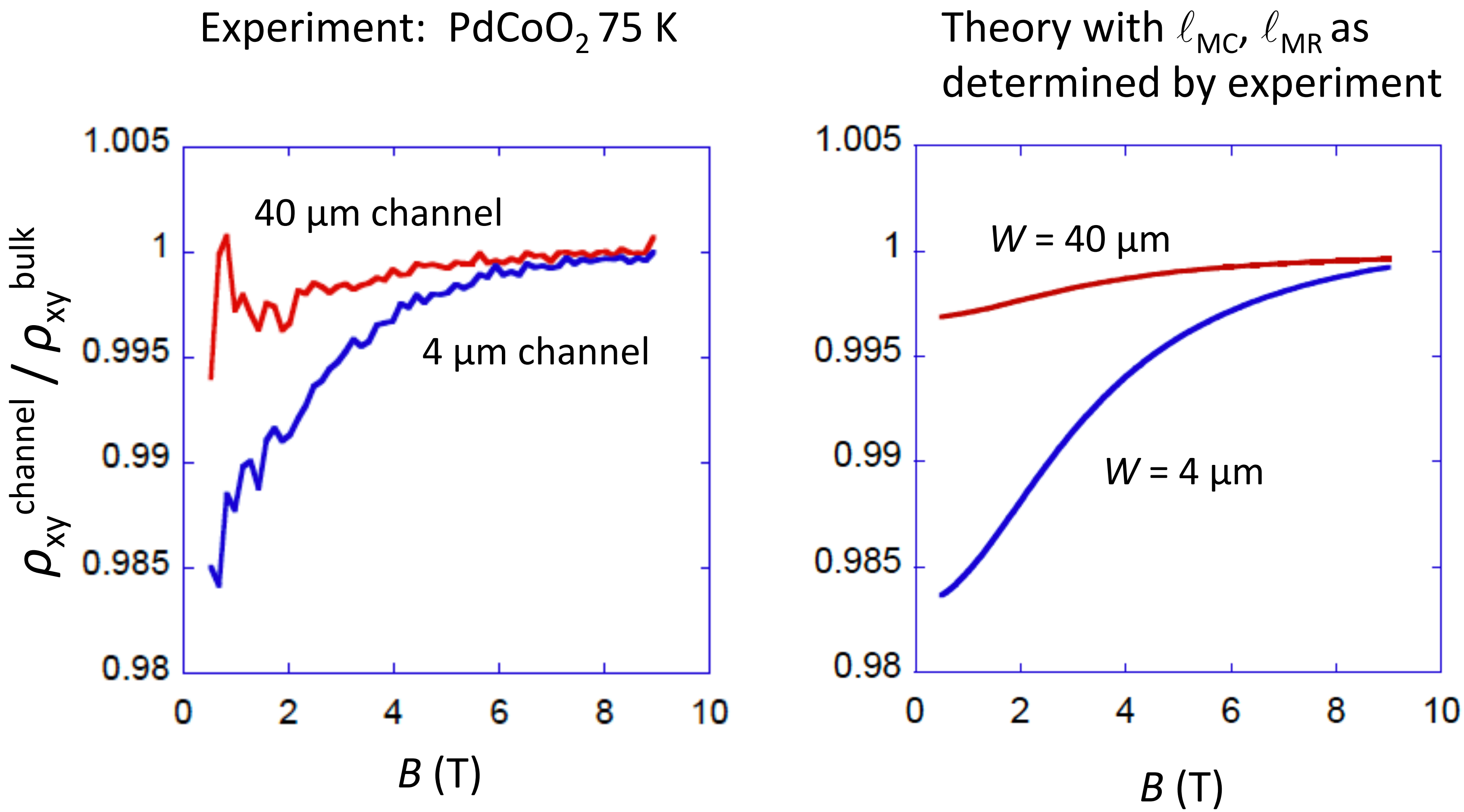
Experiment: $PdCoO_2$ at T = 100K



Navier-Stokes calculation including known W and ℓ_{MR}



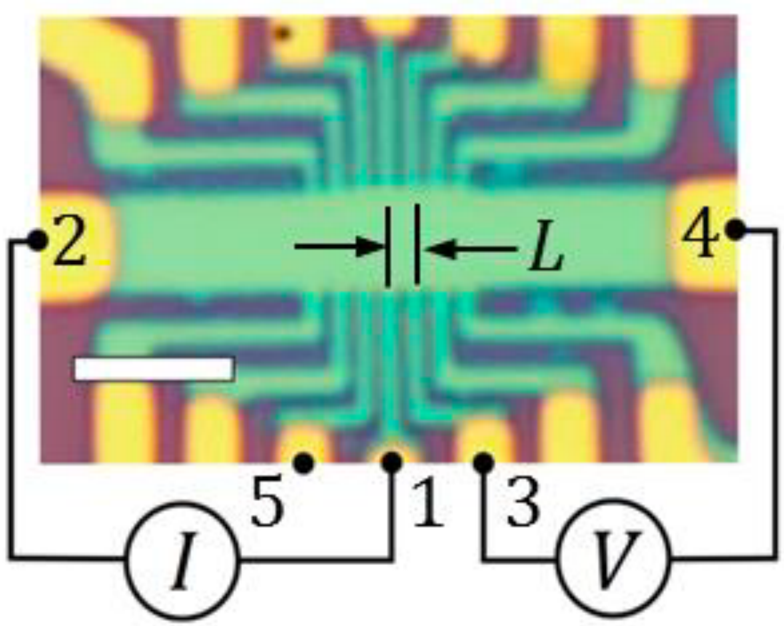
In hydrodynamic regime theory gives an excellent match to the qualitative features of the data

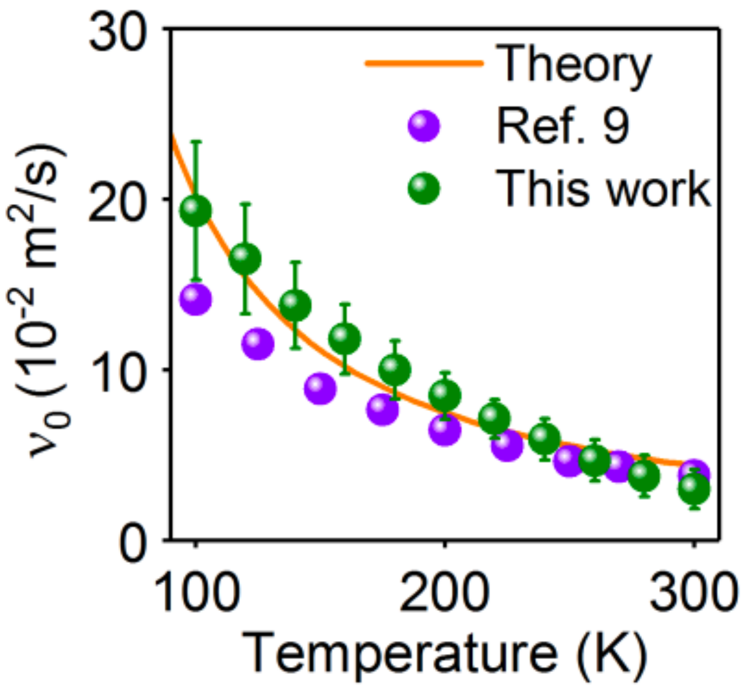


Care: kinetic calculations have recently been extended to this regime by Thomas Scaffidi and qualitatively similar predictions are made even if the momemtum-conserving scattering is turned off.

Working conclusion: We *may* be seeing a hydrodynamic signal but according to our current understanding this is not a definitive experiment.

