Fluidistic description of astrophysical and space plasmas

- Part 2 -

Daniel Gómez[1,2]

(1) Departamento de Física, Fac. Cs. Exactas y Naturales, UBA, Argentina
(2) Instituto de Astronomía y Física del Espacio, UBA-CONICET, Argentina

Two-fluid MHD

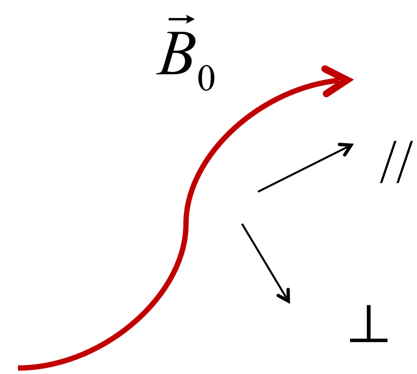
➡ MHD is a fluidistic approach to describe the large scale dynamics of plasmas.

➡ The standard approach is also known as one-fluid MHD.

➡ We are going to start from a somewhat more general approach known as two-fluid MHD, which acknowledges the presence of ions and electrons and considers kinetic effects such as Hall, electron pressure and electron inertia.

➡ Physical processes that can be addressed with MHD include:

- Magnetic reconnection
- Magnetic confinement
- Magnetic dynamo
- MHD shocks
- MHD turbulence



➡ We will also address the case of plasmas embedded in strong external magnetic fields, which allow for an approximation known as reduced MHD, both for one-fluid MHD (RMHD) and two-fluid MHD (RHMHD).

Fluid equations for multi-species plasmas

- For each species s we have (Goldston & Rutherford 1995):
  - Mass conservation
    $$\frac{\partial n_s}{\partial t} + \vec{\nabla} \cdot (n_s \vec{U}_s) = 0$$
  - Equation of motion
    $$m_s n_s \frac{d\vec{U}_s}{dt} = q_s n_s (\vec{E} + \frac{1}{c}\vec{U}_s \times \vec{B}) - \vec{\nabla} p_s + \vec{\nabla} \bullet \overleftrightarrow{\sigma}_s + \sum_{s'} \vec{R}_{ss'}$$
  - Momentum exchange rate
    $$\vec{R}_{ss'} = -m_s n_s \upsilon_{ss'} (\vec{U}_s - \vec{U}_{s'})$$
- These moving charges act as sources for electric and magnetic fields:
  - Charge density
    $$\rho_c = \sum_s q_s n_s \approx 0$$
  - Electric current density
    $$\vec{J} = \frac{c}{4\pi} \vec{\nabla} \times \vec{B} = \sum_s q_s n_s \vec{U}_s$$

Small scales: EIHMHD equations

➡ The dimensionless version, for a length scale L_0 , density n_0 and Alfven speed $v_A = B_0 / \sqrt{4\pi m_i n_0}$

$$\frac{d\vec{U}_i}{dt} = \frac{1}{\varepsilon}(\vec{E} + \vec{U}_i \times \vec{B}) - \frac{\beta}{n}\vec{\nabla}p_i - \frac{\eta}{\varepsilon n}\vec{J}$$

$$\frac{m_e}{m_i}\frac{d\vec{U}_e}{dt} = -\frac{1}{\varepsilon}(\vec{E} + \vec{U}_e \times \vec{B}) - \frac{\beta}{n}\vec{\nabla}p_e + \frac{\eta}{\varepsilon n}\vec{J}$$

where

$$\vec{J} = \vec{\nabla} \times \vec{B} = \frac{n}{\varepsilon}(\vec{U}_i - \vec{U}_e)$$

➡ We define the Hall parameter $\varepsilon = \dfrac{c}{\omega_{pi} L_0}$

as well as the plasma *beta* $\beta = \dfrac{p_0}{m_i n_0 v_A^2}$ and the electric resistivity $\eta = \dfrac{c^2 \nu_{ie}}{\omega_{pi}^2 L_0 v_A}$

➡ Adding these two equations yields:

$$\frac{d\vec{U}}{dt} = (\vec{\nabla} \times \vec{B}) \times (\vec{B} + \varepsilon_e^2 \vec{\nabla} \times \vec{J}) - \vec{\nabla}p$$

where $\vec{U} = \dfrac{m_i \vec{U}_i + m_e \vec{U}_e}{m_i + m_e}$

and $p = p_i + p_e$

$$\varepsilon_e = \sqrt{\frac{m_e}{m_i}}\,\varepsilon = \frac{c}{\omega_{pe} L_0}$$

Ideal invariants in EIHMHD

- For each species s in the incompressible and ideal limit

$$m_s n_s \left(\partial_t \vec{U}_s - \vec{U}_s \times \vec{W}_s\right) = q_s n_s (\vec{E} + \frac{1}{c}\vec{U}_s \times \vec{B}) - \vec{\nabla}(p_s + m_s n_s \frac{U_s^2}{2})$$

- Using that $\vec{J} = \frac{c}{4\pi}\vec{\nabla} \times \vec{B} = \sum_s q_s n_s \vec{U}_s$ and $E = -\frac{1}{c}\partial_t \vec{A} - \vec{\nabla}\phi$

we can readily show that energy is an ideal invariant, where

$$E = \int d^3 r \left(\sum_s m_s n_s \frac{U_s^2}{2} + \frac{B^2}{8\Pi} \right)$$

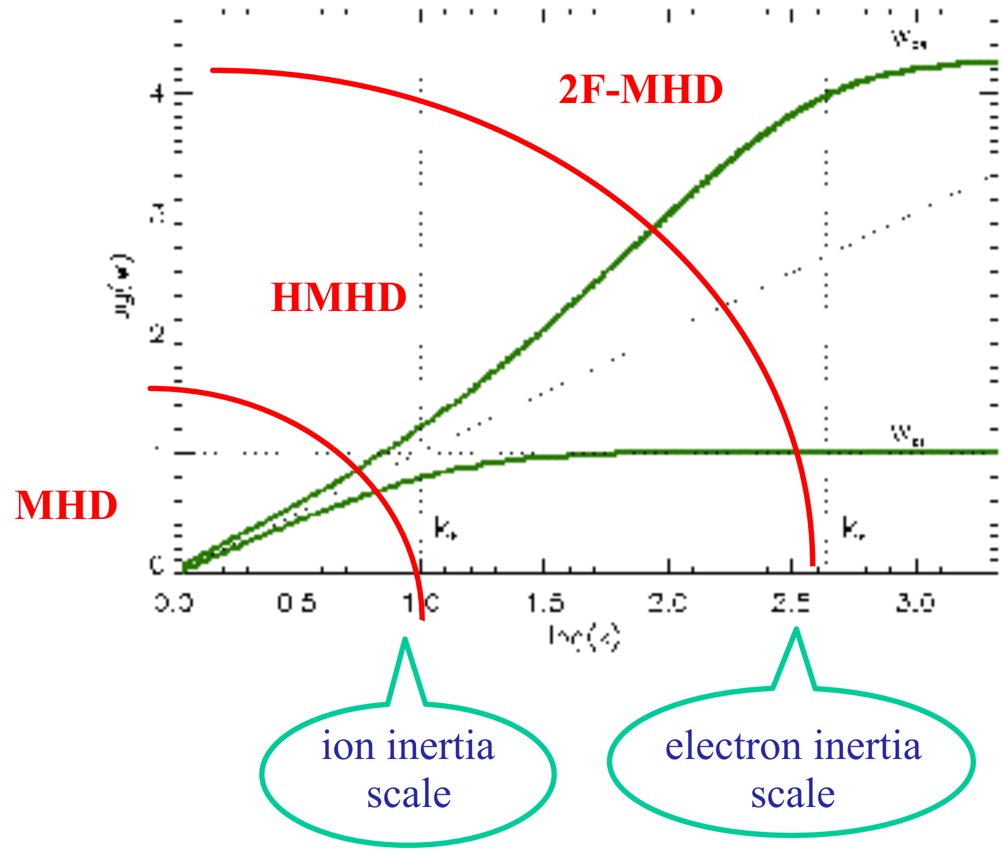
- We also have a helicity per species which is conserved, where

