Ergodic and Non-Ergodic Quantum Dynamics

(or)

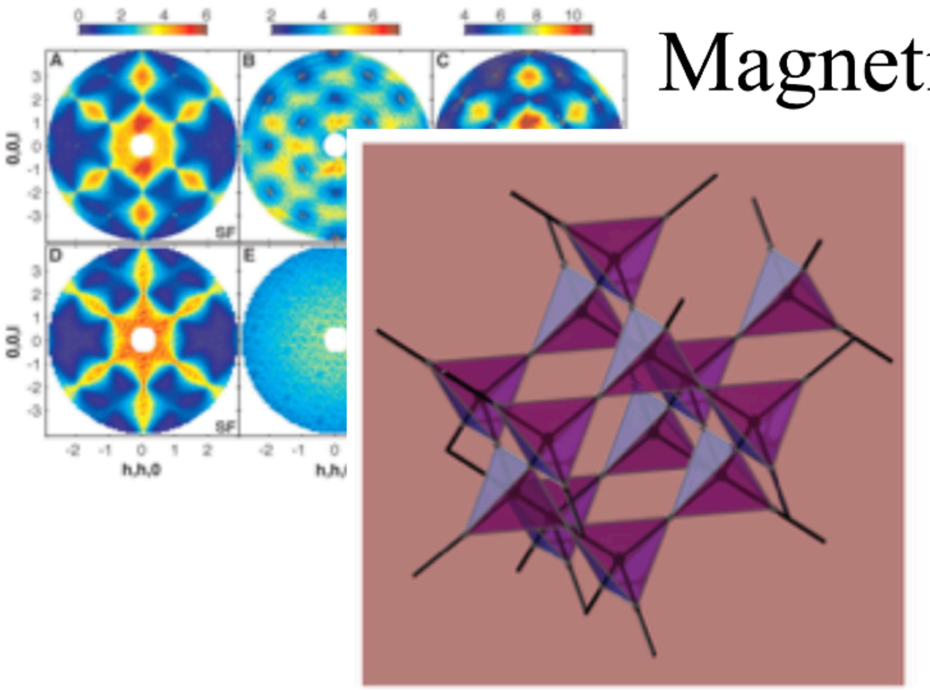
Thermalization and Localization in Many-Body Quantum Systems

Vedika Khemani

Harvard University

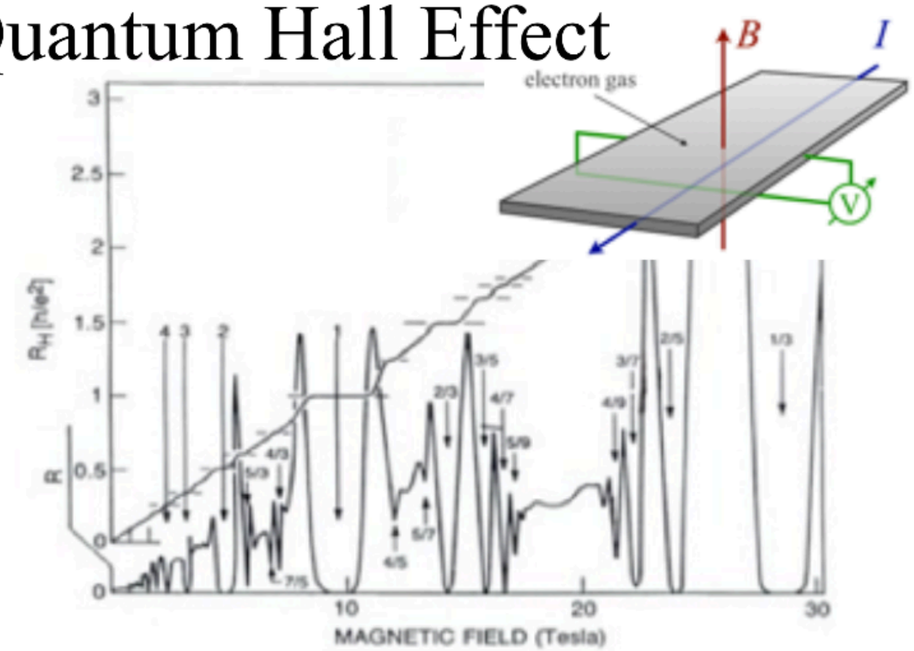
Phases of Matter in Equilibrium

Magnetism

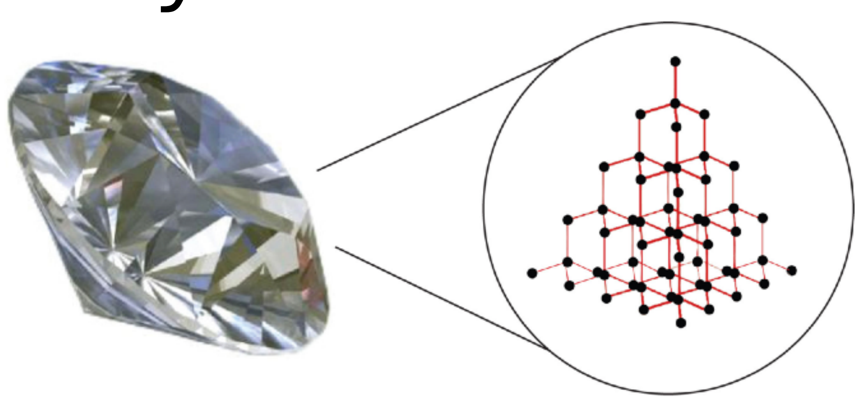


Fennell, et al. Science (2009)

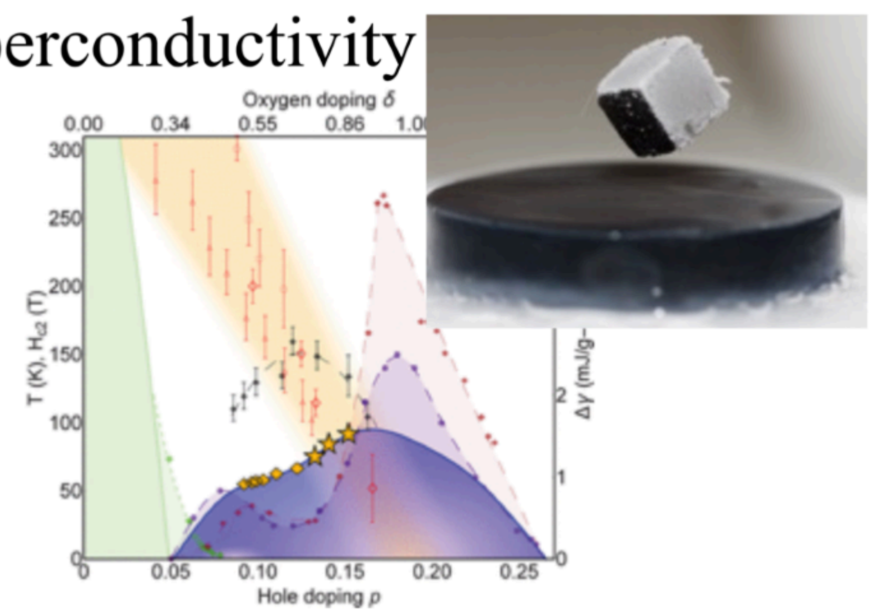
Quantum Hall Effect



Crystalline Solids



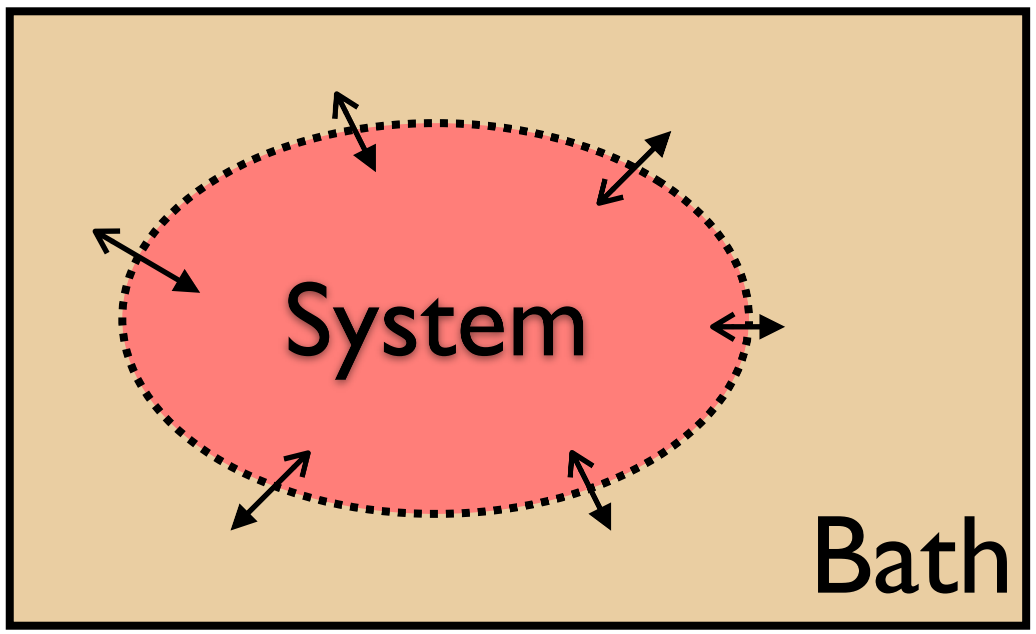
Superconductivity



Ramshaw, et al. Science (2015)

Equilibrium statistical mechanics

Two pillars of statistical mechanics: (i) Thermalization and (ii) Phase Structure

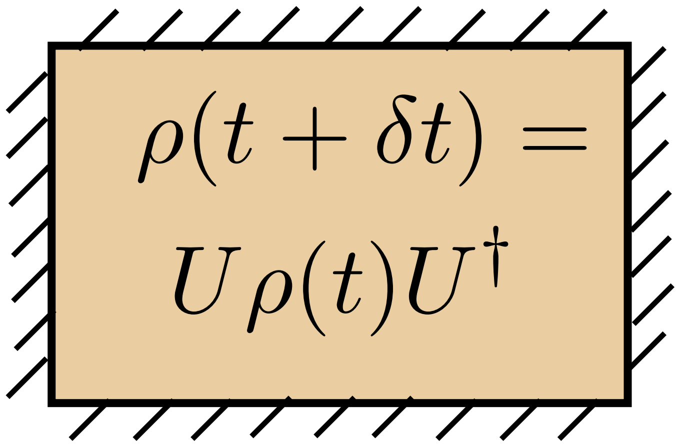


System exchanges energy/ particles with the bath and reaches thermal equilibrium at late times

$$\rho_{\mathrm{eq}}(T, \mu, ..)$$

$$\langle O \rangle = \mathrm{Tr}\left[\rho_{\mathrm{eq}} O\right]$$

What can we we say about ***isolated*** many-body quantum systems?



Dynamics of

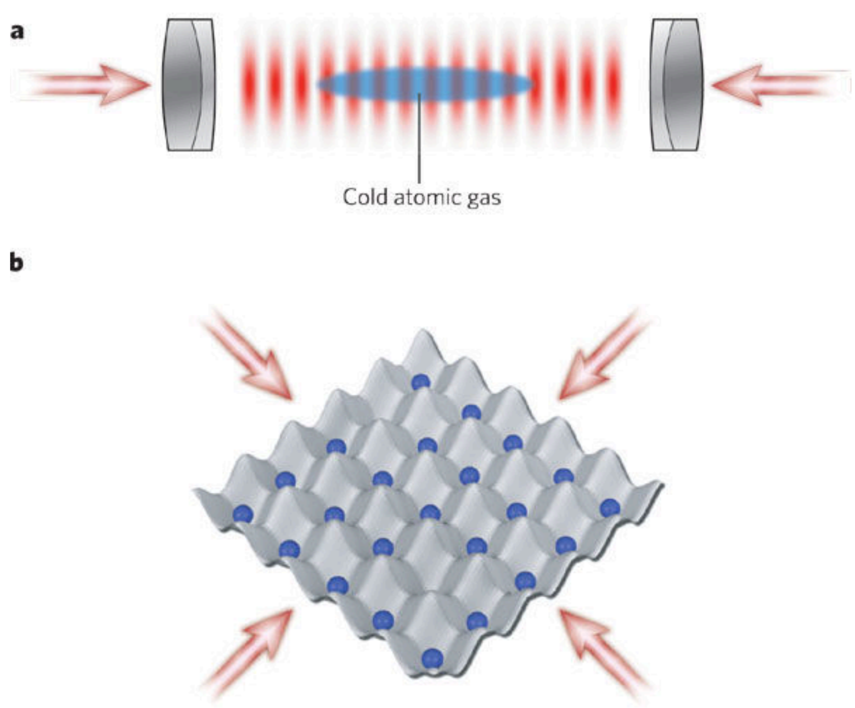
- Isolated
- Strongly interacting
- Many-body systems (spins, atoms, qubits..)