Report of the viscous Hall effect in graphene



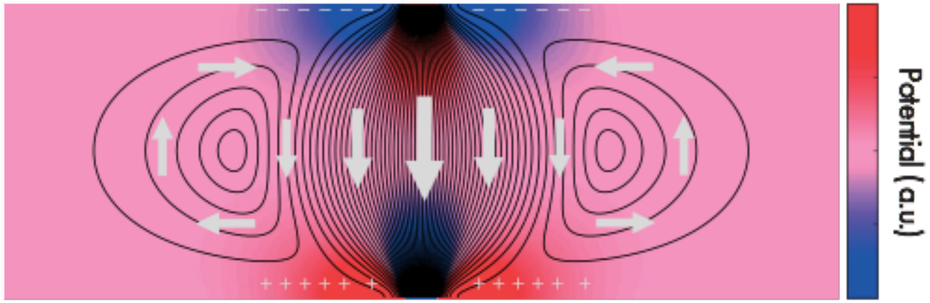


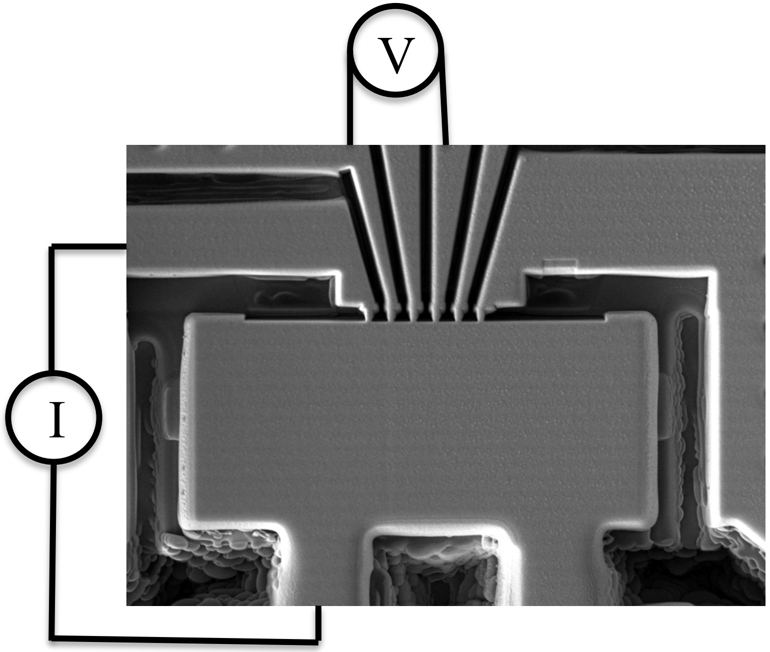
Experiment: match hydro-calculated voltage patterns to experiment and extract the *T* dependence of the shear viscosity (proportional to the momentum-conserving mean free path)

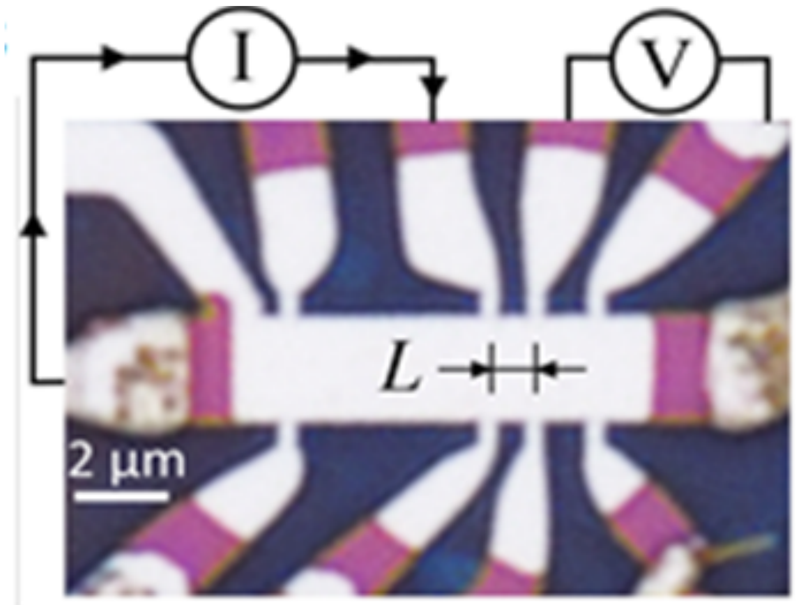
A. I. Berdyugin et al., arXiv:1806.01606