$$H_s = \int d^3 r \left(\vec{A} + \frac{c m_s}{q_s} \vec{U}_s \right) \bullet \left(\vec{B} + \frac{c m_s}{q_s} \vec{W}_s \right)$$

Normal modes in EIHMHD

- If we linearize our equations around an equilibrium characterized by a uniform magnetic field, we obtain the following dispersion relation:

$$\left(\frac{\omega}{\vec{k}\bullet\vec{B}_0}\right)^2 \pm \frac{k\varepsilon}{1+\varepsilon_e^2 k^2}\left(\frac{\omega}{\vec{k}\bullet\vec{B}_0}\right) - \frac{1}{1+\varepsilon_e^2 k^2} = 0$$



- Asymptotically, at very large k, we have two branches

$$\omega \xrightarrow[k\to\infty]{} \omega_{ce}\cos\theta$$

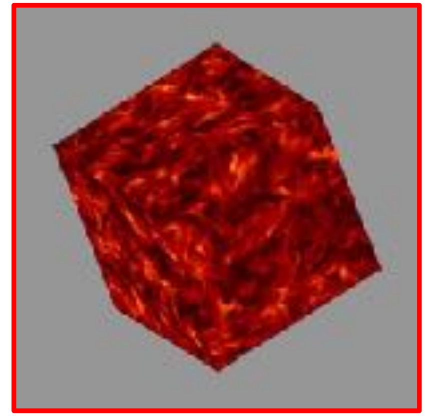
$$\omega \xrightarrow[k\to\infty]{} \omega_{ci}\cos\theta$$

while for very small k, both branches simply become Alfven modes, i.e.

$$\omega \xrightarrow[k\to 0]{} k\cos\theta$$

- Different approximations, just as one-fluid MHD, Hall-MHD and electron-inertia MHD can clearly be identified in this diagram.

Some applications



MHD

RMHD heating of solar coronal loops (Dmitruk & Gomez 1997, 1999)

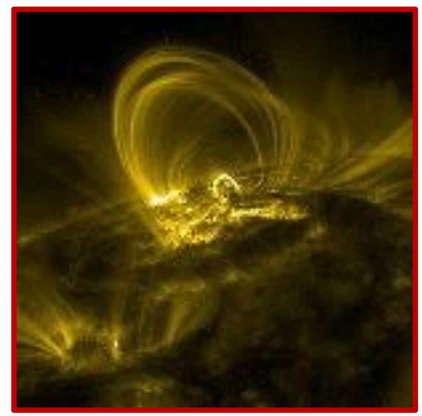
Kelvin-Helmholtz instability in the solar corona (Gomez, DeLuca & Mininni 2016)

Hall-MHD

3D HMHD turbulent dynamos. (Mininni, Gomez & Mahajan 2003, 2005; Gomez, Dmitruk & Mininni 2010)

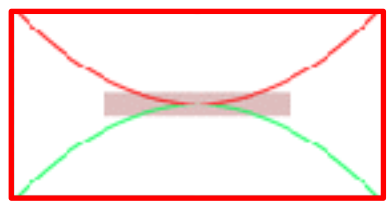
2.5 D HMHD reconnection at Earth magnetopause (Morales, Dasso & Gomez 2005, 2006)

RHMHD turbulence in the solar wind (Martin, Dmitruk & Gomez 2010, 2012)



Hall MRI in accretion disks (Bejarano, Gomez & Brandenburg 2011)

Electron inertia



1D model of perpendicular shocks (Gomez et al. 2018).

Two-fluid turbulence in the solar wind (Andres et al. 2014, 2016).

Fast reconnection in 2.5 D (Andres, Dmitruk & Gomez 2014, 2016).

Hall-MHD equations

- The dimensionless version, for a length scale L_0, density n_0 and Alfven speed $v_A = B_0 / \sqrt{4\pi m_i n_0}$

$$\frac{d\vec{U}}{dt} = \frac{1}{\varepsilon}(\vec{E} + \vec{U} \times \vec{B}) - \frac{\beta}{n}\vec{\nabla}p_i - \frac{\eta}{\varepsilon n}\vec{J} + \nu\nabla^2\vec{U} \qquad \nu = \frac{\mu}{m_i n v_A L_0}$$

$$0 = -\frac{1}{\varepsilon}(\vec{E} + \vec{U}_e \times \vec{B}) - \frac{\beta}{n}\vec{\nabla}p_e + \frac{\eta}{\varepsilon n}\vec{J}$$

where $\vec{J} = \vec{\nabla} \times \vec{B} = \frac{n}{\varepsilon}(\vec{U} - \vec{U}_e)$

- We define the Hall parameter $\varepsilon = \frac{c}{\omega_{pi} L_0}$

as well as the plasma *beta* $\beta = \frac{p_0}{m_i n_0 v_A^2}$ and the electric resistivity $\eta = \frac{c^2 \nu_{ie}}{\omega_{pi}^2 L_0 v_A}$

- Adding these two equations yields:

$$n\frac{d\vec{U}}{dt} = (\vec{\nabla} \times \vec{B}) \times \vec{B} - \beta\vec{\nabla}(p_i + p_e) + \nu\nabla^2\vec{U}$$

Hall-MHD equations

- On the other hand, using

$$\vec{E} = -\frac{1}{c}\frac{\partial \vec{A}}{\partial t} - \vec{\nabla}\phi$$

$$\vec{B} = \vec{\nabla} \times \vec{A}$$

$$\rightarrow \frac{\partial \vec{A}}{\partial t} = (\vec{U} - \frac{\varepsilon}{n}\vec{\nabla} \times \vec{B}) \times \vec{B} - \vec{\nabla}\phi + \frac{\varepsilon\beta}{n}\vec{\nabla}p_e - \frac{\eta}{n}\vec{\nabla} \times \vec{B}$$

RMHD applied to coronal loop heating

- The solar corona is a topologically complex array of loops (TRACE movie 171 A)
- Coronal loops are magnetic flux tubes with their footpoints anchored deep in the convective region.
- They confine a tenuous and hot plasma. Typical densities are n = 10^9 cm^{-3} and temperatures are T = 2-3.10^6 K.