Topic at the junction of:

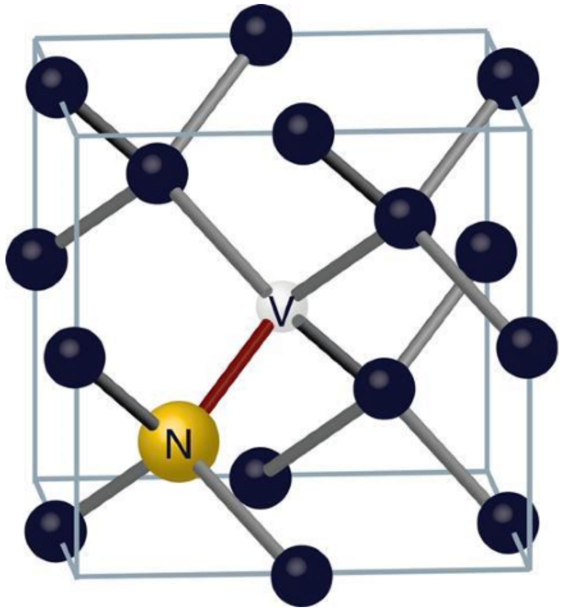
- Fundamentals of quantum statistical mechanics
- Condensed Matter
- Quantum Information/Quantum Gravity
- AMO

Well-isolated building blocks

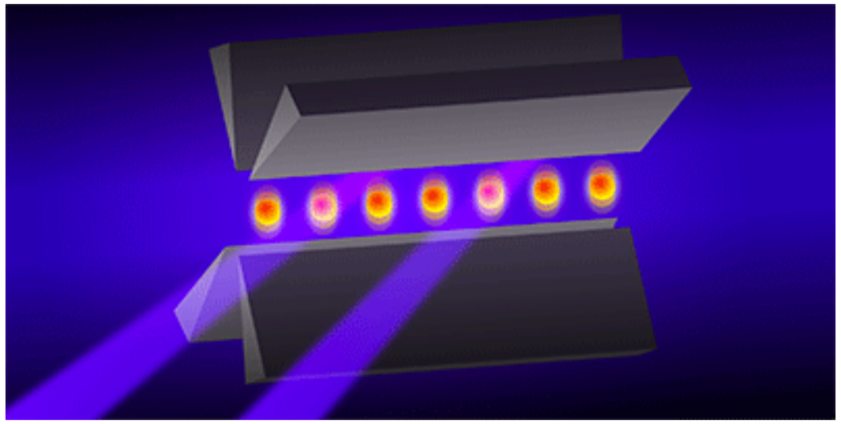
Ultracold atoms



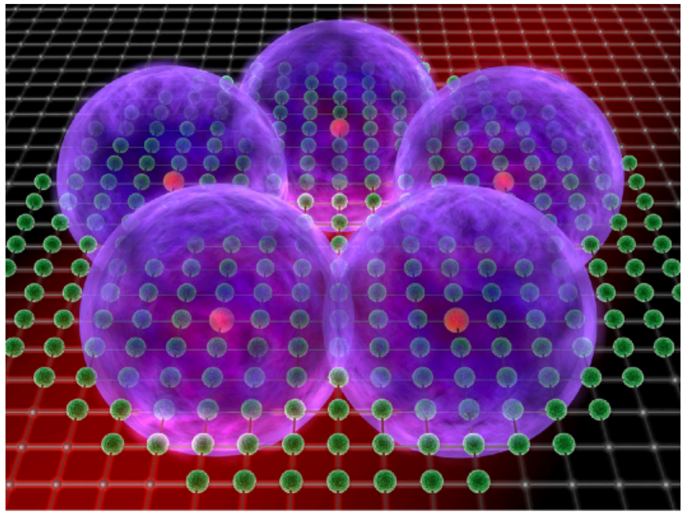
NV Centers



Trapped Ions



Rydberg atoms



$$\rho(t+\delta t) = U\rho(t)U^\dagger$$

Dynamics of

- Isolated
- Strongly interacting
- Highly excited
- Many-body systems (spins, atoms, qubits..)

Standard assumption of statistical mechanics is that this system goes to thermal equilibrium at late times.

Must this always be true?

What does "thermal equilibrium" even mean in this context?

How is thermal equilibrium reached?

Anderson 1958: First example of a system which could be many-body “localized” and fail to go to thermal equilibrium

Absence of Diffusion in Certain Random Lattices

P. W. Anderson
Bell Telephone Laboratories, Murray Hill, New Jersey
(Received October 10, 1957)

This paper presents a simple model for such processes as spin diffusion or conduction in the “impurity band.” These processes involve transport in a lattice which is in some sense random, and in them diffusion is expected to take place via quantum jumps between localized sites. In this simple model the essential randomness is introduced by requiring the energy to vary randomly from site to site. It is shown that at low enough densities no diffusion at all can take place, and the criteria for transport to occur are given.

I. INTRODUCTION

A NUMBER of physical phenomena seem to involve quantum-mechanical motion, without any particular thermal activation, among sites at which the mobile entities (spins or electrons, for example) may be localized. The clearest case is that of spin diffusion[1,2]; another might be the so-called impurity band conduction at low concentrations of impurities. In such situations we suspect that transport occurs not by motion of free carriers (or spin waves), scattered as they move through a medium, but in some sense by quantum-mechanical jumps of the mobile entities from site to site. A second common feature of these phenomena is randomness: random spacings of impurities, random interactions with the “atmosphere” of other impurities, random arrangements of electronic or nuclear spins, etc.

Our eventual purpose in this work will be to lay the foundation for a quantum-mechanical theory of transport problems of this type. Therefore, we must start with simple theoretical models rather than with the complicated experimental situations on spin diffusion or impurity conduction. In this paper, in fact, we attempt only to construct, for such a system, the simplest model we can think of which still has some expectation of representing a real physical situation reasonably well, and to prove a theorem about the model. The theorem is that at sufficiently low densities, transport does not take place; the exact wave functions are localized in a small region of space. We also obtain a fairly good estimate of the critical density at which the theorem fails. An additional criterion is that the forces be of sufficiently short range—actually, falling off as $r \longrightarrow \infty$ faster than $1/r^3$—and we derive a rough estimate of the rate of transport in the $V \propto 1/r^3$ case.

Such a theorem is of interest for a number of reasons: first, because it may apply directly to spin diffusion among donor electrons in Si, a situation in which Feher[3] has shown experimentally that spin diffusion is negligible; second, and probably more important, as an example of a real physical system with an infinite number of degrees of freedom, having no obvious oversimplification, in which the approach to equilibrium is simply impossible; and third, as the irreducible minimum from which a theory of this kind of transport, if it exists, must start. In particular, it re-emphasizes the caution with which we must treat ideas such as “the thermodynamic system of spin interactions” when there is no obvious contact with a real external heat bath.

The simplified theoretical model we use is meant to represent reasonably well one kind of experimental situation: namely, spin diffusion under conditions of

Thermalization vs. Localization

Question (Anderson 1958) : Can an isolated, strongly interacting MB system act as it's own "bath" and bring its subsystems to thermal equilibrium?

Two "generic" possibilities with a sharp dynamical distinction at late times and for large sizes

Yes: Thermalizing

↕ New kind of quantum phase transition

No: Many-Body Localized

Thermalization vs. Localization

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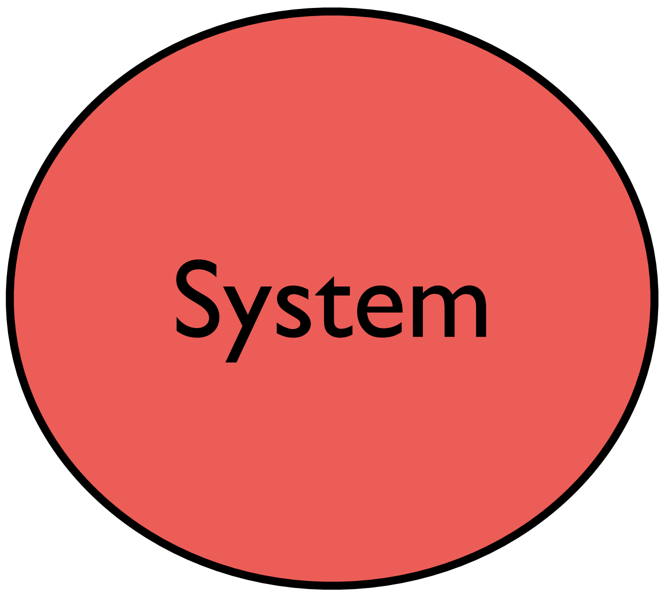
Full range of dynamical "universality classes"?

Opens up brand new possibilities for what's "allowed"

Thermalization in Isolation

Q: Can unitary time evolution bring a system to thermal equilibrium at late times?



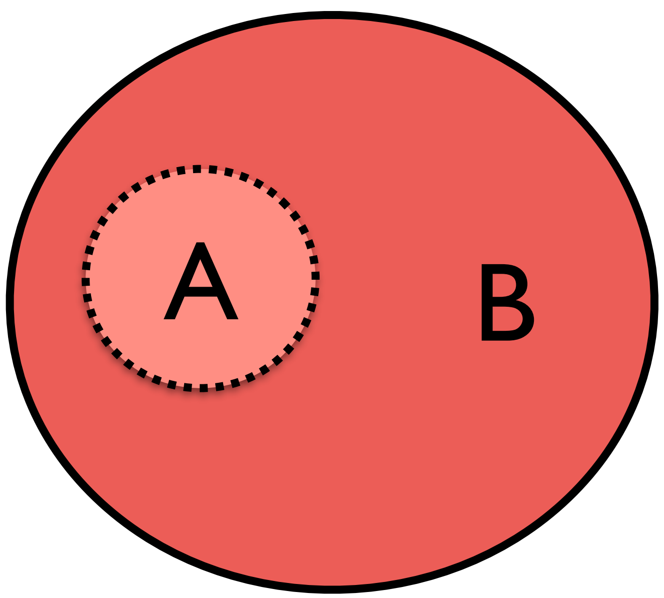
Full system remembers all details

$$|\psi(t)\rangle = U(t)|\psi_0\rangle$$

$$\rho(t) = U^\dagger(t)\rho(0)U(t)$$

Thermalization in Isolation

Q: Can unitary time evolution bring a system to thermal equilibrium at late times?



A is "observable"

Full system remembers all details

$$|\psi(t)\rangle = U(t)|\psi_0\rangle$$

System can act as its "own bath" and bring **subsystems** to thermal equilibrium

$$\rho_A(t) = \mathrm{Tr}_B |\psi(t)\rangle\langle\psi(t)|$$

Maximum entropy ensemble

$$\lim_{\substack{t\to\infty \\ B\to\infty}} \rho_A(t) = \mathrm{Tr}_B \rho_{\mathrm{eq}}(T, \mu, \cdots) = \mathrm{Tr}_B \left(\frac{e^{-\beta(H-\mu N - \cdots)}}{Z} \right)$$

$$\lim_{\substack{t\to\infty \\ B\to\infty}} \rho_A(t) = \mathrm{Tr}_B \rho_{\mathrm{eq}}(T,\mu,\cdots) \;= \mathrm{Tr}_B \left(\frac{e^{-\beta(H-\mu N-\cdots)}}{Z} \right)$$

Strong form holds for:

- All local subsystems A
- All "reasonable" initial states

Need states with sub-extensive uncertainty in all conserved quantities like energy, number so T, μ can be defined for the state

Eigenstate Thermalization Hypothesis

Berry 1987, Jensen Shankar 1985, Deutsch 1991, Srednicki 1994

Precursors form "quantum chaos" literature

If all "reasonable" initial states reach thermal equilibrium, then eigenstates of H must be thermal:

$$H|n\rangle = E_n|n\rangle$$

$$\mathrm{Tr}_B|n\rangle\langle n| = \mathrm{Tr}_B \frac{e^{-\beta_n H}}{Z}$$

Single-eigenstate microcanonical ensemble.
Each eigenstate is thermal!

Local properties of eigenstates vary smoothly with energy density

Eigenstate Thermalization Hypothesis

Berry 1987, Jensen Shankar 1985, Deutsch 1991, Srednicki 1994

Numerically verified for many MB quantum systems

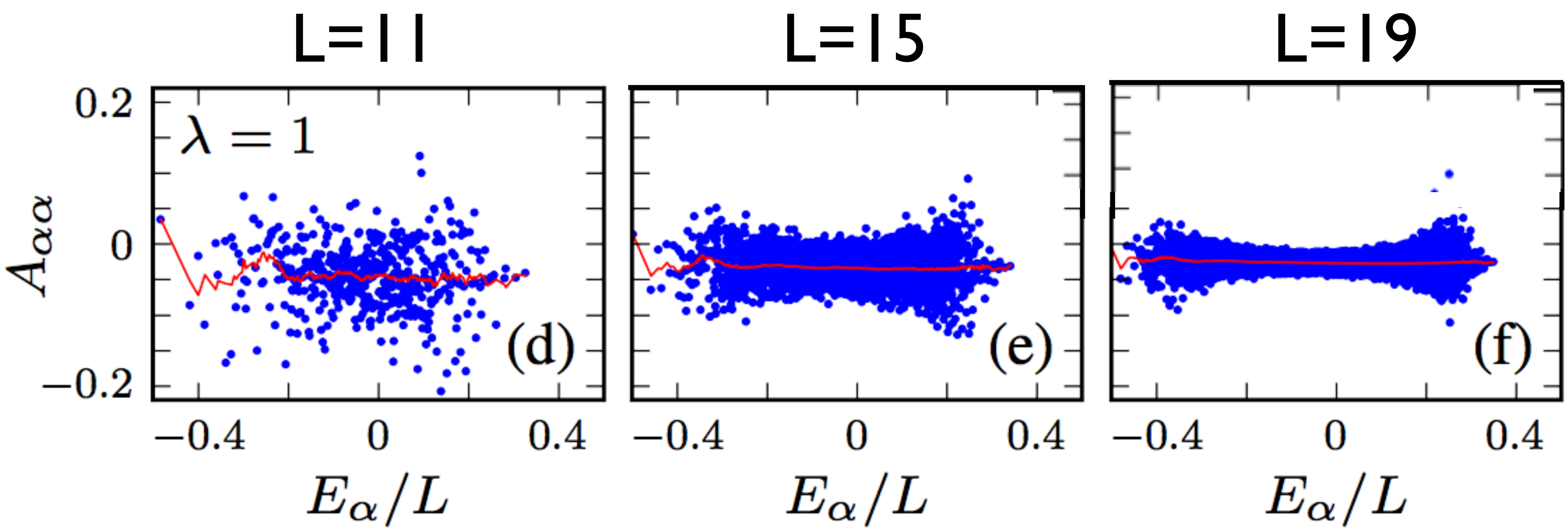
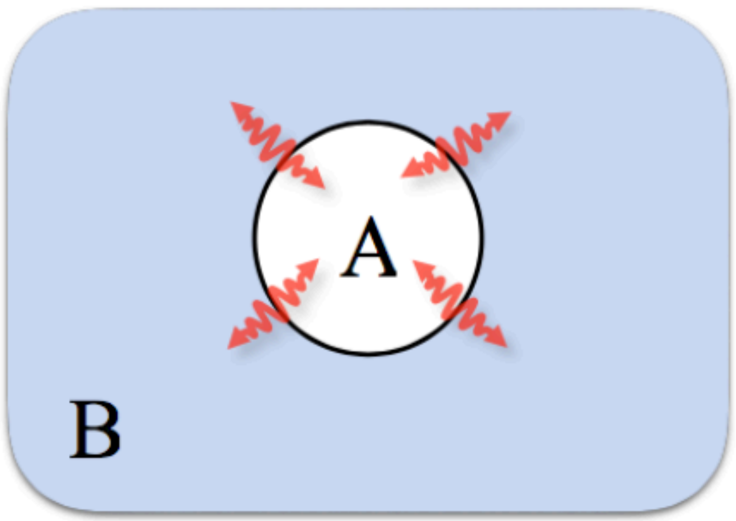