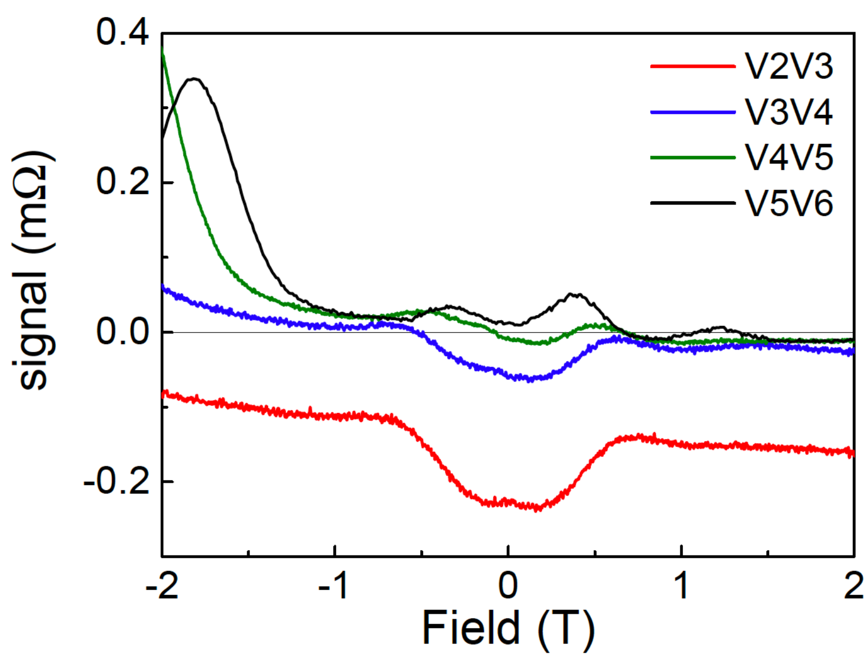
Explicit measurements in the 'graphene configuration'