- The magnetic field provides not just the confinement of the plasma, but also the energy to heat it up to coronal temperatures (Parker 1972, 1988; van Ballegooijen 1986; Einaudi et al. 1996).
- One of the key ingredients is the free energy available in the sub-photospheric convective region. Convective motions move the footpoints of fieldlines, thus building up magnetic stresses. See **Mandrini, Demoulin & Klimchuk 2000** for a comprehensive comparison between theoretical models and observations for a large number of active regions.
- However, the typical length scale of these magnetic stresses is way too large for the Ohmic dissipation to do the job, since

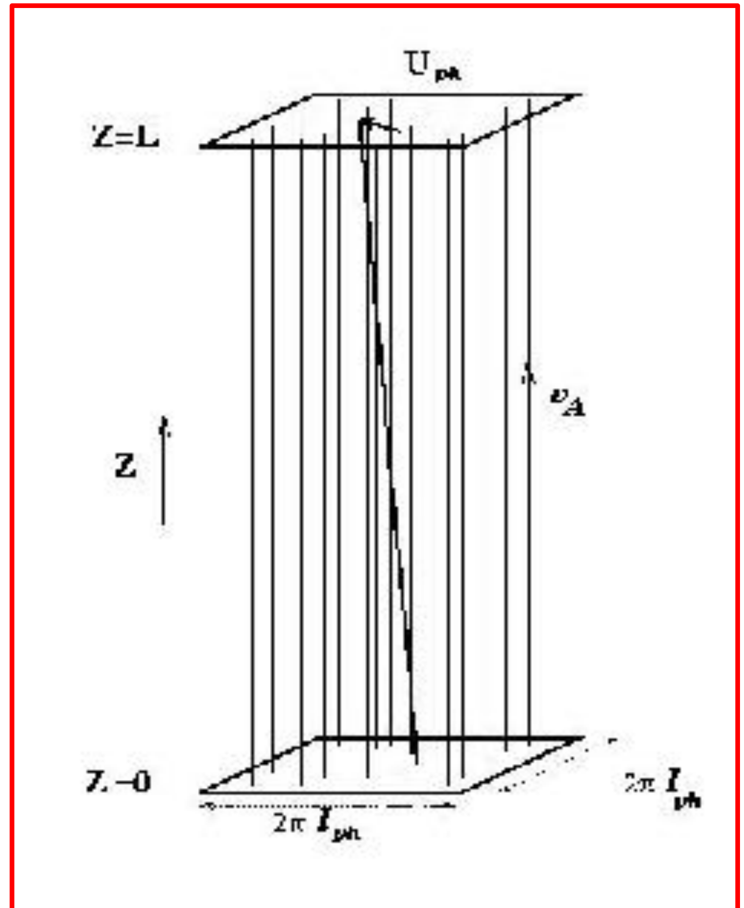
$$\tau_{diss} \approx \ell^2 / \eta$$

RMHD Equations

➡ Reduced MHD is a self-consistent approximation of the full MHD equations whenever:
(a) one component of the magnetic field is much stronger than the others and,
(b) spatial variations are smoother along than across (Strauss 1976).

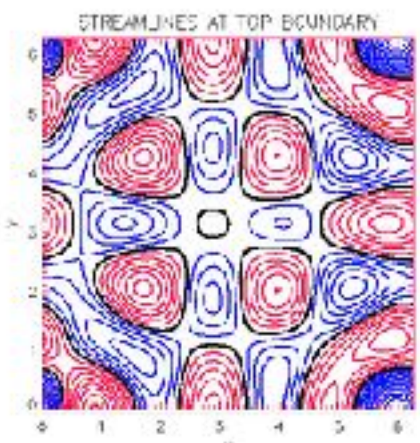
$$\partial_t a = v_A \partial_z \varphi + [\varphi, a] + \eta \nabla_\perp^2 a$$
$$\partial_t \omega = v_A \partial_z j + [\varphi, \omega] - [a, j] + \eta \nabla_\perp^2 \omega$$

$$\vec{b} = v_A \hat{z} + \vec{\nabla}_\perp \times (a\,\hat{z}) \quad , \quad \vec{u} = \vec{\nabla}_\perp \times (\varphi\,\hat{z})$$
$$\omega = -\nabla_\perp^2 \varphi \qquad , \qquad j = -\nabla_\perp^2 a$$

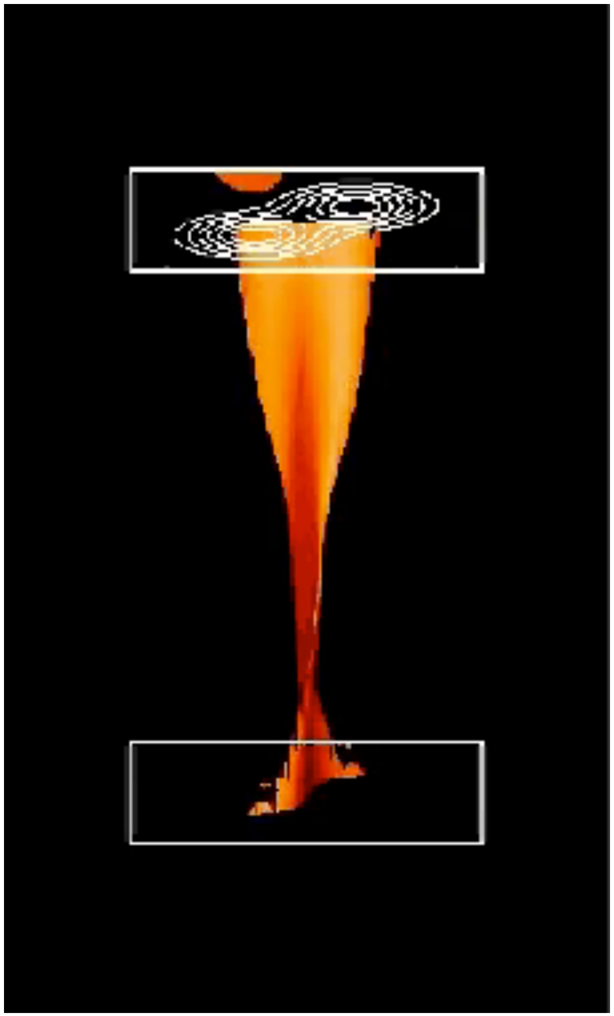


➡ These equations describe the evolution of the velocity (u) and magnetic field (b) inside the box, assuming periodic boundary conditions at the sides.