Figure from Buegeling, Moessner, Haque

Fluctuations narrow as 1/D ~ exp(-sL)

Volume law entanglement for thermal eigenstates

$$S_A = -\mathrm{Tr}\left[\rho_A \log \rho_A\right]$$



B is bath for all spins in A

$$S_A = s_{th} V_A + \ldots$$

Thermal entropy

Scales as volume of A

von-Neumann entropy of subsystems agrees with thermodynamic entropy

Scales extensively with volume of A for finite T - “volume law”

B acts as a reservoir — i.e. something to get entangled with. Conserved quantities not essential!

Figure from A. Chandran

Localization

Only "generic" exception to thermalization (that we're aware of)

Occurs in systems that are not translationally invariant

Systems retain **local** memory of initial conditions to infinitely late times! Act as "quantum memories".

New kind of phase transition between MBL to thermalizing phases

MBL can stabilize new kinds of order disallowed in equilibrium. Example: time crystals!

Local memory persists!

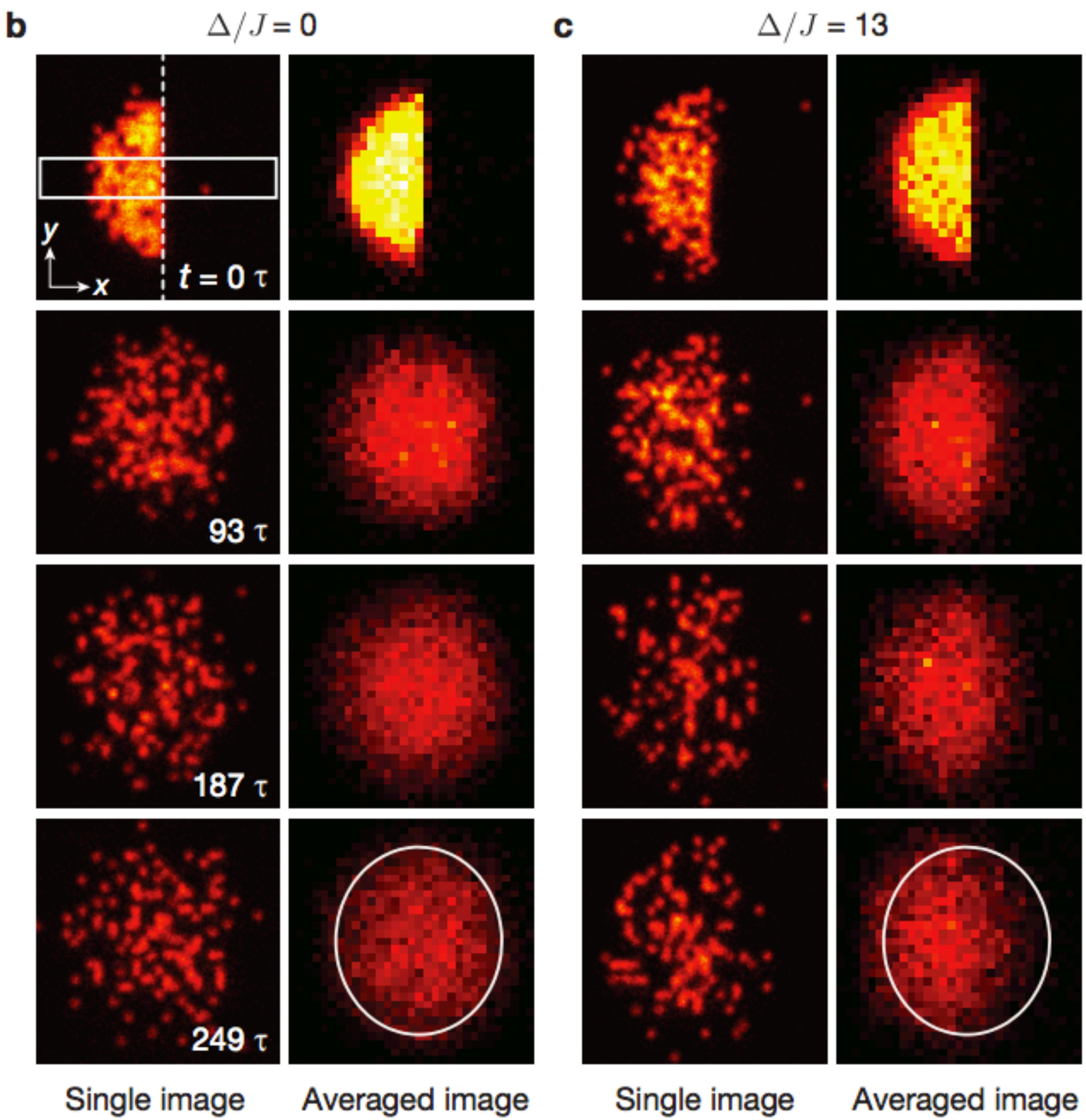


Figure from:
Choi et. al. Science (2016)

Screiber et. al. (2015)
Bordia et. al. (2015),
Smith et. al. (2015),
Kondov et. al. (2015)

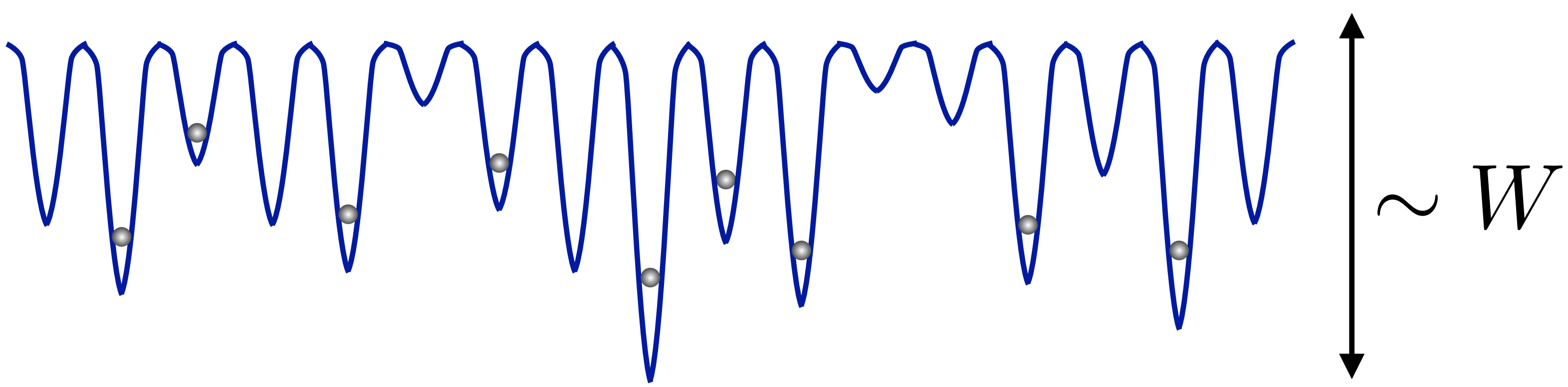
Localization

$$H = \sum_i h_i \sigma_i^z + J \sum_i (\sigma_i^x \sigma_{i+1}^x + \sigma_i^y \sigma_{i+1}^y + \sigma_i^z \sigma_{i+1}^z)$$

hopping

interactions

“Detuned” onsite fields
Can be random/quasiperiodic/…



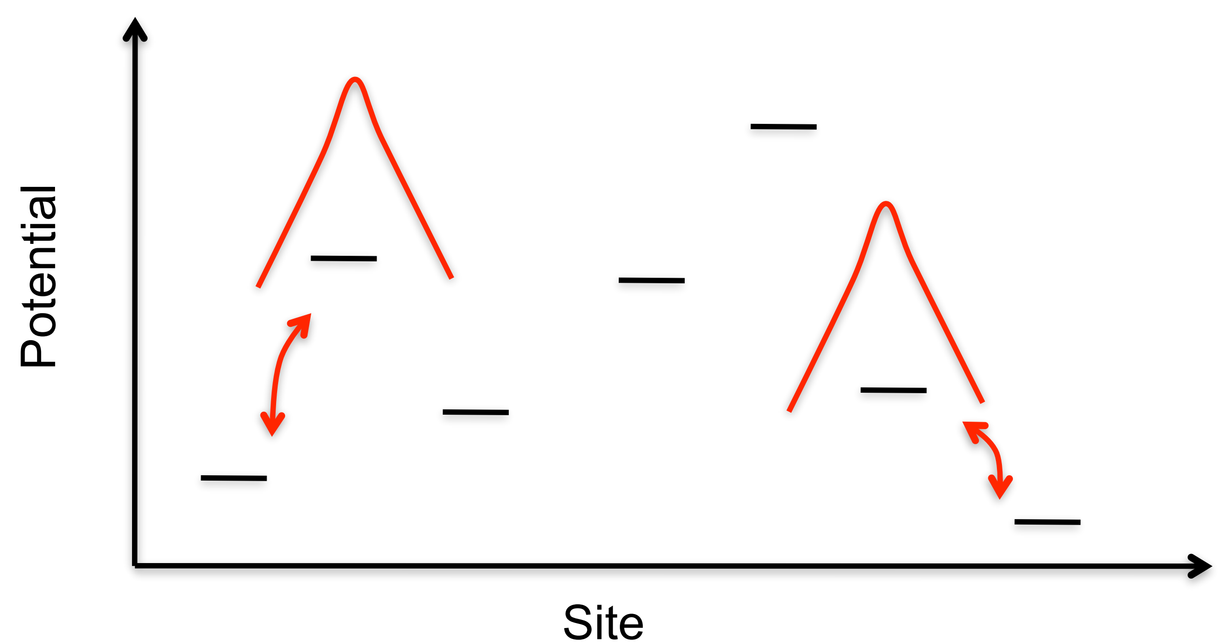
mapping to spinless fermions:
up = occupied
down = empty

Basko Aleiner Altshuler (2006)
Gyorni Mirlin Polyakov (2006)
Znidaric Prelovsek Prosen (2007)
Oganesyan Huse (2007)
Pal Huse (2010)

Single-Particle Anderson Localization

$$H = \sum_i h_i c_i^\dagger c_i + J(c_i^\dagger c_{i+1} + h.c.)$$

$$h_i \in [-W, W]$$



Locator expansion

$$J \ll W$$

Off-resonant hopping fails to hybridize sites at long-distances

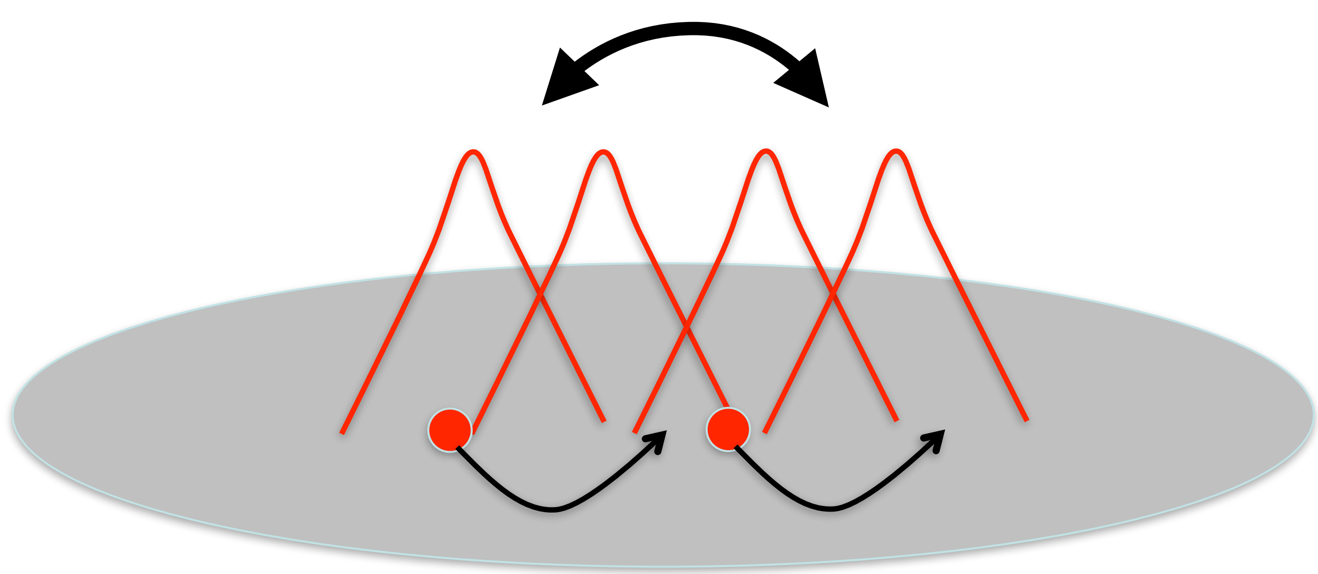
Localized $|\phi(r)|^2 \sim e^{-r/\xi}$

P.W. Anderson, Phys. Rev. (1958)