Viscous flow

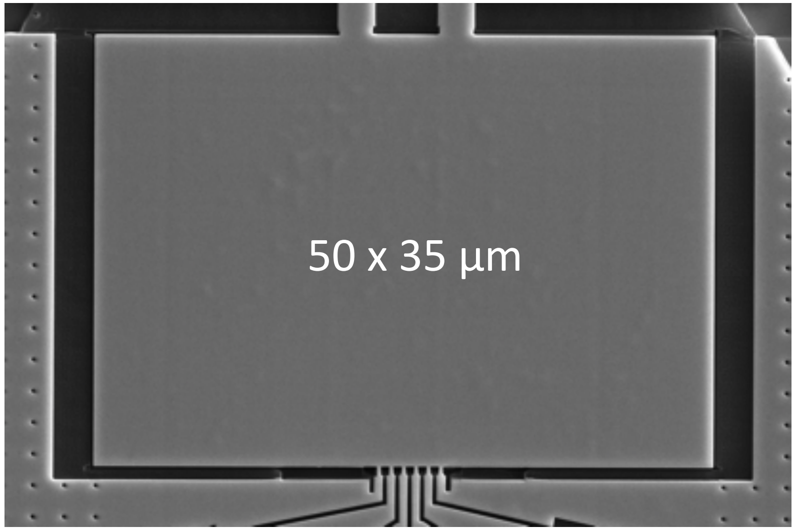


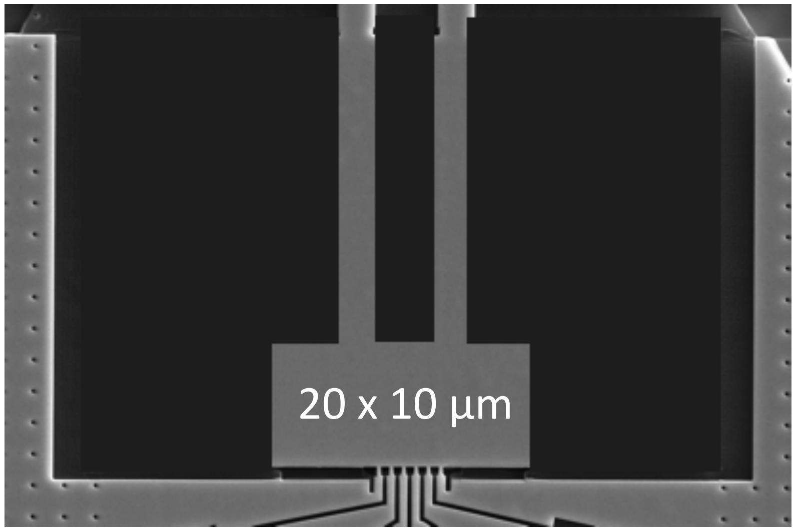


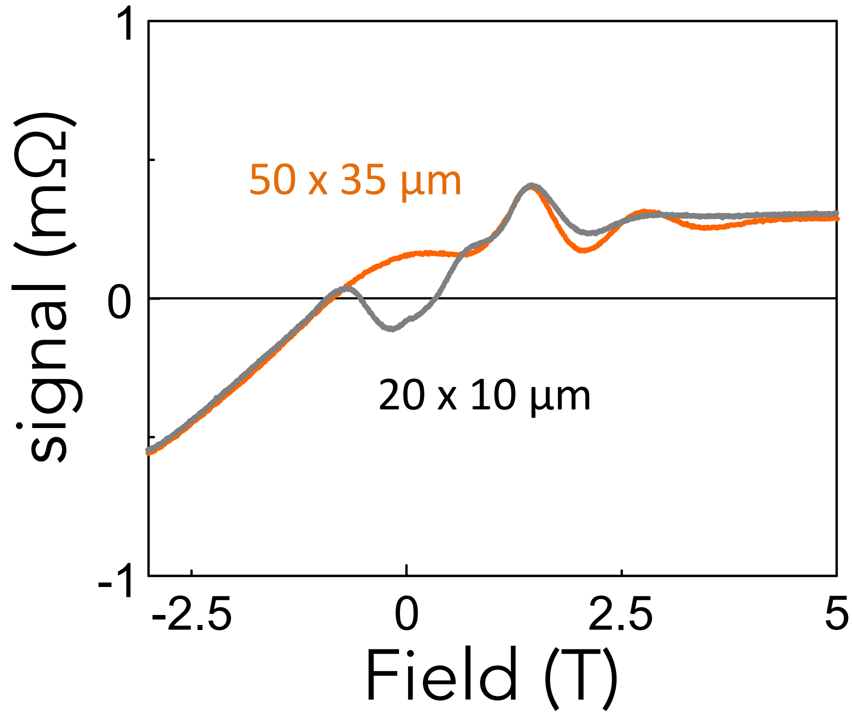




The negative voltage at zero field depends on device size



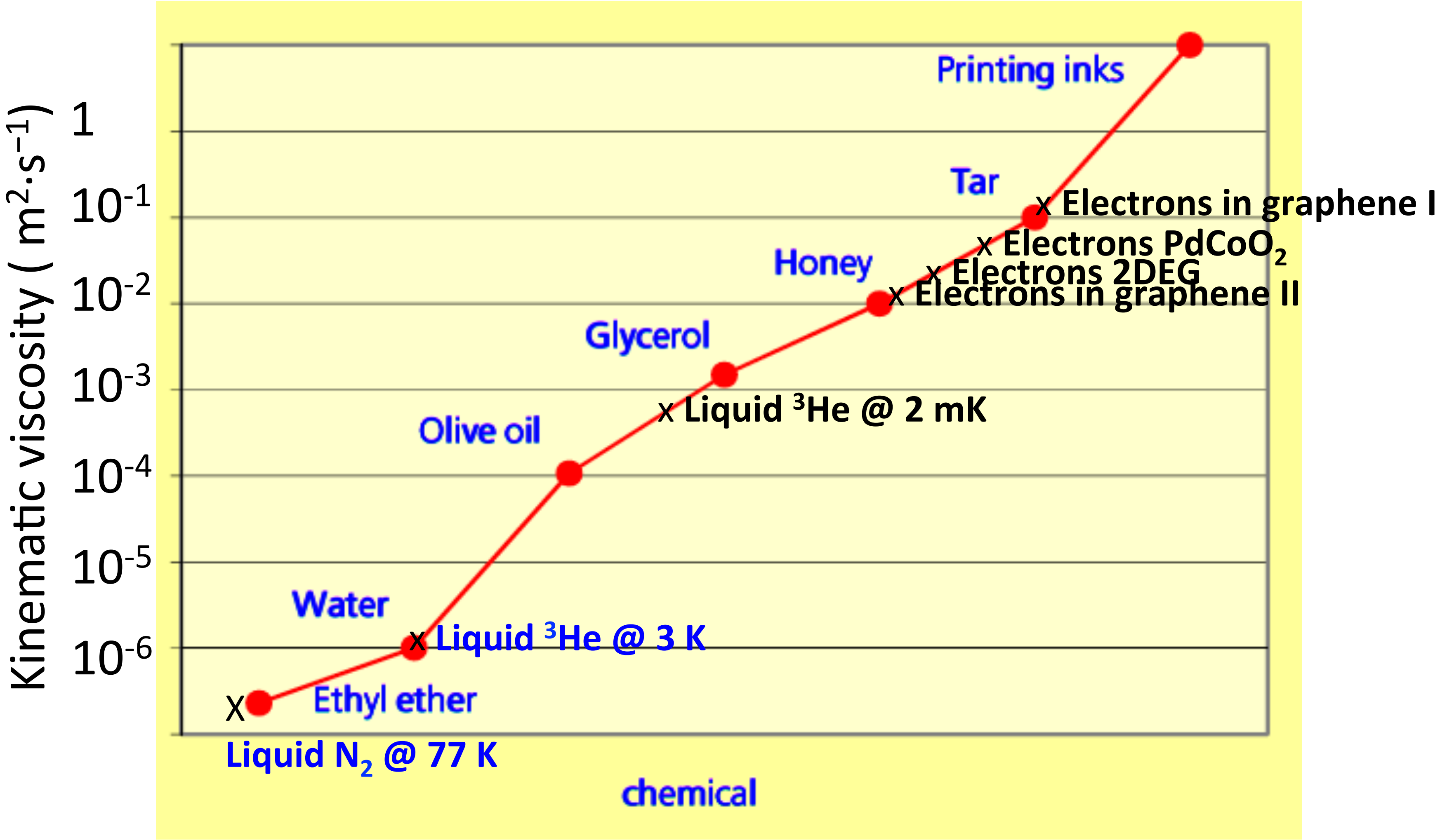




Probably simply a ballistic effect in our experiments (though not yet understood and hydrodynamic calculations are still desirable).

It would be interesting to know if sample size effects were checked in the graphene experiments.