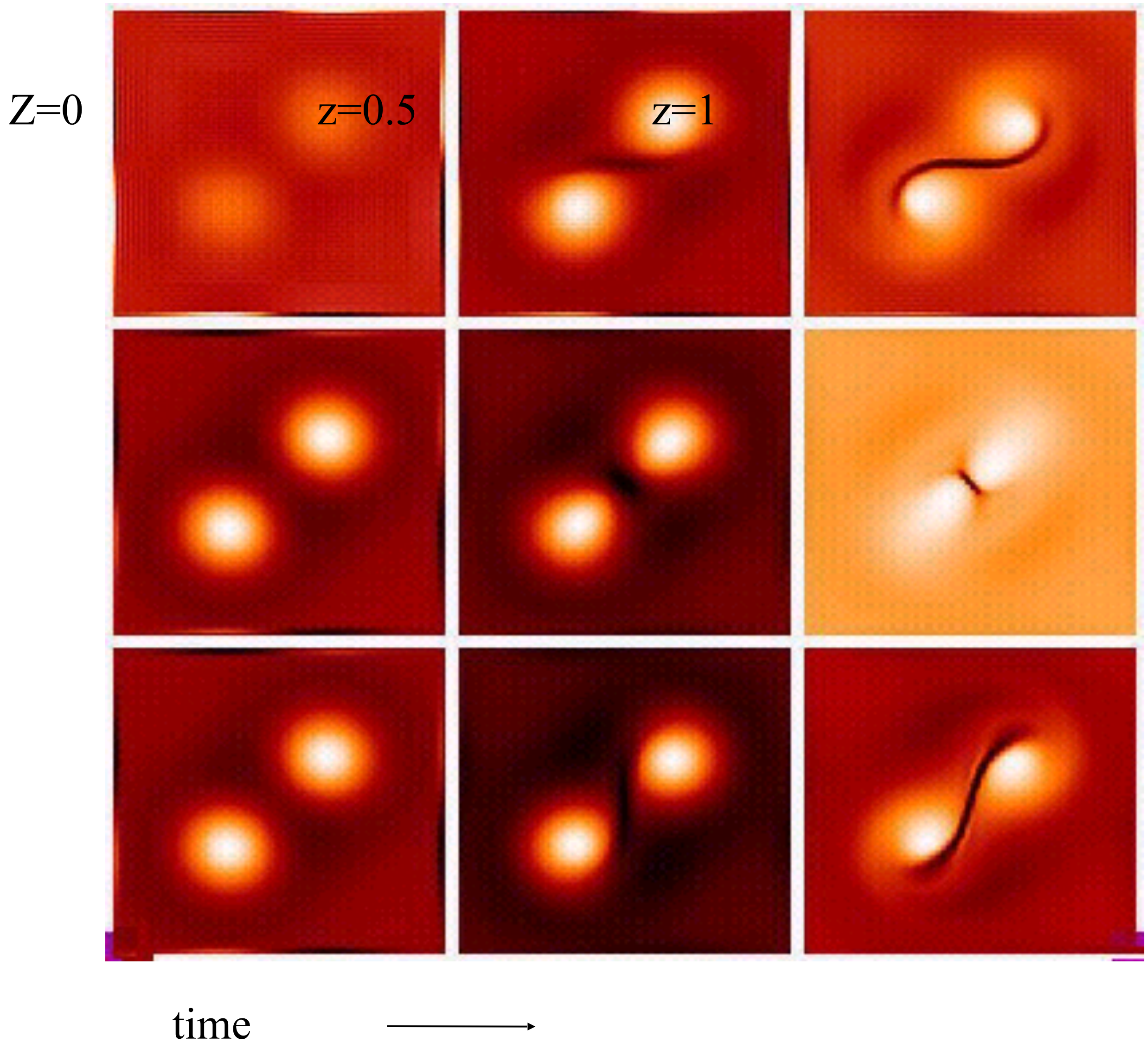
➡ We enforce stationary velocity field (U_{ph}) at the top plate.



Current density distribution



Current density



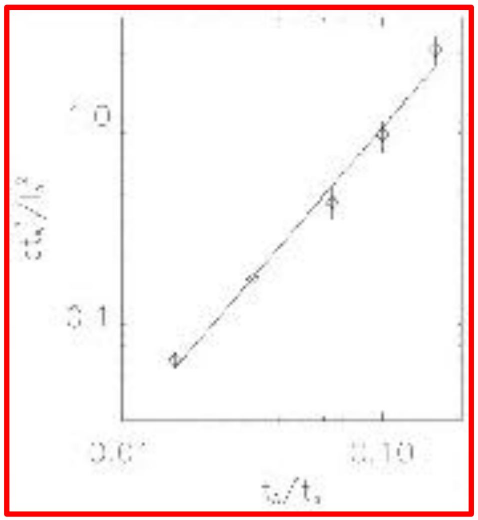
RMHD simulations

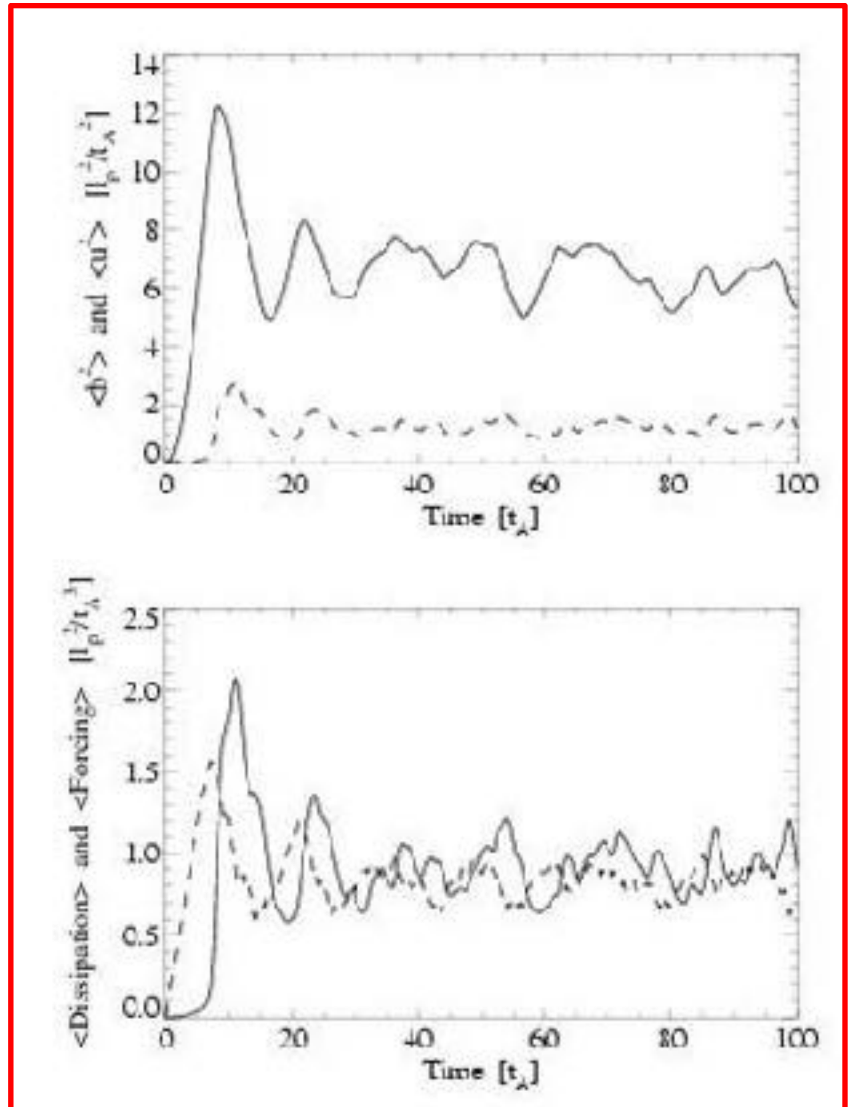
- We perform long time integrations of the RMHD equations. Lengths are in units of the photospheric motions (ℓ_{ph}) and times are in units of the Alfven time (t_A) along the loop.

- Spatial resolution is 256x256x48 and the integration time is 4000 t_A. We use a spectral scheme in the xy-plane and finite differences along z.