Many-body localization (MBL)

$$H = \sum_{\alpha} e_{\alpha} a^{\dagger}_{\alpha} a_{\alpha} + \sum_{\alpha\beta\gamma\delta} V_{\alpha\beta\gamma\delta} a^{\dagger}_{\alpha} a^{\dagger}_{\beta} a_{\gamma} a_{\delta}$$

Weak interactions fail to hybridize localized many-particle states



Basko, Aleiner, Altshuler (2006)

MBL: Simplest example

$$H = \sum_i h_i \sigma_i^z + J \sum_i (\sigma_i^x \sigma_{i+1}^x + \sigma_i^y \sigma_{i+1}^y + \sigma_i^z \sigma_{i+1}^z)$$

$J = 0 :$

$|n\rangle = |\uparrow\downarrow\uparrow\uparrow \cdots \downarrow\rangle$

Not thermal - violates ETH

Extensively many constants of motion, $\{\sigma_i^z\}$

$$[H, \sigma_i^z] = 0, \qquad [\sigma_i^z, \sigma_j^z] = 0$$

Emergent Integrability

$$H = \sum_i h_i \sigma_i^z + J \sum_i (\sigma_i^x \sigma_{i+1}^x + \sigma_i^y \sigma_{i+1}^y + \sigma_i^z \sigma_{i+1}^z)$$

$$J \ll W$$

Finite depth
local unitary

Exponentially decaying

$$H = \sum_i \tilde{h}_i \tau_i^z + \sum_{ij} \tilde{J}_{ij} \tau_i^z \tau_j^z + \sum_{ijk} \tilde{K}_{ijk} \tau_i^z \tau_j^z \tau_k^z + \ldots$$

Extensively many local integrals of motion “l-bits”

Oganesyan, Huse, Nandkishore (2014); Serbyn, Papic, Abanin (2013); Imbrie (2014)

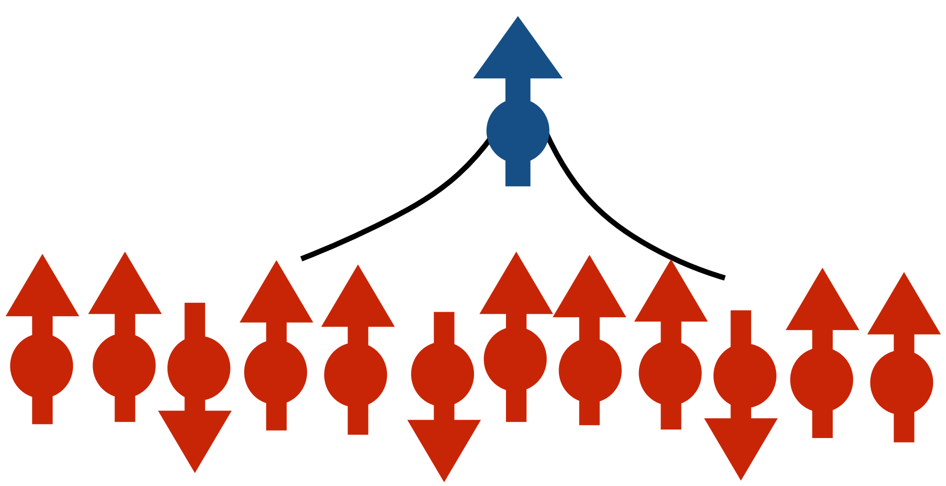
Emergent Integrability

$$H = \sum_i \tilde{h}_i \tau_i^z + \sum_{ij} \tilde{J}_{ij} \tau_i^z \tau_j^z + \sum_{ijk} \tilde{K}_{ijk} \tau_i^z \tau_j^z \tau_k^z + \ldots$$

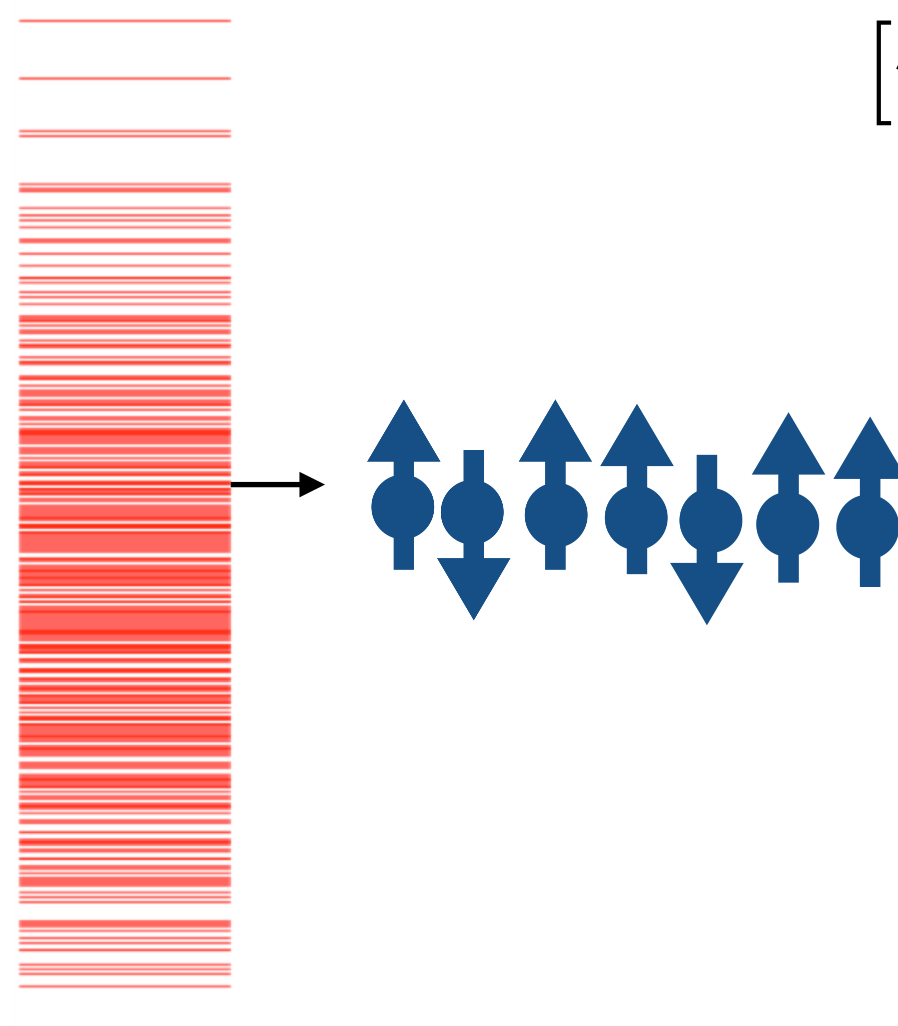
Extensively many local integrals of motion “l-bits” $[H, \tau_i^z] = 0$

$$[\tau_i^z, \tau_j^z] = 0$$

$$\tau_i^z = U^\dagger \sigma_i^z U$$



“Dressed operators”



Oganesyan, Huse, Nandkishore (2014); Serbyn, Papic, Abanin (2013); Imbrie (2014)

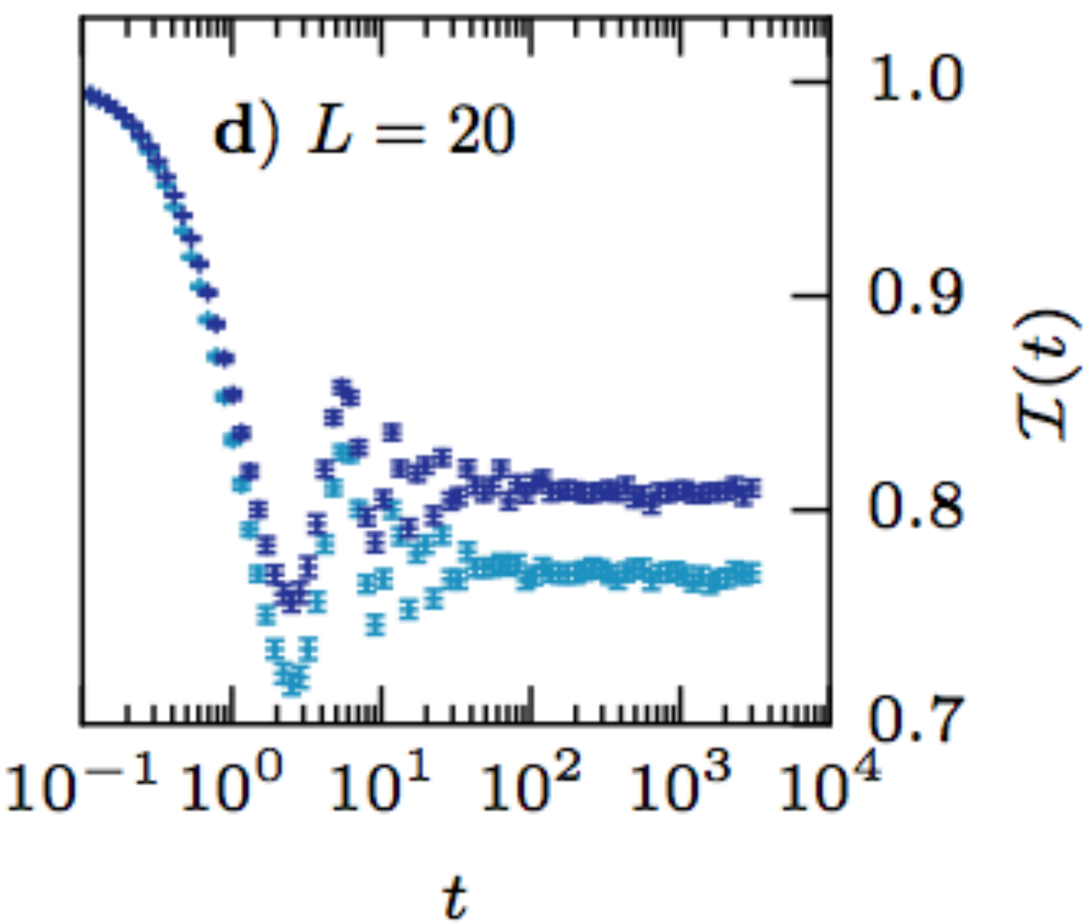
What do we *know* about the MBL phase transition?

- Existence of the MBL phase (l-bits):
    - All orders in perturbation theory including higher dimensions (Basko, Aleiner, Altshuler)
    - *Almost* proof including non-perturbative effects in one dimensional lattice models with exponentially decaying interactions (Imbrie)
    - Lots of open questions (possible non-perturbative instabilities in higher dimensions, with longer ranged interactions…) (de Roeck, Huveneers). Intermediate phases between MBL and Thermal?
- Lots of numerical evidence for existence of the thermal phase (but no proof!) — Dynamical phase transition to a thermalizing phase as function of disorder strength/ interaction strength…

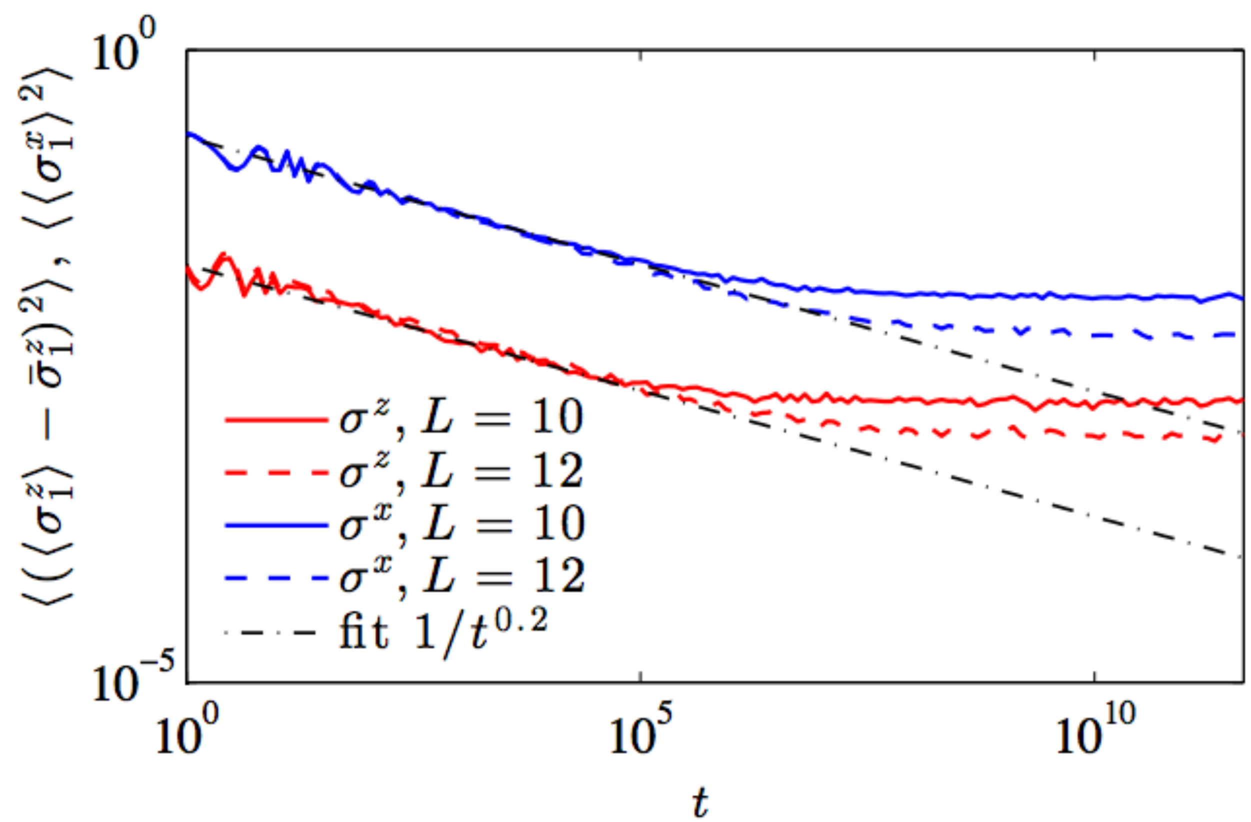
Properties: Local Memory

- Generic local operator have finite overlap with l-bits. This part doesn't decay