Viscosity of some familiar classical and quantum fluids



Electrons in graphene I: *D. A. Bandurin et al., Science **351**, 1055 (2016)*

Electrons in graphene II: *J. Crossno et al., Science **351**, 1058 (2016)*

Turbulent electron flow?

Reynolds numbers *Re* = vL/η_K

Turbulence seen at high Reynolds numbers, i.e. high drive velocity, long distances and low kinematic viscosity.

Current hydrodynamic systems feature small distances and high viscosities so need large drive velocity. Graphene more promising.

High frequency effects?

Lecture has concentrated on *dc* response but there is also interesting physics to be sought in the high frequency response.

See e.g. *R. Moessner, P. Surowka and P. Witkowski., Phys. Rev. B **97**, 161112 (2018)*

Do the known hydrodynamic electron fluids challenge the viscosity bound?

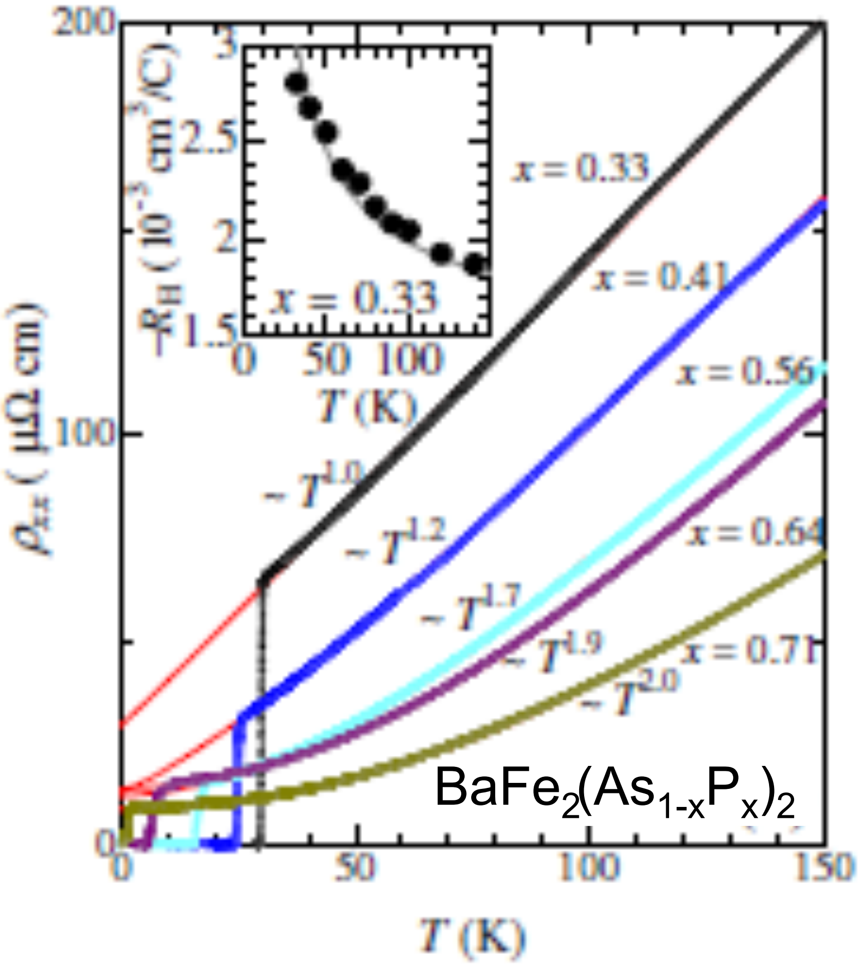
Recall the bound proposal $\frac{\eta}{s} \geq \frac{1}{4\pi}\frac{\hbar}{k_B}$