- The time averaged dissipation rate is found to scale like (Dmitruk & Gómez 1999)

$$\varepsilon \approx \frac{\rho\, \ell_{ph}^2}{t_A^3} \left(\frac{t_A}{t_{ph}}\right)^{3/2}$$





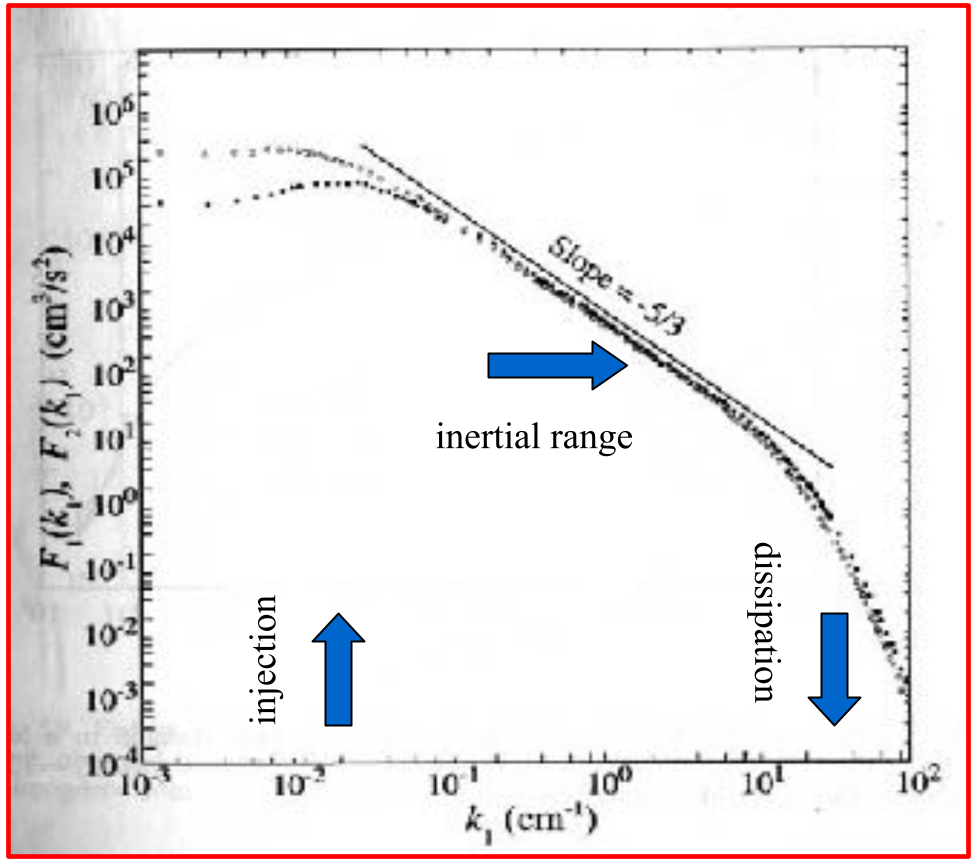
- It is essentially independent of the Reynolds number, as expected for stationary turbulence.

Stationary turbulence

- Energy cascade
  - energy flux toward high k
  - vortex breakdown

- Scale invariance
  - energy flux in k: → $\varepsilon_k \approx \frac{u_k^2}{\tau_k}$
  - energy power spectrum: → $E_k \approx \frac{u_k^2}{k}$

$$\tau_k \approx \frac{1}{k u_k}\ , \ \varepsilon_k \approx \frac{u_k^2}{\tau_k} = const.$$

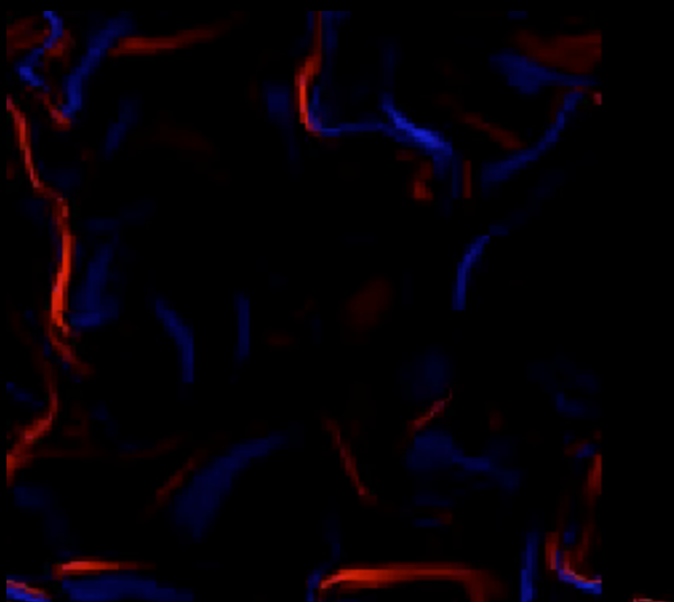


- Therefore → $E_k \approx \frac{u_k^2}{k} = \varepsilon^{\frac{2}{3}} k^{-\frac{5}{3}}$

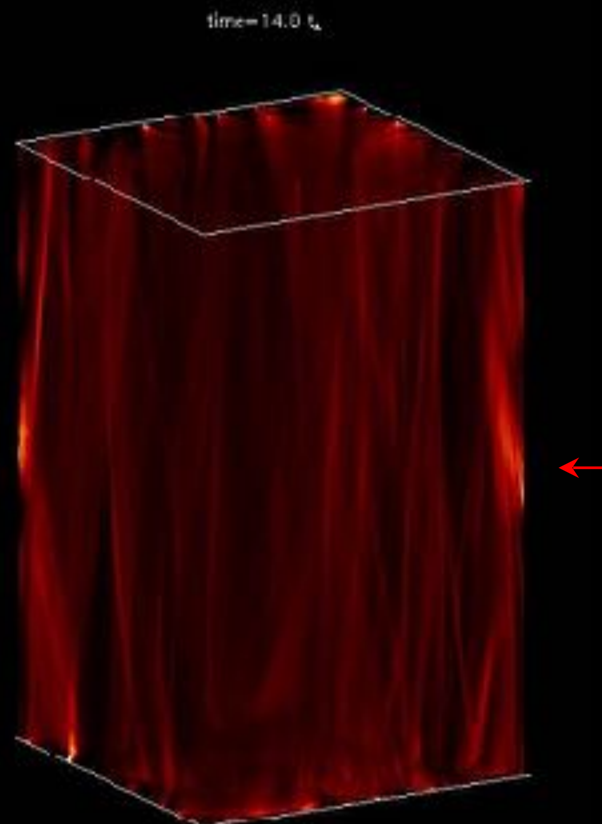
Kolmogorov spectrum (K41)

Dissipative structures: current sheets in 2D

Most of the energy dissipation takes place in current sheets. We display the current density (upflows & downflows) along the loop in a transverse cut.