- Approach to equilibrium is a slow power law because of slow "dephasing" dynamics. Serbyn, Papic, Abanin (2014);

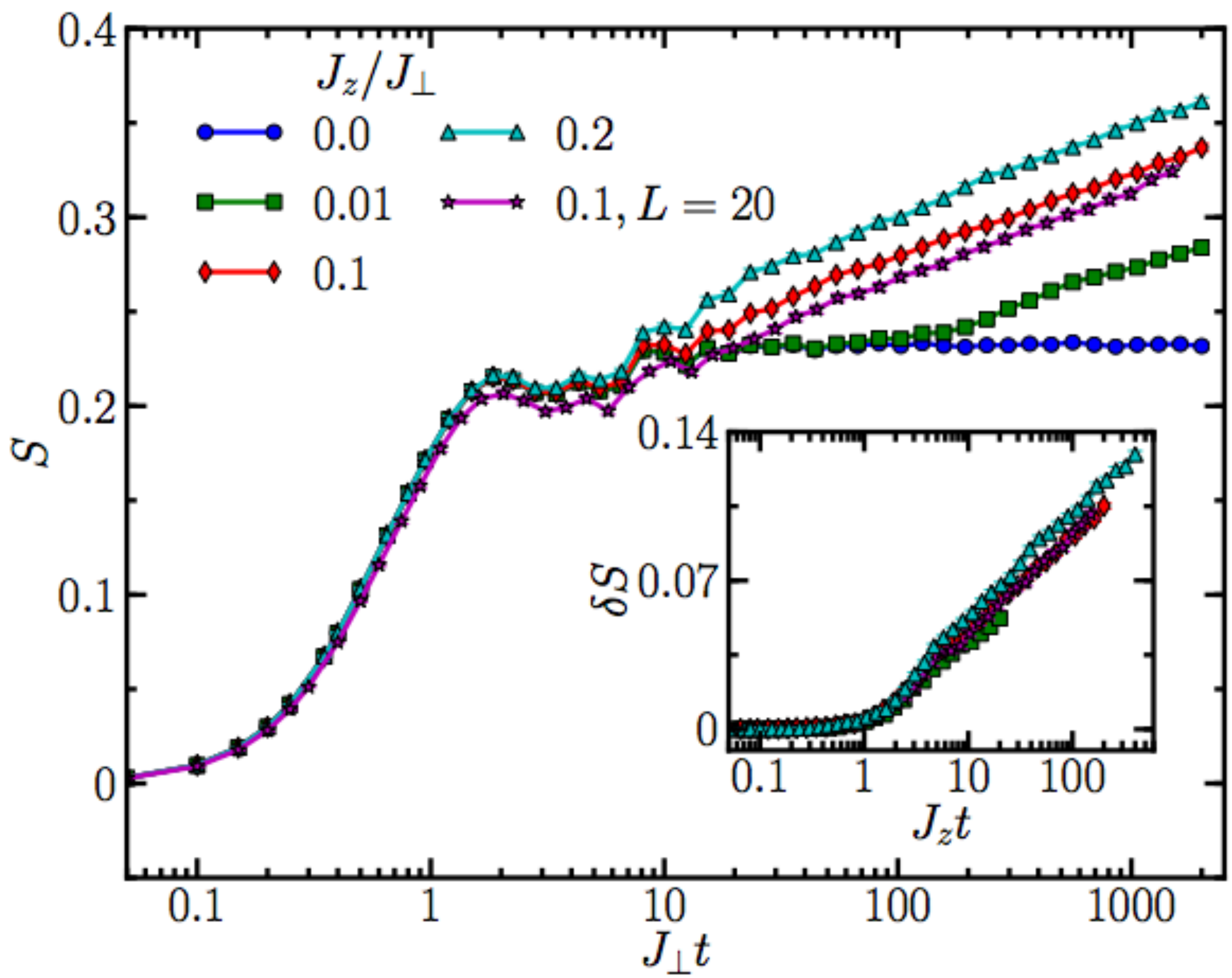


Luitz Laflorencie Alet (2017);



Properties: Log growth of entanglement

- No transport, but slow logarithmic growth due to dephasing dynamics

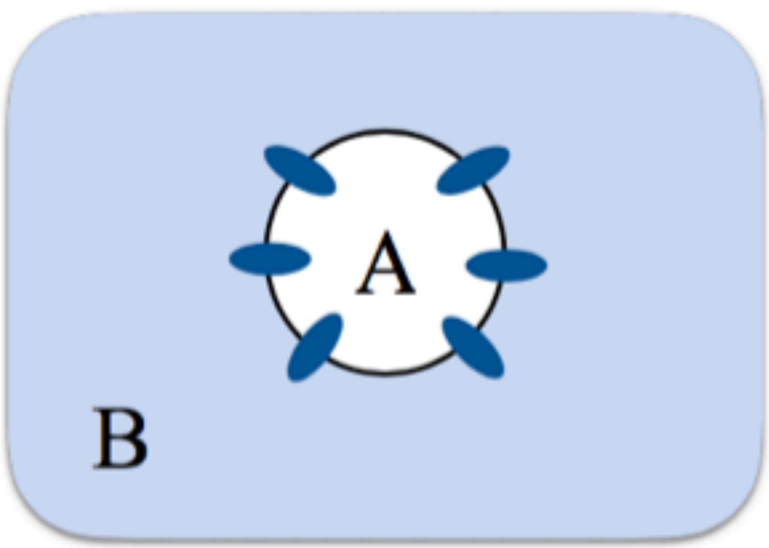


Bardarson Pollmann Moore (2012)
Znidaric Prelovsek Prosen 2007
Serbian Panic Abanin (2013)
Oganesyan Huse Nandkishore (2013)

Area law entanglement for MBL eigenstates

$$S_A = -\mathrm{Tr}\left[\rho_A \log \rho_A\right]$$

Localized



Only boundary spins correlated

$$S_A = c\partial A + \ldots$$

Scales as perimeter of A

Figure from A. Chandran

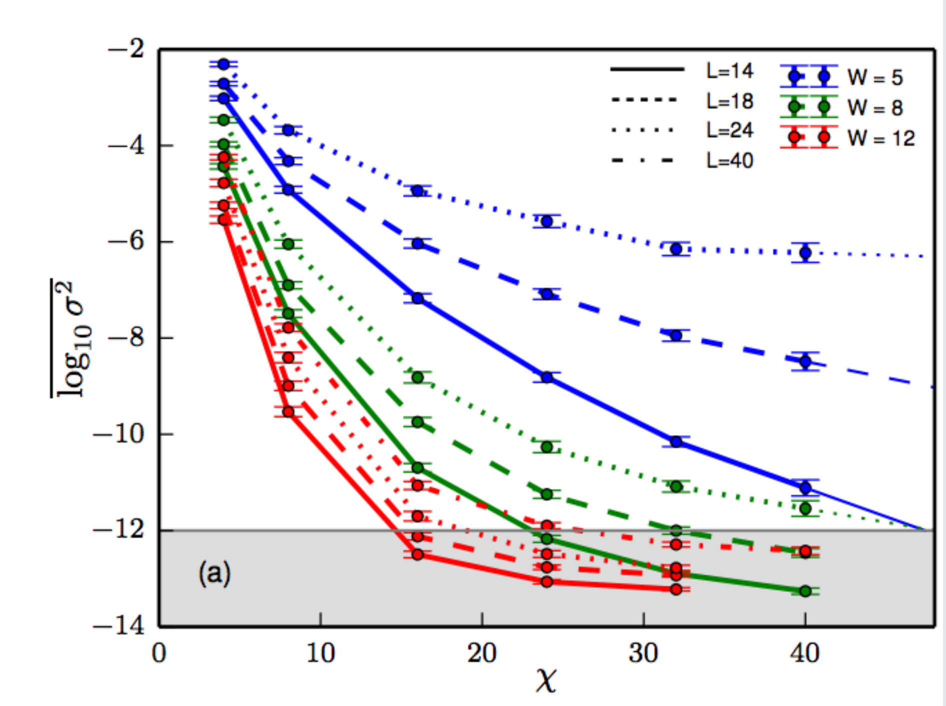
Product states have zero entanglement

"dressed" l-bits only have local correlations

Low entanglement = efficient representations

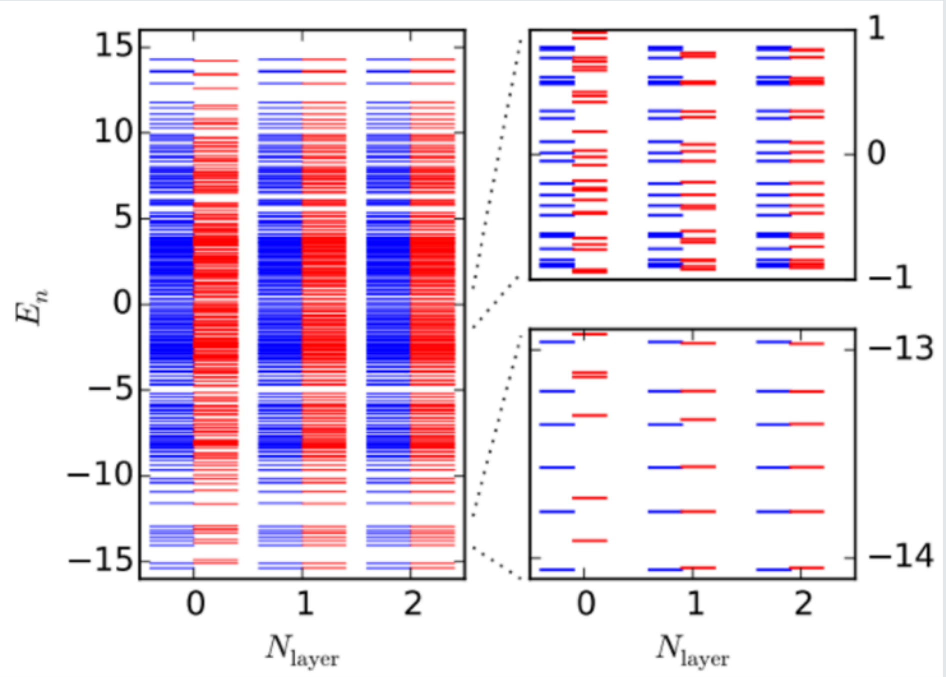
(1) DMRG-X: Obtains MPS representations of individual highly excited MBL eigenstates