Not yet: closest is graphene very near the charge neutrality point

$$\frac{\eta}{s} \approx 10\frac{\hbar}{k_B}$$

The essential issue is the Fermi velocity in graphene which is very high. It would be very interesting to study materials with similarly high scattering rates but much lower Fermi velocities.

Outstanding question – are hydrodynamics playing a role in transport in quantum critical electron systems?



S. Kasahara et al., Phys. Rev. B ***81****, 184519 (2010)*

Cuprates, pnictides, heavy fermions, organics and even some conventional metals can be tuned to show linear resistivity.

Evidence for a universal, high scattering rate when this happens.
J.A.N. Bruin, H. Sakai, R.S. Perry & A.P. Mackenzie, Science ***339****, 804 (2013)*

Is hydrodynamics playing a role in this? Unknown but, in principle, testable.

Also possible to extend in future to fully three-dimensional systems.

University of St Andrews

Max Planck Institute for Chemical Physics of Solids

Conclusions

1. Observation of hydrodynamic electron flow requires such high purity that it has only very recently been observed in naturally occurring materials.
2. The modern experiments were stimulated by modern theory, but past achievements were overlooked in the process.
3. Experiments to date have been in systems in which the electron fluid is viscous.
4. Discovery of low viscosity electron flow is not impossible; experiment will eventually determine whether it exists or not.

High scattering rate implies low viscosity: is there a lower bound on viscosity?