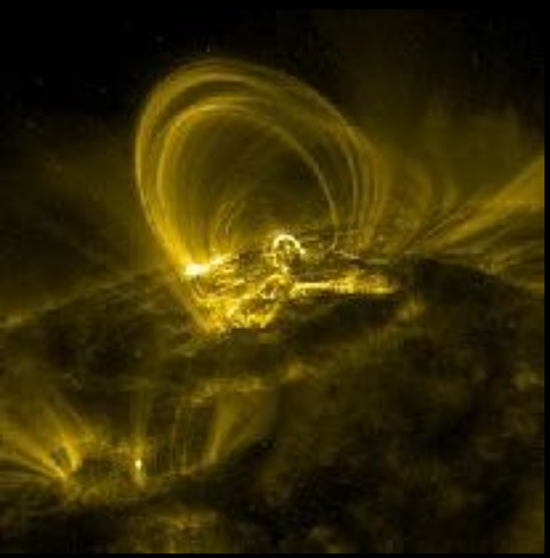
Versus height.

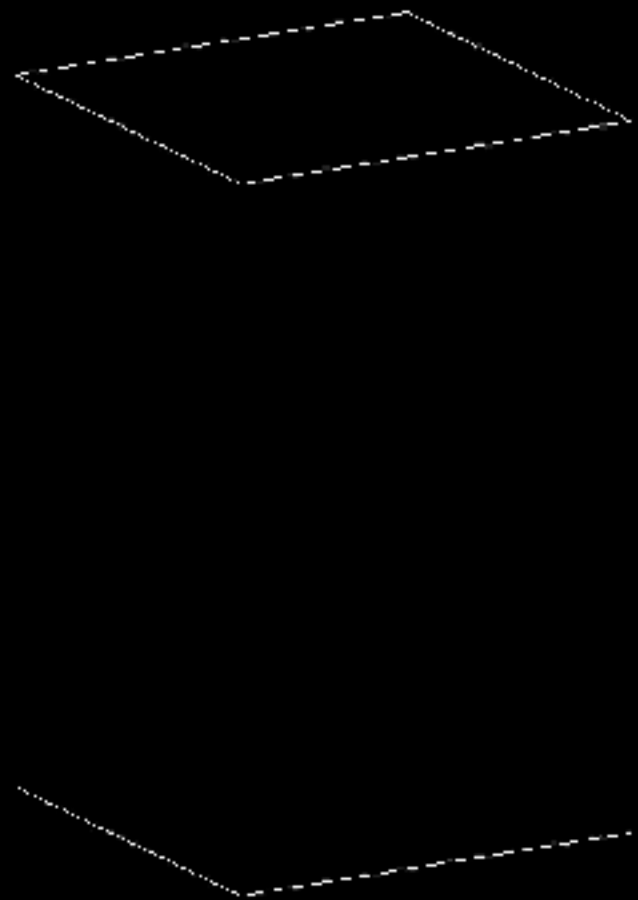


Versus time.

Dissipative structures: current sheets in 3D

- 3D distribution of the energy dissipation rate.
- We display the dissipation rate during 20 Alfven times with a cadence of 0.1 t_A.





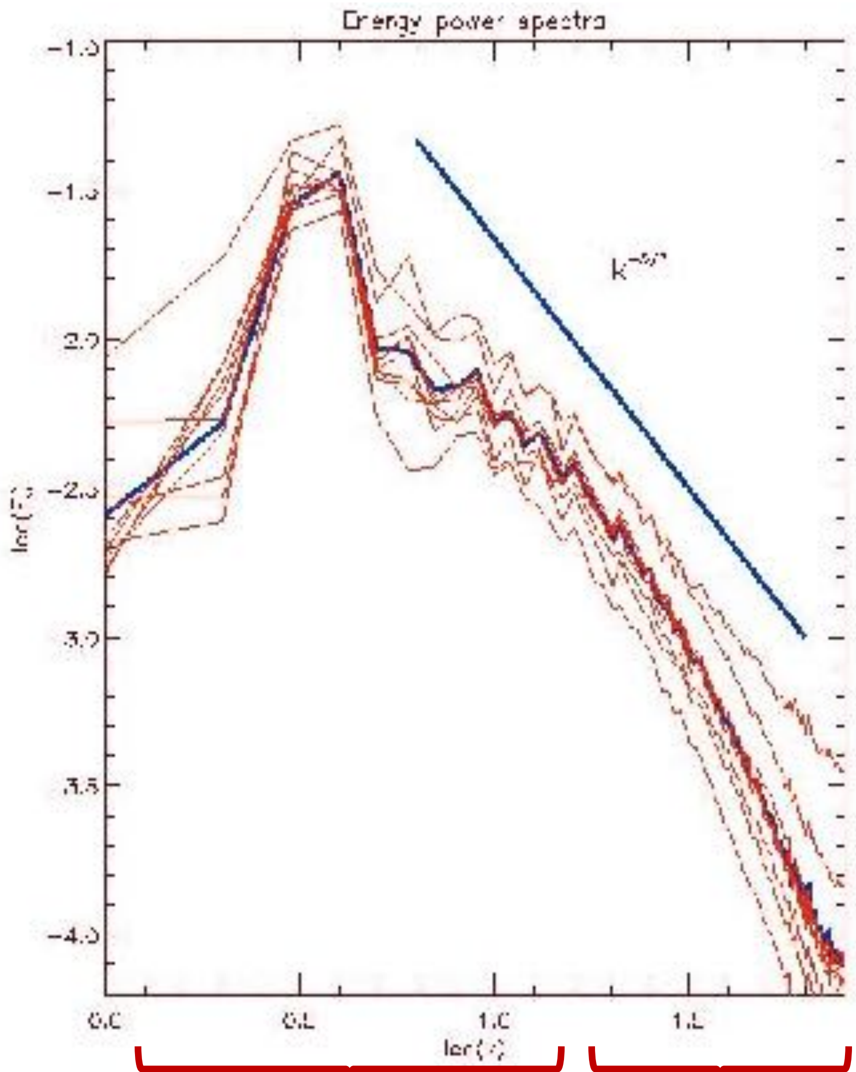
Large and small scales: Energy power spectra

➡ The energy spectra are shown here. The **red lines** correspond to ten spectra taken at different times (separated by 10 t_A). The **blue trace** is the time averaged version.

➡ The Kolmogorov slope is displayed for reference, but the moderate spatial resolution of these runs is insufficient for a serious spectral analysis.

➡ Viscosity and resistivity are large enough to spatially resolve the dissipative structures properly.

➡ In one-fluid MHD, the only kinetic effects were viscosity and resistivity. A two-fluid description would bring new physics into play.