VK, Pollmann, Sondhi



(2) VUMPO: Obtains MPO representation of finite-depth diagonalizing unitary

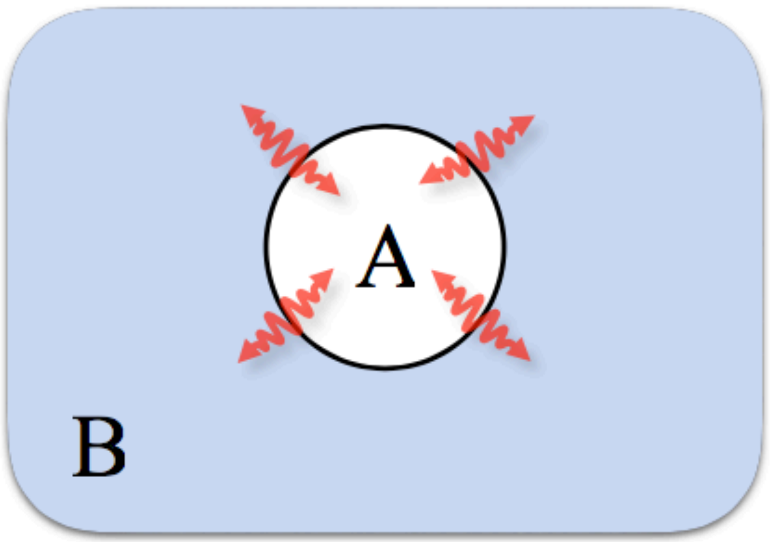
Pollmann, VK, Cirac, Sondhi



See also: Pekker Clark; Yu, Pekker Clark

Entanglement Entropy as an order parameter

Thermal



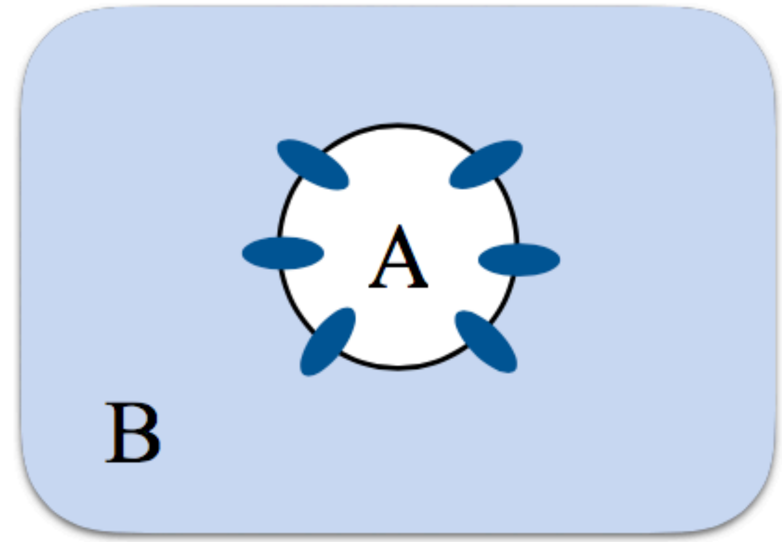
B is bath for all spins in A

$$S_A = s_{th} V_A + \dots$$

Thermal entropy

Scales as volume of A

Localized



Only boundary spins correlated

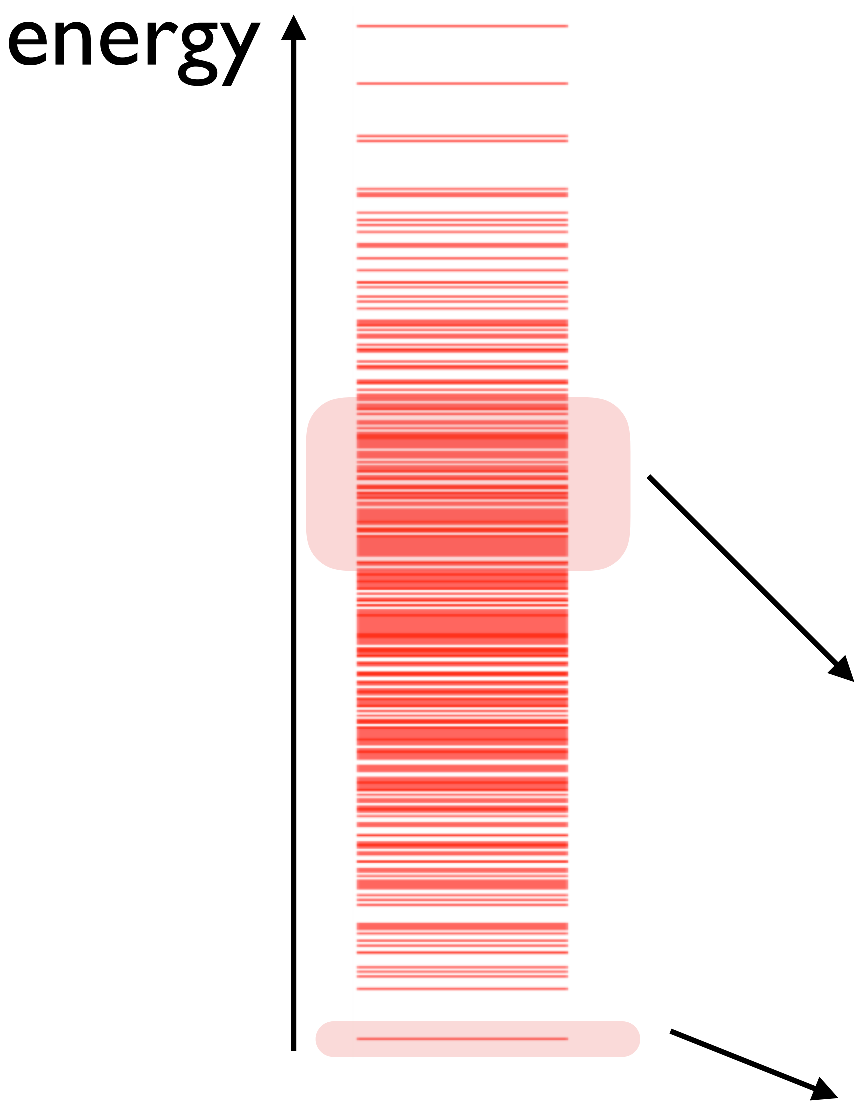
$$S_A = c\partial A + \dots$$

Scales as perimeter of A

Slide from A. Chandran

Pal, Huse (2010); Bauer, Nayak (2013)

The Efficient-Inefficient Transition

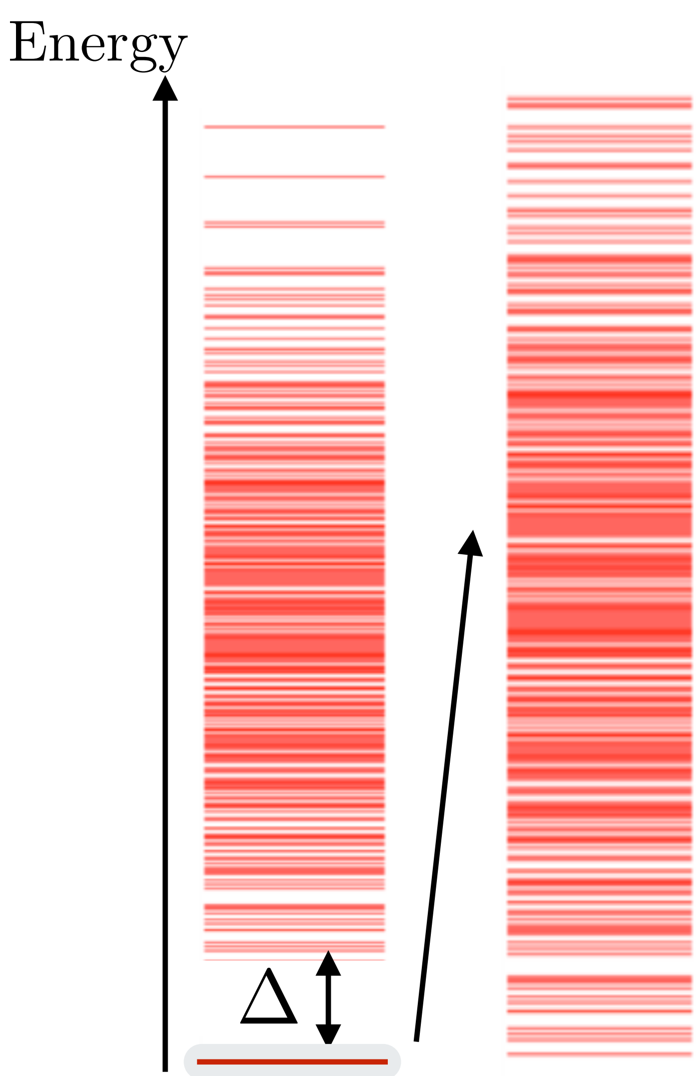


Thermal	MBL
• Quantum statistical mechanics (ETH) holds $$\mathrm{Tr}_B\|\psi\rangle\langle\psi\| = \mathrm{Tr}_B \frac{e^{-H/T}}{Z}$$	• Lies outside the framework of quantum statistical mechanics and the ETH breaks down
• Highly excited eigenstates have "volume law" entanglement.	• Highly excited eigenstates have "area law" entanglement even at infinite "temperature". MPS techniques.
• Ground states have area law (gapped) or logarithmic (gapless) entanglement	• Ground states have area law entanglement

Dynamical phase transition involving a singular rearrangement in the entanglement structure of individual highly excited MB eigenstates

Localization Protected Quantum Order

How do we think of phase structure out-of-equilibrium?



- Highly excited MBL eigenstates only have area law entanglement
- *Individual* highly excited eigenstates can display "frozen" orders that may be forbidden in equilibrium
- Experimentally measurably dynamical signatures

Equilibrium Phases ➔ Eigenstate Phases

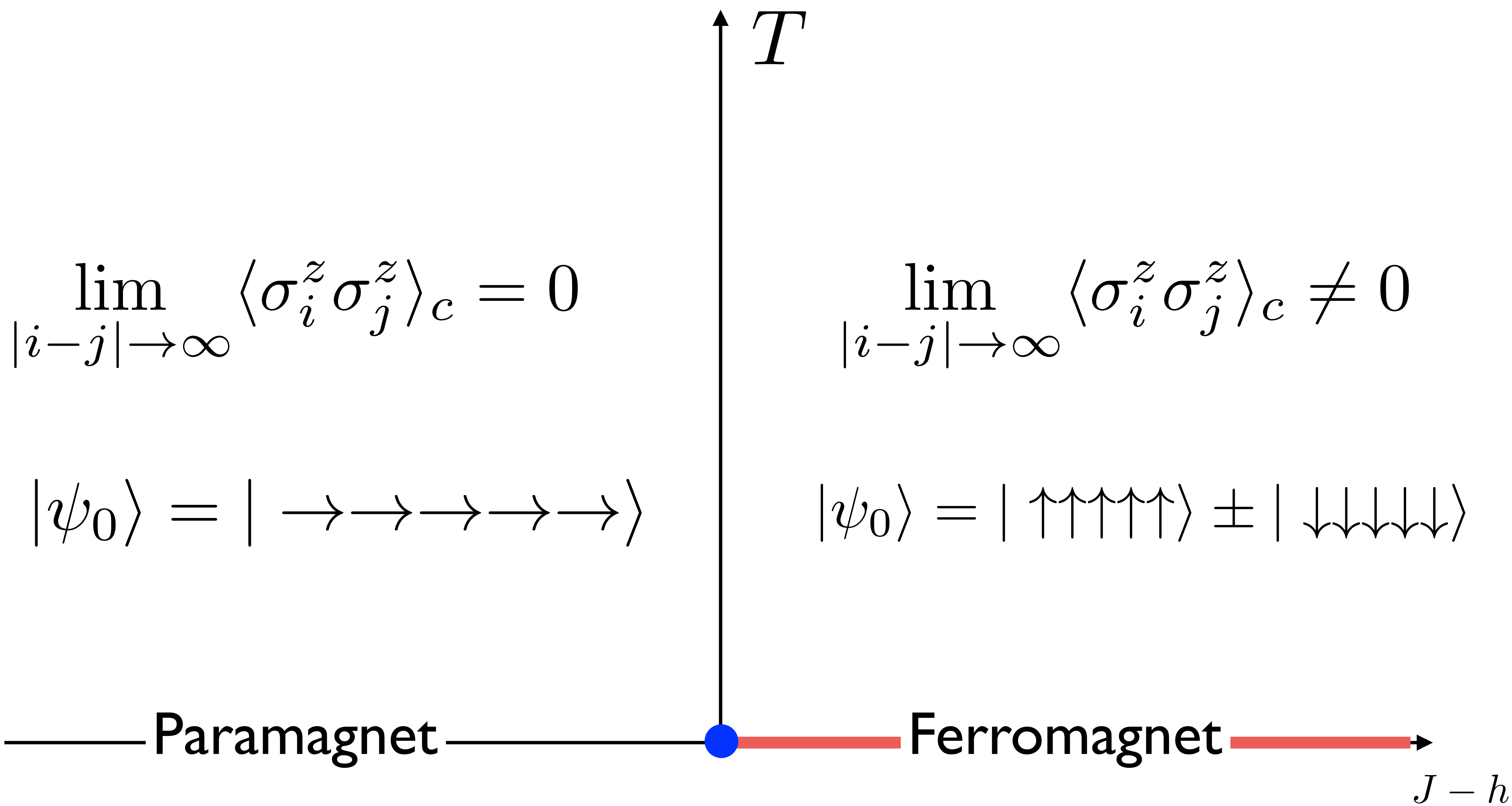
Huse et. al. (2013); Bauer Nayak (2013); Chandran, VK, Laumann, Sondhi (2014); Bahri, Altman, Vishwanath (2014)