$$\eta_K \propto v_P^2 \tau$$

$$\eta \propto \frac{nmv_P^2 \tau}{unit\ vol}$$

$$s \propto \frac{nk_B}{unit\ vol}$$

By the energy-time uncertainty principle, $(mv_P^{\ 2})\tau \geq \hbar$, so

$$\frac{\eta}{s} \geq \frac{\hbar}{k_B}$$

P. Kovtun, D.T. Son and A.O. Starinets, Phys. Rev. Lett. ***94****, 111601 (2005)*

Likely effective mass for Molenkamp ~ 0.1 m_e

Carrier density ~ 3 x 10^{11} cm^{-2}

l_{ee} ~ 1 μm

k_F ~ .013 $Å^{-1}$

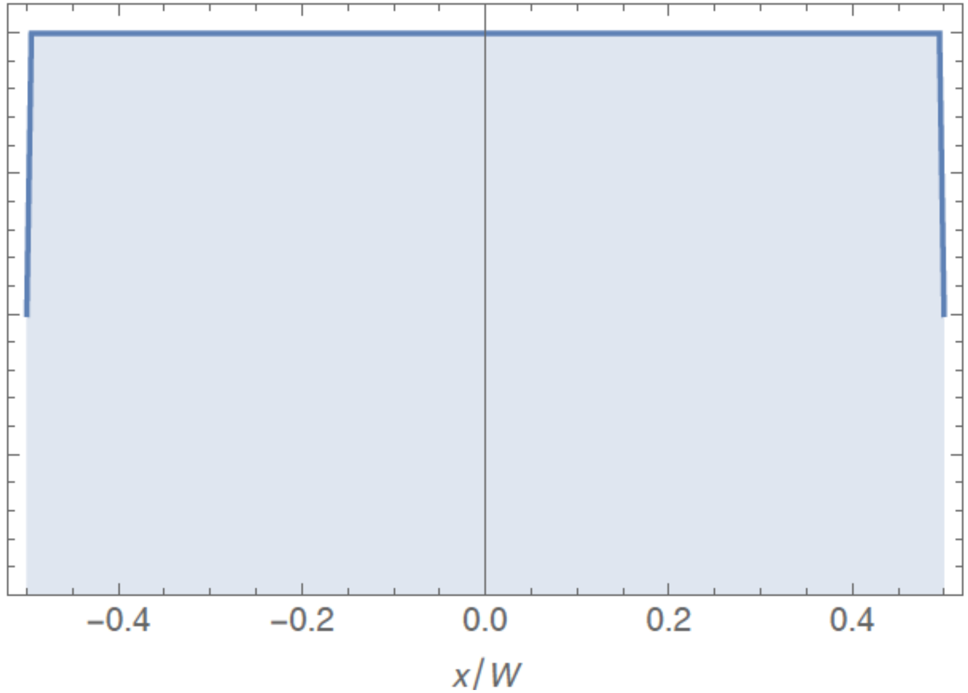
v_F ~ 1.5 x 10^5 ms^{-1}

η_K =0.23 $l_{ee}v_F$~ 0.03 m^2s^{-1}

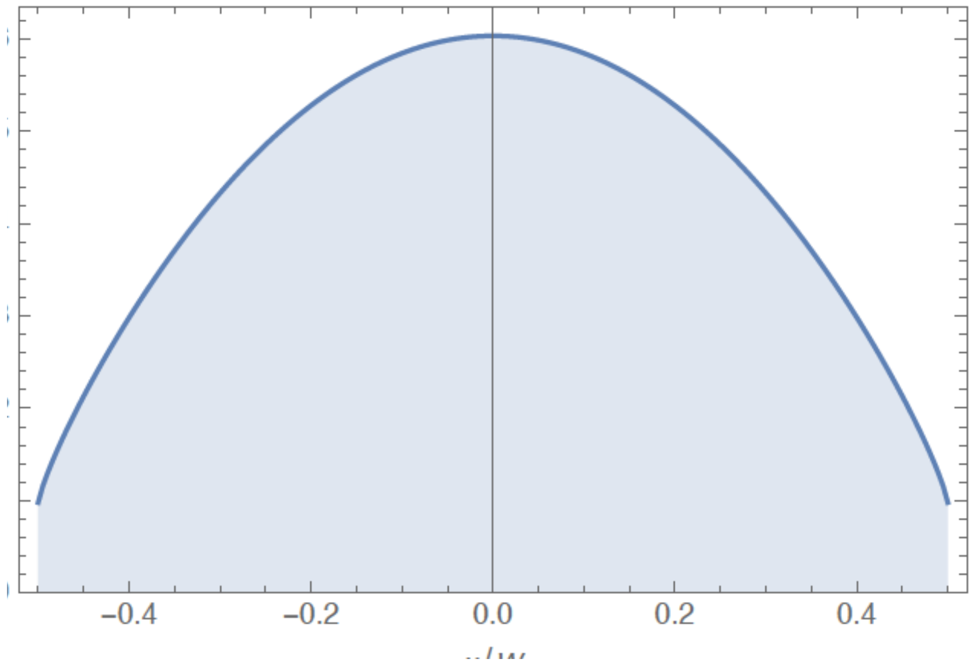
NB: In units of $\hbar/k_B$, the η in η/*s* is dynamic rather than kinematic viscosity

Transitions from laminar to turbulent flow are usually parameterised by the dimensionless Reynold's number $Re \sim vL/\eta_K$ where η_K is the kinematic viscosity. Typically one needs $Re \sim 5 \times 10^3$ for turbulent flow.

For us, $v \sim 1$ ms^{-1}, $L \sim 1 \times 10^{-3}$m and $\eta_K \sim 10^{-2}$ so we have $Re \sim 0.1$. We need about a four orders of magnitude reduction in η_K to approach turbulent regimes.



−0.4
−0.2
0.0
0.2
0.4
x/W



−0.4
−0.2
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