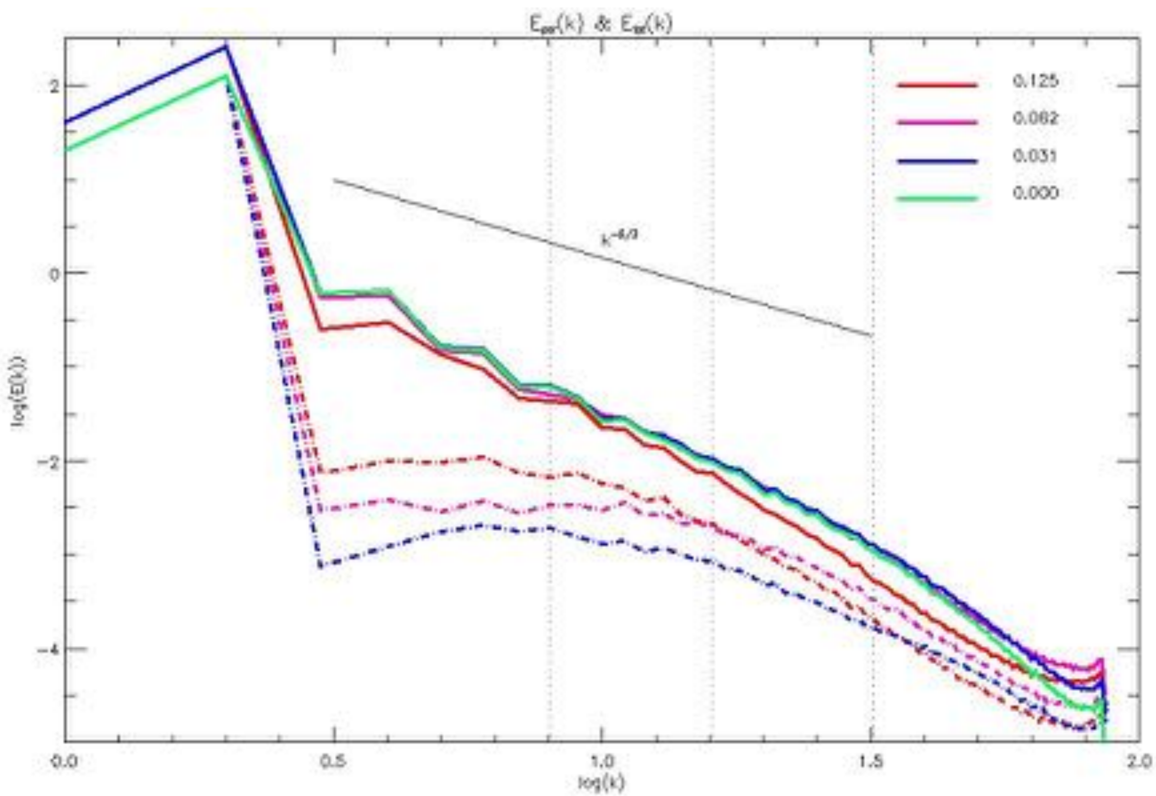


Large Scales (one-fluid MHD)

Small Scales (two-fluid MHD)

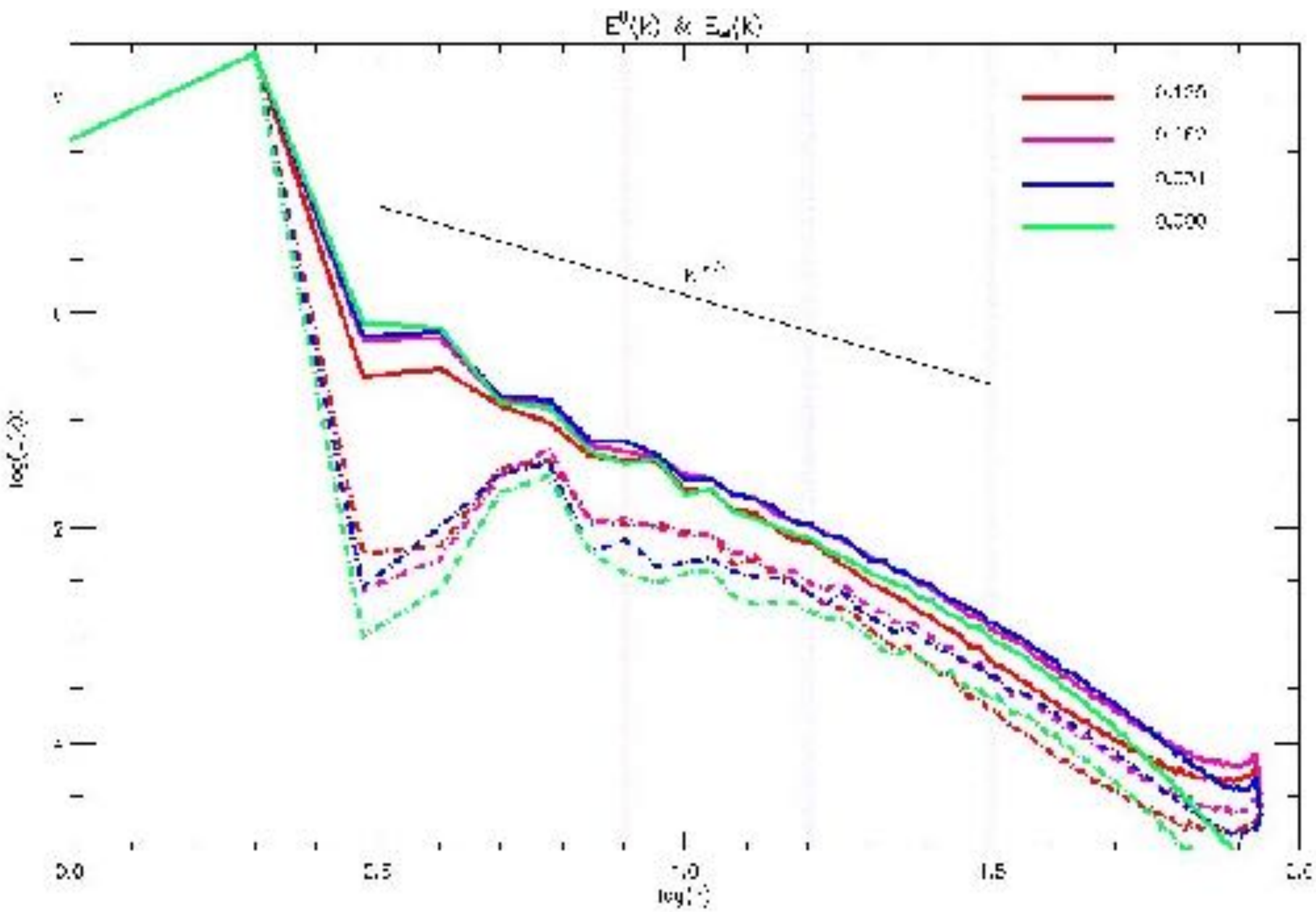
Energy spectra

- We also computed energy power spectra for different values of the Hall parameter ε .
- The Kolmogorov slope $k^{-5/3}$ is also displayed for reference.
- The dotted curves correspond to the parallel energy spectra.
- The vertical dotted lines indicate the location of the Hall scale $k_\varepsilon \cong 1/\varepsilon$ for each run.



Energy spectra

- Energy power spectra for different values of ε .
- The dotted curves are the spectra for kinetic energy.