Eigenstate Order: Ising example

1D transverse field Ising model

$$H = J\sum_i \sigma_i^z \sigma_{i+1}^z + h\sum_i \sigma_i^x$$

$$P = \prod_i \sigma_i^x$$



$$\lim_{|i-j|\to\infty} \langle \sigma_i^z \sigma_j^z \rangle_c = 0$$

$$|\psi_0\rangle = |\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow\rangle$$

$$\lim_{|i-j|\to\infty} \langle \sigma_i^z \sigma_j^z \rangle_c \neq 0$$

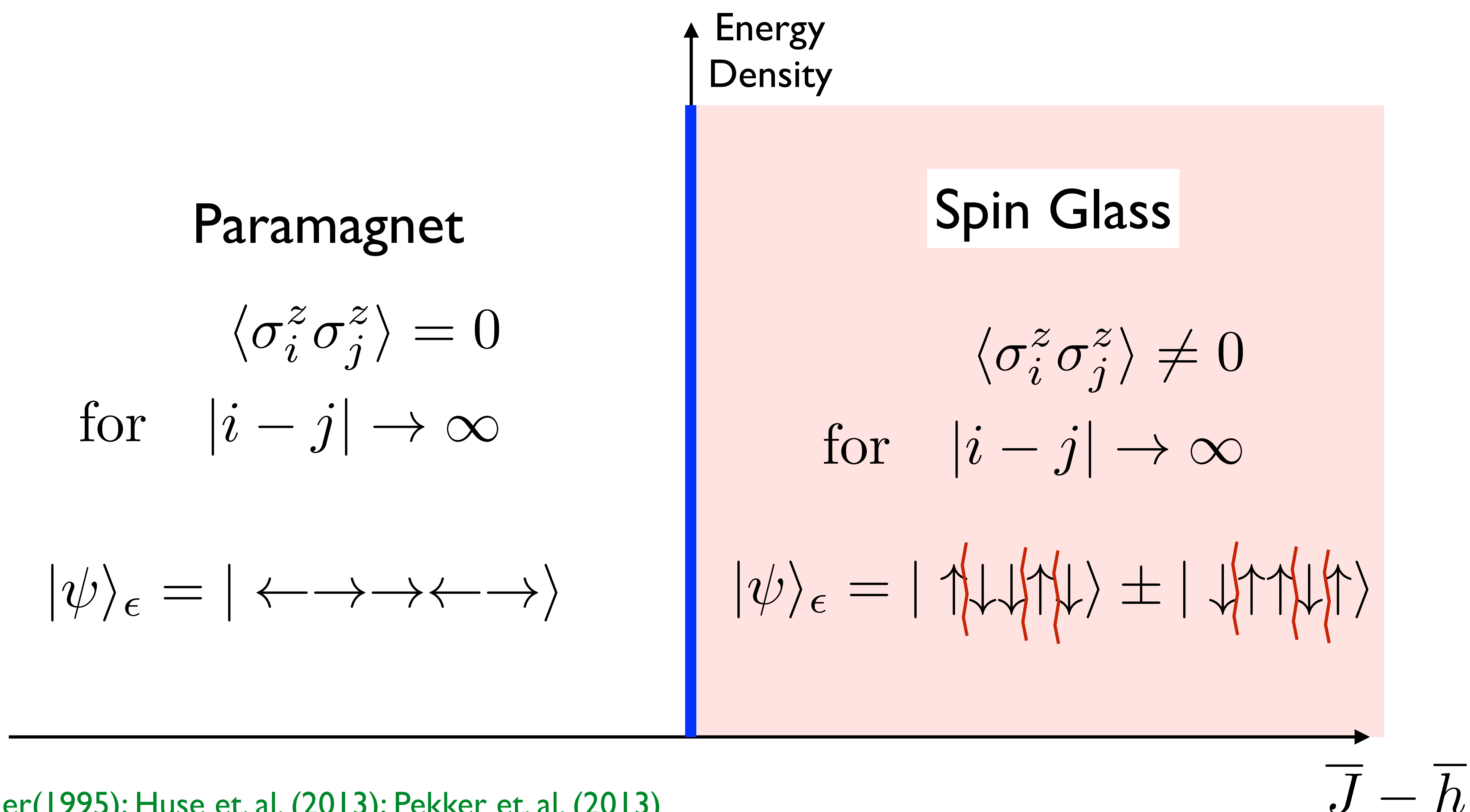
$$|\psi_0\rangle = |\uparrow\uparrow\uparrow\uparrow\uparrow\rangle \pm |\downarrow\downarrow\downarrow\downarrow\downarrow\rangle$$

Eigenstate Order: Ising example

1D transverse field Ising model

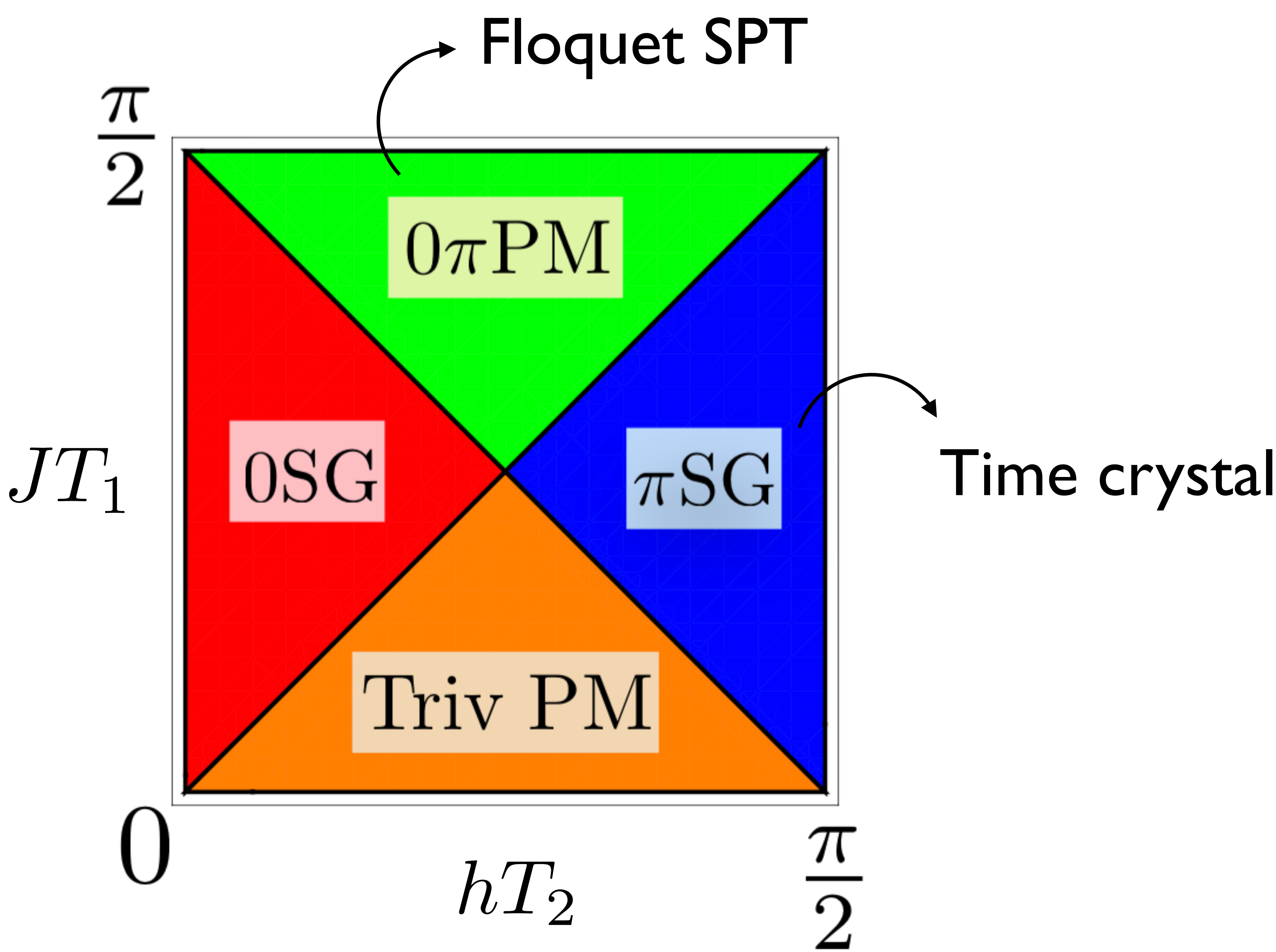
$$H = \sum_i J_i \sigma_i^z \sigma_{i+1}^z + h_i \sigma_i^x$$

$$P = \prod_i \sigma_i^x$$



D. Fisher(1995); Huse et. al. (2013); Pekker et. al. (2013)

Periodically Driven + MBL



VK Lazarides Moessner Sondhi 2015

The MBL Phase Transition

Thermalizing — MBL

W_c — W

Transport and entanglement dynamics governed by rare "Griffiths" effects. Entanglement and relaxation dynamics power laws in time

DC conductivity could be zero

Agarwal, Gopalakrishnan, Knap, Mueller, Demler; Bar Lev, Cohen, Reichman; Vosk Huse Altman; Potter, Vasseur, Parameswaran…

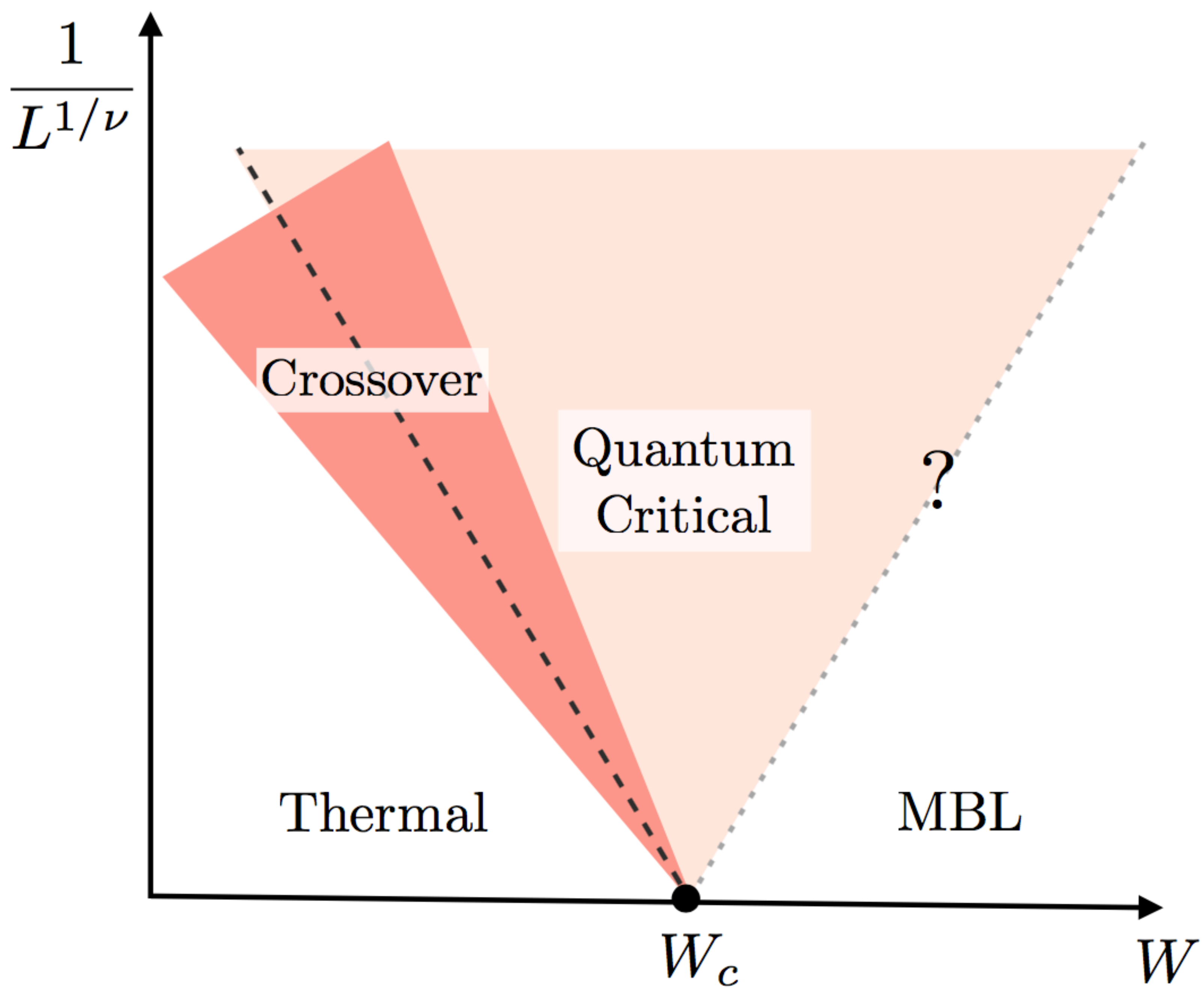
Dynamical phase transition. Visible to single eigenstate ensemble.

Lots of uncertainty about properties.

Entanglement dynamics logarithmic

DC transport zero

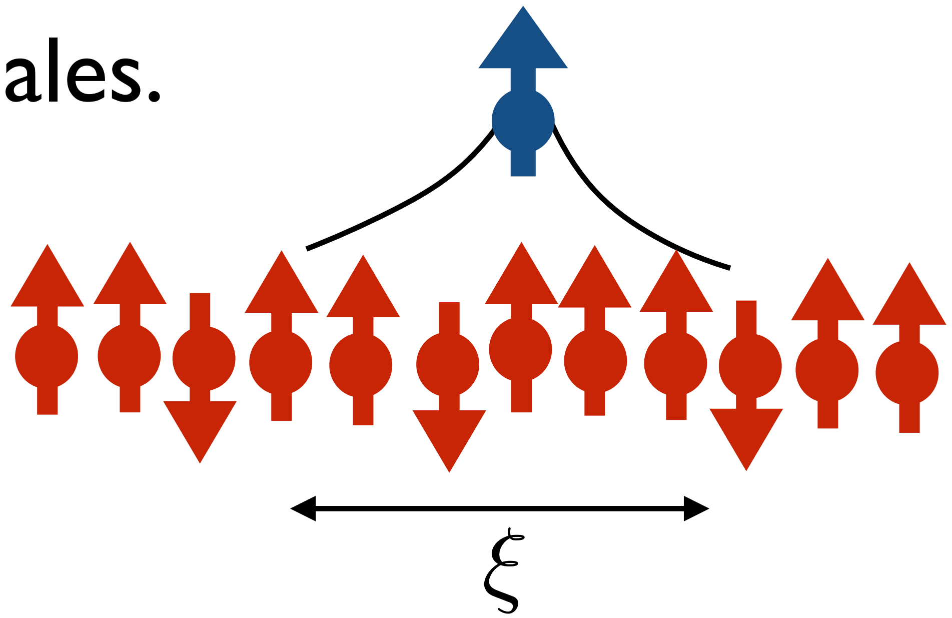
Finite-Size Critical Scaling



$$\xi \sim (W - W_c)^{-\nu}$$

Critical Entanglement

- Grover (2014) showed that if the entanglement entropy of **small** subsystems varies continuously at a direct MBL-Thermal transition, then from the strong sub-additivity of entanglement, the entanglement entropy of these subsystems looks **thermal** in the quantum critical regime.
- Very natural picture where one imagines there exists a diverging length scale ξ such that:
  - Properties probed on length scales < ξ look critical.
  - Look thermal/MBL on longer length scales.