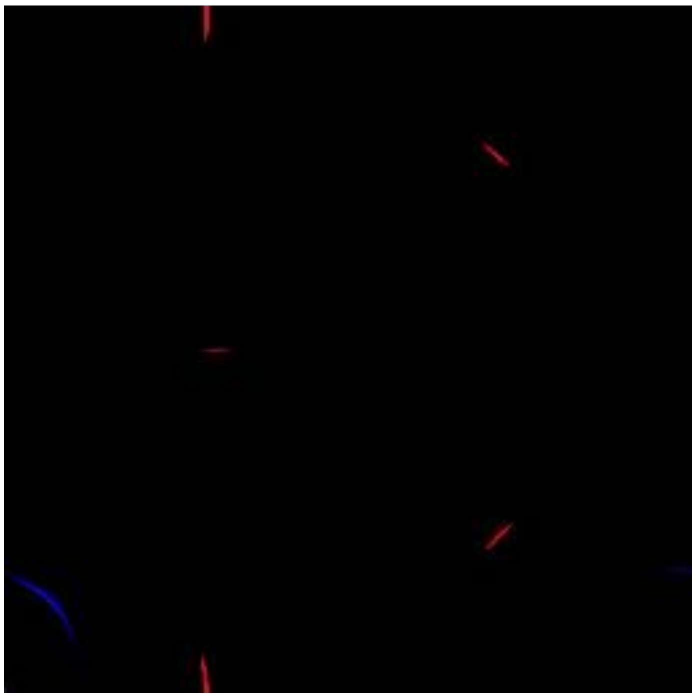


Current sheets in RHMHD

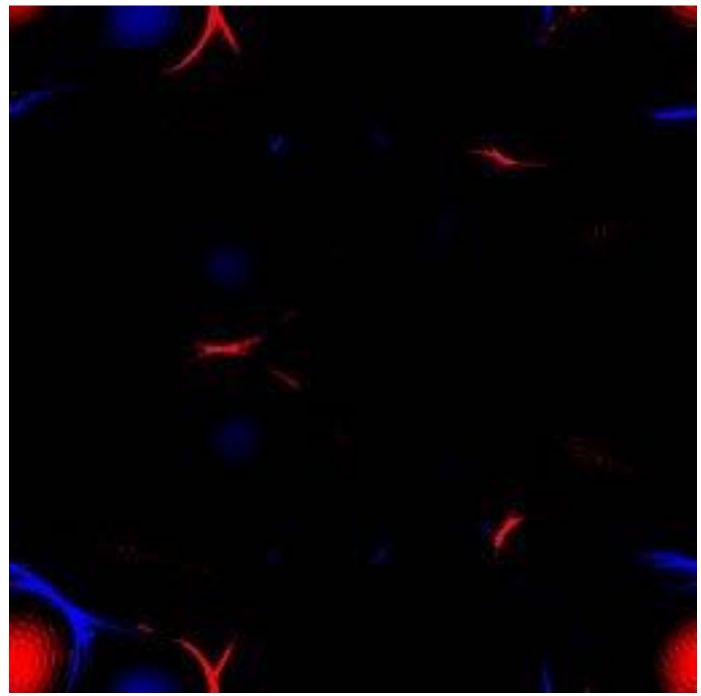
➡ Energy dissipation concentrates on very small structures known as current sheets, in which current density flows almost parallel to z.

➡ The picture shows **positive** and **negative** current density in a transverse cut at $z = \frac{1}{2}$, for pure RMHD (i.e. $\varepsilon = 0$).

➡ When the Hall effect is considered, current sheets display the typical Petschek-like structure.

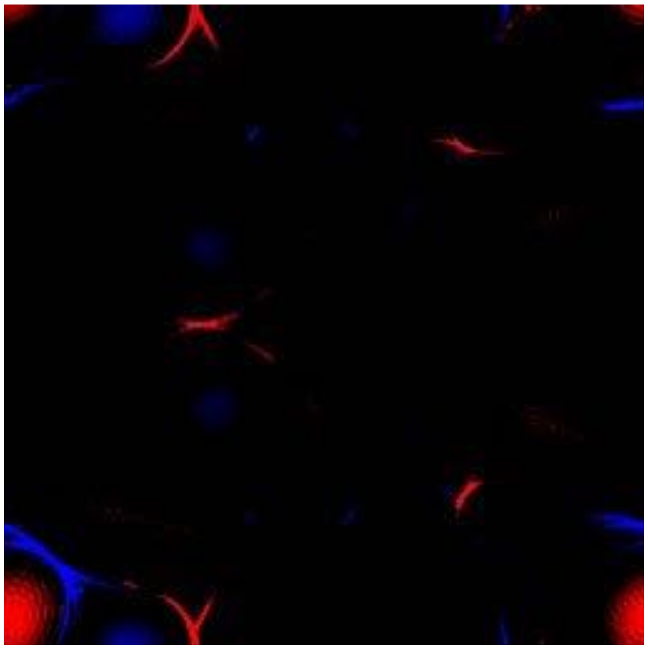


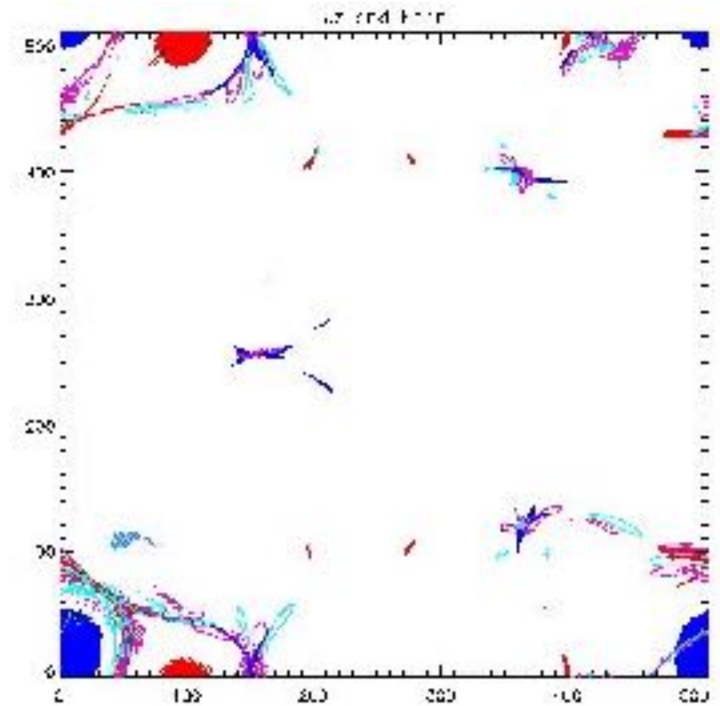
$\varepsilon = 0.0$



$\varepsilon = 0.1$

Parallel electric field





➡ One of the important new features of the Hall effect, is the presence of a parallel electric field, i.e. $E_{//} = \frac{\vec{E} \cdot \vec{B}}{|\vec{B}|}$

➡ To order α^2 it can be computed as $E_{//} = \varepsilon\,(\partial_z b - [a, b])$

and of course it can potentially accelerate particles along magnetic field lines.

➡ Current density is displayed in **red** and **blue**, while contours coloured in **light blue** and **pink** correspond to the parallel electric field.

Conclusions

- In this second lecture we introduced the **two-fluid description** as an extended version of MHD that goes beyond the ion and even the electron inertial lengths.

- Below the ion inertial length, we have the **Hall-MHD** approximation, which is an adequate theoretical framework to describe a number of astrophysical and laboratory applications.

- We also presented to so called **reduced** approximation, which is appropriate for plasmas embedded in relatively strong magnetic fields.

- We numerically integrated the Hall-MHD equations (spectral and Runge-Kutta) in the presence of a strong external magnetic field.

- As a first application, we showed RMHD simulations (no Hall effect yet) to study the internal dynamics of magnetic loops in the solar corona.

- We introduce the Hall effect and focused on its potential relevance in the dynamics of small scales and magnetic reconnection in the dissipative structures of turbulence.