Discontinuity in entanglement entropy

- Careful numerical analysis finds the critical regime actually looks localized. Thus, either no direct transition, or entanglement is discontinuous at the transition.

(VK, Lim, Sheng, Huse, PRX 2017)

Discontinuity in entanglement entropy

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- Transition driven by the proliferation of a **sparse resonant backbone** of entanglement. Just gains enough strength to thermalize the system on the thermal side of the crossover in the infinite size limit.

- Global discontinuity in presence of fully functional bath implies local discontinuity.

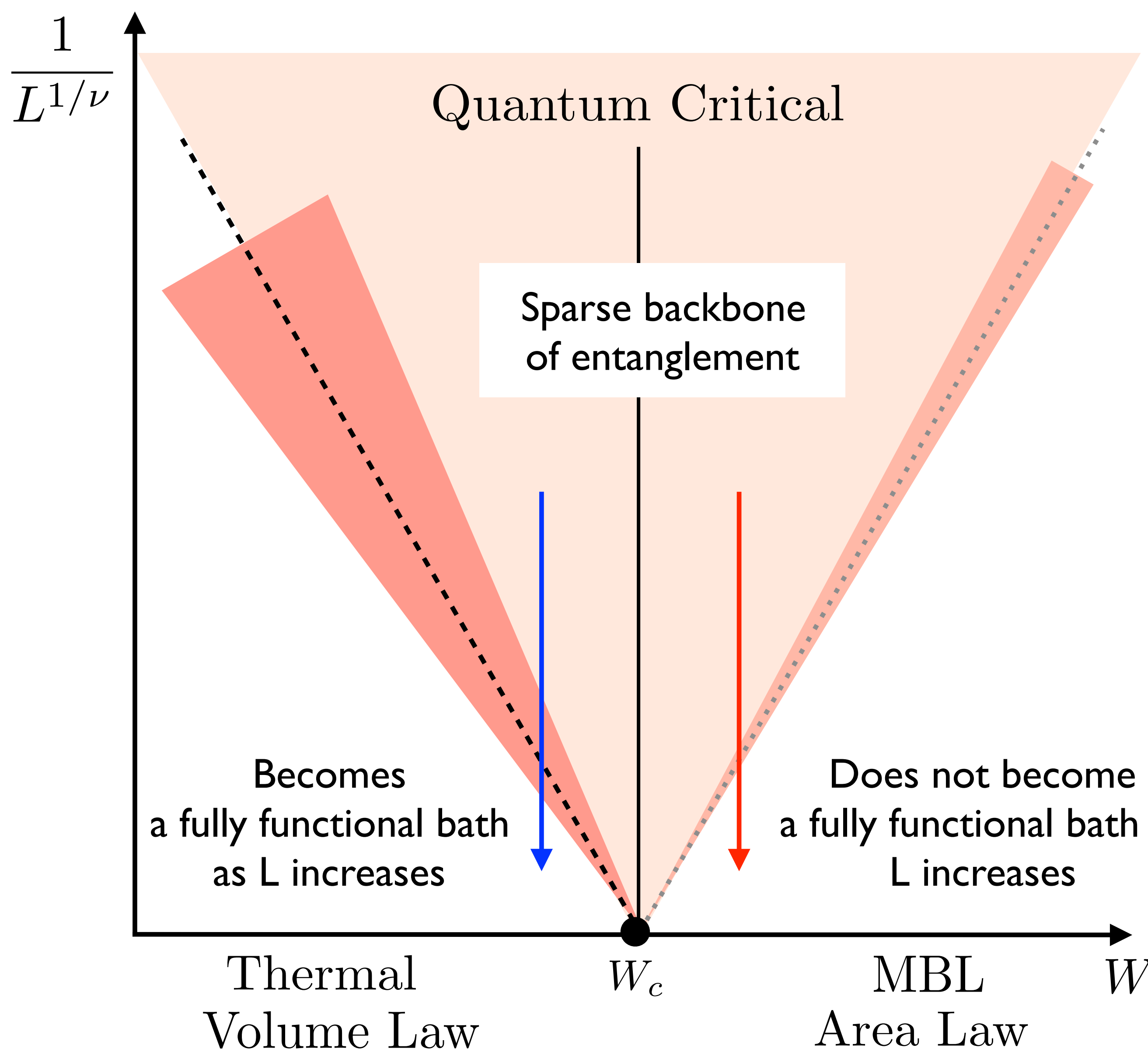
- Discontinuity subsequently verified by phenomenological strong disorder RG-like treatments (Dumitrescu, Vasseur Potter; Thierry, Huveneers, Mueller, De Roeck)

(VK, Lim, Sheng, Huse, PRX 2017)

Discontinuity in entanglement entropy

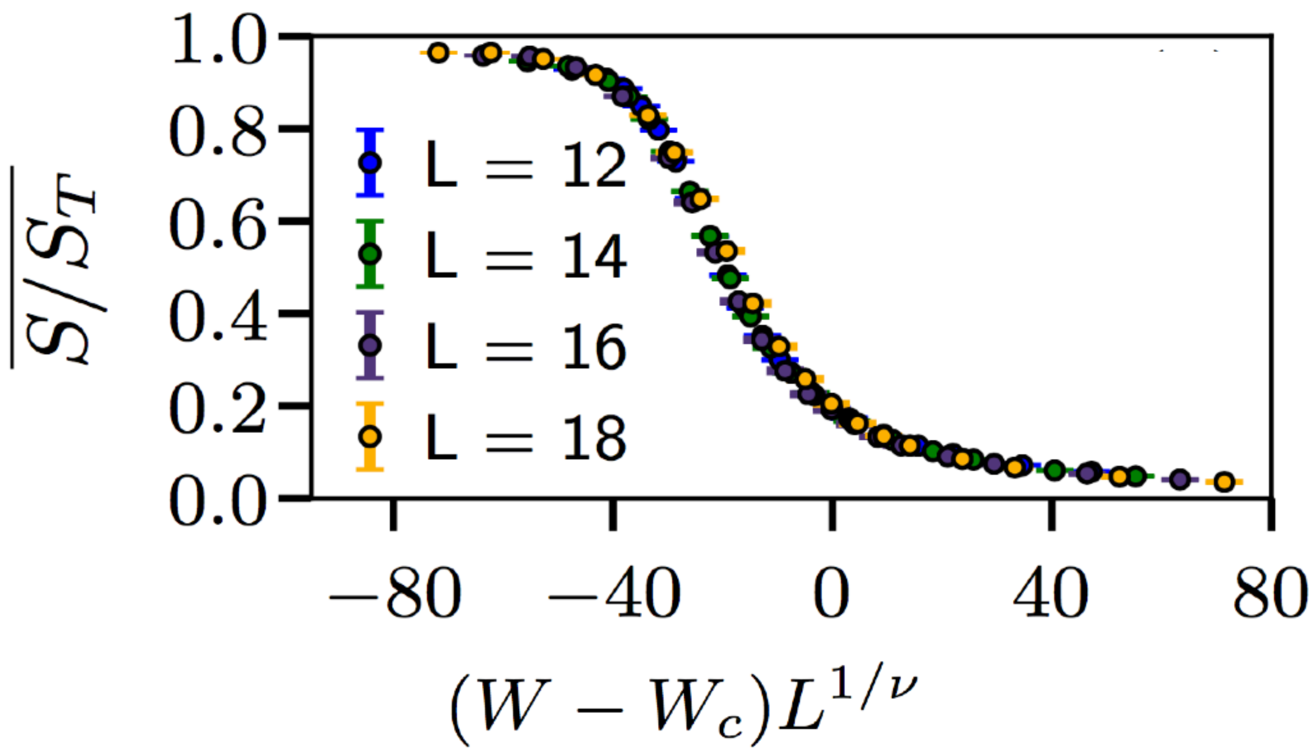
Entanglement at the transition can show a non-local dependence on the system size since an infinite thermal system can act as a bath for any finite subsystem.

$$S_A = L_A f(L^{1/\nu}(W - W_c), L_A^{1/\nu}(W - W_c))$$



(VK, Lim, Sheng, Huse, PRX 2017)

Finite-size Scaling

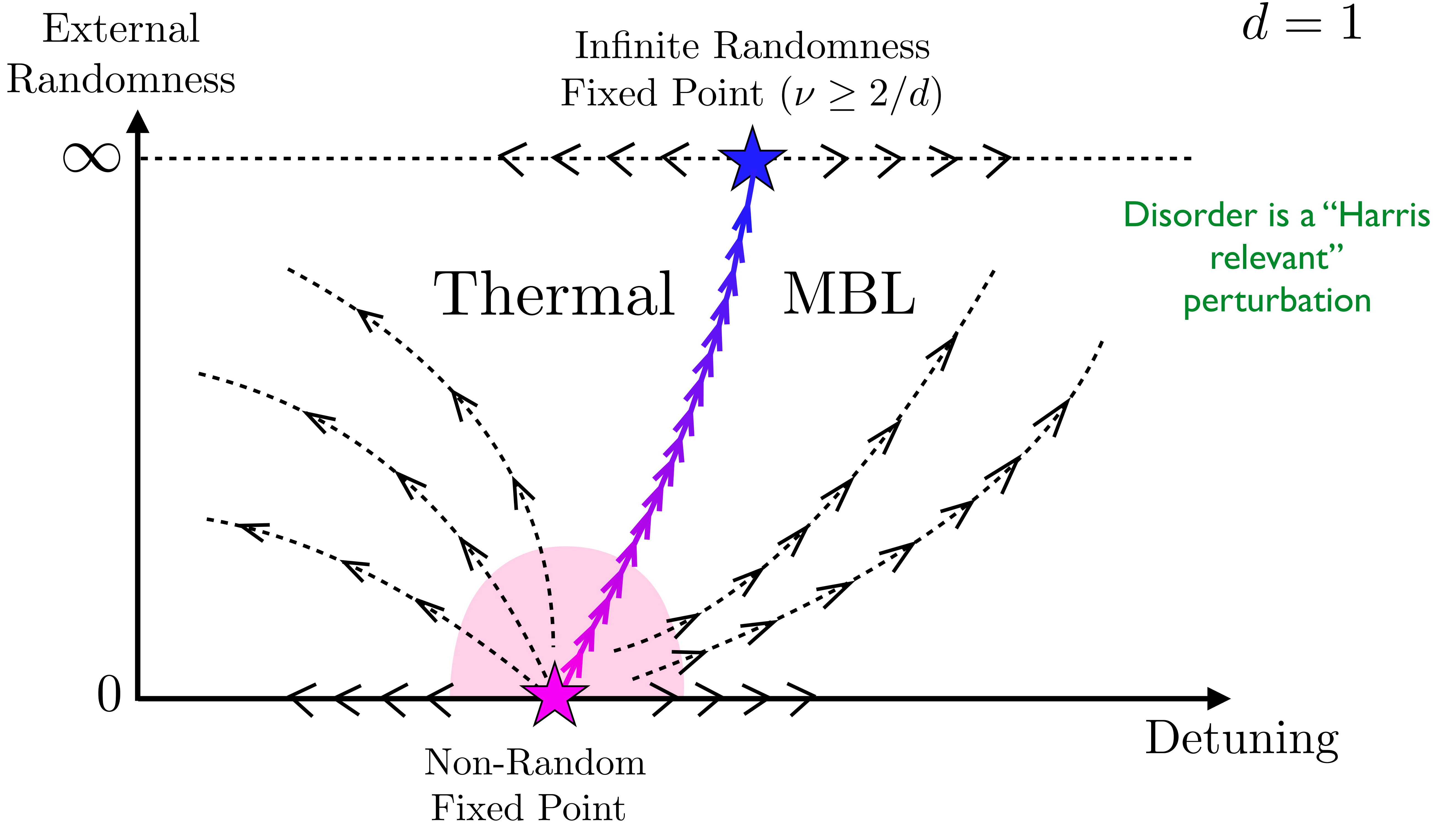


For systems with quenched randomness, asymptotically at large L, $\nu \geq 2/d$ (Harris 1974; Chayes, Chayes, Fisher, Spencer 1986; Chandran, Laumann, Oganesyan 2015)

- RG treatments find $\nu \simeq 3$ (Vosk Huse Altman; Potter Vasseur Parameswaran 2014)
- Long standing mystery: all exact diagonalization numerics show scaling collapse, but with exponent $\nu \simeq 1$ in violation of Harris
- However, numerics show similar scaling for both random and quasiperiodic models at small sizes. But random models are beginning to show a crossover into the quenched-randomness-dominated universality class.

VK, Sheng, Huse, PRL (2017)

Two Universality Classes for the MBL Transition



VK, Sheng, Huse, PRL (2017)

Open questions

- Nature of the phase transition
- New types of out-of-equilibrium phases in the MBL setting
- Possible non-perturbative instabilities of the MBL phase in higher dimensions, with longer ranged interactions, in translationally invariant systems… Role of disorder (random vs. quasiperiodic).

Question: Can an isolated MB system act as it's own "bath" and bring its subsystems to thermal equilibrium?

No: MBL Phase (localized even at infinite time)

Yes: Thermal Phase (thermalizes on accessible time scales)

Yes*: Thermal* (extremely long time scales for thermalization)

Some reviews

- Nandkishore Huse 2014
- Abanin, Altman, Bloch, Serbyn 2018
- Parameswaran Vasseur 2018

- … Rapidly evolving field!!